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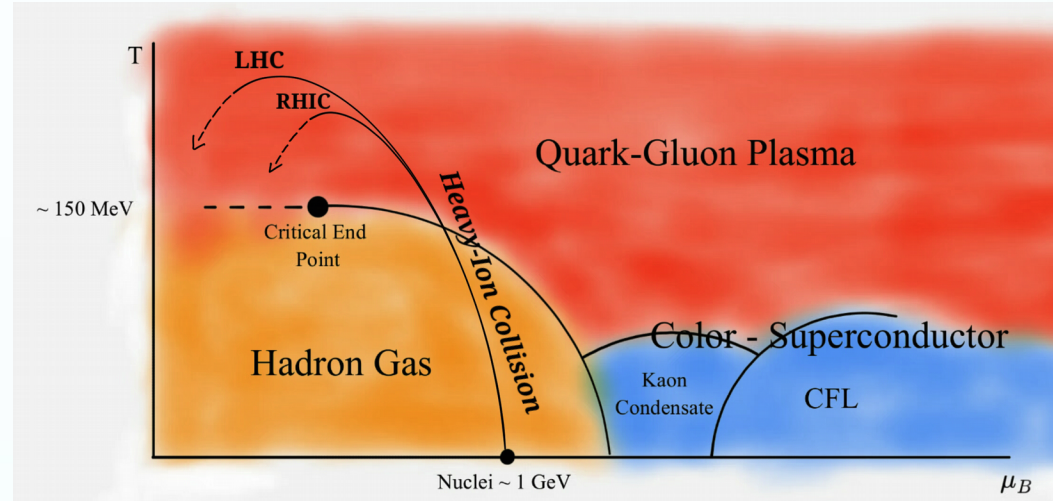
# Role of repulsive interactions in a comparison with IQCD

Erice, Italia, 17th September 2016

in collaboration with Volodymyr Vovchenko, Mark I. Gorenstein and Horst Stoecker

# The QCD phase diagram

Following the idea of asymptotic freedom, there are predictions of a new state of matter whose degrees of freedom are quarks and gluons, the so-called Quark-Gluon Plasma (QGP).



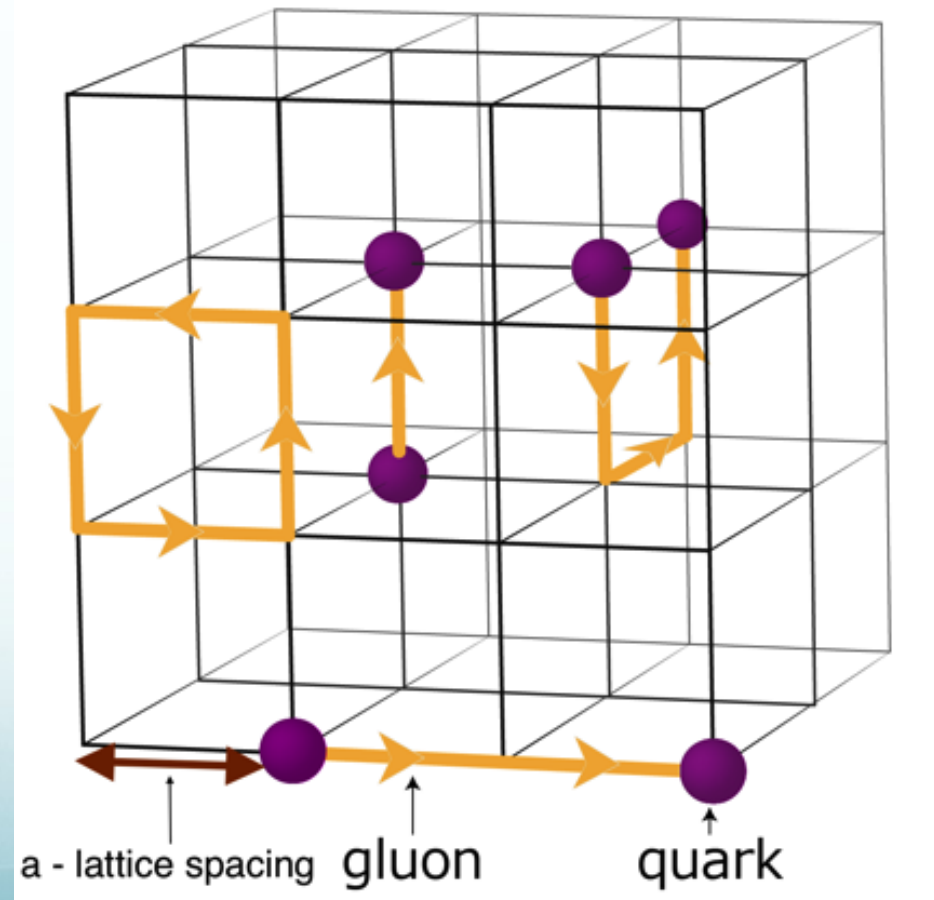
A detailed exploration of the QCD phase diagram is needed for a better understanding of the confinement mechanism which binds quarks into hadrons.

For this purpose we can use results from simulations of QCD on a lattice and from Heavy-Ion Collisions.

# Lattice QCD

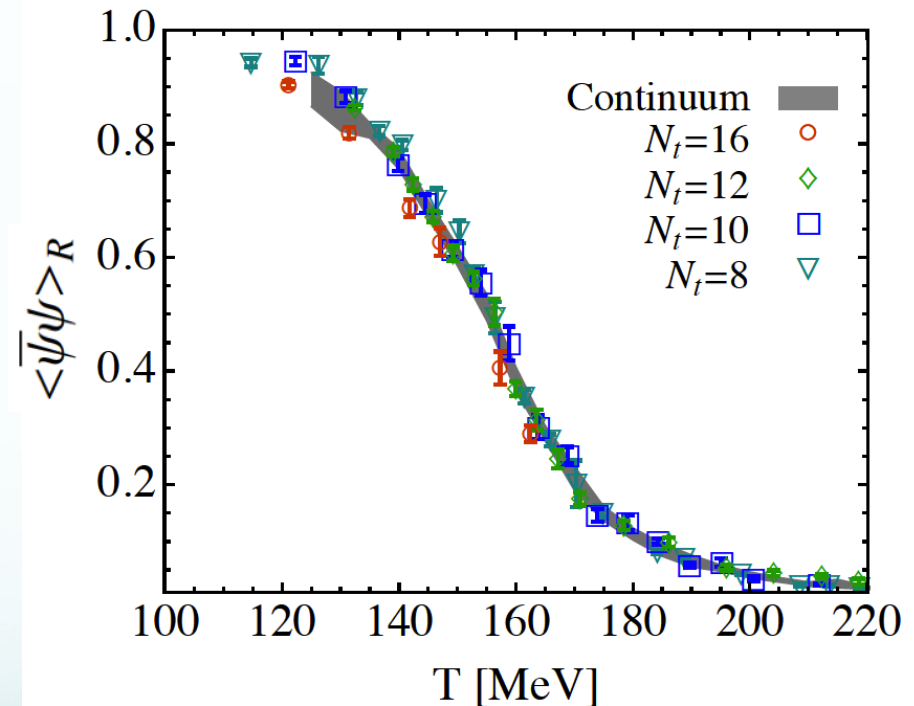
We can not solve QCD analytically in the non perturbative regime, but in the temperature and density regime of interest it is possible to simulate the theory numerically on a discretized spacetime.

Recently, these simulations have reached great precision and this allowed us to obtain very important results about the QCD.



# Results from Lattice QCD

- In the pure gauge sector the confinement is a first order transition, and the Polyakov loop is the corresponding order parameter.
- In the presence of dynamical quarks the transition turns into a smooth crossover.
- The temperature for the transition at  $\mu_B=0$  is about 150 MeV.
- We know the Equation of State of QCD with very high accuracy.
- Even for very high temperatures the QGP is still strongly interacting.

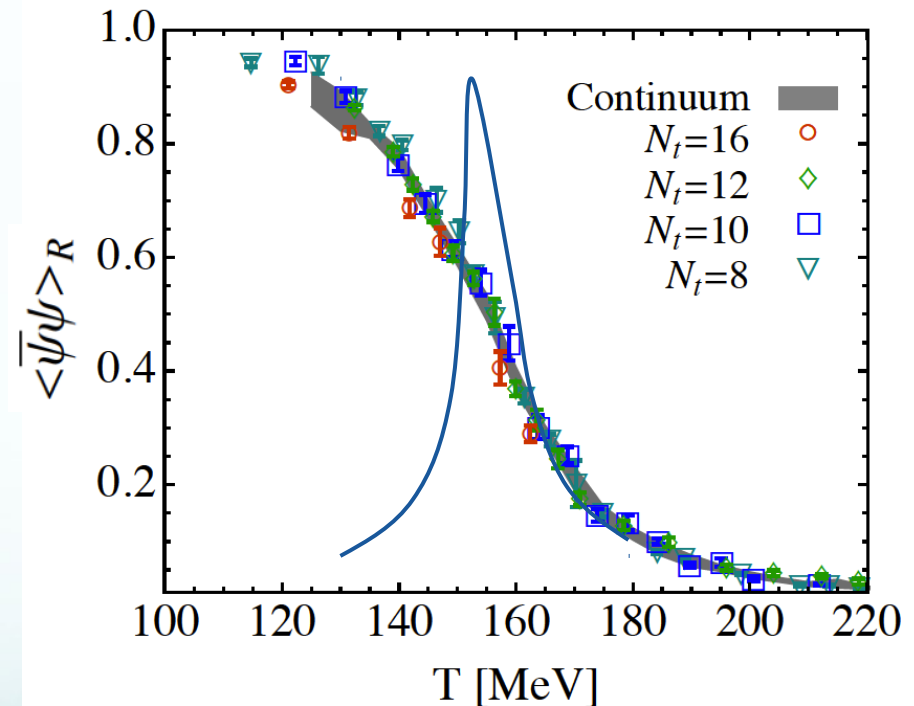


S. Borsanyi et al., JHEP 1009 (2010) 073.



# Results from Lattice QCD

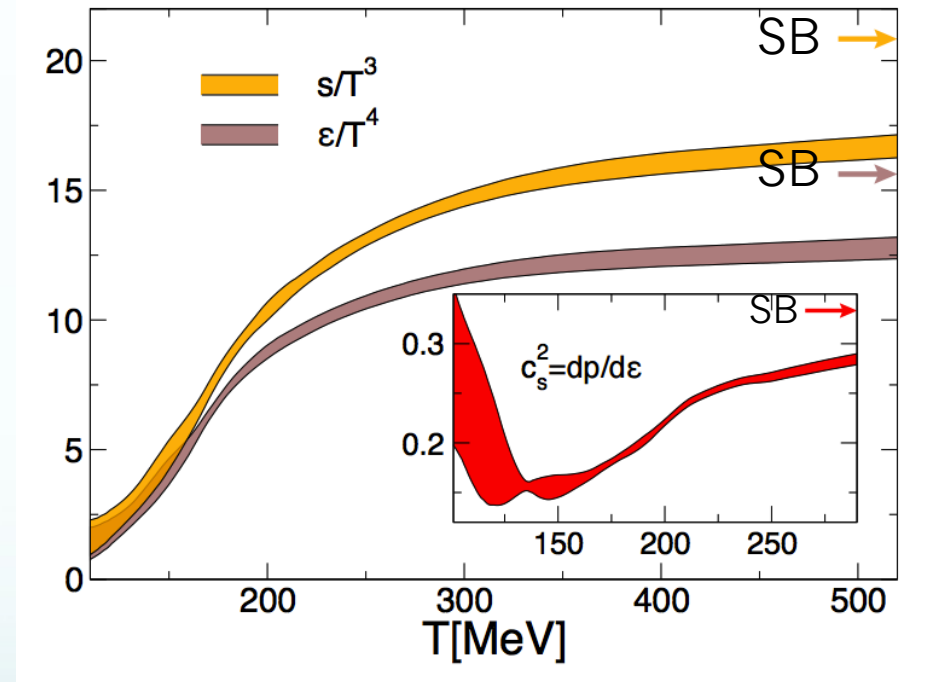
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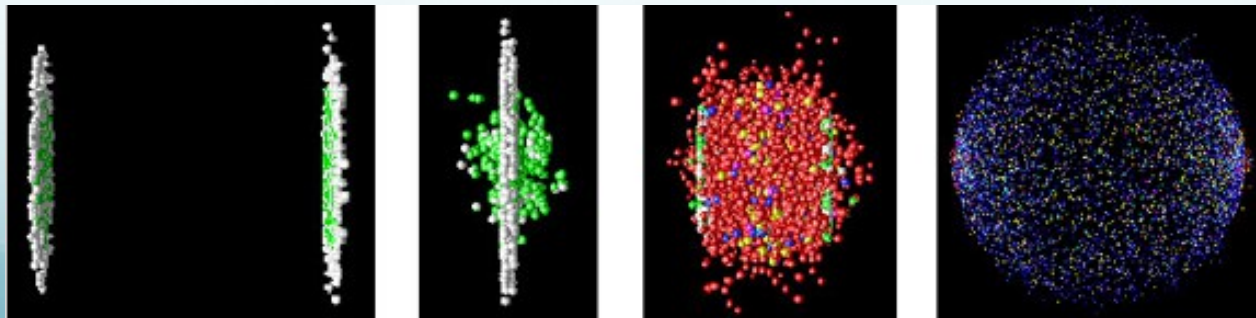
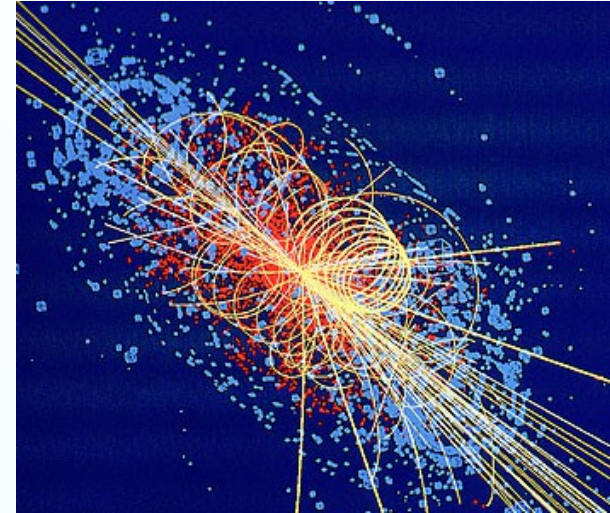


S. Borsanyi et al., Phys. Lett. B730 (2014) 99-104

# QGP in the laboratory

Experimentalists investigate the strong force via collisions between particles, starting from elementary collisions ( $e^+e^-$ ,  $pp$ ,  $p\bar{p}$ ), up to the recent Heavy-Ion collisions (HICs) performed at RHIC and LHC facilities.

The high multiplicity and energy achieved in HICs allowed us to get the first insights for the QGP formation.



# Link Theory to Experiment

The high quality of present results allows for the first time to compare several observables obtained both theoretically and experimentally, which are however affected by different constraints.

Experimental constraints:

- Finite kinematic acceptance of detectors ( $p_T$ , rapidity, etc.).
- Finite efficiency of detecting particles.

Lattice QCD constraints:

- Results in chemical and thermal equilibrium, for full acceptance.
- Sign problem: results just for vanishing or small chemical potentials.



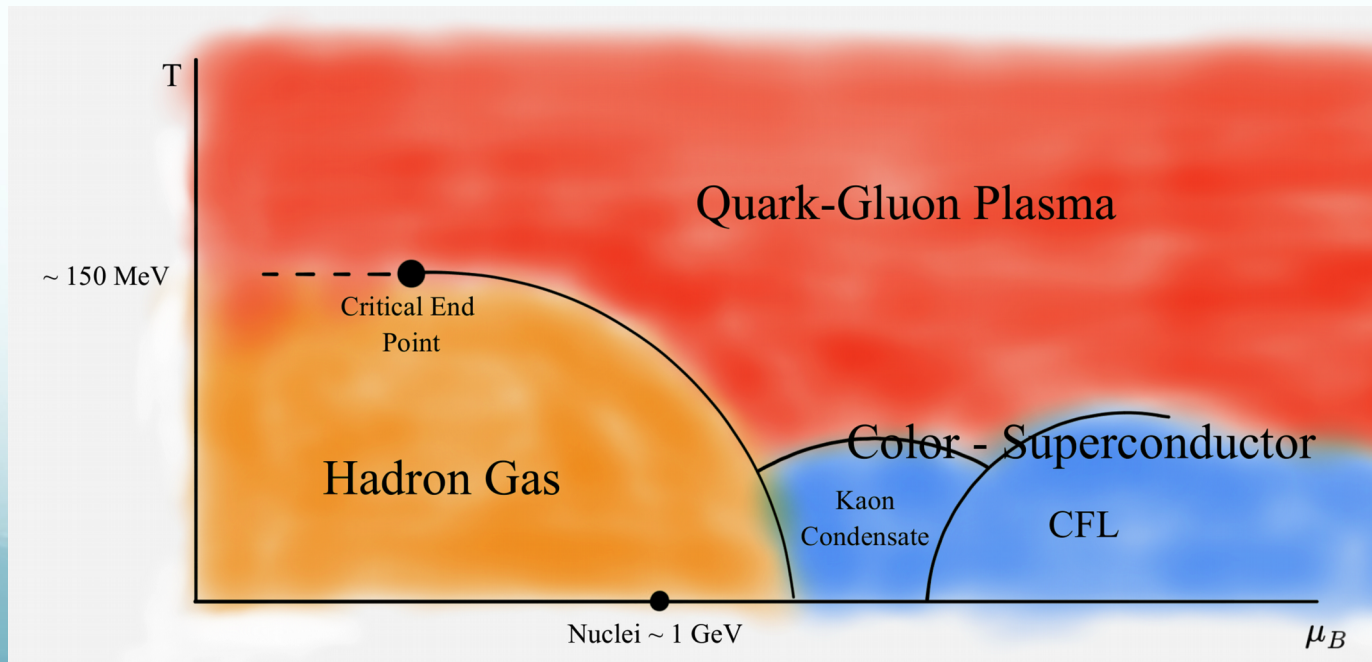
## Effective theories

They provide the link between experiment and theory, adapting to the different constraints. They can provide interesting predictions too.

# Phenomenological tools

There are different tools to analyze the different regions of the QCD phase diagram, e.g. :

- A **Quasi Particle** approach to analyze the high temperature regime down to the confinement transition.
- The **Hadron Resonance Gas model** in order to analyze the low temperature region of the phase diagram.

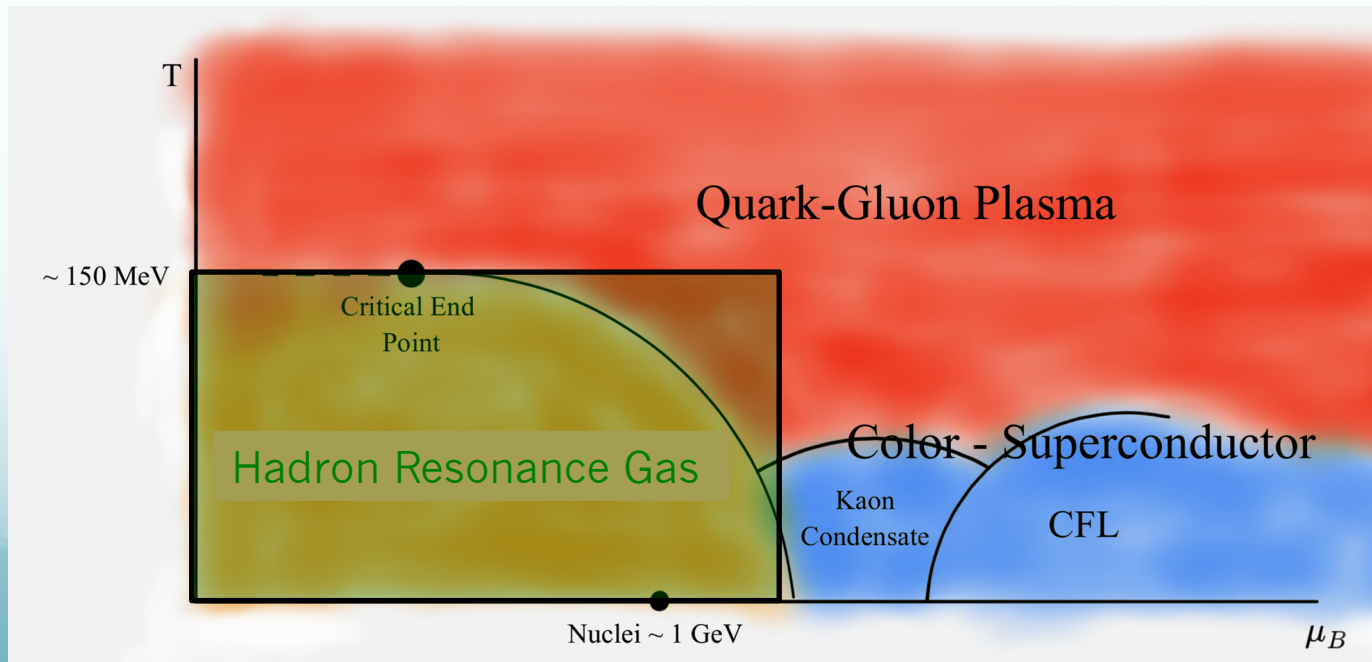




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# Hadron-Resonance Gas model

The hadronic phase (blue and green areas) can be studied by means of the Hadron-Resonance Gas (HRG) model, where resonance formation and subsequent decay mediate the attractive interactions among hadrons in the ground state.

$$p(T, \{\mu_k\}) = \sum_k (-1)^{B_k+1} \frac{d_k T}{(2\pi)^3} \int d^3\vec{p} \ln \left[ 1 + (-1)^{B_k+1} e^{-(\sqrt{\vec{p}^2 + m_k^2} - \mu_k)/T} \right]$$

$B_k$  = baryon number

$d_k$  = spin degeneracy

$m_k$  = mass

$$\mu_k = B_k \mu_B + Q_k \mu_Q + S_k \mu_S$$

Particle properties are listed by the [Particle Data Group \(PDG\)](#), which updates the list every year with the latest measured spectrum.



# Evolution of the collision in space and time

After the collision the evolution of the fireball could be the following:

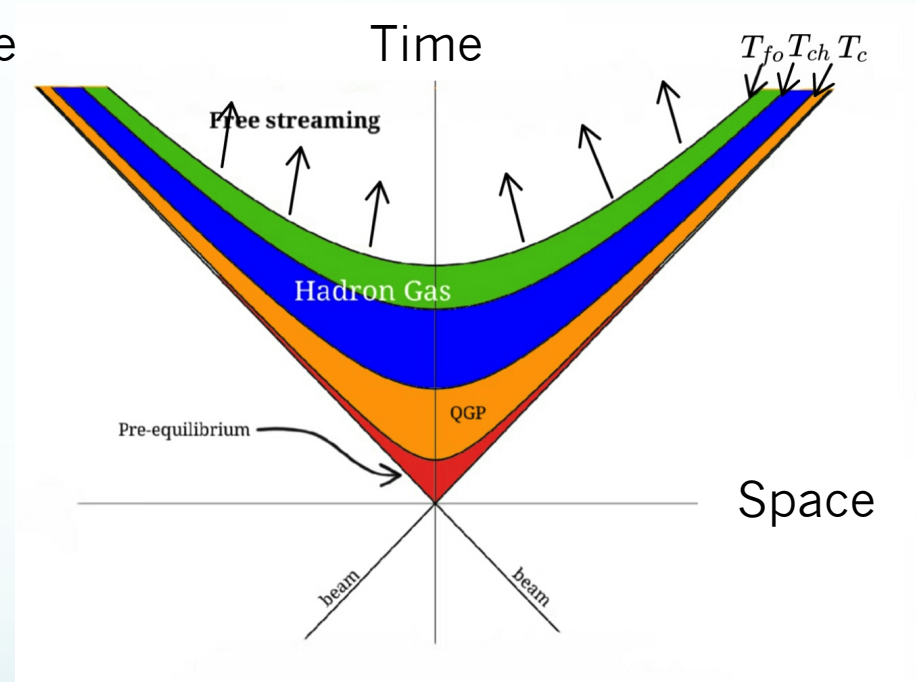
- A pre-equilibrium stage in which there is a lot of interaction driving quickly the system toward the QGP phase.

- The QGP phase in which quarks and gluons are thermalized.

- Then the particles bind into hadrons (Hadronization),  $T=T_c$ .

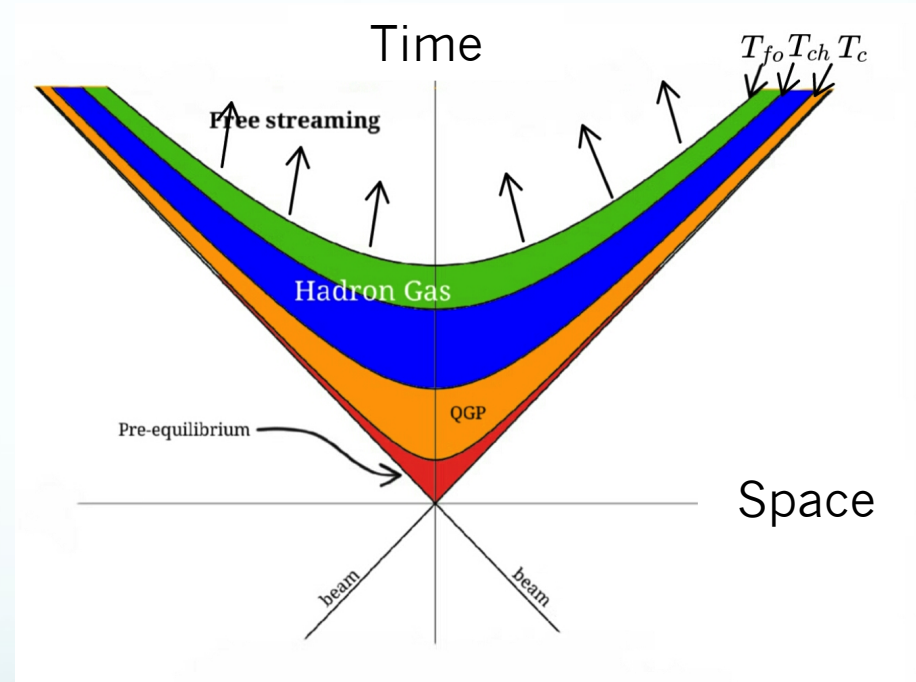
- Chemical freeze-out: the inelastic scatterings between hadrons stop, only the long-lived hadrons propagate, anything else decays,  $T=T_{ch}$ .

- Kinetic freeze-out: the elastic scatterings stop, and the hadrons reach the detector,  $T=T_{fo}$ .



# Evolution of the collision in space and time

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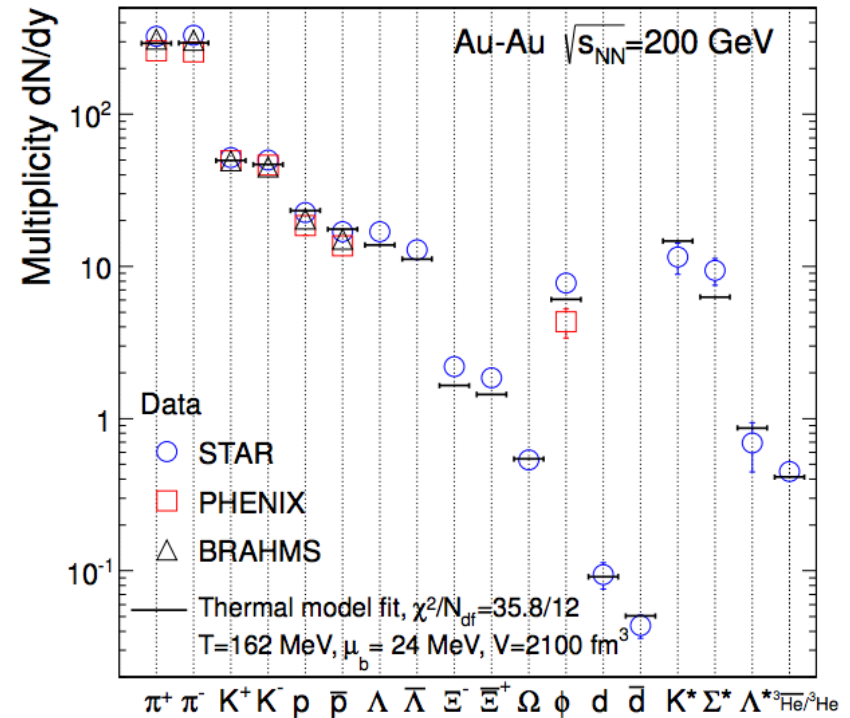


# Freeze-Out parameters from HRG

Andronic et al., Nucl.Phys. A904-905 (2013) 535c-538c

The abundancies of stable hadrons are fixed at the chemical FO. Through a thermal fit of these observables it was possible to extract  $T$  and  $\mu_B$ .

$$n_k = \frac{d_k}{(2\pi)^3} \int d^3p \frac{1}{e^{(\omega_k - \mu_k)/T} \pm 1}$$

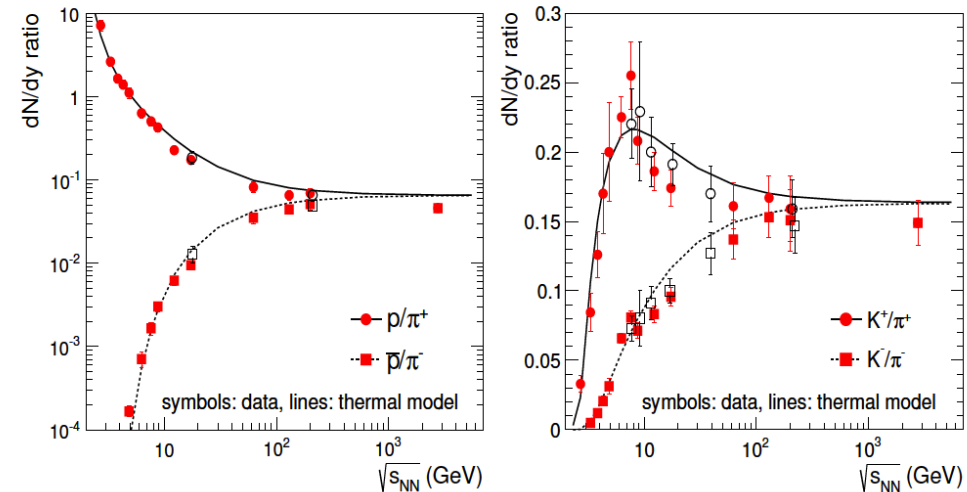


More recently, fluctuations of conserved charges have been proposed for the same purpose. The main reason is that these observables can be simulated on the lattice.

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A. Andronic, Int. J. Mod. Phys. A 29 (2014) 1430047

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# Fluctuations $\leftrightarrow$ Moments

Starting from a given partition function we define the fluctuations of a set of conserved charges as:

$$\frac{p}{T^4} = \frac{\ln \mathcal{Z}}{VT^3}$$

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} (p/T^4)}{\partial (\mu_B/T)^l \partial (\mu_S/T)^m \partial (\mu_Q/T)^n}$$

The fluctuations of conserved charges are related to the moments of the multiplicity distributions of the same charge measured in HICs.

$$\delta N = N - \langle N \rangle$$

mean:  $M = \langle N \rangle = VT^3 \chi_1,$

variance:  $\sigma^2 = \langle (\delta N)^2 \rangle = VT^3 \chi_2,$

skewness:  $S = \frac{\langle (\delta N)^3 \rangle}{\sigma^3} = \frac{VT^3 \chi_3}{(VT^3 \chi_2)^{3/2}},$

kurtosis:  $k = \frac{\langle (\delta N)^4 \rangle}{\sigma^4} - 3 = \frac{VT^3 \chi_4}{(VT^3 \chi_2)^2};$

# Fluctuations $\leftrightarrow$ Moments

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$$\frac{p}{T^4} = \frac{\ln \mathcal{Z}}{VT^3} \quad \chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} (p/T^4)}{\partial (\mu_B/T)^l \partial (\mu_S/T)^m \partial (\mu_Q/T)^n}$$

Taking ratios of these fluctuations we obtain simple quantities related to the moments of the distributions, avoiding any volume dependence.

$$\sigma^2/M = \chi_2/\chi_1$$

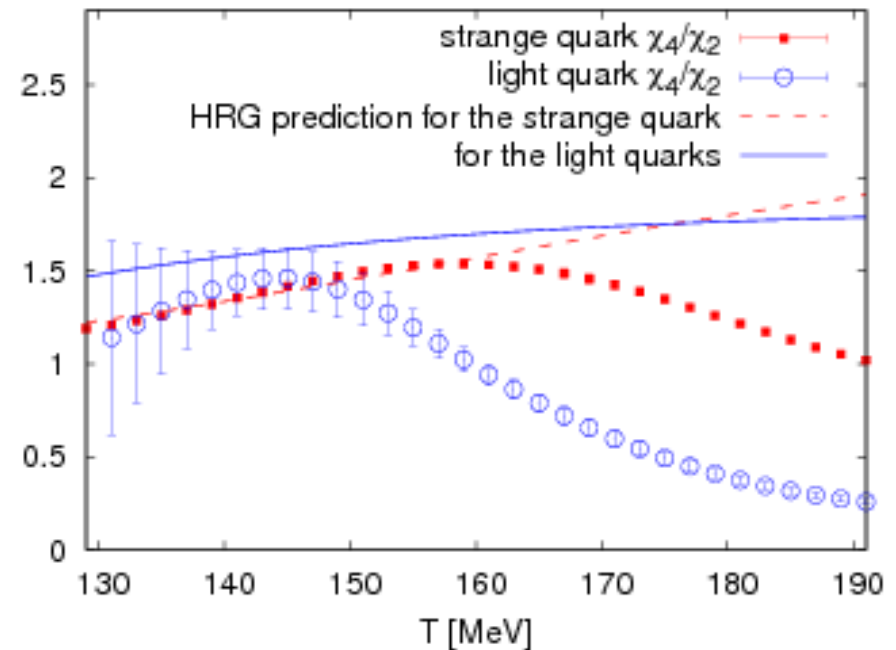
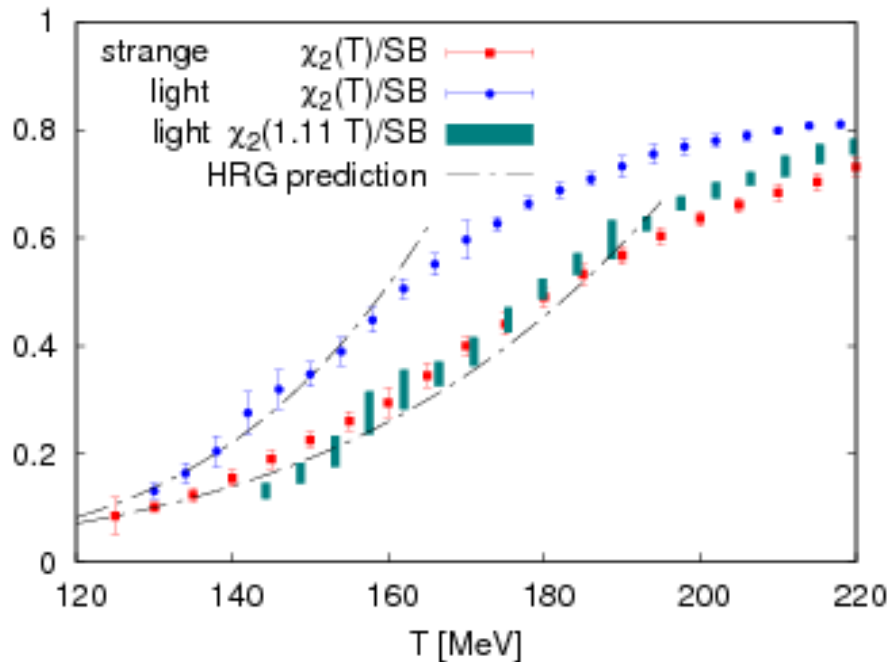
$$k\sigma^2 = \chi_4/\chi_2$$

$$S\sigma = \chi_3/\chi_2$$

$$S\sigma^3/M = \chi_3/\chi_1$$

# Fluctuations from Lattice

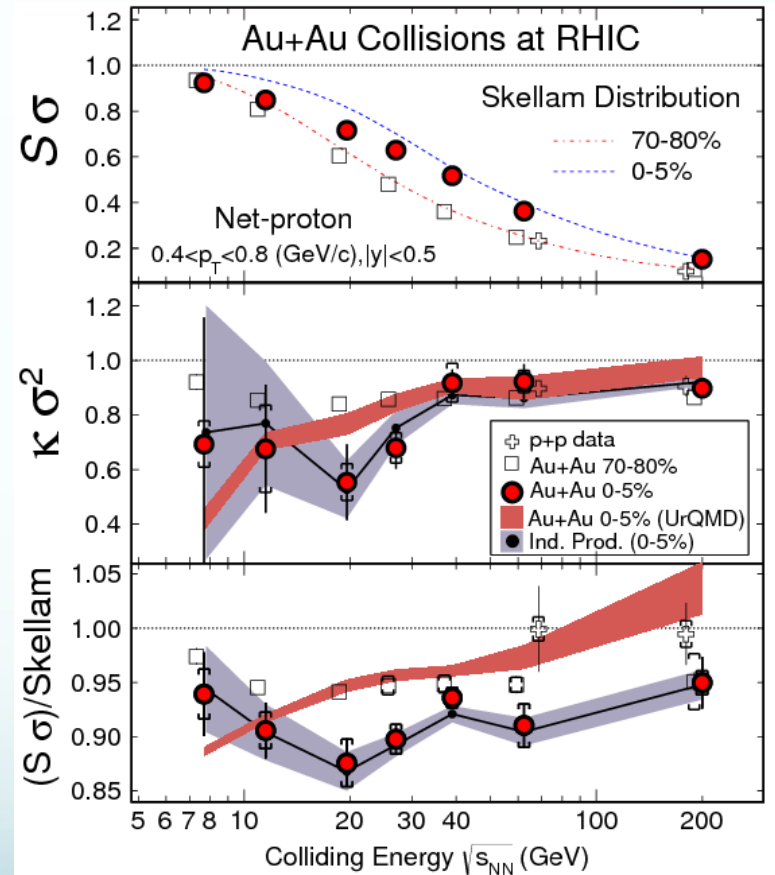
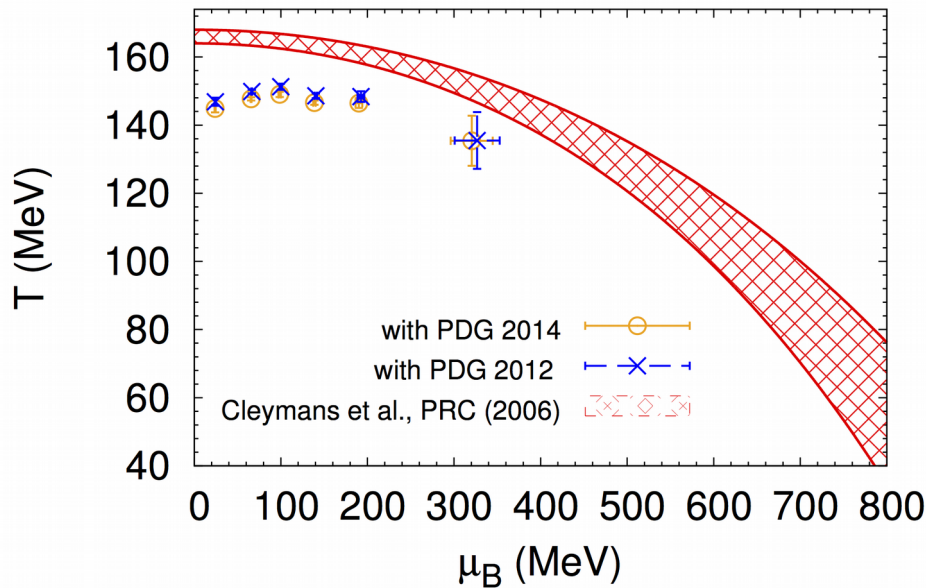
These quantities can be calculated by means of first principles simulations on the lattice, showing a remarkable agreement with HRG model predictions.





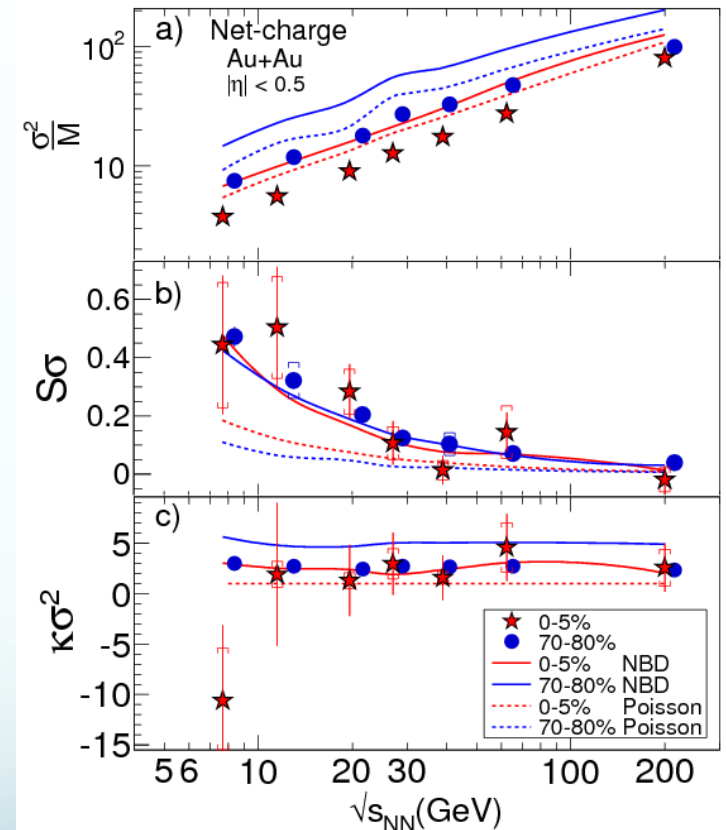
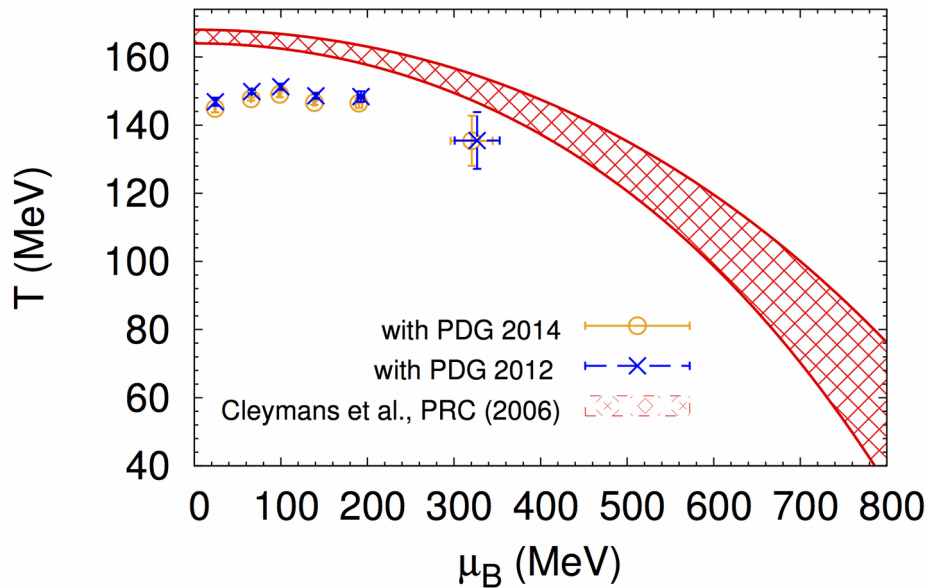
# Moments from HICs

Higher moments, for different quantum number distributions, have been measured at RHIC and analyzed within the HRG framework.



# Moments from HICs

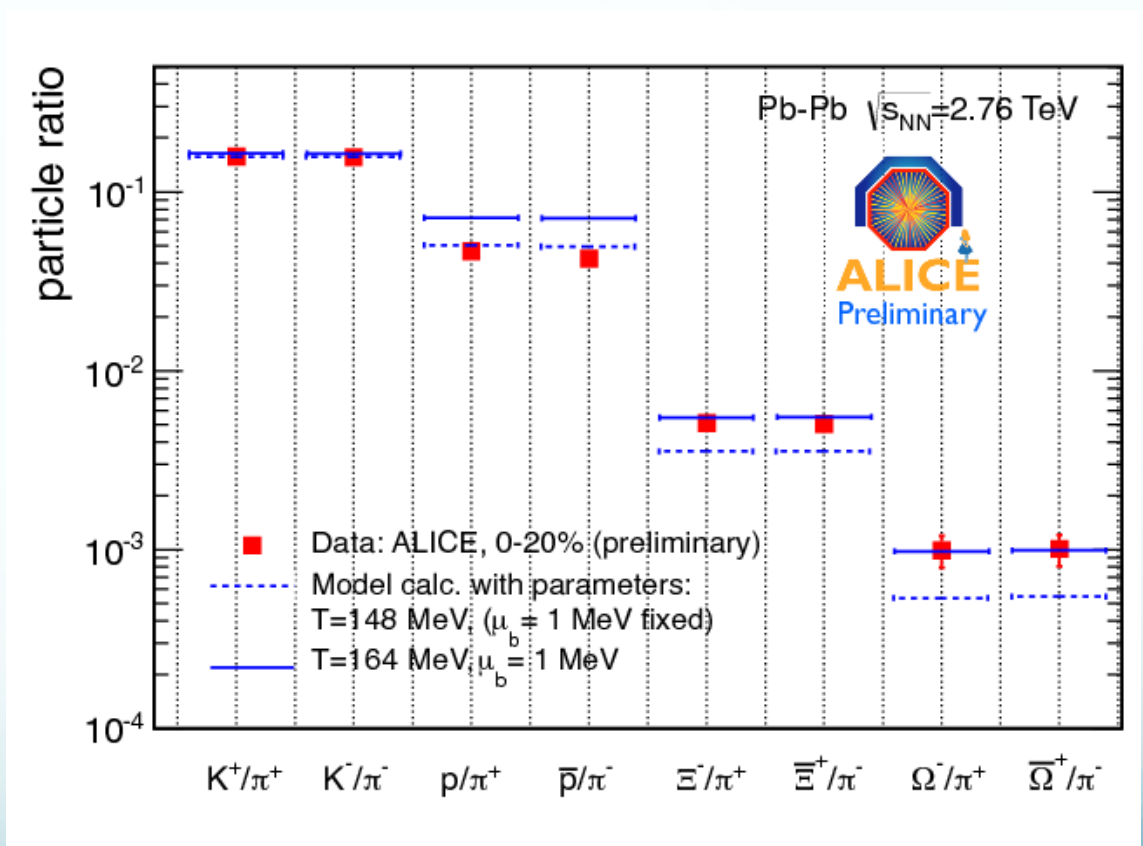
Higher moments, for different quantum number distributions, have been measured at RHIC and analyzed within the HRG framework.



# Is there a Flavour Hierarchy ?

With respect to previous results our temperature is about 20 MeV lower for the highest energy.

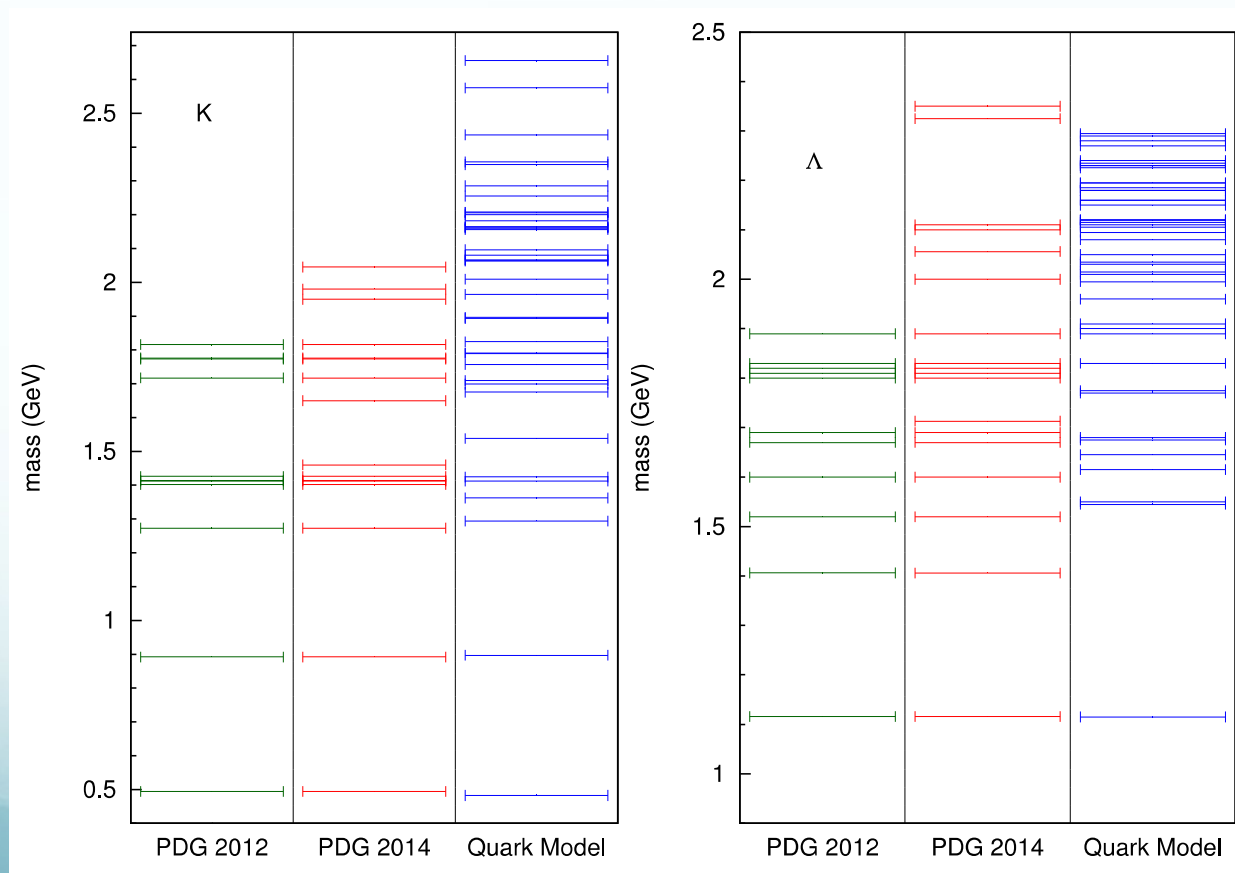
The difference may rely on a bigger FO temperature for strange particles with respect to light ones, which would otherwise drive the global fit.



# Undetected states

One of the biggest sources of uncertainty in the HRG model is the particle list.

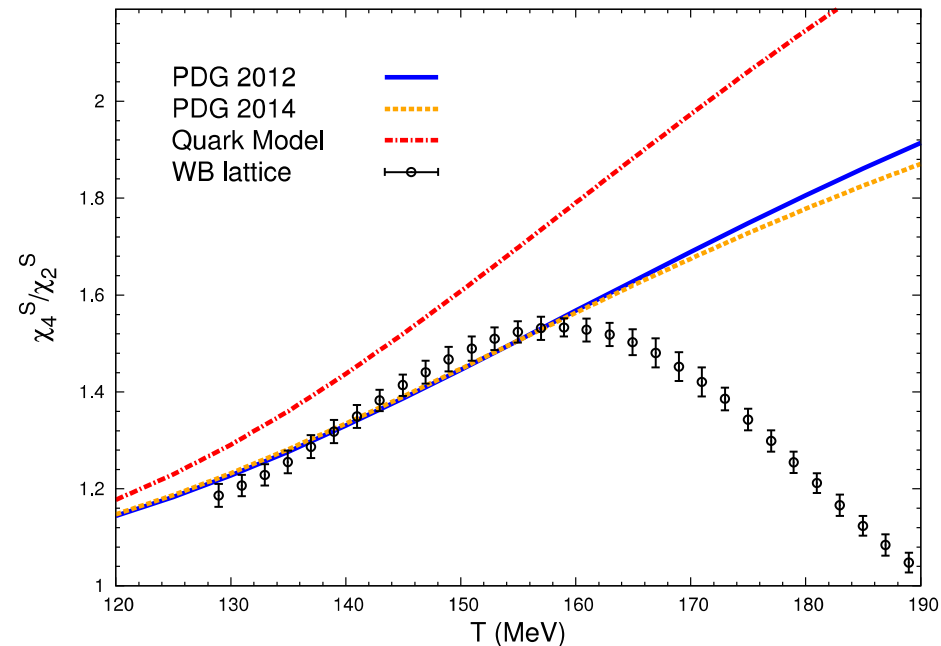
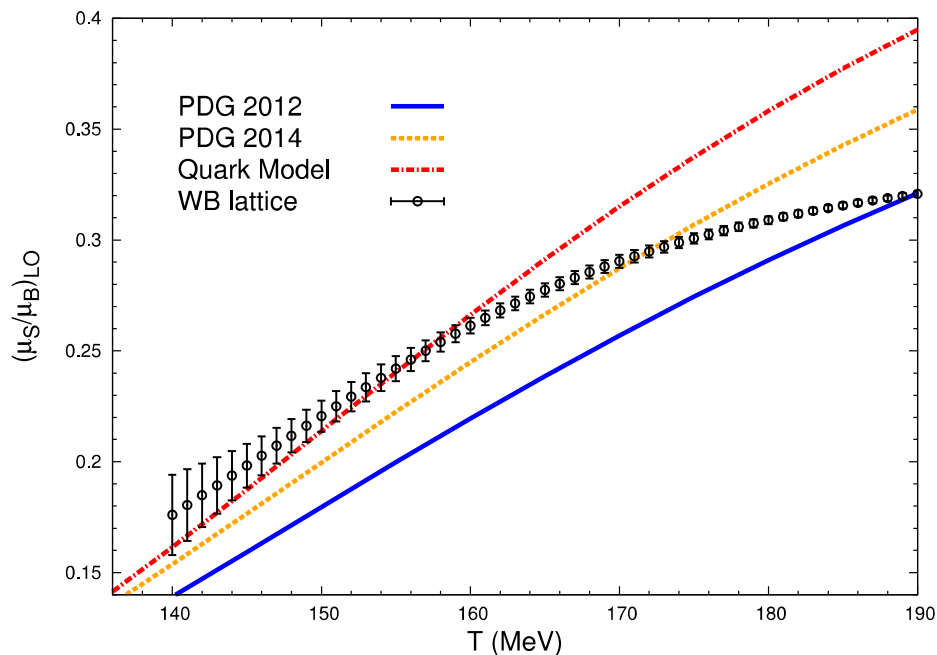
Predictions from the Quark Model may be relevant for key observables, especially in the strange sector.



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# Repulsive interactions

Repulsive interactions can be modeled in the HRG model with the introduction of an effective radius.

With this assumption particles have a finite size and occupy a portion of space which must be **Excluded** from the **Volume** of the system.

$$p_{\text{I}}(T, \vec{\mu}) = \sum_j p_j^{\text{id}}(T, \mu_j) \longrightarrow p(T, \vec{\mu}) = \sum_j p_j^{\text{id}}(T, \mu_j^*)$$

$$\mu_j^* = \mu_j - v_j p(T, \vec{\mu})$$

$$v_j = 16\pi r_j^3 / 3$$

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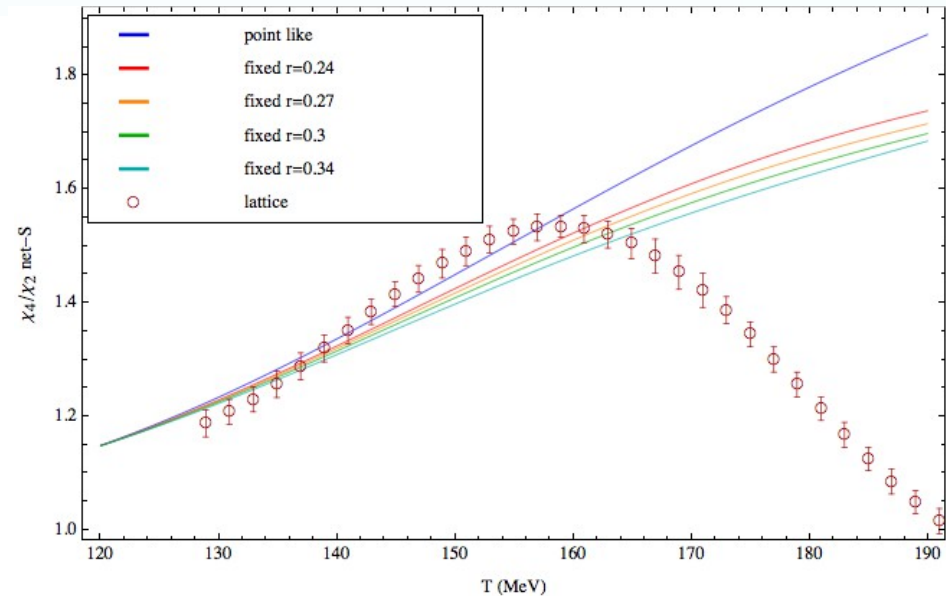
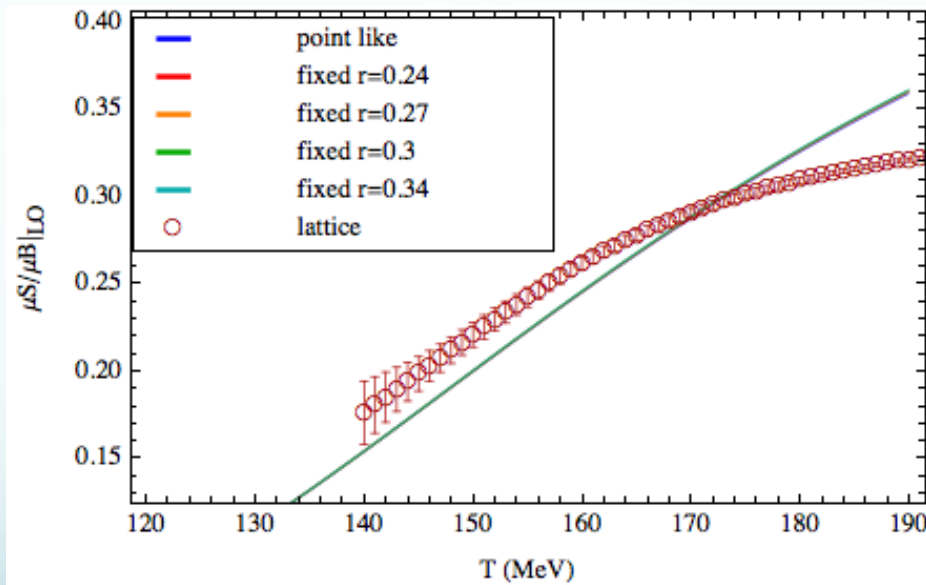
$$p_I(T, \vec{\mu}) = \sum_j p_j^{\text{id}}(T, \mu_j) \longrightarrow p(T, \vec{\mu}) = \sum_j p_j^{\text{id}}(T, \mu_j^*)$$

$$n_B(T, \vec{\mu}) = \left( \frac{\partial p}{\partial \mu_B} \right)_T = \frac{\sum_i b_i n_i^{\text{id}}(T, \mu_i^*)}{1 + \sum_j v_j n_j^{\text{id}}(T, \mu_j^*)}$$



# Excluded Volume

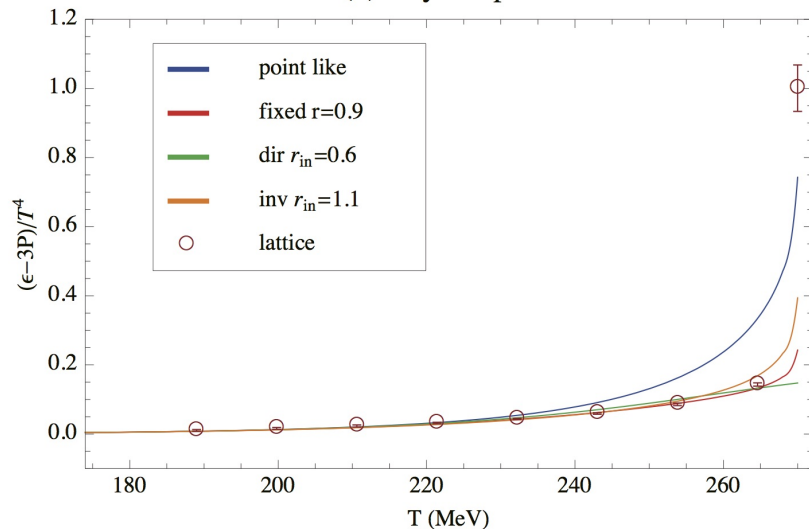
Hard core repulsions among particles may be relevant for key observables.



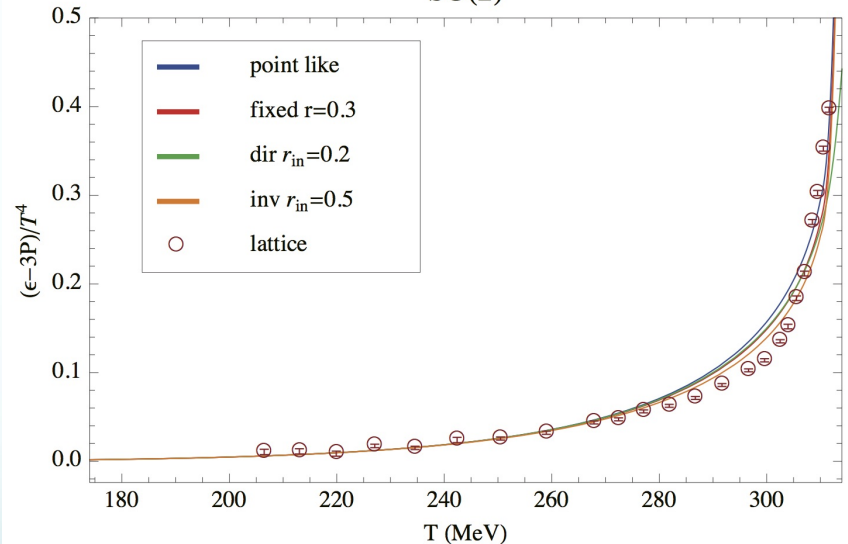
# Excluded Volume: Pure gauge

There are hints for particles with a finite size from the pure gauge sector.

SU(3) meyer-spectrum

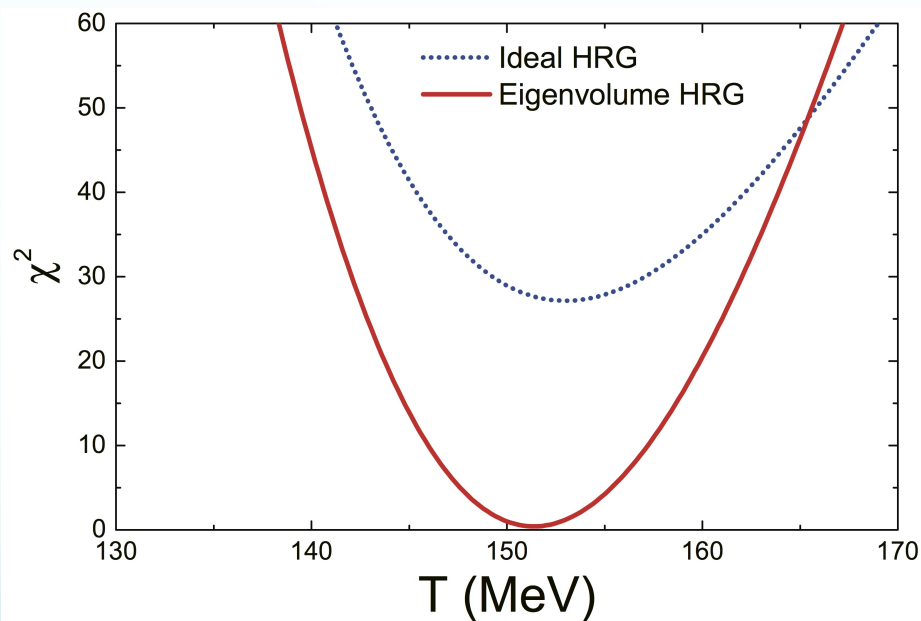


SU(2)



# Excluded Volume: particle yields

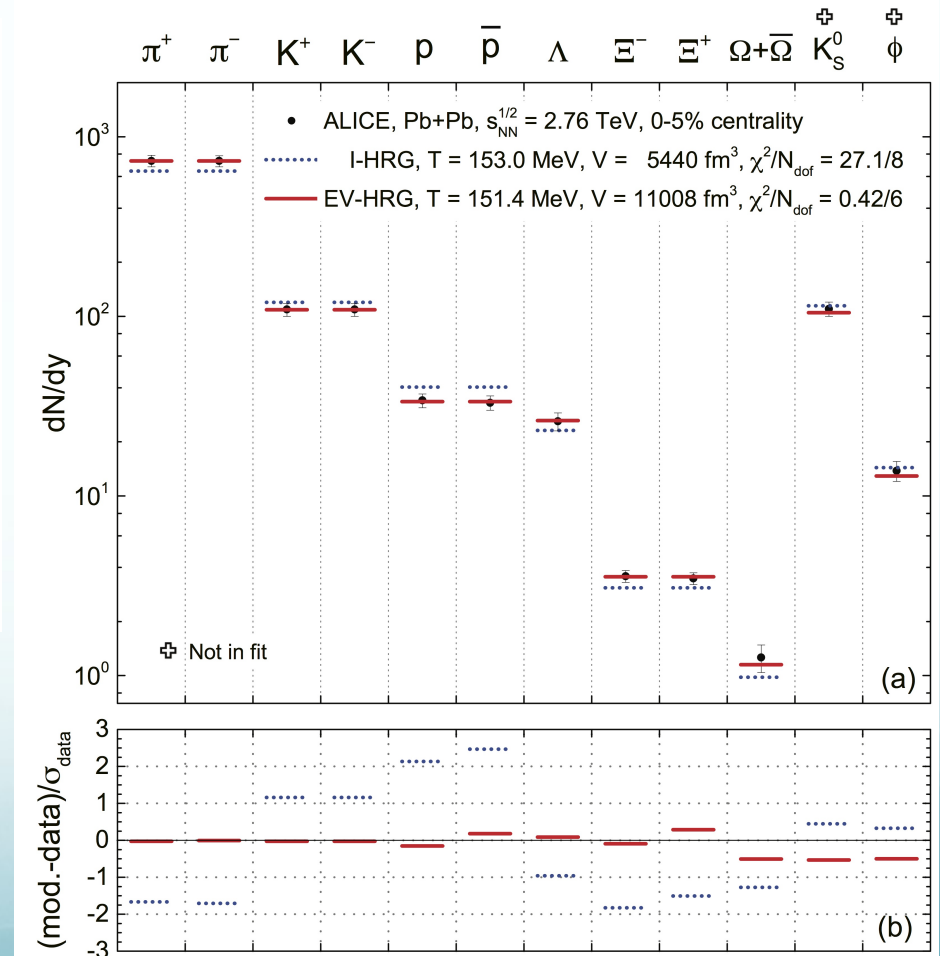
The use of a mass-dependent particle eigenvolume drastically improves the description of particle yields.



$$v_i = \alpha m_i$$

$$v_s = \gamma m_s^{-1}$$

P.A. et al., arxiv: hep-ph 1606.06542



# Excluded Volume: particle yields

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	$\chi^2/Ndf$ p.l.	$\chi^2/Ndf$	T (MeV) p.l.	T (MeV)
ALICE 0-5%	2.642537	0.0985746	152.576606	150.270412
ALICE 5-10%	4.038844	0.082681	153.855798	151.702161
ALICE 10-20%	4.831962	0.187238	156.912643	153.761281
ALICE 20-30%	5.779079	0.505264	156.269898	155.342295
ALICE 30-40%	5.290277	0.479082	156.606086	155.778665
ALICE 40-50%	4.320371	0.225175	156.901153	155.046625
ALICE 50-60%	2.528466	0.431904	153.374355	152.640780
ALICE 60-70%	2.522801	0.896884	148.338287	150.736294
ALICE 70-80%	2.480648	0.516741	150.701703	158.829787



S-inv Excluded Volume

# Excluded Volume: particle yields

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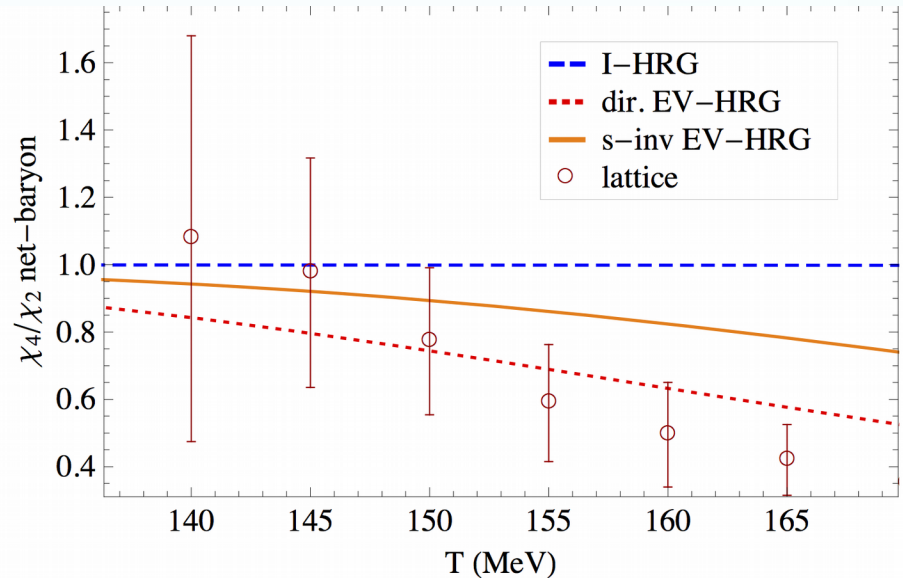
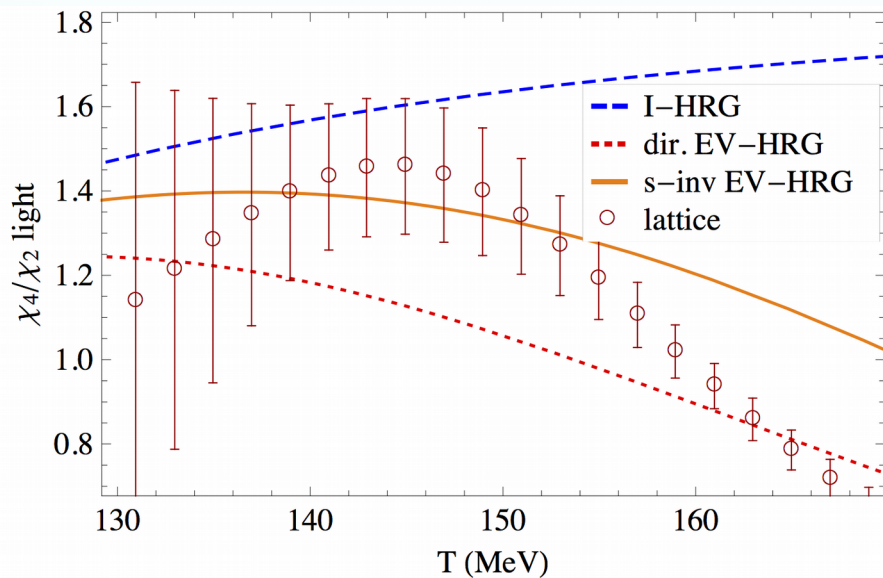
	$\chi^2/Ndf$ p.l.	$\chi^2/Ndf$	T (MeV) p.l.	T (MeV)
NA49 20GeV	5.868216	3.668726	106.448226	122.919464
NA49 30GeV	7.222598	1.269705	141.555846	136.454728
NA49 40GeV	8.077212	2.292649	139.293714	136.775614
NA49 80GeV	13.783130	4.812104	138.121797	141.917805
NA49 158GeV	5.329034	1.590537	146.535995	142.932057



S-inv Excluded Volume

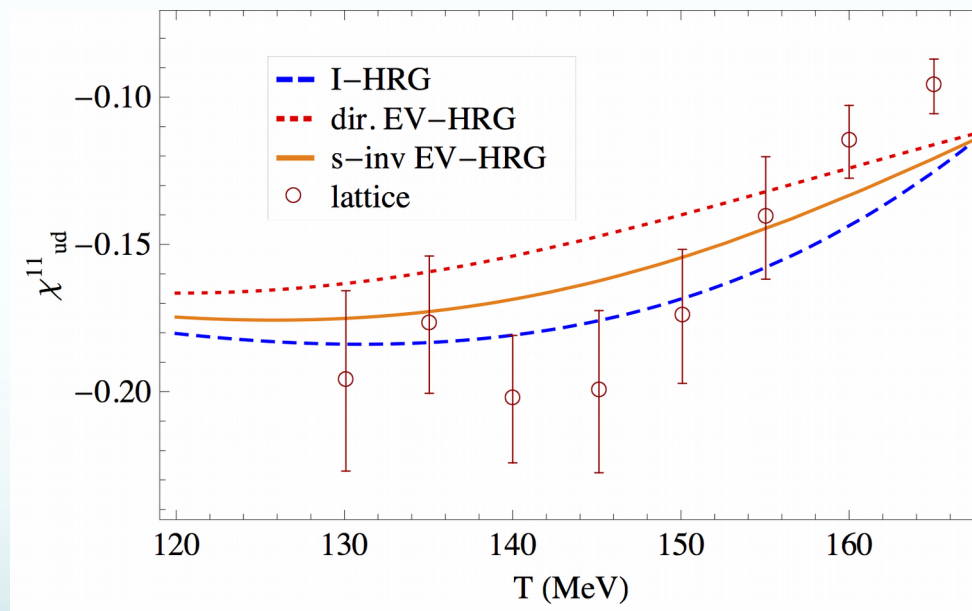
# Small strange states?

The use of an EV improves the description of lattice QCD results, and strongly suggests the presence of smaller strange particles.



# Small strange states?

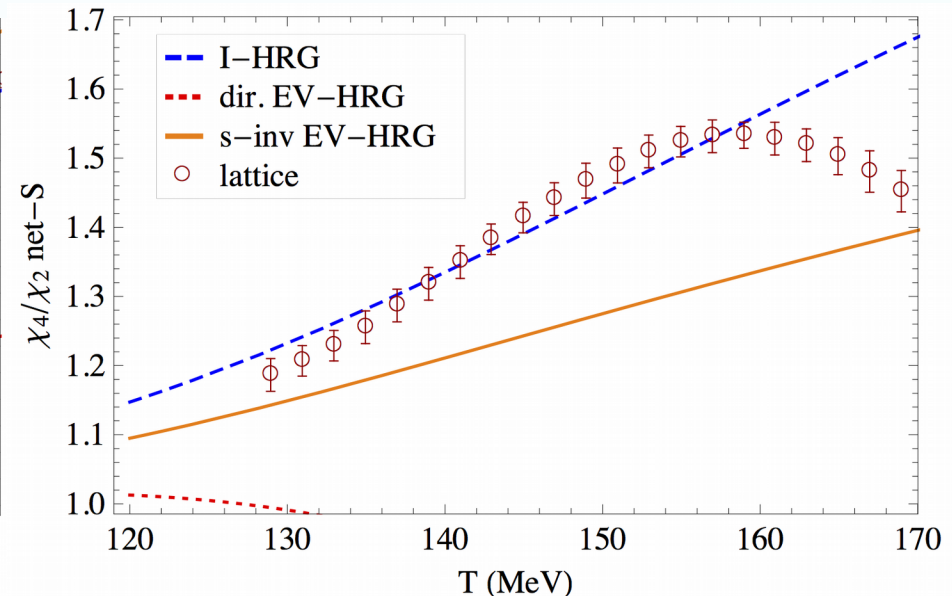
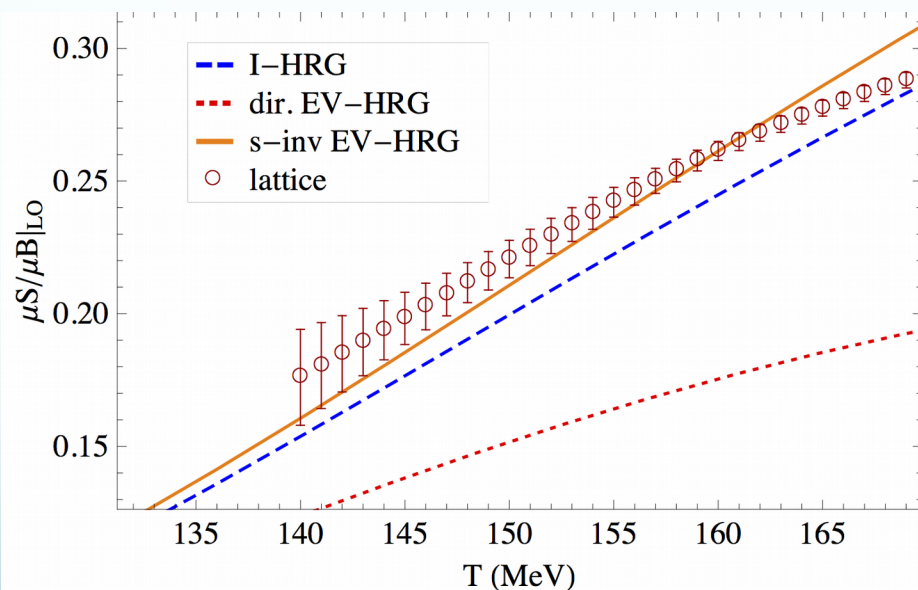
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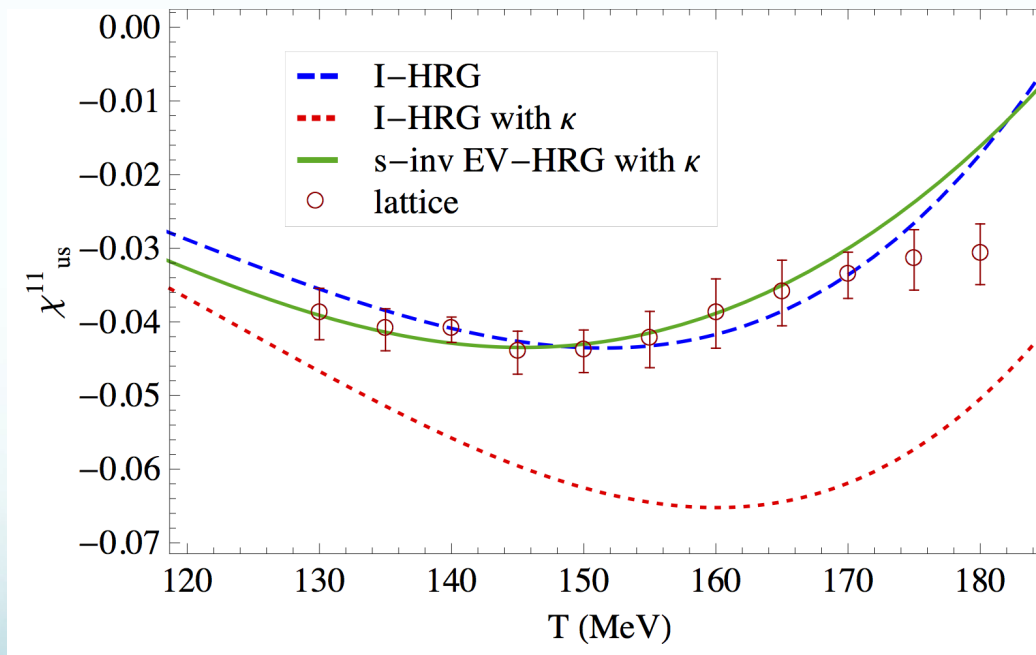
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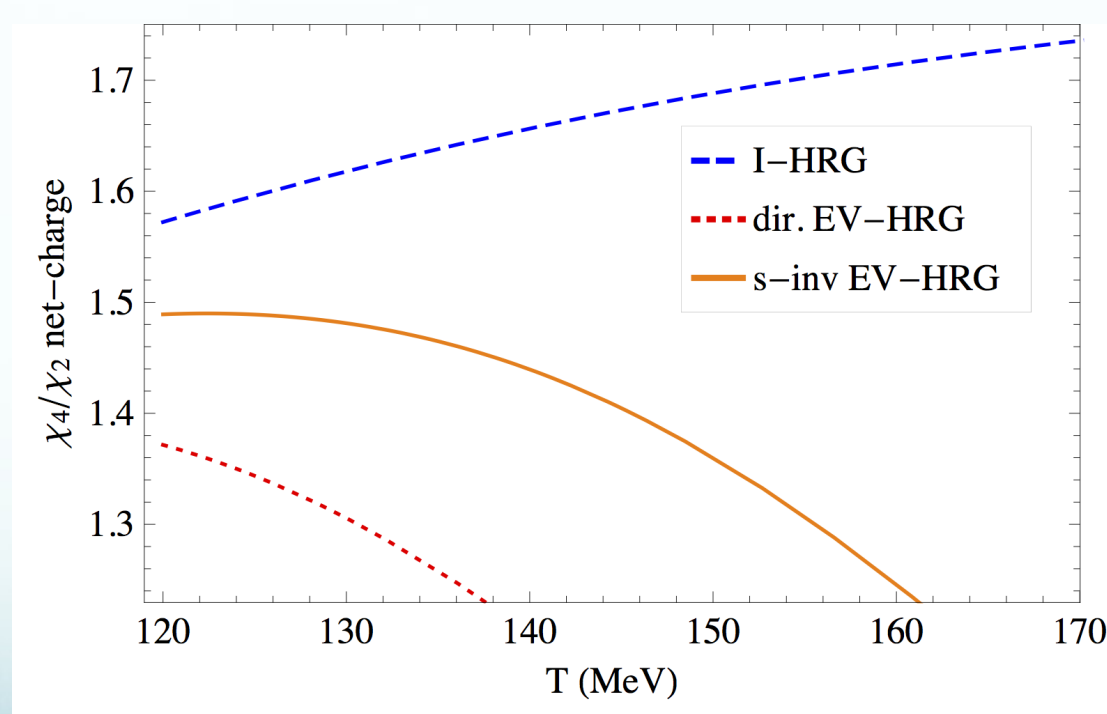
# EV vs exotic resonances

The repulsion accounted with the effective radii can counteract the presence of exotic resonances like the kappa meson, which could be relevant for specific observables.



# Excluded Volume: predictions

When available, observables connected with the electric charge could discriminate among the different possibilities



# Summary

- The point-like HRG model is not enough to describe all the available results from lattice QCD and experimental measurements.
- A flavor hierarchy in the particle freeze-out and the presence of undetected strange states are possible solutions, but their effect on all the observables must be checked.
- The inclusion of repulsive interactions helps in this direction, and is otherwise essential for the description of some key observables
- A mass and flavor dependent particle volume crucially improves the description of particle yields, being a competitive solution for the so called proton-anomaly, and of lattice QCD results.
- We are looking for observables which could definitively confirm the importance of repulsive interactions.



Thanks for your  
attention





Backup slides

# and Outlook

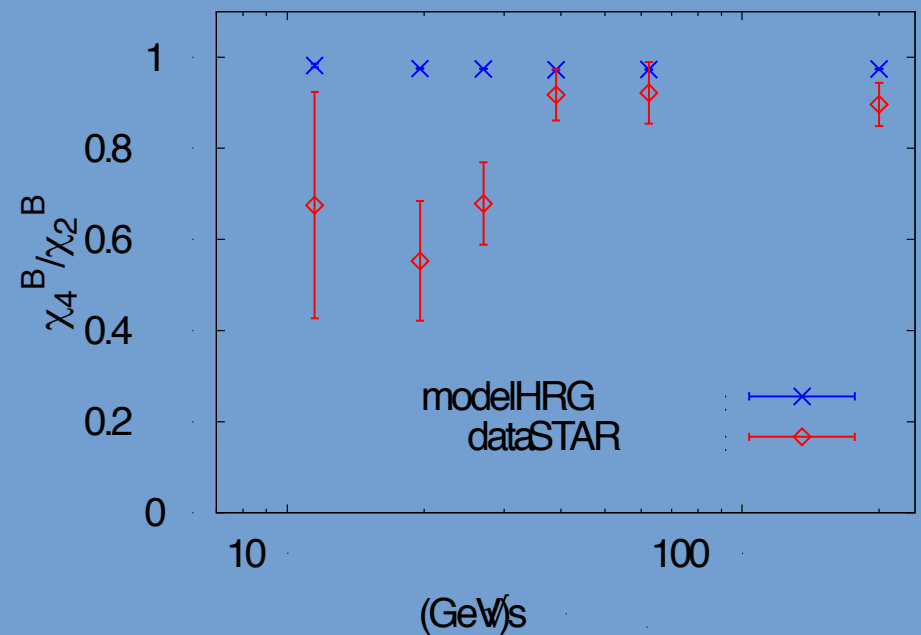
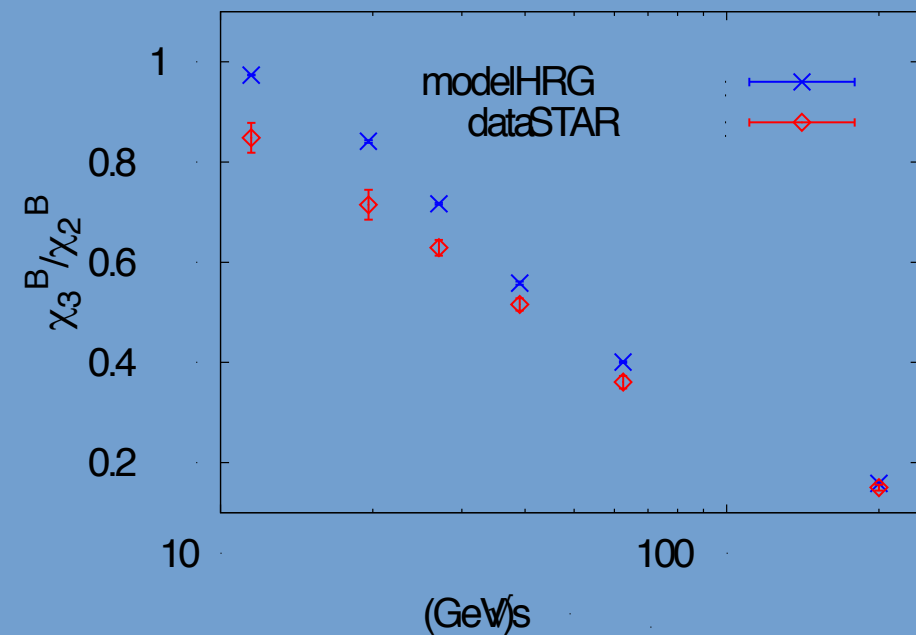
- An implementation on the quarks degrees of freedom is needed from the quasi-particle approach.
- An EoS at finite chemical potential is the next step.
- The strange sector needs further investigations, in order to clarify the role of higher mass resonances on freeze-out conditions.



# Results – Protons

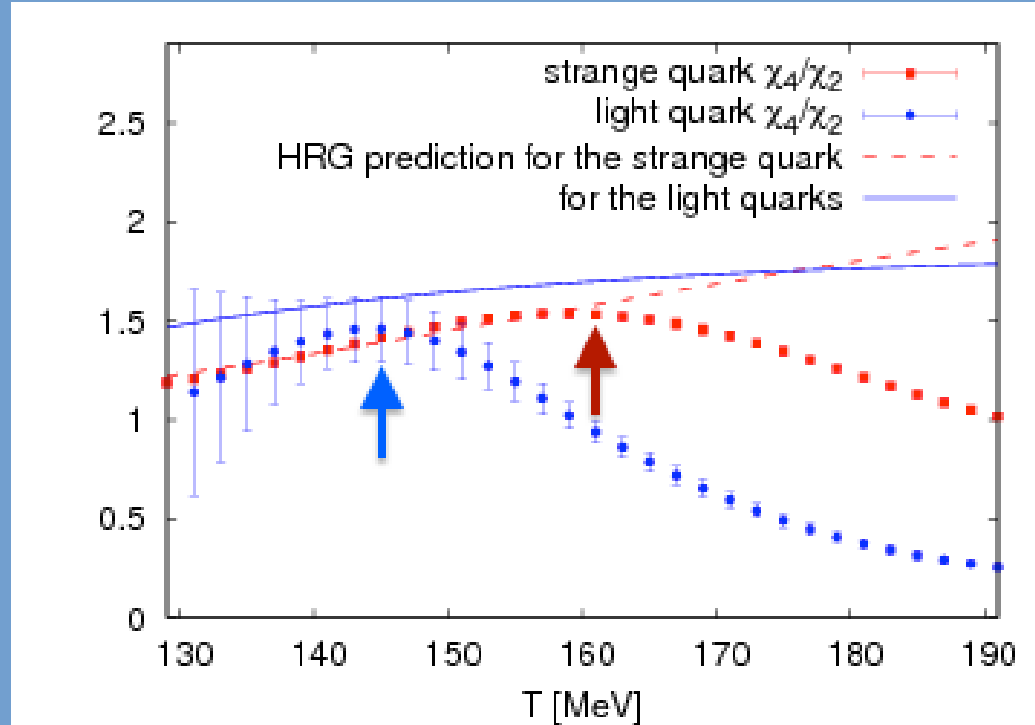
In the case of protons, we observe deviations between our predictions and the experimental values at small collision energies.

This could be due to an overestimate of the isospin randomization magnitude, or to effects related to chiral criticality.



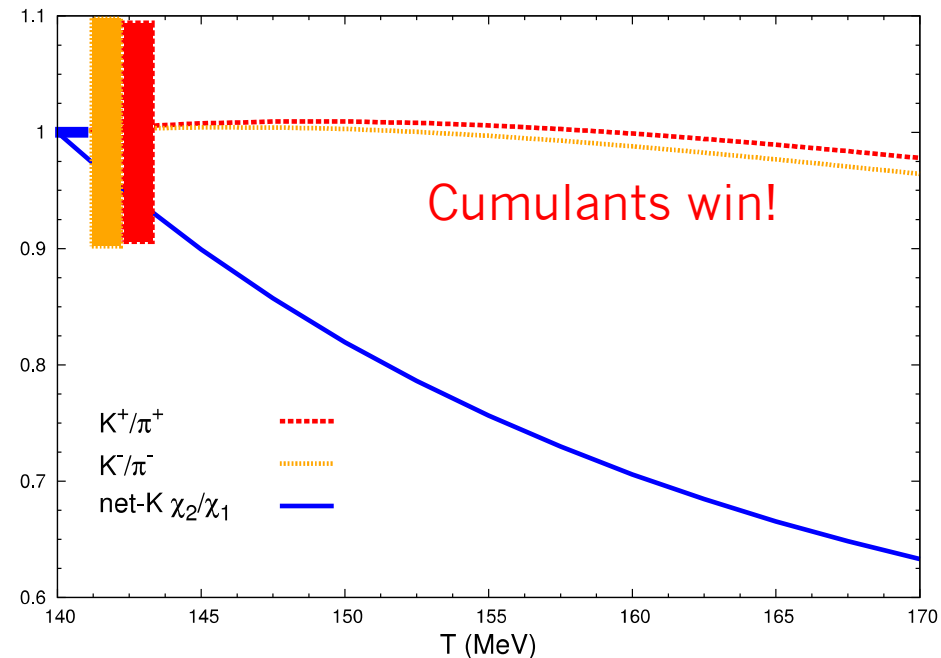
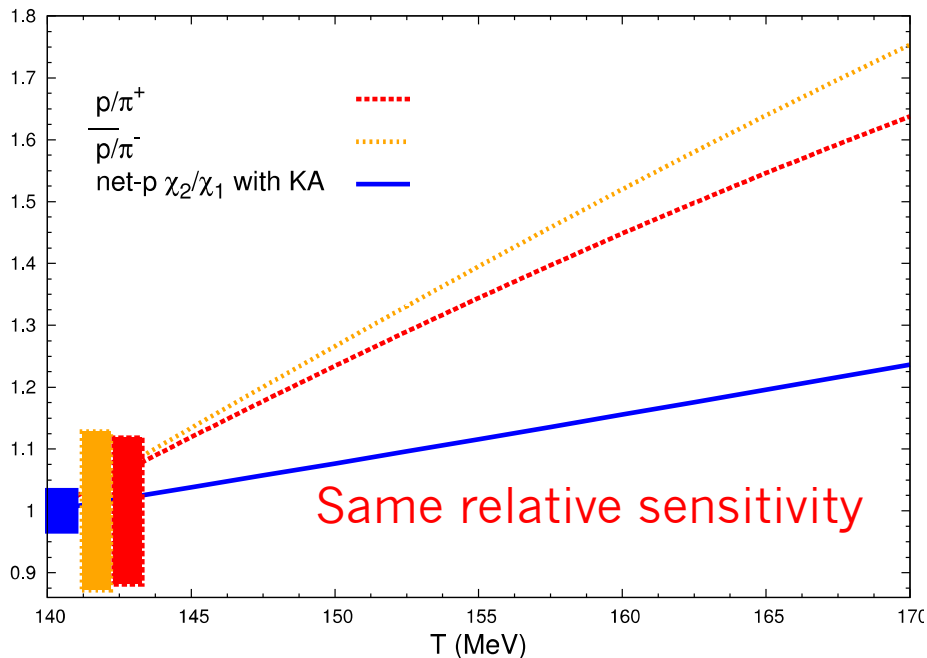
# Flavor hierarchy

We want to check whether the strange quarks freeze-out at higher temperatures than the light ones.



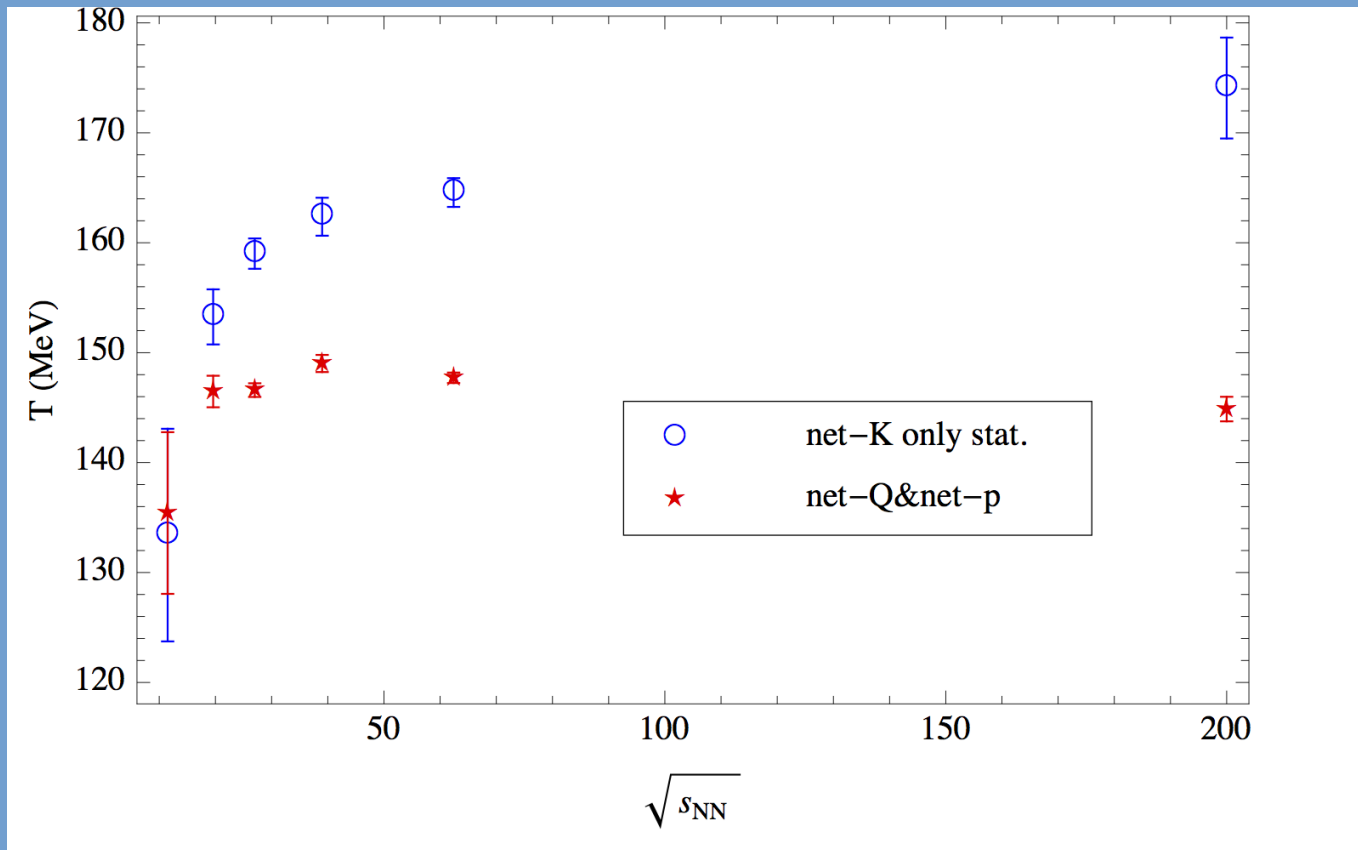
# Cumulants vs Yields

Cumulant sensitivity is affected by final state interactions, as well as finite acceptance and resonance decay.



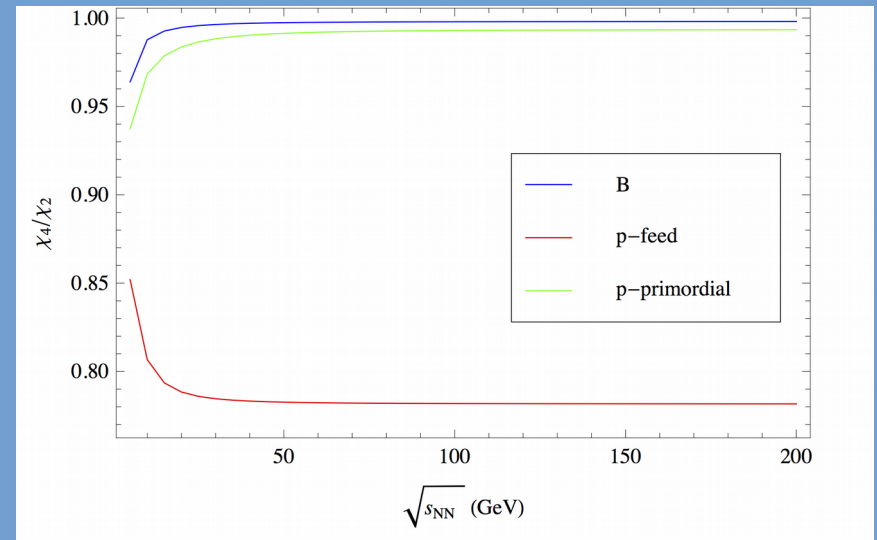
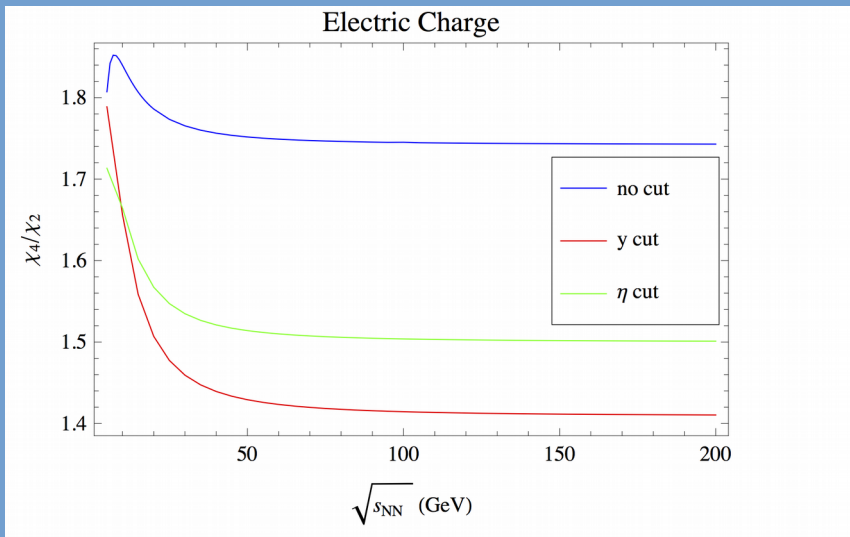
# Strangeness Freeze-Out

The FO temperature extracted from the net-Kaons cumulants (preliminary data) is about 20 MeV higher for the highest energy collision.



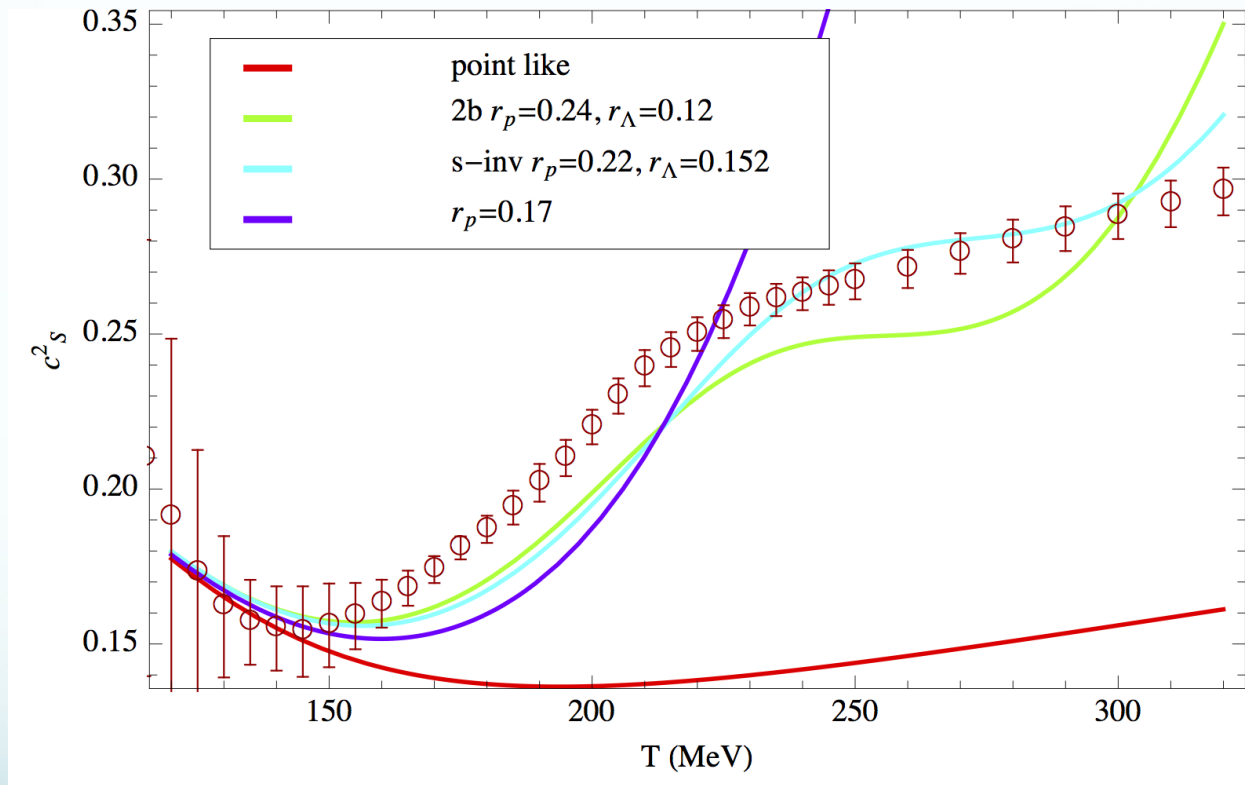
# Freeze-out: experiment

The proper implementation of the experimental setup is relevant in the HRG calculations.



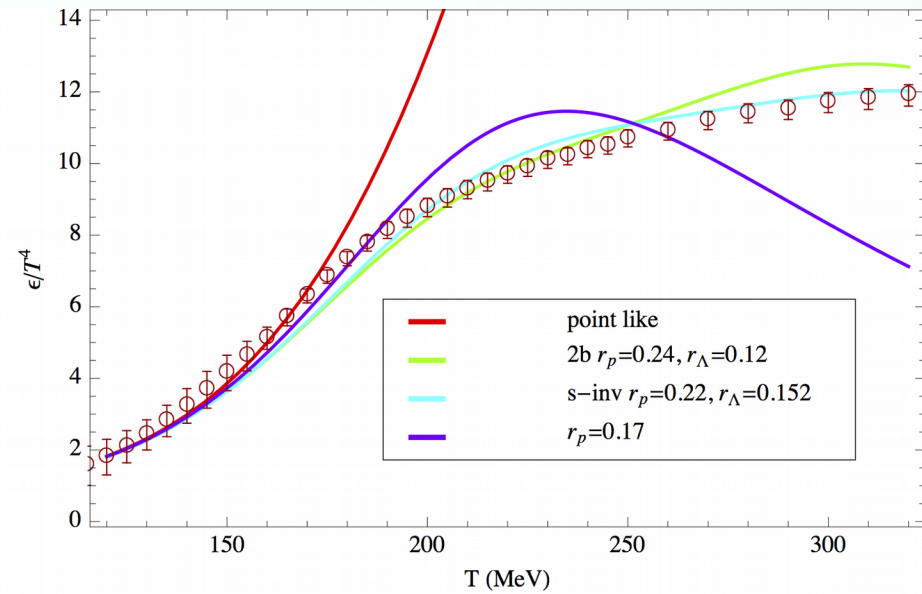
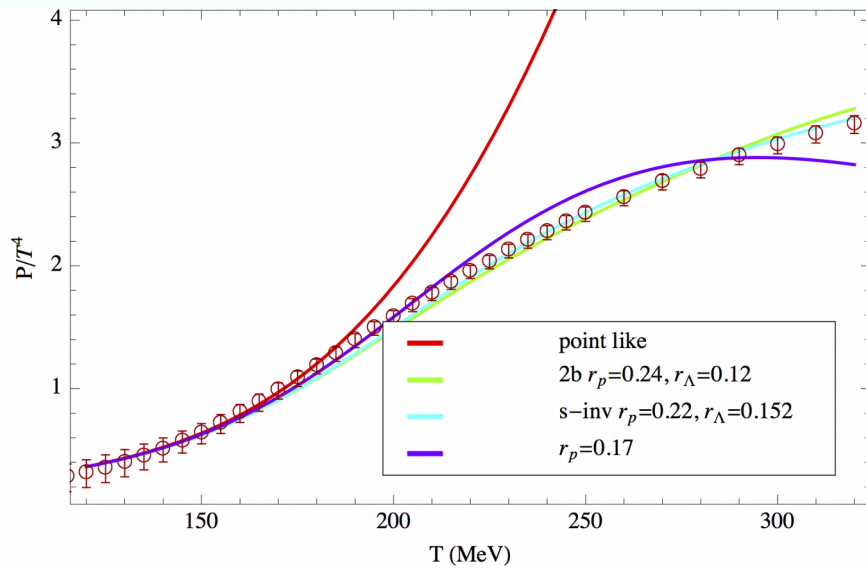
# Excluded Volume

No problem with the speed of sound



# Excluded Volume

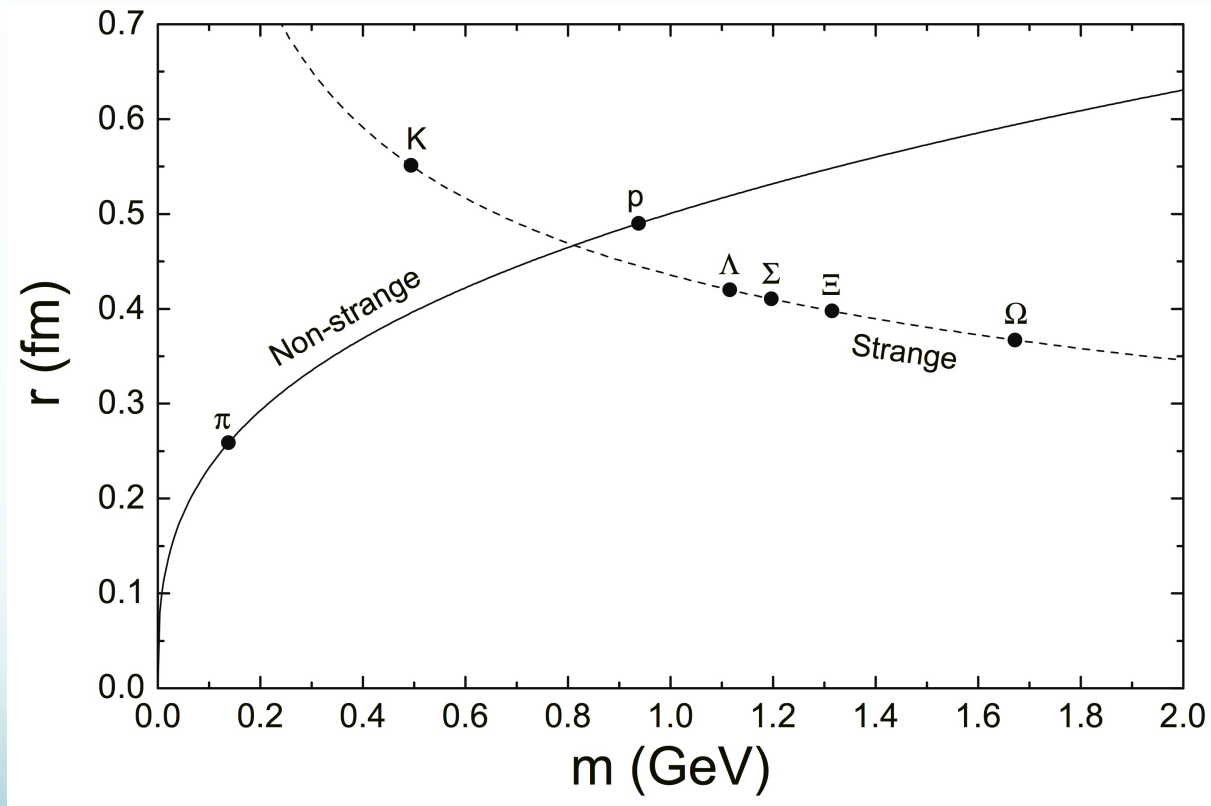
No problem with the thermodynamics





# Excluded Volume

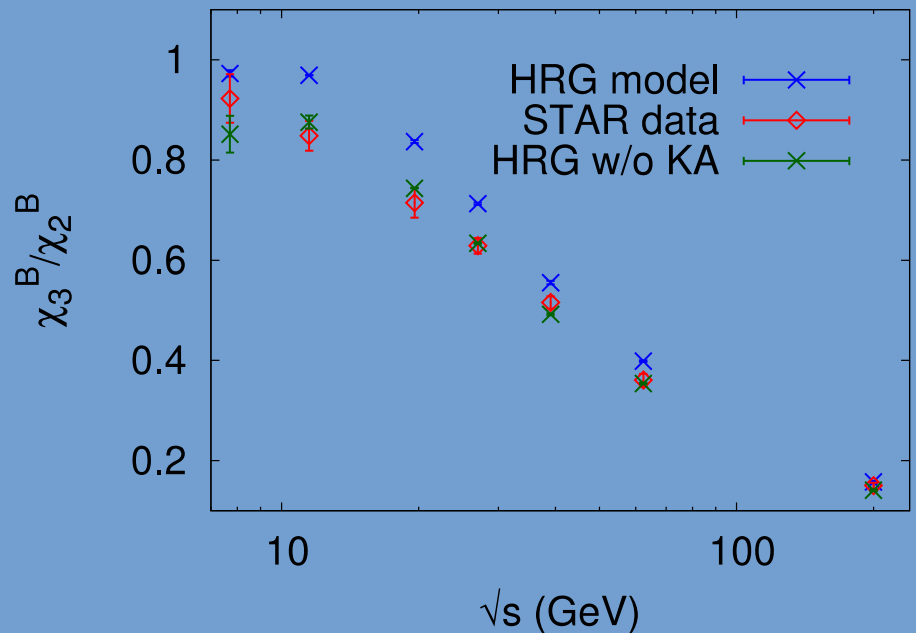
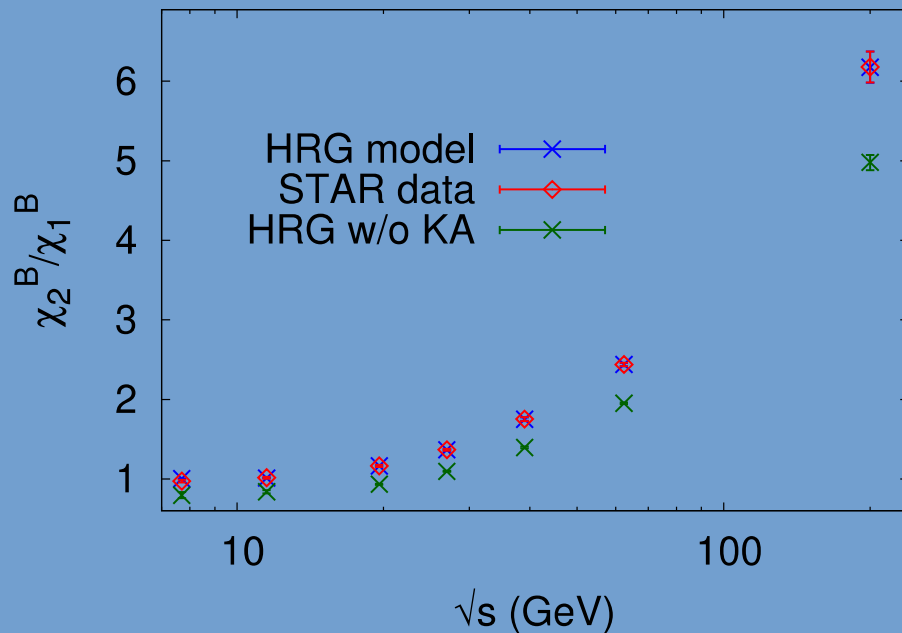
Considering some funny parametrizations ...



# Results – Protons

In the case of protons, we observe deviations between our predictions and the experimental values at small collision energies.

This could be due to an overestimate of the isospin randomization magnitude, or to effects related to chiral criticality.

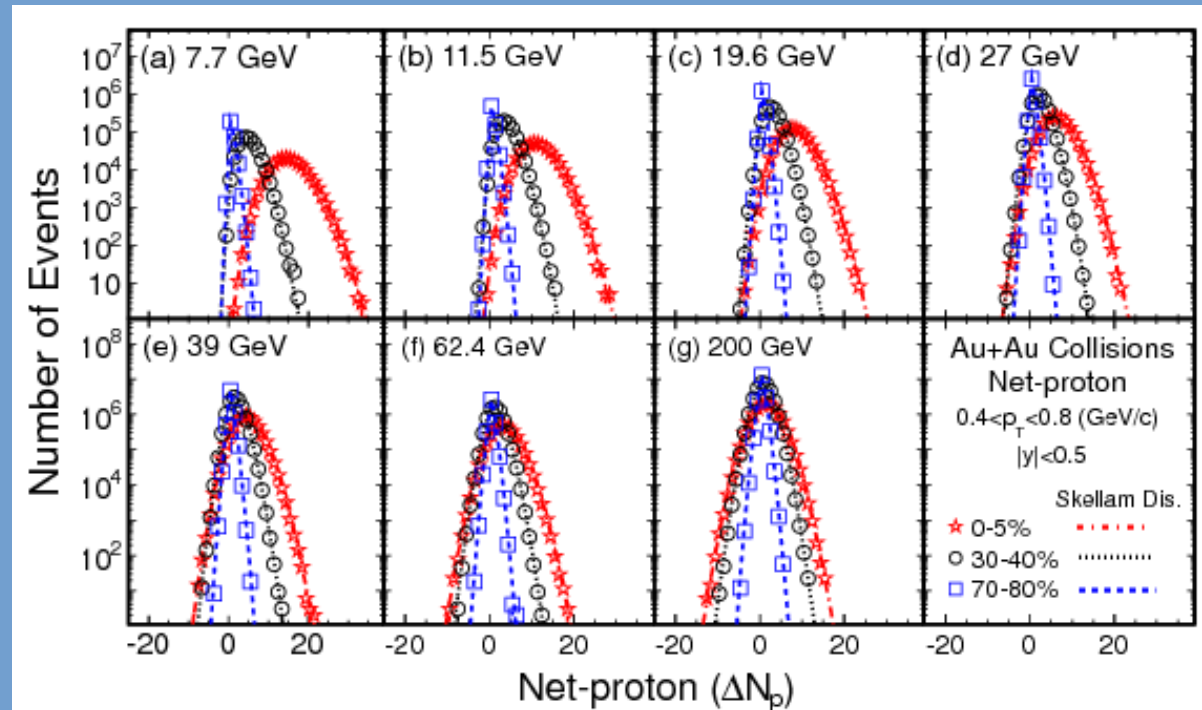


# Experimental multiplicity distribution

Measuring the net-number of conserved charges event by event we get a distribution; from this we can calculate the moments.

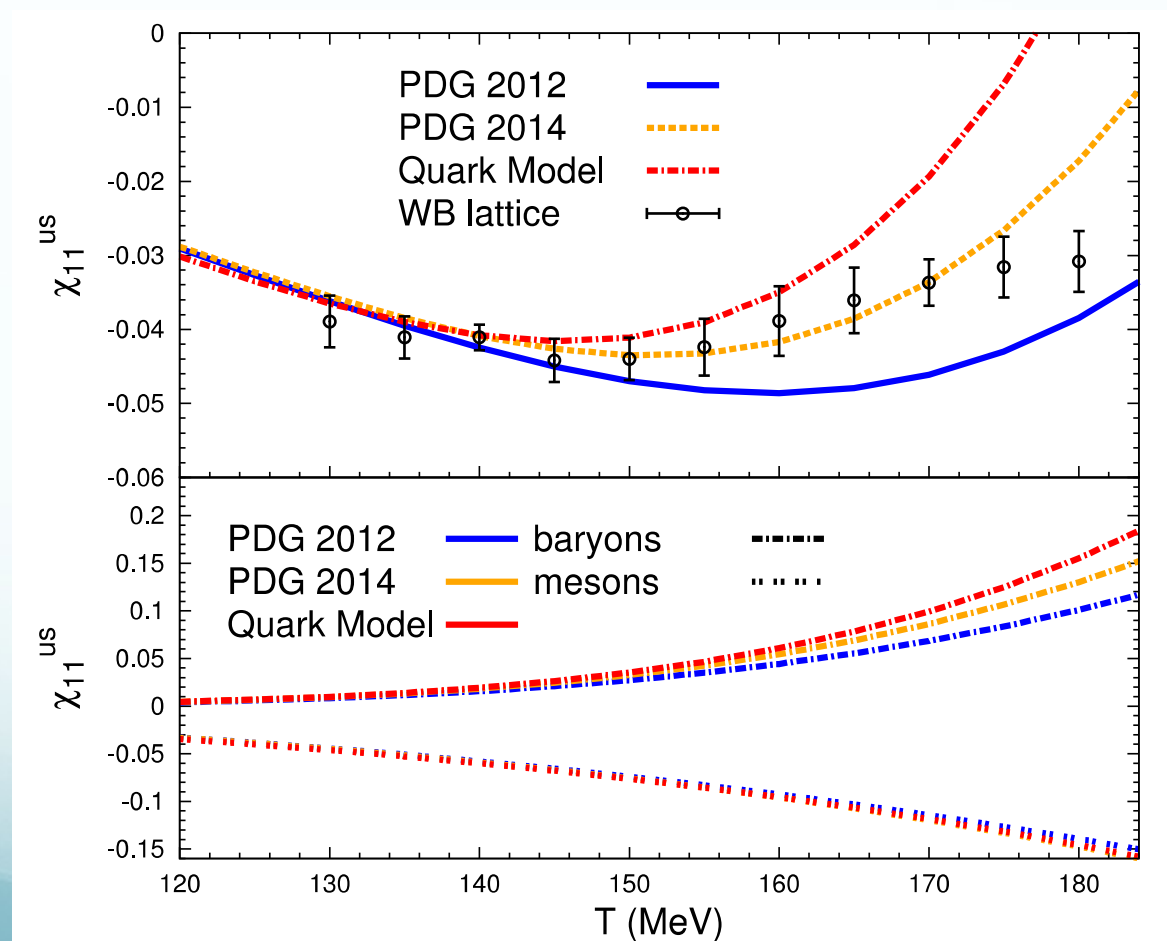
Due to exact charge conservation we should have no fluctuations in the entire system.

They come from the finite acceptance window.



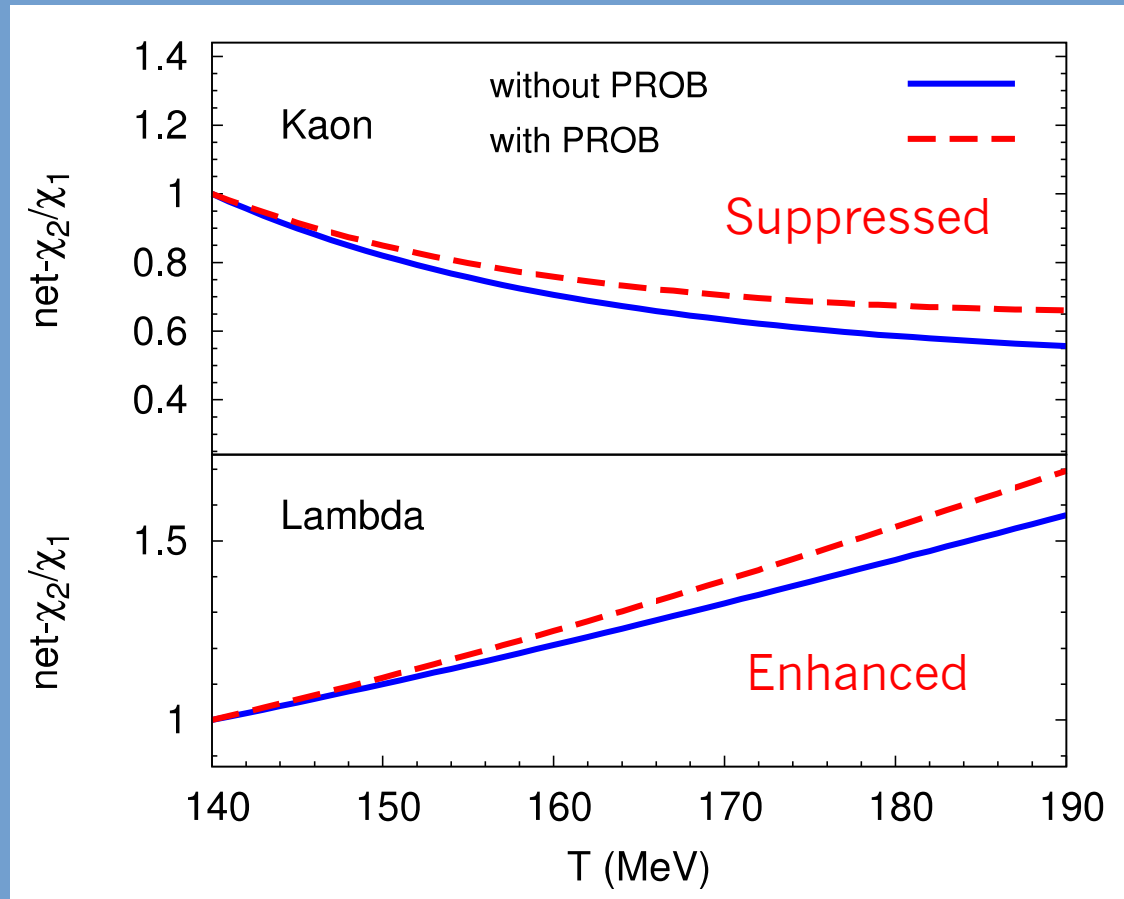
# Particle lists

We are looking for observables which, compared to lattice simulations and experimental measurements, could distinguish among the different particle content.



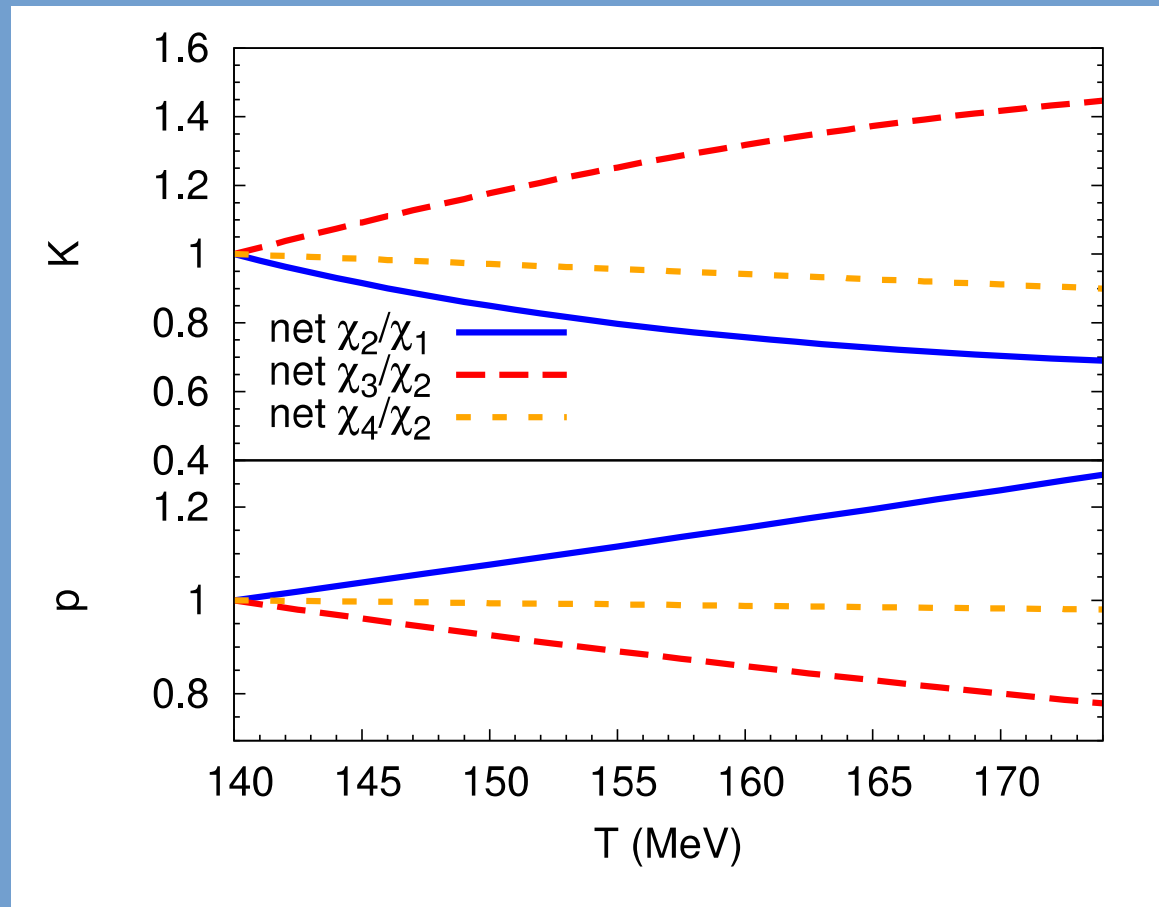
# Sensitivity of cumulants

Cumulant sensitivity is affected by final state interactions, as well as finite acceptance and resonance decay.



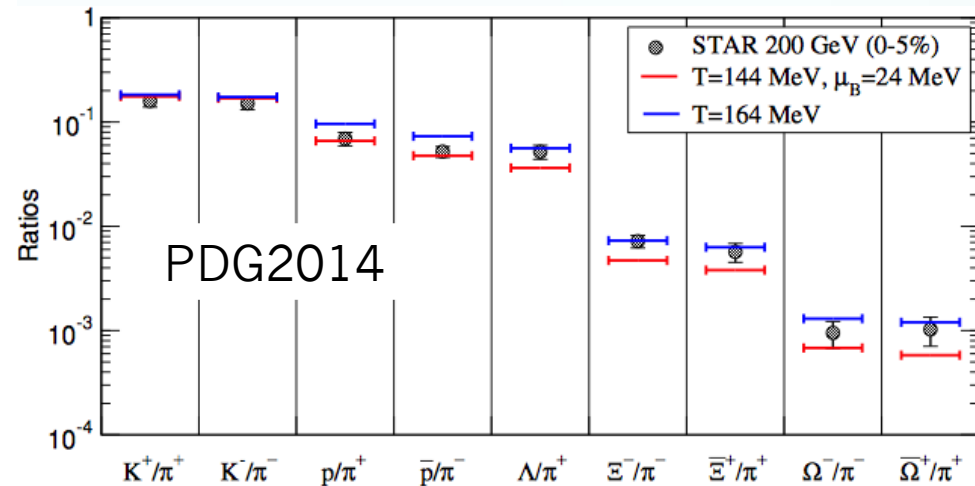
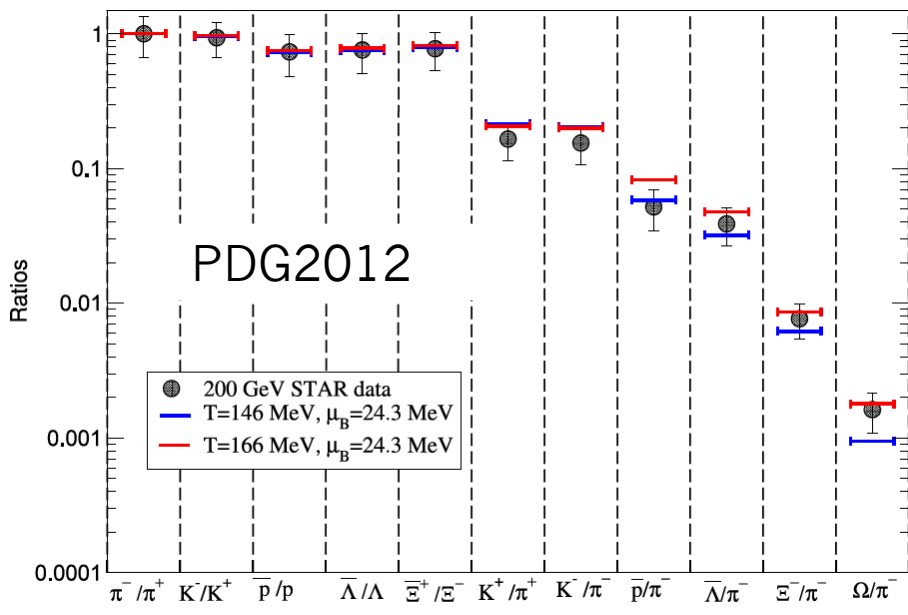
# Sensitivity of cumulants

The same sensitivity is shown by higher order cumulants.



# Particle lists

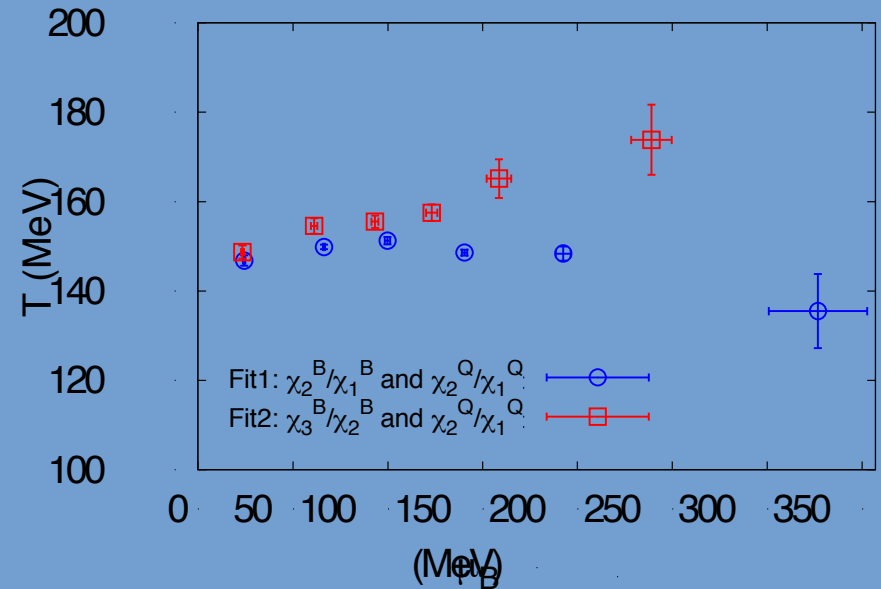
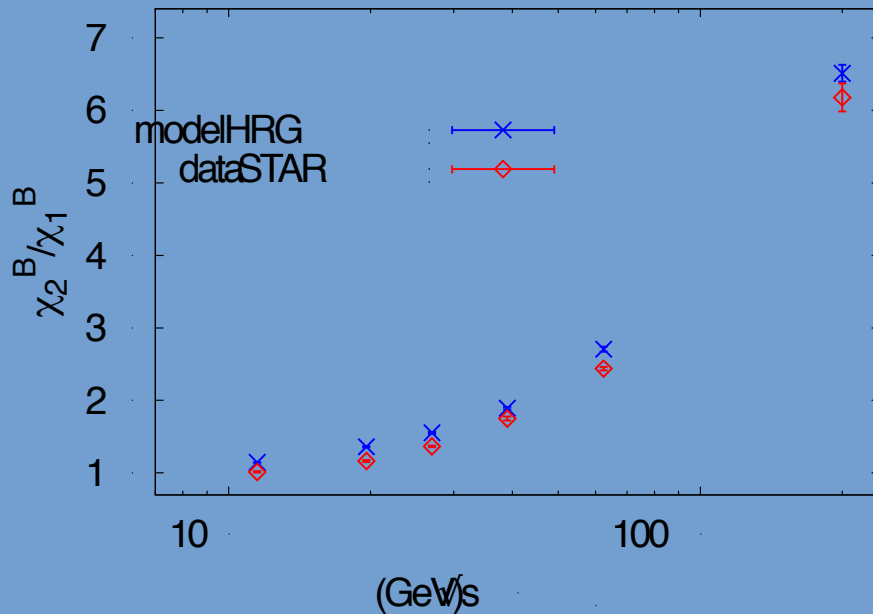
We checked that the inclusion of all the measured resonances does not modify the freeze-out parameters obtained from the analysis of cumulants, leaving the tension on particle yields unaffected.





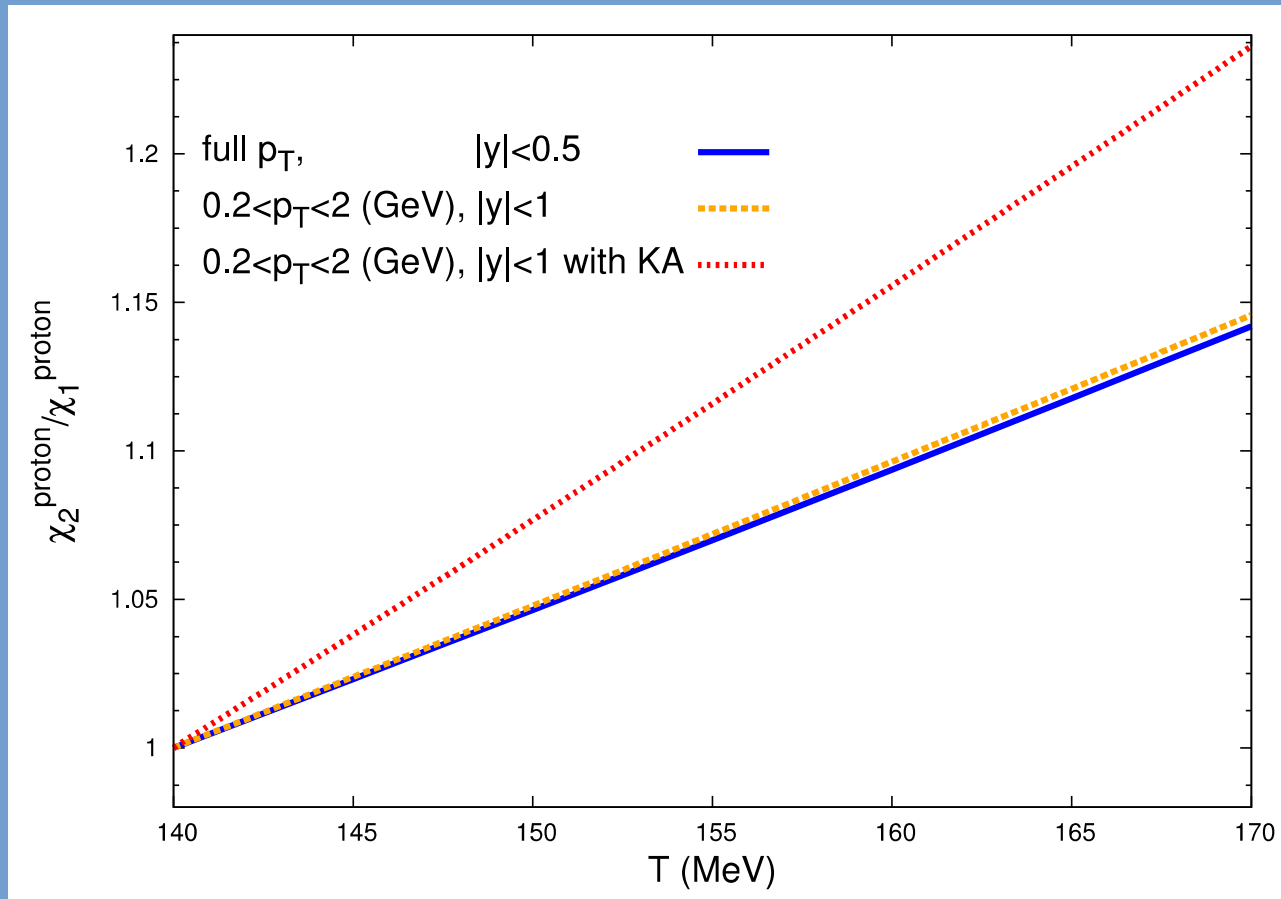
# Results – Crosscheck

Just as a crosscheck I show what happens fitting the  $\chi_3/\chi_2$  for the proton (which has still small error bars) together with the  $\chi_2/\chi_1$  for the charge. The result obtained shows a bending opposite to the one expected.



# Sensitivity of cumulants

Cumulant sensitivity is affected by final state interactions, as well as finite acceptance and resonance decay.



# QGP scales

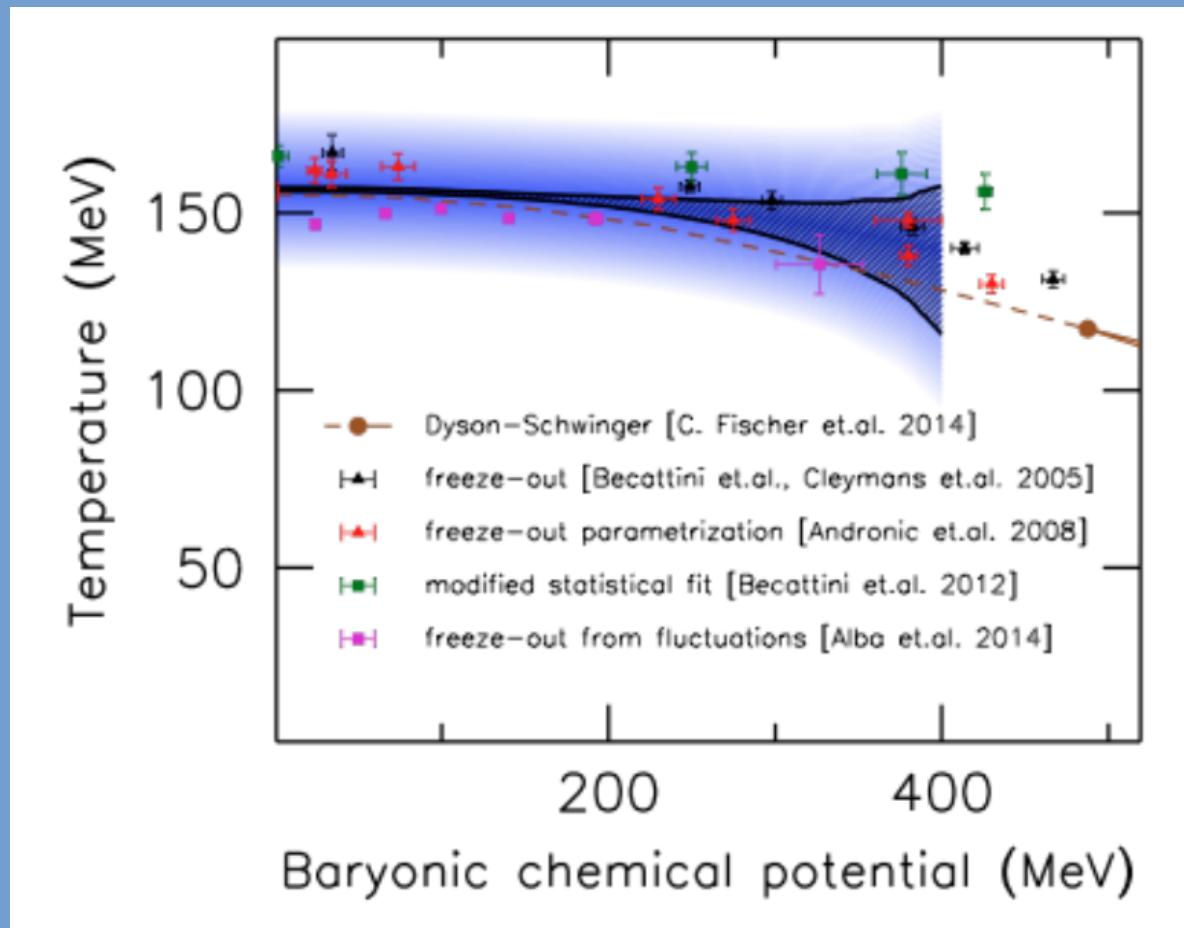
fluid	$P$ [Pa]	$T$ [K]	$\eta$ [Pa·s]	$\eta/n$ [ $\hbar$ ]	$\eta/s$ [ $\hbar/k_B$ ]
H <sub>2</sub> O	$0.1 \cdot 10^6$	370	$2.9 \cdot 10^{-4}$	85	8.2
<sup>4</sup> He	$0.1 \cdot 10^6$	2.0	$1.2 \cdot 10^{-6}$	0.5	1.9
H <sub>2</sub> O	$22.6 \cdot 10^6$	650	$6.0 \cdot 10^{-5}$	32	2.0
<sup>4</sup> He	$0.22 \cdot 10^6$	5.1	$1.7 \cdot 10^{-6}$	1.7	0.7
<sup>6</sup> Li ( $a = \infty$ )	$12 \cdot 10^{-9}$	$23 \cdot 10^{-6}$	$\leq 1.7 \cdot 10^{-15}$	$\leq 1$	$\leq 0.5$
QGP	$88 \cdot 10^{33}$	$2 \cdot 10^{12}$	$\leq 5 \cdot 10^{11}$		$\leq 0.4$

from T. Schäfer and D. Teaney (2009)

The temperature in the inner core of our Sun is about  $10^7$  K

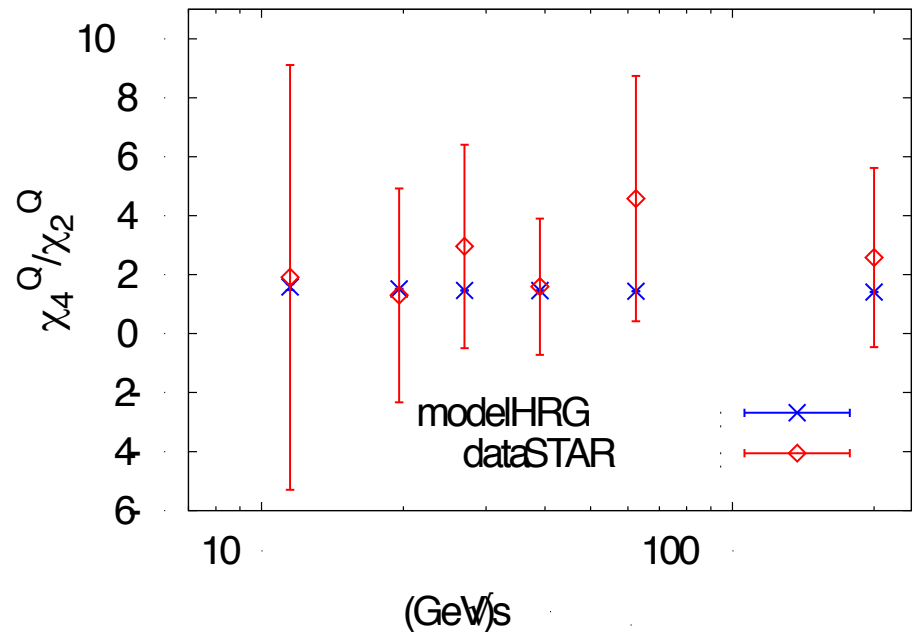
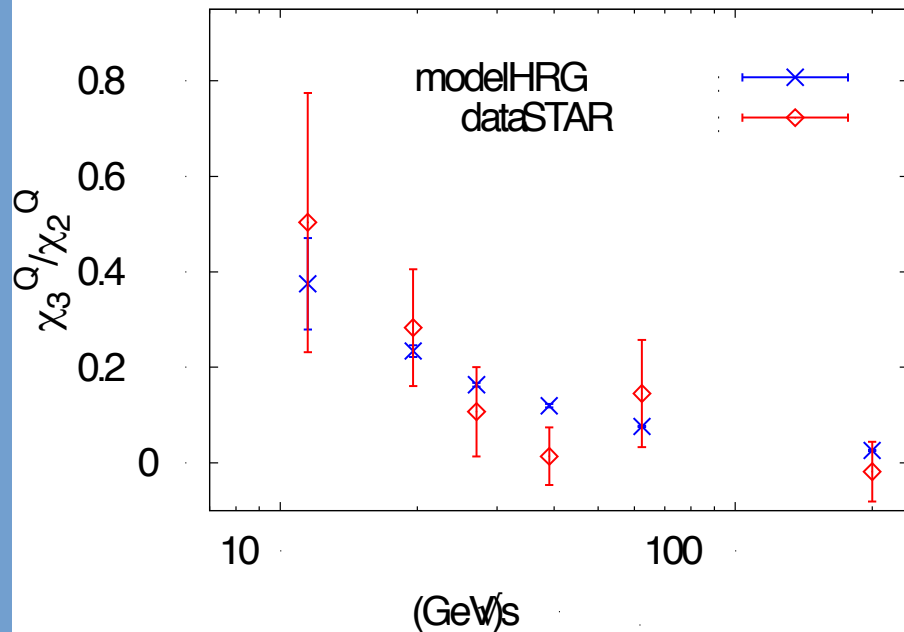
# Lattice vs HRG

My freeze-out points are below the QCD phase transition (dark blue line), as one would expect.



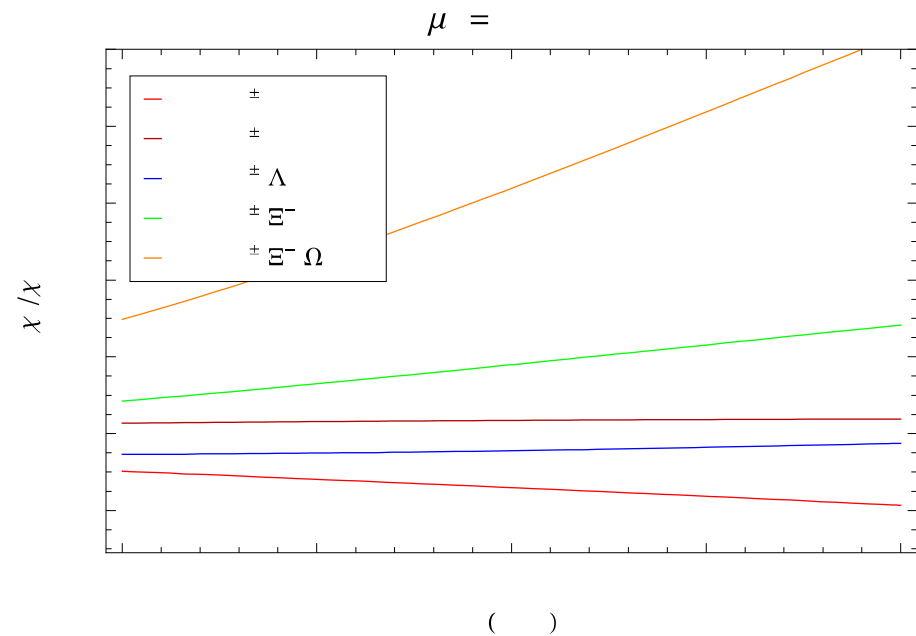
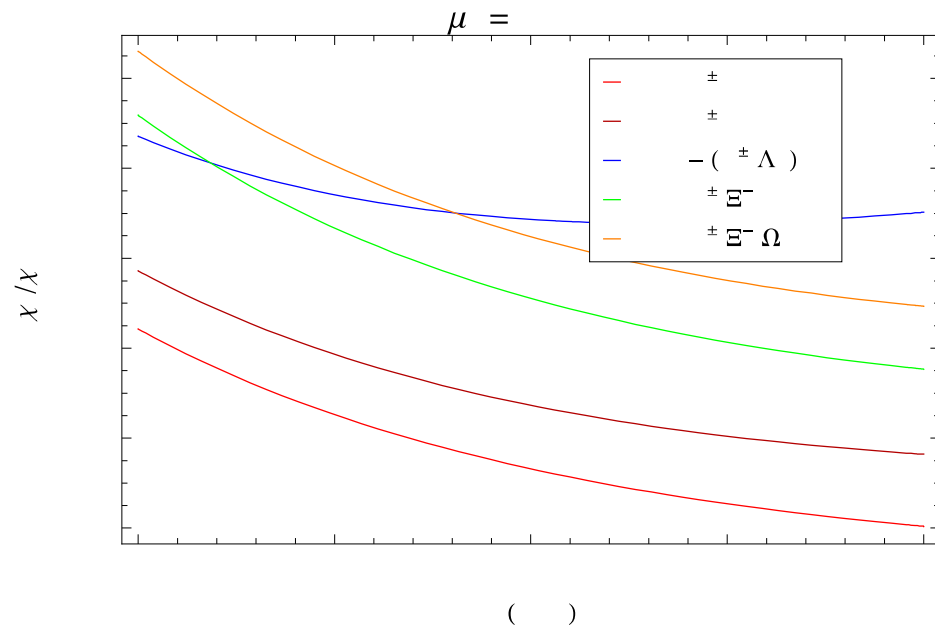
# Results – Electric charge

Using the new freeze-out parameters the higher order cumulants are in good agreement with the experimental data.



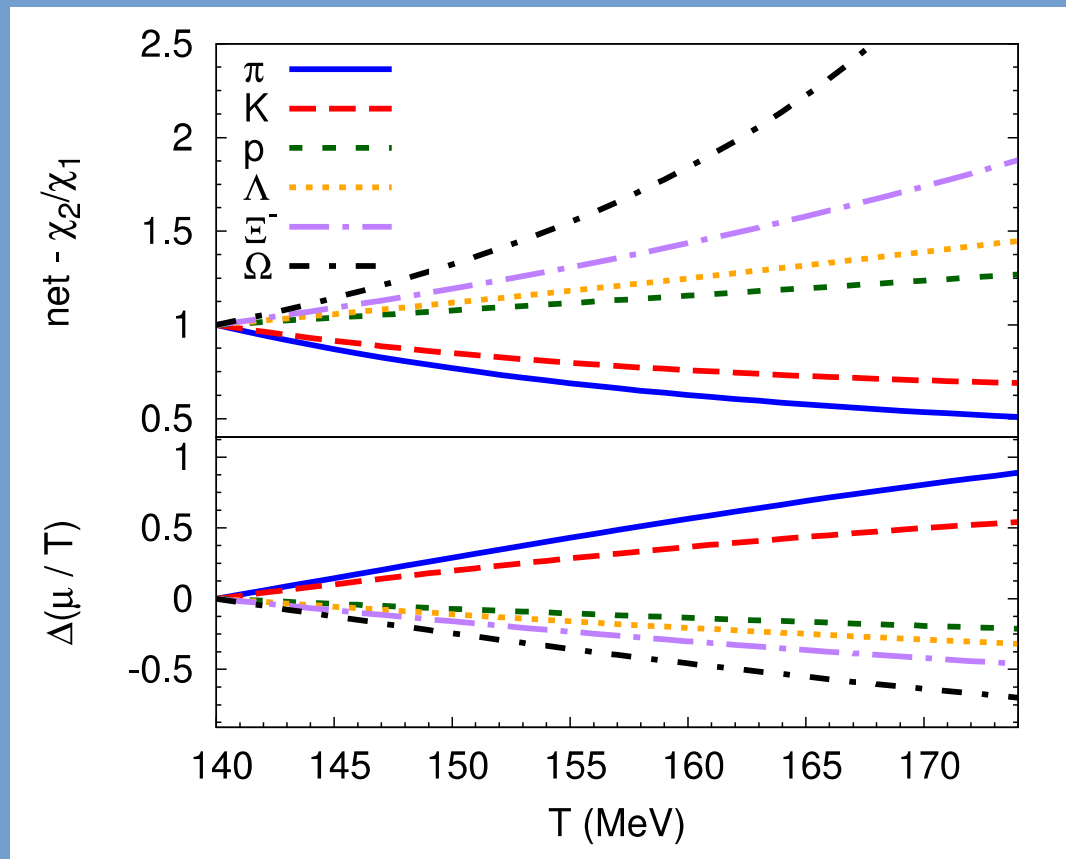
# Strangeness distribution

Multistrange baryons are difficult to be measured, and different sets of particles could give a different sensitivity to the freeze-out surface.



# Sensitivity of cumulants

The sensitivity changes with the particle species, in connection with the specific quantum numbers, whose particular combination is encoded in the single particle chemical potential.





# Polyakov loop

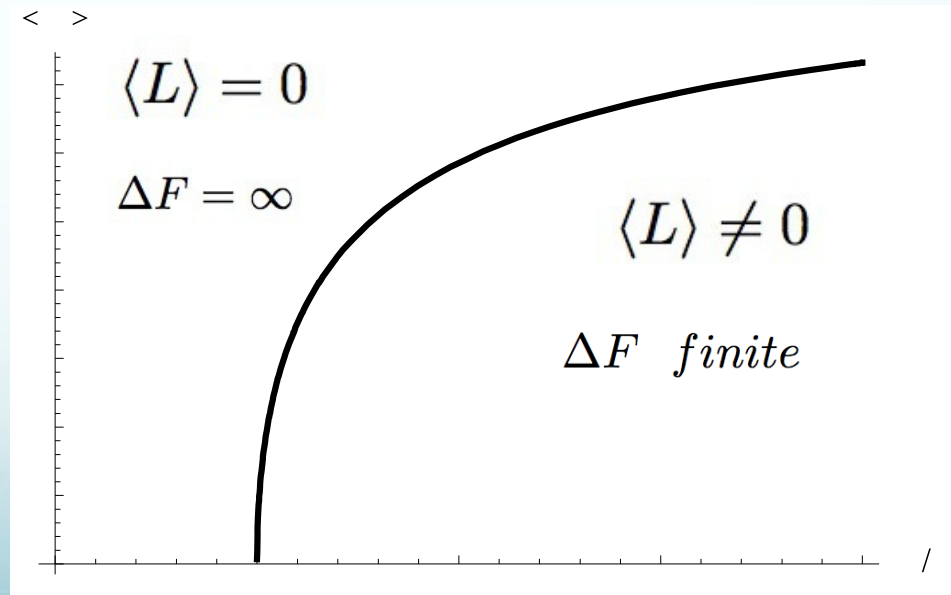
The Polyakov loop is the trace of the Polyakov line, defined in terms of the gluon field.

$$L_{\mathcal{R}}(\mathbf{x}) = \mathcal{P} \exp \left[ i \int_0^{1/T} d\tau A_4^a(\mathbf{x}, \tau) T_{a, \mathcal{R}} \right]$$

Its mean value can be related to the free energy of a massive quark in a gluon bath.

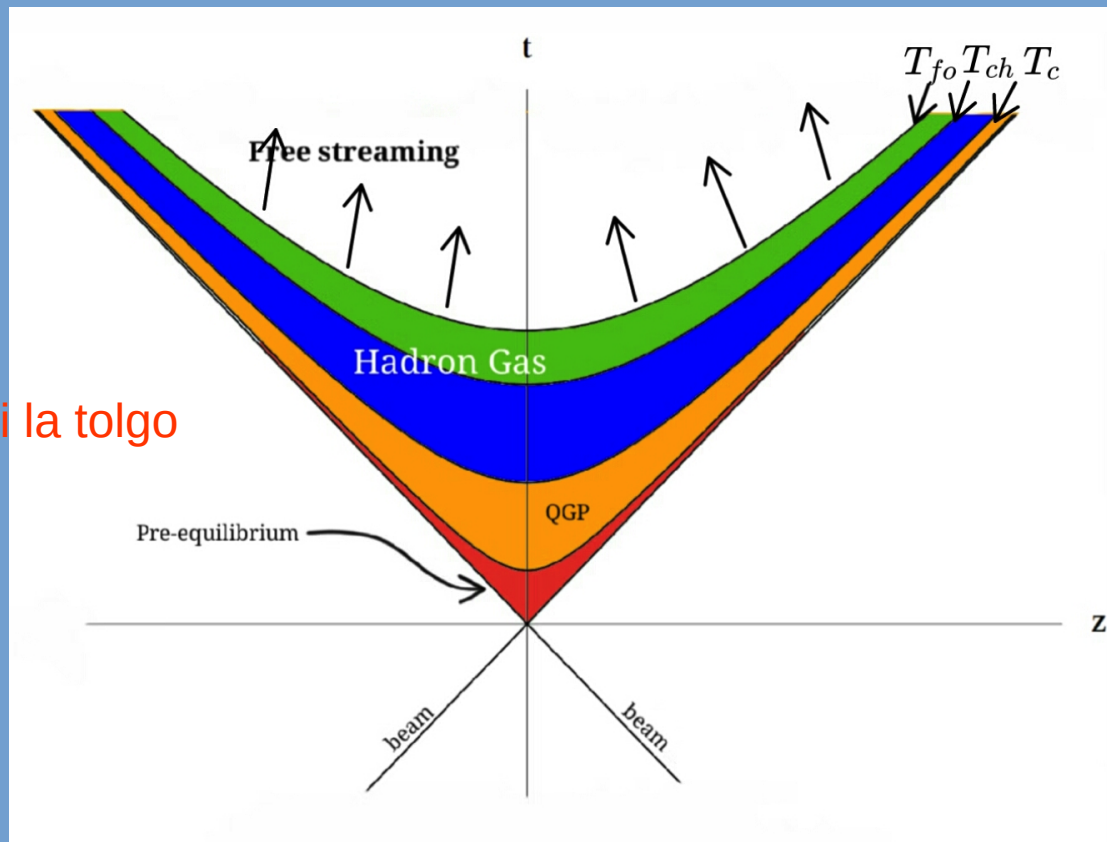
$$\Delta F \equiv F_Q - F_g$$

$$e^{-\frac{\Delta F}{T}} = \langle L(\mathbf{x}) \rangle$$



# Full vs Partial Equilibrium

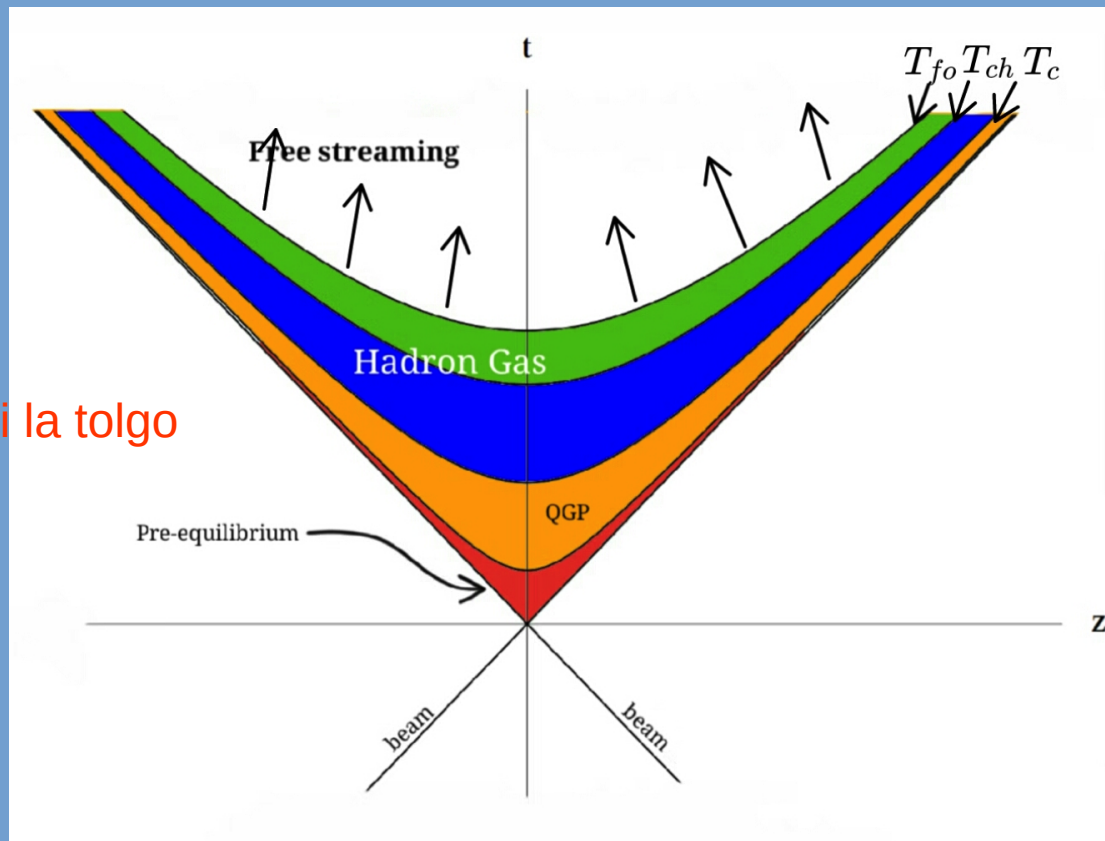
The hadronic phase (blue and green areas) can be studied by means of the Hadron-Resonance Gas (HRG) model, where resonance formation and subsequent decay mediate the interaction among hadrons in the ground state.



Questa slide magari la tolgo

# Full vs Partial Equilibrium

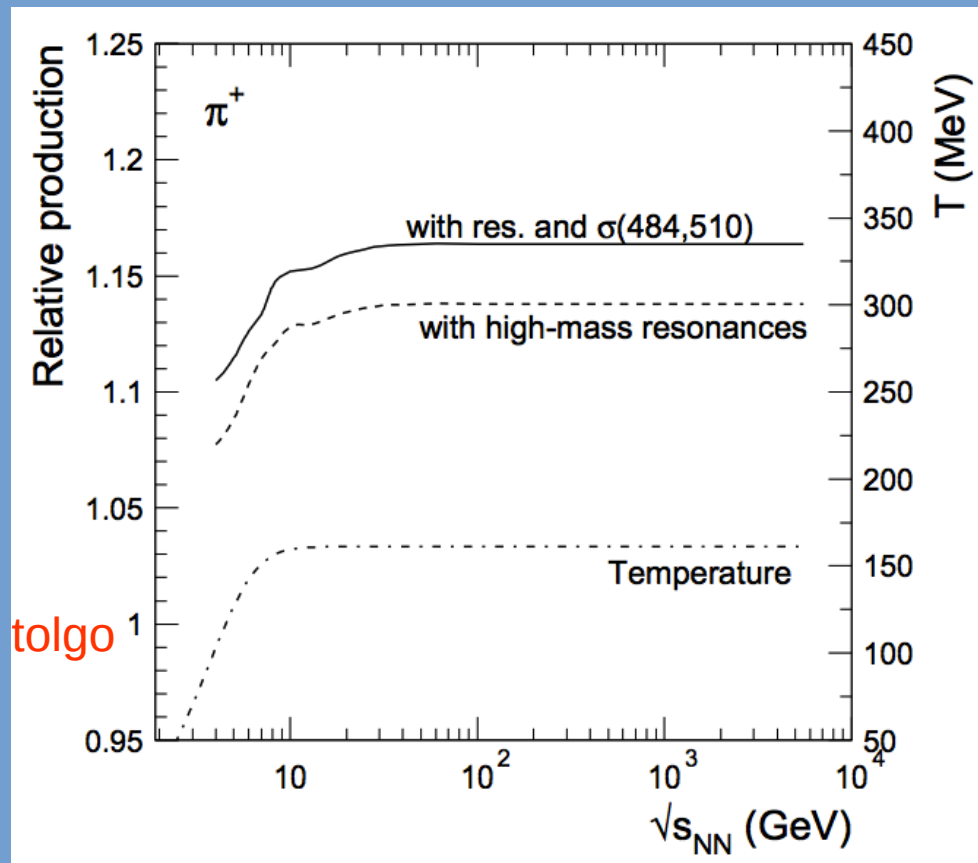
The hadronic phase (blue and green areas) can be studied by means of the Hadron-Resonance Gas (HRG) model, where resonance formation and subsequent decay mediate the interaction among hadrons in the ground state.



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# Feed from resonance decays

Resonance formation largely affects stable particle distributions, due to sequential decays chains.



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# Experimental multiplicity distribution

Measuring the net-number of conserved charges event by event we get a distribution; from this we can calculate the moments.

Due to exact charge conservation we should have no fluctuations in the entire system.

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