In-Medium Spectral Functions and Transport Coefficients of Hadrons

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International School of Nuclear Physics
Nuclear matter under extreme conditions -
Relativistic heavy-ion collisions

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Outline

I) Theoretical setup

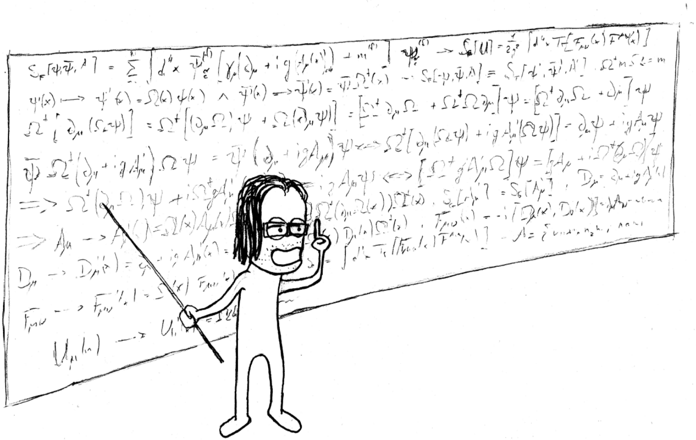
- Functional Renormalization Group (FRG)
- quark-meson model
- analytic continuation procedure

II) Results

- mesonic spectral functions
- mesonic shear viscosity and η/s

III) Summary and outlook

I) Theoretical setup



[courtesy L. Holicki]

Functional Renormalization Group

Euclidean partition function for a scalar field:

$$Z[J] = \int \mathcal{D}\varphi \, \exp\left(-S[\varphi] + \int d^4x \, J(x)\varphi(x)\right)$$

Wilson's coarse-graining: split φ into low- and high-frequency modes

$$\varphi(x) = \varphi_{q\leq k}(x) + \varphi_{q>k}(x)$$

only include fluctuations with $q > k$

$$Z[J] = \int \mathcal{D}\varphi_{q\leq k} \underbrace{\int \mathcal{D}\varphi_{q>k} \, \exp\left(-S[\varphi] + \int d^4x J(x)\varphi(x)\right)}_{Z_k[J]}$$

Functional Renormalization Group

Scale-dependent partition function can be defined as

$$Z_k[J] = \int \mathcal{D}\varphi \, \exp\left(-S[\varphi] - \Delta S_k[\varphi] + \int d^4x J(x)\varphi(x)\right)$$

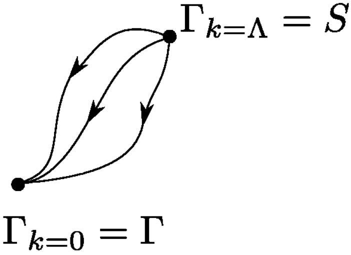
by introducing a regulator term that suppresses IR modes

$$\Delta S_k[\varphi] = \frac{1}{2} \int \frac{d^4q}{(2\pi)^4} \, \varphi(-q) R_k(q) \varphi(q)$$

Switch to scale-dependent effective action ($\phi(x) = \langle \varphi(x) \rangle$):

$$\Gamma_k[\phi] = \sup_J \left(\int d^4x \, J(x)\phi(x) - \log Z_k[J] \right) - \Delta S_k[\phi]$$

Functional Renormalization Group



[wikipedia.org/wiki/Functional_renormalization_group]

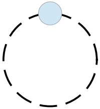
Functional Renormalization Group

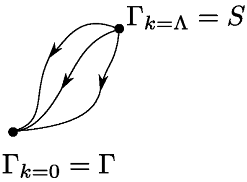
Flow equation for the effective average action Γ_k:

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left(\partial_k R_k \left[\Gamma_k^{(2)} + R_k \right]^{-1} \right)$$

[C. Wetterich, Phys. Lett. **B 301** (1993) 90]

$$\partial_k \Gamma_k \;=\; \frac{1}{2}$$





[wikipedia.org/wiki/Functional_renormalization_group]

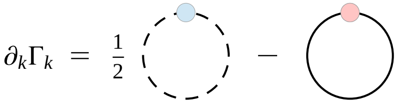
- Γ_k interpolates between bare action S at $k = \Lambda$ and effective action Γ at $k = 0$
- regulator R_k acts as a mass term and suppresses fluctuations with momenta smaller than k
- the use of 3D regulators allows for a simple analytic continuation procedure

Quark-meson model

Ansatz for the scale-dependent effective average action:

$$\Gamma_k[\bar{\psi}, \psi, \phi] = \int d^4x \left\{ \bar{\psi} \left(\partial\!\!\!/ + h(\sigma + i\vec{\tau}\vec{\pi}\gamma_5) - \mu\gamma_0 \right) \psi + \frac{1}{2}(\partial_\mu \phi)^2 + U_k(\phi^2) - c\sigma \right\}$$

- effective low-energy model for QCD with two flavors
- describes spontaneous and explicit chiral symmetry breaking
- flow equation for the effective average action:



Flow of the Effective Potential at $\mu = 0$ and $T = 0$

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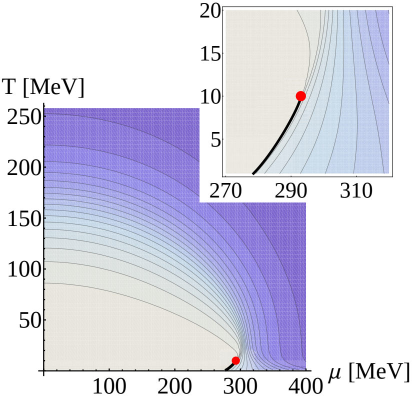
Phase diagram of the quark-meson model

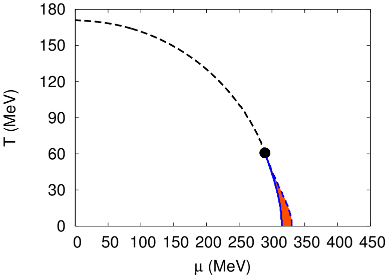
- chiral order parameter σ_0 decreases towards higher T and μ
- a crossover is observed at $T \approx 175$ MeV and $\mu = 0$
- critical endpoint (CEP) at $\mu \approx 292$ MeV and $T \approx 10$ MeV
- vacuum: $\sigma_0 = 93.5$ MeV, $m_\pi = 138$ MeV, $m_\sigma = 509$ MeV, $m_\psi = 299$ MeV



[R.-A. T., N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. **D 89**, 034010 (2014)]

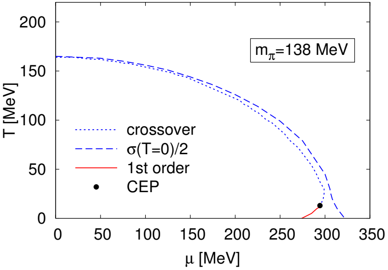
Phase diagram of the quark-meson model

mean-field



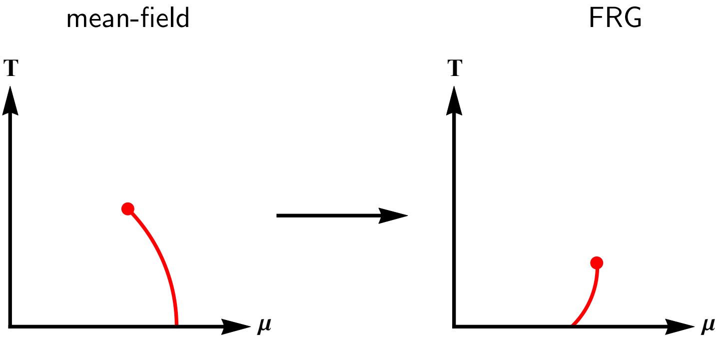
S. Carignano, M. Buballa, B.-J. Schaefer,
Phys. Rev. D 90, 014033 (2014)

FRG



T. K. Herbst, J. M. Pawlowski, B.-J. Schaefer,
Phys. Rev. D 88, 014007 (2013)

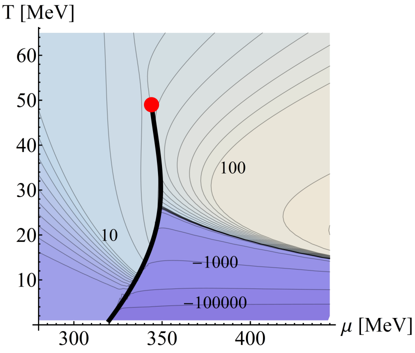
Phase diagram of the quark-meson model



$$\frac{dT_c}{d\mu_c} = -\frac{\Delta n_\psi}{\Delta s}$$

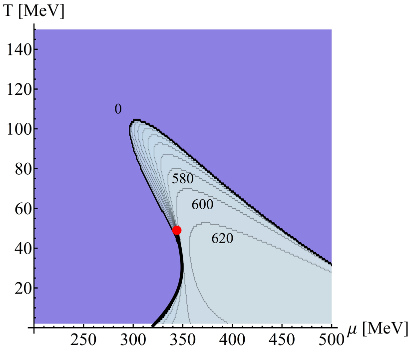
Pion condensation for the QM model - Preliminary

entropy density (FRG)



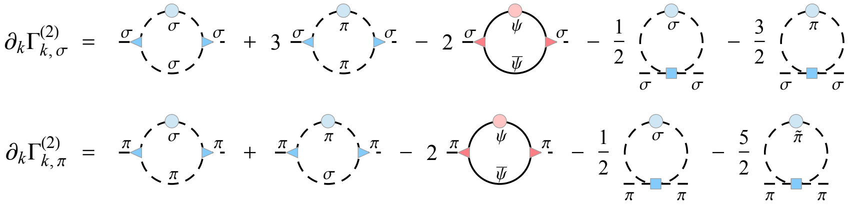
R.-A. T., B.-J. Schaefer, L. von Smekal, J. Wambach
in preparation

regime of pion condensation (FRG)



R.-A. T., B.-J. Schaefer, L. von Smekal, J. Wambach
in preparation

Flow equations for two-point functions

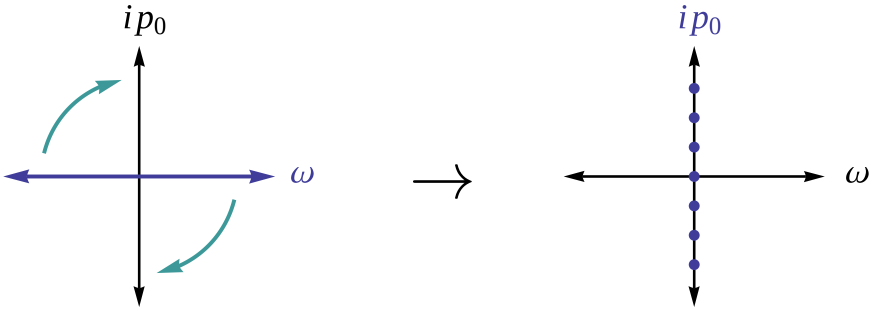


- quark-meson vertices are given by $\Gamma^{(3)}_{\bar{\psi}\psi\sigma} = h,\ \Gamma^{(3)}_{\bar{\psi}\psi\vec{\pi}} = ih\gamma^5\vec{\tau}$
- mesonic vertices from scale-dependent effective potential: $U^{(3)}_{k,\phi_i\phi_j\phi_m}$, $U^{(4)}_{k,\phi_i\phi_j\phi_m\phi_n}$
- one-loop structure and 3D regulators allow for a simple analytic continuation!

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. Rev. **D 90**, 074031 (2014)]

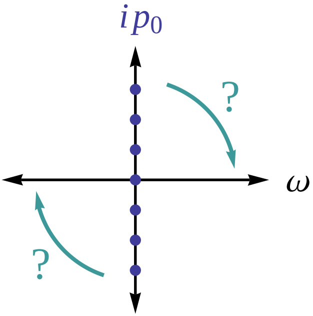
The analytic continuation problem

Calculations at finite temperature are often performed using imaginary energies:



The analytic continuation problem

Analytic continuation problem: How to get back to real energies?



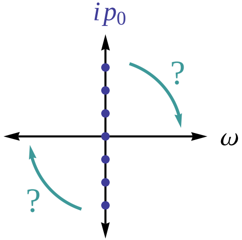
Two-step analytic continuation procedure

1) Use periodicity in external imaginary energy $ip_0 = i2n\pi T$:

$$n_{B,F}(E + ip_0) \to n_{B,F}(E)$$

2) Substitute p_0 by continuous real frequency ω:

$$\Gamma^{(2),R}(\omega, \vec{p}) = -\lim_{\epsilon\to 0} \Gamma^{(2),E}(ip_0 \to -\omega - i\epsilon, \vec{p})$$

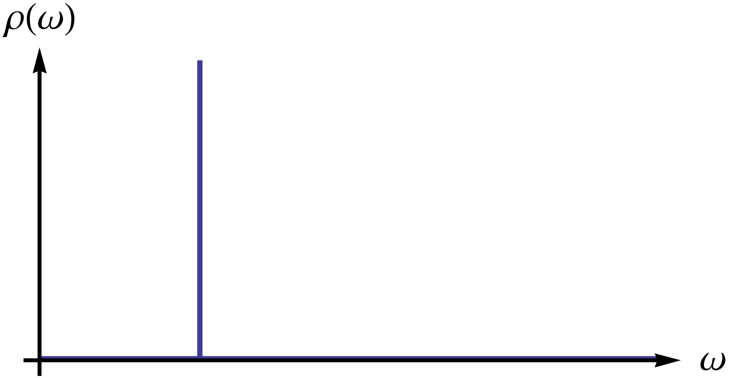


Spectral function is then given by

$$\rho(\omega, \vec{p}) = -\mathrm{Im}(1/\Gamma^{(2),R}(\omega, \vec{p}))/\pi$$

[R.-A. T., N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. **D 89**, 034010 (2014)]
[J. M. Pawlowski, N. Strodthoff, Phys. Rev. **D 92**, 094009 (2015)]
[N. Landsman and C. v. Weert, Physics Reports 145, 3&4 (1987) 141]

What is a spectral function?



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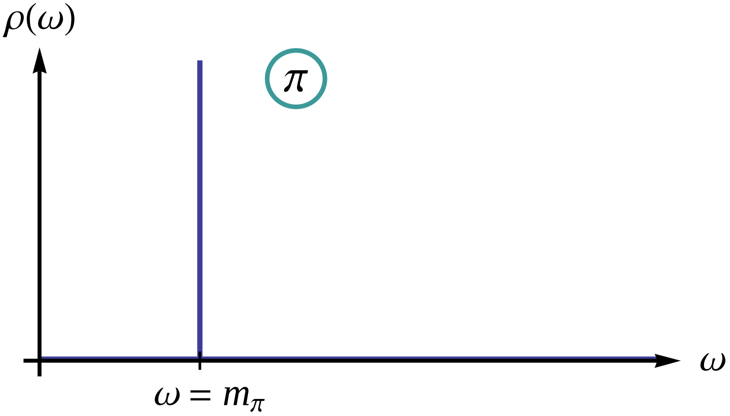

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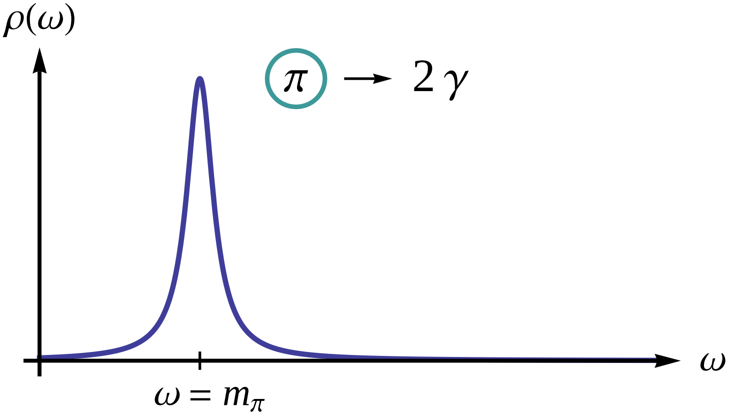

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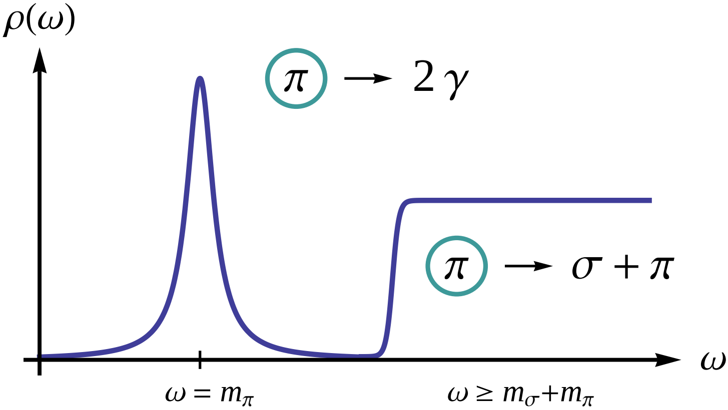


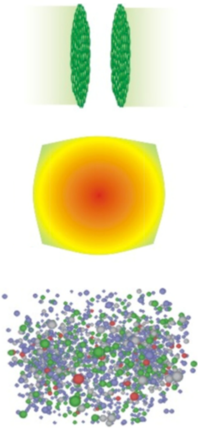
Why are spectral functions interesting?

Spectral functions determine both real-time and imaginary-time propagators,

- $$D^R(\omega) = -\int d\omega' \frac{\rho(\omega')}{\omega' - \omega - i\varepsilon}$$
- $$D^A(\omega) = -\int d\omega' \frac{\rho(\omega')}{\omega' - \omega + i\varepsilon}$$
- $$D^E(p_0) = \int d\omega' \frac{\rho(\omega')}{\omega' + ip_0}$$

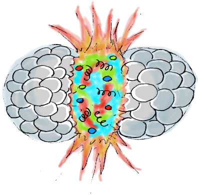
Spectral functions allow access to many observables, e.g. transport coefficients like the shear viscosity:

- $$\eta = \frac{1}{24} \lim_{\omega \to 0} \lim_{|\vec{p}| \to 0} \frac{1}{\omega} \int d^4x \; e^{ipx} \left\langle \left[T_{ij}(x), T^{ij}(0) \right] \right\rangle$$



[B. Mueller, arXiv: 1309.7616]

II) Results



[courtesy L. Holicki]

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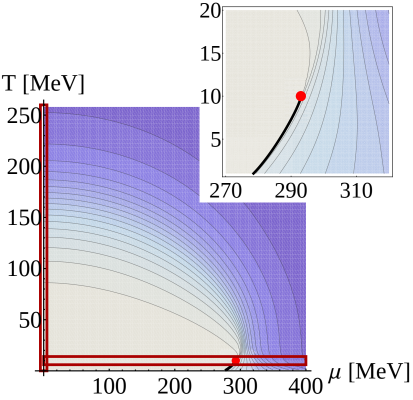
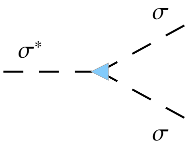
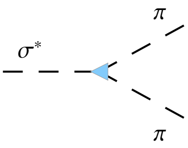
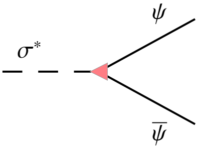


[R.-A. T., N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. **D 89**, 034010 (2014)]

Decay channels of the sigma mesons







$\sigma^* \to \sigma + \sigma$

$\sigma^* \to \pi + \pi$

$\sigma^* \to \psi + \overline{\psi}$

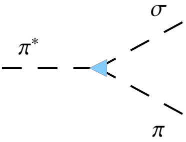
$\omega \geq \sqrt{(2m_\sigma)^2 + \vec{p}^2}$

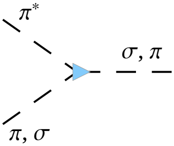
$\omega \geq \sqrt{(2m_\pi)^2 + \vec{p}^2}$

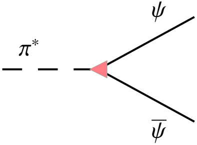
$\omega \geq \sqrt{(2m_\psi)^2 + \vec{p}^2}$

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. Rev. **D 90**, 074031 (2014)]

Decay channels of the pions







$$\pi^* \to \sigma + \pi$$

$$\pi^* + \pi \to \sigma$$

$$\pi^* + \sigma \to \pi$$

$$\pi^* \to \psi + \overline{\psi}$$

$$\omega \geq \sqrt{(m_\sigma + m_\pi)^2 + \vec{p}^{\,2}}$$

$$\omega \leq (m_\sigma - m_\pi)\sqrt{1 + \frac{\vec{p}^{\,2}}{\Delta m^2}}$$

$$\omega \leq (m_\pi - m_\sigma)\sqrt{1 + \frac{\vec{p}^{\,2}}{\Delta m^2}}$$

$$\omega \geq \sqrt{(2m_\psi)^2 + \vec{p}^{\,2}}$$

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. Rev. **D 90**, 074031 (2014)]

Flow of Sigma and Pion Spectral Function at $\mu = 0$, $T = 0$ and $\vec{p} = 0$

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Sigma and Pion Spectral Function with increasing T at $\mu = 0$ and $\vec{p} = 0$

(Loading movie...)

Sigma and Pion Spectral Function with increasing μ at $T \approx 10$ MeV and $\vec{p} = 0$

(Loading movie...)

Towards the shear viscosity

Applying the Green-Kubo formula for the shear viscosity

$$\eta = \frac{1}{24} \lim_{\omega \to 0} \lim_{|\vec{p}| \to 0} \frac{1}{\omega} \int d^4x \; e^{ipx} \left\langle \left[T_{ij}(x), T^{ij}(0) \right] \right\rangle$$

to the quark-meson model with energy-momentum tensor

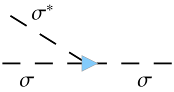
$$T^{ij}(x) = \frac{i}{2} \left(\overline{\psi} \gamma^i \partial^j \psi - \partial^j \, \overline{\psi} \gamma^i \psi \right) + \partial^j \sigma \partial^i \sigma + \partial^j \vec{\pi} \partial^i \vec{\pi}$$

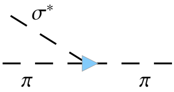
gives (dominant contribution)

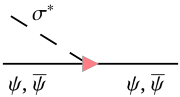
$$\eta_{\sigma,\pi} \propto \int \frac{d\omega}{2\pi} \int \frac{d^3p}{(2\pi)^3} \; p_x^2 \, p_y^2 \, n_B'(\omega) \, \rho_{\sigma,\pi}^2(\omega, \vec{p})$$

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. arXiv: 1605.00771]

Space-like processes of the sigma mesons







$\sigma^* + \sigma \to \sigma$

$\sigma^* + \pi \to \pi$

$\sigma^* + \psi \to \psi$
$\sigma^* + \overline{\psi} \to \overline{\psi}$

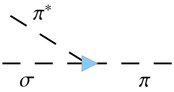
$0 \leq \omega \leq |\vec{p}|$

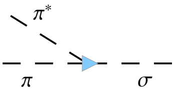
$0 \leq \omega \leq |\vec{p}|$

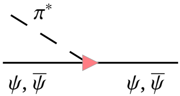
$0 \leq \omega \leq |\vec{p}|$

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. Rev. **D 90**, 074031 (2014)]

Space-like processes of the pions







$\pi^* + \sigma \to \pi$

$\pi^* + \pi \to \sigma$

$\pi^* + \psi \to \psi$

$\pi^* + \overline{\psi} \to \overline{\psi}$

$0 \leq \omega \leq |\vec{p}|$

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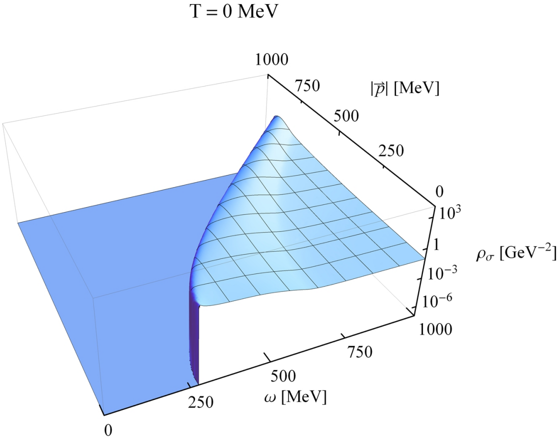
$0 \leq \omega \leq |\vec{p}|$

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. Rev. **D 90**, 074031 (2014)]

Sigma Spectral Function vs. ω and $\vec{p}$ at $\mu = 0$ and $T = 0$ MeV

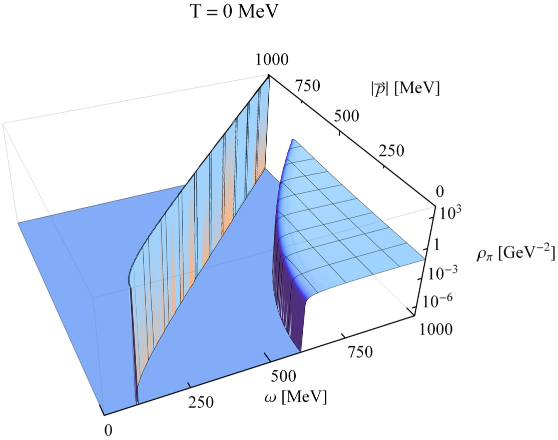
- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes



Pion Spectral Function

vs. ω and $\vec{p}$ at $\mu = 0$ and $T = 0$ MeV

- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- capture process $\pi^* + \pi \rightarrow \sigma$ is suppressed at large $\vec{p}$
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes



Sigma and Pion Spectral Function vs. ω and $\vec{p}$ at $\mu = 0$ and increasing T

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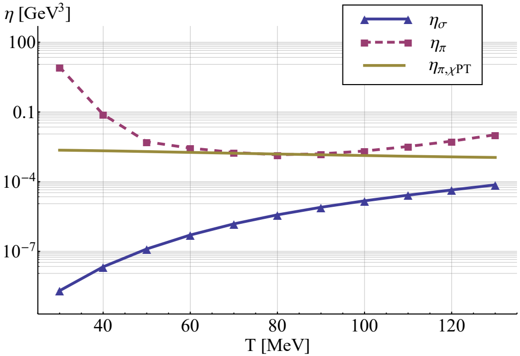
Shear viscosity at $\mu = 0$

- $\eta_{\pi,\chi\text{PT}}$: result from chiral perturbation theory

[Lang, Kaiser, Weise, EPJ **A 48**, 109 (2012)]

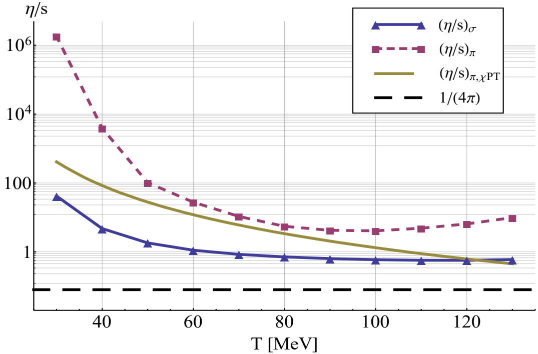
- large shear viscosity at low temperatures due to small width of the pion peak
$\rightarrow 4\pi$ processes missing



[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. arXiv: 1605.00771]

Shear viscosity over entropy density η/s at $\mu = 0$

- $\eta_{\pi,\chi\text{PT}}$: result from chiral perturbation theory
- large shear viscosity at low temperatures due to small width of pion peak $\rightarrow 4\pi$ processes missing
- η/s is always larger than the AdS/CFT limiting value of $\eta/s \geq 1/4\pi$



[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. arXiv: 1605.00771]

Summary and outlook

New method to obtain real-time quantities like spectral functions and transport coefficients at finite T and μ from the FRG:

- involves an analytic continuation from imaginary to real frequencies on the level of the flow equations
- thermodynamically consistent and symmetry-structure preserving
- feasibility of the method demonstrated by calculating meson spectral functions and η/s for the quark-meson model

Outlook:

- nucleon spectral function
- vector meson spectral functions