# *Nuclear Medium Effects in Lepton-Nucleus Deep Inelastic Scattering*

**Farhana Zaidi**

# *Nuclear Medium Effects in Lepton-Nucleus Deep Inelastic Scattering*

**Farhana Zaidi**[1]
Md. Sajjad Athar[1], Huma Haider[1], S. K. Singh[1] and I. Ruiz Simo[2]

[1]Aligarh Muslim University, India

[2]University of Valencia & IFIC, Spain

International School of Nuclear Physics
$37^{th}$ Course: Probing hadron structure with lepton and hadron beams
16-24 September 2015, Erice

# Outline

# Introduction

1. Lepton scattering experiments have been performed for a wide range of energy.
2. Lepton induced processes may be subdivided as
   1. Elastic scattering
   2. Inelastic scattering
   3. Deep Inelastic scattering(DIS)
3. Early experiments at SLAC exhibited Bjorken scaling phenomenon corresponding to DIS.
4. Structure functions are found to be independent of $Q^2$ in the asymptotic limit.
5. $Q^2$ independence of structure functions gave first evidence of quarks and gluons.

# Introduction

1. Lepton scattering experiments have been performed for a wide range of energy.
2. Lepton induced processes may be subdivided as
   1. Elastic scattering
   2. Inelastic scattering
   3. Deep Inelastic scattering(DIS)
3. Early experiments at SLAC exhibited Bjorken scaling phenomenon corresponding to DIS.
4. Structure functions are found to be independent of $Q^2$ in the asymptotic limit.
5. $Q^2$ independence of structure functions gave first evidence of quarks and gluons.

# Introduction

1. Lepton scattering experiments have been performed for a wide range of energy.
2. Lepton induced processes may be subdivided as
   1. Elastic scattering
   2. Inelastic scattering
   3. Deep Inelastic scattering(DIS)
3. Early experiments at SLAC exhibited Bjorken scaling phenomenon corresponding to DIS.
4. Structure functions are found to be independent of $Q^2$ in the asymptotic limit.
5. $Q^2$ independence of structure functions gave first evidence of quarks and gluons.

# Introduction

1. Lepton scattering experiments have been performed for a wide range of energy.
2. Lepton induced processes may be subdivided as
   1. Elastic scattering
   2. Inelastic scattering
   3. Deep Inelastic scattering(DIS)
3. Early experiments at SLAC exhibited Bjorken scaling phenomenon corresponding to DIS.
4. Structure functions are found to be independent of $Q^2$ in the asymptotic limit.
5. $Q^2$ independence of structure functions gave first evidence of quarks and gluons.

# *Introduction*

1. Lepton scattering experiments have been performed for a wide range of energy.
2. Lepton induced processes may be subdivided as
    1. Elastic scattering
    2. Inelastic scattering
    3. Deep Inelastic scattering(DIS)
3. Early experiments at SLAC exhibited Bjorken scaling phenomenon corresponding to DIS.
4. Structure functions are found to be independent of $Q^2$ in the asymptotic limit.
5. $Q^2$ independence of structure functions gave first evidence of quarks and gluons.

# *Introduction*

1. Lepton scattering experiments have been performed for a wide range of energy.
2. Lepton induced processes may be subdivided as
   1. Elastic scattering
   2. Inelastic scattering
   3. Deep Inelastic scattering(DIS)
3. Early experiments at SLAC exhibited Bjorken scaling phenomenon corresponding to DIS.
4. Structure functions are found to be independent of $Q^2$ in the asymptotic limit.
5. $Q^2$ independence of structure functions gave first evidence of quarks and gluons.

# Introduction

1. Lepton scattering experiments have been performed for a wide range of energy.
2. Lepton induced processes may be subdivided as
    1. Elastic scattering
    2. Inelastic scattering
    3. Deep Inelastic scattering(DIS)
3. Early experiments at SLAC exhibited Bjorken scaling phenomenon corresponding to DIS.
4. Structure functions are found to be independent of $Q^2$ in the asymptotic limit.
5. $Q^2$ independence of structure functions gave first evidence of quarks and gluons.

# *Introduction*

1. Lepton scattering experiments have been performed for a wide range of energy.
2. Lepton induced processes may be subdivided as
    1. Elastic scattering
    2. Inelastic scattering
    3. Deep Inelastic scattering(DIS)
3. Early experiments at SLAC exhibited Bjorken scaling phenomenon corresponding to DIS.
4. Structure functions are found to be independent of $Q^2$ in the asymptotic limit.
5. $Q^2$ independence of structure functions gave first evidence of quarks and gluons.

1. Earlier it was believed that quarks inside the nucleon are insensitive to the nuclear enviroment.

1. Earlier it was believed that quarks inside the nucleon are insensitive to the nuclear enviroment.
2. In 1982 EMC at CERN found that structure function for a nuclear target is different than for a free nucleon.

1. Earlier it was believed that quarks inside the nucleon are insensitive to the nuclear enviroment.
2. In 1982 EMC at CERN found that structure function for a nuclear target is different than for a free nucleon.

$$\frac{F_2^{Fe}}{F_2^{D}} \neq 1$$

1. Earlier it was believed that quarks inside the nucleon are insensitive to the nuclear enviroment.
2. In 1982 EMC at CERN found that structure function for a nuclear target is different than for a free nucleon.

$$\frac{F_2^{Fe}}{F_2^{D}} \neq 1$$

3. Theoretically to incorporate the nuclear medium effects many efforts has been done for the e.m. interaction but only few for the weak interaction

1. Earlier it was believed that quarks inside the nucleon are insensitive to the nuclear enviroment.
2. In 1982 EMC at CERN found that structure function for a nuclear target is different than for a free nucleon.

$$\frac{F_2^{Fe}}{F_2^{D}} \neq 1$$

3. Theoretically to incorporate the nuclear medium effects many efforts has been done for the e.m. interaction but only few for the weak interaction
4. Phenomenologically, people tried to obtain a correction factor to take into account the nuclear medium effects

1. Earlier it was believed that quarks inside the nucleon are insensitive to the nuclear enviroment.
2. In 1982 EMC at CERN found that structure function for a nuclear target is different than for a free nucleon.

$$\frac{F_2^{Fe}}{F_2^D} \neq 1$$

3. Theoretically to incorporate the nuclear medium effects many efforts has been done for the e.m. interaction but only few for the weak interaction
4. Phenomenologically, people tried to obtain a correction factor to take into account the nuclear medium effects

    1. $l^{\pm} - A$

1. Earlier it was believed that quarks inside the nucleon are insensitive to the nuclear enviroment.
2. In 1982 EMC at CERN found that structure function for a nuclear target is different than for a free nucleon.

$$\frac{F_2^{Fe}}{F_2^{D}} \neq 1$$

3. Theoretically to incorporate the nuclear medium effects many efforts has been done for the e.m. interaction but only few for the weak interaction
4. Phenomenologically, people tried to obtain a correction factor to take into account the nuclear medium effects
    1. $l^{\pm} - A$
    2. $p - A$

1. Earlier it was believed that quarks inside the nucleon are insensitive to the nuclear enviroment.
2. In 1982 EMC at CERN found that structure function for a nuclear target is different than for a free nucleon.

$$\frac{F_2^{Fe}}{F_2^{D}} \neq 1$$

3. Theoretically to incorporate the nuclear medium effects many efforts has been done for the e.m. interaction but only few for the weak interaction
4. Phenomenologically, people tried to obtain a correction factor to take into account the nuclear medium effects
    1. $l^{\pm} - A$
    2. $p - A$
    3. $\nu/\bar{\nu} - A$

1. Earlier it was believed that quarks inside the nucleon are insensitive to the nuclear enviroment.
2. In 1982 EMC at CERN found that structure function for a nuclear target is different than for a free nucleon.

$$\frac{F_2^{Fe}}{F_2^{D}} \neq 1$$

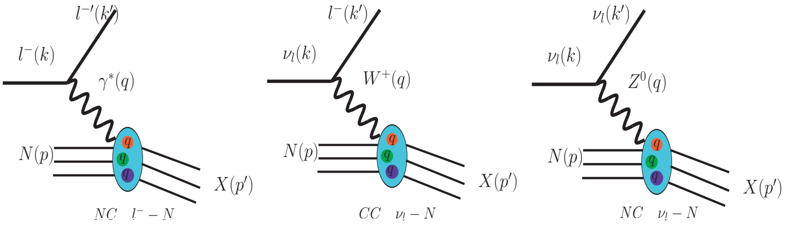
3. Theoretically to incorporate the nuclear medium effects many efforts has been done for the e.m. interaction but only few for the weak interaction
4. Phenomenologically, people tried to obtain a correction factor to take into account the nuclear medium effects
    1. $l^{\pm} - A$
    2. $p - A$
    3. $\nu/\bar{\nu} - A$
    4. DY process

## *Basic reaction for deep inelastic scattering process is*

$$l(k) + N(p) \rightarrow l'(k') + X(p')$$

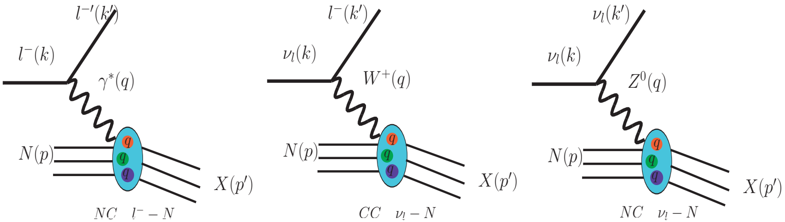
# *Basic reaction for deep inelastic scattering process is*

$$l(k) + N(p) \to l'(k') + X(p')$$



# *Basic reaction for deep inelastic scattering process is*

$$l(k) + N(p) \to l'(k') + X(p')$$




The general expression of two body scattering cross section

$$d\sigma = \frac{1}{2E_l 2E_N}(2\pi)^4\, \delta^4(k + p - k' - \sum_{i=1}^{n} p_i')\, \frac{d^3k'}{(2\pi)^3 E_{l'}} \prod_{i=1}^{n} \frac{d^3p_i'}{(2\pi)^3 E_X}\, \sum \bar{\sum} |\mathcal{M}|^2$$

## Square of matrix element is

$$|\mathcal{M}|^2 \quad \propto \quad L_{\mu\nu} W^{\mu\nu}$$

## *Square of matrix element is*

$$|\mathcal{M}|^2 \quad \propto \quad L_{\mu\nu} W^{\mu\nu}$$

The leptonic tensor $L_{\mu\nu}$

$$L_{\mu\nu} \quad = \quad 2(k_\mu\, k'_\nu + k'_\mu k_\nu - k \cdot k' g_{\mu\nu})$$

## Square of matrix element is

$$|\mathcal{M}|^2 \quad \propto \quad L_{\mu\nu} W^{\mu\nu}$$

The leptonic tensor $L_{\mu\nu}$

$$L_{\mu\nu} \quad = \quad 2(k_\mu \, k'_\nu + k'_\mu k_\nu - k \cdot k' g_{\mu\nu})$$

## In general the hadronic tensor is defined as

$$\begin{aligned} W^{\mu\nu} \quad = \quad & -g_{\mu\nu} \, W_1(\nu, Q^2) + \frac{p_\mu p_\nu}{M^2} \, W_2(\nu, Q^2) - i\varepsilon_{\mu\nu\lambda\sigma} \frac{p^\lambda q^\sigma}{2M^2} \, W_3(\nu, Q^2) \\ & + \frac{q_\mu q_\nu}{M^2} \, W_4(\nu, Q^2) + \frac{(p_\mu q_\nu + p_\nu q_\mu)}{2M^2} \, W_5(\nu, Q^2) \\ & + \frac{i(p_\mu q_\nu - p_\nu q_\mu)}{2M^2} \, W_6(\nu, Q^2) \end{aligned}$$

## *By contraction of hadronic tensor with $L_{\mu\nu}$*

$$W_3(\nu, Q^2) \rightarrow 0\,, \quad W_6(\nu, Q^2) \rightarrow 0$$

## *By contraction of hadronic tensor with $L_{\mu\nu}$*

$$W_3(\nu, Q^2) \rightarrow 0\,, \qquad W_6(\nu, Q^2) \rightarrow 0$$

## *Conservation of current leads to*

$$\begin{aligned} q_\mu W^{\mu\nu} &= 0 \\ W_4(\nu, Q^2) &= \frac{-2p\cdot q}{q^2} W_2(\nu, Q^2) \\ W_5(\nu, Q^2) &= \frac{M^2}{q^2} W_1(\nu, Q^2) + \left(\frac{p\cdot q}{q^2}\right)^2 W_2(\nu, Q^2) \end{aligned}$$

## *By contraction of hadronic tensor with $L_{\mu\nu}$*

$$W_3(\nu, Q^2) \to 0\,, \qquad W_6(\nu, Q^2) \to 0$$

## *Conservation of current leads to*

$$\begin{aligned} q_\mu W^{\mu\nu} &= 0 \\ W_4(\nu, Q^2) &= \frac{-2p\cdot q}{q^2} W_2(\nu, Q^2) \\ W_5(\nu, Q^2) &= \frac{M^2}{q^2} W_1(\nu, Q^2) + \left(\frac{p\cdot q}{q^2}\right)^2 W_2(\nu, Q^2) \end{aligned}$$

## *Therefore, we are left with only $W_1(\nu, Q^2)$ and $W_2(\nu, Q^2)$ and the hadronic tensor is*

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1(\nu, Q^2) + \left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right) \frac{W_2(\nu, Q^2)}{M^2}$$

## *Double differential scattering cross section for em interaction*

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{\pi\alpha^2}{4E_l^3 E_l' sin^4\left(\frac{\theta}{2}\right)} \left\{ 2sin^2\left(\frac{\theta}{2}\right) W_1(\nu, Q^2) + cos^2\left(\frac{\theta}{2}\right) W_2(\nu, Q^2) \right\}$$

## Double differential scattering cross section for em interaction

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{\pi\alpha^2}{4E_l^3 E_l' sin^4\left(\frac{\theta}{2}\right)}\left\{2sin^2\left(\frac{\theta}{2}\right)W_1(\nu,Q^2) + cos^2\left(\frac{\theta}{2}\right)W_2(\nu,Q^2)\right\}$$

## Bjorken variables are

$$x = \frac{Q^2}{2M\nu}\,; \quad y = \frac{\nu}{E_l}$$

## Double differential scattering cross section for em interaction

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{\pi\alpha^2}{4E_l^3 E_l' sin^4\left(\frac{\theta}{2}\right)} \left\{ 2sin^2\left(\frac{\theta}{2}\right) W_1(\nu, Q^2) + cos^2\left(\frac{\theta}{2}\right) W_2(\nu, Q^2) \right\}$$

## Bjorken variables are

$$x = \frac{Q^2}{2M\nu}\ ; \quad y = \frac{\nu}{E_l}$$

$$\begin{aligned} \frac{d\sigma}{dxdy} &= \frac{d\sigma}{dQ^2 d\nu} \frac{dQ^2 d\nu}{dxdy} \\ \frac{dQ^2 d\nu}{dxdy} &= 2MyE_l^2 \end{aligned}$$

## Double differential scattering cross section for em interaction

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{\pi\alpha^2}{4E_l^3 E_l' sin^4\left(\frac{\theta}{2}\right)} \left\{ 2sin^2\left(\frac{\theta}{2}\right) W_1(\nu, Q^2) + cos^2\left(\frac{\theta}{2}\right) W_2(\nu, Q^2) \right\}$$

## Bjorken variables are

$$x = \frac{Q^2}{2M\nu}\,; \quad y = \frac{\nu}{E_l}$$

$$\begin{aligned} \frac{d\sigma}{dxdy} &= \frac{d\sigma}{dQ^2 d\nu} \frac{dQ^2 d\nu}{dxdy} \\ \frac{dQ^2 d\nu}{dxdy} &= 2MyE_l^2 \end{aligned}$$

$$\frac{d\sigma}{dxdy} = 2ME_l^2 y \frac{\pi\alpha^2}{4E_l^3 E_l' sin^4(\frac{\theta}{2})} \left[ W_2(\nu, Q^2) \frac{E_l}{E_l'} \left(1 - y - \frac{Mxy}{2E_l}\right) + 2W_1(\nu, Q^2) \left(xy \frac{M}{2E_l'}\right) \right]$$

The nucleon structure functions $W_i(\nu, Q^2)$ are redefined as

$$
\begin{aligned}
M\, W_1(\nu, Q^2) &= F_1(x) \\
\nu\, W_2(\nu, Q^2) &= F_2(x)
\end{aligned}
$$

The nucleon structure functions $W_i(\nu, Q^2)$ are redefined as

$$\begin{aligned} M\, W_1(\nu, Q^2) &= F_1(x) \\ \nu\, W_2(\nu, Q^2) &= F_2(x) \end{aligned}$$

Differential scattering cross section may also be written as

$$\frac{d^2\sigma}{dx\,dy} = \frac{8ME_l\pi\alpha^2}{Q^4}\left(y^2 x F_1(x) + \left[(1 - y - \frac{xyM}{2E_l})\right] F_2(x)\right)$$

The nucleon structure functions $W_i(\nu, Q^2)$ are redefined as

$$\begin{aligned} M\ W_1(\nu, Q^2) &= F_1(x) \\ \nu\ W_2(\nu, Q^2) &= F_2(x) \end{aligned}$$

Differential scattering cross section may also be written as

$$\frac{d^2\sigma}{dx\,dy} = \frac{8ME_l\pi\alpha^2}{Q^4}\left(y^2 xF_1(x) + \left[(1 - y - \frac{xyM}{2E_l})\right]F_2(x)\right)$$

Electromagnetic structure functions

$$\begin{aligned} F_2(x) &= \sum_i e_i^2 x f_i(x) \\ F_1(x) &= \frac{F_2(x)}{2x} \end{aligned}$$

The nucleon structure functions $W_i(\nu, Q^2)$ are redefined as

$$\begin{aligned} M\, W_1(\nu, Q^2) &= F_1(x) \\ \nu\, W_2(\nu, Q^2) &= F_2(x) \end{aligned}$$

Differential scattering cross section may also be written as

$$\frac{d^2\sigma}{dx\, dy} = \frac{8ME_l\pi\alpha^2}{Q^4}\left(y^2 x F_1(x) + \left[(1 - y - \frac{xyM}{2E_l})\right] F_2(x)\right)$$

Electromagnetic structure functions

$$\begin{aligned} F_2(x) &= \sum_i e_i^2 x f_i(x) \\ F_1(x) &= \frac{F_2(x)}{2x} \end{aligned}$$

where $xf_i(x)$ is the probability of finding a quark of flavor $i$ carrying a momentum fraction $x$ of nucleon's momentum.

The nucleon structure functions $W_i(\nu, Q^2)$ are redefined as

$$\begin{aligned} M\ W_1(\nu, Q^2) &= F_1(x) \\ \nu\ W_2(\nu, Q^2) &= F_2(x) \end{aligned}$$

Differential scattering cross section may also be written as

$$\frac{d^2\sigma}{dx\,dy} = \frac{8ME_l\pi\alpha^2}{Q^4}\left(y^2 x F_1(x) + \left[(1 - y - \frac{xyM}{2E_l})\right] F_2(x)\right)$$

Electromagnetic structure functions

$$\begin{aligned} F_2(x) &= \sum_i e_i^2 x f_i(x) \\ F_1(x) &= \frac{F_2(x)}{2x} \end{aligned}$$

where $xf_i(x)$ is the probability of finding a quark of flavor $i$ carrying a momentum fraction $x$ of nucleon's momentum.

$$\begin{aligned} F_2^{ep} &= \frac{4\,x}{9}\,(u_v + u_s + \bar{u}_s + c + \bar{c} + ....) + \frac{x}{9}\,(d_v + d_s + \bar{d}_s + s + \bar{s} + ....) \\ F_2^{en} &= \frac{x}{9}\,(u_v + u_s + \bar{u}_s + s + \bar{s} + ....) + \frac{4\,x}{9}\,(d_v + d_s + \bar{d}_s + c + \bar{c} + ....) \end{aligned}$$

*Parton distribution functions have been parametrized by many groups*

1. MRST/MSTW
2. GJR/GRV
3. ALEKHIN
4. CTEQ
5. ............

*We have used CTEQ6.6 PDFs in our numerical calculations.*

# *Target Mass Correction*

1. *At finite $Q^2$ heavy quarks are produced like strange, charm, etc.*

# *Target Mass Correction*

1. *At finite $Q^2$ heavy quarks are produced like strange, charm, etc.*
2. *Heavy quark mass can not be ignored as compared to target nucleon mass*

# *Target Mass Correction*

1. *At finite $Q^2$ heavy quarks are produced like strange, charm, etc.*
2. *Heavy quark mass can not be ignored as compared to target nucleon mass*
3. *Heavy quarks production modifiy the scattering kinematics, therefore*

# *Target Mass Correction*

1. *At finite $Q^2$ heavy quarks are produced like strange, charm, etc.*
2. *Heavy quark mass can not be ignored as compared to target nucleon mass*
3. *Heavy quarks production modifiy the scattering kinematics, therefore*

$$x \rightarrow \xi$$

$$\xi = \frac{2x}{1+\sqrt{1+4\mu x^2}},$$

# *Target Mass Correction*

1. *At finite $Q^2$ heavy quarks are produced like strange, charm, etc.*
2. *Heavy quark mass can not be ignored as compared to target nucleon mass*
3. *Heavy quarks production modifiy the scattering kinematics, therefore*

$$x \rightarrow \xi$$

$$\xi = \frac{2x}{1+\sqrt{1+4\mu x^2}}, \qquad \mu = \frac{M^2}{Q^2}$$

# *Target Mass Correction*

1. *At finite $Q^2$ heavy quarks are produced like strange, charm, etc.*
2. *Heavy quark mass can not be ignored as compared to target nucleon mass*
3. *Heavy quarks production modifiy the scattering kinematics, therefore*

$$x \rightarrow \xi$$

$$\xi = \frac{2x}{1+\sqrt{1+4\mu x^2}}, \qquad \mu = \frac{M^2}{Q^2}$$

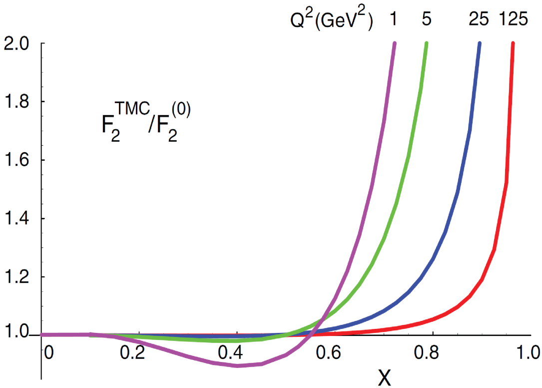
4. *This effect is known as Target mass correction*
5. *TMC is effective at low $Q^2$ and high $x$*

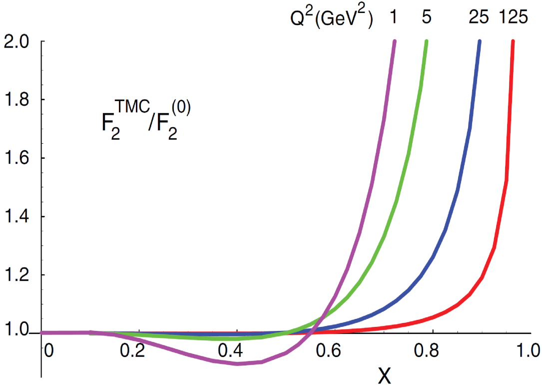
1. *In this region PDFs are not very well determined*
2. *Hence to precisely determine the PDFs TMC should be taken into account*
3. *Structure function depend on the dimensionless variable x*
4. *Therefore, nucleon structure function with TMC is given by*

$$F_2^{TMC}(x,Q^2) \approx \frac{x^2}{\xi^2\gamma^3}F_2(\xi)\left(1+\frac{6\mu x\xi}{\gamma}(1-\xi)^2\right)$$

$$\gamma = \sqrt{1+\frac{4M^2x^2}{Q^2}}$$

Schienbein et al. JPG 35 (2008) 053101


Q^2(GeV^2) 1 5 25 125
2.0
1.8
1.6
1.4
1.2
1.0
F_2^TMC/F_2^(0)
0
0.2
0.4
0.6
0.8
1.0
X




Schienbein et al. JPG 35 (2008) 053101

# *Next-to-Leading Order(NLO)*

*"Structure functions in DIS and their scale evolution are closely related to the origins of quantum chromodynamics (QCD)."*

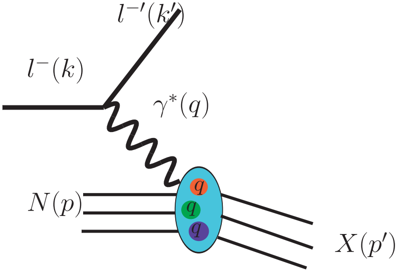
The expression for structure function $F_2$ can be expressed as a function of the PDFs by

$$x^{-1}F_2 = \sum_{f=q,g} C_2 \otimes f$$

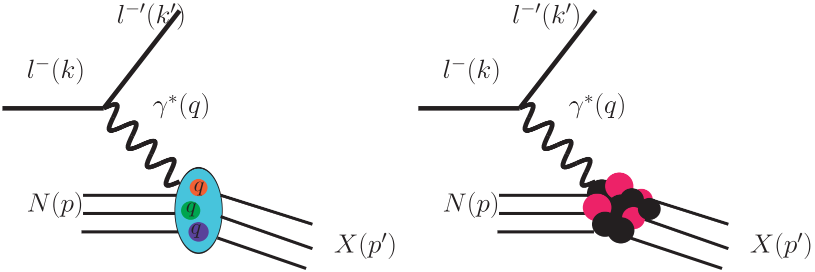
where $C_2$ is the coefficient function for the quarks and gluons, $\otimes$ symbols is for the Mellin convolution which turns into a simple multiplication in N-space and $f$ represents the quark and gluon distributions.
Vermaseren and van Neerven et al.NPB 724(2005)3
van Neerven and Vogt NPB 568(2000)263.

# *If we look inside the nucleus*

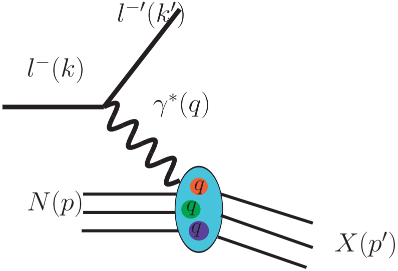
# *If we look inside the nucleus*

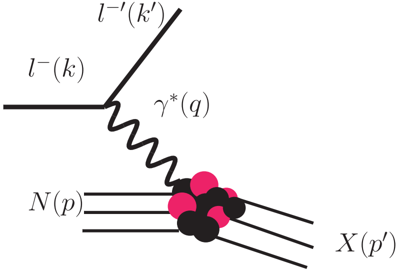


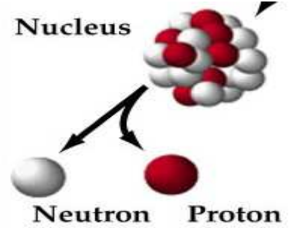
# *If we look inside the nucleus*



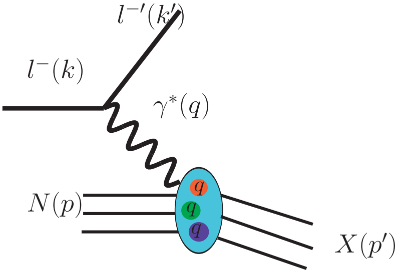
# *If we look inside the nucleus*

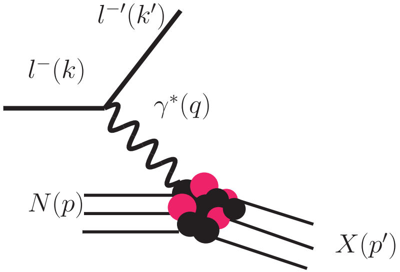


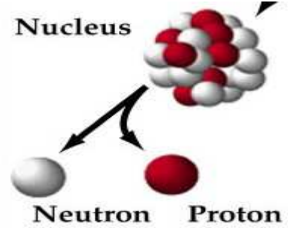



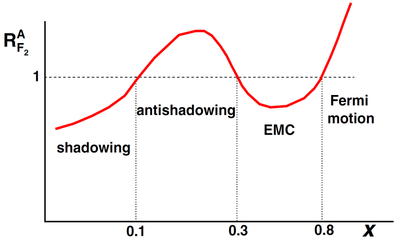



# *If we look inside the nucleus*



We have incorporated the following NME in the present calculation

1. Fermi motion

We have incorporated the following NME in the present calculation

1. Fermi motion
2. Pauli blocking

We have incorporated the following NME in the present calculation

1. Fermi motion
2. Pauli blocking
3. Nucleon correlations

We have incorporated the following NME in the present calculation

1. Fermi motion
2. Pauli blocking
3. Nucleon correlations
4. Pion and rho meson cloud contributions

We have incorporated the following NME in the present calculation

1. Fermi motion
2. Pauli blocking
3. Nucleon correlations
4. Pion and rho meson cloud contributions
5. Shadowing and antishadowing

   Kulagin and Petti PRD76,094033, 2007

# Medium effects in lepton-A scattering

- Kinematic effect which arises as the struck nucleon is not at rest but is moving with a Fermi momentum in the rest frame of the nucleus.
- Dynamic effect which arises due to the strong interaction of the initial nucleon in the nuclear medium.

*In a nuclear medium for em interaction the expression for the cross section is written as:*

$$\frac{d^2\sigma^A}{d\Omega' dE'} = \frac{\alpha^2}{q^4}\frac{|\vec{k}'|}{|\vec{k}|} L^{\mu\nu} W^A_{\mu\nu},$$

# *Medium effects in lepton-A scattering*

- Kinematic effect which arises as the struck nucleon is not at rest but is moving with a Fermi momentum in the rest frame of the nucleus.
- Dynamic effect which arises due to the strong interaction of the initial nucleon in the nuclear medium.

*In a nuclear medium for em interaction the expression for the cross section is written as:*

$$\frac{d^2\sigma^A}{d\Omega' dE'} = \frac{\alpha^2}{q^4}\frac{|\vec{k}'|}{|\vec{k}|} L^{\mu\nu} W^A_{\mu\nu},$$

# *Medium effects in lepton-A scattering*

- Kinematic effect which arises as the struck nucleon is not at rest but is moving with a Fermi momentum in the rest frame of the nucleus.
- Dynamic effect which arises due to the strong interaction of the initial nucleon in the nuclear medium.

*In a nuclear medium for em interaction the expression for the cross section is written as:*

$$\frac{d^2\sigma^A}{d\Omega' dE'} = \frac{\alpha^2}{q^4}\,\frac{|\vec{k}'|}{|\vec{k}|}\,L^{\mu\nu}\,W^A_{\mu\nu},$$

## Nuclear hadronic tensor:

$$W_{\mu\nu}^{A} = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) W_1^A(\nu, Q^2) + \frac{W_2^A(\nu, Q^2)}{M_A^2}\left(p_\mu - \frac{p.q}{q^2}\, q_\mu\right)\left(p_\nu - \frac{p.q}{q^2}\, q_\nu\right)$$

**$W_i^A(\nu, Q^2)$ are redefined as:**

$$\begin{aligned} M_A W_1^A(\nu, Q^2) &= F_1^A(x) \\ \nu W_2^A(\nu, Q^2) &= F_2^A(x) \end{aligned}$$

## Nuclear hadronic tensor:

$$W^A_{\mu\nu} = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) W^A_1(\nu, Q^2) + \frac{W^A_2(\nu, Q^2)}{M_A^2}\left(p_\mu - \frac{p.q}{q^2}\, q_\mu\right)\left(p_\nu - \frac{p.q}{q^2}\, q_\nu\right)$$

$W^A_i(\nu, Q^2)$ **are redefined as:**

$$\begin{aligned} M_A W^A_1(\nu, Q^2) &= F^A_1(x) \\ \nu W^A_2(\nu, Q^2) &= F^A_2(x) \end{aligned}$$

- We use a relativistic nucleon spectral function to describe the momentum distribution of nucleons in nuclei.
- The spectral function has been calculated using Lehmann's representation for the relativistic nucleon propagator.
- Nuclear many body theory is used to calculate it for an interacting Fermi sea in nuclear matter.
- A local density approximation is then applied to translate these results to finite nuclei.

- We use a relativistic nucleon spectral function to describe the momentum distribution of nucleons in nuclei.
- The spectral function has been calculated using Lehmann's representation for the relativistic nucleon propagator.
- Nuclear many body theory is used to calculate it for an interacting Fermi sea in nuclear matter.
- A local density approximation is then applied to translate these results to finite nuclei.

- We use a relativistic nucleon spectral function to describe the momentum distribution of nucleons in nuclei.
- The spectral function has been calculated using Lehmann's representation for the relativistic nucleon propagator.
- Nuclear many body theory is used to calculate it for an interacting Fermi sea in nuclear matter.
- A local density approximation is then applied to translate these results to finite nuclei.

- We use a relativistic nucleon spectral function to describe the momentum distribution of nucleons in nuclei.
- The spectral function has been calculated using Lehmann's representation for the relativistic nucleon propagator.
- Nuclear many body theory is used to calculate it for an interacting Fermi sea in nuclear matter.
- A local density approximation is then applied to translate these results to finite nuclei.

The probability per unit time for the incoming lepton to collide with nucleons when traveling through nuclear matter:

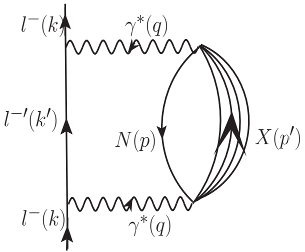
$$\Gamma = -\frac{2m_l}{|\vec{k}|}\text{Im}\Sigma$$

The probability per unit time for the incoming lepton to collide with nucleons when traveling through nuclear matter:

$$\Gamma = -\frac{2m_l}{|\vec{k}|}\mathrm{Im}\Sigma$$

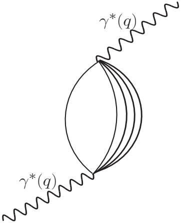
The cross section $\sigma$ for lepton-scattering from an element of volume $d^3r$ and surface dS in the nucleus:

$$\begin{aligned} d\sigma &= \Gamma dt dS = \Gamma \frac{dt}{dl} dS dl = \Gamma \frac{1}{v} dV \\ &= \Gamma \, \frac{E_l(\vec{k})}{|\vec{k}|} \, dV = -\frac{2m_l}{|\vec{k}|} \, \mathrm{Im}\,\Sigma \, d^3 r \,. \end{aligned}$$



## *Lepton self energy $\Sigma(k)$ for em interaction is written as:*

$$-i\Sigma(k) = \int \frac{d^4q}{(2\pi)^4} \bar{u}_l(\vec{k})\, ie\gamma^\mu\, i\frac{\not{k}' + m}{k'^2 - m^2 + i\epsilon}\, ie\gamma^\nu u_l(\vec{k}) \frac{-ig_{\mu\rho}}{q^2}\, (-i)\, \Pi^{\rho\sigma}(q)\, \frac{-ig_{\sigma\nu}}{q^2}$$




photon self-energy $\Pi^{\mu\nu}(q)$ in the nuclear medium:

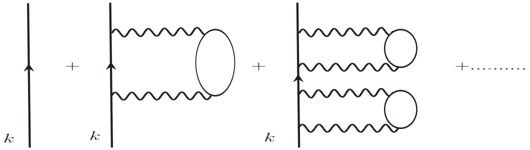
$$\Pi^{\mu\nu}(q) \quad = \quad e^2 \int \frac{d^4p}{(2\pi)^4} G(p) \sum_X \sum_{s_p, s_l} \prod_{i=1}^N \int \frac{d^4p'_i}{(2\pi)^4} \prod_l G_l(p'_l) \prod_j D_j(p'_j)$$

$$< X|J^\mu|H >< X|J^\nu|H >^* (2\pi)^4\, \delta^4(q + p - \sum_{i=1}^N p'_i)$$

Relativistic Dirac propagator $G^0(p_0,\vec{p})$ for a free nucleon:

$$G^0(p_0,\vec{p}) = \frac{M}{E(\vec{p})}\left\{\frac{\sum_r u_r(p)\bar{u}_r(p)}{p^0 - E(\vec{p}) + i\varepsilon} + \frac{\sum_r v_r(-p)\bar{v}_r(-p)}{p^0 + E(\vec{p}) - i\varepsilon}\right\}$$

The nucleon propagator in the interacting Fermi sea is obtained by making a perturbative expansion of $G(p^0,p)$ in terms of $G^0(p^0,p)$ by retaining the positive energy contributions only:




$$\begin{aligned} G(p_0,\vec{p}) &= \frac{M}{E(\vec{p})}\frac{\sum_r u_r(p)\bar{u}_r(p)}{(p^0 - E(\vec{p}) + i\varepsilon)} + \\ &\quad \left(\frac{M}{E(\vec{p})}\right)^2 \frac{1}{(p^0 - E(\vec{p}) + i\varepsilon)} \sum \frac{\sum_r u_r(p)\bar{u}_r(p)}{(p^0 - E(\vec{p}) + i\varepsilon)} + \ldots \\ &= \frac{M}{E(\vec{p})}\frac{\sum_r u_r(p)\bar{u}_r(p)}{\left(p^0 - E(\vec{p}) + i\varepsilon\frac{M}{E(\vec{p})}\Sigma\right)} \end{aligned}$$

## *This allows us to write the relativistic nucleon propagator in a nuclear medium in terms of the spectral functions of holes and particles as:*

$$G(p^0,\vec{p}) = \frac{M}{E(\vec{p})}\sum_r u_r(\vec{p})\bar{u}_r(\vec{p}) \left[\int_{-\infty}^{\mu} d\omega \frac{S_h(\omega,\vec{p})}{p^0-\omega-i\varepsilon} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega,\vec{p})}{p^0-\omega+i\varepsilon}\right]$$

## *This allows us to write the relativistic nucleon propagator in a nuclear medium in terms of the spectral functions of holes and particles as:*

$$G(p^0,\vec{p}) = \frac{M}{E(\vec{p})} \sum_r u_r(\vec{p})\bar{u}_r(\vec{p}) \left[ \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega,\vec{p})}{p^0-\omega-i\varepsilon} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega,\vec{p})}{p^0-\omega+i\varepsilon} \right]$$

for $p^0 \leq \mu$

$$S_h(p^0,\vec{p}) = \frac{1}{\pi} \frac{\frac{M}{E(\vec{p})} Im\Sigma(p^0,\vec{p})}{(p^0 - E(\vec{p}) - \frac{M}{E(\vec{p})} Re\Sigma(p^0,\vec{p}))^2 + (\frac{M}{E(\vec{p})} Im\Sigma(p^0,\vec{p}))^2}$$

for $p^0 > \mu$

$$S_p(p^0,\vec{p}) = -\frac{1}{\pi} \frac{\frac{M}{E(\vec{p})} Im\Sigma(p^0,\vec{p})}{(p^0 - E(\vec{p}) - \frac{M}{E(\vec{p})} Re\Sigma(p^0,\vec{p}))^2 + (\frac{M}{E(\vec{p})} Im\Sigma(p^0,\vec{p}))^2}$$

P. Fernandez de Cordoba and E. Oset, Phys. Rev. C 46 (1992) 1697

E. Marco, E. Oset and P. Fernandez de Cordoba, Nucl. Phys. A 611 (1996) 484

# *Local Density Approximation*

In the local density approximation reaction takes place at a point $r$, lying inside a volume $d^3r$ with local density $\rho_p(r)$ and $\rho_n(r)$ corresponding to the proton and neutron densities

$$\begin{aligned}\rho_p(r) &= \frac{Z}{A}\rho(r) \\ \rho_n(r) &= \frac{A-Z}{A}\rho(r)\end{aligned}$$

# *Local Density Approximation*

In the local density approximation reaction takes place at a point $r$, lying inside a volume $d^3r$ with local density $\rho_p(r)$ and $\rho_n(r)$ corresponding to the proton and neutron densities

$$\begin{aligned}\rho_p(r) &= \frac{Z}{A}\rho(r)\\ \rho_n(r) &= \frac{A-Z}{A}\rho(r)\end{aligned}$$

Fermi momentum of the nucleon is

$$\begin{aligned}p_{F_p} &= (3\pi^2\rho_p(\vec{r}))^{1/3}\\ p_{F_n} &= (3\pi^2\rho_n(\vec{r}))^{1/3}\end{aligned}$$

# *Local Density Approximation*

In the local density approximation reaction takes place at a point $r$, lying inside a volume $d^3r$ with local density $\rho_p(r)$ and $\rho_n(r)$ corresponding to the proton and neutron densities

$$\begin{aligned}\rho_p(r) &= \frac{Z}{A}\rho(r)\\ \rho_n(r) &= \frac{A-Z}{A}\rho(r)\end{aligned}$$

Fermi momentum of the nucleon is

$$\begin{aligned}p_{F_p} &= (3\pi^2\rho_p(\vec{r}))^{1/3}\\ p_{F_n} &= (3\pi^2\rho_n(\vec{r}))^{1/3}\end{aligned}$$

This leads to the spectral functions for the protons and neutrons to be the function of local Fermi momentum given by

$$2\int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega,\vec{p},p_{F_{p,n}}(\vec{r}))\, d\omega = \rho_{p,n}(\vec{r})$$

$$4\int d^3r \int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega,\vec{p},\rho(r))\, d\omega = A$$

## *Nuclear hadronic tensor:*

In the LDA, the nuclear hadronic tensor can be written as a convolution of nucleonic hadronic tensor with the hole spectral function

$$W^A_{\alpha\beta} = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} dp^0 \frac{M}{E(\vec{p})} S_h(p^0, \vec{p}, \rho(r)) W^N_{\alpha\beta}(p,q)$$

## *Nuclear hadronic tensor:*

In the LDA, the nuclear hadronic tensor can be written as a convolution of nucleonic hadronic tensor with the hole spectral function

$$W^A_{\alpha\beta} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} dp^0 \frac{M}{E(\vec{p})} S_h(p^0, \vec{p}, \rho(r)) W^N_{\alpha\beta}(p,q)$$

Taking the xx component

$$W^N_{xx} = \left(\frac{q_x q_x}{q^2} - g_{xx}\right) W^N_1 + \frac{1}{M^2}\left(p_x - \frac{p.q}{q^2}\,q_x\right)\left(p_x - \frac{p.q}{q^2}\,q_x\right) W^N_2$$

## *Nuclear hadronic tensor:*

In the LDA, the nuclear hadronic tensor can be written as a convolution of nucleonic hadronic tensor with the hole spectral function

$$W^A_{\alpha\beta} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} dp^0 \frac{M}{E(\vec{p})} S_h(p^0, \vec{p}, \rho(r)) W^N_{\alpha\beta}(p,q)$$

Taking the xx component

$$W^N_{xx} = \left(\frac{q_x q_x}{q^2} - g_{xx}\right) W^N_1 + \frac{1}{M^2}\left(p_x - \frac{p.q}{q^2}\, q_x\right)\left(p_x - \frac{p.q}{q^2}\, q_x\right) W^N_2$$

Chosing $\vec{q}$ along the z-axis

$$W^N_{xx}(\nu_N, Q^2) = W^N_1(\nu_N, Q^2) + \frac{1}{M^2} p_x^2 W^N_2(\nu_N, Q^2)$$

Similarily taking xx component of nuclear hadronic tensor

$$W^A_{xx}(\nu_A, Q^2) = W^A_1(\nu_A, Q^2) = \frac{F^A_1(x_A)}{AM}$$

Similarily taking xx component of nuclear hadronic tensor

$$W^A_{xx}(\nu_A, Q^2) = W^A_1(\nu_A, Q^2) = \frac{F^A_1(x_A)}{AM}$$

## $F_1(x) = M\ W_1(\nu, Q^2),\ \ F_2(x) = \nu\ W_2(\nu, Q^2)$

$$\frac{F^A_{_1}(x_A)}{AM} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\,\frac{M}{E(\vec{p})}\int_{-\infty}^{\mu} dp^0 S_h(p^0, \vec{p}, \rho(\vec{r})) \times \left[\frac{F^N_1(x_N)}{M} + \frac{1}{M^2}{p_x}^2\frac{F^N_2(x_N)}{\nu}\right]$$

## *By using Callan-Gross relation* $2xF_1(x) = F_2(x)$

$$
\begin{aligned}
F_2^A(x_A) &= 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\vec{p})}\int_{-\infty}^{\mu} dp^0\, S_h(p^0,\vec{p},\rho(\vec{r}))\frac{x}{x_N} \\
&\quad \times \left(1+\frac{2x_N p_x^2}{M\nu_N}\right) F_2^N(x_N)
\end{aligned}
$$

## *By using Callan-Gross relation* $2xF_1(x) = F_2(x)$

$$F_2^A(x_A) = 4\int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\vec{p})} \int_{-\infty}^{\mu} dp^0 \, S_h(p^0, \vec{p}, \rho(\vec{r})) \frac{x}{x_N} \times \left(1 + \frac{2x_N p_x^2}{M\nu_N}\right) F_2^N(x_N)$$

where

$$x_A = \frac{x}{A},$$
$$x_N = \frac{Q^2}{2(p^0q^0 - p_zq_z)}$$

# Effect of nuclear medium on Callan Gross relation

To obtain $F_2(x)$ independently

$$
\begin{aligned}
W_{zz}^N &= \left(\frac{q_z^2}{q^2} - g_{zz}\right) W_1^N + \frac{1}{M^2}\left(p_z - \frac{p.q}{q^2}\, q_z\right)^2 W_2^N \\
&= \frac{q_0^2}{q^2}\, W_1^N + \frac{1}{M^2}\left(\frac{(p_z q^2 - p.q\; q_z)^2}{q^4}\right) W_2^N
\end{aligned}
$$

$$
W_{zz}^A(\nu_A, Q^2) = \left(\frac{q_z^2}{q^2} - g_{zz}\right) W_1^A + \frac{1}{M_A^2}\left(-\frac{p_A.q}{q^2}\, q_z\right)^2 W_2^A
$$

$F_2(x) = \nu W_2(\nu, Q^2)$

*Finally, we obtain*

$$
\begin{aligned}
F_2^A(x_A) \quad &= \quad 2\sum_{p,n}\int d^3r\int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\vec{p})}\int_{-\infty}^{\mu} dp^0 S_h^{p,n}(p^0,\vec{p},\rho_{p,n}(\vec{r}))\, F_2^N(x_N) \\
&\times \quad \left[\frac{Q^2}{q_z^2}\left(\frac{p^2 - p_z^2}{2M^2}\right) + \frac{(p.q)^2}{M^2\nu^2}\left(\frac{p_z\, Q^2}{p.q q_z} + 1\right)^2 \frac{q_0 M}{p_0\, q_0 - p_z\, q_z}\right]
\end{aligned}
$$

This expression of $F_2^A(x_A)$ is obtained without applying Callan-Gross relation.

# $\pi$ *and* $\rho$ *mesons contribution to the nuclear structure function*

## *For mesons cloud contribution*

$$2\pi\frac{M}{E(\vec{p})}S_h(p_0,\vec{p})W_N^{\alpha\beta}(p,q) \rightarrow 2ImD(p)\theta(p_0)W_\pi^{\alpha\beta}(p,q)$$

Pion propagator in the nuclear medium

$$D(p) = [{p_0}^2 - \vec{p}^{\,2} - m_\pi^2 - \Pi_\pi(p_0,\vec{p})]^{-1}$$

with

$$\Pi_\pi = \frac{f^2/m_\pi^2 F^2(p)\vec{p}^{\,2}\Pi^*}{1 - f^2/m_\pi^2 V_L'\Pi^*}$$

$\pi NN$ form factor

$$F(p) = (\Lambda^2 - m_\pi^2)/(\Lambda^2 + \vec{p}^{\,2})$$

Similar to nucleonic case

$$W^{\mu\nu}_{A,\pi} = 3 \int d^3r \int \frac{d^4p}{(2\pi)^4}\, \theta(p_0)(-2)\, ImD(p)\, 2m_\pi W^{\mu\nu}_\pi(p,q)$$

*Factor 3 $\Rightarrow$ Three Charged states of pion*

For pion excess in nuclear medium

$$ImD(p) \;\rightarrow\; \delta ImD(p) \equiv ImD(p) - \rho\, \frac{\partial ImD(p)}{\partial\rho}\Big|_{\rho=0}$$

which leads to

$$F^A_{1,\pi}(x_\pi) \;=\; -6AM \int d^3r \int \frac{d^4p}{(2\pi)^4} \theta(p_0)\, \delta ImD(p)\, 2m_\pi \;\times \left[\frac{F_{1\pi}(x_\pi)}{m_\pi} + \frac{|\vec{p}|^2 - p_z^2}{2(p_0\, q_0 - p_z q_z)} \frac{F_{2\pi}(x_\pi)}{m_\pi}\right]$$

## *Contribution from rho meson*

Propagator for *rho* meson

$$D_\rho(p) = [{p_0}^2 - \vec{p}^{\,2} - m_\rho^2 - \Pi_\rho^*(p_0, \vec{p})]^{-1}$$

with irreducible $\rho$ self-energy

$$\Pi_\rho^* = \frac{f^2/m_\rho^2 C_\rho F_\rho^2(p) \vec{p}^{\,2} \Pi^*}{1 - f^2/m_\rho^2 V_T' \Pi^*}$$

$\rho NN$ form factor

$$F_\rho(p) = (\Lambda_\rho^2 - m_\rho^2)/(\Lambda_\rho^2 + \vec{p}^{\,2})$$

*Finally,*

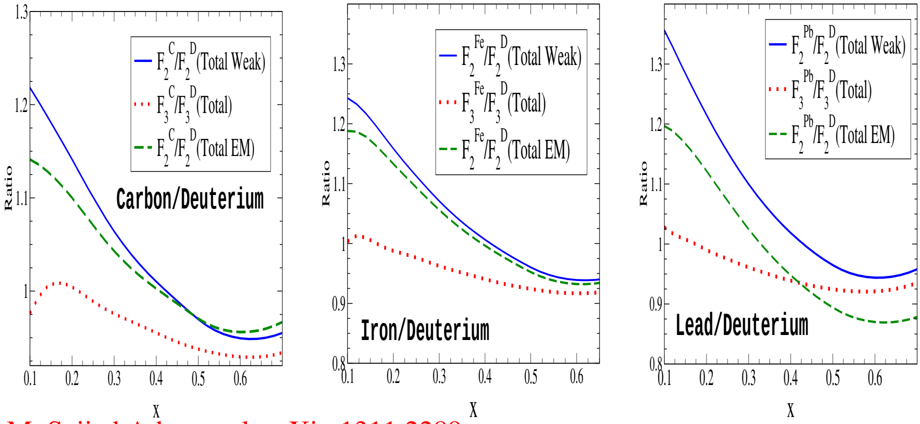
$$\begin{aligned} F^A_{1,\rho}(x_\rho) &= -12AM \int d^3r \int \frac{d^4p}{(2\pi)^4} \theta(p_0)\, \delta Im D_\rho(p)\, 2m_\rho \times \\ & \left[ \frac{F_{1\rho}(x_\rho)}{m_\rho} + \frac{|\vec{p}|^2 - p_z^2}{2(p_0\, q_0 - p_z\, q_z)} \frac{F_{2\rho}(x_\rho)}{m_\rho} \right] \end{aligned}$$

## Structure functions for $\pi$ and $\rho$ mesons without using Callan-Gross relation:

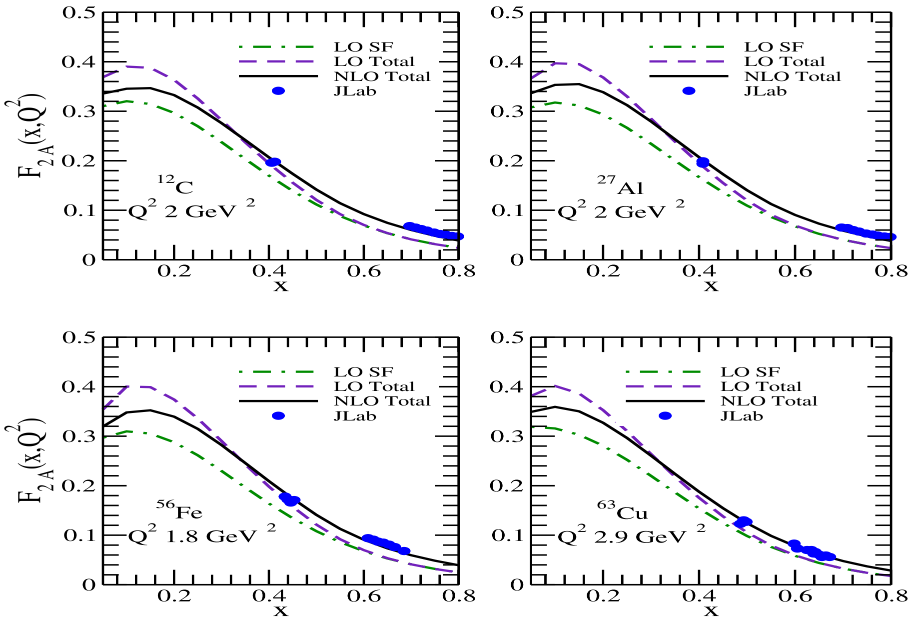
$$F^A_{2,\pi}(x_\pi) = -6\int d^3r \int \frac{d^4p}{(2\pi)^4}\,\theta(p_0)\,\delta ImD(p)\,2m_\pi\,\frac{m_\pi}{p_0 - p_z\,\gamma}\times$$
$$\left[\frac{Q^2}{q_z^2}\left(\frac{|\vec{p}|^2 - p_z^2}{2m_\pi^2}\right) + \frac{(p_0 - p_z\,\gamma)^2}{m_\pi^2}\left(\frac{p_z\,Q^2}{(p_0 - p_z\,\gamma)q_0 q_z} + 1\right)^2\right] F_{2\pi}(x_\pi)$$

$$F^A_{2,\rho}(x_\rho) = -12\int d^3r \int \frac{d^4p}{(2\pi)^4}\,\theta(p_0)\,\delta ImD_\rho(p)\,2m_\rho\,\frac{m_\rho}{p_0 - p_z\,\gamma}\times$$
$$\left[\frac{Q^2}{q_z^2}\left(\frac{|\vec{p}|^2 - p_z^2}{2m_\rho^2}\right) + \frac{(p_0 - p_z\,\gamma)^2}{m_\rho^2}\left(\frac{p_z\,Q^2}{(p_0 - p_z\,\gamma)q_0 q_z} + 1\right)^2\right] F_{2\rho}(x_\rho)$$

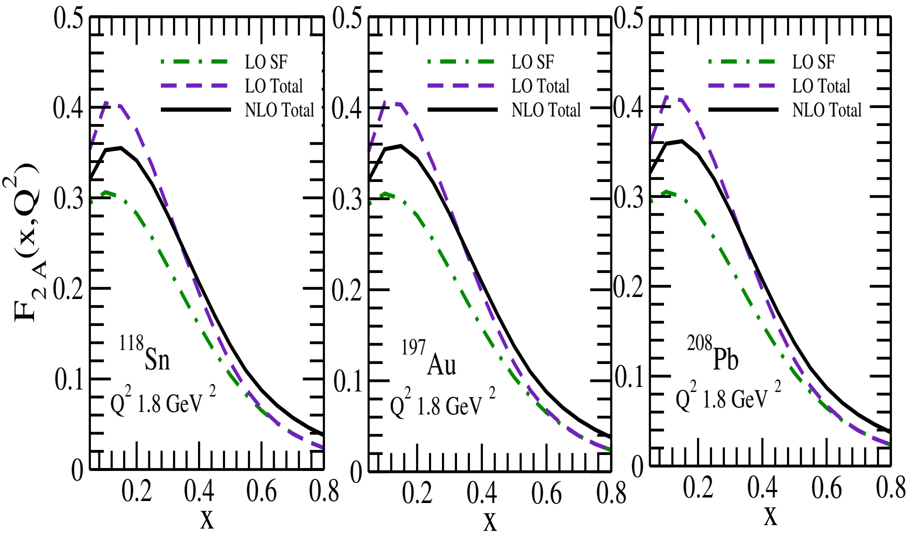
# *Ratio of Structure functions in Weak and E.M. cases*



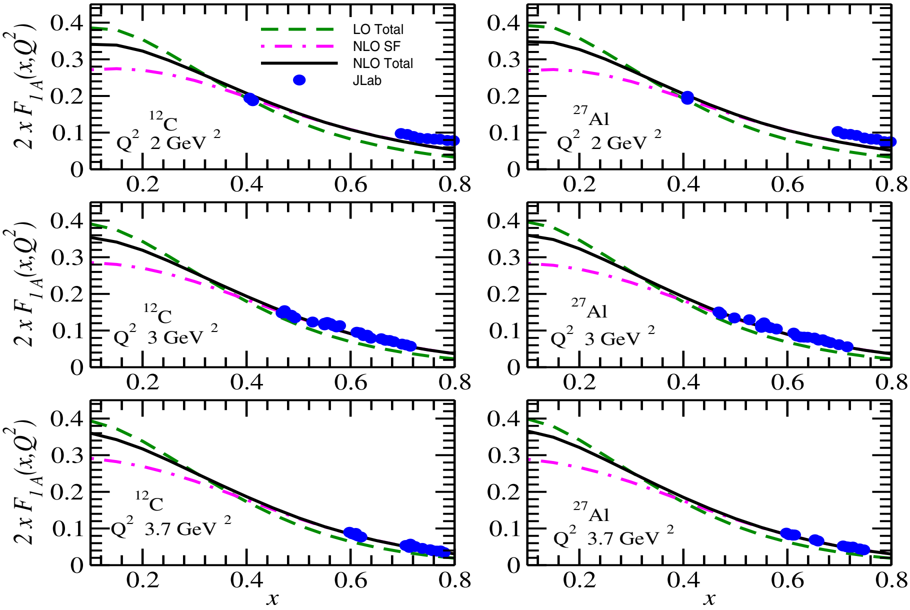

M. Sajjad Athar et al. arXiv:1311.2289



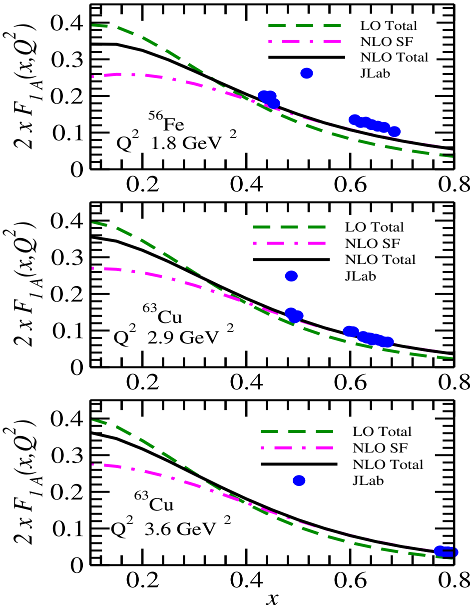

NPA 943 58 (2015), V. Mamyan arXiv:1202.1457



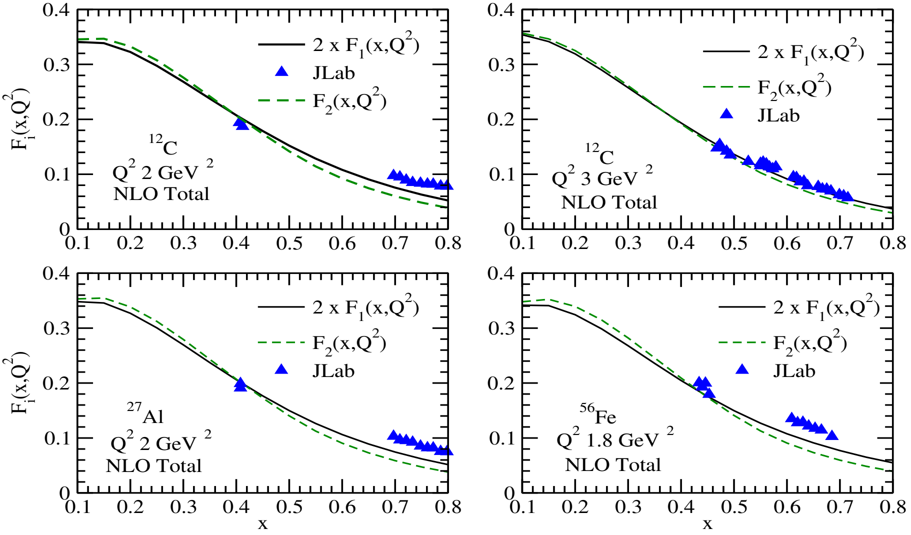
0.5
0.4
0.3
0.2
0.1
0
$F_{2A}(x,Q^2)$
LO SF
LO Total
NLO Total
$^{118}$Sn
$Q^2$ 1.8 GeV $^2$
$^{197}$Au
$Q^2$ 1.8 GeV $^2$
$^{208}$Pb
$Q^2$ 1.8 GeV $^2$
0.2
0.4
0.6
0.8
X



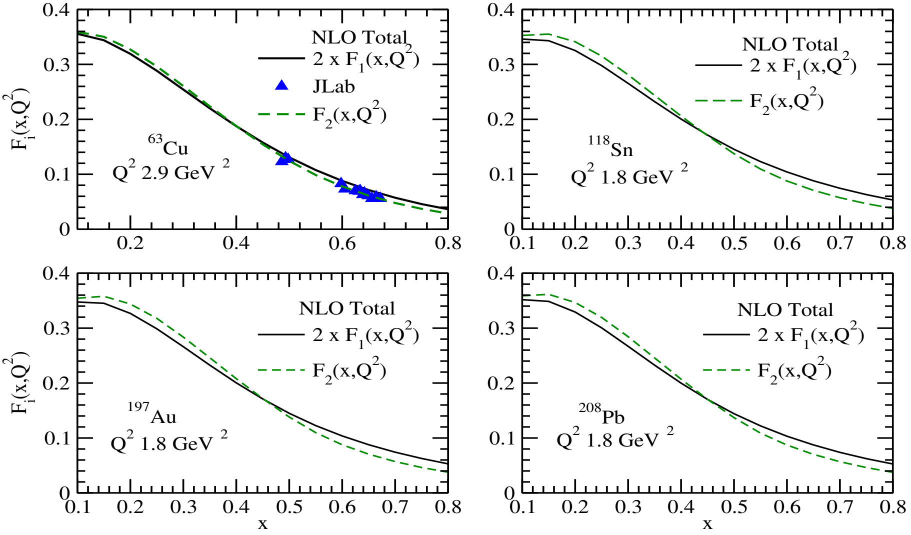

NPA 943 58 (2015), V. Mamyan arXiv:1202.1457



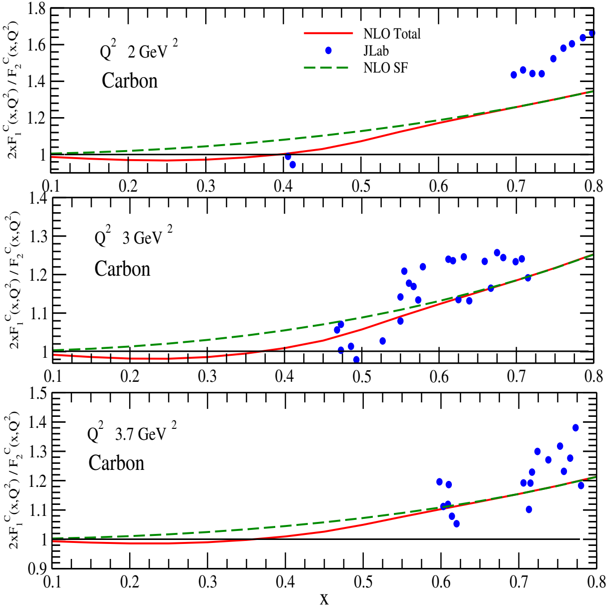

NPA 943 58 (2015), V. Mamyan arXiv:1202.1457




NPA 943 58 (2015), V. Mamyan arXiv:1202.1457

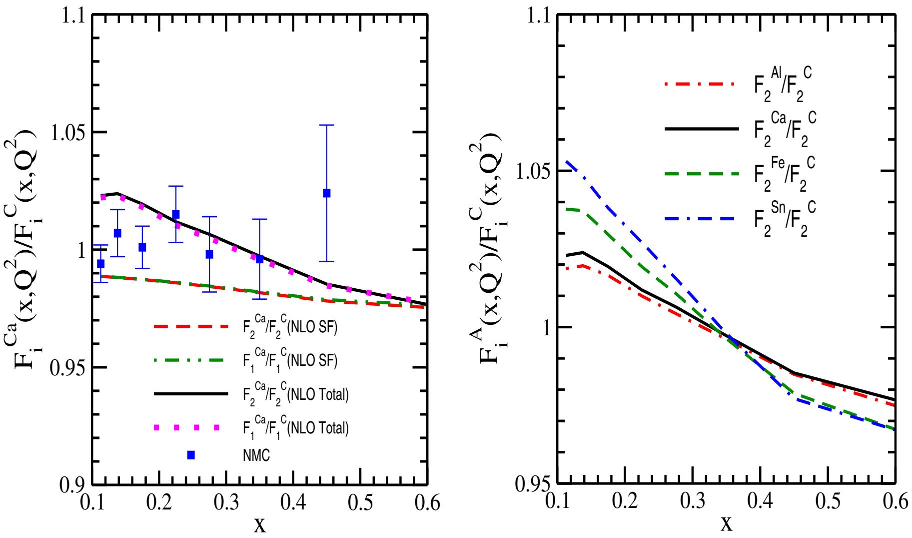



NPA 943 58 (2015), V. Mamyan arXiv:1202.1457




NPA 943 58 (2015), V. Mamyan arXiv:1202.1457

# Nuclear dependence in $\frac{F_i^A(x,Q^2)}{F_i^C(x,Q^2)}$




NPA 943 58 (2015), Nucl. Phys. B 441 3 (1995)

# *Percantage difference for $F_2$ in different nuclei*

| Nucleus | $Q^2(GeV^2)$ | $\lvert\frac{LO_{SF}-LO_T}{LO_T}\rvert\%$ | $\lvert\frac{LO_T-NLO_T}{NLO_T}\rvert\%$ | $\lvert\frac{NLO_T-NLO_{WOS}}{NLO_{WOS}}\rvert\%$ |
|---|---|---|---|---|
| $^{12}$C(x=0.2) | 2 | 18 | 9 | ≈2 |
| (x=0.4) | | 12 | 6 | 1 |
| (x=0.8) | | 0.16 | 38 | 0.25 |
| $^{27}$Al(x=0.2) | 2 | 20 | ≈9 | 2 |
| (x=0.4) | | 14 | 6 | 1 |
| (x=0.8) | | 0.2 | 38 | 0.3 |
| $^{56}$Fe(x=0.2) | 1.8 | 23 | 10 | 2 |
| (x=0.4) | | 17 | 5 | 1.3 |
| (x=0.8) | | 0.24 | 37 | 0.37 |
| $^{63}$Cu(x=0.2) | 2.9 | 20 | 7.7 | 1.9 |
| (x=0.4) | | 12 | 6.5 | 1 |
| (x=0.8) | | 0.18 | 38.7 | 0.3 |
| $^{118}$Sn(x=0.2) | 1.8 | 24.6 | 9.8 | 2.3 |
| (x=0.4) | | 18.5 | 5.3 | 1.3 |
| (x=0.8) | | 0.26 | 37 | 0.4 |
| $^{197}$Au(x=0.2) | 1.8 | 25 | 9.7 | 2.6 |
| (x=0.4) | | 19.8 | 5 | 1.5 |
| (x=0.8) | | 0.3 | 37 | 0.46 |
| $^{208}$Pb(x=0.2) | 1.8 | 26 | 9.6 | 2.4 |
| (x=0.4) | | 19 | 5 | 1.4 |
| (x=0.8) | | 0.28 | 37 | 0.43 |

# *Percantage difference for* $|\frac{2xF_1 - F_2}{2xF_1}|$ *in different nuclei*

| Nucleus | $Q^2(GeV^2)$ | $\lvert\frac{2xF_1-F_2}{2xF_1}\rvert\%$ |
|---|---|---|
| $^{12}$C(x=0.2) | 2 | $\approx$ 3 |
| (x=0.7) | | $\approx$ 21 |
| (x=0.2) | 3 | $\approx$ 2 |
| (x=0.7) | | $\approx$ 16 |
| (x=0.2) | 3.7 | 1.4 |
| (x=0.7) | | $\approx$ 13 |
| $^{56}$Fe(x=0.2) | 1.8 | $\approx$ 5 |
| (x=0.7) | | $\approx$ 23 |
| (x=0.2) | 5 | 1.3 |
| (x=0.7) | | 11 |
| (x=0.2) | 10 | $\approx$ 0.6 |
| (x=0.7) | | $\approx$ 6 |
| (x=0.2) | 20 | $\approx$ 0.3 |
| (x=0.7) | | $\approx$ 3 |
| $^{63}$Cu(x=0.2) | 2.9 | 2.5 |
| (x=0.7) | | $\approx$ 17 |
| (x=0.2) | 3.6 | 2 |
| (x=0.7) | | $\approx$ 14 |

# Conclusions

1. We find that the effect of nuclear medium is also quite important even for DIS.

# *Conclusions*

1. We find that the effect of nuclear medium is also quite important even for DIS.
2. We find that $F_2^A(x, Q^2)$ is different from $2xF_1^A(x, Q^2)$ that means Callan Gross relation deviates at nuclear level.

# Conclusions

1. We find that the effect of nuclear medium is also quite important even for DIS.
2. We find that $F_2^A(x,Q^2)$ is different from $2xF_1^A(x,Q^2)$ that means Callan Gross relation deviates at nuclear level.
3. We compare our results with the JLab and NMC data and found them in good agreement.

# Conclusions

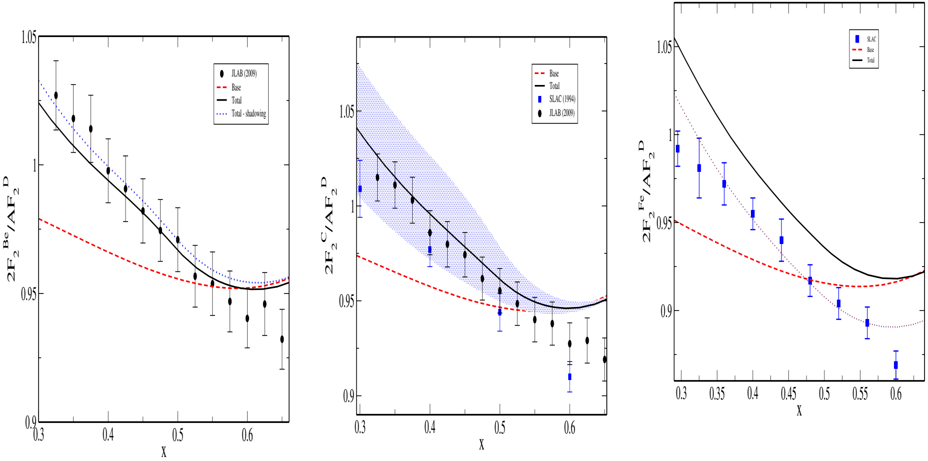
1. We find that the effect of nuclear medium is also quite important even for DIS.
2. We find that $F_2^A(x,Q^2)$ is different from $2xF_1^A(x,Q^2)$ that means Callan Gross relation deviates at nuclear level.
3. We compare our results with the JLab and NMC data and found them in good agreement.
4. The present work will make useful predictions for the future experiments in the low $x$ and moderate $Q^2$.


Grazie

# BACK UP SLIDES

# *Electromagnetic Nuclear Structure Function*

$$\frac{2F_2^A}{AF_2^D}(A = Be, C, Fe) \text{ vs x}$$




8

M Sajjad Athar et al. NPA 857 29 (2011)

## *Theoretical Study*

1. Along with the experimental efforts theoretical groups also performed calculations in order to understand the nuclear medium effects for the e.m. DIS process.

## *Theoretical Study*

1. Along with the experimental efforts theoretical groups also performed calculations in order to understand the nuclear medium effects for the e.m. DIS process.
2. Our group at Aligarh performed calculations by taking into account the dynamic origin of nuclear medium effects.

## *Theoretical Study*

1. Along with the experimental efforts theoretical groups also performed calculations in order to understand the nuclear medium effects for the e.m. DIS process.
2. Our group at Aligarh performed calculations by taking into account the dynamic origin of nuclear medium effects.

**Aligarh group:**

NPA 943 58 (2015)

NPA 940 138 (2015)

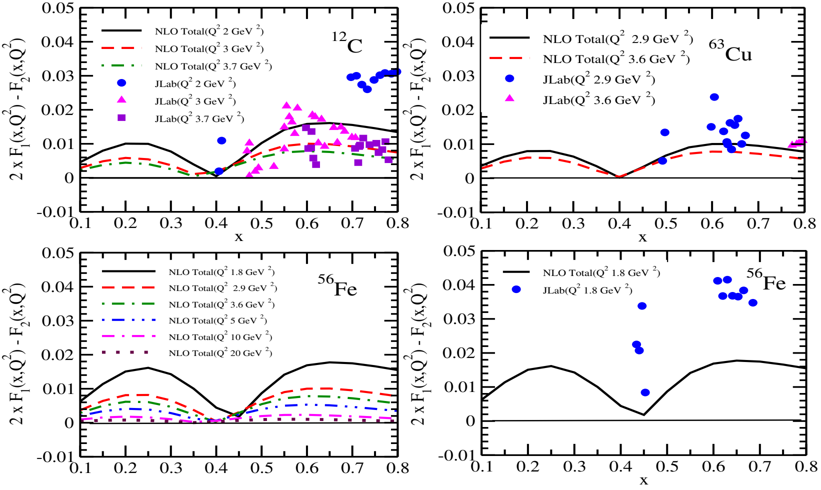
PRC 87 035502 (2013)

PRC 85 055201 (2012)

PRC 84 054610 (2011)

NPA 857 29 (2011)

PLB 668 133 (2008)



2 x F$_1$(x,Q$^2$) - F$_2$(x,Q$^2$)
x
NLO Total(Q$^2$ 2 GeV $^2$)
NLO Total(Q$^2$ 3 GeV $^2$)
NLO Total(Q$^2$ 3.7 GeV $^2$)
JLab(Q$^2$ 2 GeV $^2$)
JLab(Q$^2$ 3 GeV $^2$)
JLab(Q$^2$ 3.7 GeV $^2$)
$^{12}$C
NLO Total(Q$^2$ 2.9 GeV $^2$)
NLO Total(Q$^2$ 3.6 GeV $^2$)
JLab(Q$^2$ 2.9 GeV $^2$)
JLab(Q$^2$ 3.6 GeV $^2$)
$^{63}$Cu
NLO Total(Q$^2$ 1.8 GeV $^2$)
NLO Total(Q$^2$ 2.9 GeV $^2$)
NLO Total(Q$^2$ 3.6 GeV $^2$)
NLO Total(Q$^2$ 5 GeV $^2$)
NLO Total(Q$^2$ 10 GeV $^2$)
NLO Total(Q$^2$ 20 GeV $^2$)
$^{56}$Fe
NLO Total(Q$^2$ 1.8 GeV $^2$)
JLab(Q$^2$ 1.8 GeV $^2$)
$^{56}$Fe

# *Shadowing and antishadowing effects*

"Significant at low-x and low-$Q^2$"

The shadowing suppression at small x occurs due to coherent multiple scattering inside the nucleus of a quark-anti quark pair coming from the virtual boson with destructive interference of the amplitudes. The shadowing effect is important at low $x$ and low $Q^2$ .

The anti-shadowing effect is due to constructive interference of the multiple scattering amplitudes. This effect is also important at low x but greater than $x$ region of shadowing.

For the shadowing and antishadowing effects, Glauber-Gribov multiple scattering model has been used following the work of Kulagin and Petti. PRD76(2007)094033.

# *Shadowing and antishadowing effects*

"Significant at low-x and low-$Q^2$"

The shadowing suppression at small x occurs due to coherent multiple scattering inside the nucleus of a quark-anti quark pair coming from the virtual boson with destructive interference of the amplitudes. The shadowing effect is important at low $x$ and low $Q^2$ .

The anti-shadowing effect is due to constructive interference of the multiple scattering amplitudes. This effect is also important at low x but greater than $x$ region of shadowing.

For the shadowing and antishadowing effects, Glauber-Gribov multiple scattering model has been used following the work of Kulagin and Petti. PRD76(2007)094033.

# *Shadowing and antishadowing effects*

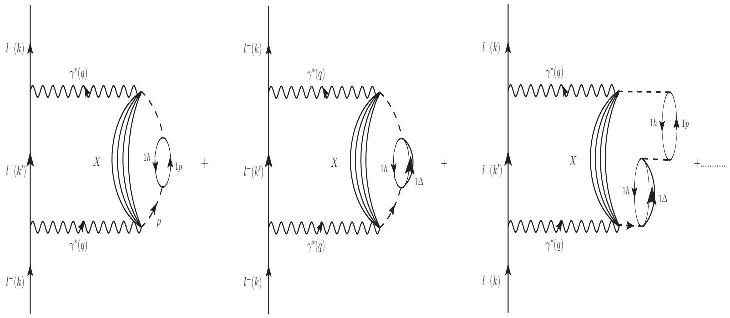
"Significant at low-x and low-$Q^2$"

The shadowing suppression at small x occurs due to coherent multiple scattering inside the nucleus of a quark-anti quark pair coming from the virtual boson with destructive interference of the amplitudes. The shadowing effect is important at low $x$ and low $Q^2$ .

The anti-shadowing effect is due to constructive interference of the multiple scattering amplitudes. This effect is also important at low x but greater than $x$ region of shadowing.

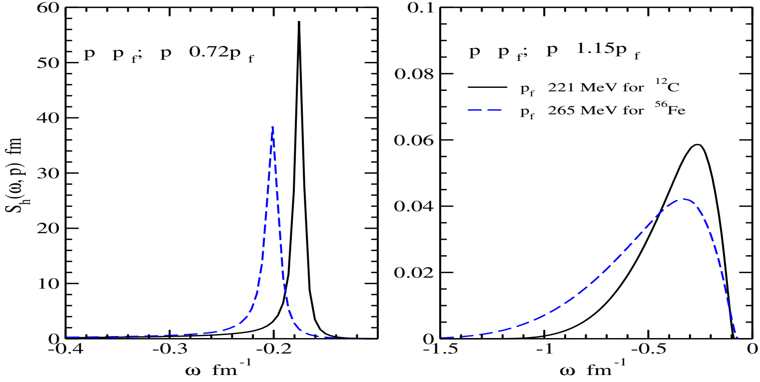
For the shadowing and antishadowing effects, Glauber-Gribov multiple scattering model has been used following the work of Kulagin and Petti. PRD76(2007)094033.

# π and ρ mesons contribution to the nuclear structure function




1. More effective for heavier nuclei
2. Contributes to the intermediate $x$ region
3. Implemented following the many body field theoretical approach
4. Using parametrization by Gluck et al.

# *Nature of Spectral Function*




- Behave like a $\delta$ function for $p < p_F$,
- Long range for $p > p_F$.

## Hole and particle spectral function fulfills

$$\int_{-\infty}^{\mu} dp_0 \, S_h(p_0,\vec{p}) + \int_{\mu}^{\infty} dp_0 \, S_p(p_0,\vec{p}) = 1$$

Hole and particle spectral function fulfills

$$\int_{-\infty}^{\mu} dp_0 \, S_h(p_0, \vec{p}) + \int_{\mu}^{\infty} dp_0 \, S_p(p_0, \vec{p}) = 1$$

When interactions are not present

$$\begin{aligned} \Sigma^N(p) &= 0 \\ G(p) &\to G^0(p) \end{aligned}$$

Hole and particle spectral function fulfills

$$\int_{-\infty}^{\mu} dp_0 \, S_h(p_0, \vec{p}) + \int_{\mu}^{\infty} dp_0 \, S_p(p_0, \vec{p}) = 1$$

When interactions are not present

$$\begin{aligned} \Sigma^N(p) &= 0 \\ G(p) &\rightarrow G^0(p) \end{aligned}$$

Therefore,

$$S_h(p_0, \vec{p}) = S_p(p_0, \vec{p}) = \delta(p_0 - E(\vec{p}))$$

$$\begin{aligned} \int_{-\infty}^{\mu} dp_0 \, S_h(p_0, \vec{p}) &= \int_{-\infty}^{\mu} dp_0 \, \delta(p_0 - E(\vec{p})) = \begin{cases} 1 & \text{if} \quad \mu > E(\vec{p}) \\ 0 & \text{if} \quad \mu < E(\vec{p}) \end{cases} \\ \int_{\mu}^{\infty} dp_0 \, S_p(p_0, \vec{p}) &= \int_{\mu}^{\infty} dp_0 \, \delta(p_0 - E(\vec{p})) = \begin{cases} 1 & \text{if} \quad \mu < E(\vec{p}) \\ 0 & \text{if} \quad \mu > E(\vec{p}) \end{cases} \end{aligned}$$

## *Chemical potential is defined as*

$$\begin{aligned} \mu &= M + \varepsilon_F \\ \text{and} \quad p_0 &= M + \omega \end{aligned}$$

$$\begin{aligned} \int_{-\infty}^{\mu} dp_0\, S_h(p_0, \vec{p}) &= \int_{-\infty}^{\mu - M} d\omega\, \delta(\omega + M - E(\vec{p})) \\ &= \begin{cases} 1 & ; \mu - M > E(\vec{p}) - M \Rightarrow \varepsilon_F > \varepsilon(\vec{p}) \\ 0 & ; \mu - M < E(\vec{p}) - M \Rightarrow \varepsilon_F < \varepsilon(\vec{p}) \end{cases} \\ \int_{\mu}^{\infty} dp_0\, S_p(p_0, \vec{p}) &= \int_{\mu - M}^{\infty} d\omega\, \delta(\omega + M - E(\vec{p})) \\ &= \begin{cases} 1 & ; \mu - M < E(\vec{p}) - M \Rightarrow \varepsilon_F < \varepsilon(\vec{p}) \\ 0 & ; \mu - M > E(\vec{p}) - M \Rightarrow \varepsilon_F > \varepsilon(\vec{p}) \end{cases} \end{aligned}$$

## *Hole spectral function*

With Fermi energy

$$\varepsilon_F = \frac{p_F^2}{2M}$$

we have

$$\begin{aligned} \int_{-\infty}^{\mu} dp_0\, S_h(p_0, \vec{p}) &= \theta(p_F - |\vec{p}|) \equiv n_0(\vec{p}) \\ \int_{\mu}^{\infty} dp_0\, S_p(p_0, \vec{p}) &= \theta(|\vec{p}| - p_F) \equiv 1 - n_0(\vec{p}) \end{aligned}$$

Hole spectral function is

1. $\mathcal{P}$ of removing a nucleon from correlated ground state
2. $\mathcal{P}$ of finding the nucleons with an energy $p_0 < E < p_0 + dp_0$