Jet Physics Phenomenology in High Energy Factorisation

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Work in collaboration with
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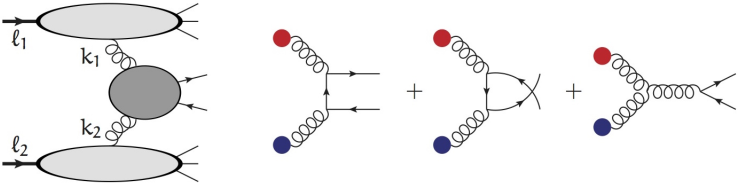
Collinear Factorisation: the standard approach

Collinear factorisation *(J. Collins, D. Soper, G. Sterman, 80's)*



$$\sigma_{h_1,h_2\to q\bar{q}} = \int dx_1\, dx_2\, f_g(x_1)\, f_g(x_2)\, \hat{\sigma}_{gg}\left(\frac{m^2}{x_1 x_2 s}\right)$$

where the f_g's are the PDFs, obeying DGLAP evolution equations.

To be applied if: $Q >> \Lambda^2$ (QCD scale 200 MeV)

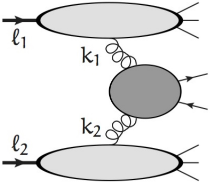
Momentum parameterization:

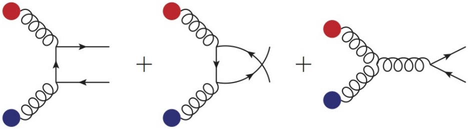
$$k_1^\mu = x_1\, l_1^\mu \quad , \quad k_2^\mu = x_2\, l_2^\mu \qquad l_i^2 = 0 \Leftrightarrow k_i^2 = 0$$

Proved only for Drell-Yan ($q\,\bar{q} \Rightarrow l^-\, l^+$) and
Inclusive Deep Inelastic Scattering ($e^-\, p \Rightarrow e^-\, X$)

High-Energy-Factorisation: more degrees of freedom

High-Energy-factorisation *(Catani,Ciafaloni,Hautmann, 1991 / Collins,Ellis, 1991)*





$$\sigma_{h_1,h_2\to q\bar{q}} = \int d^2k_{1\perp}\,d^2k_{2\perp}\,\frac{dx_1}{x_1}\,\frac{dx_2}{x_2}\,f_g(x_1,k_{1\perp})\,f_g(x_2,k_{2\perp})\,\hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$

where the f_g's are the gluon densities, obeying BFKL, BK, CCFM evolution equations (replace DGLAP).

Non negligible transverse momentum $\Leftrightarrow$ to small x physics.

With time, more insight into the structure of the proton...

To be applied in the regime: $s >> M^2 \sim k_\perp^2$

Momentum parameterization:

$$k_1^\mu = x_1\,l_1^\mu + k_{1\perp}^\mu \quad , \quad k_2^\mu = x_2\,l_2^\mu + k_{2\perp}^\mu$$

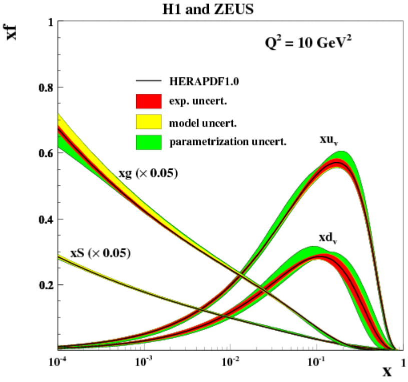
$$l_i^2 = 0, \quad l_i\cdot k_i = 0, \quad k_i^2 = -k_{i\perp}^2, \quad i = 1,2$$

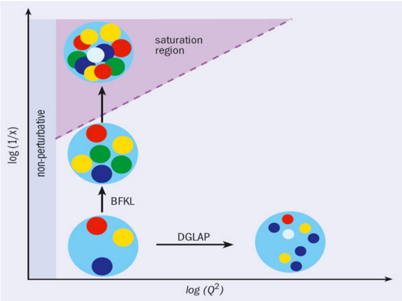
Small x-physics @ the LHC ?

Growth of the gluon distribution function for small x

$\Rightarrow$ Non linear effects expected: see BK equation (restores unitarity w.r.t. BFKL)

Unprecedented energies open the window for hard scattering with low-x effects !



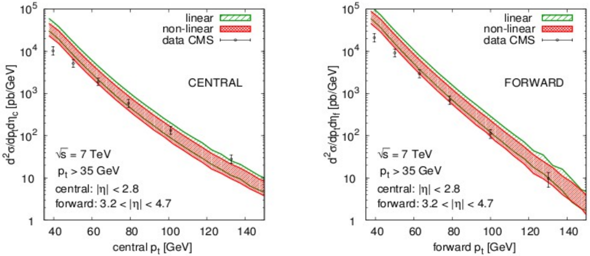


One of the ultimate goals: prove saturation !

Hybrid factorization, (Deak, Hautmann, Jung, Kutak, '09):

$$\sigma_{h_1,h_2\to q\bar{q}} = \int d^2 k_{1\perp}\, dx_1\, dx_2\, \mathcal{F}(x_1, k_{1\perp}, \mu)\, f(x_2, \mu)\, \hat{\sigma}\, (x_1, x_2, k_{1\perp}, \mu)$$

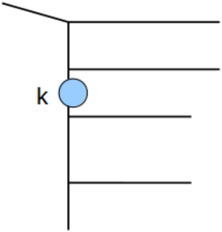
Kutak, Sapeta, Phys.Rev. D86 (2012) 094043, central-forward dijets production:



- Reasonable agreement with data
- **Fully differential hybrid factorization formula for dijet production derived from Color-Glass-Condensate effective theory:**
  Kotko, Kutak, Marquet, Petreska, van Hameren, JHEP 1509 (2015) 106

Our PDFs: the prescription

Kimber, Martin, Ryskin prescription, '01 :



$$
\begin{aligned}
{}^{\mu}T_s(\mu^2, k^2) &= exp\left(- \int_{\mu^2}^{k^2} \frac{dk'^2}{k'^2} \frac{\alpha_s(k'^2)}{2\pi} \right. \\
&\qquad \left. \times \sum_{a'} \int_0^{1-\Delta} dz' P_{aa'}(z') \right) \\
\Delta &= \frac{\mu}{\mu + k}, \quad \mu = \text{hard scale} \\
\mathcal{F}(x, k^2, \mu^2) &\sim \partial_{\lambda^2} \left(T_s(\lambda^2, \mu^2)\, x\, g(x, \lambda^2) \right) \big|_{\lambda^2 = k^2}
\end{aligned}
$$

DLC 2016 (Double Log Coherence)

K. Kutak, R. Maciula, M.S., A. Szczurek, A. van Hameren, JHEP 1604 (2016) 175

Gauge invariant off-shell amplitudes

Problem: general partonic processes must be described by gauge invariant amplitudes
$\Rightarrow$ ordinary Feynman rules are not enough !

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ONE IDEA:

on-shell amplitudes are gauge invariant, so off-shell gauge-invariant amplitudes could be got by embedding them into on-shell processes...

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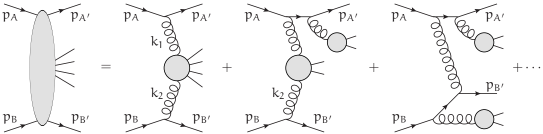
...first result...: 1) For off-shell gluons: represent g^* as coming from a $\bar{q}qg$ vertex, with the quarks taken to be on-shell



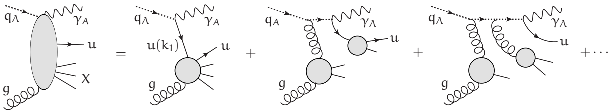
- embed the scattering of the off-shell gluons in the scattering of two quark pairs carrying momenta $p_A^\mu = k_1^\mu$, $p_B^\mu = k_2^\mu$, $p_{A'}^\mu = 0$, $p_{B'}^\mu = 0$
- Assign the spinors $|p_1\rangle, |p_1]$ to the A-quark and the propagator $\frac{i\,\not{p}_1}{p_1 \cdot k}$ instead of $\frac{i\not{k}}{k^2}$ to the propagators of the A-quark carrying momentum k; same thing for the B-quark line.
- ordinary Feynman elsewhere and factor $x_1\sqrt{-k_\perp^2/2}$ to match to the collinear limit

K. Kutak, P. Kotko, A. van Hameren, JHEP 1301 (2013) 078

Prescription for off-shell quarks

... and second result:

2) for off-shell quarks: represent q^* as coming from a $\gamma\bar{q}q$ vertex, with a 0 momentum and $\bar{q}$ on shell (and vice-versa)



- embed the scattering of the quark with whatever set of particles in the scattering of an auxiliary quark-photon pair, q_A and γ_A carrying momenta $p^\mu_{q_A} = k^\mu_1$, $p^\mu_{\gamma_A} = 0$
- Let q_A-propagators of momentum k be $\frac{i\,\not{p}_1}{p_1\cdot k}$ and assign the spinors $|p_1\rangle, |p_1]$ to the A-quark.
- Assign the polarization vectors $\epsilon^\mu_+ = \frac{\langle q|\gamma^\mu|p_1]}{\sqrt{2}\langle p_1 q\rangle}$, $\epsilon^\mu_- = \frac{\langle p_1|\gamma^\mu|q]}{\sqrt{2}[p_1 q]}$ to the auxiliary photon, with q a light-like auxiliary momentum.
- Multiply the amplitude by $x_1\sqrt{-k^2_{1\perp}/2}$ and use ordinary Feynman rules everywhere else.

K. Kutak, T. Salwa, A. van Hameren, Phys.Lett. B727 (2013) 226-233

One left issue: huge slowness for many legs

The diagrammatic approach is too slow to allow for the computation of amplitudes containing more than 4 particles in a reasonable time.

Computing scattering amplitudes in Yang-Mills theories via ordinary Feynman diagrams: soon overwhelming !

Number of Feynman diagrams at tree level on-shell:

# of gluons	4	5	6	7	8	9	10
# of diagrams	4	25	220	2485	34300	559405	10525900

And there are even more with the proposed method for amplitudes with off-shell particles due to the gauge-restoring terms.

A method to efficiently compute helicity amplitudes: **BCFW recursion relation**

Britto, Cachazo, Feng, Nucl.Phys. B715 (2005) 499-522
Britto, Cachazo, Feng, Witten, Phys.Rev.Lett. 94 (2005) 181602

BCFW recursion relation

Two very simple ideas for tree level amplitudes:

1. Cauchy's residue theorem: if the amplitude is formally treated as a function of a complex variable z and if it is rational and vanishes for $z \to \infty$, then the integral extended to an infinite contour enclosing all poles vanishes

$$\lim_{z \to \infty} \mathcal{A}(z) = 0 \Rightarrow \frac{1}{2\pi i} \oint dz \, \frac{\mathcal{A}(z)}{z} = 0$$

   implying that the value at $z = 0$ (physical amplitude) can be determined as a sum of the residues at the poles:

$$\mathcal{A}(0) = -\sum_i \frac{\lim_{z \to z_i} [(z - z_i) \, f(z)]}{z_i}$$

   where z_i is the location of the i-th pole
2. Unitarity: Poles in Yang-Mills tree level amplitudes can only be due to gluon propagators dividing the n-point amplitude into two on-shell sub-amplitudes with $k+1$ and $n-k+1$ gluons $\Rightarrow$ it is all about finding the proper way to "complexify" an amplitude.

BCFW applies to* color-ordered partial amplitudes*, for which the kinematics and gauge structure are factorised like

$$\mathcal{M}_n = g^{n-2} \sum_{\sigma \in S_n/Z_n} \mathrm{Tr}(T_{\sigma(1)} \dots T_{\sigma(n)})\, \mathcal{A}(g_{\sigma(1)}, \dots, g_{\sigma(n)})$$

To properly "complexify" $\mathcal{A}$: for helicities $(h_i, h_j) = (-, +)$

$$\begin{aligned} p_i &\to \hat{p}_i \equiv p_i - z\, p_j \\ p_j &\to \hat{p}_j \equiv p_j + z\, p_i \end{aligned}$$

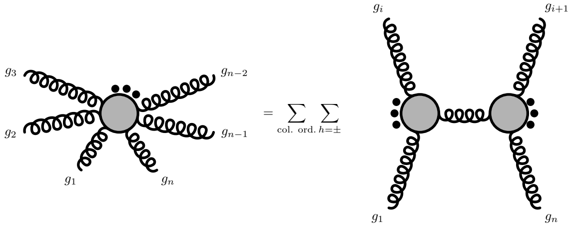
- On-shell conditions, gauge invariance and momentum conservation preserved throughout.
- the most serious issue is the behaviour for $z \to \infty$, but either a result derived with twistor methods (*Cachazo,Svrcek and Witten JHEP 0409 (2004) 006*) or a smart choice of reference lines always allow to overcome the problem, so that $\lim_{z\to\infty} \mathcal{A}(z) = 0$ holds

Amazingly simple recursive relation:

any tree-level color-ordered amplitude is the sum of residues of the poles it develops when it is made dependent on a complex variable as above.
Such residues are simply products of color-ordered lower-point amplitudes evaluated at the pole times an intermediate propagator.
Shifted particles are always on opposite sides of the propagator.

$$\mathcal{A}(g_1,\ldots,g_n) = \sum_{i=2}^{n-2} \sum_{h=+,-} \mathcal{A}(g_1,\ldots,g_i,\hat{P}^h)\,\frac{1}{(p_1+\cdots+p_i)^2}\,\mathcal{A}(-\hat{P}^{-h},g_{i+1},\ldots,g_n)$$

$$z_i = \frac{(p_1+\cdots+p_i)^2}{[1|p_1+\cdots+p_i|n\rangle}$$ location of the pole corresponding for the "i-th" partition



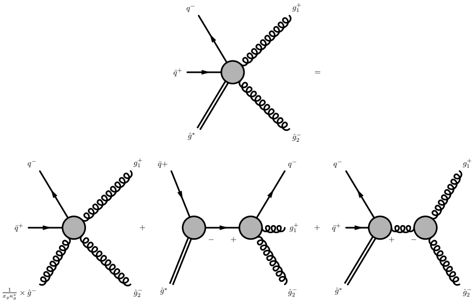
It is natural to ask whether something like a BCFW recursion relation exists with off-shell particles. For off shell, gluons, the answer was first found in *A. van Hameren, JHEP 1407 (2014) 138*

$$\mathcal{A}(0) = \sum_{s=g,f}\left(\sum_{p}\sum_{h=+,-}\mathrm{A}^{s}_{p,h} + \sum_{i}\mathrm{B}^{s}_{i} + \mathrm{C}^{s} + \mathrm{D}^{s}\right),$$

- $\mathrm{A}^{g/f}_{p,h}$ are due to the poles which appear in the original BCFW recursion for on-shell amplitudes. The pole appears because one of the intermediate virtual gluon, whose shifted momentum squared $K^2(z)$ goes on-shell.
- $\mathrm{B}^{g/f}_{i}$ are due to the poles appearing in the propagator of auxiliary eikonal quarks. This means $p_i \cdot \hat{K}(z) = 0$ for $z = -\frac{2\,p_i\cdot K}{2\,p_i\cdot e}$. $\hat{K}$ is the momentum flowing through the eikonal propagator.
- $\mathrm{C}^{g/f}$ and $\mathrm{D}^{g/f}$ show up us the first/last shifted particle is off-shell and their external propagator develops a pole.
  The external propagator for off-shell particles is necessary to ensure

$$\lim_{z\to\infty}\mathcal{A}(z) = 0$$

From 30 diagrams to...



$$\mathcal{A}(g^*, \bar{q}^+, q^-, g_1^+, g_2^-) = \frac{1}{\kappa_g^*} \frac{[\bar{q}1]^3 \langle 2g\rangle^4}{[\bar{q}q]\langle g|\not{p}_2 + \not{k}_g|1]\langle 2|\not{k}_g\ (\not{k}_g + \not{p}_2)\,|g]\langle 2|\not{k}_g|\bar{q}]}$$

$$+ \frac{1}{\kappa_g} \frac{1}{(k_g + p_{\bar{q}})^2} \frac{[g\bar{q}]^2\langle 2q\rangle^3\langle 2|\not{k}_g + \not{p}_{\bar{q}}|g]}{\langle 1q\rangle\langle 12\rangle\left\{(k_g + p_{\bar{q}})^2[\bar{q}g]\langle 2q\rangle - \langle 2|\not{k}_g + \not{p}_{\bar{q}}|g]\langle q|\not{k}_g|\bar{q}]\right\}}$$

$$+ \frac{\langle gq\rangle^3[g1]^4}{\langle\bar{q}q\rangle[12][g2]\langle q|\not{p}_1 + \not{p}_2|g]\langle g|\not{p}_1 + \not{p}_2|g]\langle g|\not{k}_g + \not{p}_2|1]}$$

Outline of results on scattering amplitudes and outlook

- It is necessary to understand which shifts are legitimate in the off-shell case, i.e. for which choices $\lim_{z\to\infty} \mathcal{A}(z) = 0$. We provide a full classification of the possibilities. Explicit results discussed thoroughly in ***A. van Hameren, M.S. JHEP 1507 (2015) 010*** .
- Numerical cross-checks are always successful. They were performed cross checked with a program implementing Berends-Giele recursion relation, *A. van Hameren, M. Bury, Comput.Phys.Commun. 196 (2015) 592-598*
- **Upcoming**: generalisation of off-shell BCFW to full High Energy Factorisation (two off-shell partons) and detailed kinematical study of $2 \to 2$ matrix elements.
- **SWITCHOFF**, a *Mathematica* program for the automatic recursive computation of tree level amplitudes in HEF (pre-announced here year ago): **NOW IN PREPARATION** !
- **Long-Term:** push HEF to NLO. Andreas van Hameren and Oleksandr Gituliar are working on that...

Introducing Double Parton Scattering

For a review of DPS: Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089

DPS $\equiv$ the simultaneous occurrence of two partonic hard scatterings in the same proton-proton collision

Double parton scattering cross section:

$$\sigma^D = \mathcal{S} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2, b; t_1, t_2)\, \Gamma_{kl}(x_1', x_2', b; t_1, t_2)\, \hat{\sigma}(x_1, x_1')\, \hat{\sigma}(x_2, x_2')\, dx_1 dx_2 dx_1' dx_2' d^2 b$$

Usual assumption: separation of longitudinal and transverse DOFs:

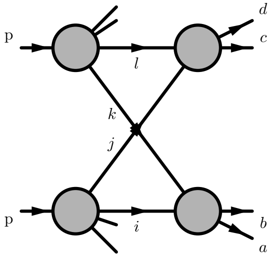
$$\Gamma_{ij}(x_1, x_2, b; t_1, t_2) = D_h^{ij}(x_1, x_2; t_1, t_2)\, F^{ij}(b) = D_h^{ij}(x_1, x_2; t_1, t_2)\, F(b)$$

- Longitudinal correlations, most often ignored or assumed to be negligible, especially at small x : $D_h^{ij}(x_1, x_2; t_1, t_2) = D^i(x_1; t_1)\, D^j(x_2; t_2)$
- Transverse correlation, assumed to be independent of the parton species, taken into account via $\sigma_{eff}^{-1} = \int d^2 b\, F(b)^2 \approx 15 mb$ (CDF and D0)

Usual final kind-of-crafty formula:

$$\sigma^D = \frac{\mathcal{S}}{\sigma_{eff}} \sum_{i_1, j_1, k_1, l_1; i_2, j_2, k_2, l_2} \sigma(i_1 j_1 \to k_1 l_1) \times \sigma(i_2 j_2 \to k_2 l_2)$$

Introducing Double Parton Scattering



$$k^{\mu}_{i/k} = x_{i/k}\, l^{\mu}_{i/k} + k^{\mu}_{i/k\perp}$$
$$k^{\mu}_{l/j} = x_{l/j}\, l^{\mu}_{l/j} + k^{\mu}_{l/j\perp}$$
$$\sqrt{s} = 7/8\, TeV \quad \text{or} \quad 13/14\, TeV$$

Double Parton Scattering LHC was already known as a potentially mischievous child for other reasons:

- For two hard-enough scattering to take place, x's or the transferred momentum must be large...
- ...which implies that the c.o.m. energy should be large enough...
- ...which implies, considering the known PDFs behaviour, that two **very high energy** scatterings (read: transverse momentum cuts higher tan $50 - 60 GeV$ in the final state are going to miss it)
- But the lower the cuts, the farther we are from the perturbative region, so it is good to go beyond pure LO.

"Conjectured" formulas for 2 and 4 jets production:

$$
\begin{aligned}
\sigma_{2-jets} &= \sum_{i,j} \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\, d^2k_{T1} d^2k_{T2}\, \mathcal{F}_i(x_1,k_{T1},\mu_F)\,\mathcal{F}_j(x_2,k_{T2},\mu_F) \\
&\times \frac{1}{2\hat{s}} \prod_{l=i}^{2} \frac{d^3k_l}{(2\pi)^3 2E_l} \Theta_{2-jet}\,(2\pi)^4\,\delta\left(P-\sum_{l=1}^{2} k_i\right) \overline{|\mathcal{M}(i^*,j^*\to 2\,\text{part.})|^2} \\
\sigma_{4-jets} &= \sum_{i,j} \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\, d^2k_{T1} d^2k_{T2}\, \mathcal{F}_i(x_1,k_{T1},\mu_F)\,\mathcal{F}_j(x_2,k_{T2},\mu_F) \\
&\times \frac{1}{2\hat{s}} \prod_{l=i}^{4} \frac{d^3k_l}{(2\pi)^3 2E_l} \Theta_{4-jet}\,(2\pi)^4\,\delta\left(P-\sum_{l=1}^{4} k_i\right) \overline{|\mathcal{M}(i^*,j^*\to 4\,\text{part.})|^2}
\end{aligned}
$$

- PDFs and matrix elements well defined.
- No rigorous factorisation proof around (not even in the collinear case, actually)
- Reasonable description of data justifies this formula *a posteriori*

Our framework

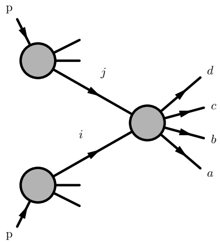
AVHLIB (A. van Hameren) : https://bitbucket.org/hameren/avhlib

- complete Monte Carlo program for tree-level calculations
- any process within the Standard Model
- any initial-state partons on-shell or off-shell
- employs numerical Dyson-Schwinger recursion to calculate helicity amplitudes
- automatic phase space optimization

- **Flavour scheme**: $N_f = 5$
- **Running** α_s from the MSTW68cl PDF sets
- **Massless quarks approximation** $E_{cm} = 7/8\,TeV \Rightarrow m_{q/\bar{q}} = 0.$
- **Scale** $\mu_R = \mu_F \equiv \mu = \frac{H_T}{2} \equiv \frac{1}{2}\sum_i p_T^i$, (sum over final state particles)

We don't take into account correlations in DPS: $D(x_1, x_2, \mu) = f(x_1, \mu)\, f(x_2, \mu)$.
There are attempts to go beyond this approximation:
Golec-Biernat, Lewandowska, Snyder, M.S., Stasto, Phys.Lett. B750 (2015) 559-564
Rinaldi, Scopetta, Traini, Vento, JHEP 1412 (2014) 028
k_T**-dependence** $\Rightarrow$ **see Golec-Biernat's talk at Diffraction 2016**

4-jet production: Single Parton Scattering (SPS)



We take into account all the (according to our conventions) 20 channels.

Here q and q' stand for different quark flavours in the initial (final) state.

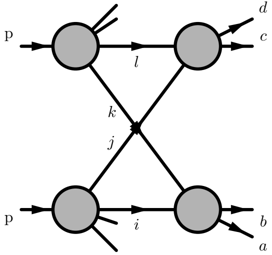
We do not introduce K factors, amplitudes@LO.

$\sim$ 95 % of the total cross section

There are 19 different channels contributing to the cross section at the parton-level:

$$gg \to 4g\,, gg \to q\bar{q}\,2g\,, qg \to q\,3g\,, q\bar{q} \to q\bar{q}\,2g\,, qq \to qq\,2g\,, qq' \to qq'\,2g\,,$$
$$gg \to q\bar{q}q\bar{q}\,, gg \to q\bar{q}q'\bar{q}'\,, qg \to qgq\bar{q}\,, qg \to qgq'\bar{q}'\,,$$
$$q\bar{q} \to 4g\,, q\bar{q} \to q'\bar{q}'\,2g\,, q\bar{q} \to q\bar{q}q\bar{q}\,, q\bar{q} \to q\bar{q}q'\bar{q}'\,, q\bar{q} \to q'\bar{q}'q'\bar{q}'\,,$$
$$q\bar{q} \to q'\bar{q}'q''\bar{q}''\,, qq \to qqq\bar{q}\,, qq \to qqq'\bar{q}'\,, qq' \to qq'q\bar{q}\,,$$

4-jet production: Double parton scattering (DPS)



$$\sigma = \sum_{i,j,a,b;k,l,c,d} \frac{\mathcal{S}}{\sigma_{eff}} \sigma(i,j \to a,b)\, \sigma(k,l \to c,d)$$

$$\mathcal{S} = \begin{cases} 1/2 & \text{if} \quad i\,j = k\,l \quad \text{and} \quad a\,b = c\,d \\ 1 & \text{if} \quad i\,j \neq k\,l \quad \text{or} \quad a\,b \neq c\,d \end{cases}$$

$$\sigma_{eff} = 15\,mb\,, \text{(CDF, D0 and LHCb collaborations)}\,,$$

Experimental data may hint at different values of σ_{eff}; main conclusions not affected

In our conventions, 9 channels from $2 \to 2$ SPS events,

$$\begin{aligned} \#1 &= gg \to gg\,, \quad \#6 = u\bar{u} \to d\bar{d} \\ \#2 &= gg \to u\bar{u}\,, \quad \#7 = u\bar{u} \to gg \\ \#3 &= ug \to ug\,, \quad \#8 = uu \to uu \\ \#4 &= gu \to ug\,, \quad \#9 = ud \to ud \\ \#5 &= u\bar{u} \to u\bar{u} \end{aligned}$$

$\Rightarrow$ 45 channels for the DPS; only 14 contribute to $\geq 95\%$ of the cross section :

$$(1,1), (1,2), (1,3), (1,4), (1,8), (1,9), (3,3)$$
$$(3,4), (3,8), (3,9), (4,4), (4,8), (4,9), (9,9)$$

Hard jets

We reproduce all the LO results (only SPS) for $p\,p \to n\,jets\,, \quad n = 2, 3, 4$ published in
BlackHat collaboration, Phys.Rev.Lett. 109 (2012) 042001
S. Badger et al., Phys.Lett. B718 (2013) 965-978

Asymmetric cuts for hard central jets

$$p_T \geq 80\,\text{GeV}\,, \quad \text{for leading jet}$$
$$p_T \geq 60\,\text{GeV}\,, \quad \text{for non leading jets}$$
$$|\eta| \leq 2.8\,, \quad R = 0.4$$

PDFs set: MSTW2008LO@68cl

$$\sigma(\geq 2\,\text{jets}) = 958^{+316}_{-221} \quad \sigma(\geq 3\,\text{jets}) = 93.4^{+50.4}_{-30.3} \quad \sigma(\geq 4\,\text{jets}) = 9.98^{+7.40}_{-3.95}$$

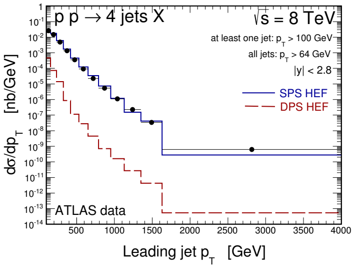
Cuts are too hard to pin down DPS and/or benefit from HEF: 4-jet case

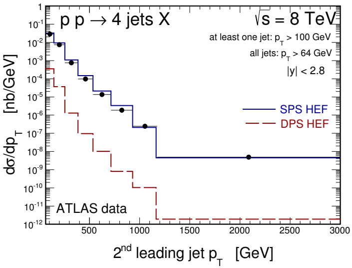
$$\text{Collinear case} \quad \begin{cases} 9.98^{+7.40}_{-3.95} & SPS \\ 0.094^{+0.06}_{-0.036} & DPS \end{cases} \qquad \text{HEF case} \quad \begin{cases} 10.0^{+6.9}_{-5.3} & SPS \\ 0.05^{+0.054}_{-0.029} & DPS \end{cases}$$

Differential cross section

Most recent ATLAS paper on 4-jet production in proton-proton collision:
ATLAS, JHEP 1512 (2015) 105

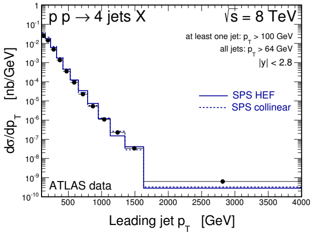
$$p_T \geq 100\,\text{GeV}\,, \quad \text{for leading jet}$$
$$p_T \geq 64\,\text{GeV}\,, \quad \text{for non leading jets}$$
$$|\eta| \leq 2.8\,, \quad R = 0.4$$

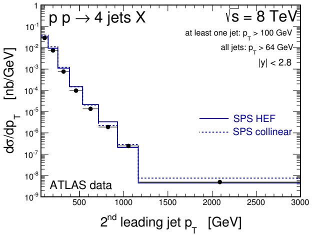


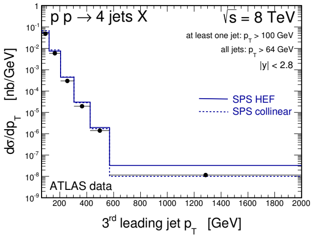


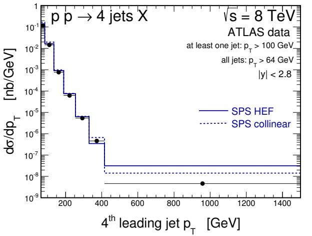
- All channels included and running α_s @ NLO
- Good agreement with data
- DPS effects are manifestly too small for such hard cuts: this could be expected.

Comparing collinear factorization and HEF









Collinear factorization performs slightly better for intermediate values and HEF does a better job for the last bins, except for the 4th jet.

DPS effects in collinear and HEF

Inspired by Maciula, Szczurek, Phys.Lett. B749 (2015) 57-62

DPS effects are expected to become significant for lower p_T cuts, like the ones of the CMS collaboration, Phys.Rev. D89 (2014) no.9, 092010

$$p_T(1,2) \geq 50\,\text{GeV}\,, \quad p_T(3,4) \geq 20\,\text{GeV}\,, \quad |\eta| \leq 4.7\,, \quad R = 0.5$$

$$\begin{aligned} \text{CMS collaboration :} \quad & \sigma_{tot} = 330 \pm 5\,(\text{stat.}) \pm 45\,(\text{syst.})\,nb \\ \text{LO collinear factorization :} \quad & \sigma_{SPS} = 697\,nb\,, \quad \sigma_{DPS} = \mathbf{125\,nb}\,, \quad \sigma_{tot} = 822\,nb \\ \text{LO HEF } k_T\text{-factorization :} \quad & \sigma_{SPS} = 548\,nb\,, \quad \sigma_{DPS} = \mathbf{33\,nb}\,, \quad \sigma_{tot} = 581\,nb \end{aligned}$$

In HE factorization DPS gets suppressed and does not dominate at low p_T

Counterintuitive result from well-tested perturbative framework
$\Rightarrow$ phase space effect ?

Higher order corrections to 2-jet production

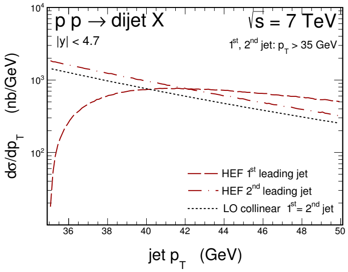


Figure: The transverse momentum distribution of the leading (long dashed line) and subleading (long dashed-dotted line) jet for the dijet production in HEF.

NLO corrections to 2-jet production suffer from instability problem when using symmetric cuts: Frixione, Ridolfi, Nucl.Phys. B507 (1997) 315-333

Symmetric cuts rule out from integration final states in which the momentum imbalance due to the initial state non vanishing transverse momenta gives to one of the jets a lower transverse momentum than the threshold.

ATLAS data vs. theory (nb) @ LHC7 for 2,3,4 jets. Cuts are defined in Eur.Phys.J. C71 (2011) 1763; theoretical predictions from Phys.Rev.Lett. 109 (2012) 042001

#jets	ATLAS	LO	NLO
2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	$1193(3)^{+130}_{-135}$
3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	$54.5(0.5)^{+2.2}_{-19.9}$
4	$4.3 \pm 0.04^{+1.4}_{-0.79} \pm 0.24$	$9.98(0.01)^{+7.40}_{-3.95}$	$5.54(0.12)^{+0.08}_{-2.44}$

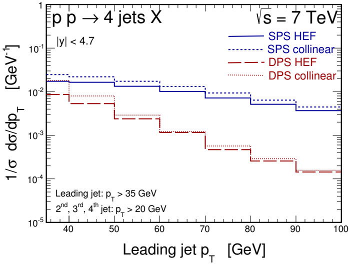
Reconciling HE and collinear factorisation: asymmetric p_T cuts

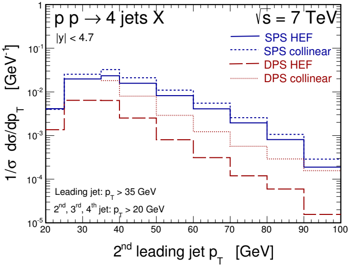
In order to open up wider region of soft final states and thereof expected that the DPS contribution increases

$$p_T(1) \geq 35\text{GeV}\,, \quad p_T(2,3,4) \geq 20\text{ GeV}\,, |\eta| < 4.7\,, \quad \Delta R > 0.5$$

LO collinear factorization : $\sigma_{SPS} = 1969\, nb\,, \quad \sigma_{DPS} = 514\, nb\,, \quad \sigma_{tot} = 2309\, nb$

LO HEF k_T-factorization : $\sigma_{SPS} = 1506\, nb\,, \quad \sigma_{DPS} = 297\, nb\,, \quad \sigma_{tot} = 1803\, nb$



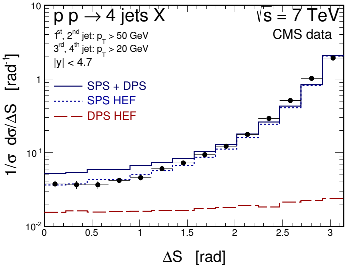


DPS dominance pushed to even lower p_T but restored in HE factorization as well

A supposed smoking gun: do we really see DPS ?

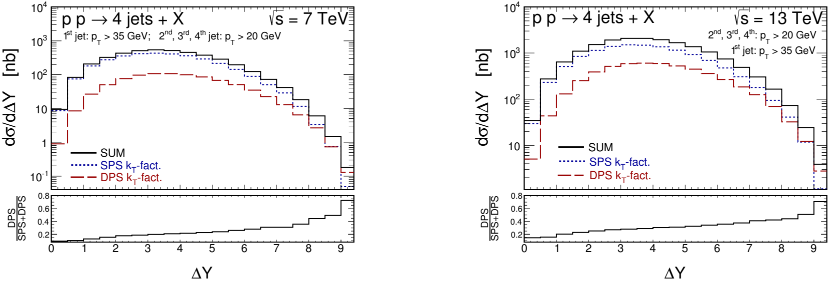
$$\Delta S = \arccos\left(\frac{\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}}) \cdot \vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})}{|\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}})| \cdot |\vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})|}\right), \quad \vec{p}_T(j_i, j_k) = p_{T,i} + p_{T,j}$$

We roughly describe the data via pQCD effects within our HEF approach which are (equally partially) described by parton-showers and soft MPIs by CMS.
CMS collaboration Phys.Rev. D89 (2014) no.9, 092010



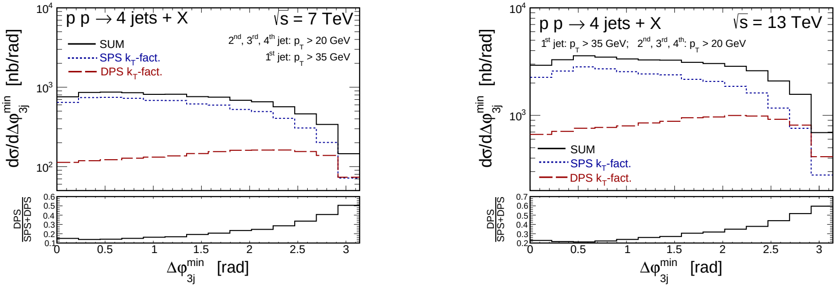
Pinning down double parton scattering: large rapidity separation

K. Kutak, R. Maciula, M.S., A. Szczurek, A. van Hameren,
Phys.Rev. D94 (2016) no.1, 014019



- It is interesting to look for kinematic variables which could make DPS apparent.
- The maximum rapidity separation in the four jet sample is one such variable, especially at 13 GeV.
- for $\Delta Y > 6$ the total cross section is dominated by DPS.

Pinning down double parton scattering: $\Delta\phi_3^{min}$ - azimuthal separation



- Definition: $\Delta\phi_3^{min} = min_{i,j,k[1,4]}\left(\left|\phi_i - \phi_j\right| + \left|\phi_j - \phi_k\right|\right)\ , \quad i \neq j \neq k$
- Proposed by ATLAS in *JHEP 12 105 (2015)* for high p_T analysis
- High values favour configurations closer to back-to-back, i.e. DPS
- For $\Delta\phi_3^{min} \geq \pi/2$ the total cross section is dominated by DPS

Summary and conclusions

- The problem of the recursive computation of tree-level amplitudes in kt-factorization was completely solved for any number of legs in massless QCD
- We have a complete framework for the evaluation of cross sections from amplitudes with off-shell quarks and TMDs via KMR procedure obtained from NLO collinear PDFs
- HE factorisation reproduces well ATLAS data @ 7 and 8 TeV for hard central inclusive 4-jet production. Essential agreement with collinear predictions.
- HE factorisation smears out the DPS contribution to the cross section for less central jet, pushing the DPS-dominance region to lower p_T, but asymmetric cuts are in order: initial state transverse momentum generates asymmetries in the p_T of final state jet pairs.
- We proposed an experimental analysis with cuts which are *asymmetric and soft* and a set of observables which would help pinning down DPS more effectively
- Further insight into High Energy Factorisation will come by matching our Monte Carlo with initial and final state parton showers: to be done asap, with **Marcin Bury, Hannes Jung, Krzysztof Kutak, Andreas van Hameren.**

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Thank you for your attention !

Evolution with energy of PDFs

Single parton scattering cross section:

$$\sigma^S = \sum_{i,j} \int dx_1\, dx_2\, D_i(x_1, \mu_F)\, D_j(x_2, \mu_F) \times \frac{1}{2\hat{s}} \prod_{l=i}^{4} \frac{d^3 k_l}{(2\pi)^3 2E_l} (2\pi)^4\, \delta\,(P_i - P_f)\, \overline{|\mathcal{M})|^2}$$

- Parton emission with $k_\perp \in [\Lambda_{QCD}, Q]$ makes single PDFs (sPDFs) scale-dependent
- Evolution is described by the well known DGLAP equations:

$$\frac{\partial}{\partial lnQ^2} D_f(x, Q) = \frac{\alpha_s(Q)}{2\pi} \sum_{f'} \int_x^1 \frac{du}{u}\, \mathcal{P}_{ff'}\left(\frac{x}{u}\right)\, D_{f'}(u, Q)$$

- Initial conditions at an initial scale Q_0 for DGLAP equations are known very well from several groups' fits. For this talk we stick to the Durham MSTW2008 parameterization, Martin, Stirling, Thorne, Watt, Eur.Phys.J. C63 (2009) 189-285 :

$$D_f(x, Q_0) = \sum_i A^i_f\, x^{\alpha^i_f}\, (1-x)^{\beta^i_f} \quad \textbf{Dirichlet distributions}$$

Why factorised ansatz cannot hold: evolution

Factorisation of the dPDFs cannot hold for several reasons:

- For $t1 = t2 \equiv t$ an evolution equation at LLA similar to DGLAP exists:
  Kirschner, Phys. Lett. B84 (1979) 266
  Shelest, Snigiriev, Zinovjev, Phys. Lett. B113 (1982) 325

$$\frac{\partial}{\partial \ln Q^2} D_{f_1 f_2}(x_1, x_2, Q) = \frac{\alpha_s(Q)}{2\pi} \sum_{f'} \left\{ \int_{x_1}^{1-x_2} \frac{du}{u} \mathcal{P}_{f_1 f'}\left(\frac{x_1}{u}\right) D_{f' f_2}(u, x_2, Q) \right.$$
$$\left. + \int_{x_2}^{1-x_1} \frac{du}{u} \mathcal{P}_{f_2 f'}\left(\frac{x_2}{u}\right) D_{f_1 f'}(x_1, u, Q) + \frac{1}{x_1 + x_2} \mathcal{P}^R_{f' \to f_1 f_2}\left(\frac{x_1}{x_1 + x_2}\right) D_{f'}(x_1 + x_2, Q) \right\}$$

$\mathcal{P}_{f_1 f_2} =$ Altarelli-Parisi splitting function

$\mathcal{P}^R_{f \to f_1 f_2} =$ real emission part of the Altarelli-Parisi splitting function

Evolution predicts violation of factorised form of the ansatz

Why factorised ansatz cannot hold: sum rules

- $x_1 + x_2 \leq 1$ constraint not taken into account
- Sum rules are badly violated by a factorised ansatz: the probability of finding a second quark of flavour *a* **must** be correlated to the probability of finding a first one Gaunt, Stirling, JHEP 1003 (2010) 005

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1 D_{f_1 f_2}(x_1, x_2) = (1 - x_2) D_{f_2}(x_2)\,,$$

$$\int_0^{1-x_2} dx_1 \left\{ D_{q f_2}(x_1, x_2) - D_{\bar{q} f_2}(x_1, x_2) \right\} = (N_q - \delta_{f_2 q} + \delta_{f_2 \bar{q}}) D_{f_2}(x_2)$$

where $q = u, d, s$ and $N_u = 2, N_d = 1, N_s = 0$.
Similar equation for $1 \leftrightarrow 2$; symmetry preserved by evolution.

Our approach for this work:
build initial conditions using the sum rules as constraints;
then solve evolution equarion...

Solving the constraints in the pure gluon case

- MSTW08 parameterisation at $Q_0 = 1 GeV$: all parameters known

$$D_g(x) = \sum_{k=1}^{3} A_k \, x^{\alpha_k} \, (1-x)^{\beta}$$

- Hypothesis: Dirichlet distributions linear combinations for the dGDF:

$$D_{gg}(x_1, x_2) = \sum_{i=1}^{3} N_k \, (x_1 x_2)^{a_k} \, (1 - x_1 - x_2)^{b_k}$$

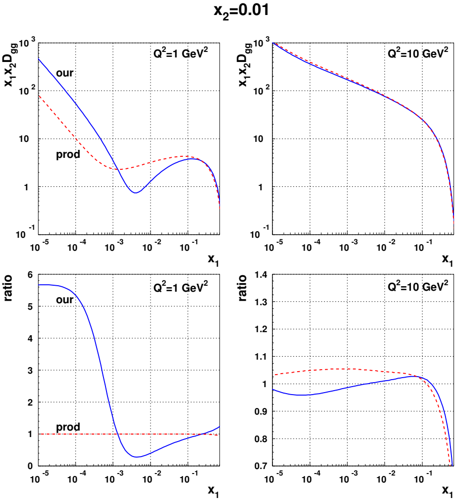
- Only sum rule for gluons is the momentum one

$$\sum_{f_1} \int_0^{1-x_2} dx_1 \, x_1 \, D_{f_1 f_2}(x_1, x_2) = (1 - x_2) \, D_{f_2}(x_2)$$

- After solving, one ends up with the very simple constraints

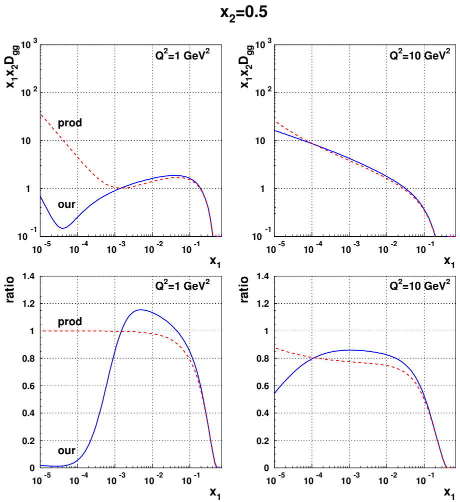
$$a_k = \alpha_k \,, \quad 2\,a_k + b_k + 3 = \alpha_k + \beta + 2 \,, \quad N_k \, \Gamma(2 + a_k) \, \Gamma(1 + b_k) = A_k \, \Gamma(2 + \beta)$$

Evolution of the dGDF: $x_2 = 0.01$



- ratio $= \frac{D_{gg}(x_1, x_2)}{D_g(x_1)\, D_g(x_2)}$
- prod $=$
  $D_g(x_1)\, D_g(x_2)\, \frac{(1-x_2-x_2)^2}{(1-x_1)^2\,(1-x_2)^2}$
  Gaunt,Stirling.
  Respects sum rules only approximately.
- Evolution washes out difference w.r.t. factorised case

Evolution of the dGDF: $x_2 = 0.5$



K. Golec-Biernat, E. Lewandowska, M.S., A. M. Stasto, Z. Snyder, Phys.Lett. B750 (2015) 559-564

Obstruction to including quarks

It is possible to include valence sum rules and extend the system to include quarks. Then the generalised expansion of a sPDF in terms of Dirichlet distributions is now

$$D_f(x) = \sum_k A_k \, x^{\alpha_k} \, (1-x)^{\beta_k}$$

- Reduces to a straightforward linear system in Mellin space
- Apparently $(2N_f+1)(N_f+1)$ equations for the same number of normalisation constants
- The system contains N_f redundant equations $\Rightarrow$ needs further assumptions
- Solving, for instance, k by k implies $\beta_k^{f_2} + \alpha_k^{f_1} = \beta_k^{f_1} + \alpha_k^{f_2}$, **manifestly violated by MSTW08** (does not work with other sets either)

Why: sPDFs simply do not contain enough information to fully determine dPDFs

Attempts based on a generalised valon model are at present underway:
W. Broniowski, K. Golec-Biernat, E. Ruiz Arriola, arXiv:1602.00254, C15-09-21.
In this approach, one first tries to reproduce known sPDFs from a light-cone Fock-space expansion at low energies. So far successful for the pion.
dPDFs in a few steps?

Conclusions on DPDFs

- Factorised ansatz are not enough for Double Parton Scattering description
- A program to build explicitly dPDFs exploiting sum rules was successful for the pure gluon case
- For small longitudinal momentum fractions, the solution is never factorisable. Evolution washes this out significantly for high energies.
- Including quarks in this framework is still a challenge, because the resulting parameterisation of sPDFs does not quite fit the results in the literature. Attempts with light-front approaches are underway at present.

Backup: derivation of the prescription for off-shell gluons, 1

Auxiliary vectors (complex in general):
$$\begin{cases} p_3^\mu = \frac{1}{2}\langle p_2|\gamma^\mu|p_1] \\ p_4^\mu = \frac{1}{2}\langle p_1|\gamma^\mu|p_2] \\ p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0 \\ p_{1,2}\cdot p_{3,4} = 0\,, \quad p_1\cdot p_2 = -p_3\cdot p_4 \end{cases}$$

Auxiliary momenta:
$$\begin{cases} p_A^\mu = (\Lambda + x_1)p_1^\mu - \frac{p_4\cdot k_{1\perp}}{p1\cdot p_2}p_3^\mu\,, \quad p_{A'}^\mu = \Lambda p_1^\mu + \frac{p_3\cdot k_{1\perp}}{p1\cdot p_2}p_4^\mu \\ p_B^\mu = (\Lambda + x_2)p_2^\mu - \frac{p_3\cdot k_{2\perp}}{p1\cdot p_2}p_4^\mu\,, \quad p_{B'}^\mu = \Lambda p_2^\mu + \frac{p_4\cdot k_{2\perp}}{p1\cdot p_2}p_3^\mu \end{cases}$$

For any Λ:
$$\begin{cases} p_A^\mu - p_{A'}^\mu = x_1\, p_1^\mu + k_{1\perp}^\mu \\ p_B^\mu - p_{B'}^\mu = x_2\, p_2^\mu + k_{2\perp}^\mu \\ p_A^2 = p_{A'}^2 = p_B^2 = p_{B'}^2 = 0 \end{cases}$$

Backup: derivation of the prescription for off-shell gluons, 2

Momentum flowing through a propagator of an auxiliary quark line:

$$k^\mu = (\Lambda + x_k) p_1^\mu + y_k \, p_2^\mu + k_\perp$$

Final step: remove complex components taking the $\Lambda \to \infty$ limit.

$$\frac{\not{k}}{k^2} = \frac{(\Lambda + x_k)\not{p}_1 + y_k \not{p}_2 + \not{k}}{2(\Lambda + x_k) y_k p_1 \cdot p_2 + k_\perp^2} \xrightarrow{\Lambda \to \infty} \frac{\not{p}_1}{2 \, y_k p_1 \cdot p_2} = \frac{\not{p}_1}{2 p_1 \cdot k}$$

...and the factor $x_1 \sqrt{-k_\perp^2/2}$ is to match the collinear limit.

In agreement with Lipatov's effective action

Lipatov Nucl.Phys. B452 (1995) 369-400

Antonov, Lipatov, Kuraev, Cherednikov, Nucl.Phys. B721 (2005) 111-135