QCD with strong magnetic fields and the lowest Landau level

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G. Bali, G. Endrődi, Z. Fodor, M. Giordano, F. Gruber, S. Katz,
T. Kovács, S. Krieg, F. Pittler, A. Schäfer, K. Szabó, J. Wellnhofer

[1111.4956 – in progress]

Heisenberg-Programm
Deutsche Forschungsgemeinschaft

QCD with strong magnetic fields

- early universe ($\sqrt{eB} \approx 2$ GeV)
- heavy-ion collisions ($\sqrt{eB} \approx 0.1 \ldots 0.5$ GeV)
- magnetised neutron stars ($\sqrt{eB} \approx$ MeV, 10^{15} G)

B acts primarily on the quarks with their el. charges $+\frac{2}{3}e$ or $-\frac{1}{3}e$

Idealisation

QCD in thermal equilibrium, magn. field B static and homogeneous

$\Rightarrow$ a new parameter

- lattice simulations without sign problem

  ab initio phenomenology

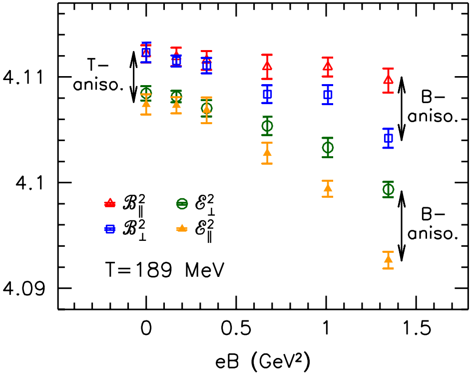
- testing ground for low-energy approaches to QCD
  and expectations from free fermions in magnetic fields

  - Landau level picture

  do we understand and correctly model effects of B in QCD?

The obvious: B causes anisotropies

- ⟨ nonabelian field strengths ⟩ : Bali, FB, Endrődi, Gruber, Schäfer 13

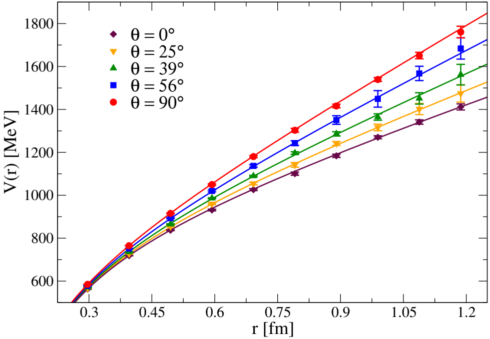


- free case at one-loop with constant $\mathcal{F}_{\mu\nu}$: a la Euler-Heisenberg 36

$$\text{quark eff. action}\Big|_{\mathcal{O}(B^2,\mathcal{F}^2_{\mu\nu})} \sim B^2\,\mathrm{Tr}\Big(-3\underbrace{\mathcal{B}^2_\parallel}_{\text{cheapest }\checkmark} - \mathcal{B}^2_\perp - \mathcal{E}^2_\perp + \tfrac{5}{2}\underbrace{\mathcal{E}^2_\parallel}_{\text{most expensive }\checkmark}\Big)$$

$\mathcal{E}_{\parallel}$ smaller $\Rightarrow$ less confinement along B? yes

- string tension smaller along B — Bonati, D'Elia et al. 14, 16



- free picture:

  charged particles on orbits perp. to B — Lorentz force, Larmor radius

  = stronger confined than along B

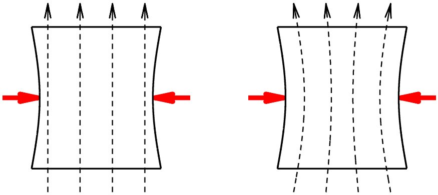
The not-so-obvious: pressure in the presence of B

p = response to change in volume resp. individual extensions $L_{x,y,z}$

- in compressing/expanding one may keep B or its flux Φ constant

$\vec{B} = B\vec{e}_z \uparrow$:

Bali et al. 13



B scheme

Φ scheme

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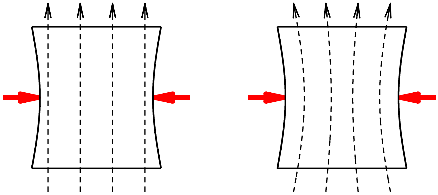


B scheme

Φ scheme

$$p_z^{(\Phi)} = p_z^{(B)}\,, \qquad p_x^{(\Phi)} = p_x^{(B)} + \underbrace{\frac{T}{V}\frac{\partial \log Z}{\partial B}}_{eM}\underbrace{\frac{\partial B}{\partial \log L_x}\bigg|_\Phi}_{-B}$$

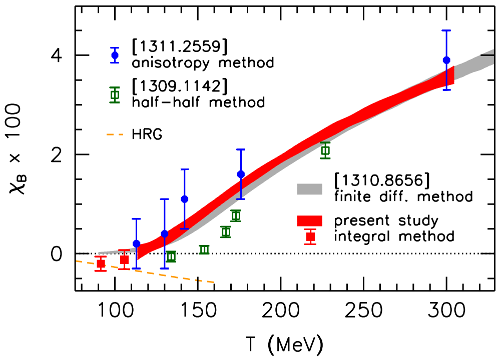
▸ more details

Φ scheme = frozen B lines: perfectly conducting plasmas

& lattice (Φ quantized in finite volume, like momentum)

$\Rightarrow$ magnetisation M, magn. susceptibility $\chi = \frac{\partial M}{\partial(eB)}\Big|_{B=0} = -\frac{\partial^2 f}{\partial(eB)^2}\Big|_{B=0}$

- magnetic susceptibility from various lattice studies:



Levkova, DeTar 13

Bonati, D'Elia, Mariti, Negro, Sanfilippo 13

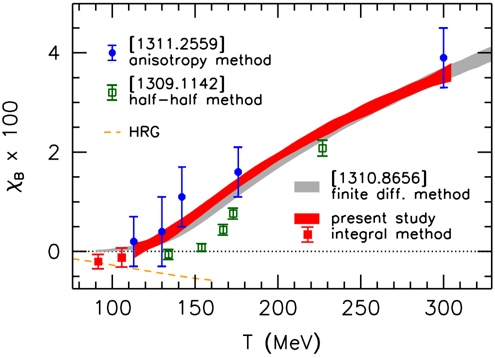
Bali, FB, Endrődi, Schäfer 13

Bali, FB, Endrődi, Katz, Schäfer, 14

QCD is

- paramagnetic at high T
- "almost diamagnetic" at low T

- magnetic susceptibility from various lattice studies:



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QCD is

- paramagnetic at high T
- "almost diamagnetic" at low T

in agreement with

- perturbation theory
- HRG (pions etc.)

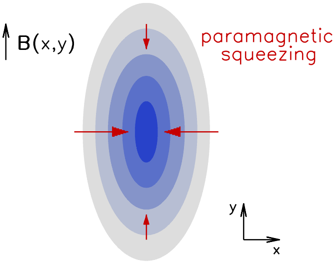
Endrődi 13

Towards phenomenology

'paramagnetic squeezing' — Bali, FB, Endrődi, Schäfer 13

- in HICs the magnetic field in the transverse plane is anisotropic

  & paramagnetic quark matter (at high T) minimises the free enegy when located in regions of large B (actually $|B|$)



gradient $\Rightarrow$ force that might have an impact on the elliptic flow

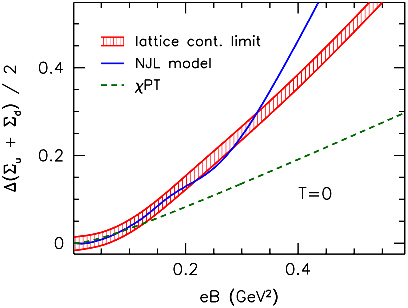
- taken into account in hydro simulations — Pu, Roy, Rezzolla, Rischke 16

  Pang, Endrődi, Petersen 16

The expected: B catalyses the condensate

- at $T = 0$ — Müller, Schramm[2] 92; Gusynin, Miransky, Shovkovy 96
- change of condensate: $\left[\bar{\psi}\psi(B) - \bar{\psi}\psi(0)\right]m$ (divergence-free)

  Bali, FB, Endrődi, Fodor, Katz, Schäfer 12



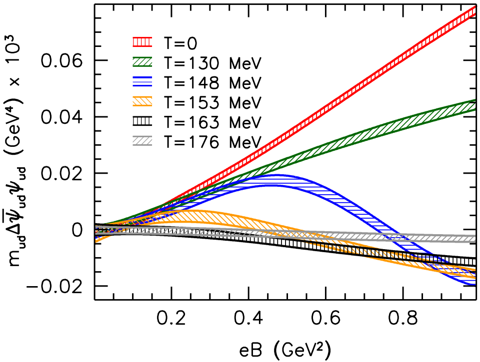
- compared to NJL model and χPT — Gatto, Ruggieri 10

  Cohen, McGady, Werbos 07, Andersen 12

  - Landau picture: zero modes with degeneracy prop. to B
    thus B enhances low mode density $\sim \bar{\psi}\psi$ — Banks-Casher

The unexpected: inverse magnetic catalysis

- at crossover: Bali, FB, Endrődi, Fodor, Katz, Krieg, Schäfer, Szabó 11



- missed at higher-than-physical masses and in many non-lattice approaches D'Elia, Mukherjee, Sanfilippo 10
  - non-monotonic behavior ⇒ no simple picture

- inverse catalysis in gluon action (plaquette), too

  Bali, FB, Endrődi, Gruber, Schäfer 13

  secondary effect revealing the backreaction on the sea quarks (on the quark determinant in lattice configurations)

  both condensates contribute to interaction measure/trace anomaly

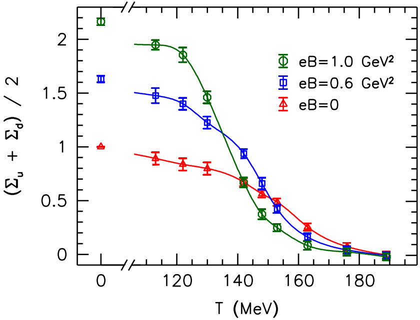
- inverse catalysis is a sea effect FB, Endrődi, Kovács 13

  split the quark condensate into a sum of sea and valence part (approximately)

  sea part changes sign at crossover more details

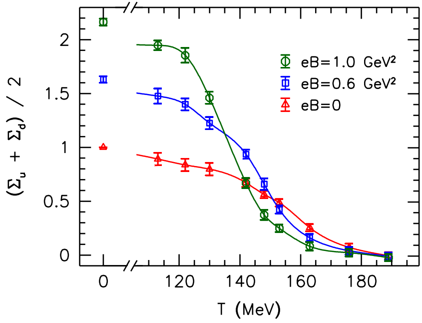
The phase diagram

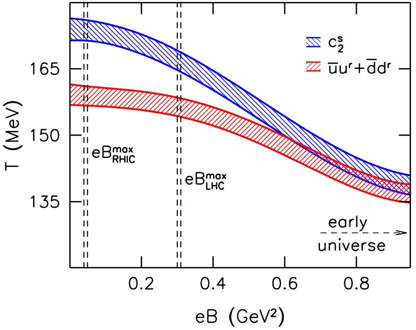
- condensate again:



(watch inflection points)

- condensate again:





(watch inflection points)

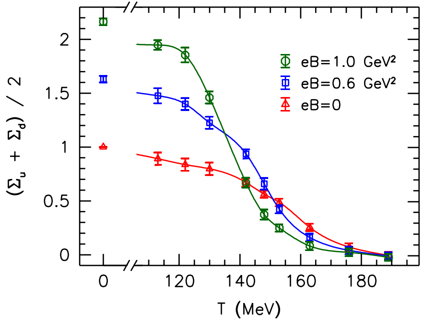
light condensate, strange susc.

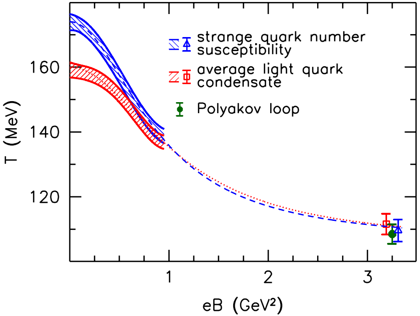
- T_c decreases with B missed before

Bali, FB, Endrődi, Fodor, Katz, Krieg, Schäfer, Szabó 11

The phase diagram

- condensate again:





(watch inflection points)

Endrődi 15

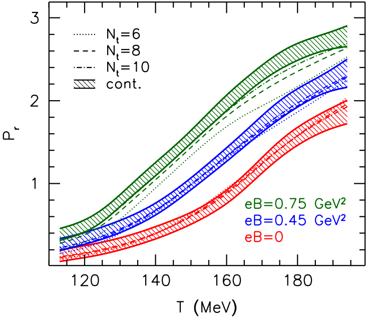
- T_c decreases with B

  missed before

The Polyakov loop

- monotonic in B

FB, Endrődi, Kovács 13



(hard to determine $T_c(B)$ from inflection points)

- free energies a la Euler-Heisenberg with constant Polyakov loop:

F(confined)-F(deconfined) $\nearrow$ with $B \Rightarrow B$ suppresses confinement $\checkmark$

equation of state

Landau level picture

= magnetic field acts on otherwise free particles

- fermions in 2d, continuum: Landau 30

$$-\not{D}^2_{m=0} \to \lambda^2 = |qB| \cdot \underbrace{(2n + 1 + 2s_z)}_{k\;=\;0,1,\ldots} \qquad n = 0, 1, \ldots \qquad 2s_z = \pm 1$$

- lowest Landau level (LLL):

  zero eigenvalue $k = 0$

  degeneracy = magn. flux Φ = $|qB| \times$ area (times N_c)

  ↑ quantised in finite volume

- content of the index theorem in 2d

  with 'topological charge' $\int d^2x\, F = \Phi$ and the role of definite chirality of the zero modes played by spin projection $2s_z = -1$

- note: scalar eigenvalues

$$|qB|\underbrace{(2n+1+0)}_{k=1,\ldots}$$

  - mass of charged pions increases as $\sqrt{m^2+|eB|}$ quantitatively ✓ since weakly coupled

- note: vector eigenvalues

$$|qB|\underbrace{(2n+1\pm 2)}_{k=-1,\ldots}$$

  - mass of charged rhos decreases with B ✓

back to quarks as fermions

many potential obstructions against the LL picture:

- lattice discretisation

  ⇒ fractal structure — Hofstadter 76

- 4d

  ⇒ add momenta $p_z^2 + p_t^2$

  ⇒ Landau levels spread in sorted 4d eigenvalues

- QCD = strong interactions

  ⇒ picture too simplistic?

  condensed matter [quantum Hall etc.] = weaker interaction

Outline for the remainder of the talk

FB, Endrődi, Giorgano, Katz, Kovács, Pittler, Wellnhofer in prep.

- 2d spectra
- transfer to 4d
- is LLL dominating the condensate?

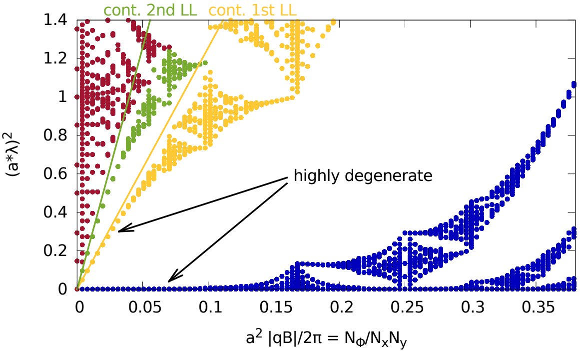
`technicality:`

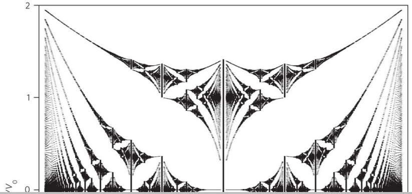
on QCD configurations with $B = 0$: only valence effect so far

Spectra in 2d

cont. free → lattice free

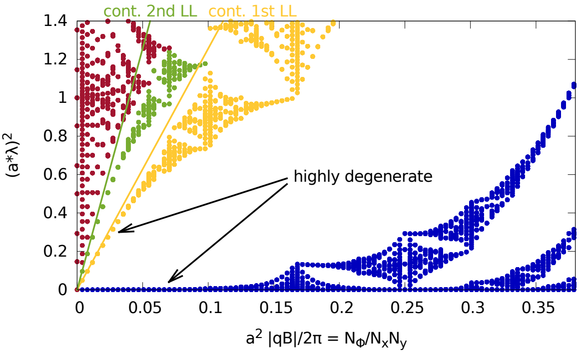
$\lambda^2 = k \cdot |qB|$ $\lambda^2(B)$ fractal





Spectra in 2d

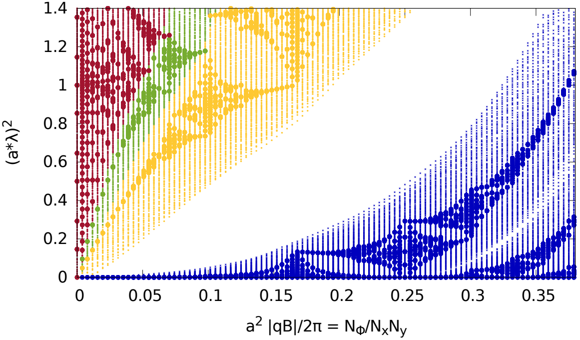
cont. free	$\rightarrow$	lattice free
$\lambda^2 = k \cdot \lvert qB \rvert$		$\lambda^2(B)$ fractal



both axis in lattice units: continuum limit in the lower left corner

Spectra in 2d

lattice free	$\rightarrow$	lattice QCD on 2d slices
$\lambda^2(B)$ fractal		λ^2 washed out, nondegenerate

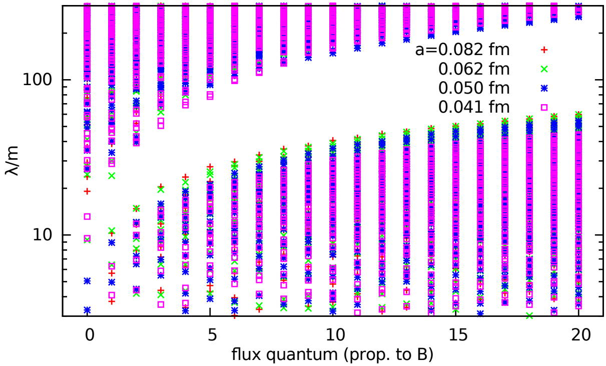


⇒ in QCD a 2d gap between the lowest and the first LL remains

protected by 2d index theorem

Continuum limit of the 2d gap

- measure eigenvalues λ in units of light quark mass m since λ/m is a renormalisation group invariant



$\Rightarrow$ the 2d QCD gap is physical, i.e. survives the continuum limit

Spin

- along $F_{xy} = B$:

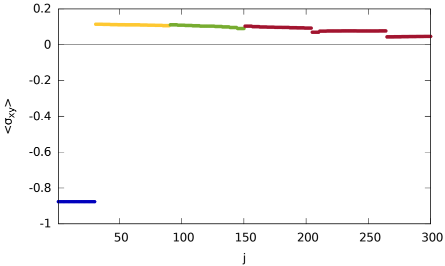
$$\sigma_{xy} := \frac{1}{2i}\left[\gamma_x, \gamma_y\right] \overset{\text{chiral rep.}}{=} \text{Pauli}_3 \otimes 1_2$$

- continuum free:
  σ_{xy} commutes with $\not{D}^2$
  measurable with eigenvalues $\pm 1 = 2s_z$
  LLL signature: $2s_z = -1$
  higher LL's: $2s_z = \pm 1$

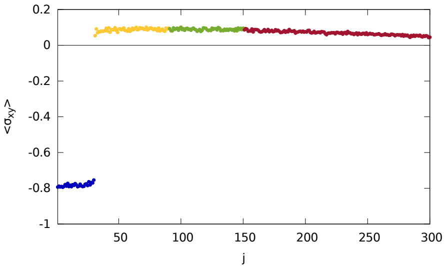
- lattice:
  analogue of σ_{xy} does not commute with $\not{D}^2$
  but off-diagonal elements are found to be small

- diagonal elements $\langle \phi_j | \sigma_{xy} | \phi_j \rangle$:

lattice free:



lattice QCD on 2d slice:



$\Rightarrow$ indeed close to -1 just in the LLL index range = below the gap

(finite volume: σ_{xy} traceless similar to γ_5 in 4d)

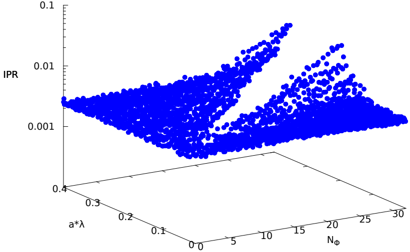
upshot: in 2d the LLL is present even with QCD interactions

Intermezzo: Localisation

- the inverse participation ratio:

$$\text{IPR}[\phi] := \sum_{\text{sites}} |\phi|^4$$

  large for localised modes, small for extended (wave-like) modes
- finding for the 2d modes:



⇒ modes localised near the gap vs. extended in the bulk

as in Anderson localisation (the bulk being a band)

and QCD at high T Garcia-Garcia, Osborn 06; Kovács 10, …

Transfer to 4d

- gap in 4d Dirac operator?

  only for huge values of B

  even in free case p_z and p_t mix Landau levels

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  even in free case p_z and p_t mix Landau levels

- 2+2d separation:

$$\not{D} = \underbrace{\sum_{\mu=x,y} \gamma^\mu D_\mu}_{} + \sum_{\mu=z,t} \gamma^\mu D_\mu \quad \to \text{4d modes } \psi_\lambda \text{ to eigenvalue } i\lambda$$

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$\rightarrow$ 2d modes ϕ_j to index j

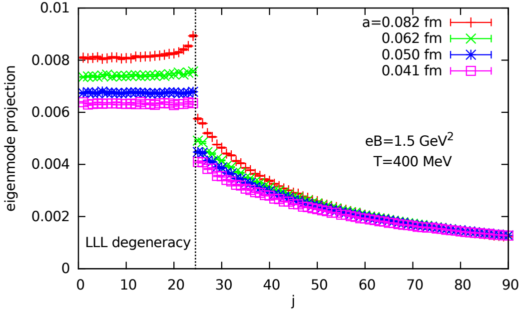
$\phi_j(x,y)$ taken on all (z,t)-slices form a 4d basis

- use $\bigcup_{\text{slices}} \phi_j$ with $j \in [1, \Phi \times N_c]$ (LLL-degeneracy) to test for LLL

- overlap of 4d mode ψ_λ with such a 2d mode:

$$\sum_{\text{slices}} |\langle \phi_j | \psi_\lambda \rangle_{\text{slice}}|^2$$

for certain λ/m-interval as a function of index j:



jump at index $j = N_c \times \Phi$ = LLL degeneracy, continuum limit ✓

increasing curves for higher λ: overlap with higher LL's

$\Rightarrow$ LLL encoded in low 4d modes $\Rightarrow$ way to determine LLL dominance

LLL projection in the condensate

light quark condensate:

$$\langle \bar{\Psi}\Psi \rangle = \quad \mathrm{Tr}\, \frac{2m}{-\not{D}^2 + m^2} = \sum_\lambda \frac{2m}{\lambda^2 + m^2}$$

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renormalisation:

- additive divergence cancels in the change

$$\Delta\langle\bar{\Psi}\Psi\rangle := \langle\bar{\Psi}\Psi\rangle(B) - \langle\bar{\Psi}\Psi\rangle(0)$$

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---

technicality: $B = 0$ subtraction for $\langle\bar{\Psi}\Psi\rangle_{\mathrm{LLL}}$

use the same number of 2d modes to project although no B-field = condensate that “was there before” in the same eigenvalue range

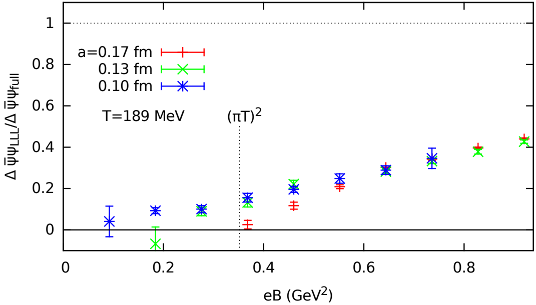
alternative: gradient flow to some flow time ($\approx$ UV cut-off)

- multiplicative divergence cancels in ratios

$$\frac{\Delta\langle\bar{\Psi}\Psi\rangle_{\mathrm{LLL}}}{\Delta\langle\bar{\Psi}\Psi\rangle}$$

- at $T \gtrsim T_c$:

preliminary



slow increase towards 1, note free case at $T = 0$: $1 + \mathcal{O}(\frac{1}{\log(qB/m^2)})$

⇒ LLL significant when $qB \gtrsim (\pi T)^2$, i.e. when B the largest scale but does not dominate

note that in 2d LLL dominates

Summary and Outlook

QCD with external magnetic fields is

- anisotropic
- paramagnetic
- chirally restored/deconfining at lower temperatures
  inverse magnetic catalysis of condensates around T_c
- dominated by the lowest Landau level?
  - 2d spectra: gap in eigenvalues and jump in spin at the LLL degeneracy ✓
  - transfer to 4d: carefully define a LLL projection (and renorm.)
  - light quark condensate: LLL significant for $qB \gtrsim (\pi T)^2$, but dominant only for very large B
  - temperature dependence, sea effect

benchmark for non-lattice approaches

Back-up: anisotropic pressure

- with the free energy and its density

$$F = -T \log Z \,, \qquad f = -\frac{T}{V} \log Z$$

the pressure is defined as

$$p_i = -\frac{L_i}{V} \frac{dF}{dL_i} = -L_i \frac{df}{dL_i}$$

- for a homogeneous system the free energy is extensive with a *B*-dependent density:

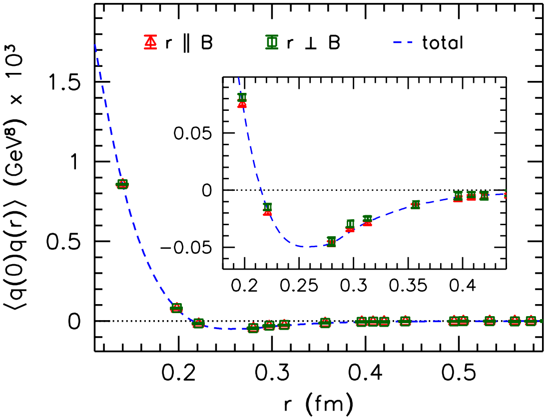
$$F = L_x L_y L_z f(B) = L_x L_y L_z f\Big(\frac{\Phi}{L_x L_y}\Big)$$

- thus

$$p_{x,y}^{(B)} = p_z^{(B)} = p_z^{(\Phi)} = -f \quad \neq \quad p_{x,y}^{(\Phi)} = -f + f' \ldots$$

Back-up: anisotropy in topology?

correlator of topological charge $\langle q(0)q(r)\rangle$ with r along vs. perp. to B:



basically the same

Back-up: valence-sea splitting

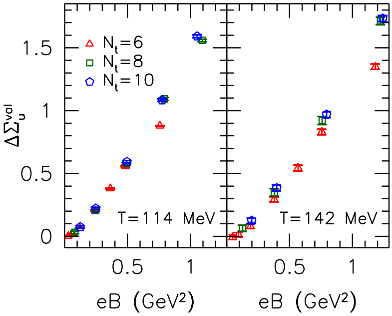
$$\bar\psi\psi^{\text{full}} = \int DA\, e^{-S_g - S_q[B]}\, \text{Tr}(\not{D}[B]+m)^{-1} / Z[B]$$

$$\bar\psi\psi^{\text{val}} = \int DA\, e^{-S_g - S_q[0]}\, \text{Tr}(\not{D}[B]+m)^{-1} / Z[0] \qquad B \text{ in observable}$$

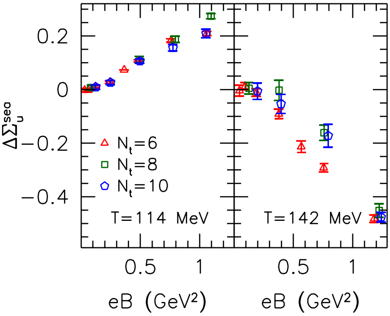
$$\bar\psi\psi^{\text{sea}} = \int DA\, e^{-S_g - S_q[B]}\, \text{Tr}(\not{D}[0]+m)^{-1} / Z[B] \qquad B \text{ in config. generation}$$

to lowest order in B: $\bar\psi\psi^{\text{full}} \simeq \bar\psi\psi^{\text{val}} + \bar\psi\psi^{\text{sea}}$ D'Elia, Negro 11

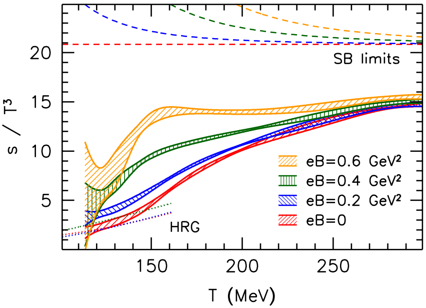
valence



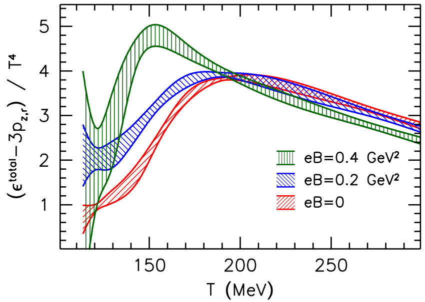
sea: changes sign around T_c



entropy

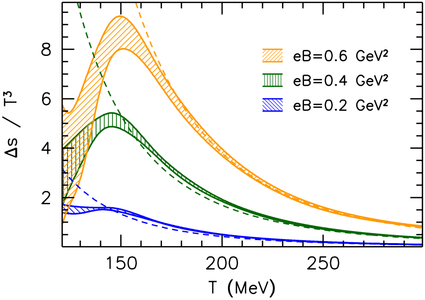


interaction measure/trace anomaly

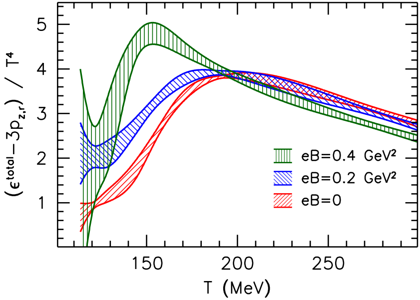


Back-up: Equation of state with B

entropy change



interaction measure/trace anomaly



very close to Stefan-Boltzmann limit