New dynamic critical phenomena in nuclear and quark superfluids

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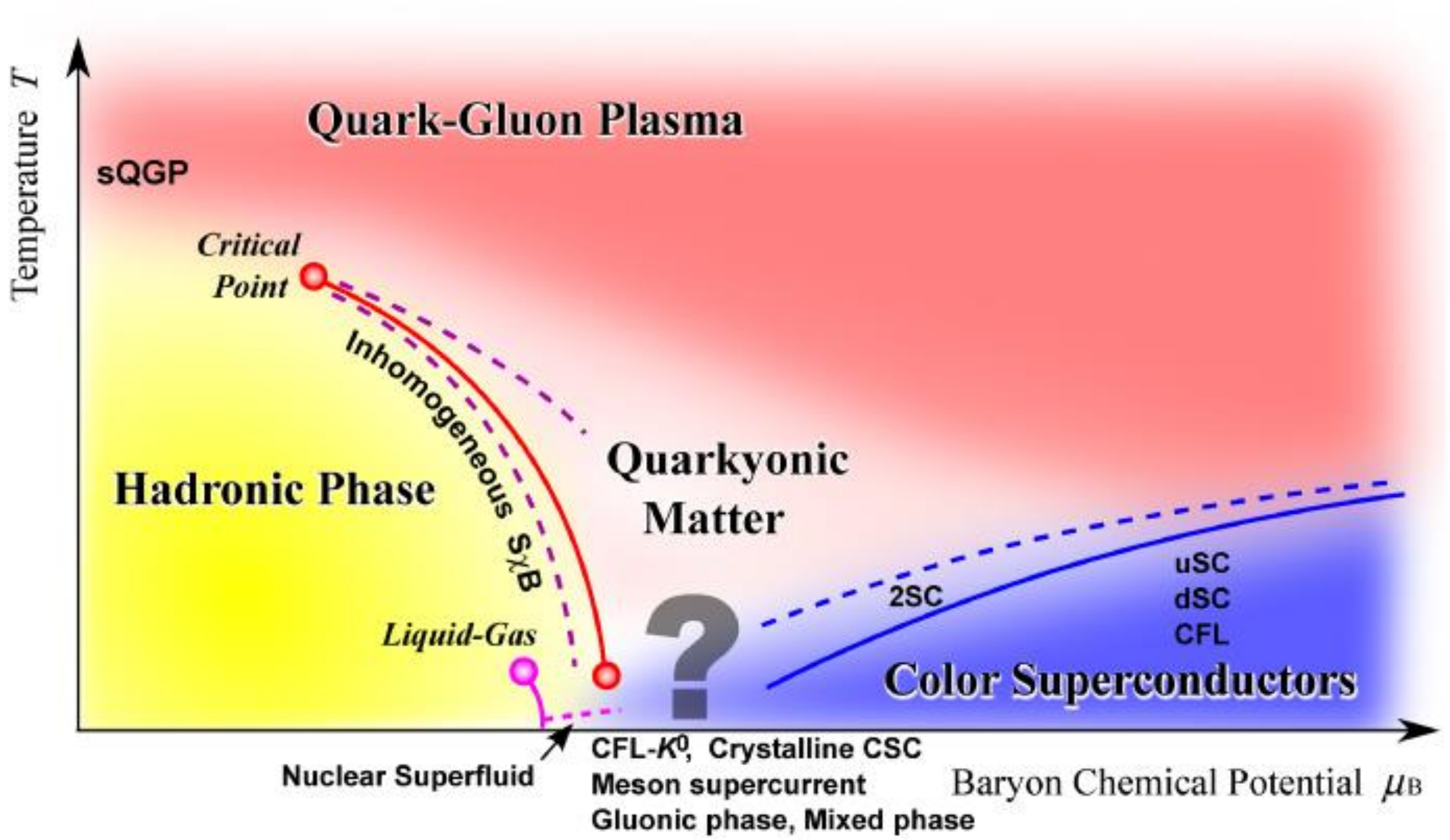
In collaboration with Naoki Yamamoto

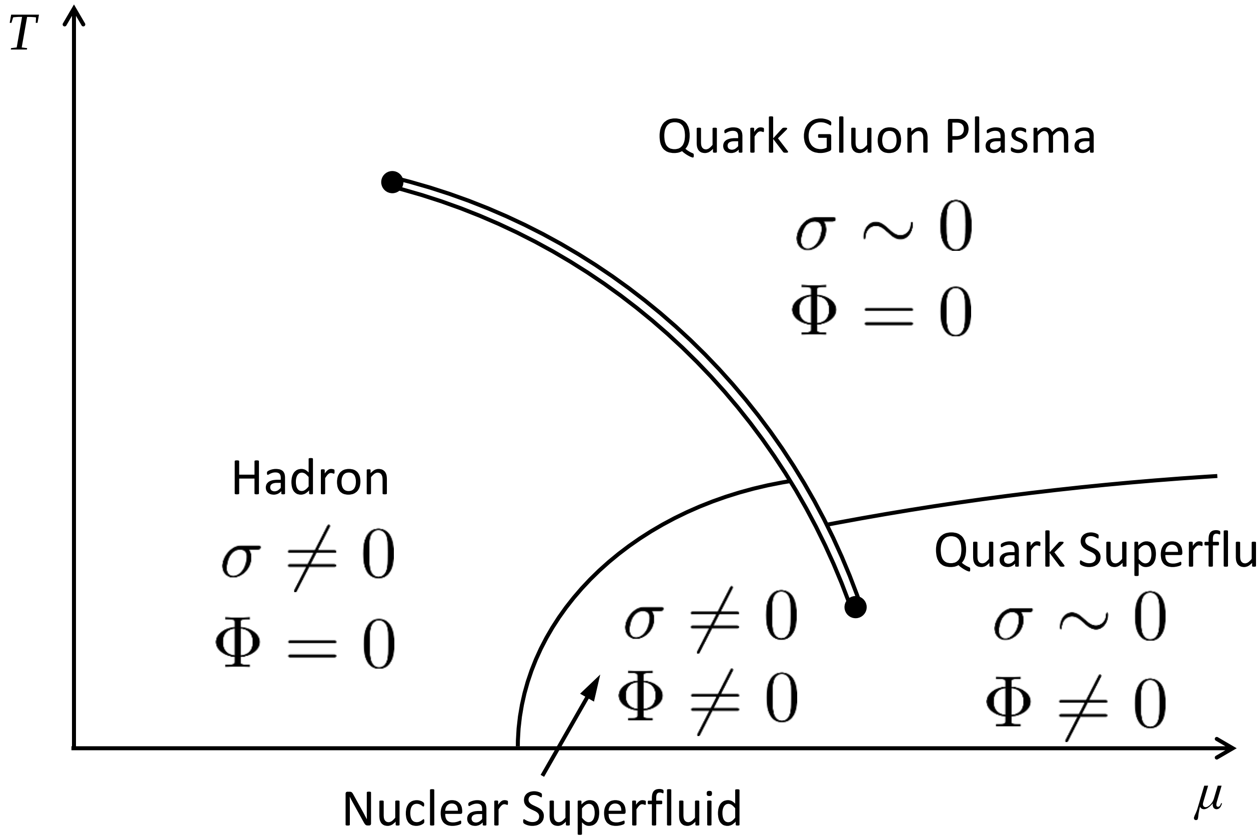
NS and N. Yamamoto, to appear

Phase diagram of QCD



Fukushima, Hatsuda, Rept. Prog. Phys. (2010)

Phases of QCD



T

Quark Gluon Plasma

$\sigma \sim 0$

$\Phi = 0$

Hadron

$\sigma \neq 0$

$\Phi = 0$

$\sigma \neq 0$

$\Phi \neq 0$

Quark Superfluid

$\sigma \sim 0$

$\Phi \neq 0$

Nuclear Superfluid

μ

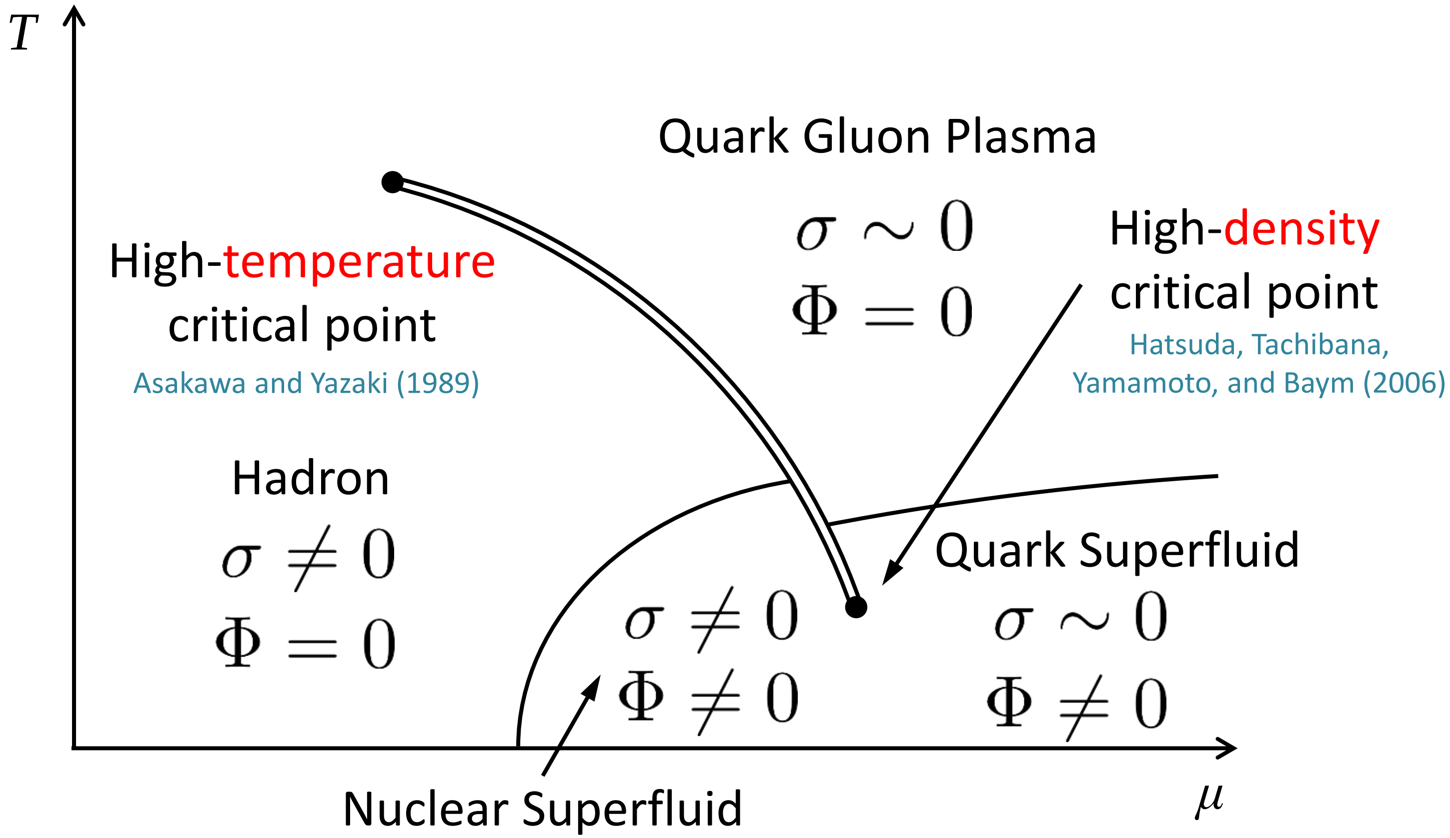
Chiral condensate: $\sigma \equiv \langle \bar{q}q \rangle$

$\cancel{\mathrm{SU}(N_f)_R \times \mathrm{SU}(N_f)_L}$

Diquark condensate: $\Phi \equiv \langle qq \rangle$

$\cancel{\mathrm{U}(1)_\mathrm{B}}$

QCD critical points



Quark Gluon Plasma

$\sigma \sim 0$
$\Phi = 0$

High-temperature
critical point
Asakawa and Yazaki (1989)

High-density
critical point
Hatsuda, Tachibana,
Yamamoto, and Baym (2006)

Hadron
$\sigma \neq 0$
$\Phi = 0$

$\sigma \neq 0$
$\Phi \neq 0$

Quark Superfluid
$\sigma \sim 0$
$\Phi \neq 0$

Nuclear Superfluid

Chiral condensate: $\sigma \equiv \langle \bar{q} q \rangle$

$\cancel{\mathrm{SU}(N_f)_R \times \mathrm{SU}(N_f)_L}$

Diquark condensate: $\Phi \equiv \langle q q \rangle$

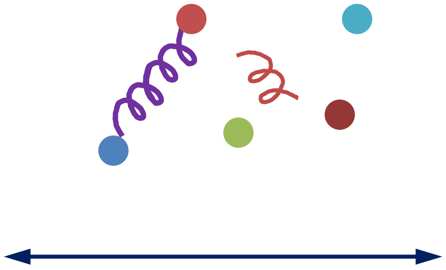
$\cancel{\mathrm{U}(1)_\mathrm{B}}$

Classification of critical points

Universality class	High-T critical point	High-n_B critical point
Static	3D Ising	?
Dynamic Hohenberg and Halperin (1977)	Model H Fujii (2003), Son and Stephanov (2004)	?

What are the universality classes of the high-n_B critical Point ?

Universality class



$$\xi \to \infty$$

$$\langle \sigma(\boldsymbol{r}) \sigma(0) \rangle \sim e^{-r/\xi}$$

Universality class

$$\xi \to \infty$$

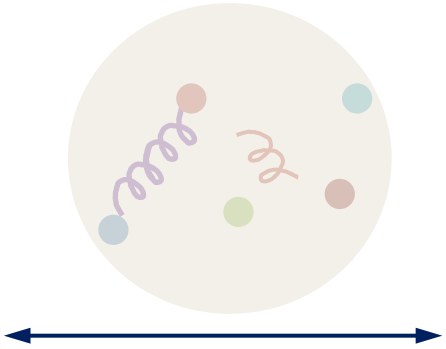
$$\langle \sigma(\boldsymbol{r})\sigma(0) \rangle \sim e^{-r/\xi}$$

Coarse graining

Hydrodynamic variables:

- Order parameters
- Conserved quantities
- Nambu-Goldstone modes

Universality class



$$\xi \to \infty$$

$$\langle \sigma(\boldsymbol{r})\sigma(0)\rangle \sim e^{-r/\xi}$$

Coarse graining

Hydrodynamic variables:

- Order parameters
- Conserved quantities
- Nambu-Goldstone modes

Symmetries

Classification based on hydrodynamic variables and symmetries

Results

Universality class	High-T critical point	High-n_B critical point
Static	3D Ising	3D Ising
Dynamic Hohenberg and Halperin (1977)	Model H Fujii (2003), Son and Stephanov (2004)	New class

New dynamic universality class

beyond the conventional Hohenberg-Halperin's classification

Outline

1

High-density critical point

- Hydrodynamic variables
- Symmetries

2

Statics

- Ginzburg-Landau theory

3

Dynamics

- Langevin equation

High-density critical point

- Hydrodynamic variables

Diquark condensate

σ

Chiral condensate

n

Baryon number density

$\theta : \ \Phi \sim e^{i\theta}$

Superfluid phonon

T^{00}

Energy density

T^{0i}

Momentum density

- Symmetries

$$\mathrm{SU}(N_f)_R \times \mathrm{SU}(N_f)_L \times \mathrm{U}(1)_\mathrm{B} \qquad C \ P \ T$$

Statics

$$F[\sigma,n,\theta]=\int d\boldsymbol{r}\left[\frac{a}{2}(\boldsymbol{\nabla}\sigma)^2+b\boldsymbol{\nabla}\sigma\cdot\boldsymbol{\nabla}n+\frac{c}{2}(\boldsymbol{\nabla}n)^2+\frac{d}{2}(\boldsymbol{\nabla}\theta)^2+V(\sigma,n)\right]$$

$$V(\sigma,n)=\frac{A}{2}\sigma^2+B\sigma n+\frac{C}{2}n^2$$

- Expansion dictated by the symmetries
- θ is irrelevant to the statics.

$$F[\sigma,n,\theta]=\ F_{\mathrm{MF}}[\sigma,n]+F_{\mathrm{MF}}[\theta]+\#\sigma^2(\boldsymbol{\nabla}\theta)^2+\cdots$$

decoupled due to *T* symmetry

derivative coupling
due to U(1) symmetry

Statics

$$F[\sigma, n, \theta] = \int d\boldsymbol{r} \left[\frac{a}{2}(\boldsymbol{\nabla}\sigma)^2 + b\boldsymbol{\nabla}\sigma \cdot \boldsymbol{\nabla} n + \frac{c}{2}(\boldsymbol{\nabla} n)^2 + \frac{d}{2}(\boldsymbol{\nabla}\theta)^2 + V(\sigma, n) \right]$$

$$V(\sigma, n) = \frac{A}{2}\sigma^2 + B\sigma n + \frac{C}{2}n^2$$

- Expansion dictated by the symmetries

$$\langle \sigma(\boldsymbol{r})\sigma(0) \rangle = \frac{1}{4\pi r} e^{-r/\xi} \qquad \xi \sim \frac{1}{\sqrt{AC - B^2}} \to \infty$$

$$\chi_B \equiv \frac{\partial n}{\partial \mu} = T \left\langle n^2 \right\rangle_{\boldsymbol{q}\to 0} \sim \xi^{2-\eta} \qquad (\eta = 0.04)$$

Same universality class of high-T critical point

Dynamics

- Langevin equation for $x_i \equiv \sigma, n, \theta$

$$\dot{x}_i(\boldsymbol{r}, t) = -\gamma_{ij} \frac{\delta F}{\delta x_j} - \int d\boldsymbol{r}' \left[x_i(\boldsymbol{r}), x_j(\boldsymbol{r}')\right] \frac{\delta F}{\delta x_j(\boldsymbol{r}')} + \text{noise term}$$

dissipative non-dissipative

$\gamma_{ij} = \gamma_{ji}$ Onsager's principle $[\theta(\boldsymbol{r}), n(\boldsymbol{r}')] = \delta(\boldsymbol{r} - \boldsymbol{r}')$

$$\gamma_{ij}(\boldsymbol{q}) = \gamma_{ij}^{(0)} + \gamma_{ij}^{(1)} \boldsymbol{q}^2 + O(\boldsymbol{q}^4)$$

Dynamics

- Langevin equation for $x_i \equiv \sigma, n, \theta$

$$\dot{x_i}(\boldsymbol{r}, t) = -\gamma_{ij} \frac{\delta F}{\delta x_j} - \int d\boldsymbol{r}' \left[x_i(\boldsymbol{r}), x_j(\boldsymbol{r}') \right] \frac{\delta F}{\delta x_j(\boldsymbol{r}')} + \text{noise term}$$

dissipative non-dissipative

$$\dot{\sigma}(\boldsymbol{r}) = -\Gamma_{\sigma\sigma} \frac{\delta F}{\delta \sigma(\boldsymbol{r})} + \Gamma_{\sigma n} \boldsymbol{\nabla}^2 \frac{\delta F}{\delta n(\boldsymbol{r})}$$

$$\dot{n}(\boldsymbol{r}) = \Gamma_{\sigma n} \boldsymbol{\nabla}^2 \frac{\delta F}{\delta \sigma(\boldsymbol{r})} + \Gamma_{nn} \boldsymbol{\nabla}^2 \frac{\delta F}{\delta n(\boldsymbol{r})} - \int d\boldsymbol{r}' [n(\boldsymbol{r}), \theta(\boldsymbol{r}')] \frac{\delta F}{\delta \theta(\boldsymbol{r}')}$$

$$\dot{\theta}(\boldsymbol{r}) = -\Gamma_{\theta\theta} \frac{\delta F}{\delta \theta(\boldsymbol{r})} - \int d\boldsymbol{r}' [\theta(\boldsymbol{r}), n(\boldsymbol{r}')] \frac{\delta F}{\delta n(\boldsymbol{r}')}$$

Dynamics

- Langevin equation for $x_i \equiv \sigma, n, \theta$

$$\dot{x_i}(\boldsymbol{r},t) = -\gamma_{ij}\frac{\delta F}{\delta x_j} - \int d\boldsymbol{r}' \left[x_i(\boldsymbol{r}), x_j(\boldsymbol{r}')\right] \frac{\delta F}{\delta x_j(\boldsymbol{r}')} + \text{noise term}$$

dissipative non-dissipative

- Leading order of $\boldsymbol{q}$:

$$\begin{pmatrix} i\omega - \Gamma_{\sigma\sigma}A - (\Gamma_{\sigma\sigma}a + \Gamma_{\sigma n}B)\boldsymbol{q}^2 & -\Gamma_{\sigma\sigma}B - (\Gamma_{\sigma\sigma}b + \Gamma_{\sigma n}C)\boldsymbol{q}^2 & 0 \\ -(\Gamma_{\sigma n}A + \Gamma_{nn}B)\boldsymbol{q}^2 & i\omega - (\Gamma_{\sigma n}B + \Gamma_{nn}C)\boldsymbol{q}^2 & d\boldsymbol{q}^2 \\ -B - b\boldsymbol{q}^2 & -C - c\boldsymbol{q}^2 & i\omega - \Gamma_{\theta\theta}d\boldsymbol{q}^2 \end{pmatrix} \begin{pmatrix} \sigma \\ n \\ \theta \end{pmatrix} = 0$$

- Hydrodynamic modes: $\omega = -i\Gamma_{\sigma\sigma}$ $\boxed{\omega^2 = c_s^2\boldsymbol{q}^2}$

Dynamic critical phenomena

- Speed of phonon

$$c_s = \sqrt{\frac{d}{\chi_B}} \to 0$$

“Critical slowing down”

- Dynamic critical exponent $\omega = c_s |\boldsymbol{q}| \sim \xi^{-z}$

$$z = 2 - \frac{\eta}{2}$$

New dynamic universality class
beyond Hohenberg-Halperin’s classification

Why the universality class is new?

Compare with the other critical points:

- High-T critical point

  Due to superfluid phonon associated with U(1) symmetry

- Superfluid transition of ^{4}He

  Because characteristic order parameters are different.

Superfluid gap
of superfluid helium 4

v.s.

Chiral condensate
of high-n_B critical point

Future heavy-ion collisions


- Dynamic critical phenomena distinguish the high-T and high-n_B critical points.
- Observation of high- n_B critical point would provide the indirect evidence of the superfluidity in QCD.

Conclusion

- We found the new dynamic universality class beyond the conventional Hohenberg-Halperin's classification.

Universality class	High-T critical point	High-n_B critical point
Static	3D Ising	3D Ising
Dynamic Hohenberg and Halperin (1977)	Model H Fujii (2003), Son and Stephanov (2004)	New class

Back up slides

Canonical conjugate

D. T. Son, hep-ph/0204199

S. Weinberg, *The quantum theory of elds. Vol. 2*

- Microscopic theory

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_0 - \mu\bar{q}\gamma^0 q \\ &= \mathcal{L}_0 - A_\mu(x)\bar{q}\gamma^\mu q\end{aligned} \qquad A^\mu \equiv (\mu, \mathbf{0})$$

- Gauge transformation

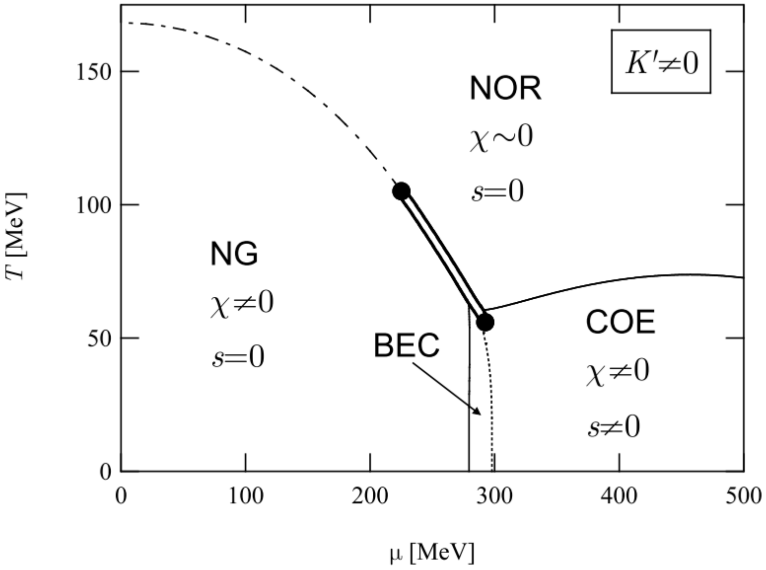
$$A_\mu \to A_\mu - \partial_\mu \alpha \qquad \theta \to \theta + \alpha$$

- Effective theory

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}(\dot{\theta} + \mu, \boldsymbol{\nabla}\theta)$$

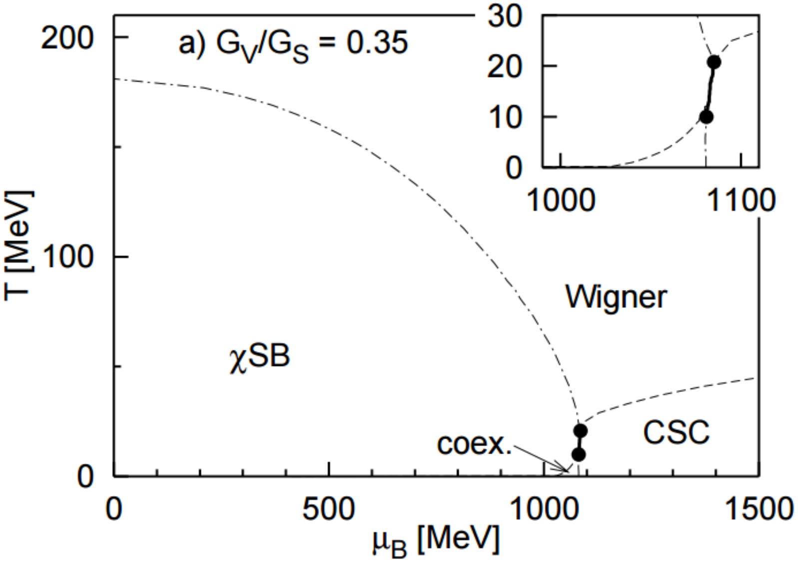
$$\boxed{n \equiv \frac{\delta \mathcal{L}}{\delta \mu} = \frac{\delta \mathcal{L}}{\delta \dot{\theta}}}$$

High-density critical point in NJL model



$N_f\ = 3$ with axial anomaly

Abuki, Baym, Hatsuda, Yamamoto (2010)

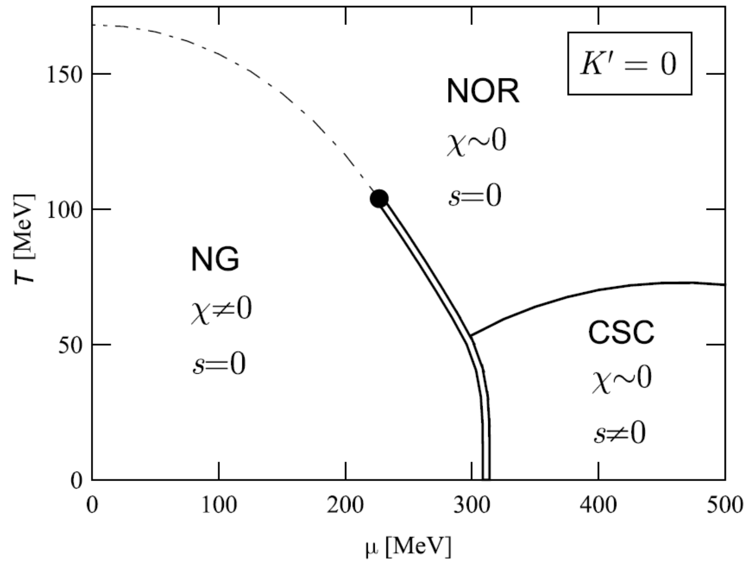


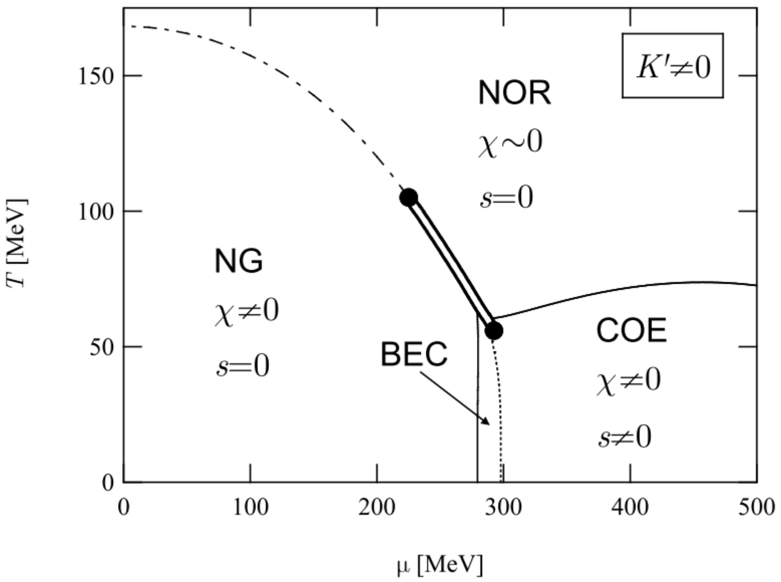
$N_f\ = 2$ with vector interaction

Kitazawa, Koide, Kunihiro, Nemoto (2002)

No-superfludity in 2SC phase

High-density critical point in NJL model





$$\mathcal{L} = \overline{q}(i\gamma_\mu \partial^\mu - m_q + \mu\gamma_0)q + \mathcal{L}^{(4)} + \mathcal{L}^{(6)}$$

$$\mathcal{L}^{(6)} \ni K' \left(\mathrm{Tr}[(d_\mathrm{R}^\dagger d_\mathrm{L})\phi] \right)$$

$$(d_R)_{ai} \equiv \epsilon_{abc}\epsilon_{ijk}(q_R)_b^j C (q_R)_c^k$$
$$\phi_{ij} \equiv (\bar{q}_R)_a^j (q_L)_a^i$$

With energy and momentum

- Speed of phonon

$$\boxed{c_s^2 = \frac{V_{\pi\pi} V_{\theta\theta} T^3 s_{\mathrm{eq}}^2}{\kappa_{nn}\chi_{nn} + 2\kappa_{n\varepsilon}\chi_{n\varepsilon} + \kappa_{\varepsilon\varepsilon}\chi_{\varepsilon\varepsilon}}}$$

$V_{\pi\pi},\ V_{\theta\theta}$: Curvatures in the free energy

$\kappa_{nn},\ \kappa_{n\varepsilon},\ \kappa_{\varepsilon\varepsilon}$: Thermodynamic quantities (no singularity)

$\chi_{nn} \sim \chi_{n\varepsilon} \sim \chi_{\varepsilon\varepsilon} \sim \xi^{2-\eta}$: Susceptibilities

Superfluid transition of ^{4}He

- Speed of phonon

$$c_s = \sqrt{\frac{\rho_s}{c_\mathrm{p}}}$$

Hohenberg and Halperin (1977)

$\rho_s \sim \xi^{-1}$: Stiffness constant

$c_\mathrm{p} \sim \xi^{\frac{\alpha}{\nu}}$: Specific heat at constant pressure

- Critical exponent

$$z = \frac{3}{2} - \frac{\alpha}{2\nu}$$