Vacuum-fluctuation effects on inhomogeneous chiral condensates

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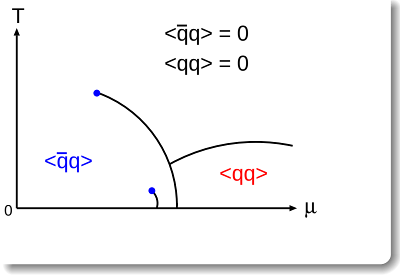
Motivation

- QCD phase diagram (standard picture):



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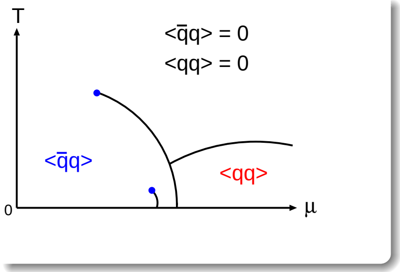

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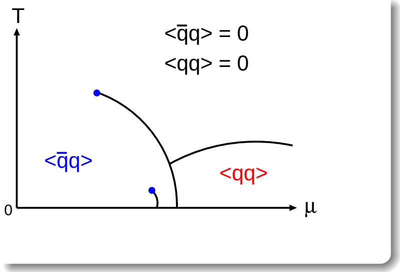

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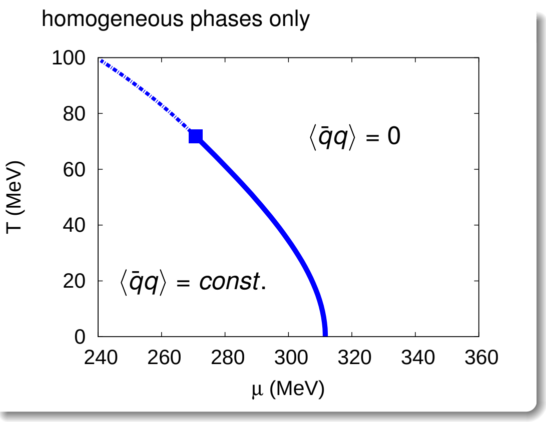
- How about non-uniform phases ?

NJL-model studies



[D. Nickel, PRD (2009)]

NJL-model studies

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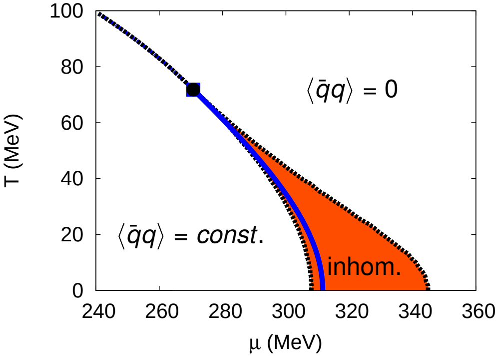
including inhomogeneous phase



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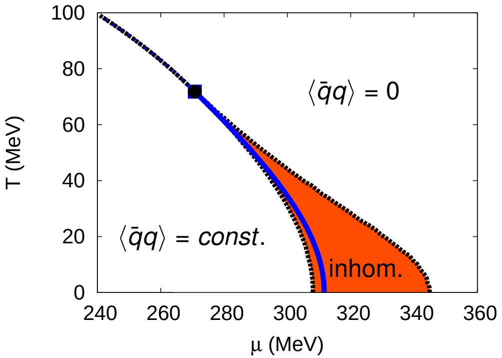

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- 1st-order phase boundary completely covered by the inhomogeneous phase!
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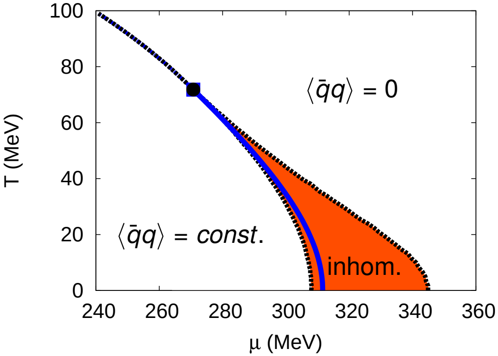

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  [D. Nickel, PRL (2009)]
- Inhomogeneous phase rather robust under model extensions and variations:
  - vector interactions
  - Polyakov-loop dynamics
  - including strange quarks
  - isospin imbalance
  - magnetic fields

[MB, S. Carignano, PPNP (2015)]

Model limitations and open questions

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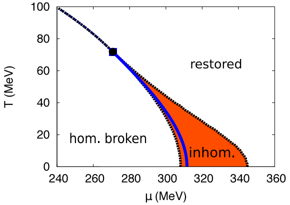

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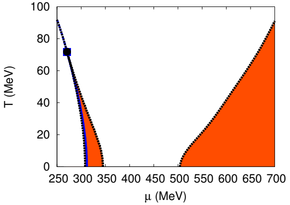

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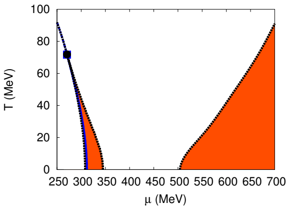

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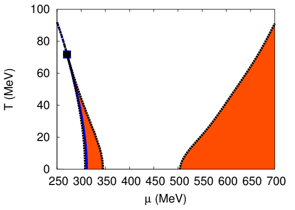

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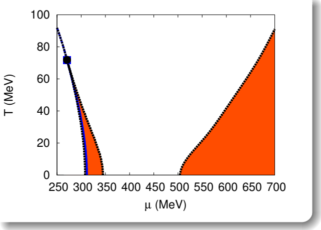

  $\Rightarrow$ use renormalizable model: QM model

- Long-term goal:
  Study role of fluctuations beyond mean field (FRG)

Quark-meson model

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- Lagrangian: $\mathcal{L}_{\rm QM} = \mathcal{L}_{mes} + \mathcal{L}_q$
  - $\mathcal{L}_{mes} = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - U(\sigma,\vec{\pi})\,,$

    $U(\sigma,\vec{\pi}) = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - h\sigma,$ chiral limit: $h = 0$
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- Mean-field approximation:

$$\sigma(x) \to \langle\sigma(x)\rangle \equiv S(\vec{x}),\quad \pi_a(x) \to \langle\pi_a(x)\rangle \equiv P(\vec{x})\,\delta_{a3}$$

  - $S(\vec{x})$, $P(\vec{x})$ time independent classical fields
  - Retain space dependence!

Mean-Field Approximation

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- Mean-field Lagrangian: $\mathcal{L}_{MF} = \mathcal{L}_{mes}^{MF} + \bar{\psi}\,\mathcal{S}^{-1}\,\psi$
  - mesonic part: $\mathcal{L}_{mes}^{MF} = -\frac{1}{2}\left((\vec{\nabla}S)^2 + (\vec{\nabla}P)^2\right) - U(S,P)$ (entirely classical)
  - quark part bilinear in ψ and $\bar{\psi}$ $\Rightarrow$ quark fields can be integrated out!
  - inverse dressed quark propagator:

    $\mathcal{S}^{-1}(x) = i\partial\!\!\!/ - g\left(S(\vec{x}) + i\gamma_5\tau_3 P(\vec{x})\right) \equiv \gamma^0\left(i\partial_0 - H_{MF}[S,P]\right)$

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- Thermodynamic potential: $\Omega_{MF} = \Omega_{mes} + \Omega_q$
  - $\Omega_{mes} = \frac{1}{V}\int_V d^3x\,\left(\frac{1}{2}\left((\vec\nabla\sigma)^2 + (\vec\nabla\vec\pi)^2\right) + U(\sigma,\vec\pi)\right)$ straightforward
  - $\Omega_q = -\frac{T}{V}\mathbf{Tr}\,\mathrm{Log}\left[\frac{1}{T}\left(i\partial_0 - H_{MF} + \mu\right)\right]$

    $= -\frac{1}{V}\sum_\lambda\left[\frac{E_\lambda - \mu}{2} + T\log\left(1 + e^{\frac{E_\lambda-\mu}{T}}\right)\right]$,

    E_λ = eigenvalues of $H_{MF}[S,P]$ $\Rightarrow$ in general difficult

Condensate modulations

- Difficulty: H_{QM} is nondiagonal in momentum space

  $\rightarrow$ has been diagonalized only for certain condensate functions so far

- Generalized constituent quark mass functions: $M(\vec{x}) := g\big(S(\vec{x}) + iP(\vec{x})\big)$

- Modulations with analytically known eigenvalues:
  - homogeneous matter: $M = const.$
  - chiral density wave (CDW): $M(z) = \Delta e^{iqz}$
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- With this one gets:

$$\Omega_q = -\int_0^\infty dE\,\rho(E;\{S,P\})\Big\{E + T\log\Big[1 + e^{-\frac{E-\mu}{T}}\Big] + T\log\Big[1 + e^{-\frac{E+\mu}{T}}\Big]\Big\}$$

  - $\rho(E;\{S,P\})$: analytically known density of states

Standard and extended MFA

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  neglect the vacuum part completely ("no-sea approximation")
  - standard procedure for a long time
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- This talk: eMFA study of inhomogeneous (and homogeneous) phases
  [S. Carignano, MB, B.-J. Schaefer, PRD (2014); S. Carignano, MB, W. Elkamhawy, PRD (2016)]

Fixing the parameters

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- Fit g, λ, and v to three vacuum "observables" : $M_{vac} = g\langle\sigma\rangle$, m_σ, f_π

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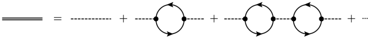
- Correct procedure

  meson propagator with loop corrections:



$$D_j(q^2) = \frac{1}{q^2 - m^2_{j,tree} + g^2\Pi_j(q^2) + i\epsilon} = \frac{Z_j}{q^2 - m^2_{j,pole} + i\epsilon} + \text{reg. terms}, \qquad j = \sigma, \pi$$

$$\rightarrow \qquad m_\sigma \equiv m_{\sigma,pole}, \qquad f_\pi = \frac{M_{vac}}{g_{\pi,\text{ren}}} = \frac{1}{\sqrt{Z_\pi}}\langle\sigma\rangle$$

Fixing the parameters

- $g^2 = \frac{M_{vac}^2}{f_\pi^2 + \frac{1}{2} M_{vac}^2 L_2(0)}, \quad \lambda = 2g^2 \frac{m_\sigma^2}{4M_{vac}^2} \left[1 - \frac{1}{2} g^2 \left(1 - \frac{4M_{vac}^2}{m_\sigma^2}\right) L_2(m_\sigma^2)\right], \quad v^2 = \frac{M_{vac}^2}{g^2} - \frac{g^2 L_1}{\lambda};$

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- "Hands-on renormalization":
  - Regularize L_1 and L_2 (here: Pauli-Villars).
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- “Hands-on renormalization”:
  - Regularize L_1 and L_2 (here: Pauli-Villars).
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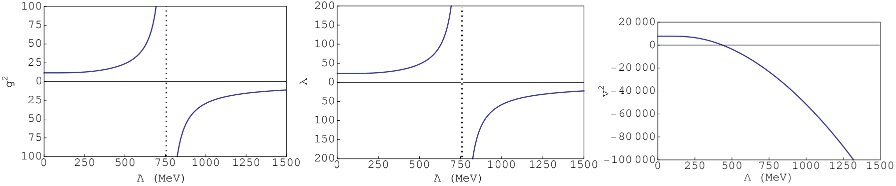
- Results for M_{vac} = 300 MeV, $\quad f_\pi$ = 88 MeV, $\quad m_\sigma$ = 600 MeV:



Phase diagram for homogeneous matter

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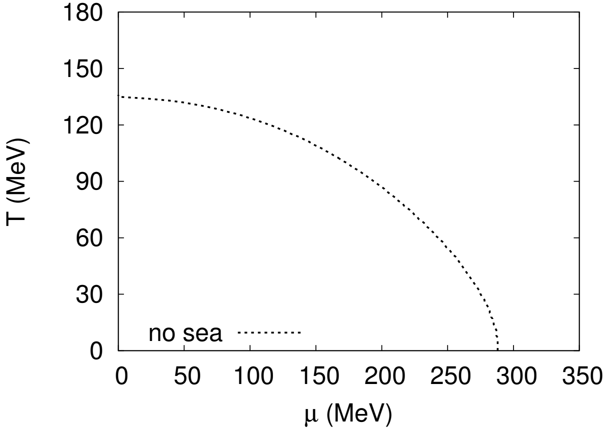

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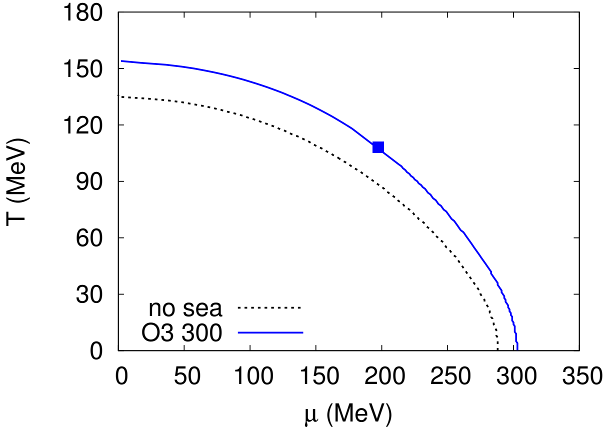

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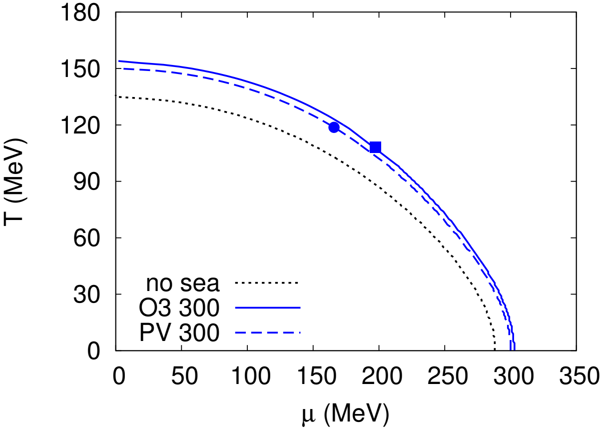

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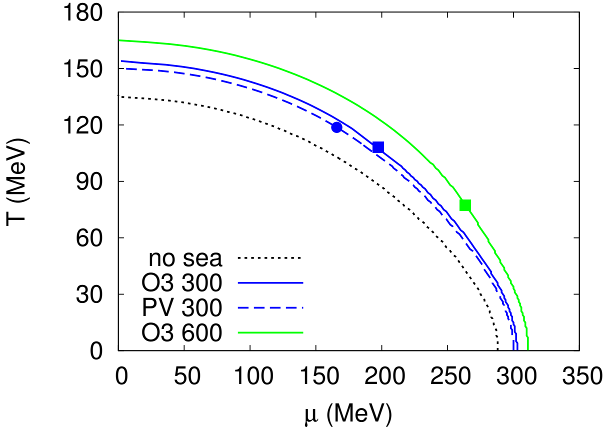

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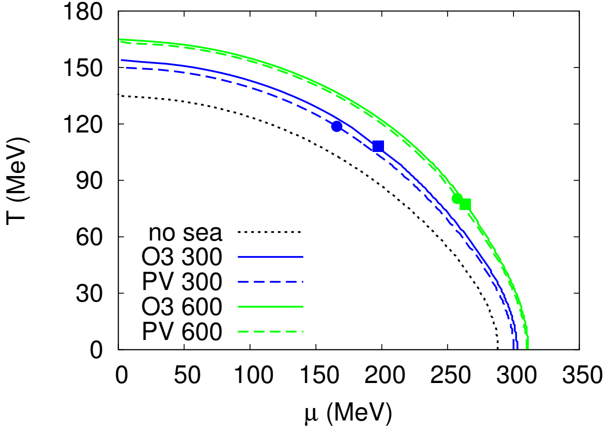

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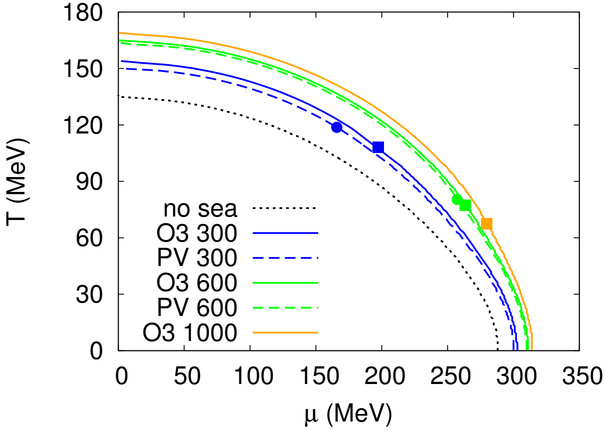

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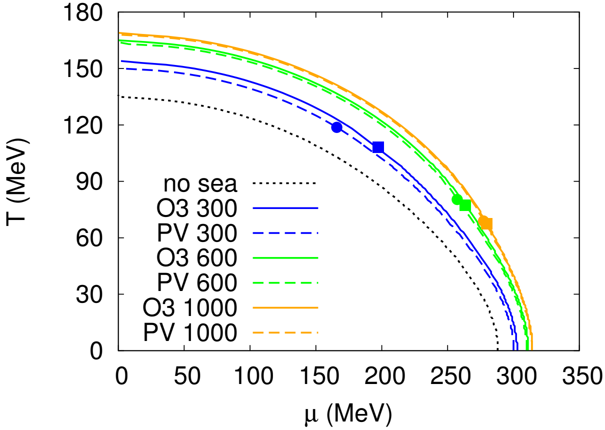

Phase diagram for homogeneous matter

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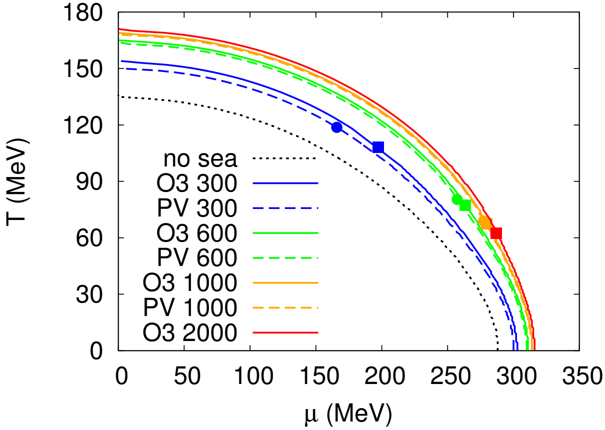

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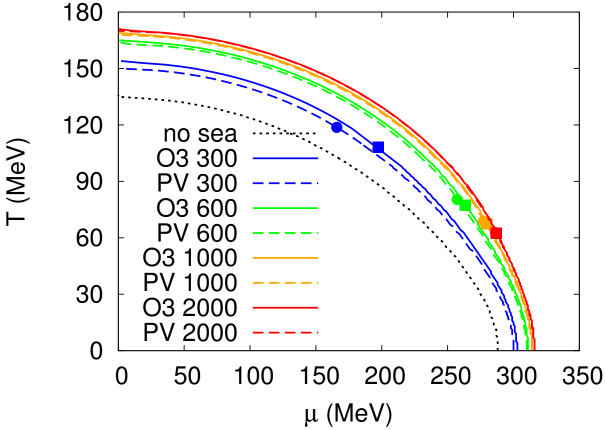

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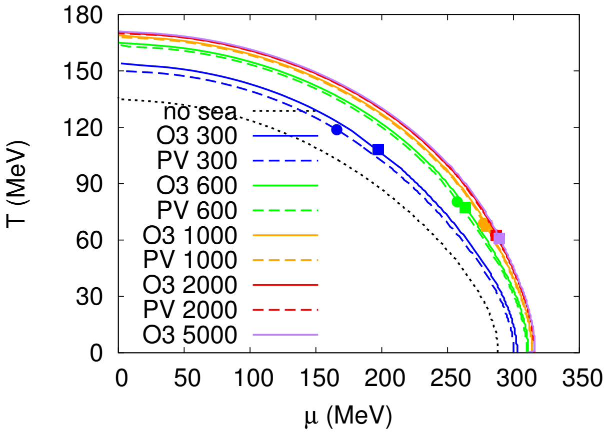

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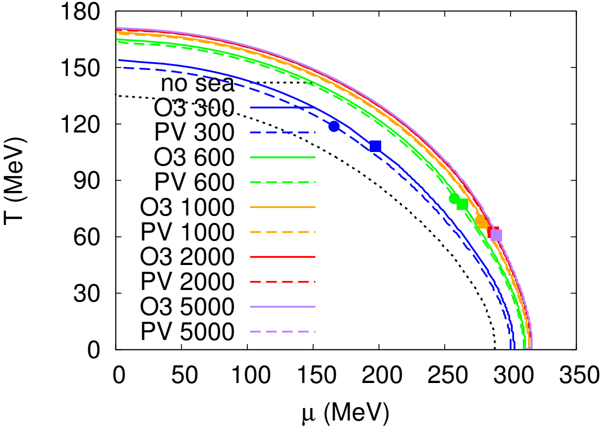

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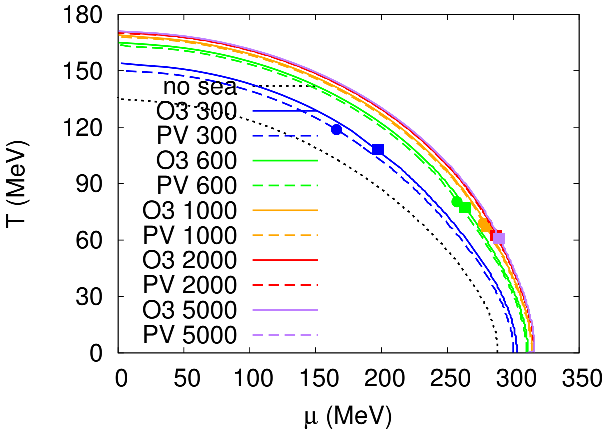

- Convergence reached at $\Lambda \approx 2$ GeV.

Phase diagram for inhomogeneous matter

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- CDW modeluation: $M(\vec{x}) \equiv g\big(S(\vec{x}) + iP(\vec{x})\big) = \Delta e^{iqz}$

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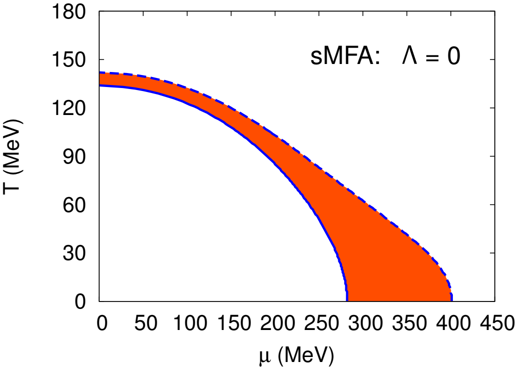

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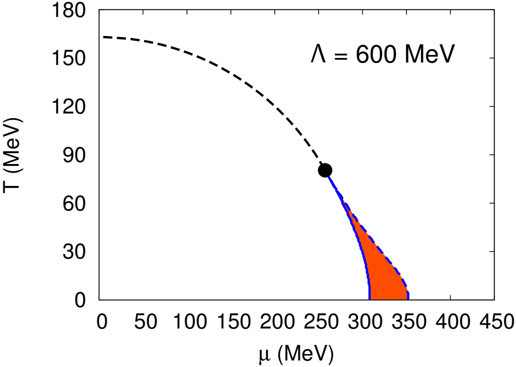

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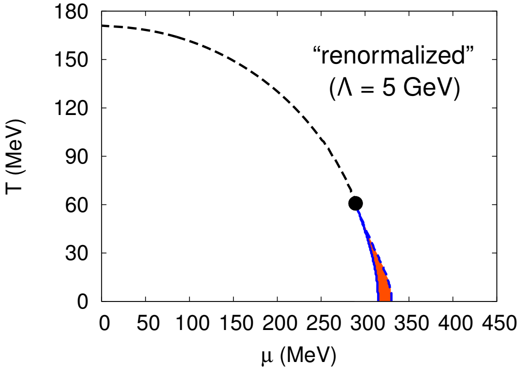

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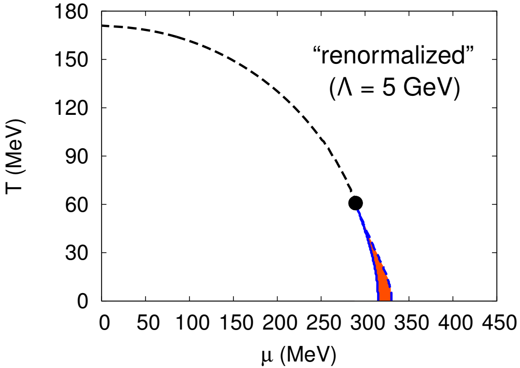

- inhomogeneous island gets smaller but survives.
- Lifshitz point $\hat{=}$ critical point

Ginzburg-Landau analysis

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- Expansion of the thermodynamic potential:

$$\Omega(M) = \Omega(0) + \frac{1}{V}\int d^3x \, \left\{ \frac{1}{2}\gamma_2 |M(\vec{x})|^2 + \frac{1}{4}\gamma_{4,a}|M(\vec{x})|^4 + \frac{1}{4}\gamma_{4,b}|\nabla M(\vec{x})|^2 + ... \right\},$$

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- QM-model: $\gamma_{4,a} = \gamma_{4,b}$ if $m_\sigma = 2M_{vac}$ (as always in NJL!),

  but in general CP and LP do not coincide.

Sigma-mass dependence

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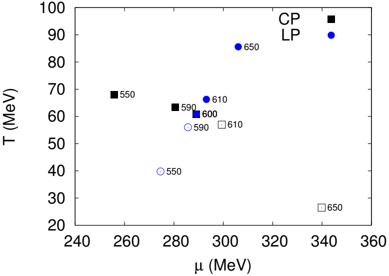
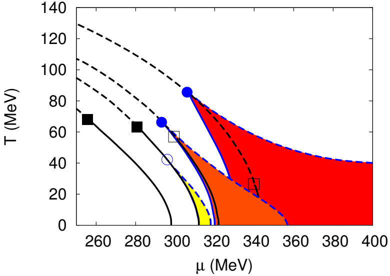
- GL results and phase diagrams for m_σ = 550, 590, 610, 650 MeV:





- Size of the inhomogeneous phase very sensitive to m_σ!

Different parameter fixing

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- "RP": $f_\pi = \frac{\langle\sigma\rangle}{\sqrt{Z_\pi}}, \quad m_\sigma = m_{\sigma,pole}$ "BC": $f_\pi = \langle\sigma\rangle, \quad m_\sigma = m_{\sigma,curv}$

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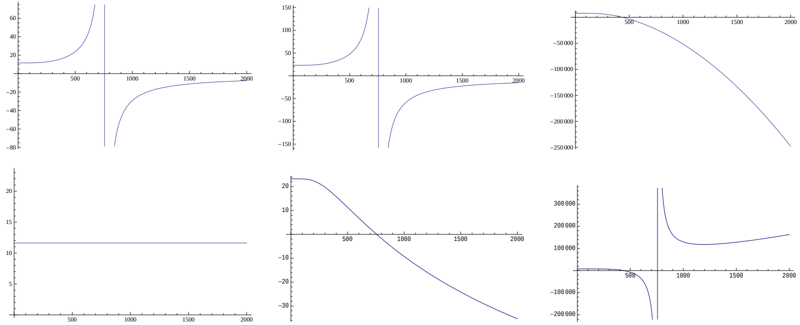
- Parameters:

g^2 λ v^2



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- Homogeneous phase diagram:
  - identical for $m_\sigma = 2M_{vac}$ (moderate differences for $m_\sigma \neq 2M_{vac}$)
  - reason: Ω_{hom} depends only on the ratio $\frac{\lambda}{g^4}$

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  - reason: Ω_{hom} depends only on the ratio $\frac{\lambda}{g^4}$
- Ginzburg-Landau:
  - γ_2 and $\gamma_{4,a}$ UV finite in both schemes
  - $\gamma_2^{RP} - \gamma_2^{BC} = -M_{vac}^2\,\eta(m_\sigma^2)$, $\gamma_{4,a}^{RP} - \gamma_{4,a}^{BC} = \eta(m_\sigma^2)$.
  - $\eta(m_\sigma^2)$ UV finite, vanishes for $m_\sigma = 2M_{vac}$

Different parameter fixing

- Inhomogeneous phase diagram: qualitatively different!

Different parameter fixing

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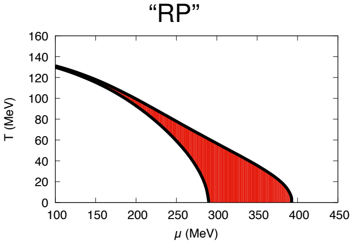
- Inhomogeneous phase diagram: Λ = 200 MeV



Different parameter fixing

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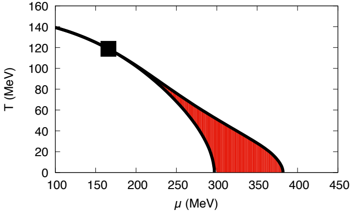
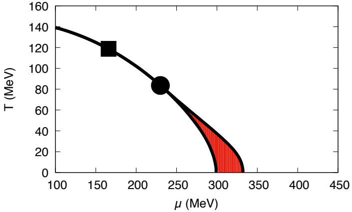
- Inhomogeneous phase diagram: $\Lambda = 300$ MeV

“RP”



“BC”



Different parameter fixing

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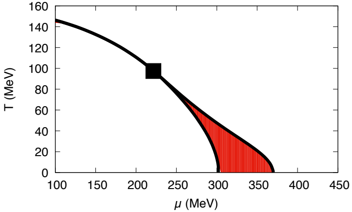
- Inhomogeneous phase diagram: Λ = 400 MeV

“RP”



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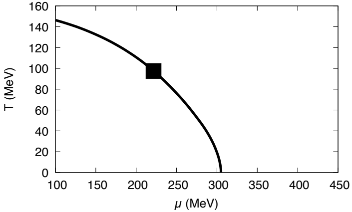

Different parameter fixing

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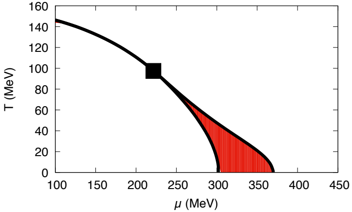
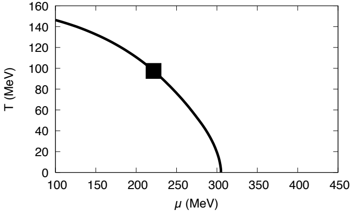


“BC”



Different parameter fixing

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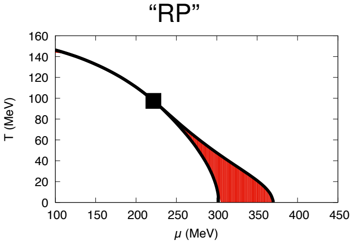

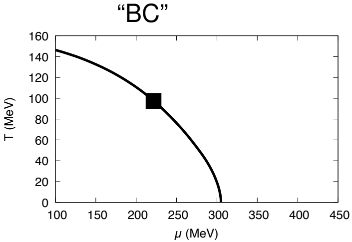

- Ginzburg-Landau:
  - $\gamma_{4,b} = \frac{2}{g^2} - L_2(0)\big|_{M=0} +$ UV finite terms
  - RP: $2/g^2 = L_2(0) +$ finite $\Rightarrow$ UV divergences cancel
  - BC: $g = const.$ $\Rightarrow$ UV divergence persists

Different parameter fixing

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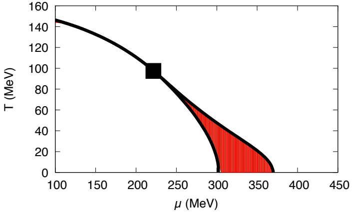
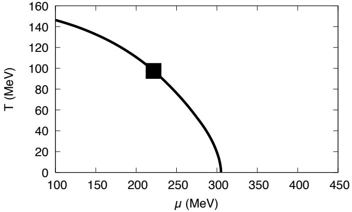


“BC”



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  - RP: $2/g^2 = L_2(0) +$ finite $\Rightarrow$ UV divergences cancel
  - BC: $g = const.$ $\Rightarrow$ UV divergence persists $\Rightarrow$ no renormalized limit!

Different parameter fixing

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- So far we discussed:
  - "RP": $f_\pi = \frac{\langle\sigma\rangle}{\sqrt{Z_\pi}}, \quad m_\sigma = m_{\sigma,pole}$
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- How about the two other possibilities?
  - "BP": $f_\pi = \langle\sigma\rangle, \quad m_\sigma = m_{\sigma,pole}$
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  Does not even work for homogeneous phases:
  - $\gamma_2^{BP} = \frac{m_\sigma^2}{4} L_2(0) +$ fiinite $\Rightarrow$ chiral symmetry gets never restored
  - $\gamma_2^{RC} = -\frac{m_\sigma^2}{4} L_2(0) +$ fiinite $\Rightarrow$ chiral symmetry is never broken

Fate of the inhomogeneous continent

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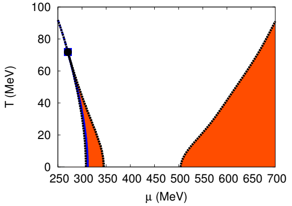
NJL model



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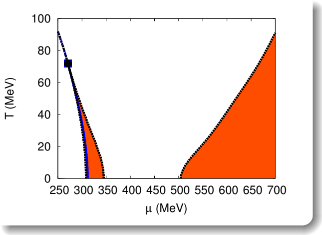
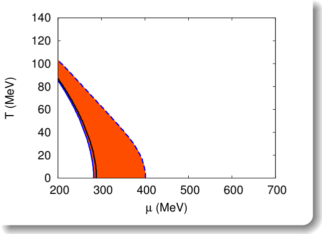


QM model, $\Lambda = 0$



- no continent

Fate of the inhomogeneous continent

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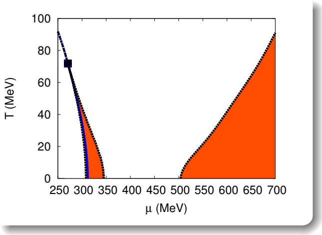
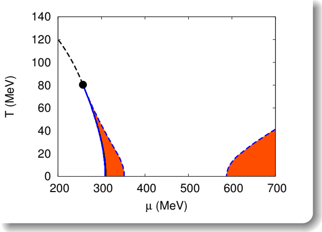


QM model, Λ = 600 MeV



- The continent appears ...

Fate of the inhomogeneous continent

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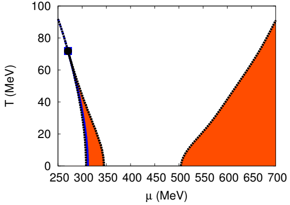

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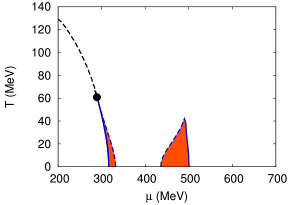

- ... and melts away

Fate of the inhomogeneous continent

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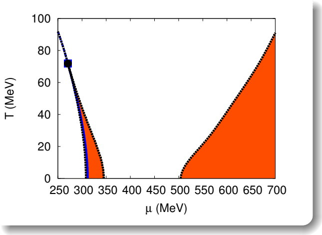

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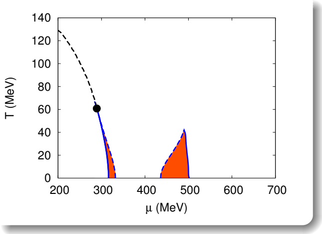

- ... and melts away

- Problem: Ω_{MF} not bounded from below
  - w.r.t. large amplitudes Δ when $\lambda < 0$
  - w.r.t. large wave numbers q when $g^2 < 0$

Vacuum instabilities

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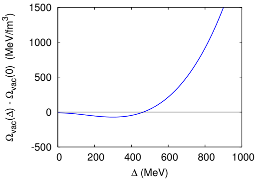
- Thermodynamic potential for $T = \mu = 0$

Λ = 600 MeV



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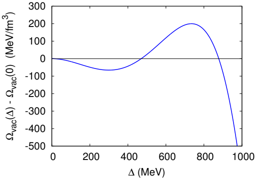

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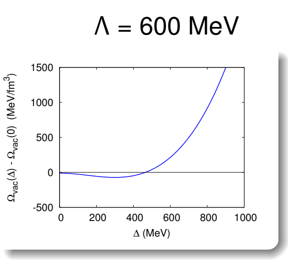

- known instability [Skokov et al., PRD 2010]
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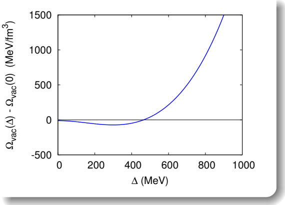

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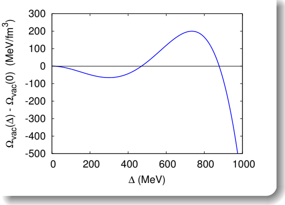

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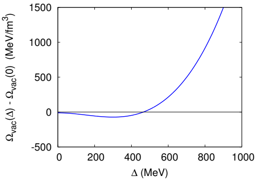

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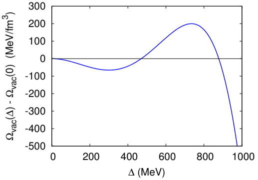

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  "symptomatic of the renormalized one-loop approximation" [Coleman,Weinberg, PRD (1973)]. The inclusion of higher order loop contributions is known to cure this problem".
- similar behavior in *q* direction [Broniowski, Kutschera, PLB (1990)]
- Can the problem be cured by including bosonic fluctuations ($\rightarrow$ FRG)?

Summary

- Inhomogeneous phases in the QCD phase diagram should be considered!

Summary

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- Inhomogeneous phases in the QCD phase diagram should be considered!
- Investigation of inhomogeneous phases in the QM model with fermionic vacuum fluctuations (Dirac sea):
  - Increasing cutoff: Inhomogeneous phase shrinks, but generally survives
  - High sensitivity to the sigma-meson mass;
    $m_\sigma = 2M_{vac} \;\Rightarrow\; LP = CP$
  - Consistent loop corrections to m_σ and f_π crucial
  - Vacuum instabilities w.r.t. large amplitudes and wave numbers

Outlook

- Towards including bosonic fluctuations:
  - Repeat present analysis of fermionic fluctuations within the FRG framework [Carignano, Schaefer, MB; work in progress]
  - Identify phase boundary of the inhomogeneous phase as onset of p-wave pion condensation [$\rightarrow$ talk by R.A. Tripolt]:

$$D_{\pi}^{-1}(\omega = 0, |\vec{p}| = q) = 0$$

  - Analyze bosonic excitation spectrum in the inhomogeneous phase

    [$\rightarrow$ M. Schramm, next talk]