Institut für Theoretische Physik

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

The quark susceptibility in a generalized dynamical quasiparticle model

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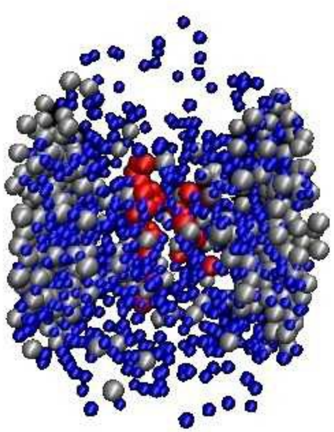
for the PHSD group

Erice, 18.09.2016



HGS-HIRe for FAIR
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H-QM | Helmholtz Research School Quark Matter Studies

PHSD

Overview

- QCD equation of state
- Thermodynamics in the quasiparticle limit
- Selfenergy and transport
- Generalized quasiparticle model
- Transport coefficients and susceptibility
- Hadronic equation of state

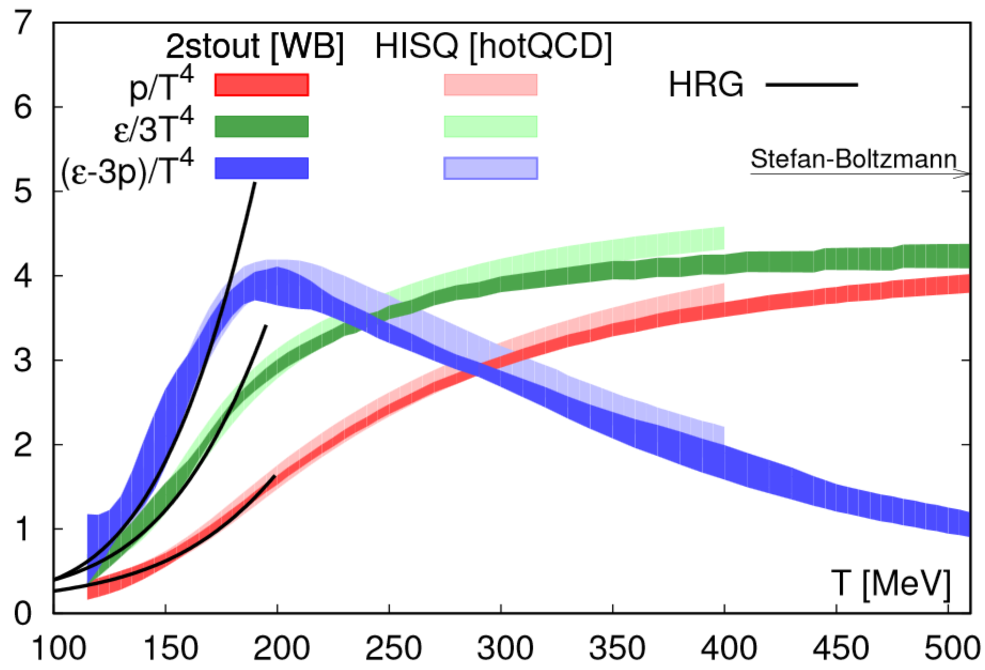
Lattice QCD

- Different lattice EoS's start to converge.
- Agreement on the QCD-EoS.

Open problems:

- No lattice calculations for large μ_B.
- No calculations out of equilibrium.

Use effective models!



Wuppertal-Budapest: Phys. Lett. B 370 (2014) 99-104
HotQCD: Phys. Rev. D 90, 094503

Quasiparticle thermodynamics 4

- **Idea: treat partons as dynamical quasiparticles.**

Propagator with effective mass M and width γ:

$$G(\omega,\mathbf{p}) = \frac{-1}{\omega^2 - \mathbf{p}^2 - M^2 + 2i\gamma\omega} = \frac{-1}{\omega^2 - \mathbf{p}^2 - \Sigma}$$

$$A(\omega,\mathbf{p}) = \frac{2\gamma\omega}{(\omega^2 - \mathbf{p}^2 - M^2)^2 + 4\gamma^2\omega^2}$$

- **Grand canonical potential in propagator representation:**

$$\beta\Omega[D,S] = \frac{1}{2}\mathrm{Tr}[\ln D^{-1} - \Pi D] - \mathrm{Tr}[\ln S^{-1} + \Sigma S] + \Phi[D,S]$$

with selfenergies $\frac{\delta\Phi}{\delta D} = \frac{1}{2}\Pi \qquad \frac{\delta\Phi}{\delta S} = -\Sigma$

$\Phi[D,S]$ **has no contribution to entropy or density.**

J.P. Blaizot, E. Iancu and A. Rebhan, Phys. Rev. D 63 (2001) 065003

Entropy density

- **Entropy consists of a pole and an interaction term.**

Pole term is the entropy of a noninteracting gas:

$$s^{(0)} = \frac{1}{2\pi^2}\int_0^\infty dk\; k^2 \left(\frac{\omega_k-\mu}{T} n_{B/F}(\omega_k) - S\ln\left(1 - Se^{-(\omega_k-\mu)/T}\right)\right)$$

Interaction term contains the width γ and vanishes in the on-shell limit γ→0:

$$\Delta s = \int_{d^4k} \frac{\partial n_{B/F}(\omega)}{\partial T}\left(2\gamma\omega\frac{\omega^2-\mathbf{p}^2-M^2}{(\omega^2-\mathbf{p}^2-M^2)^2+4\gamma^2\omega^2} - \arctan\left(\frac{2\gamma\omega}{\omega^2-\mathbf{p}^2-M^2}\right)\right)$$

A. Peshier, PRD 70 (2004) 034016; J. Phys. G31(2005) S371

Effective mass and width

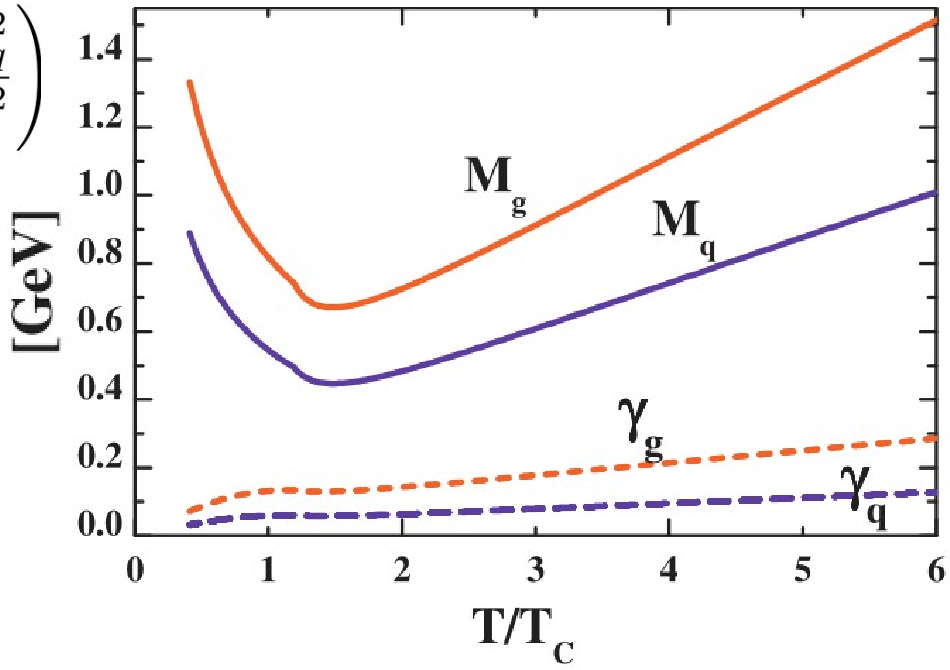
- **Motivated by HTL**

$$M_g^2 = \frac{g^2}{6}\left(\left(N_c + \frac{1}{2}N_f\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right)$$

$$M_{q,\bar{q}}^2 = \frac{N_c^2 - 1}{8N_c} g^2 \left(T^2 + \frac{\mu_q^2}{\pi^2}\right)$$

$$\gamma_g = \frac{1}{3} N_c \frac{g^2 T}{8\pi} \ln\left(1 + \frac{2c}{g^2}\right)$$

$$\gamma_{q,\bar{q}} = \frac{1}{3}\frac{N_c^2 - 1}{2N_c}\frac{g^2 T}{8\pi} \ln\left(1 + \frac{2c}{g^2}\right)$$



The width is fixed by correlators.

$$g^2(T, T_c) = \frac{48\pi^2}{(11N_c - 2N_f)\ln(\lambda^2((T - T_s)/T_c)^2)}$$

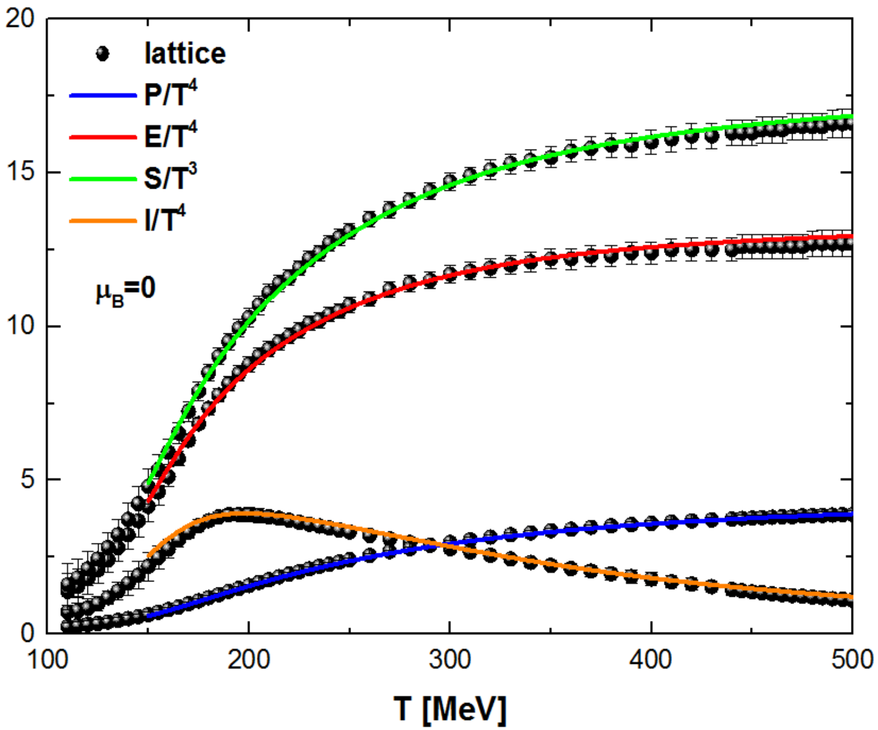
A. Peshier, W. Cassing, PRL 94 (2005) 172301;
Cassing, NPA 791 (2007) 365: NPA 793 (2007)

Effective coupling

- 1) Parametrisation of the effective coupling:

$$g^2(T,T_c) = \frac{48\pi^2}{(11N_c - 2N_f)\ln(\lambda^2((T-T_s)/T_c)^2)}$$

- 2) or use EoS as input: $g^2(S/S_{SB}) = g_0 \cdot ((S/S_{SB})^c - 1)^d$



- **Finite chemical potential**

Scaling Hypothesis:

$$T^* = \sqrt{T^2 + \frac{\mu^2}{\pi^2}} \qquad T_c(\mu) = T_c\sqrt{1 - \alpha\ \mu^2}$$

$$\alpha \approx 8.79\ \text{GeV}^{-2}$$

Consistent with lattice curvature

$$\kappa_{DQPM} \approx 0.0122 \qquad \kappa = 0.013(2)$$

Selfenergies

Effective mass and width define the parton selfenergies:

$$\Sigma = M^2 - 2i\gamma\omega$$

Selfenergies allow for a transport description in the Kadanoff-Baym framework!

Solve with extended testparticle ansatz:

$$F_{XP} = iG^<(X,P) \sim \sum_{i=1}^{N} \delta^{(3)}(\mathbf{X} - \mathbf{X}_i(t))\delta^{(3)}(\mathbf{P} - \mathbf{P}_i(t))\delta(P_0 - \epsilon_i(t))$$

Σ defines the dynamics:

$$\frac{d\mathbf{X}_i}{dt} = \frac{1}{2\epsilon_i}\left[2\mathbf{P}_i + \nabla_{P_i}\mathrm{Re}\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \mathbf{P}_i^2 - M_0^2 - \mathrm{Re}\Sigma^{ret}_{(i)}}{\Gamma_{(i)}}\nabla_{P_i}\Gamma_{(i)}\right]$$

$$\frac{d\mathbf{P}_i}{dt} = -\frac{1}{2\epsilon_i}\left[\nabla_{X_i}\mathrm{Re}\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \mathbf{P}_i^2 - M_0^2 - \mathrm{Re}\Sigma^{ret}_{(i)}}{\Gamma_{(i)}}\nabla_{X_i}\Gamma_{(i)}\right]$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{2\epsilon_i}\left[\frac{\partial\mathrm{Re}\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \mathbf{P}_i^2 - M_0^2 - \mathrm{Re}\Sigma^{ret}_{(i)}}{\Gamma_{(i)}}\frac{\partial\Gamma_{(i)}}{\partial t}\right]$$

PHSD

- **DQPM+KB allow a transport treatment of partons.**
- **Hadronic transport can be done in KB too: <u>HSD</u>**
- **Hadrons+partons are treated in the same framework.**

Unified transport approach: <u>PHSD</u>

- **Successful description of heavy-ion collisions from SPS to top RHIC and LHC energies.**
- **More on monday by Eduard Seifert and on wednesday by Alessia Palmese + Wolfgang Cassing!**

=> <u>DQPM is a valid description of partonic matter</u>

Lattice QCD at finite μ

- **Sign problem prevents simulations for finite μ.**

$$P(T,\{\mu_i\}) = \frac{T}{V}\ln Z(T,\{\mu_i\})$$

- **Pressure is obtained via Taylor expansion:**

$$\frac{P(T,\{\mu_i\})}{T^4} = \frac{P(T,\{0\})}{T^4} + \frac{1}{2}\sum_{i,j}\frac{\mu_i\mu_j}{T^2}\chi_2^{ij}$$

$$\chi_2^{ij} = \frac{T}{V}\frac{1}{T^2}\frac{\partial^2 \ln Z}{\partial\mu_i\partial\mu_j}\Big|_{\mu_i=\mu_j=0}$$

Lattice EoS at finite μ is controlled by the susceptibilities χ.

Particle density

- Particle density follows in the same way as the entropy density:

$$n = -\frac{1}{V}\frac{\partial \Omega}{\partial \mu} = -\frac{1}{V}\left(\left.\frac{\partial \Omega}{\partial \mu}\right|_{D,S} + \underbrace{\frac{\partial \Omega}{\partial D}}_{=0}\frac{\partial D}{\partial \mu} + \underbrace{\frac{\partial \Omega}{\partial S}}_{=0}\frac{\partial S}{\partial \mu} \right)$$

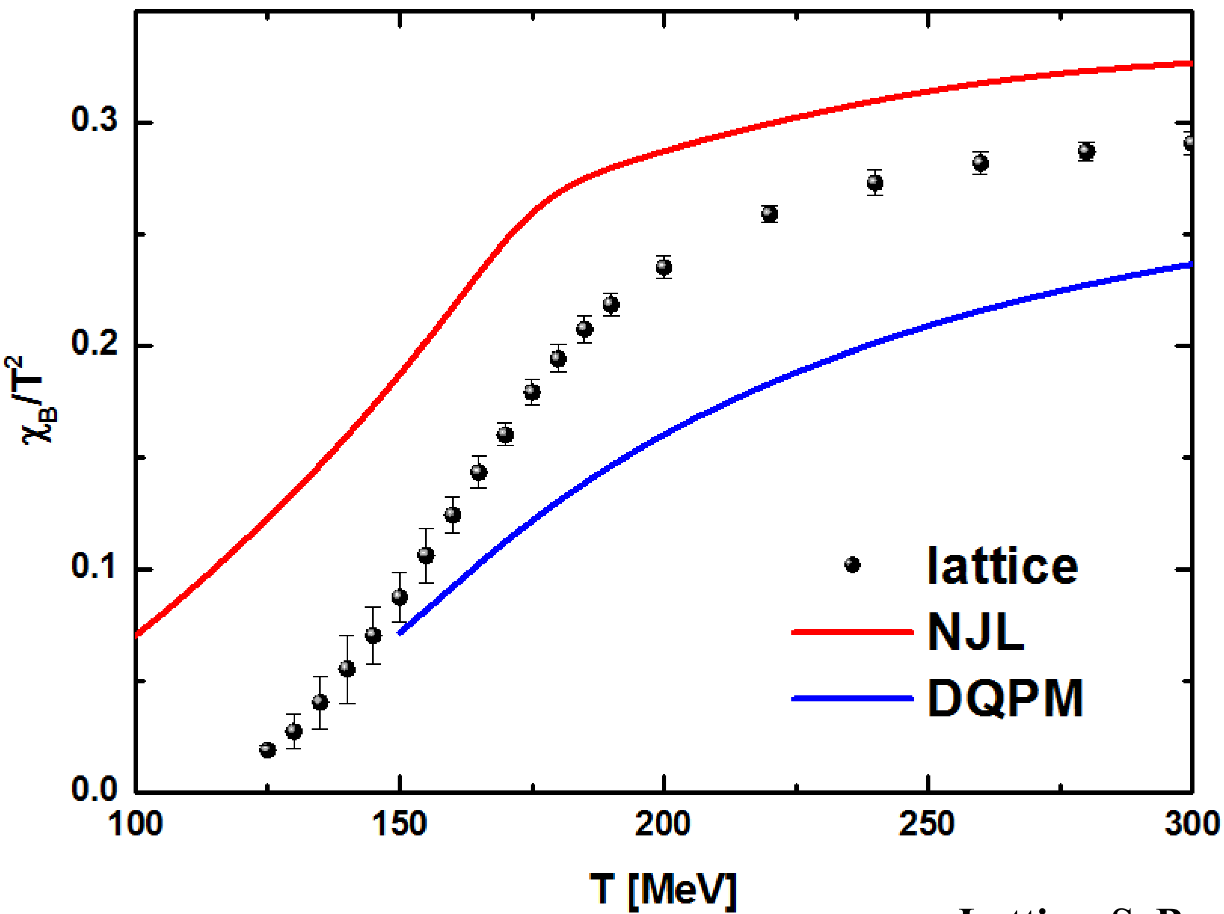
- Same systematics as the entropy:

$$n^{(0)} = \int \frac{d^3k}{(2\pi)^3}\left(n_F - n_F^{(A)}\right)$$

$$\Delta n = \int_{d^4k} \frac{\partial n_F(\omega)}{\partial \mu}\left(2\gamma\omega\frac{\omega^2-\mathbf{p}^2-M^2}{(\omega^2-\mathbf{p}^2-M^2)^2+4\gamma^2\omega^2} - \arctan\left(\frac{2\gamma\omega}{\omega^2-\mathbf{p}^2-M^2}\right)\right)$$
$$+ \int_{d^4k} \frac{\partial n_F^{(A)}(\omega)}{\partial \mu}\left(2\gamma\omega\frac{\omega^2-\mathbf{p}^2-M^2}{(\omega^2-\mathbf{p}^2-M^2)^2+4\gamma^2\omega^2} - \arctan\left(\frac{2\gamma\omega}{\omega^2-\mathbf{p}^2-M^2}\right)\right)$$

Susceptibilities

- First glimpse on finite chemical potentials.
- Contains only informations from quarks.



$$\chi_L = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_L^2}\Big|_{\mu_L=0}$$

DQPM quarks appear too heavy!

NJL quarks seem too light!

Lattice: S. Borsanyi, *et al.*, JHEP 1208 (2012) 053

„Non-perturbative“ QCD

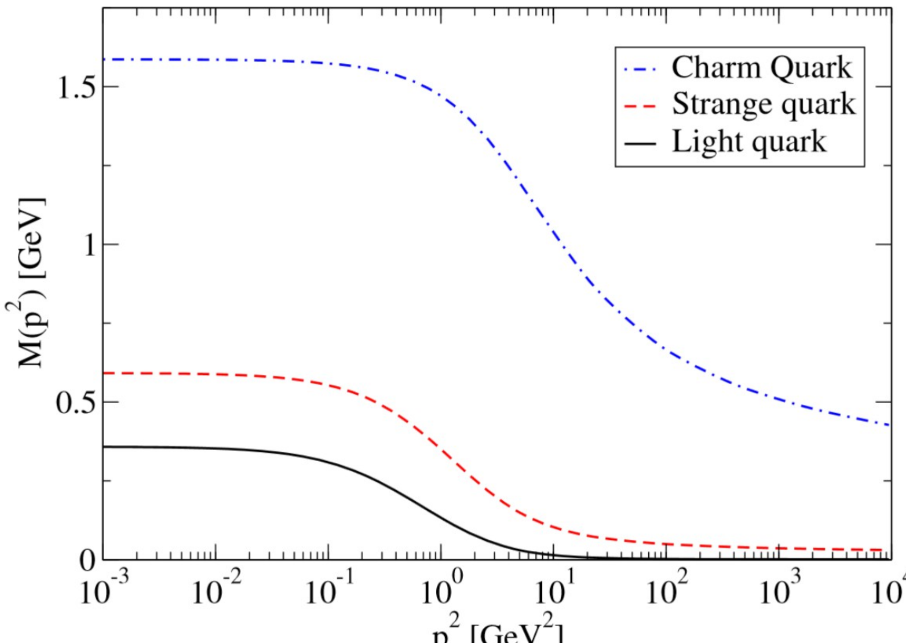
- Heavy partons in the perturbative regime.

- Dynamical chiral symmetry breaking: The quark masses have to drop for higher energies to reach the perturbative limit!

We introduce a correction factor to model CSR:

$$h(\Lambda, p) = \frac{1}{\sqrt{1 + \Lambda \cdot p^2 \cdot (T_c/T)^2}}$$

$$M \to M(p) = M \cdot h(\Lambda, p)$$
$$\gamma \to \gamma(p) = \gamma \cdot h(\Lambda, p)$$



- Propagator remains analytic in the upper half plane.

- Transport realisation stays valid!

C. S. Fischer, *et al.*, Phys. Rev. D 90 (2014) 3, 034022

Masses and width

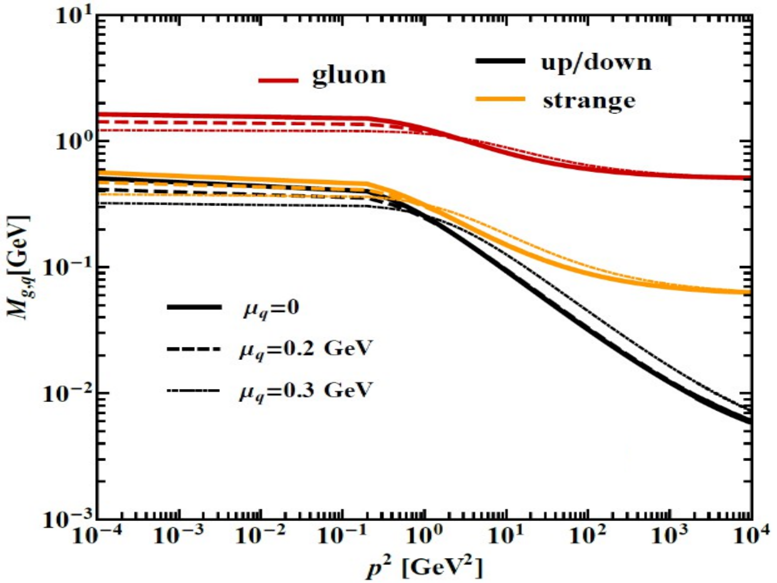
$$M_g(T,\mu_q,p) = \left(\frac{3}{2}\right)\left[\frac{g^2(T^\star/T_c(\mu_q))}{6}\left[\left(N_c+\frac{N_f}{2}\right)T^2+\frac{N_c}{2}\sum_q\frac{\mu_q^2}{\pi^2}\right]\right]^{1/2}\times h(\Lambda_g,p)+m_{\chi g}$$

$$M_{q,\bar q}(T,\mu_q,p) = \left[\frac{N_c^2-1}{8N_c}\,g^2(T^\star/T_c(\mu_q))\left[T^2+\frac{\mu_q^2}{\pi^2}\right]\right]^{1/2}\times h(\Lambda_q,p)+m_{\chi q}$$

$$\gamma_g(T,\mu_q,p) = N_c\,\frac{g^2(T^\star/T_c(\mu_q))}{8\pi}\,T\,\ln\left(\frac{2c}{g^2(T^\star/T_c(\mu_q))}+1.1\right)^{3/4}\times h(\Lambda_g,p)$$

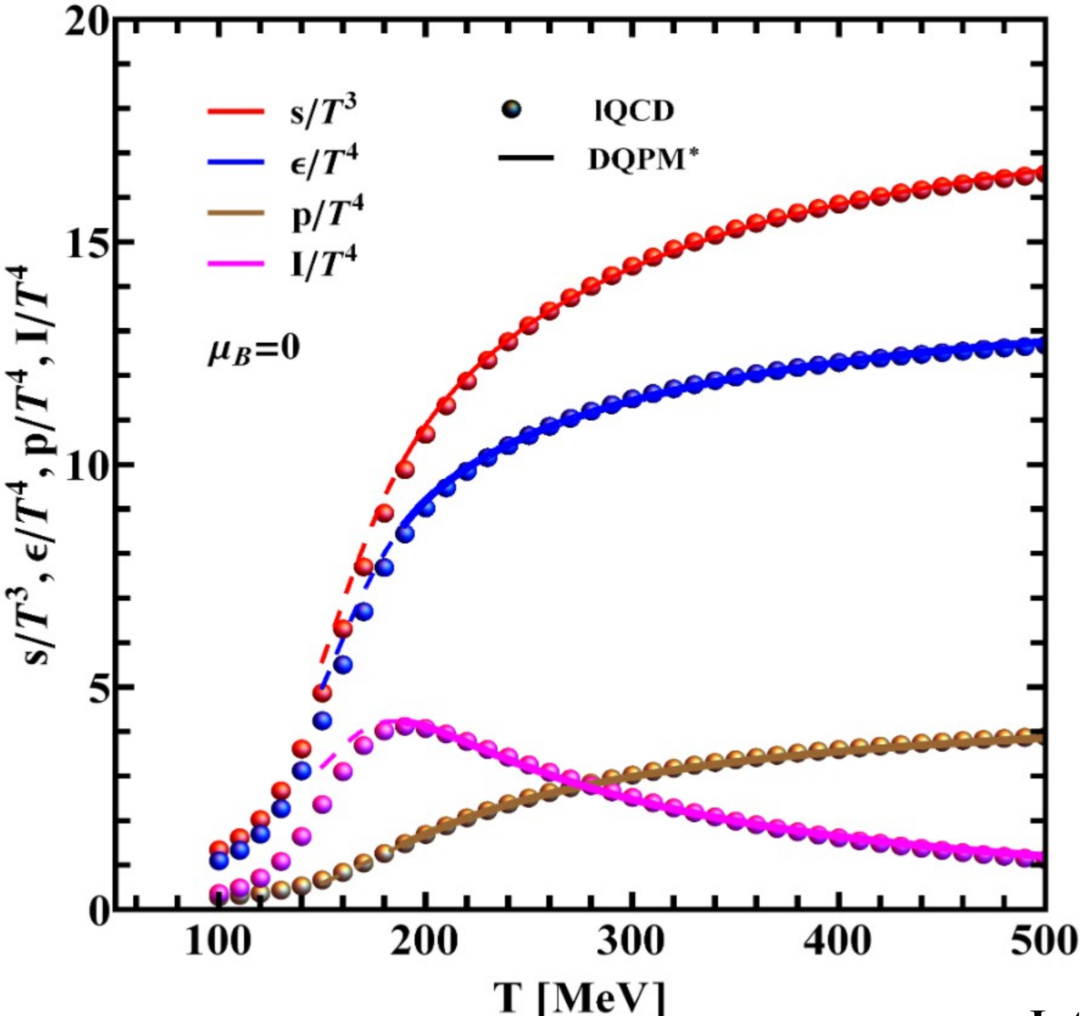
$$\gamma_{q,\bar q}(T,\mu_q,p) = \frac{N_c^2-1}{2N_c}\,\frac{g^2(T^\star/T_c(\mu_q))}{8\pi}\,T\ln\left(\frac{2c}{g^2(T^\star/T_c(\mu_q))}+1.1\right)^{3/4}\times h(\Lambda_q,p)$$

- **DQPM* uses momentum dependent selfenergies.**
- **EoS and susceptibilty in good agreement with lQCD.**



DQPM* EoS

- **EoS by thermodynamic relations:**



$$P(T) = \int_0^T S(T')dT'$$

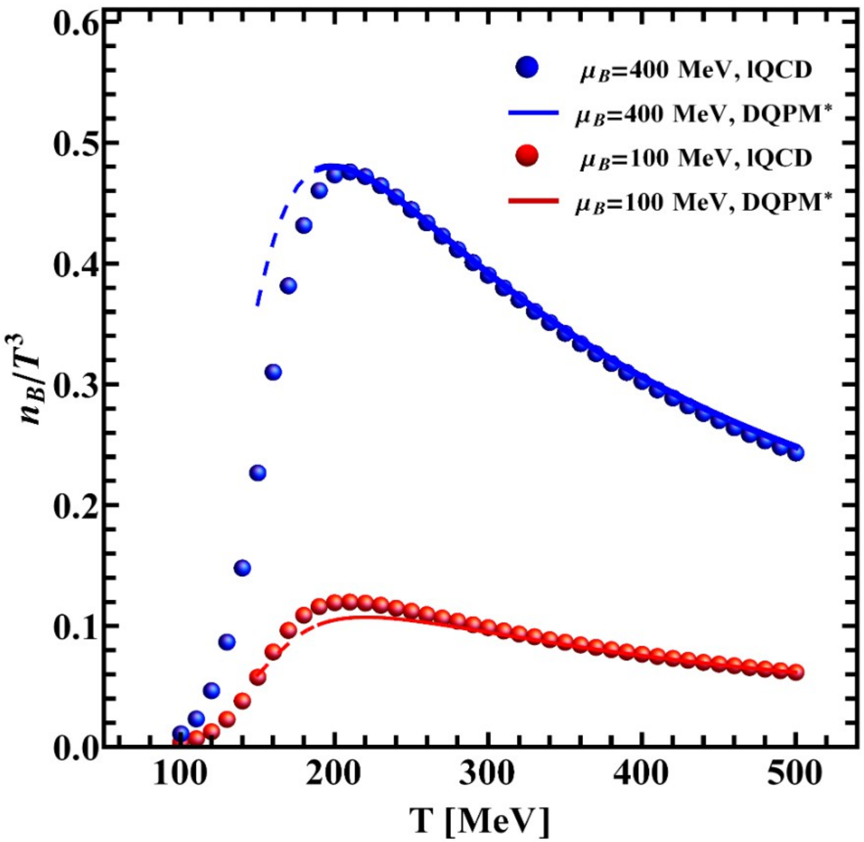
$$E = TS - P + \mu N$$

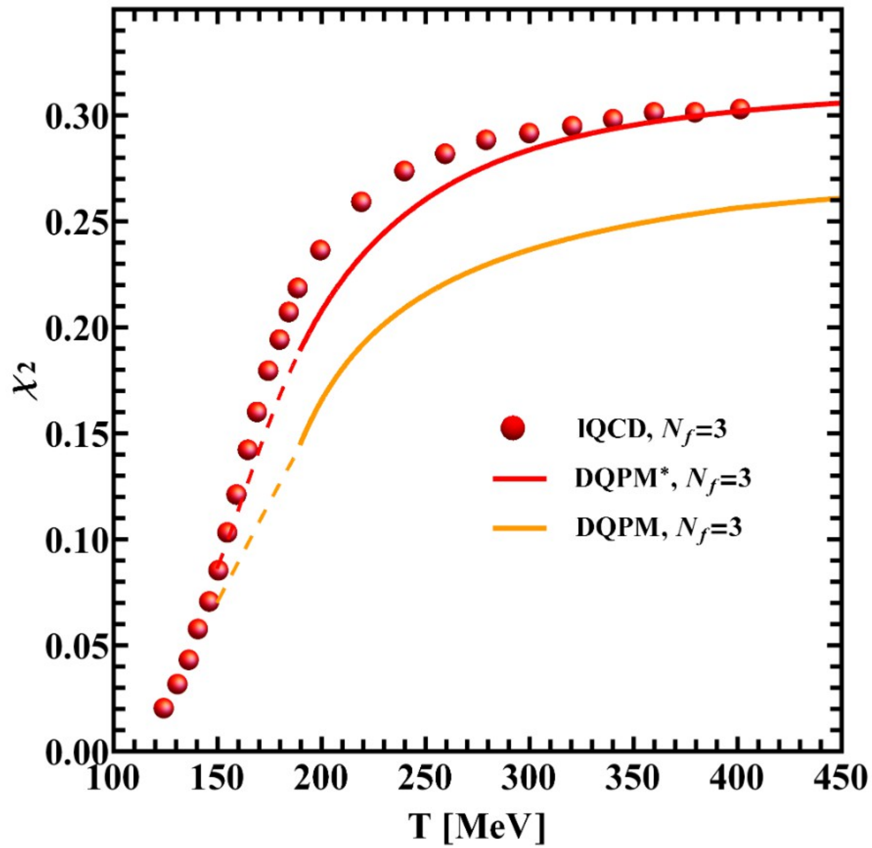
Momentum dependent DQPM* reproduces the EoS at T > 170 MeV.

Phys. Rev. C93 (2016) no. 4, 044914
Int. J. Mod. Phys. E25 (2016) no. 07, 1642003

DQPM* at finte μ

- Default quasiparticle models fail to describe χ_B.
- Running selfenergies improve the susceptibility:





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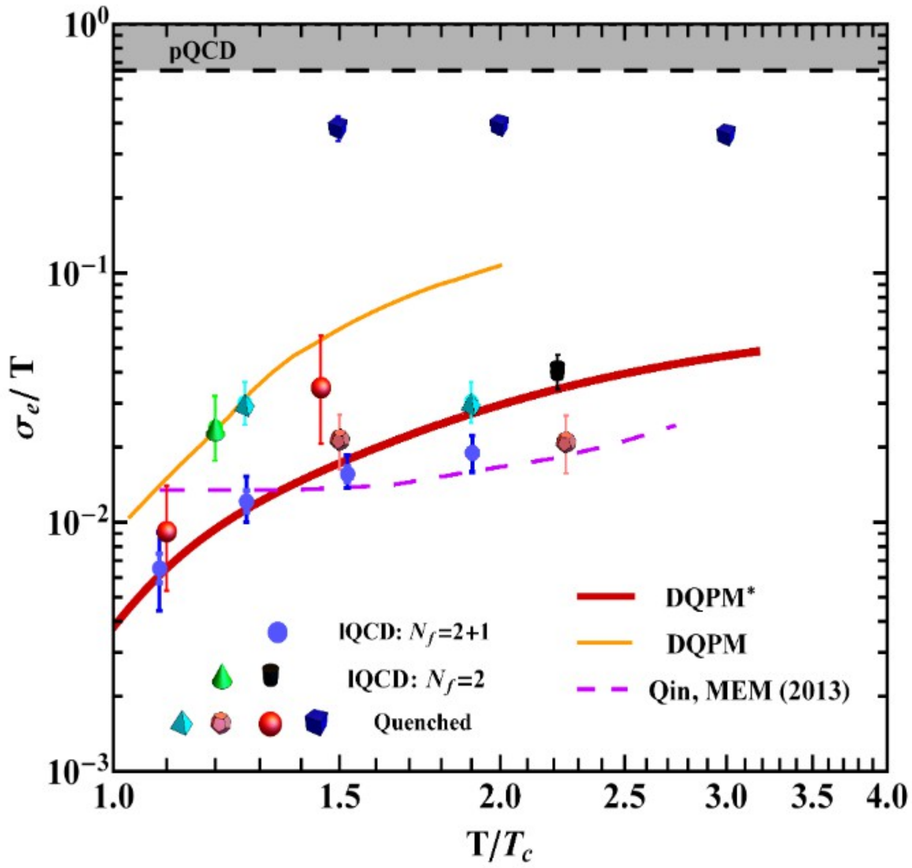
Transport coefficients

- The width so far is not well fixed by the EoS.
- Use transport coefficients

Electric conductivity in relaxation time approach:

$$\sigma_e(T,\mu_q) = \sum_{f,\bar{f}}^{u,d,s} \frac{e_f^2 \; n_f^{\text{off}}(T,\mu_q)}{\bar{\omega}_f(T,\mu_q) \; \bar{\gamma}_f(T,\mu_q)}$$



Conductivity probes only the quark width γ_f.

Phys. Rev. C93 (2016) no. 4, 044914
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Transport coefficients

- **BW-spectralfunction in relaxation time approximation**

Off-shell bulk and shear viscosity:

Probes quarks and gluons!

$$\eta(T,\mu_q) = \frac{1}{15T} d_g \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \omega \; \bar{\tau}_g(T,\mu_q) \; f_g(\omega/T) \times \rho_g(\omega,\boldsymbol{p}) \frac{\boldsymbol{p}^4}{\omega^2} \Theta(P^2)$$

$$+ \frac{1}{15T} \frac{d_q}{6} \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \omega \left[\sum_q^{u,d,s} \bar{\tau}_q(T,\mu_q) f_q((\omega-\mu_q)/T) \rho_q(\omega,\boldsymbol{p}) \right.$$

$$\left. + \sum_{\bar{q}}^{\bar{u},\bar{d},\bar{s}} \bar{\tau}_{\bar{q}}(T,\mu_q) f_{\bar{q}}((\omega+\mu_q)/T) \; \rho_{\bar{q}}(\omega,\boldsymbol{p}) \right] \frac{\boldsymbol{p}^4}{\omega^2} \Theta(P^2)$$

Integrate only over timelike part of the spectralfunction!

Transport coefficients

- **BW-spectralfunction in relaxation time approximation**

Off-shell bulk and shear viscosity:

Probes quarks and gluons!

$$\zeta(T,\mu_q) = \frac{1}{9T} d_g \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \omega\ \bar{\tau}_g(T,\mu_q)\ f_g(\omega/T)\ \rho_g(\omega,\boldsymbol{p})\ \Theta(P^2) \frac{1}{\omega^2}\ F_g(\omega,\boldsymbol{p})$$

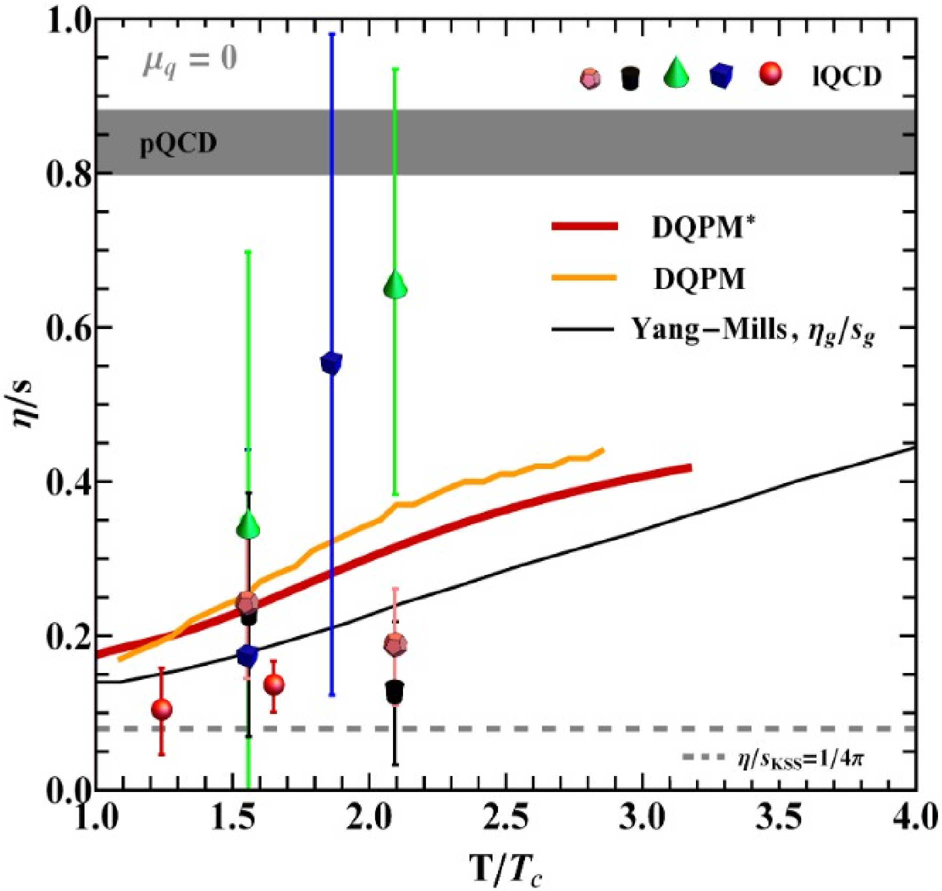
$$+ \frac{1}{9T} \frac{d_q}{6} \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \omega \left[\sum_q^{u,d,s} \bar{\tau}_q(T,\mu_q) f_q((\omega-\mu_q)/T) \rho_q(\omega,\boldsymbol{p}) \right.$$

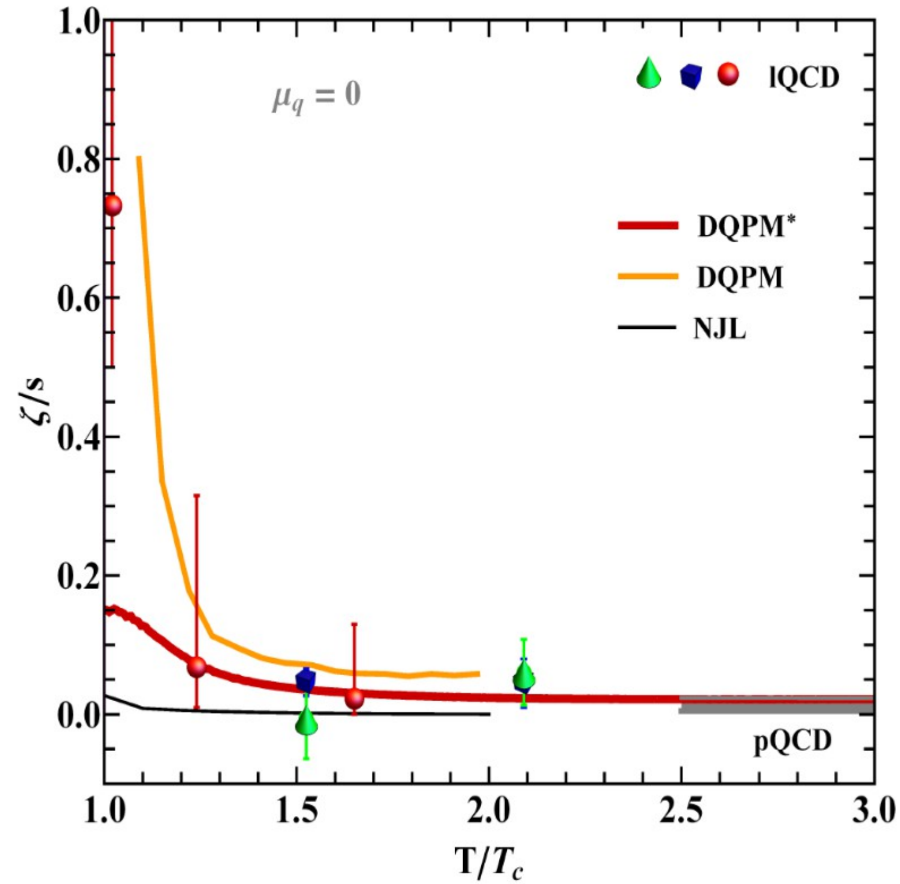
$$\left. + \sum_{\bar{q}}^{\bar{u},\bar{d},\bar{s}} \bar{\tau}_{\bar{q}}(T,\mu_q) f_{\bar{q}}((\omega+\mu_q)/T)\ \rho_{\bar{q}}(\omega,\boldsymbol{p}) \right] \Theta(P^2) \frac{1}{\omega^2}\ F_q(\omega,\boldsymbol{p})$$

$$F_i(\omega,\boldsymbol{p}) = \left[\boldsymbol{p}^2 - 3c_s^2 \left(\omega^2 - T^2 \frac{dM_i^2}{dT^2} \right) \right]^2$$

Transport coefficients

- Transport coefficients are sensitive to the width.
- Matching to lattice justifies functional form:





Phys. Rev. C93 (2016) no. 4, 044914

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Hadronic EoS including interacting nucleons

- **Nonlinear Walecka interaction for nucleons:**

$$\mathcal{L}_B = \bar{\Psi}\left(i\gamma_\mu \partial^\mu - M\right)\Psi$$

$$\mathcal{L}_M = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega^\mu \omega_\mu$$

$$\mathcal{L}_{int} = g_\sigma \bar{\Psi}\sigma\Psi - g_\omega \bar{\Psi}\gamma^\mu \omega_\mu \Psi$$

- **σ-interaction connects to chiral symmetry restoration:**

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}B\sigma^3 + \frac{1}{4}C\sigma^4$$

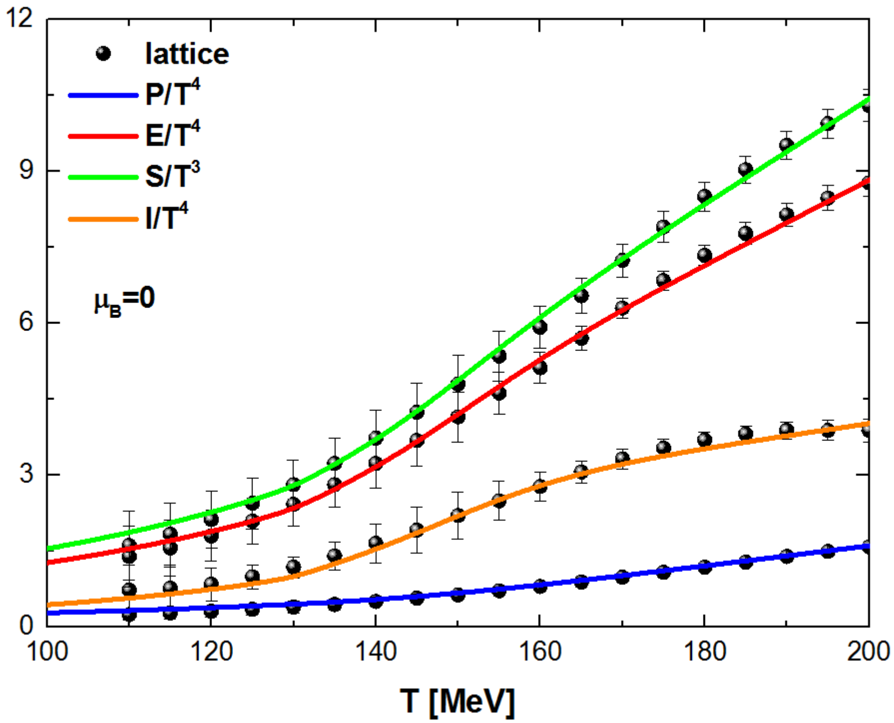
$$\frac{\partial U}{\partial \sigma} = g_\sigma \rho_s \qquad \frac{\langle \bar{q}q\rangle}{\langle \bar{q}q\rangle_0} = 1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho_s$$

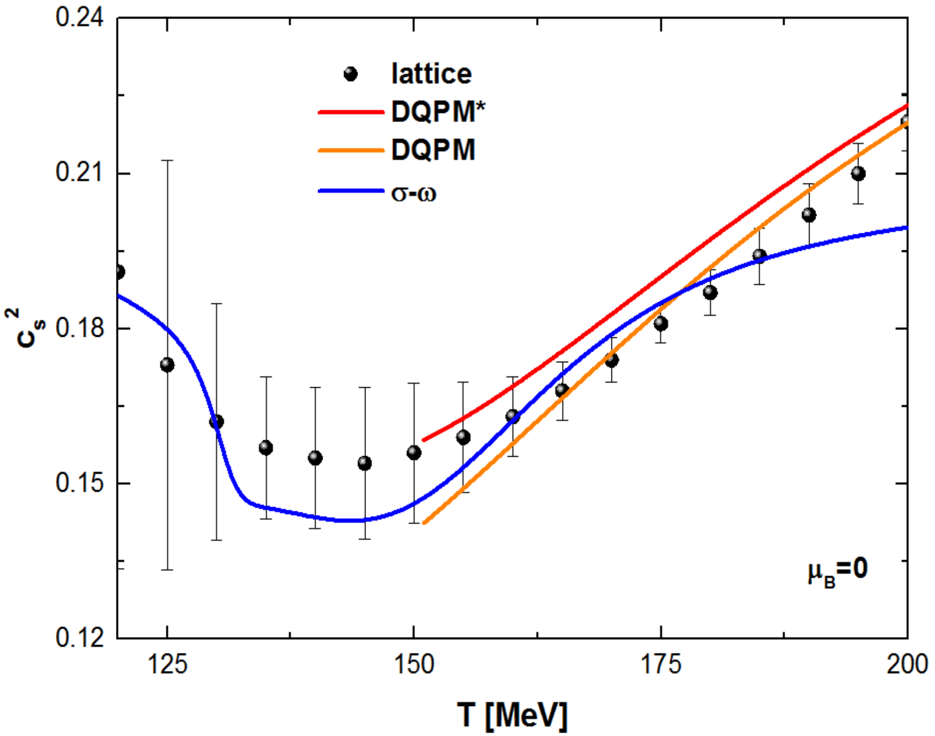
- **More about chiral symmetry restoration on wednesday from Alessia Palmese**

Hadronic EoS, μ_B=0

- Include important baryons with strong interactions and mesons as noninteracting particles.

- Resulting EoS describes hadronic part of the EoS:



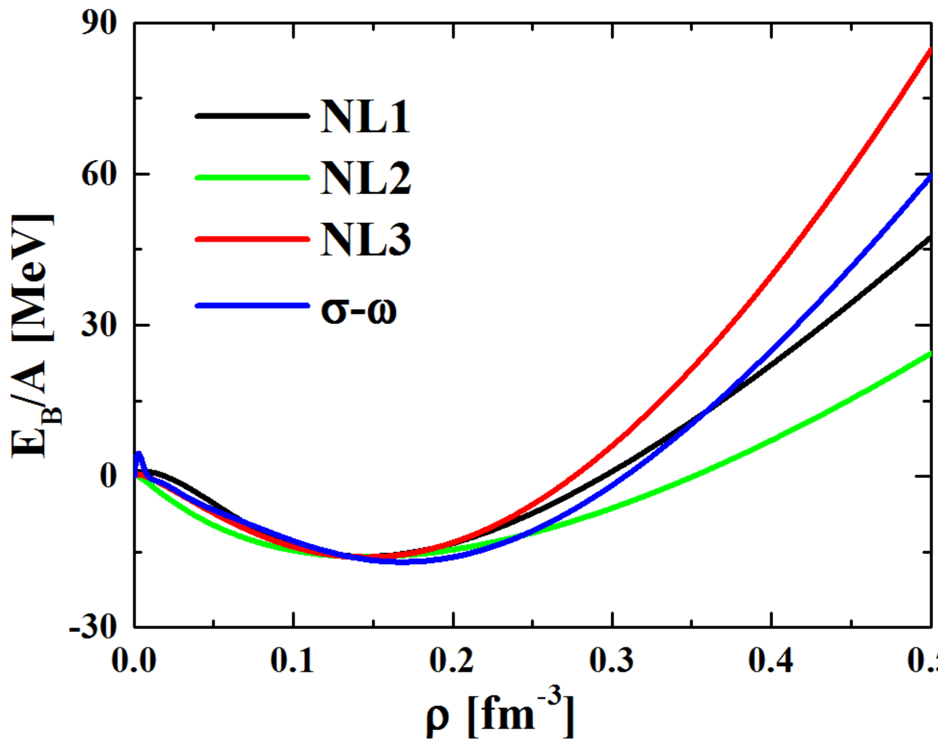


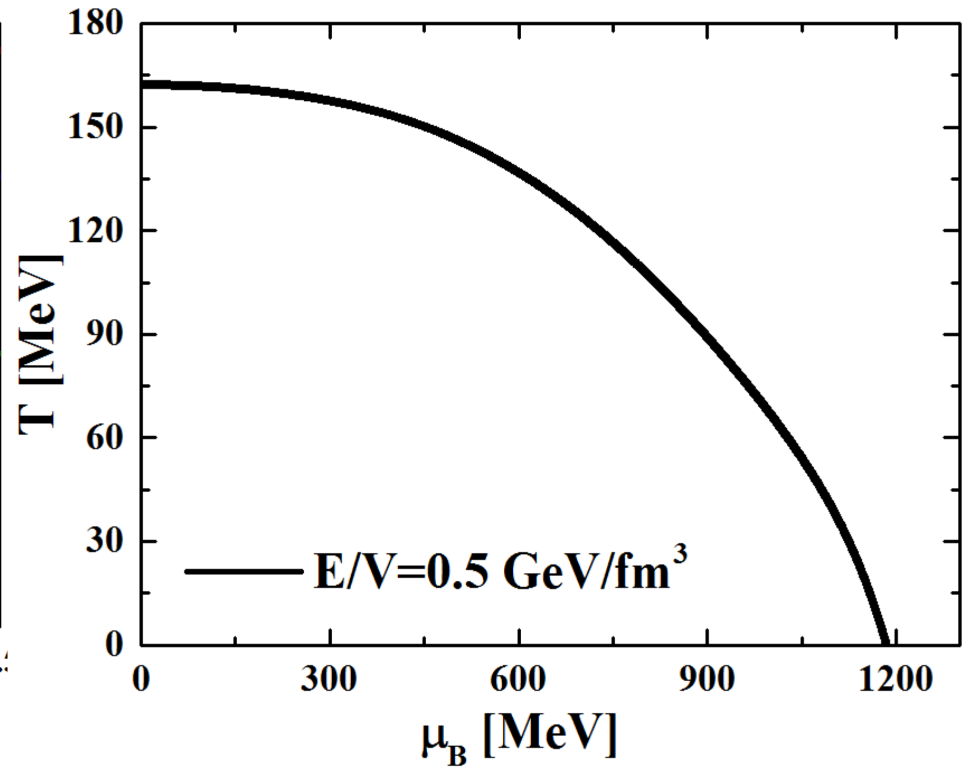
Here only interacting nucleons, generalization possible.

Hadronic EoS, T=0

- Nuclear EoS defines the vector interaction.
- Density dependent vector coupling: $m_\omega^2 \omega^\mu = \Gamma(\rho_B)\rho_B$

Walecka + HRG consistent with nuclear and lattice EoS.





Summary

- **DQPM* defines parton propagators**
- **Propagator enables transport in KB-framework**
- **Susceptibilities challenge quasiparticle models**
- **Mom. dep. Selfenergies reproduce EoS + χ_B**
- **Width is controlled by transport coefficients**

DQPM* is in line with lQCD EoS and correlators.

- **Hadronic EoS is controlled by Walecka interaction,**

Walecka+HRG is in line with nuclear and lattice EoS.

PHSD group 2016

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