# Decays of heavy mesons in the framework of covariant quark model


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# Overview

- **Motivation**
  - New high-luminosity machines, new measurements.
  - New physics?
- **Covariant quark model for mesons**
  - Lagrangian.
  - Compositeness condition.
  - Confinement.
- **Processes and results**
  - $B_s \to J/\Psi + \eta^{(\prime)}$.
  - $B \to K^{(*)}\mu^+\mu^-$, $B \to K^{(*)}\bar{\nu}\nu$, $B \to K^*\gamma$.
  - Short mention of other meson, baryons and tetraquark results.
- **Conclusion**
  - Summary.
  - Outlook.

# Motivation

- **Experiment & theory**
  - New high-luminosity machines: rare flavor-changing processes observed & measured.
  - Some heavy meson (baryon) decays: sensitiveness to hypothetical new heavy particles in Feynman diagrams for many NP scenarios.
  - Increasing experimental information, for $B \to K^* \mu^+ \mu^-$ (BABAR, Belle, CDF, LHCb) angular distributions measured.
  - Standard model confirmed with some tensions (~3σ).

- **Covariant quark model**
  - Model dependence (form factors) cannot be fully eliminated even with smartly defined observables (asymmetries, ratios) $\Rightarrow$ CQM.
  - Fully relativistic Lagrangian-based approach to hadronic interactions with a non-local quark current definition.
  - Standard QFT techniques and limited number of parameters.
  - Nice agreement with experimental data.

# Covariant quark model

- **Lagrangian (mesons)**

$$\mathrm{L}_{\mathrm{int}} = g_H \cdot H(x) \cdot J_H(x)$$

$$J_M(x) = \int dx_1 \int dx_2 \, F_M(x, x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1) \, \Gamma_M \, q^a_{f_2}(x_2)$$

$$F_H(x, x_1, \ldots, x_n) = \delta\left(x - \sum_{i=1}^{n} w_i x_i\right) \Phi_H\left(\sum_{i<j}((x_i - x_j)^2\right)$$

$$w_i = m_i / \sum_{j=1}^{n} m_j \qquad \bar{\Phi}_H(-k^2) = \exp\left(k^2/\Lambda_H^2\right)$$

- **Free parameters**
  - Constituent quark masses [4], hadron-size related parameters [N] and universal cut-off [1] (N+5 in total). Numerical values from fits to data.

    In GeV: $m_{u,d}$= 0.235, $m_s$= 0.424, $m_c$= 2.16, $m_b$= 5.09, $\lambda_{\text{cut-off}}$= 0.181, $\Lambda_\pi$ = 0.87, $\Lambda_K$= 1.04, …

- **Compositeness condition**
  - Hadrons are made up of quarks → renormalization constant $Z_H^{1/2}$ can be interpreted as the matrix element between the physical state and the corresponding bare state. $Z_H^{1/2}$ =0 → physical state does not contain bare state and is properly described as a bound state. Couplings $g_H$ are eliminated as free parameters.

[ A. Salam, Nuovo Cim. 25, 224 (1962), S. Weinberg, Phys. Rev. 130, 776 (1963) ]

$$\mathrm{Z_H} = 1 - \frac{3\mathrm{g_H^2}}{4\pi^2}\tilde{\Pi}'_{\mathrm{H}}\left(\mathrm{m_H^2}\right) = 0$$

($\Pi_H$ – meson mass operator)

# Computation techniques & infrared confinement

- **Feynman graph evaluation**
  - General form $$\Pi(p_1,...,p_j) = \int\!\![d^4k]^\ell \prod_{i_1=1}^{m} \Phi_{i_1+n}\left(-K^2_{i_1+n}\right) \prod_{i_3=1}^{n} S_{i_3}(\tilde{k}_{i_3} + \tilde{p}_{i_3})$$

    *j* external momenta
    *l* loop integrations
    *m* vertices *n* quark propagators
    $\tilde{k}_i$ -linear combination of loop momenta $k_i$ $\tilde{p}_i$ -linear combination of external momenta $p_i$

    $$K^2_{i_1+n} = \sum_{i_2}(\tilde{k}^{(i_2)}_{i_1+n} + \tilde{p}^{(i_2)}_{i_1+n})^2$$

  - Schwinger representation and integration (over loops and Schwinger parameters)

$$\tilde{S}_q(k) = \left(m + \hat{k}\right)\int_0^\infty d\alpha\, e^{[-\alpha(m^2-k^2)]}$$

$$\int d^4k\, P(k)e^{2kr} = \int d^4k\, P\left(\frac{1}{2}\frac{\partial}{\partial r}\right)e^{2kr} = P\left(\frac{1}{2}\frac{\partial}{\partial r}\right)\int d^4k\, e^{2kr}$$

$$\int_0^\infty d^n\alpha\, P\left(\frac{1}{2}\frac{\partial}{\partial r}\right)e^{-\frac{r^2}{a}} = \int_0^\infty d^n\alpha\, e^{-\frac{r^2}{a}}P\left(\frac{1}{2}\frac{\partial}{\partial r} - \frac{r}{a}\right),\quad r = r(\alpha_i),\quad a = a(\Lambda_H, \alpha_i)$$

- **Infrared cut-off**

$$\Pi = \int_0^\infty d^n\alpha\, F(\alpha_1,\cdots,\alpha_n) = \int_0^{\infty \to \frac{1}{\lambda^2}} dt\, t^{n-1}\int_0^1 d^n\alpha\, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big)F(t\alpha_1,\ldots,t\alpha_n)$$

  - Π becomes smooth function, thresholds in quark loop diagrams and corresponding branch points removed. Universal value $\lambda_{\text{cut-off}} = 0.181$, numerical integration.

# Form factors and weak decays

- **Observables expressed via form factors**

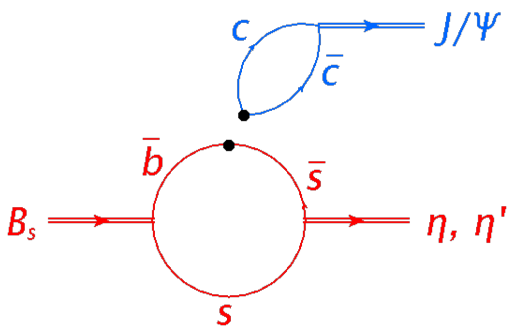
$$\left\langle P'_{[\bar{q}_3,q_2]}(p_2)\,|\bar{q}_2 O^\mu q_1|P'_{[\bar{q}_3,q_1]}(p_1)\right\rangle = F_+(q^2)\,P^\mu + F_-(q^2)\,q^\mu$$

$$\left\langle P'_{[\bar{q}_3,q_2]}(p_2)\,|\bar{q}_2(\sigma^{\mu\nu}q_\nu)\,q_1|P'_{[\bar{q}_3,q_1]}(p_1)\right\rangle = \frac{i}{m_1+m_2}\left(q^2P^\mu - q\cdot P q^\mu\right)F_T(q^2)$$

$$\langle V_{[\bar{q}_3,q_2]}(p_2,\epsilon_2)\,|\bar{q}_2 O^\mu q_1|P_{[\bar{q}_3,q_1]}(p_1)\rangle = \frac{\epsilon^\dagger_\nu}{m_1+m_2}\Big[-g^{\mu\nu}P\cdot q A_0(q^2) + P^\mu P^\nu A_+(q^2) + q^\mu P^\nu A_-(q^2) + i\varepsilon^{\mu\nu\alpha\beta}P_\alpha q_\beta V(q^2)\Big]$$

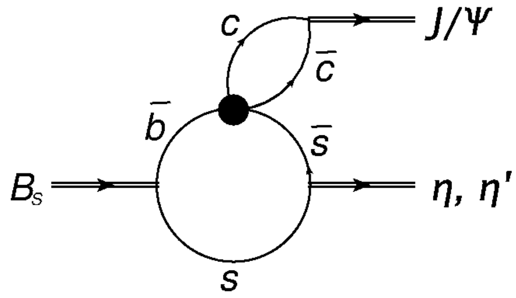
$$\langle V_{[\bar{q}_3,q_2]}(p_2,\epsilon_2)\,|\bar{q}_2\left[\sigma^{\mu\nu}q_\nu\left(1+\gamma^5\right)\right]q_1|P_{[\bar{q}_3,q_1]}(p_1)\rangle = \epsilon^\dagger_\nu\Big[-\left(g^{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)P\cdot q\; a_0(q^2) + \left(P^\mu P^\nu - q^\mu P^\nu\frac{p\cdot q}{q^2}\right)a_+(q^2) + i\varepsilon^{\mu\nu\alpha\beta}P_\alpha q_\beta\; g(q^2)\Big]$$

- **Flavor transitions**
  - Effective theory (Wilson coefficients) used to describe quark flavor transition
  - Factorization: convolution of form factor and expression proportional to decay constant



# $B_s \to J/\psi + \eta^{(')}$

- **$B_s \to J/\psi + \eta$ and $B_s \to J/\psi + \eta'$:**
  - Measured by Belle [PRL 108, 181808 (2012)] and LHCb [Nucl. Phys. B867 (2013)547]
  - Light-strange quark mixing




$$B_S^0 : \; s\bar{b} \qquad \eta : \; \frac{1}{\sqrt{2}} \sin\delta(u\bar{u} + d\bar{d}) - \cos\delta(s\bar{s}) \qquad \eta' : \; \frac{1}{\sqrt{2}} \cos\delta(u\bar{u} + d\bar{d}) + \sin\delta(s\bar{s})$$

$$\mathcal{L}_\eta(x) = g_\eta \eta(x) \iint dx_1 dx_2 \delta\left(x - \frac{1}{2}x_1 - \frac{1}{2}x_2\right) \phi_\eta\left[(x_1 - x_2)^2\right] \times \left\{\frac{1}{\sqrt{2}} \cos(\delta)\left[\bar{u}(x_1)\, i\gamma^5 u(x_2) + \bar{d}(x_1)\, i\gamma^5 d(x_2)\right] - \sin(\delta)\left[\bar{s}(x_1)\, i\gamma^5 s(x_2)\right]\right\}$$

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \sum_i C_i Q_i \qquad Q_1 = (\bar{c}_{a_1} b_{a_2})_{V-A} (\bar{s}_{a_2} c_{a_1})_{V-A} \qquad Q_2 = \ldots$$

$$(\bar{\psi}\psi)_{V-A} = \bar{\psi} O^\mu \psi, \; O^\mu = \gamma^\mu (1 - \gamma^5) \qquad (\bar{\psi}\psi)_{V+A} = \bar{\psi} O_+^\mu \psi, \; O_+^\mu = \gamma^\mu (1 + \gamma^5)$$

# $B_s \to J/\psi + \eta^{(\prime)}$

- **Model over-constrained**

$$\eta \to \gamma\gamma \qquad \eta' \to \gamma\gamma \qquad \rho \to \eta\gamma \qquad \varphi \to \eta\gamma \qquad \varphi \to \eta'\gamma$$
$$\mathrm{B_d} \to \mathrm{J}/\psi\ \eta \qquad \mathrm{B_d} \to \mathrm{J}/\psi\ \eta' \qquad \omega \to \eta\gamma \qquad \eta' \to \omega\gamma$$

- **Results (x10$^{-4}$)**

$$\mathcal{B}_{\mathrm{CQM}}\,(\mathrm{J}/\psi\ \eta) = 4.67$$
$$\mathcal{B}_{\mathrm{Belle}}\,(\mathrm{J}/\psi\ \eta) = 5.10 \pm 1.12$$
$$\mathcal{B}_{\mathrm{CQM}}\,(\mathrm{J}/\psi\ \eta') = 4.04$$
$$\mathcal{B}_{\mathrm{Belle}}\,(\mathrm{J}/\psi\ \eta') = 3.71 \pm 0.95$$

$$R = \frac{\Gamma(J/\psi + \eta')}{\Gamma(J/\psi + \eta)} = \begin{cases} 0.73 \pm 0.14 \pm 0.02 & \text{Belle} \\ 0.90 \pm 0.09^{+0.06}_{-0.02} & \text{LHCb} \end{cases}$$

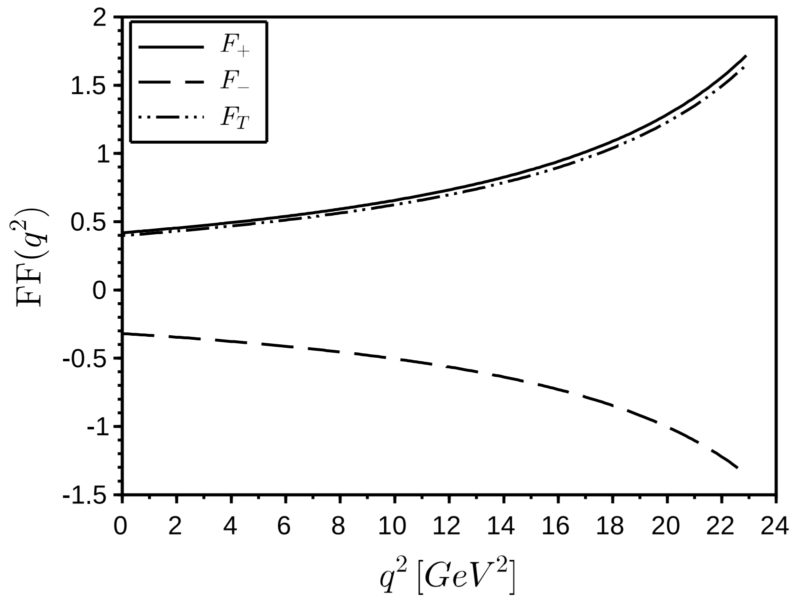
$$R^{\,\mathrm{theor}} = \underbrace{\frac{|\mathbf{q}_{\eta'}|^3}{|\mathbf{q}_{\eta}|^3} \tan^2\delta}_{\approx 1.04} \times \underbrace{\left(\frac{F_+^{B_s\eta'}}{F_+^{B_s\eta}}\right)^2}_{\approx 0.83} \approx 0.86.$$

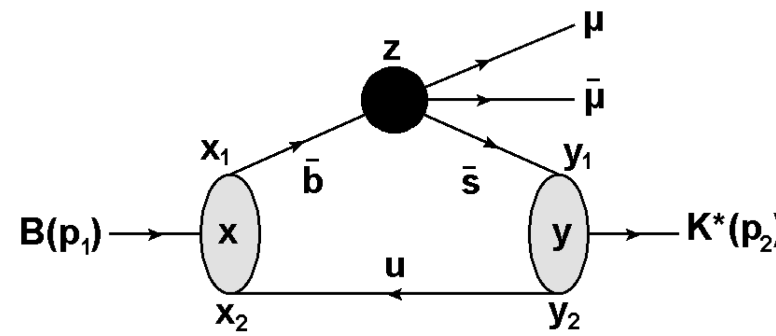
**q** - momentum of the outgoing particles in the rest frame of the decaying particle.

[S. Dubnička, A. Z. Dubničková, M. A. Ivanov and A. Liptaj Phys. Rev. D 87, 074201 (2013)]

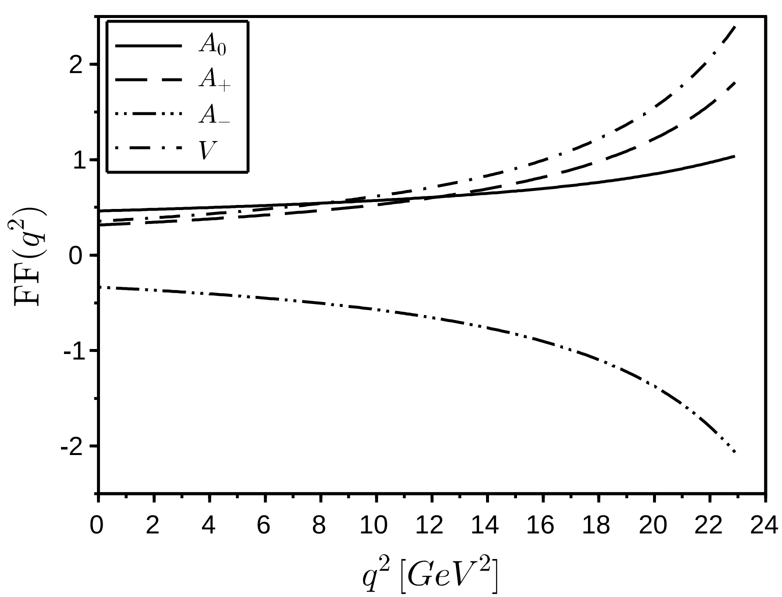
# $B \to K^{(*)}\bar{l}l$

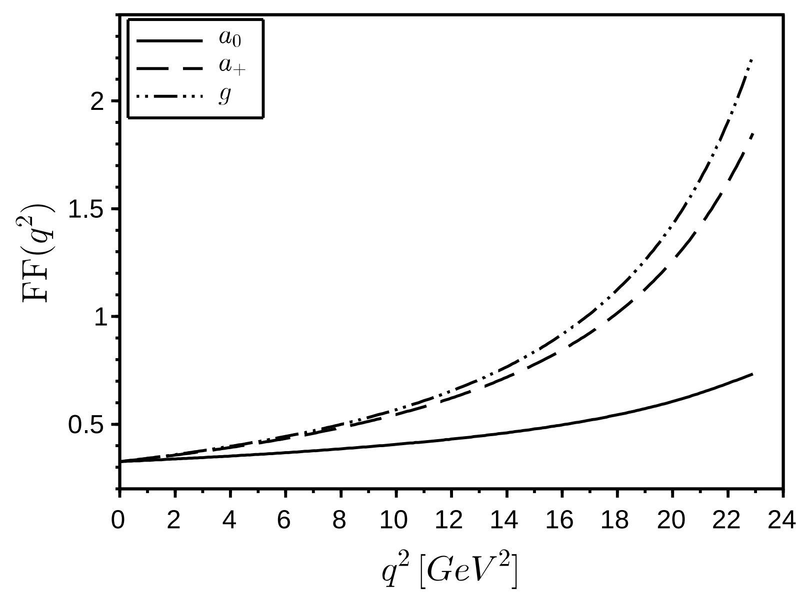
- **$B \to K$ form factors**



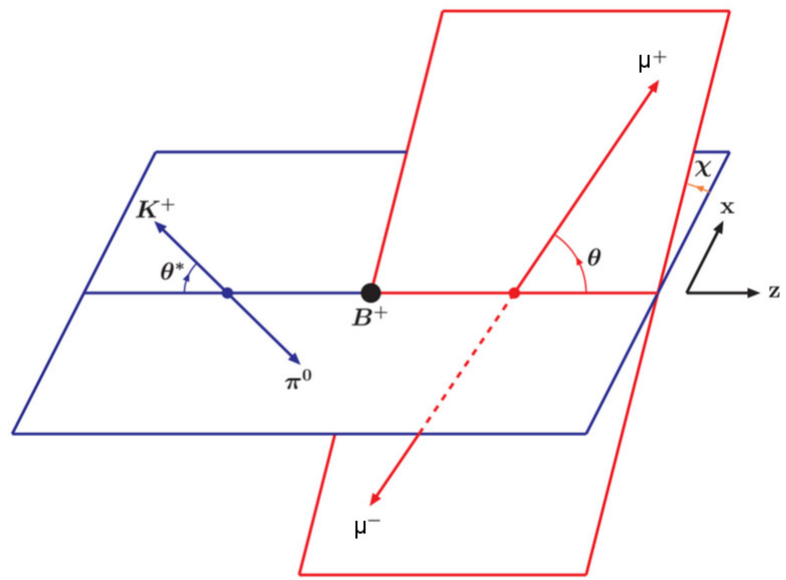

- **$B \to K^*$ form factors**



# Kinematics

- **Amplitude calculation**
  - Helicity basis – hadronic and leptonic tensor evaluated in different frames
  - Hadronic tensor parametrized through form factors




$$L^{(k)}(m,n) = \epsilon^{\mu}(m)\epsilon^{\dagger\nu}(n)L^{(k)}_{\mu\nu}$$

$$H^{ij}(m,n) = \epsilon^{\dagger\mu}(m)\epsilon^{\nu}(n)H^{ij}_{\mu\nu}$$

$$H^{ij}(m,n) = H^{i}(m)H^{\dagger j}(n)$$

- **scalar (K)**

$$H^i(t) = \frac{1}{\sqrt{q^2}}\left[Pq\mathcal{F}^i_+ + q^2\mathcal{F}^i_-\right]$$

$$H^i(\pm) = 0$$

$$H^i(0) = \frac{1}{m_1+m_2}\frac{1}{2m_2\sqrt{q^2}}$$

- **vector (K*)**

$$H^i(t) = \frac{1}{m_1+m_2}\frac{m_1}{m_2}\frac{|\mathbf{p_2}|}{\sqrt{q^2}}\left[Pq(-A^i_0 + A^i_+) + q^2A^i_-\right]$$

$$H^i(\pm) = \frac{1}{m_1+m_2}\left(-PqA^i_0 \pm 2m_1|\mathbf{p_2}|V^i\right)$$

$$H^i(0) = \frac{1}{m_1+m_2}\frac{1}{2m_2\sqrt{q^2}}$$

$$\times\left[-Pq(m_1^2 - m_2^2 - q^2)A^i_0 + 4m_1^2|\mathbf{p_2}|^2A^i_+\right]$$

# Full differential distribution $B \to K^*(\to K\pi)\mu^+\mu^-$

$$
\begin{aligned}
\frac{d\Gamma(B \to K^*(\to K\pi)\,\bar{\mu}\mu)}{dq^2\, d(\cos\theta)\,(d\chi/2\pi)\, d(\cos\theta^*)} = \mathrm{Br}(K^* \to K\pi) \times \Bigg\{ & \frac{3}{8}(1+\cos^2\theta)\cdot\frac{3}{4}\sin^2\theta^*\cdot\frac{1}{2}\left(\frac{d\Gamma_{U_{11}}}{dq^2}+\frac{d\Gamma_{U_{22}}}{dq^2}\right) \\
& +\frac{3}{4}\sin^2\theta\cdot\frac{3}{2}\cos^2\theta^*\cdot\frac{1}{2}\left(\frac{d\Gamma_{L_{11}}}{dq^2}+\frac{d\Gamma_{L_{22}}}{dq^2}\right)-\frac{3}{4}\sin^2\theta\cdot\cos 2\chi\cdot\frac{3}{4}\sin^2\theta^*\cdot\frac{1}{2}\left(\frac{d\Gamma_{T_{11}}}{dq^2}+\frac{d\Gamma_{T_{22}}}{dq^2}\right) \\
& -\frac{9}{16}\sin 2\theta\cdot\cos\chi\cdot\sin 2\theta^*\cdot\frac{1}{2}\left(\frac{d\Gamma_{I_{11}}}{dq^2}+\frac{d\Gamma_{I_{22}}}{dq^2}\right)+v\cdot\Bigg[-\frac{3}{4}\cos\theta\cdot\frac{3}{4}\sin^2\theta^*\cdot\frac{d\Gamma_{P_{12}}}{dq^2} \\
& +\frac{9}{8}\sin\theta\cdot\cos\chi\cdot\sin 2\theta^*\cdot\frac{1}{2}\left(\frac{d\Gamma_{A_{12}}}{dq^2}+\frac{d\Gamma_{A_{21}}}{dq^2}\right)-\frac{9}{16}\sin\theta\cdot\sin\chi\cdot\sin 2\theta^*\cdot\left(\frac{d\Gamma_{II_{12}}}{dq^2}+\frac{d\Gamma_{II_{21}}}{dq^2}\right)\Bigg] \\
& +\frac{9}{32}\sin 2\theta\cdot\sin\chi\cdot\sin 2\theta^*\cdot\left(\frac{d\Gamma_{IA_{11}}}{dq^2}+\frac{d\Gamma_{IA_{22}}}{dq^2}\right)+\frac{9}{32}\sin^2\theta\cdot\sin 2\chi\cdot\sin^2\theta^*\cdot\left(\frac{d\Gamma_{IT_{11}}}{dq^2}+\frac{d\Gamma_{IT_{22}}}{dq^2}\right) \\
& +\frac{3}{4}\sin^2\theta\cdot\frac{3}{4}\sin^2\theta^*\cdot\frac{1}{2}\cdot\frac{d\tilde{\Gamma}_{U_{11}}}{dq^2}-\frac{3}{8}(1+\cos^2\theta)\cdot\frac{3}{4}\sin^2\theta^*\cdot\frac{d\tilde{\Gamma}_{U_{22}}}{dq^2} \\
& +\frac{3}{2}\cos^2\theta\cdot\frac{3}{2}\cos^2\theta^*\cdot\frac{1}{2}\cdot\frac{d\tilde{\Gamma}_{L_{11}}}{dq^2}-\frac{3}{4}\sin^2\theta\cdot\frac{3}{2}\cos^2\theta^*\cdot\frac{d\tilde{\Gamma}_{L_{22}}}{dq^2} \\
& +\frac{3}{4}\sin^2\theta\cdot\cos 2\chi\cdot\frac{3}{4}\sin^2\theta^*\cdot\left(\frac{d\tilde{\Gamma}_{T_{11}}}{dq^2}+\frac{d\tilde{\Gamma}_{T_{22}}}{dq^2}\right)+\frac{9}{8}\sin 2\theta\cdot\cos\chi\cdot\sin 2\theta^*\cdot\frac{1}{2}\left(\frac{d\tilde{\Gamma}_{I_{11}}}{dq^2}+\frac{d\tilde{\Gamma}_{I_{22}}}{dq^2}\right) \\
& +\frac{3}{2}\cos^2\theta^*\cdot\frac{1}{4}\frac{d\tilde{\Gamma}_{S_{22}}}{dq^2}-\frac{9}{16}\,,\sin 2\theta\cdot\sin\chi\cdot\sin 2\theta^*\cdot\left(\frac{d\Gamma_{IA_{11}}}{dq^2}+\frac{d\Gamma_{IA_{22}}}{dq^2}\right) \\
& -\frac{9}{16}\sin^2\theta\cdot\sin 2\chi\cdot\sin^2\theta^*\cdot\left(\frac{d\Gamma_{IT_{11}}}{dq^2}+\frac{d\Gamma_{IT_{22}}}{dq^2}\right)\Bigg\}
\end{aligned}
$$

$$\frac{d\tilde{\Gamma}^{ij}_X}{dq^2} = \frac{2m_\mu^2}{q^2}\frac{d\Gamma^{ij}_X}{dq^2} \qquad \frac{d\Gamma^{ij}_X}{dq^2} = \frac{G_F^2}{(2\pi)^3}\left(\frac{\alpha|\lambda_t|}{2\pi}\right)^2\frac{|\mathbf{p_2}|q^2 v}{12m_1^2}H^{ij}_X \qquad H^{ij}_X \to \text{Bilinear combination of } \mathrm{H}^{\mathrm{i}}$$

# Observables

- **Searching for**
  - Small model dependence (on hadronic physics, form factors).
  - Sensitivity to new physics (at short distance).
  - Experimental accessibility (clear signature, high cross-section, small backgrounds).
  - Ratios, asymmetries, asymmetry ratios...

- **Chosen observables ($B \to K^* \mu^+ \mu^-$)**
  - Separate integration (numerator/denominator) over relevant $q^2$ range (bin size).

$$\frac{d\Gamma}{dq^2} = \frac{1}{2}\left(\frac{d\Gamma_U^{11}}{dq^2} + \frac{d\Gamma_U^{22}}{dq^2} + \frac{d\Gamma_L^{11}}{dq^2} + \frac{d\Gamma_L^{22}}{dq^2}\right) + \frac{1}{2}\frac{d\tilde{\Gamma}_U^{11}}{dq^2} - \frac{d\tilde{\Gamma}_U^{22}}{dq^2} + \frac{1}{2}\frac{d\tilde{\Gamma}_L^{11}}{dq^2} - \frac{d\tilde{\Gamma}_L^{22}}{dq^2} + \frac{3}{2}\frac{d\tilde{\Gamma}_S^{22}}{dq^2}$$

$$F_L = \frac{\int dq^2 \quad H_L^{11} + H_L^{22}}{\int dq^2 \quad H_L^{11} + H_L^{22} + H_U^{11} + H_U^{22}} \qquad A_{FB} = -\frac{3}{2}\frac{\int dq^2 \quad H_P^{12}}{\int dq^2 \quad H_L^{11} + H_L^{22} + H_U^{11} + H_U^{22}}$$

$$P_1 = -2\frac{\int dq^2 \quad \beta_l^2[dT^{11} + dT^{22}]}{\int dq^2 \quad \beta_l^2[dU^{11} + dU^{22}]} \qquad P_2 = -\frac{\int dq^2 \quad \beta_l dP^{12}}{\int dq^2 \quad \beta_l^2[dU^{11} + dU^{22}]} \qquad P_3 = -\frac{\int dq^2 \quad \beta_l^2[dIT^{11} + dIT^{22}]}{\int dq^2 \quad \beta_l^2[dU^{11} + dU^{22}]}$$

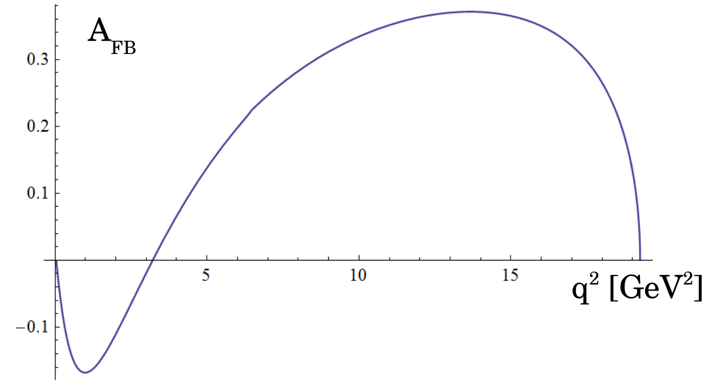
$$P_4' = 2\frac{\int dq^2 \quad \beta_l^2[dI^{11} + dI^{22}]}{N} \qquad P_5' = -2\frac{\int dq^2 \quad \beta_l[dA^{12} + dA^{21}]}{N} \qquad P_8 = 2\frac{\int dq^2 \quad \beta_l^2[dIA^{11} + dIA^{22}]}{N}$$

$$dX^{ij} = \frac{d\Gamma_X^{ij}}{dq^2} \qquad N = \sqrt{\int dq^2 \beta_l^2 [dU^{11} + dU^{22}] \cdot \int dq^2 \beta_l^2 [dL^{11} + dL^{22}]} \qquad \beta_l = \sqrt{\frac{1 - 4m_\mu^2}{q^2}}$$

# Results

- **Branching fractions**

| | CQM | Data [PDG] |
|---|---|---|
| $\mathcal{B}\left[B \to K^* \mu^+ \mu^-\right]$ | $1.27 \times 10^{-6}$ | $(1.05 \pm 0.10) \times 10^{-6}$ |
| $\mathcal{B}\left[B \to K \mu^+ \mu^-\right]$ | $7.18 \times 10^{-7}$ | $(3.4 \pm 0.5) \times 10^{-7}$ |
| $\mathcal{B}\left[B \to K^* \nu \bar{\nu}\right]$ | $1.36 \times 10^{-5}$ | $< 5.5 \times 10^{-5}$ |
| $\mathcal{B}\left[B \to K \nu \bar{\nu}\right]$ | $0.60 \times 10^{-5}$ | $< 4.9 \times 10^{-5}$ |
| $\mathcal{B}\left[B \to K^* \gamma\right]$ | $3.74 \times 10^{-5}$ | $(4.21 \pm 0.18) \times 10^{-5}$ [HFAG coll.] |




- **$A_{FB}$ and $F_L$**
  in the 1 GeV$^2$ < q$^2$ < 6 GeV$^2$ region ($B \to K^* \mu^+ \mu^-$ only)

| | $\langle A_{FB} \rangle$ | $\langle F_L \rangle$ |
|---|---|---|
| [1] | $0.26^{+0.27}_{-0.30} \pm 0.07$ | $0.67 \pm 0.23 \pm 0.05$ |
| [2] | $-0.06^{+0.13}_{-0.14} \pm 0.04$ | $0.55 \pm 0.10 \pm 0.03$ |
| [3] | $0.29^{+0.20}_{-0.23} \pm 0.07$ | $0.69^{+0.19}_{-0.21} \pm 0.08$ |
| CQM | $0.022$ | $0.75$ |

$$\frac{1}{\Gamma} \frac{d^2\Gamma}{d\,cos\theta_l\, dq^2} = \frac{3}{4} F_L (1 - cos^2\theta_l) + \frac{3}{8}(1 - F_L)(1 + cos^2\theta_l) + A_{FB} cos\theta_l$$


[1] Belle Coll., Phys. Rev. Lett. 103, 171801 (2009)
[2] LHCb Coll., Phys. Rev. Lett. 108, 181806 (2012)
[3] CDF Coll., Phys. Rev. Lett. 108, 081807 (2012)
[4] S. Descotes-Genon *et al.*, JHEP 1305, 137 (2013)

# Binned results

| | $F_L$ | | | | |
|---|---|---|---|---|---|
| Bin ($GeV^2$) | [1] | [2] | [3] | [4] | CQM |
| 1.00–2.00 | - | - | - | $0.605^{+0.179+0.021}_{-0.229-0.024}$ | 0.782623 |
| 0.00–2.00 | $0.29^{+0.21}_{-0.18}\pm0.02$ | $0.00\ ^{+0.13}_{-0.00} \pm 0.02$ | $0.30^{+0.16}_{-0.16} \pm 0.02$ | $0.323^{+0.198+0.019}_{-0.178-0.020}$ | 0.53665 |
| 2.00–4.30 | $0.71^{+0.24}_{-0.24}\pm0.05$ | $0.77 \pm 0.15 \pm 0.03$ | $0.37^{+0.25}_{-0.24} \pm 0.10$ | $0.754^{+0.128+0.015}_{-0.198-0.018}$ | 0.790552 |
| 4.30–8.68 | $0.64^{+0.23}_{-0.24}\pm0.07$ | $0.60\ ^{+0.06}_{-0.07} \pm 0.01$ | $0.68^{+0.15}_{-0.17} \pm 0.09$ | $0.634^{+0.175+0.022}_{-0.216-0.022}$ | 0.602306 |
| 10.09–12.89 | $0.17^{+0.17}_{-0.15}\pm0.03$ | $0.41 \pm 0.11 \pm 0.03$ | $0.47^{+0.14}_{-0.14} \pm 0.03$ | $0.482^{+0.163+0.014}_{-0.208-0.013}$ | 0.424467 |
| 14.18–16.00 | $-0.15^{+0.27}_{-0.23}\pm0.07$ | $0.37 \pm 0.09 \pm 0.05$ | $0.29^{+0.14}_{-0.13} \pm 0.05$ | $0.396^{+0.141+0.004}_{-0.241-0.004}$ | 0.359567 |
| >16.00 | $0.12^{+0.15}_{-0.13}\pm0.02$ | $0.26\ ^{+0.10}_{-0.08} \pm 0.03$ | $0.20^{+0.19}_{-0.17} \pm 0.05$ | $0.357^{+0.074+0.003}_{-0.133-0.003}$ | 0.338756 |
| 1.00–6.00 | $0.67^{+0.23}_{-0.23}\pm0.05$ | $0.55 \pm 0.10 \pm 0.03$ | $0.69^{+0.19}_{-0.21} \pm 0.08$ | $0.703^{+0.149+0.017}_{-0.212-0.019}$ | 0.747141 |
| | $A_{FB}$ | | | | |
| 1.00–2.00 | - | - | - | $-0.212^{+0.11+0.014}_{-0.144-0.015}$ | -0.146603 |
| 0.00–2.00 | $0.47^{+0.26}_{-0.32}\pm0.03$ | $-0.15 \pm 0.20 \pm 0.06$ | $-0.35^{+0.26}_{-0.23} \pm 0.10$ | $-0.136^{+0.048+0.016}_{-0.045-0.016}$ | -0.122687 |
| 2.00–4.30 | $0.37^{+0.25}_{-0.24} \pm 0.10$ | $0.05\ ^{+0.16}_{-0.20} \pm 0.04$ | $0.29^{+0.32}_{-0.35} \pm 0.15$ | $-0.081^{+0.054+0.008}_{-0.068-0.009}$ | -0.00593019 |
| 4.30–8.68 | $0.45^{+0.15}_{-0.21}\pm0.15$ | $0.27\ ^{+0.06}_{-0.08} \pm 0.02$ | $0.01^{+0.20}_{-0.20} \pm 0.09$ | $0.220^{+0.138+0.014}_{-0.112-0.016}$ | 0.219059 |
| 10.09–12.89 | $0.43^{+0.18}_{-0.20}\pm0.03$ | $0.27\ ^{+0.11}_{-0.13} \pm 0.02$ | $0.38^{+0.16}_{-0.19} \pm 0.09$ | $0.371^{+0.150+0.010}_{-0.164-0.011}$ | 0.356071 |
| 14.18–16.00 | $0.70^{+0.16}_{-0.22}\pm0.10$ | $0.47\ ^{+0.06}_{-0.08} \pm 0.03$ | $0.44^{+0.18}_{-0.21} \pm 0.10$ | $0.404^{+0.199+0.005}_{-0.191-0.005}$ | 0.362603 |
| >16.00 | $0.66^{+0.11}_{-0.16}\pm0.04$ | $0.16\ ^{+0.11}_{-0.13} \pm 0.06$ | $0.65^{+0.17}_{-0.18} \pm 0.16$ | $0.360^{+0.205+0.004}_{-0.172-0.005}$ | 0.293887 |
| 1.00–6.00 | $0.26^{+0.27}_{-0.30}\pm0.07$ | $-0.06\ ^{+0.13}_{-0.14} \pm 0.04$ | $0.29^{+0.20}_{-0.23} \pm 0.07$ | $-0.035^{+0.036+0.008}_{-0.033-0.009}$ | 0.0222029 |
| | $\mathcal{B}(10^{-7})$ | | | | |
| 1.00–2.00 | - | - | - | $0.437^{+0.345+0.026}_{-0.148-0.023}$ | 0.510043 |
| 0.00–2.00 | $1.46^{+0.40}_{-0.35}\pm0.11$ | $0.61 \pm 0.12 \pm 0.06$ | - | $1.446^{+1.537+0.057}_{-0.561-0.054}$ | 1.39569 |
| 2.00–4.30 | $0.86^{+0.31}_{-0.27}\pm0.07$ | $0.34 \pm 0.09 \pm 0.02$ | - | $0.904^{+0.664+0.061}_{-0.314-0.055}$ | 1.12945 |
| 4.30–8.68 | $1.37^{+0.47}_{-0.42}\pm0.39$ | $0.69 \pm 0.08 \pm 0.05$ | - | $2.674^{+2.326+0.156}_{-0.973-0.145}$ | 2.66943 |
| 10.09–12.89 | $2.24^{+0.44}_{-0.40}\pm0.19$ | $0.55 \pm 0.09 \pm 0.07$ | - | $2.344^{+2.814+0.069}_{-1.100-0.063}$ | 2.1427 |
| 14.18–16.00 | $1.05^{+0.29}_{-0.26}\pm0.08$ | $0.63 \pm 0.11 \pm 0.05$ | - | $1.290^{+2.122+0.013}_{-0.815-0.013}$ | 1.38883 |
| >16.00 | $2.04^{+0.27}_{-0.24}\pm0.16$ | $0.50 \pm 0.08 \pm 0.05$ | - | $1.450^{+2.333+0.015}_{-0.922-0.015}$ | 1.71453 |
| 1.00–6.00 | $1.49^{+0.45}_{-0.40}\pm0.12$ | $0.42 \pm 0.06 \pm 0.03$ | - | $2.155^{+1.646+0.138}_{-0.742-0.123}$ | 2.58066 |

# Binned results

| Bin ($GeV^2$) | $\langle P_1 \rangle$ [4] | $\langle P_1 \rangle$ CQM | $\langle P_2 \rangle$ [4] | $\langle P_2 \rangle$ CQM |
|---|---|---|---|---|
| 1–2 | $0.007^{+0.008+0.054}_{-0.005-0.051}$ | $-0.0115773$ | $0.399^{+0.022+0.006}_{-0.023-0.008}$ | $0.46981$ |
| 0.1–2 | $0.007^{+0.007+0.043}_{-0.004-0.044}$ | $0.0108792$ | $0.172^{+0.009+0.018}_{-0.009-0.018}$ | $0.219029$ |
| 2.00–4.30 | $-0.051^{+0.010+0.045}_{-0.009-0.045}$ | $-0.266563$ | $0.234^{+0.058+0.015}_{-0.085-0.016}$ | $0.0192036$ |
| 4.30–8.68 | $-0.117^{+0.002+0.056}_{-0.002-0.052}$ | $-0.372456$ | $-0.407^{+0.048+0.008}_{-0.037-0.006}$ | $-0.369719$ |
| 10.09–12.89 | $-0.181^{+0.278+0.032}_{-0.361-0.029}$ | $-0.470412$ | $-0.481^{+0.08+0.003}_{-0.005-0.002}$ | $-0.413794$ |
| 14.18–16.00 | $-0.352^{+0.696+0.014}_{-0.467-0.015}$ | $-0.614669$ | $-0.449^{+0.136+0.004}_{-0.041-0.004}$ | $-0.37829$ |
| 16.00–19 | $-0.603^{+0.589+0.009}_{-0.315-0.009}$ | $-0.777736$ | $-0.374^{+0.151+0.004}_{-0.126-0.004}$ | $-0.296817$ |
| 1.00–6.00 | $-0.055^{+0.009+0.040}_{-0.008-0.042}$ | $-0.26338$ | $0.084^{+0.057+0.019}_{-0.076-0.019}$ | $-0.0596227$ |

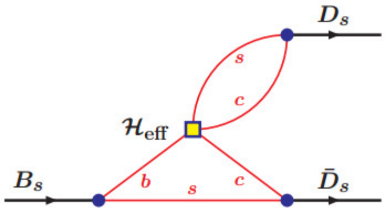
| Bin ($GeV^2$) | $\langle P_3 \rangle$ [4] | $\langle P_3 \rangle$ CQM | $\langle P'_4 \rangle$ [4] | $\langle P'_4 \rangle$ CQM |
|---|---|---|---|---|
| 1–2 | $-0.003^{+0.001+0.027}_{-0.002-0.024}$ | $0.00435836$ | $-0.160^{+0.040+0.013}_{-0.031-0.013}$ | $0.141964$ |
| 0.1–2 | $-0.002^{+0.001+0.02}_{-0.001-0.023}$ | $0.00159832$ | $-0.342^{+0.026+0.018}_{-0.019-0.017}$ | $-0.153449$ |
| 2.00–4.30 | $-0.004^{+0.001+0.022}_{-0.003-0.022}$ | $0.00454996$ | $0.569^{+0.070+0.020}_{-0.059-0.021}$ | $0.892132$ |
| 4.30–8.68 | $-0.001^{+0.000+0.027}_{-0.001-0.027}$ | $0.00224737$ | $1.003^{+0.014+0.024}_{-0.015-0.029}$ | $1.13376$ |
| 10.09–12.89 | $0.003^{+0.000+0.014}_{-0.001-0.015}$ | $0.00151139$ | $1.082^{+0.140+0.014}_{-0.144-0.017}$ | $1.20871$ |
| 14.18–16.00 | $0.004^{+0.000+0.002}_{-0.001-0.002}$ | $0.00101528$ | $1.161^{+0.190+0.007}_{-0.332-0.007}$ | $1.26991$ |
| 16.00–19 | $0.003^{+0.001+0.001}_{-0.001-0.001}$ | $0.00068909$ | $1.263^{+0.119+0.004}_{-0.248-0.004}$ | $1.33254$ |
| 1.00–6.00 | $-0.003^{+0.001+0.020}_{-0.002-0.022}$ | $0.00355465$ | $0.555^{+0.065+0.018}_{-0.055-0.019}$ | $0.832529$ |

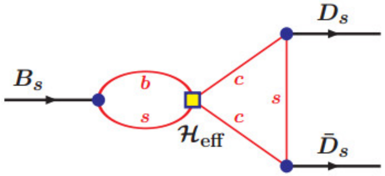
| Bin ($GeV^2$) | $\langle P'_5 \rangle$ [4] | $\langle P'_5 \rangle$ CQM | $\langle P_8 \rangle$ | $\langle P_8 \rangle$ CQM |
|---|---|---|---|---|
| 1–2 | $0.387^{+0.047+0.014}_{-0.063-0.015}$ | $0.258474$ | - | $-0.0388866$ |
| 0.1–2 | $0.533^{+0.028+0.017}_{-0.036-0.020}$ | $0.495414$ | - | $-0.0327505$ |
| 2.00–4.30 | $-0.334^{+0.095+0.02}_{-0.111-0.019}$ | $-0.423802$ | - | $-0.025576$ |
| 4.30–8.68 | $-0.872^{+0.043+0.03}_{-0.029-0.029}$ | $-0.704599$ | - | $-0.0113325$ |
| 10.09–12.89 | $-0.893^{+0.223+0.018}_{-0.110-0.017}$ | $-0.697185$ | - | $-0.00595051$ |
| 14.18–16.00 | $-0.779^{+0.328+0.010}_{-0.363-0.009}$ | $-0.600105$ | - | $-0.00285195$ |
| 16.00–19 | $-0.601^{+0.282+0.008}_{-0.367-0.007}$ | $-0.449369$ | - | $-0.0014646$ |
| 1.00–6.00 | $-0.349^{+0.086+0.019}_{-0.098-0.017}$ | $-0.394563$ | - | $-0.0228404$ |

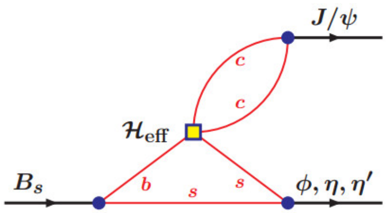
# Other meson-related results

- **$B_s$ nonleptonic decays**
  - Branching ratios (%) [M. A. Ivanov, et. al., Phys. Rev., D85:034004, 2012.]

| Process | CQM | PDG |
|---|---|---|
| $B_s \to D_s^- D_s^+$ | 1.65 | $1.04^{+0.29}_{-0.26}$ |
| $B_s \to D_s^- D_s^{*+} + D_s^{*-} D_s^+$ | 2.40 | $2.8 \pm 1.0$ |
| $B_s \to D_s^{*-} D_s^{*+}$ | 3.18 | $3.1 \pm 1.4$ |
| $B_s \to J/\psi\phi$ | 0.16 | $0.14 \pm 0.05$ |



# Summary of selected baryon and tetraquark results

- **Lagrangians**

$$J_B(x) = \int dx_1 \int dx_2 \int dx_3\, F_B(x, x_1, x_2, x_3) \times \Gamma_1\, q^{a_1}_{f_1}(x_1) \left( q^{a_2}_{f_2}(x_2) C\, \Gamma_2\, q^{a_3}_{f_3}(x_3) \right) \cdot \varepsilon^{a_1 a_2 a_3}$$

$$J_T(x) = \int dx_1 \ldots \int dx_4\, F_T(x, x_1, \ldots, x_4) \times \left( q^{a_1}_{f_1}(x_1)\, C\Gamma_1\, q^{a_2}_{f_2}(x_2) \right) \cdot \left( \bar{q}^{a_3}_{f_3}(x_3)\, \Gamma_2 C\, \bar{q}^{a_4}_{f_4}(x_4) \right) \cdot \varepsilon^{a_1 a_2 c} \varepsilon^{a_3 a_4 c}$$

- **Baryons**
  - Nucleons

    [ T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and P. Santorelli, Phys. Rev. D 87, 074031 (2013) ]

| Quantity | CQM | PDG |
|---|---|---|
| $\mu_p$ (in n.m.) | 2.96 | 2.793 |
| $\mu_n$ (in n.m.) | -1.83 | -1.913 |
| $r^p_E$ (fm) | 0.805 | $0.8768 \pm 0.0069$ |
| $< r^2_E >^n$ (fm$^2$) | -0.121 | $-0.1161 \pm 0.0022$ |
| $r^p_M$ (fm) | 0.688 | $0.777 \pm 0.013 \pm 0.010$ |
| $r^n_M$ (fm) | 0.685 | $0.862^{+0.009}_{-0.008}$ |

  - Rare baryon decays $\Lambda_b \to \Lambda \ell^+ \ell^-$

    **$B(\Lambda_b \to \Lambda \mu^+ \mu^-) = 1.0 \times 10^{-6}$**

    **CDF $(1.73 \pm 0.69) \times 10^{-6}$**

    **LHCb $(0.96 \pm 0.25) \times 10^{-6}$**

    [ T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, P. Santorelli, Phys. Rev. D 87 074031 (2013) ]

- **Tetraquark X(3872)**
  - Molecule interpretation

$$\left. \frac{\Gamma(X_l \to \gamma J/\psi)}{\Gamma(X_l \to J/\psi + \pi\pi)} \right|_{\rm CQM} = 0.15 \pm 0.03$$

$$\frac{\Gamma(X \to \gamma J/\psi)}{\Gamma(X \to 2\pi)} = \begin{cases} 0.14 \pm 0.05 & \text{Belle} \\ 0.22 \pm 0.06 & \text{BaBar} \end{cases}$$

[ M. A. Ivanov et. al., Phys. Rev. D 84, 014006 (2011) ]
[ S. Dubnička, A. Z. Dubničková, M. A. Ivanov and J. G. Körner, Phys. Rev. D 81, 114007 (2010) ]

# Conclusion

- **Summary**
  - ➔ Heavy particle decays: active field with discovery potential, data quantity increasing.
  - ➔ Effort to minimize hadronic uncertainty by a clever choice of observables.
  - ➔ Yet, hadronic effect cannot be fully removed ⇒ CQM.
  - ➔ CQM – relativistic, Lagrangian-based with limited number of free parameters, well suited for heavy hadron decays.
  - ➔ Model results roughly agree with experimental data.

- **Outlook**
  - ➔ Further processes can be evaluated and agreement with the SM checked.
    [ $B \to \mu^+\mu^-$, $B_s^0 \to K_S^0 K^*(892)^0$ ]

**Thank for your attention!**