

**EbyE fluctuations in the EKRT
pQCD+saturation+hydrodynamics model:
Determining QCD matter shear viscosity in
ultrarelativistic A+A collisions**

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Reviewing the results from:

Phys.Rev. C93 (2016) 024907, arXiv:1505.02677 [hep-ph]

Phys.Rev. C93 (2016) 014912, arXiv:1511.04296 [hep-ph]

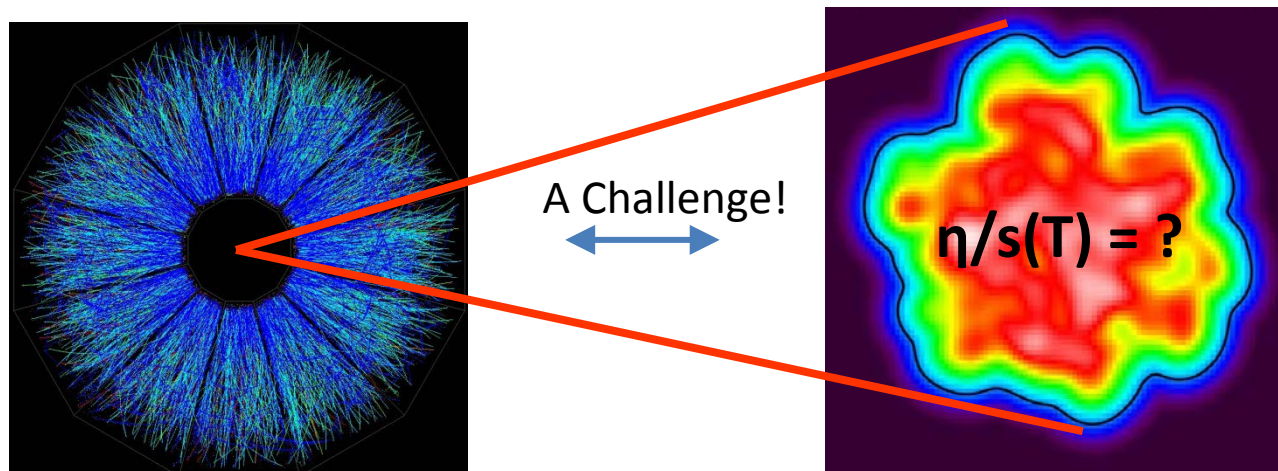
Talk plan

1. Background and some history

- original EKRT model

2. Recent developments in EKRT [Phys.Rev. C93 (2016) 024907]

- NLO pQCD, EbyE viscous hydro framework
- **Comparison with LHC & RHIC data $\rightarrow \eta/s(T)$**



3. Predictions for the 5.02 TeV Pb+Pb LHC run

- **Compare** [Phys.Rev. C93 (2016) 014912] **with measurements**

1. Background: pQCD + saturation + hydro = EKRT model

[KJE, Kajantie, Ruuskanen, Tuominen, hep-ph/9909456, NPB570 (2000) 379]

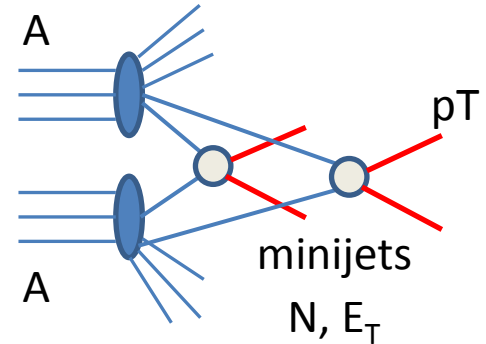
Collinear factorization + pQCD

→ # few-GeV gluon "minijets" in A+A, y at ΔY , $p_T \geq p_0$

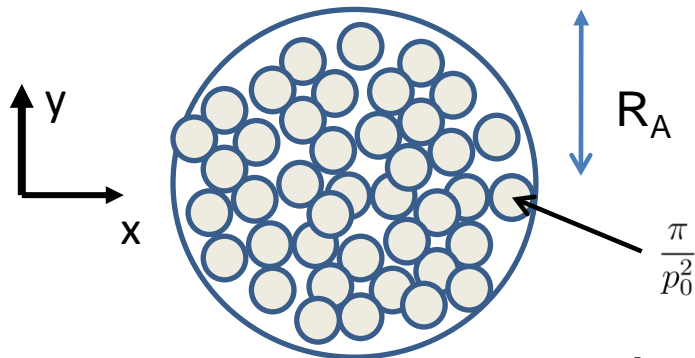
$$N_{AA}(p_0, \sqrt{s}, \mathbf{0}, \Delta Y) = 2T_{AA}(\mathbf{0})\sigma_{\text{jet}}(p_0, \sqrt{s}, \Delta Y, A)$$

$$T_{AA}(\mathbf{b}) = \int d^2\mathbf{s} T_A(\mathbf{s}) T_A(\mathbf{b} - \mathbf{s}) \quad T_A(\mathbf{b}) = \int dz n_A(r)$$

$$\sigma_{\text{jet}}(p_0, \sqrt{s}, \Delta Y, A) = K \frac{1}{2} \sum_{\substack{ijkl= \\ g, q, \bar{q}}} \int_{\Delta Y}^{\frac{p_0^2}{\Delta Y}} dp_T^2 dy_1 dy_2 x_1 f_{i/A}(x_1, Q^2) x_2 f_{j/A}(x_2, Q^2) \frac{d\hat{\sigma}^{ij \rightarrow kl}}{d\hat{t}}, \quad \text{pQCD + nuclear PDFs}$$



Collision geometry

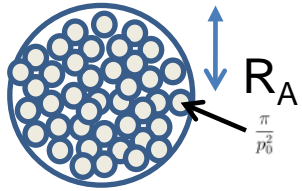


Saturation of minijet (= gluon) production

when $p_0 = 1...2$ GeV: lower- p_T gluons conjectured to be not relevant due to fusion of produced gluons

[Concept of saturation introduced by Gribov, Levin&Ryskin '83, Mueller&Qiu '86, and in the CGC framework by McLerran&Venugopalan '94, and noticed by us in '96]

[Original EKRT model]

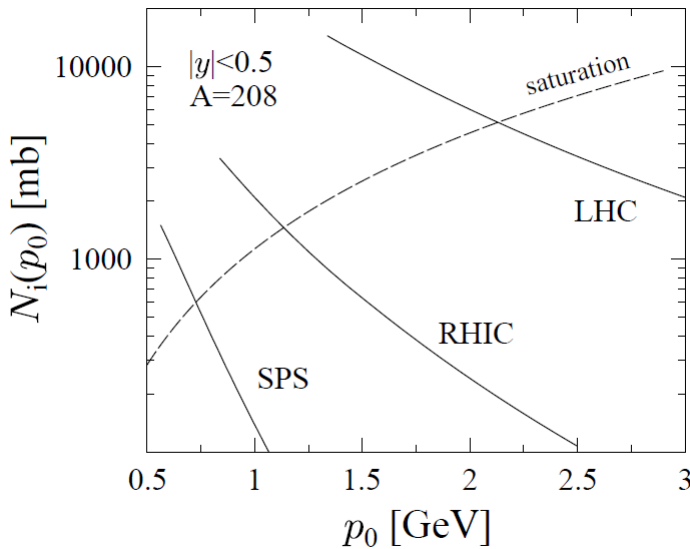


Saturation of gluon production in A+A at b=0 in $\Delta Y=1$ when

$$N_{AA}(p_0, \sqrt{s}, \mathbf{0}, \Delta Y = 1) \times \frac{\pi}{p_0^2} = \pi R_A^2$$

→ $p_{\text{sat}} = p_0(\mathbf{vs}, \mathbf{A})$
 → minijet $N_i(p_{\text{sat}})$ and $E_T(p_{\text{sat}})$

[EKRT, hep-ph/9909456, NPB570 (2000) 379]



$$dN/dy \propto A^{0.92} s^{0.19 \dots 0.2}$$

- QGP **forms early**: $\tau_i = 1/p_{\text{sat}} = 0.1 \dots 0.2$ fm
- the produced QGP looks "thermal" in E_T/N :
 - may assume early thermalization, $\tau_0 = \tau_i$
 - **initial conditions for ideal hydro (1 D BJ)**

$$N_i(p_{\text{sat}}) \text{ or } E_T(p_{\text{sat}}) \rightarrow S_i = S_f \rightarrow N_f$$

- **Predicted very definite scaling laws, before any(!) RHIC data**

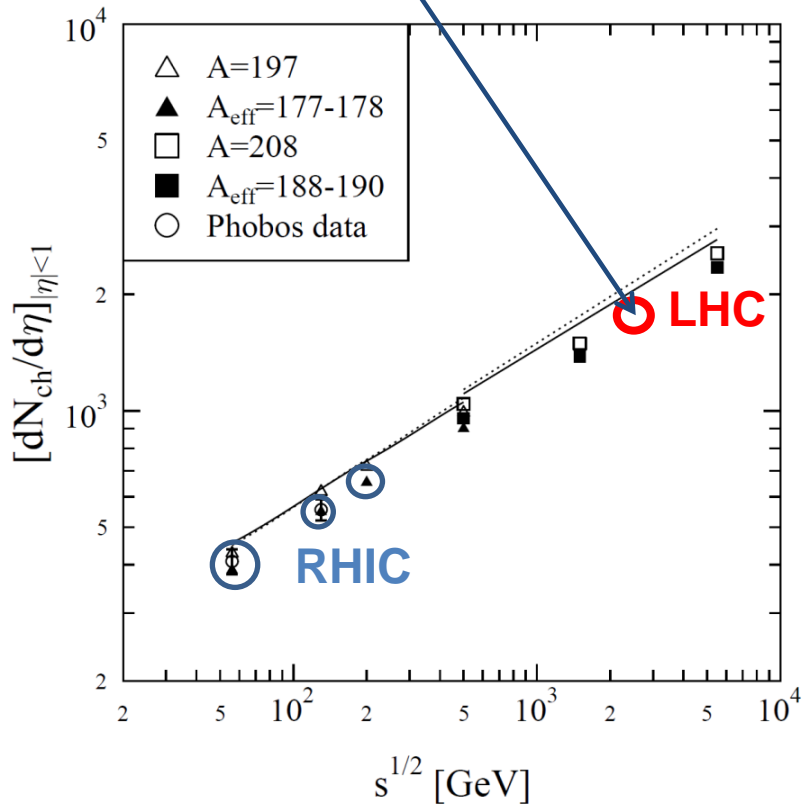
[EKRT, hep-ph/9909456, NPB570 (2000) 379]

Hard scaling $A^{4/3}$ was tamed to $\sim A$ & Power law in cms-energy !

[Comment in 2001 in CERN (=after the first RHIC data) "It can never be a power-law in s!"]

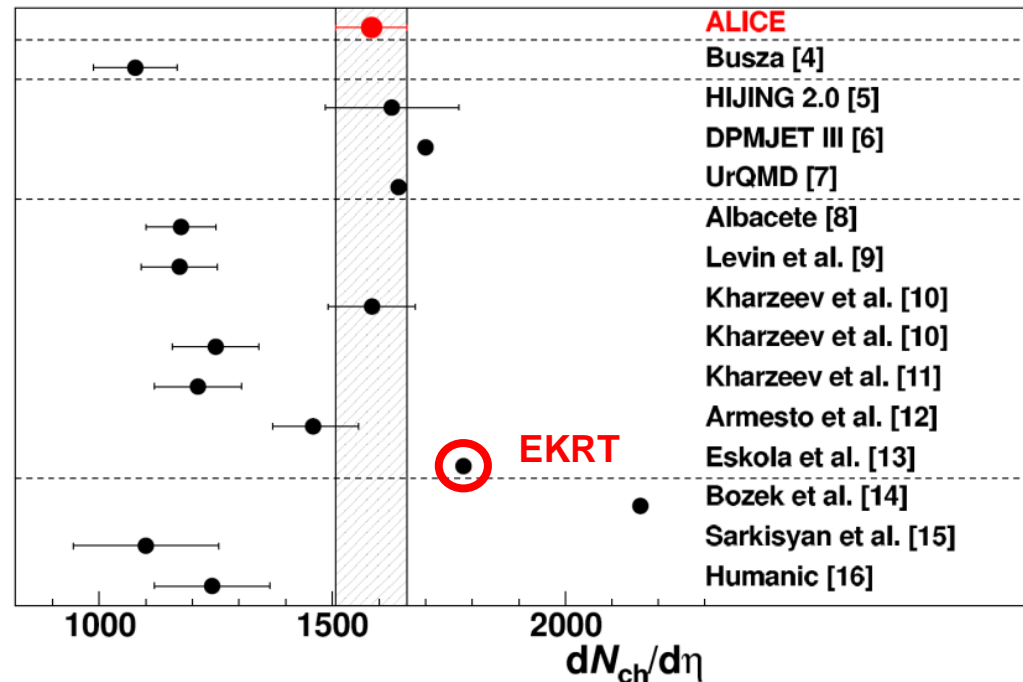
A more detailed EKRT-model prediction

- ideal 1+1 D hydro, LO nPDFs + pQCD partly NLO [KJE, Tuominen, Phys.Rev. D63 (2001) 114006] [KJE,Ruuskanen,Räsänen, Tuominen,NPA696:715,2001]
- made **before** the 200 GeV RHIC data
- $dN_{ch}/d\eta = 1782$ for 2.76 ATeV



ALICE, PRL105:252301, **2010**

1584 ± 76



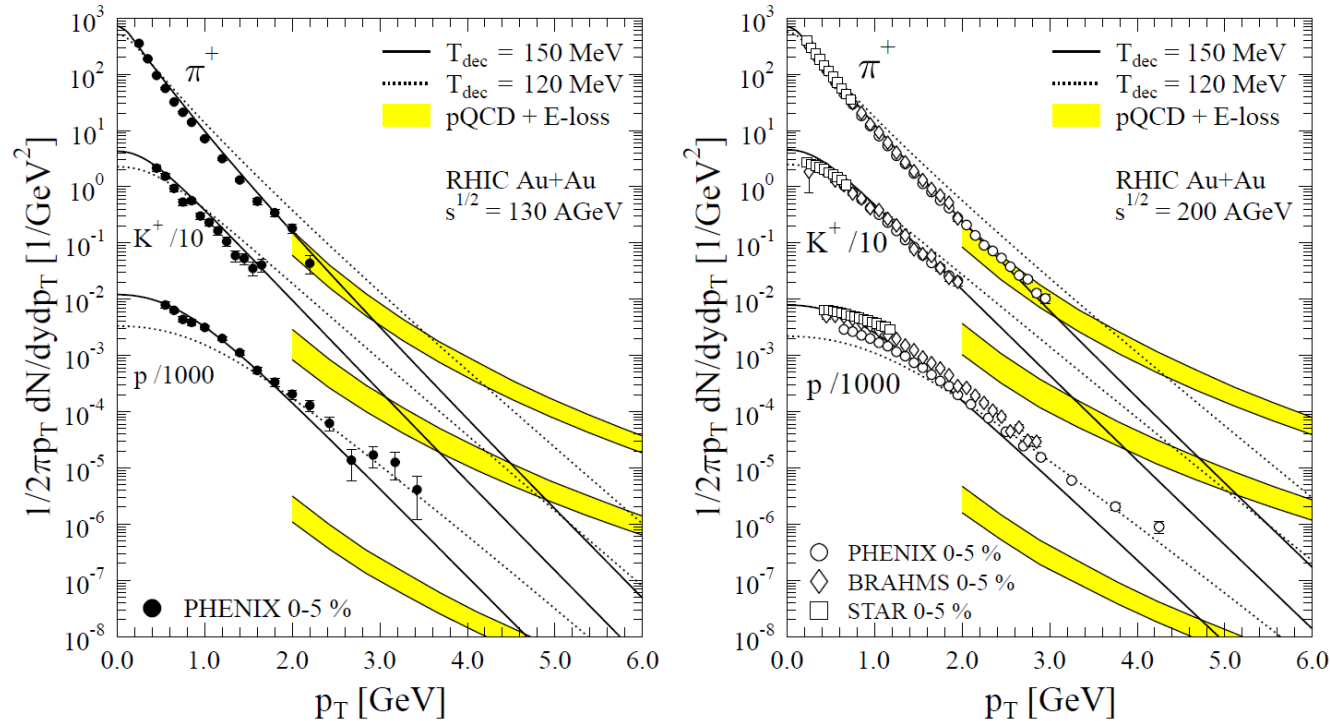
EKRT correct within ~7%

Our prediction 2560 for the max cms-energy was VERY LOW AT THE TIME, << 8000, but after RHIC 200 GeV data, suddenly **WE** were on the **high-side** of predictions!

[Miklos in 2007 at CERN: "I'm glad your knees don't wobble!" (but they did...)]

Successful framework tests in Au+Au at RHIC (still with **ideal hydro**)

[KJE, Honkanen, Niemi, Ruuskanen, Räsänen, PRC72 (2005) 044904]



EKRT observation: To get these p_T spectra right means a factor **THREE(!) reduction from the computed $E_{\text{Tinitial}}(p_{\text{sat}})$ to the measured E_{Tfinal} = **PRESSURE at work during the hydrodynamic stage!****

OK but how about the detailed properties of QCD matter...?

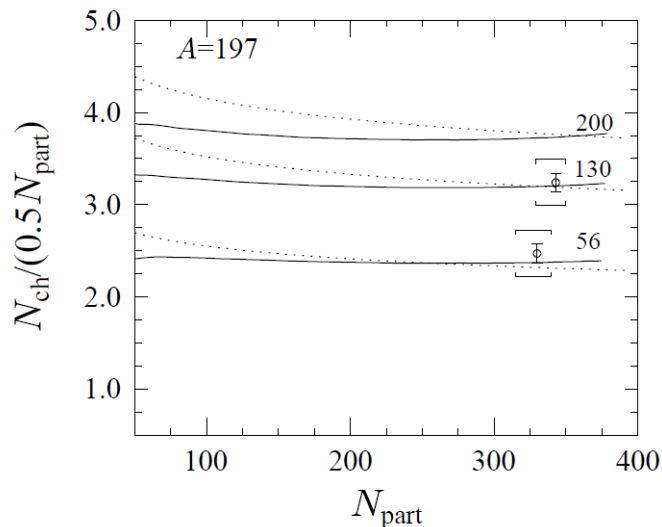
[Original EKRT model]

History remark: Centrality dependence in EKRT was in fact **not** a problem !

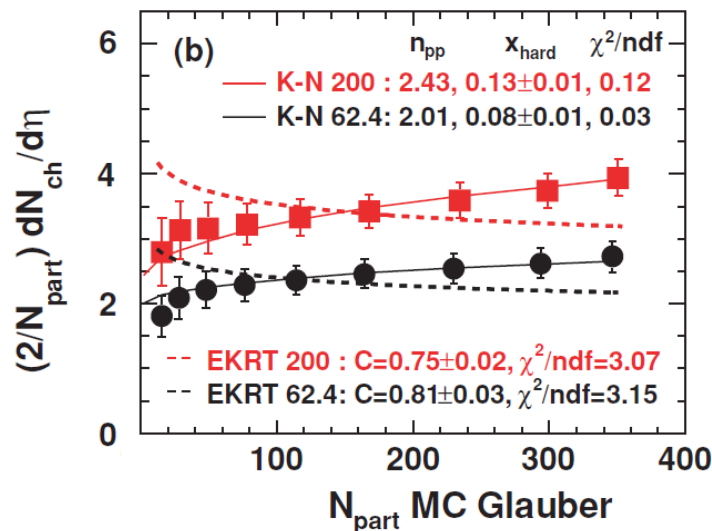
Localizing the model for non-central collisions:

$$\rho_{\text{sat}} = \rho_0(\mathbf{v}_s, \mathbf{A}, \mathbf{x}, \mathbf{y})$$

[KJE, Kajantie, Tuominen, Phys.Lett. B497 (2001) 39]



[STAR, PRC79, 034909 (2009)]



EKRT prediction: $N_{\text{ch}}/N_{\text{part}}$ vs N_{part} flat or even slightly rising towards non-central A+A, seemed **not** agree with data...

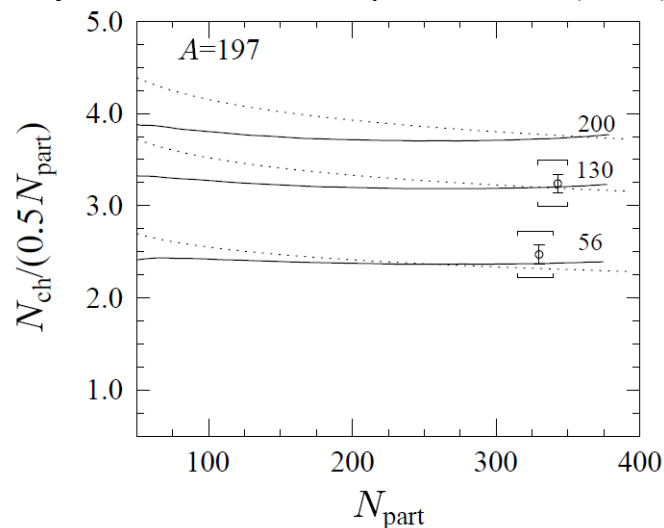
[Original EKRT model]

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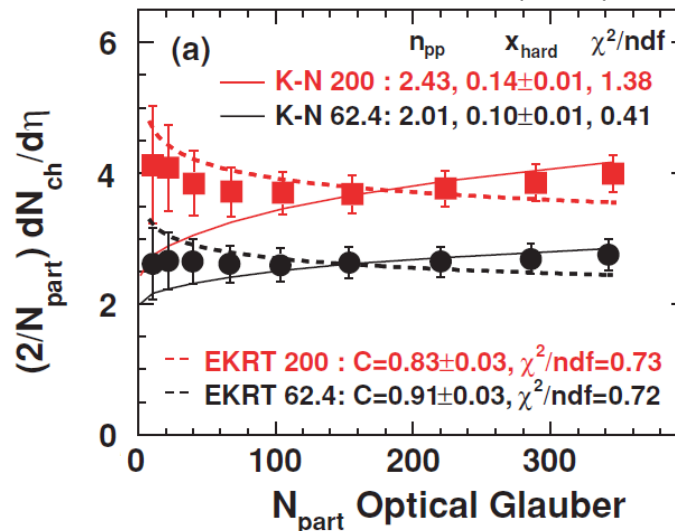
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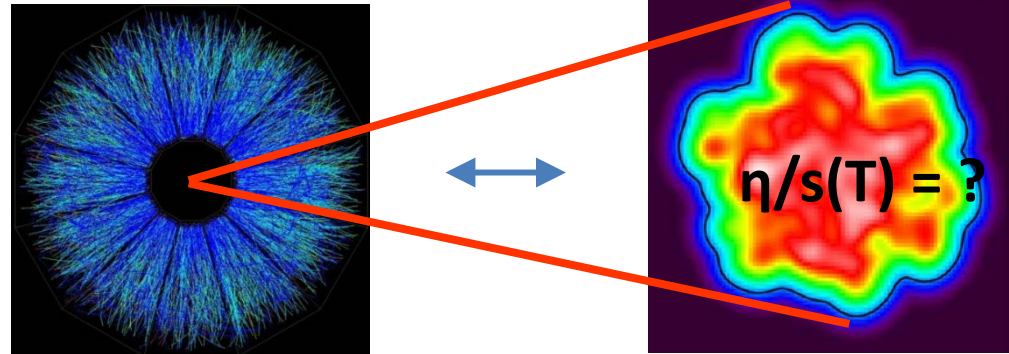
EKRT prediction $N_{\text{ch}}/N_{\text{part}}$ vs N_{part} is flat or even slightly rising towards non-central A+A, did agree with data when the same optical Glauber model was used for Npart !

[This thorough STAR analysis revived also my interest in the more detailed EKRT studies discussed next]

2. Recent developments in EKRT

- full NLO pQCD & improved minijet ET definition
- new angle to saturation
- viscous 2+1 D hydro
- **EbyE framework**

[Phys.Rev. C87 (2013) 044904]
[Phys.Lett. B731 (2014) 126]
[**Phys.Rev. C93 (2016) 024907**]



Basic idea:

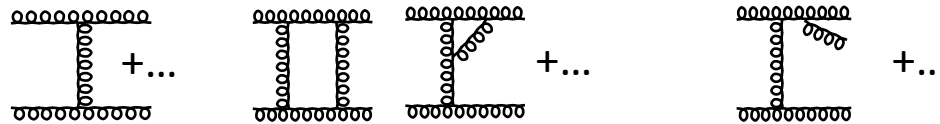
1. Compute minijet (=gluons, p_T = a few GeV) ET production in A+A, using **NLO perturbative QCD** + **saturation** conjecture for ET
2. Describe space-time evolution of QCD matter with **viscous hydrodynamics** initialized with **fluctuating** pQCD+saturation initial conditions, **event by event**
3. Compare with LHC & RHIC data for bulk (=low p_T) observables, to
 - **pin down QCD matter $\eta/s(T)$**
 - **test the initial state calculation and its predictive power**
 - **study the applicability region of viscous hydro**

Minijet ET production in A+A and Δy from NLO pQCD

$$\frac{dE_T(p_0, \sqrt{s}, \Delta y, \mathbf{s}, \mathbf{b})}{d^2\mathbf{s}} = T_A(\mathbf{s} + \mathbf{b}/2)T_A(\mathbf{s} - \mathbf{b}/2)\sigma\langle E_T\rangle_{p_0, \Delta y}$$

- $\mathbf{s}=(x,y)$ transverse position, $\Delta y =$ unit rapidity window
- $T_A T_A$ accounts for nuclear collision geometry
- Collinear factorization, NLO pQCD in $2 \rightarrow 2$ and $2 \rightarrow 3$ parton scatterings:

$$\sigma\langle E_T\rangle_{p_0, \Delta y} = \int d[\text{PS}]_2 \frac{d\sigma^{2 \rightarrow 2}}{d[\text{PS}]_2} S_2 + \int d[\text{PS}]_3 \frac{d\sigma^{2 \rightarrow 3}}{d[\text{PS}]_3} S_3$$



$$\frac{d\sigma^{2 \rightarrow n}}{d[\text{PS}]_n} \sim \sum_{g, q, \bar{q}} f_{i/A}(x_1, Q^2, \mathbf{s}) \otimes f_{j/A}(x_2, Q^2, \mathbf{s}) \otimes |\mathcal{M}|^2$$

- $f_{i/A} =$ **spatially dependent** nPDFs (see next page)
- UV-renormalized $|\mathcal{M}|^2$ [Ellis, Sexton, Nucl. Phys. B269 (1986) 445, Paatelainen's PhD thesis]
- Ellis-Kunszt-Soper subtraction method [see KS, Phys. Rev. D46 (1992) 192; KJE, Tuominen, Phys. Rev. D63 (2001) 114006] with IR/CL safe **measurement functions** S_2 & S_3 to define the minijet ET in NLO

Spatially dependent nPDFs

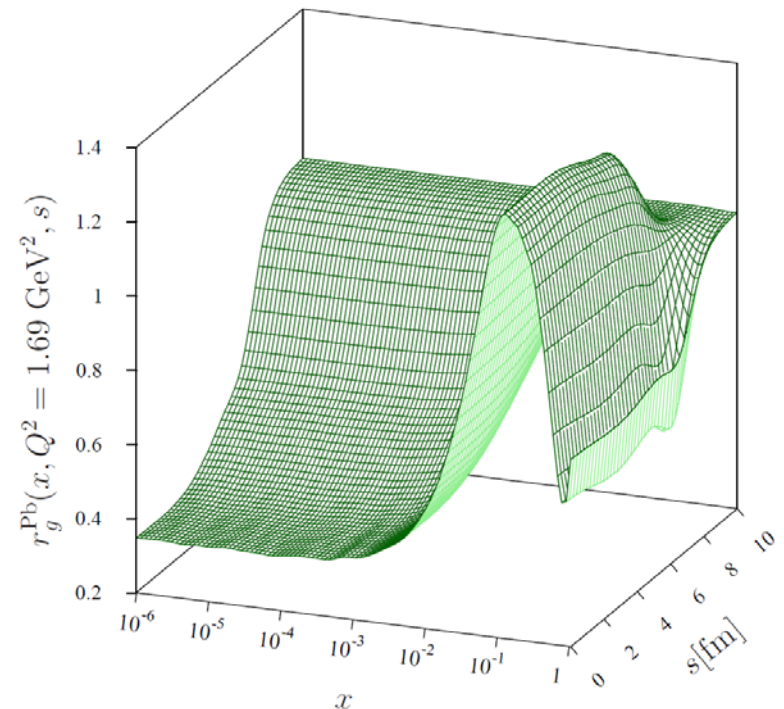
$$f_{i/A}(x, Q^2, \mathbf{s}) \equiv r_i^A(x, Q^2, \mathbf{s}) \otimes f_i^p(x, Q^2)$$

- $f_i^p(x, Q^2)$: free proton PDFs from CTEQ6M
- $r_i^A(x, Q^2, \mathbf{s})$: **spatially dependent** NLO **EPS09s** nuclear PDF modifications from [Helenius et al, JHEP 1207 (2012) 073], where the A-dependences of the EPS09 nPDF modification factors $R_i^A(x, Q^2)$ were converted into \mathbf{s} -dependences via

$$r_i^A(x, Q^2, \mathbf{s}) = 1 + \sum_{j=1}^n c_j^i(x, Q^2) [T_A(\mathbf{s})]^j$$

$$R_i^A(x, Q^2) \equiv \frac{1}{A} \int d^2\mathbf{s} T_A(\mathbf{s}) r_i^A(x, Q^2, \mathbf{s})$$

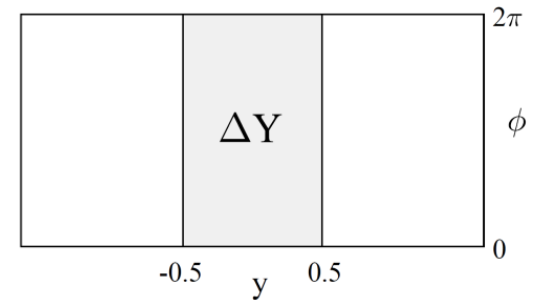
Nuclear modifications are strongest near the center of a nucleus, and **weaken towards the edge**



Measurement functions for computing minijet ET in NLO

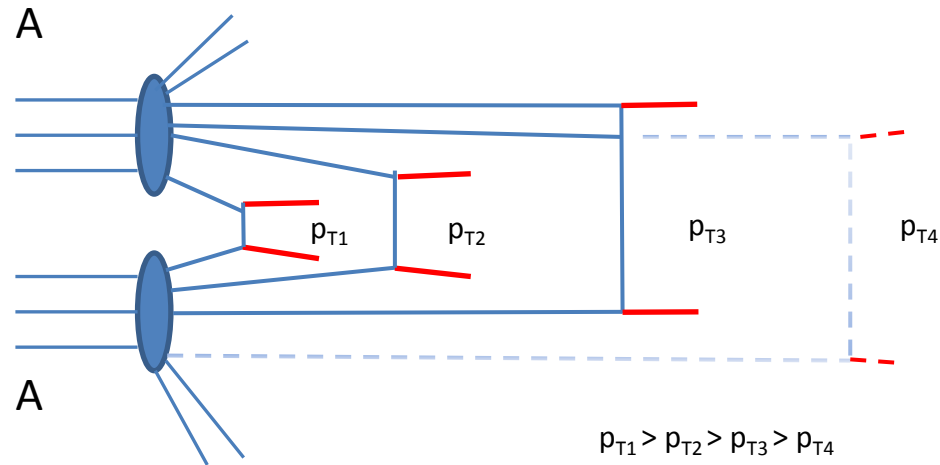
$$S_n = \underbrace{\left[\sum_{i=1}^n p_{T,i} \Theta(y_i \in \Delta y) \right]}_{\text{Minijet } E_T \in \Delta y} \times \underbrace{\Theta \left(\sum_{i=1}^n p_{T,i} \geq 2p_0 \right)}_{\text{Hard scat. of partons}} \times \underbrace{\Theta(E_{T,n} \geq \beta \times p_0)}_{\text{Minimum } E_T \in \Delta y}$$

- analogous to jet definition; S_n define
 - the minijet ET in Δy
 - what we mean by hard scattering (p_0)
 - what is the min ET we may require in Δy



- IR & CL safeness: $S_3 \rightarrow S_2$ at IR & CL limits
- **any** β in $[0,1]$ is **OK** \rightarrow leave β as a free parameter since the minijet ET is **not** a direct observable
 [Paatelainen, KJE, Holopainen, Tuominen, Phys. Rev. C87 (2013) 4, 044904]
- with these S_n + given nPDFs, minijet NLO ET computation is **well defined!**

A new angle to saturation [Paatelainen,KJE,Holopainen,Tuominen, PRC87 (2013) 044904]
 instead of fusion of produced gluons, we conjecture saturation to happen when
 E_T from $3 \rightarrow 2$, $4 \rightarrow 2, \dots$ processes becomes of the same order as the E_T from $2 \rightarrow 2$:

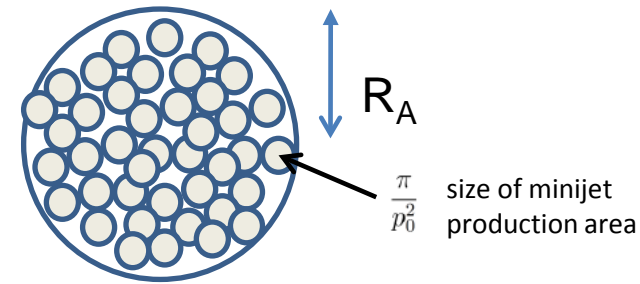


$$\frac{dE_T}{d^2sdy}(2 \rightarrow 2) \sim \frac{dE_T}{d^2sdy}(3 \rightarrow 2) \sim \text{H.O.}$$

$$(T_{AG_A})^2 \frac{\alpha_s^2}{p_0} \sim (T_{AG_A})^3 \left(\frac{\alpha_s}{p_0} \right)^3 \Rightarrow T_{AG_A} \sim \frac{p_0^2}{\alpha_s} \Rightarrow \frac{dE_T}{d^2sdy} \sim p_0^3$$

⇒ **Saturation criterion** like in the original EKRT but now consistently **for ET**:

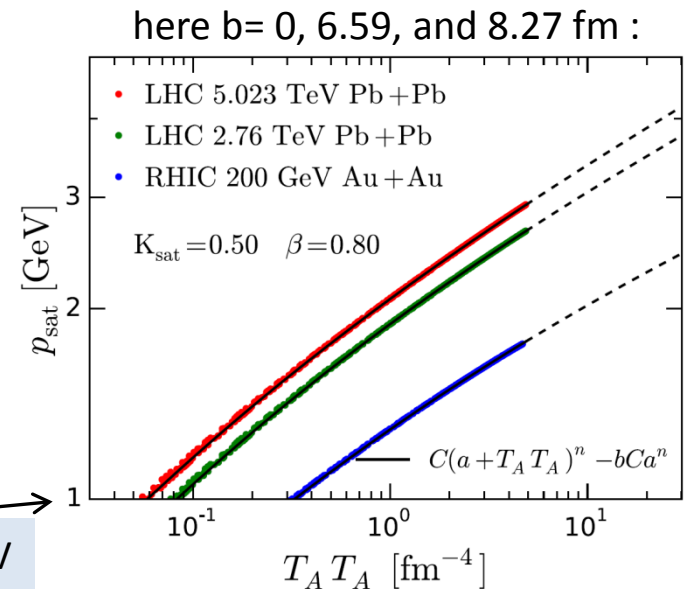
$$\underbrace{\frac{dE_T}{d^2\mathbf{s}}(p_0, \sqrt{s}, \dots, \beta)}_{= \text{NLO pQCD part}} = \left(\frac{K_{\text{sat}}}{\pi} \right) p_0^3 \Delta y$$



$$\Rightarrow p_0 = p_{\text{sat}}(\sqrt{s_{NN}}, A, \mathbf{b}, \mathbf{s}; \beta, K_{\text{sat}}) \longleftarrow K_{\text{sat}} \text{ from data}$$

Key observation: p_{sat} scales with $T_A T_A$:
This enables the EbyE framework for us...

[Paatelainen, KJE, Niemi, Tuominen, Phys.Lett. B731 (2014) 126]



$\min(p_{\text{sat}}) = 1 \text{ GeV}$

[Niemi, KJE, Paatelainen, Tuominen, Phys.Rev. C93 (2016) 014912]

NLO EKRT EbyE framework [Niemi, KJE, Paatelainen, Phys.Rev. C93 (2016) 024907]

- Nucleon positions in A: sample **WS distribution**
- Around each nucleon, set a **gluon cloud of transverse density**

$$T_n(s) = \frac{1}{2\pi\sigma^2} e^{-s^2/2\sigma^2}$$

$\sigma = 0.43$ fm from HERA $\gamma^* p \rightarrow J/\psi + p$ data

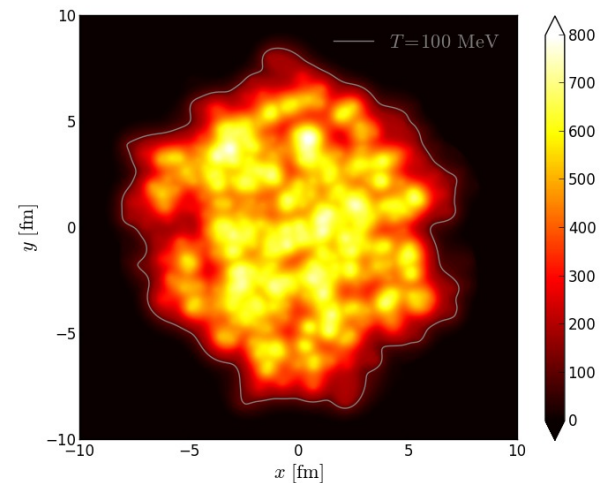
\Rightarrow **Overlap functions $T_{A1}(x,y)$ & $T_{A2}(x,y)$** $\Rightarrow p_{\text{sat}} = p_{\text{sat}}(T_{A1} * T_{A2})$ (*)

\Rightarrow Local energy density at $\tau_{\text{sat}} = 1/p_{\text{sat}}(\mathbf{s})$

$$\epsilon(\mathbf{s}, \tau_{\text{sat}}) = \frac{dE_T(p_{\text{sat}}, \dots, \beta)}{d^2\mathbf{s}} \frac{1}{\tau_{\text{sat}}(\mathbf{s}) \Delta y} = \frac{K_{\text{sat}}}{\pi} p_{\text{sat}}(\mathbf{s})^4$$

- "Pre-thermal" evolution from $\tau_{\text{sat}} = 1/p_{\text{sat}}(x,y)$ to $1/p_{\text{sat}}^{\text{min}} = \mathbf{0.2 fm/c}$ here done simply with 1 D Bjorken hydro at each (x,y) (we tested also free streaming, both OK)
- Below $p_{\text{sat}}^{\text{min}} = 1$ GeV, connect smoothly to BC profile

run 2+1 D viscous hydro EbyE



(*) Parametrization of $p_{\text{sat}}(T_{A1}T_{A2})$ vs (K_{sat}, β) is available for public use in Phys.Rev. C93 (2016) 024907

Viscous Hydrodynamics [Niemi et al]

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (P + \cancel{\Pi})(g^{\mu\nu} - u^\mu u^\nu) + \underline{\pi^{\mu\nu}}$$

Neglect heat conductivity & bulk viscosity;
 Keep **shear viscosity η** ;
 2nd order dissipative relativistic hydrodyn.

$$\partial_\mu T^{\mu\nu} = 0 \quad + \quad \text{transient fluid-dynamics EoM for } \pi^{\mu\nu}$$

[Denicol, Niemi, Molnar, Rischke, PRD85(2012)114047]
 + s95p-PCE-v1 QCD EoS: $T_{\text{chem}} = 175 \text{ MeV}$; $T_{\text{fo}} = 100 \text{ MeV}$
 [Huovinen, Petreczky, NPA837(2010)26]

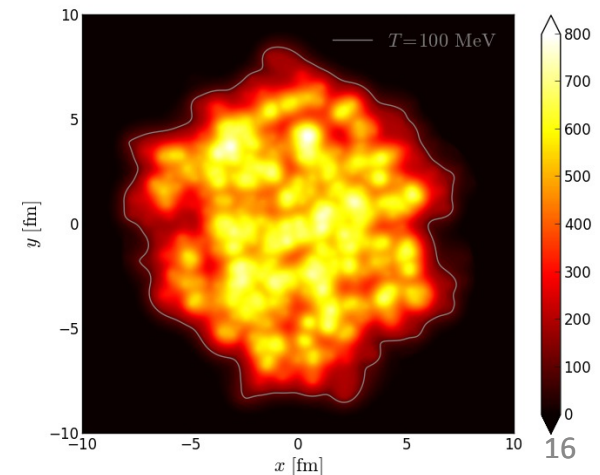
Viscosity effects:

- reduced flow-velocity gradients **during the evolution**
- **entropy increase** from initial to final state
- Non-equilibrium particle distributions **on the freeze-out surface**

$$f_i(x, p) = f_{0i}(x, p) + \delta f_i = f_{0i}(x, p) \left[1 + \frac{p_{i\mu} p_{i\nu} \pi^{\mu\nu}}{2T^2(e + P_0)} \right]$$

Initial conditions in our EbyE case:

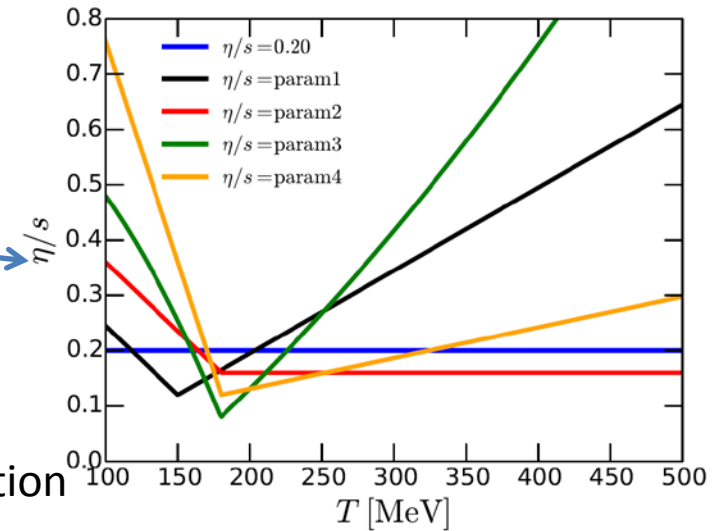
- The **computed** EbyE-fluctuating energy densities $\epsilon(x, y, \tau_i = 0.2 \text{ fm})$
- Initial $\mathbf{v}_T = 0$
- Initial $\pi^{\mu\nu} = 0$



Comparison with LHC and RHIC data

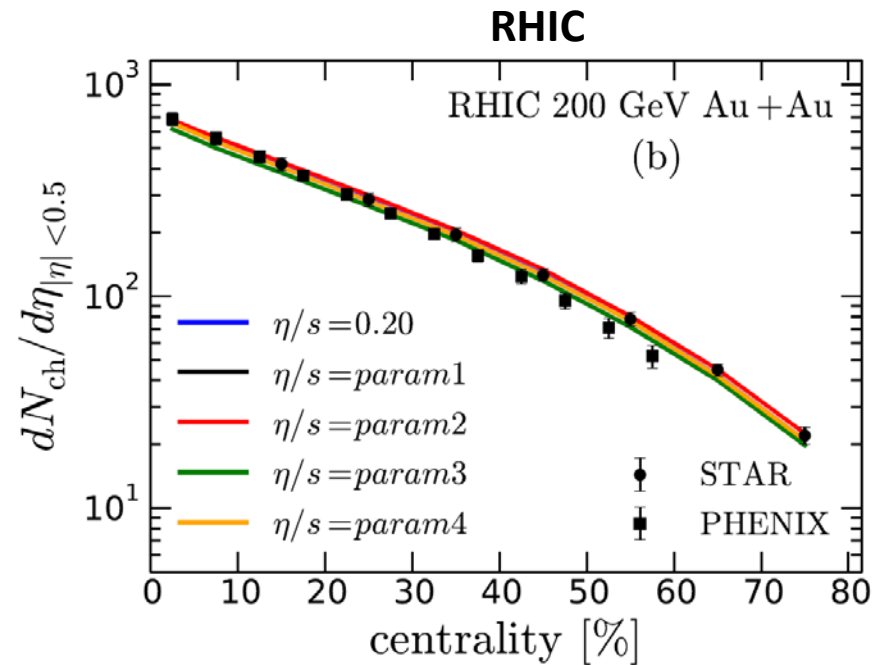
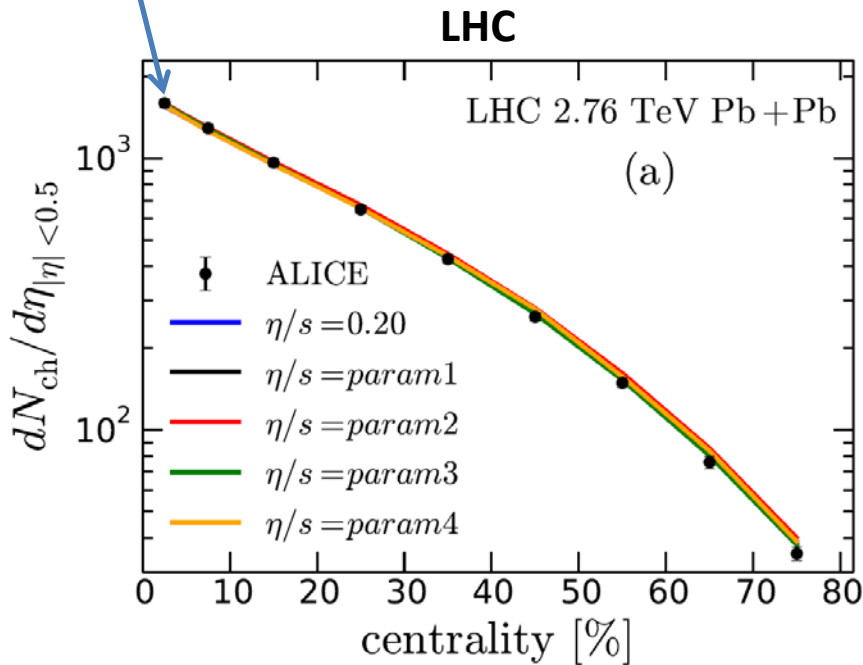
Map the possible T dependence of $\eta/s(T)$ with these parametrizations, reproducing the measured v_2 at LHC

[Niemi, KJE, Paatelainen, Phys.Rev. C93 (2016) 024907]



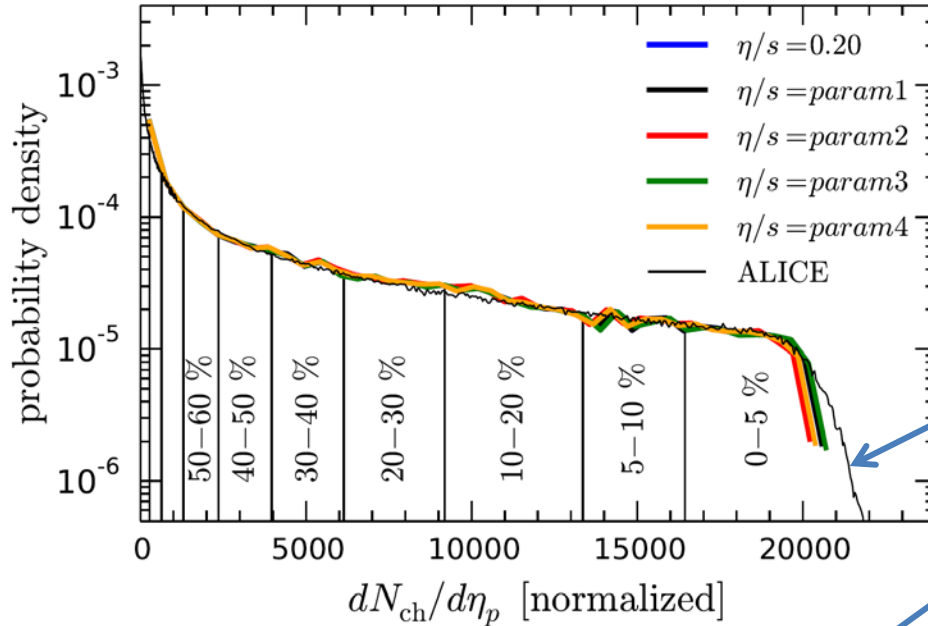
Centrality dependence of N_{ch} comes out correctly;

-- only one LHC-point (K_{sat} & β) is fitted, the rest is prediction

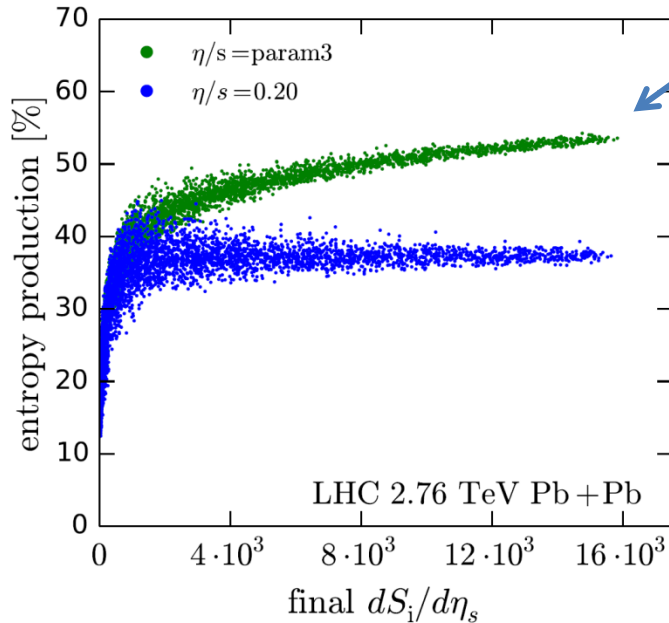


Our **<initial transverse densities>** [width of the gluon cloud!] are under control, but essentially no constraints for $\eta/s(T)$ from this observable

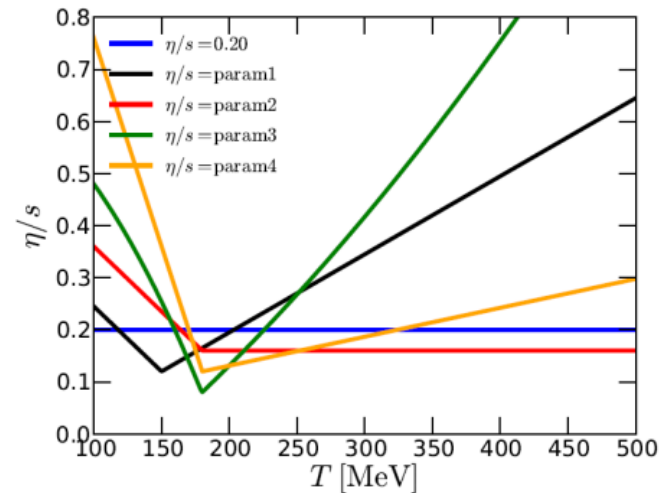
Our centrality classification is under control



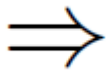
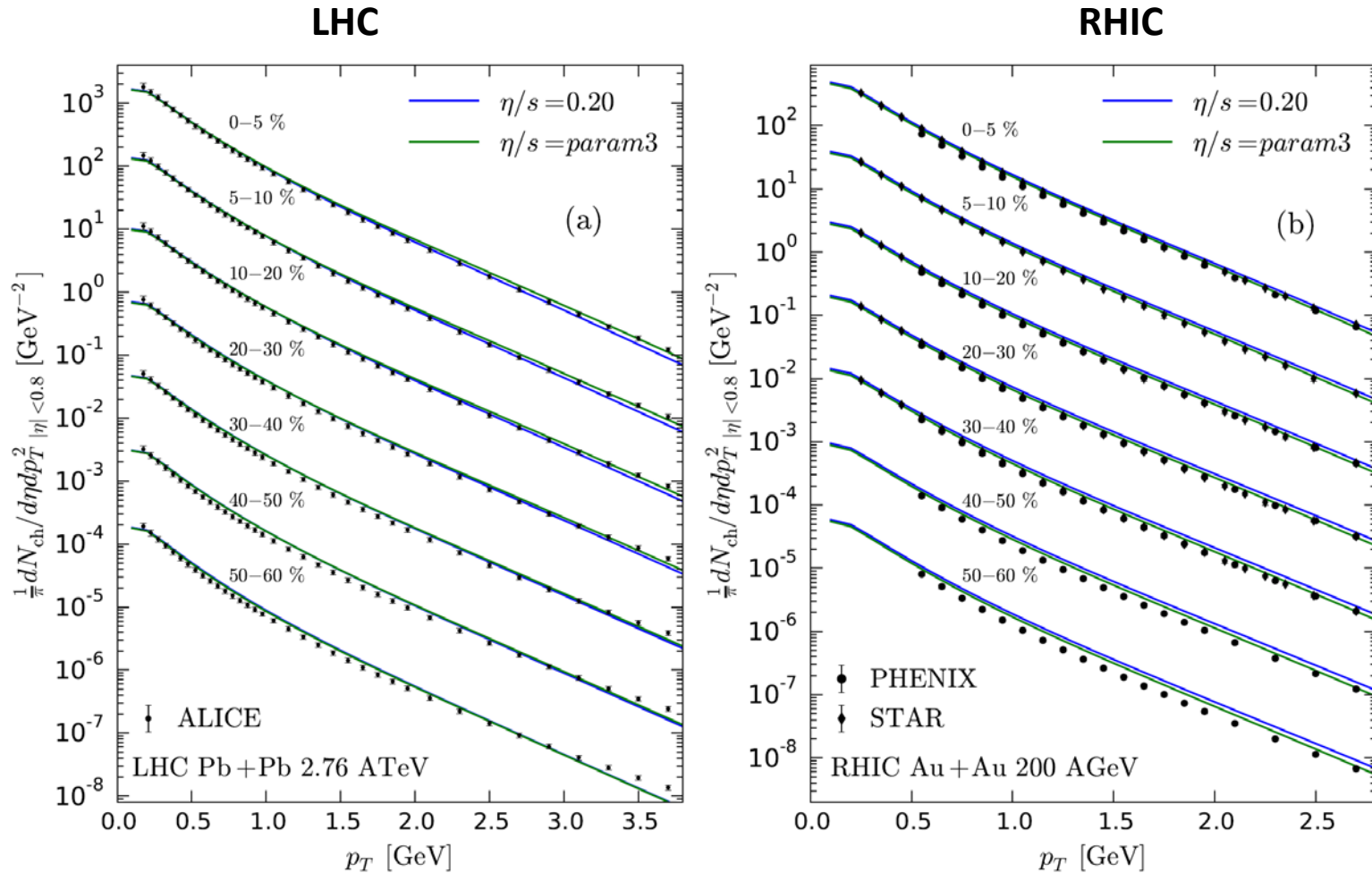
We do **not** yet include dynamical fluctuations of p_{sat} , hence we **do not (should not!)** reproduce the highest-multiplicity tail



Entropy increases from initial to final state the more the QGP viscosity is!

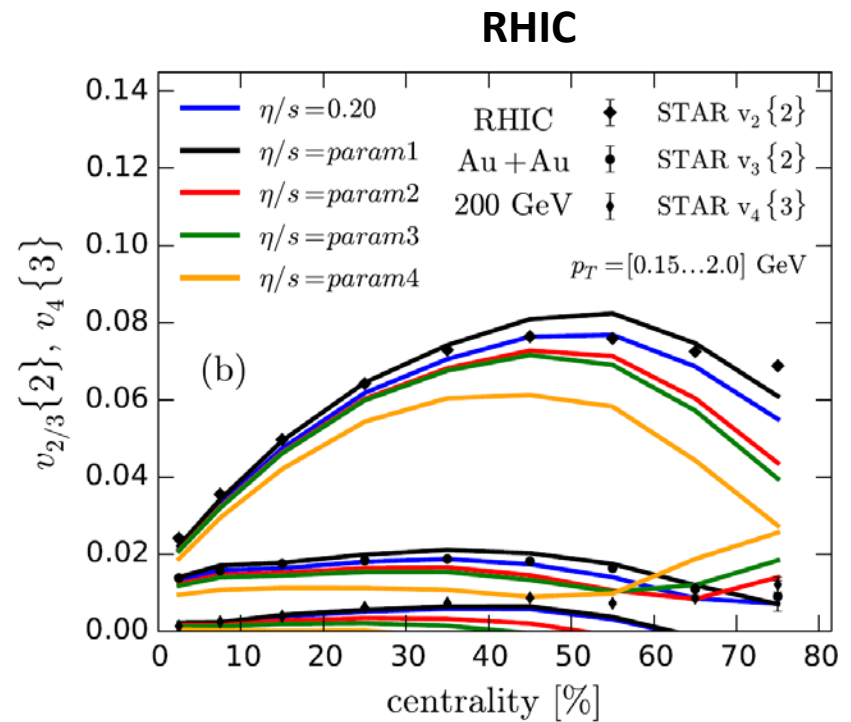
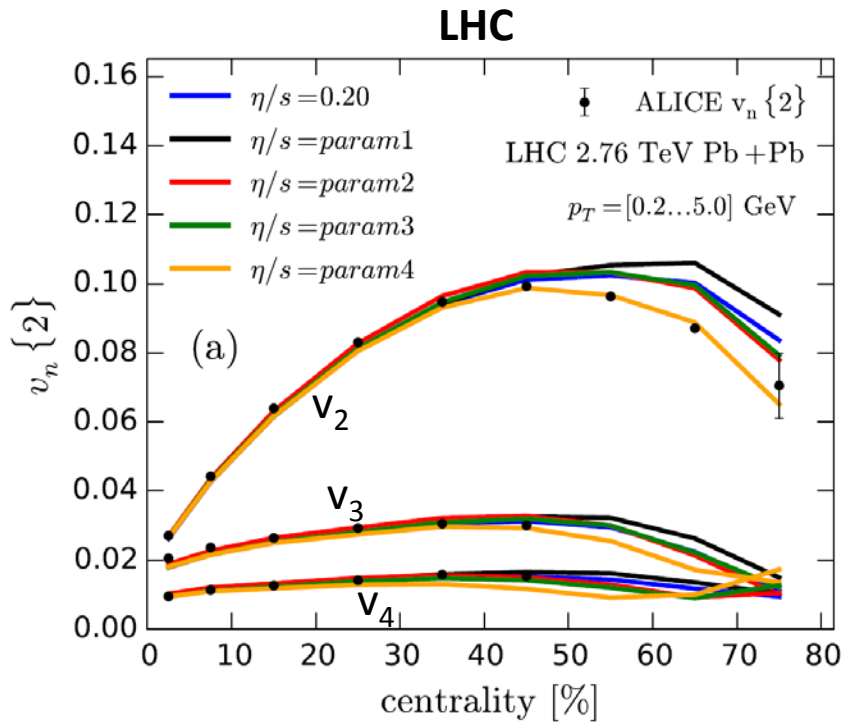


Centrality dependence of charged-hadron p_T spectra ~OK



Our **QCD matter EoS is under (sufficient) control** but essentially no constraints for $\eta/s(T)$ from here, either

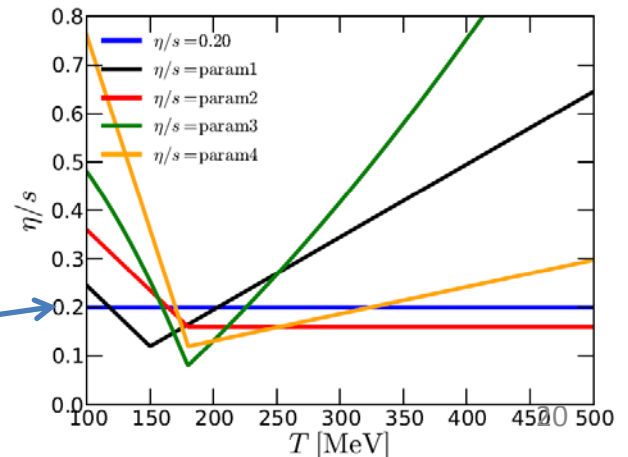
Centrality dependence of 2,3-particle cumulant flow coefficients v_n



LHC v_n s well reproduced by **all** these $\eta/s(T)$

Simultaneous LHC & RHIC analysis very important!

Constraints for $\eta/s(T)$:
Small $\eta/s(T)$ in the HRG seems favored



Relative EbyE fluctuations of elliptic flow at LHC come out beautifully

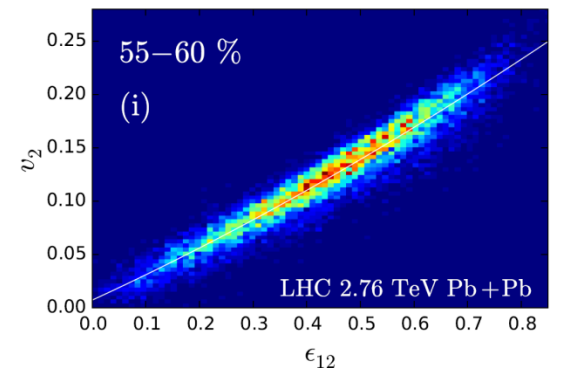
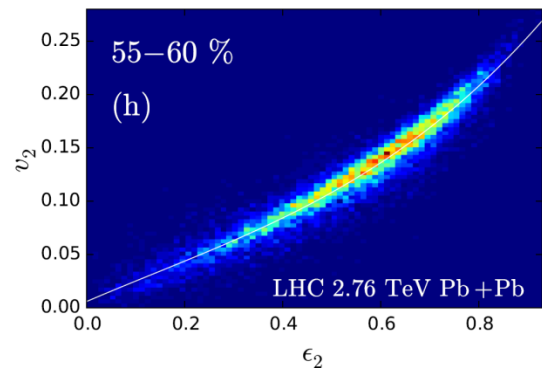
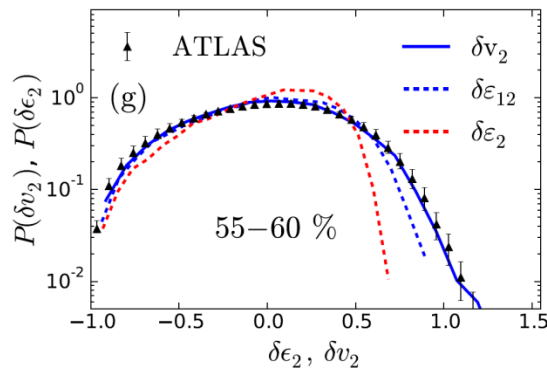
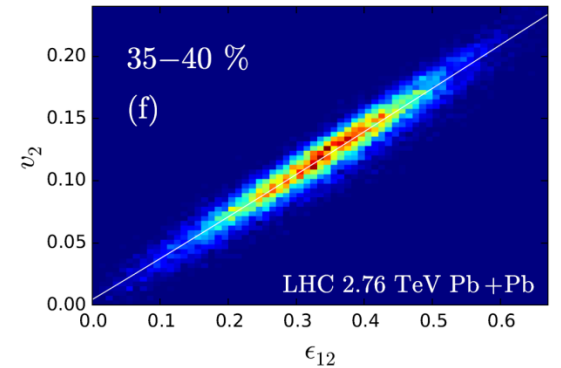
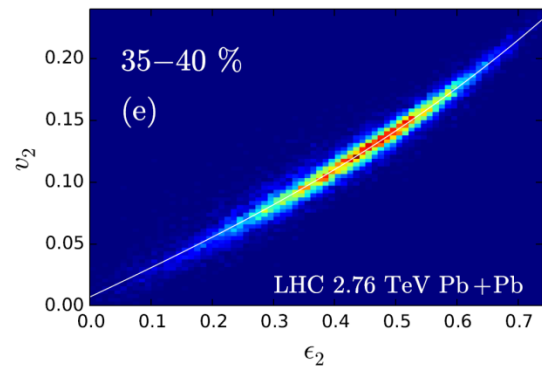
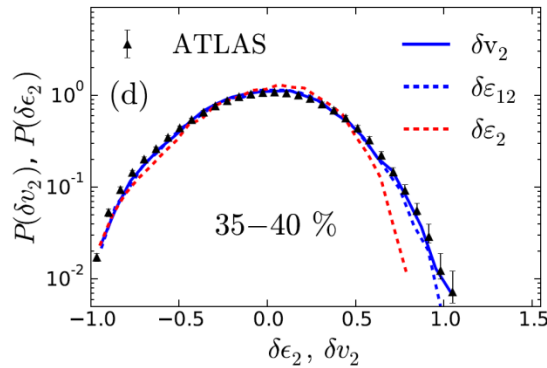
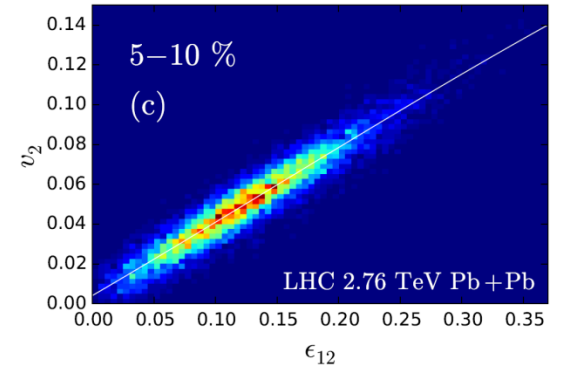
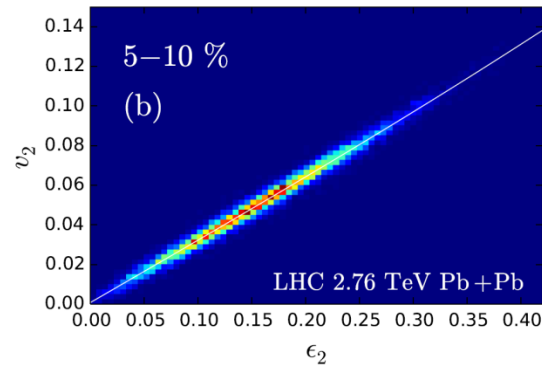
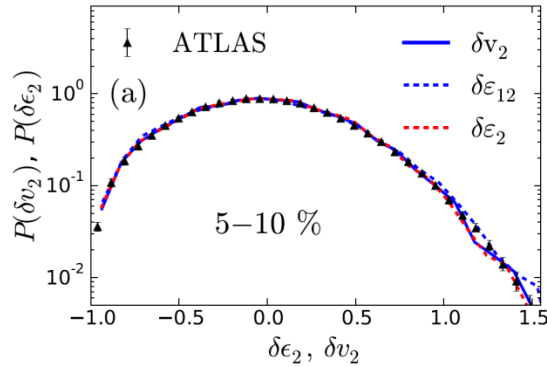
$$\delta v_n = \frac{v_n - \langle v_n \rangle_{ev}}{\langle v_n \rangle_{ev}}$$

$$\delta \epsilon_n = \frac{\epsilon_n - \langle \epsilon_n \rangle_{ev}}{\langle \epsilon_n \rangle_{ev}}$$

$$\epsilon_2 = \epsilon_{2,2}$$

$$\epsilon_{m,n} e^{in\Psi_{m,n}} = -\{r^m e^{in\phi}\} / \{r^m\}$$

$$\{\dots\} = \int dx dy e(x, y, \tau_0) (\dots)$$

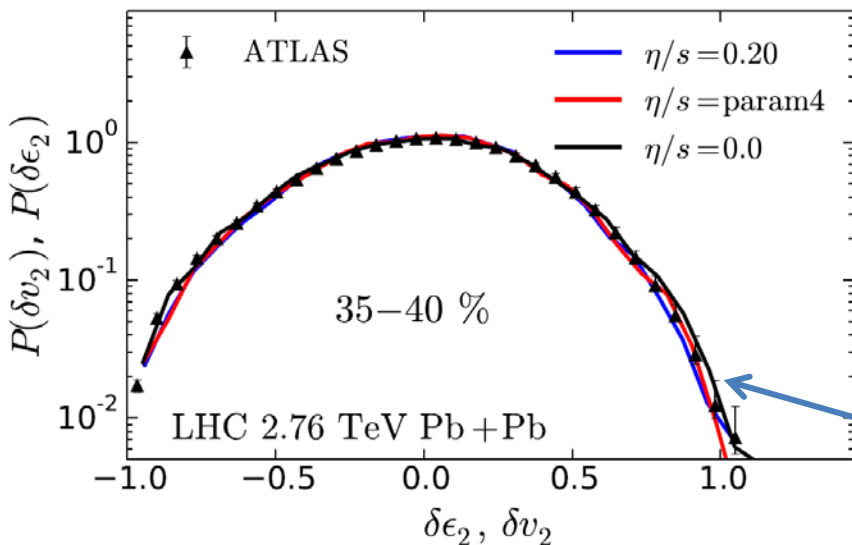
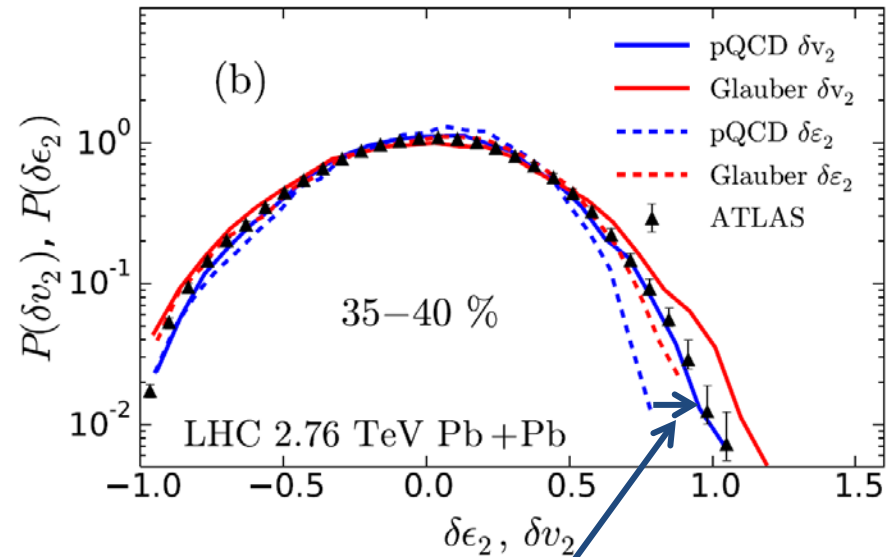
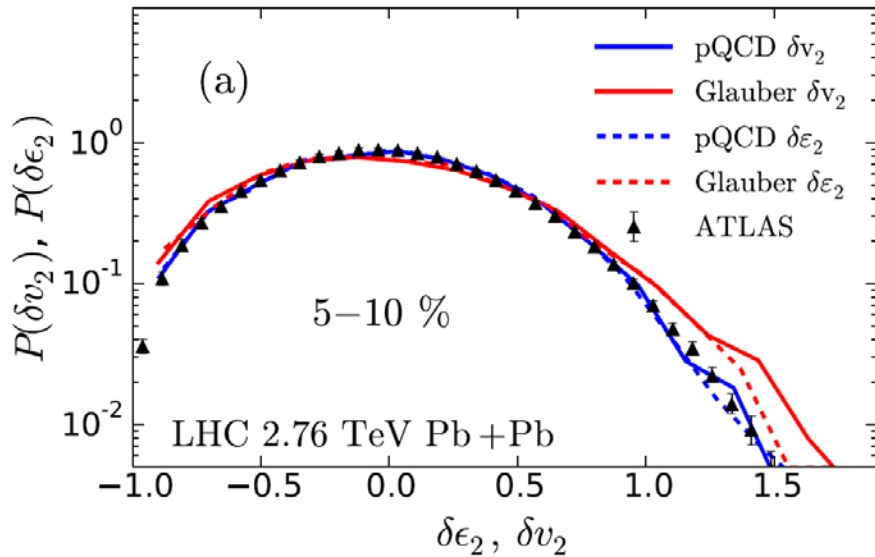


Relative EbyE fluctuations of elliptic flow at LHC come out beautifully

$$\delta v_n = \frac{v_n - \langle v_n \rangle_{ev}}{\langle v_n \rangle_{ev}}$$

$$\delta \epsilon_n = \frac{\epsilon_n - \langle \epsilon_n \rangle_{ev}}{\langle \epsilon_n \rangle_{ev}}$$

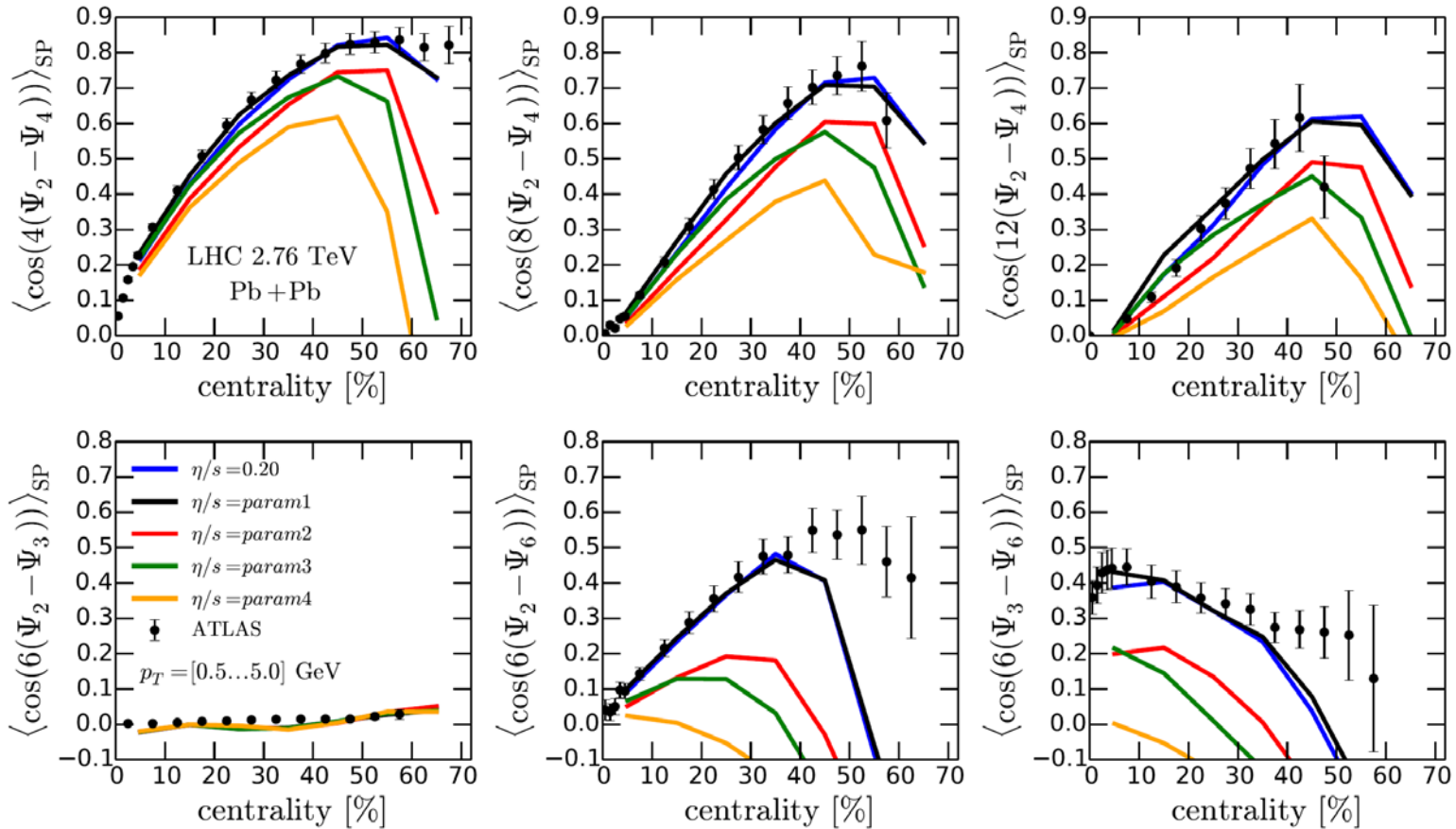
$$\epsilon_2 = \epsilon_{2,2}$$



- To reproduce these measurements, **need EbyE hydro**: Initial spatial asymmetry correlates **nonlinearly** with final state momentum asymmetry!

- **No sensitivity to $\eta/s(T)$**
- **Constraint to the initial state**
- **Our initial states are in control**

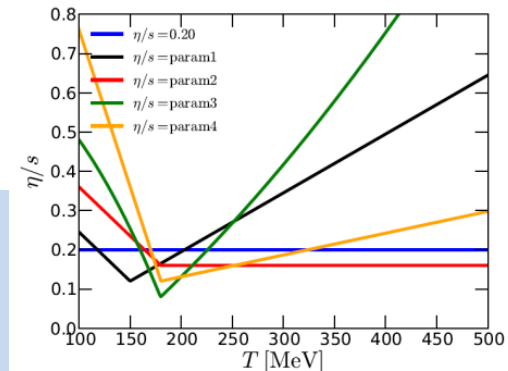
Correlations of 2 Event-plane angles also OK, for centralities < 40-50%



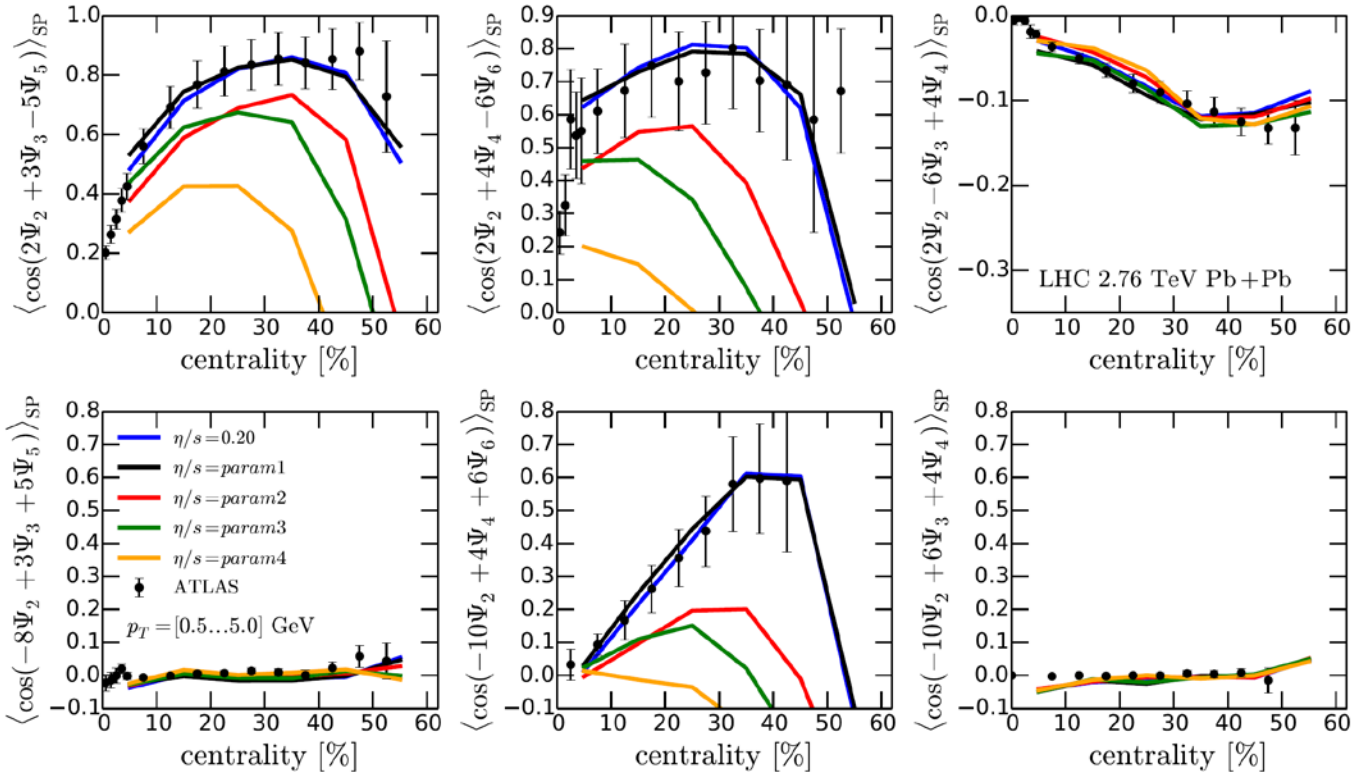
LHC

$$\langle \cos(k_1 \Psi_1 + \dots + n k_n \Psi_n) \rangle_{SP} \equiv \frac{\langle v_1^{|k_1|} \dots v_n^{|k_n|} \cos(k_1 \Psi_1 + \dots + n k_n \Psi_n) \rangle_{ev}}{\sqrt{\langle v_1^{2|k_1|} \rangle_{ev} \dots \langle v_n^{2|k_n|} \rangle_{ev}}}$$

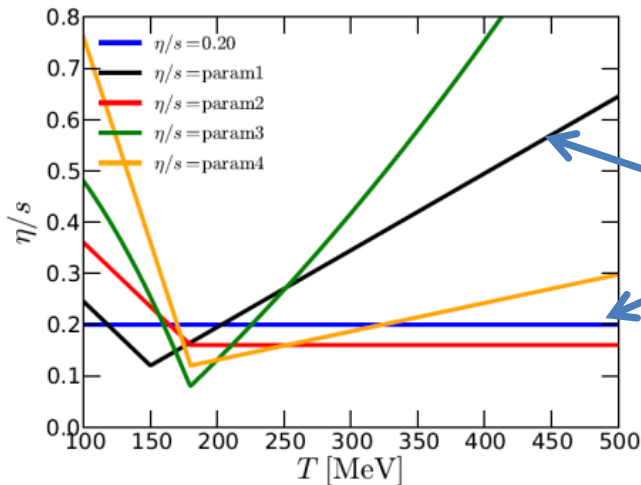
Especially since $P(\delta v_n)$ constrain our ISs independently of η/s , these correlations **give further constraints for $\eta/s(T)$** and simultaneously **test the validity** of the EbyE viscous framework!



Even the correlations of 3(!) Event-plane angles similarly OK, for centralities < 40-50%



LHC

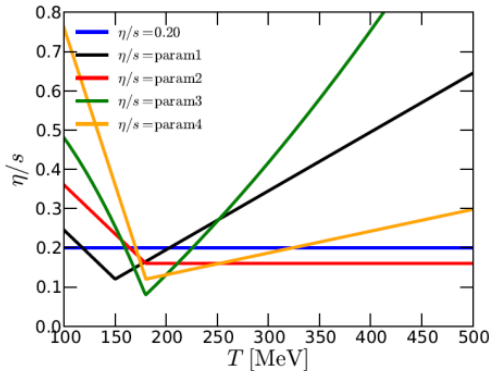


Remarkably, **the same two $\eta/s(T)$ parametrizations** that explain the RHIC v_n 's — the black and blue in the fig. — work best also at LHC! For these, also the viscous hydro seems best under control...

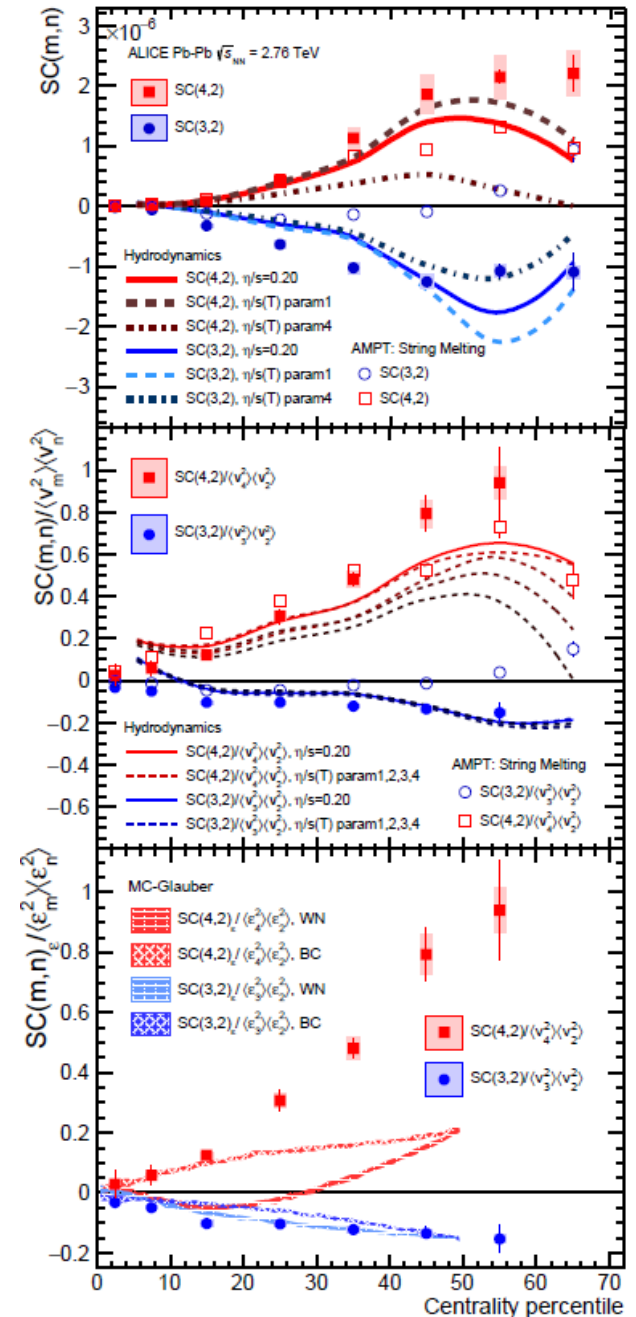
Symmetric 2-harmonic 4-particle (!) cumulants from ALICE

$$\begin{aligned} \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle_c &= \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle \\ &\quad - \langle\langle \cos[m(\varphi_1 - \varphi_2)] \rangle\rangle \langle\langle \cos[n(\varphi_1 - \varphi_2)] \rangle\rangle \\ &= \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle, \end{aligned}$$

ALICE, arXiv:1604.07663 [nucl-ex]
EKRT results from H.Niemi



The same two best $\eta/s(T)$ parametrizations [0.2 & param1] work best also here!

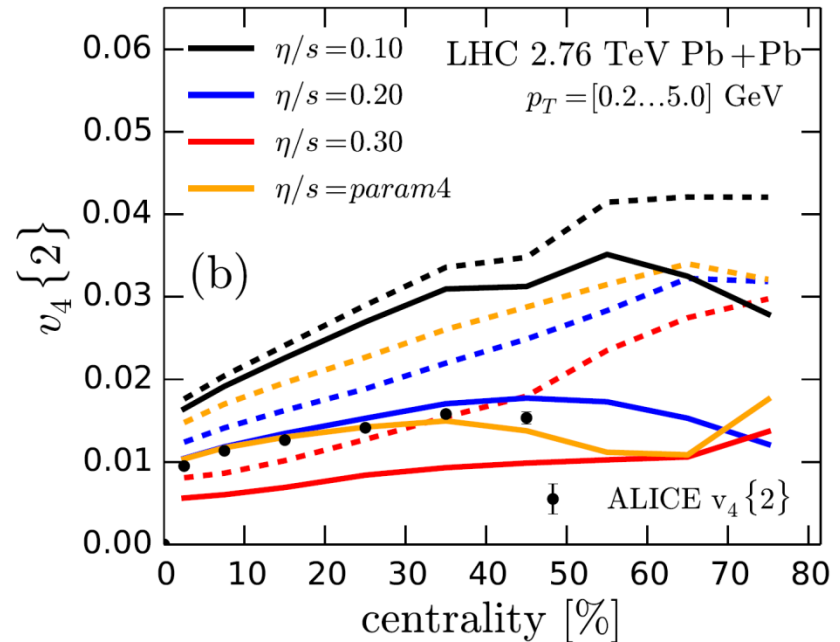
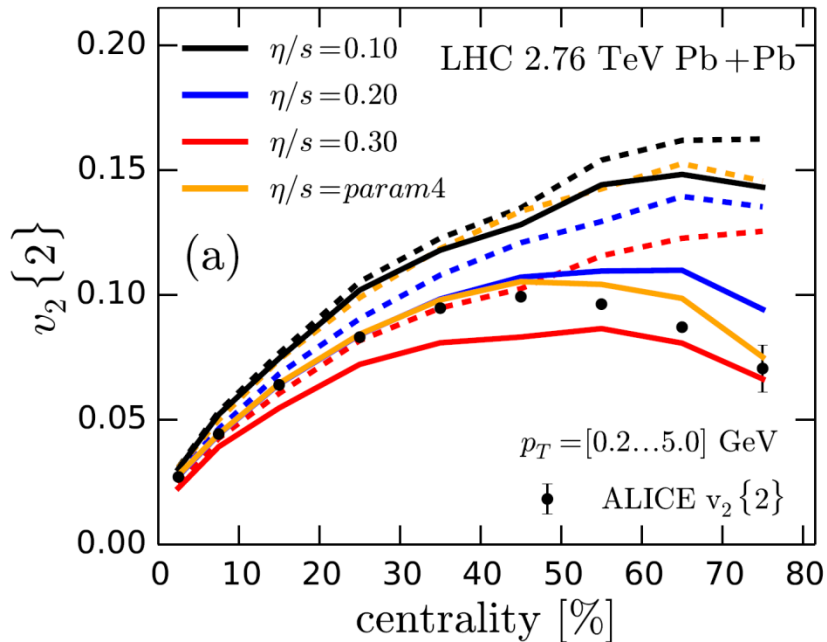


Applicability region of viscous hydro: magnitude of δf corrections?

$$f_i(x, p) = f_{0i}(x, p) + \delta f_i = f_{0i}(x, p) \left[1 + \frac{p_{i\mu} p_{i\nu} \pi^{\mu\nu}}{2T^2(e + P_0)} \right]$$

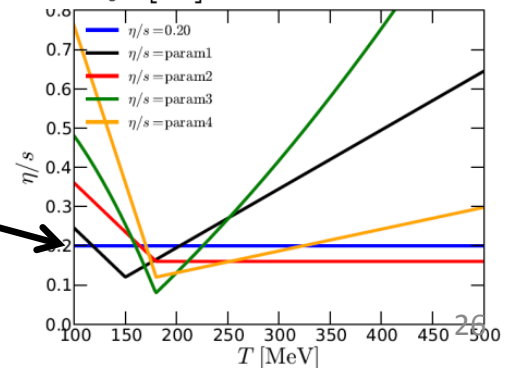
Solid lines = with δf

Dashed = without δf

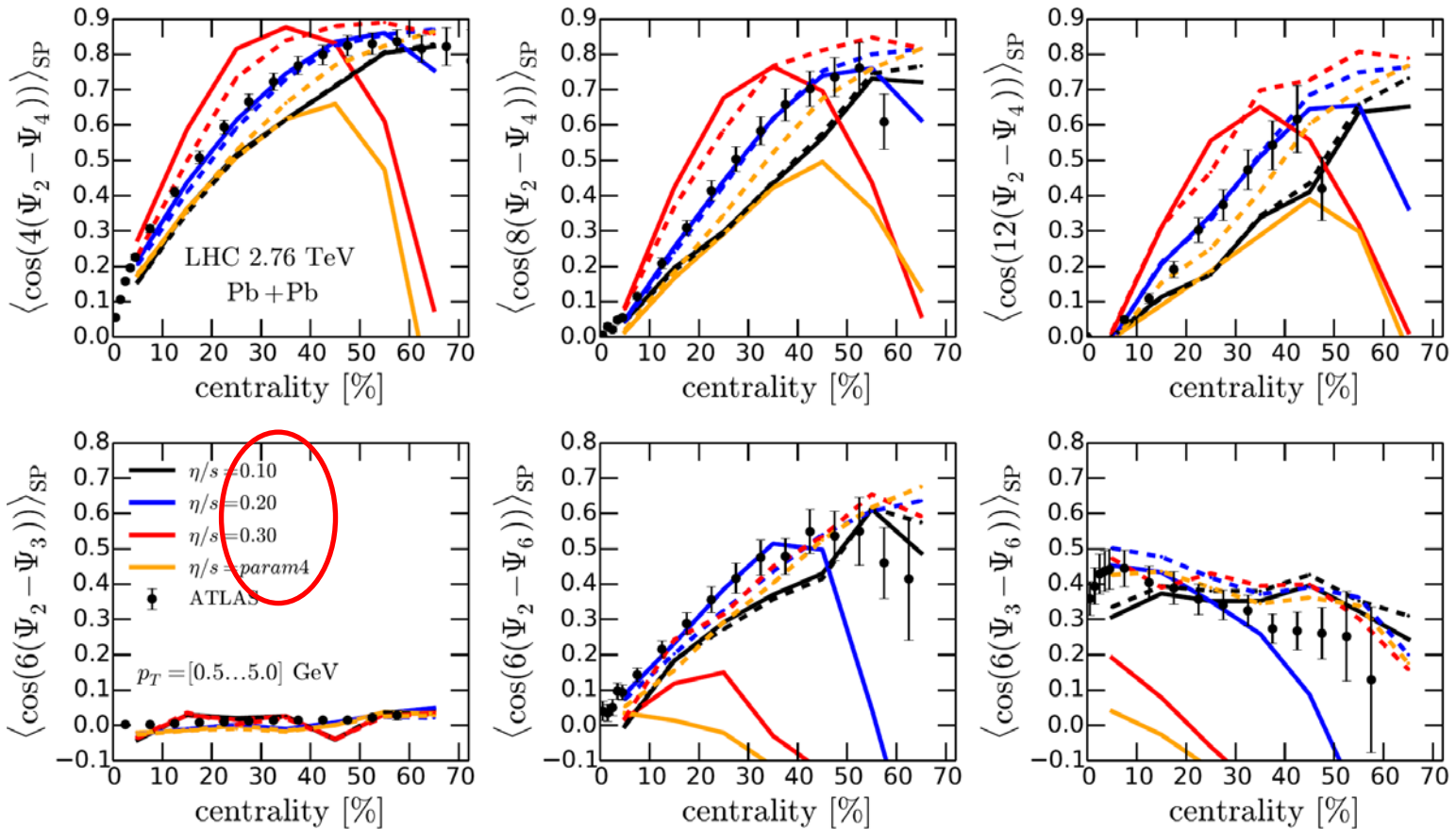


(decays not included in these figures)

Smallest hadronic viscosities (blue&black) work best:
 δf effects in v_n remain small up to 40-50 % centralities

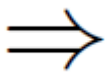


Applicability region of dissipative hydro: magnitude of δf corrections?

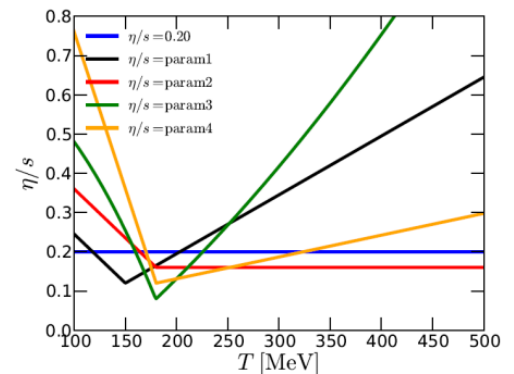


Solid lines = with δf

Dashed = without δf

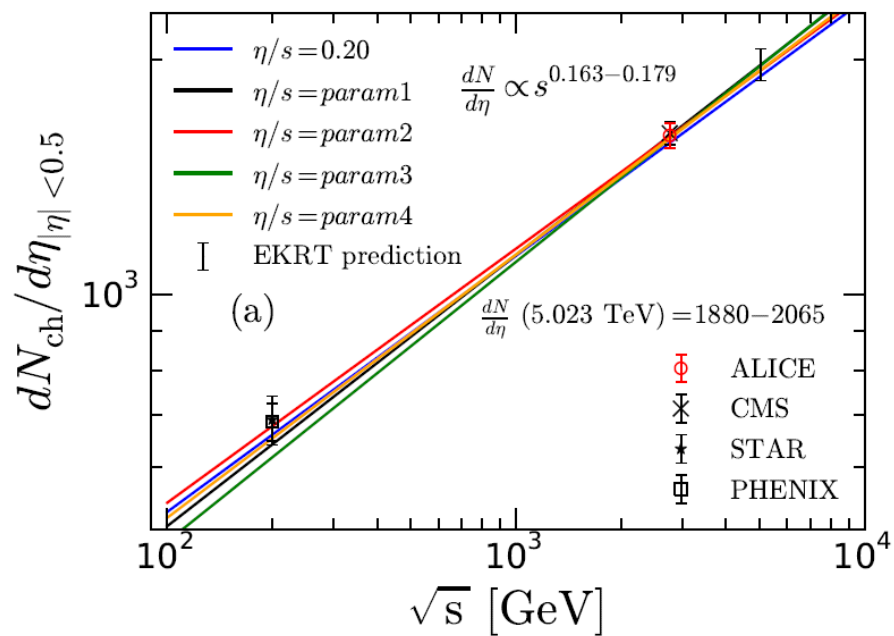
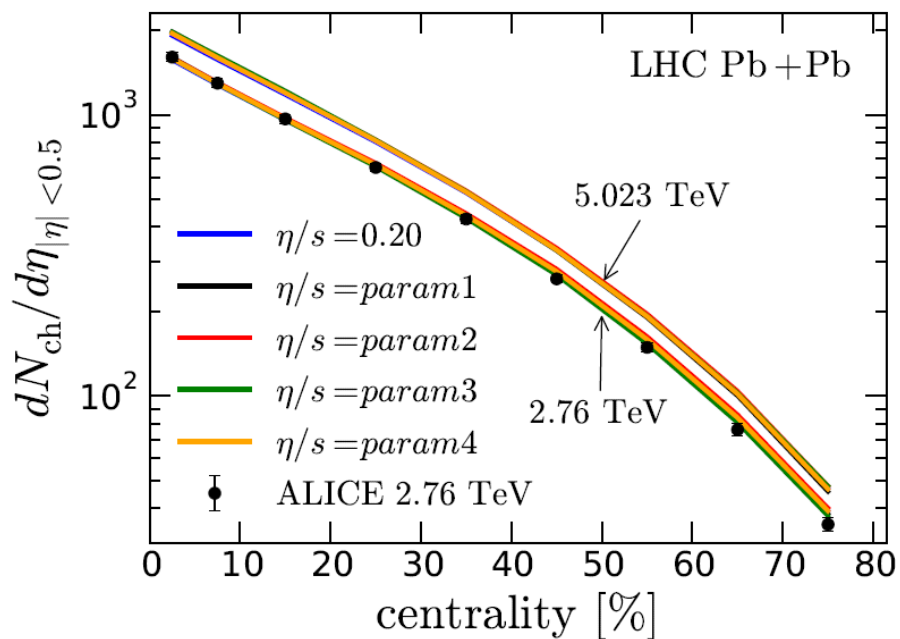
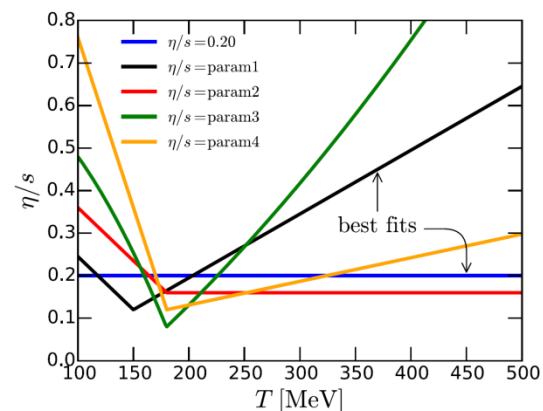


δf effects remain (mostly) small from central to semi-central collisions:
constraints for η/s in the applicability region of hydro!



3. Predictions for the 5.02 TeV Pb+Pb LHC run

[Phys.Rev. C93 (2016) 014912, arXiv:1511.04296 [hep-ph]]



0-5% central:

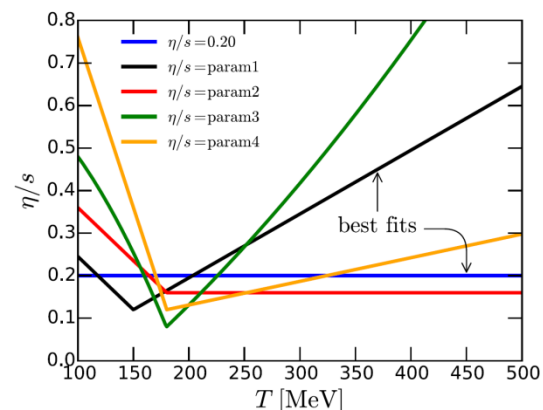
EKRT

$$\left. \frac{dN_{\text{ch}}}{d\eta} \right|_{|\eta| \leq 0.5} = 1876 \dots 2046$$

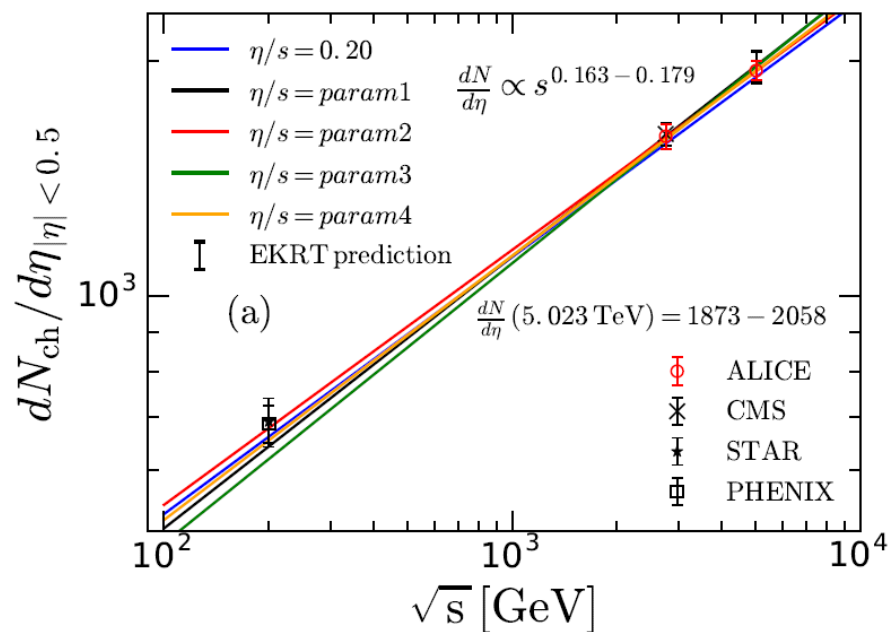
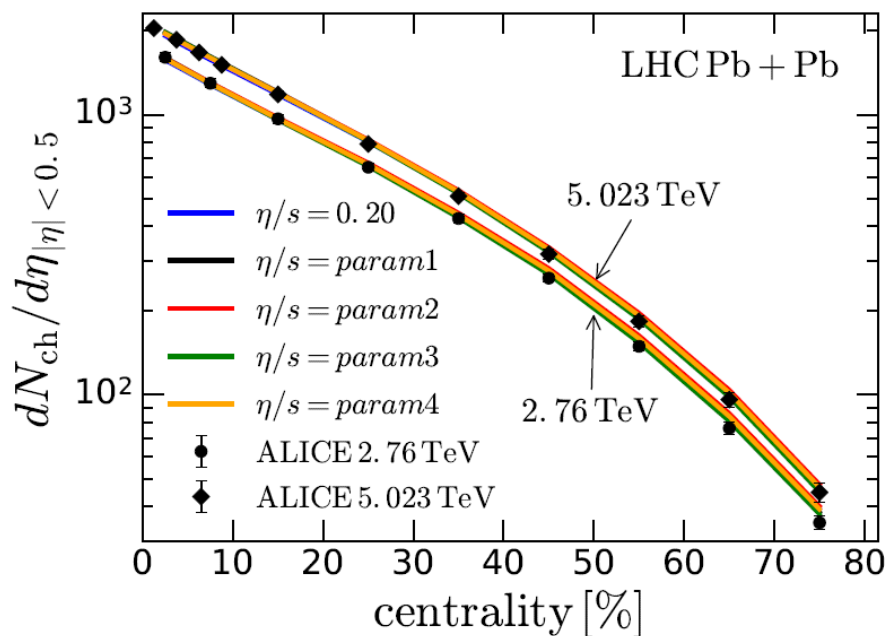
blue ... black

$$\left. \frac{dN_{\text{ch}}}{d\eta} \right|_{|\eta| \leq 0.5} \propto s^{0.164 \dots 0.174}$$

3. Predictions for the 5.02 TeV Pb+Pb LHC run



[Phys.Rev. C93 (2016) 014912, arXiv:1511.04296 [hep-ph]]



vs. 5.02 TeV ALICE data [Phys.Rev.Lett. 116 (2016) 222302, arXiv:1512.06104 [nucl-ex]]

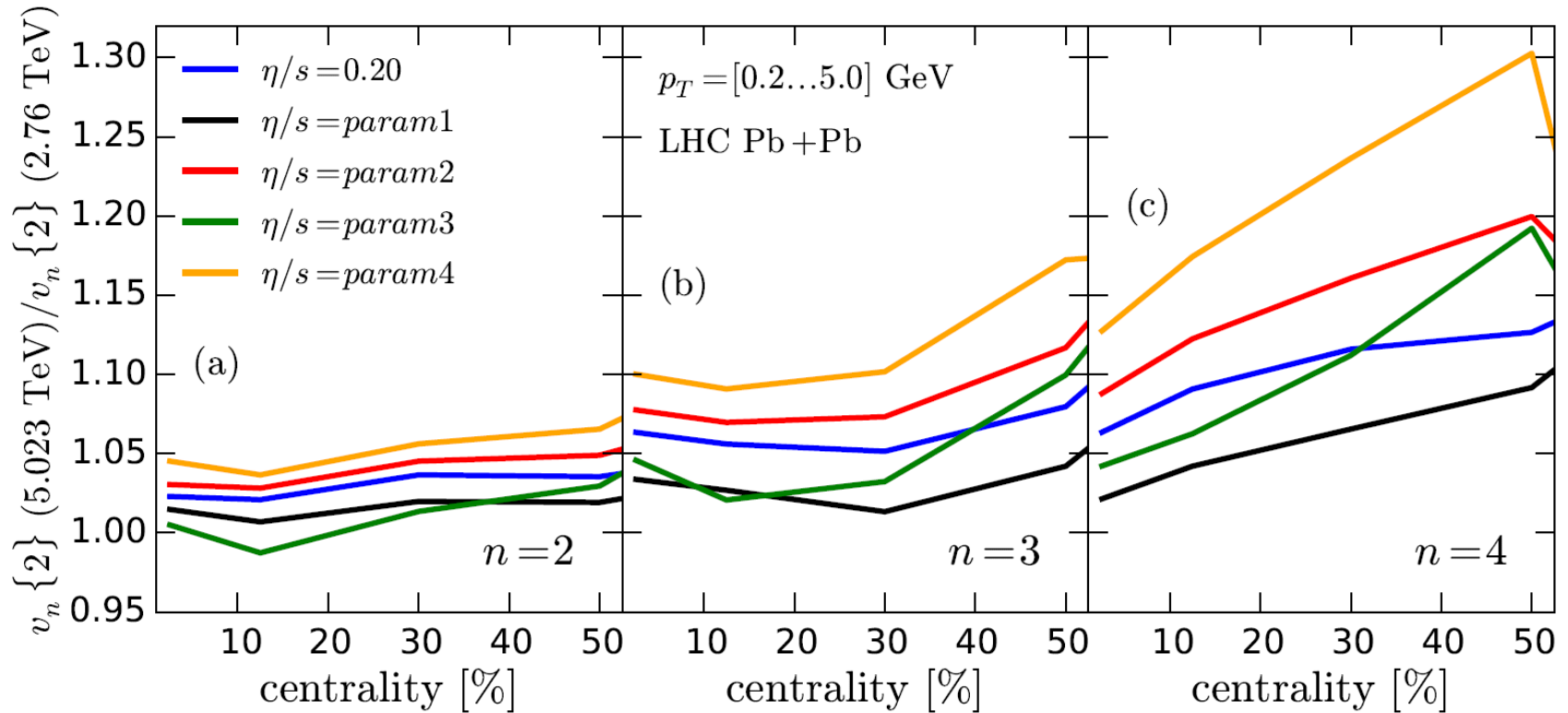
0-5% central: EKRT $\left. \frac{dN_{\text{ch}}}{d\eta} \right|_{|\eta| \leq 0.5} = 1876 \dots 2046$
blue ... black

ALICE: 1943 ± 54

Ratio of the flow coefficients $v_n\{2\}$ at 5.02 TeV and 2.76 TeV

[Phys.Rev. C93 (2016) 014912, arXiv:1511.04296 [hep-ph]]

EKRT prediction...

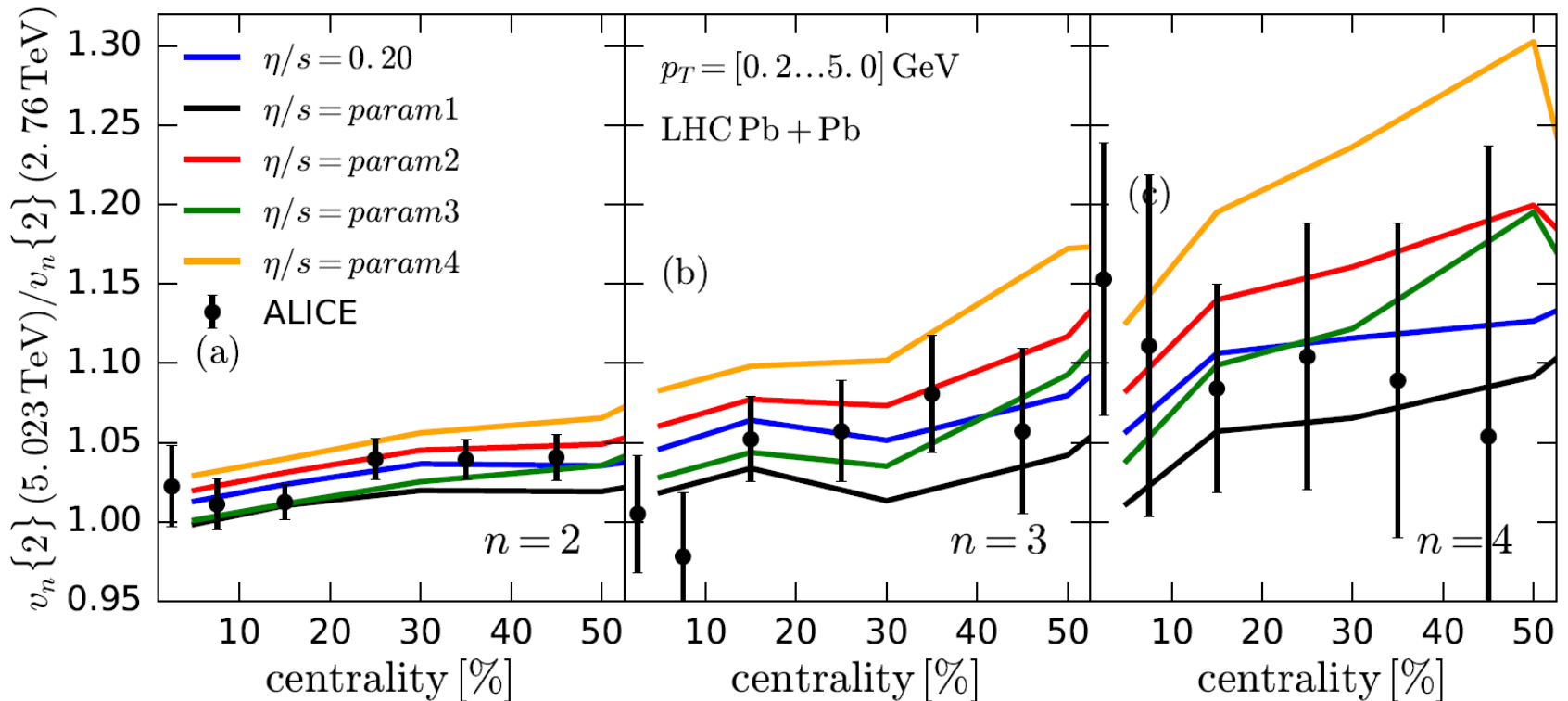


- Higher harmonics $n > 2$ more sensitive to $\eta/s(T)$
- Further constraints for $\eta/s(T)$

Ratio of the flow coefficients $v_n\{2\}$ at 5.02 TeV and 2.76 TeV

[Phys.Rev. C93 (2016) 014912, arXiv:1511.04296 [hep-ph]]

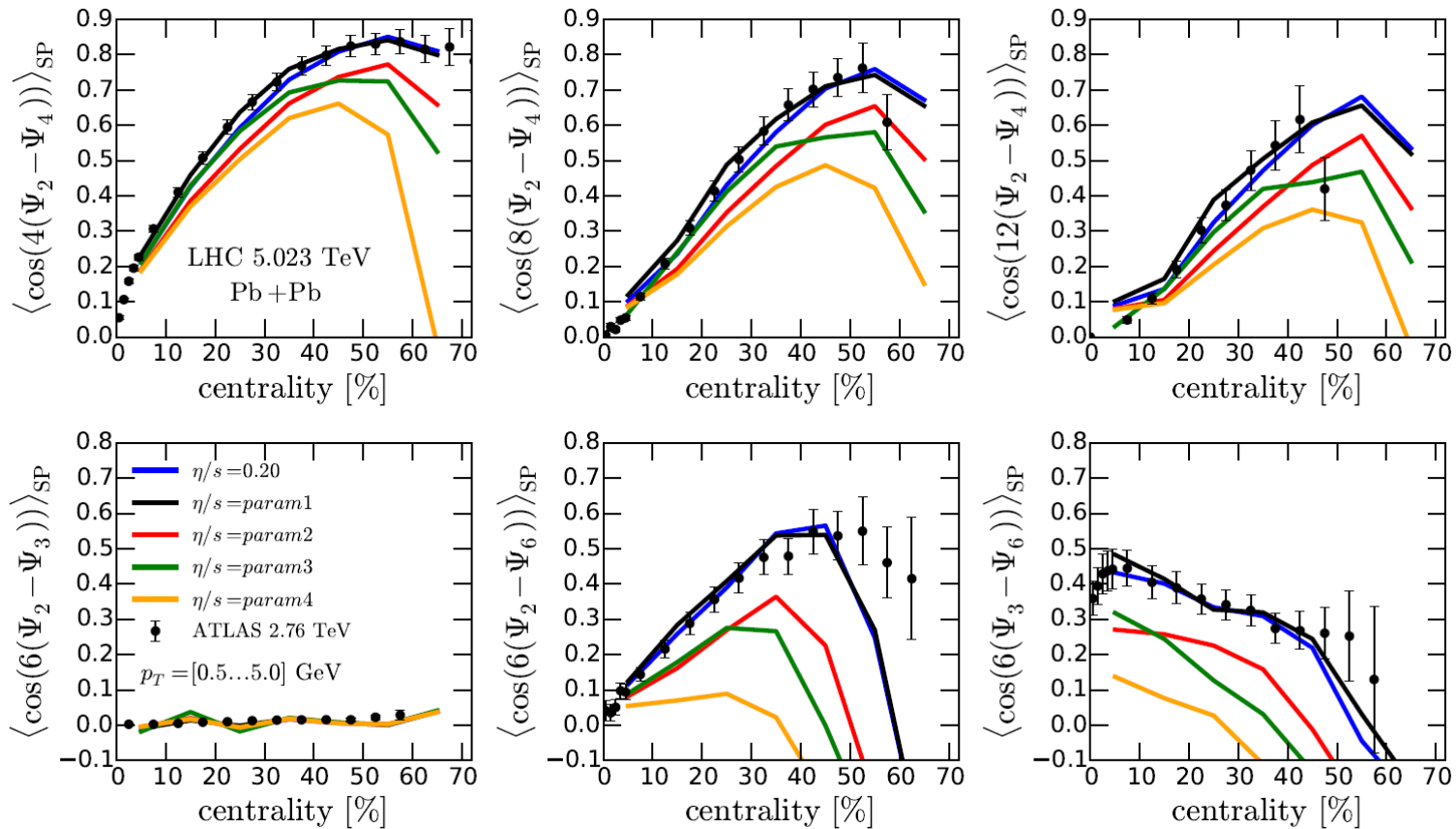
EKRT prediction vs ALICE data [Phys.Rev.Lett. 116 (2016) 132302]



- Higher harmonics $n > 2$ more sensitive to $\eta/s(T)$
- Further constraints for $\eta/s(T)$

Correlations of two EP angles for charged hadrons in 5.02 TeV Pb+Pb

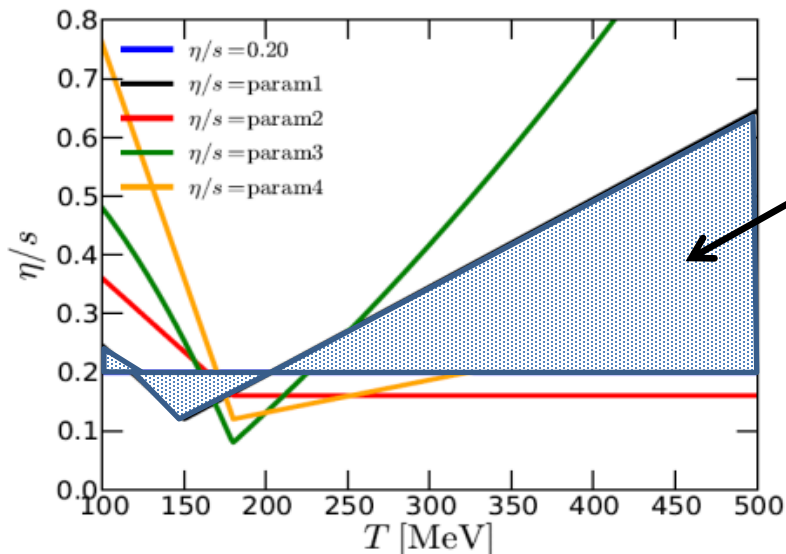
EKRT 5.02 TeV prediction [Phys.Rev. C93 (2016) 014912, arXiv:1511.04296 [hep-ph]]
 vs. **ATLAS 2.76 TeV data** [Phys. Rev. C 90 (2014) 2, 024905]



**Very similar to the 2 EP correlations at 2.76 TeV,
 i.e. similar constraining power for $\eta/s(T)$ as at 2.76 TeV**

Conclusions & outlook

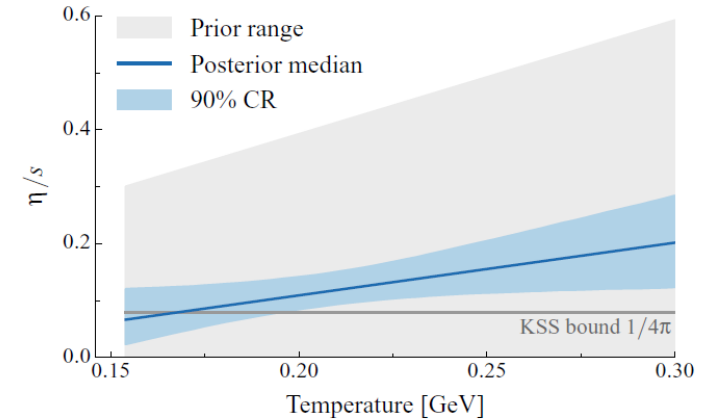
- **NLO-improved pQCD+saturation+viscous hydro *EbyE framework*** (EKRT model)
 - explains consistently the LHC and RHIC bulk observables in URHIC
 - has clear predictive power in cms energy, centrality, A
 - enables estimation of the **QCD matter $\eta/s(T)$** and its uncertainties
- Now there starts to be **enough orthogonal data constraints** available from LHC and RHIC, for (i) pinning down the initial conditions,
 - (ii) probing the validity of the framework and
 - (iii) probing & determining the QCD matter $\eta/s(T)$
 - a **simultaneous LHC and RHIC multiobservable analysis** is required!



Our "best" estimate currently for $\eta/s(T)$ but this is **not yet a true error band** — **statistical global analysis needed**

- Similar η/s magnitudes also from
 - **IP-glasma ISs**: $\eta/s = 0.12$ (RHIC) ... 0.2 (LHC) [Gale, Jeon, Schenke, Tribedy, Venugopalan, Phys. Rev. Lett. 110 (2013) 012302]
 - **MCG/MC-KLN+VISHNU**: $0.08 < \text{const. } \eta/s < 0.2$ [Song, Bass, Heinz, Hirano, Shen, Phys. Rev. Lett. 106, 192301 (2011)]

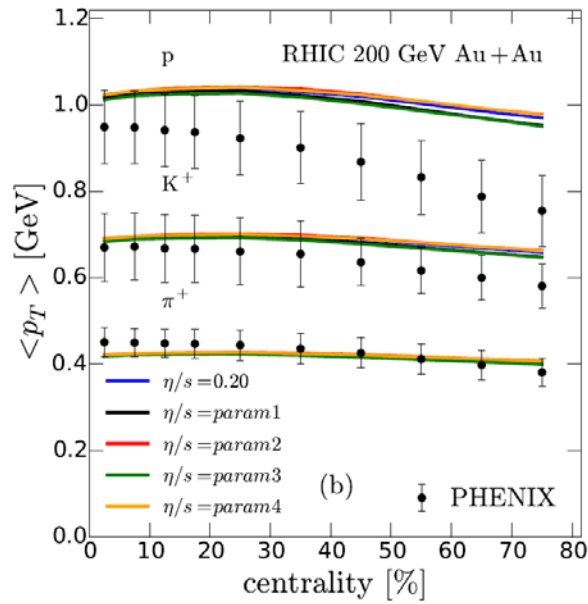
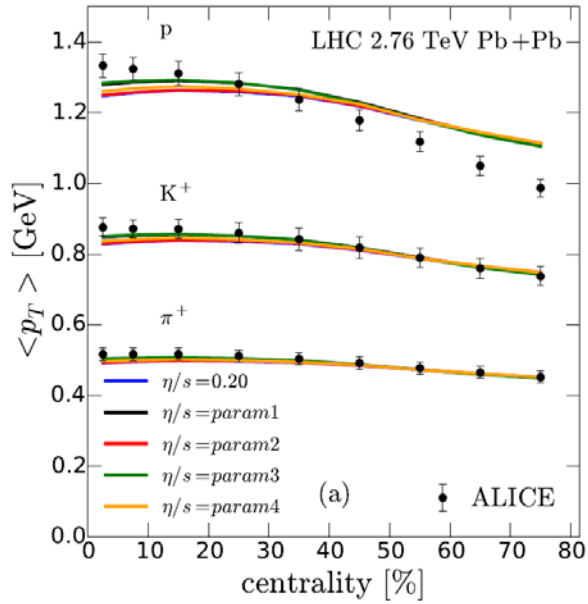
- First attempt towards a statistical global analysis of data in [Bernhard, Moreland, Bass, Liu, Heinz, Phys.Rev. C94 (2016) 024907]
 - supports EKRT (and IP-glasma)-type initial states
 - $\eta/s(T)$ trend similar to EKRT
 - indications of bulk viscosity(?)



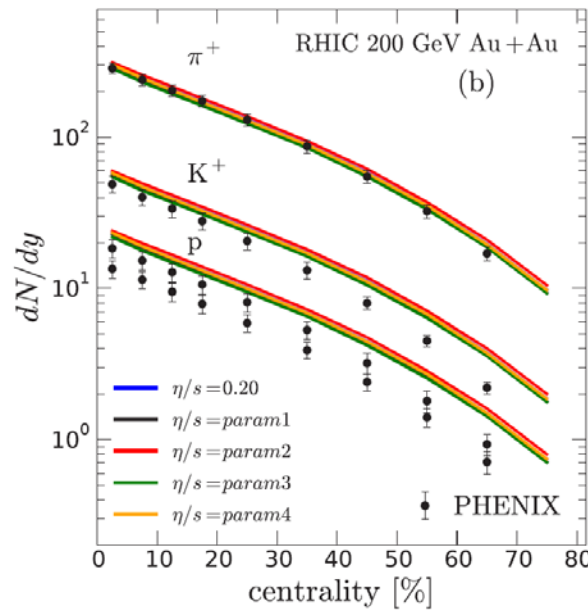
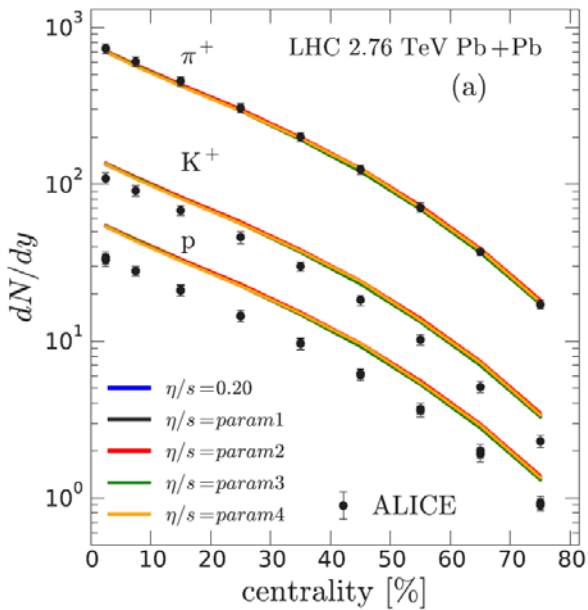
Next in EKRT:

- Include dynamical fluctuations of p_{sat} → EKRT predictions in **p+A collisions** (?)
- Develop a **global analysis** of these observables → **a statistical error band to $\eta/s(T)$**
- Improve the description of "pre-thermal" evolution [Eff.Kin.Th./BAMPS]
- Study also **bulk viscosity** effects, see e.g.
 - Ryu et al, Phys. Rev. Lett. 115 (2015) 132301
 - Bernhard, et al., Phys.Rev. C94 (2016) 024907
- Need a "MC-EKRT" event generator to study also y -dependent observables

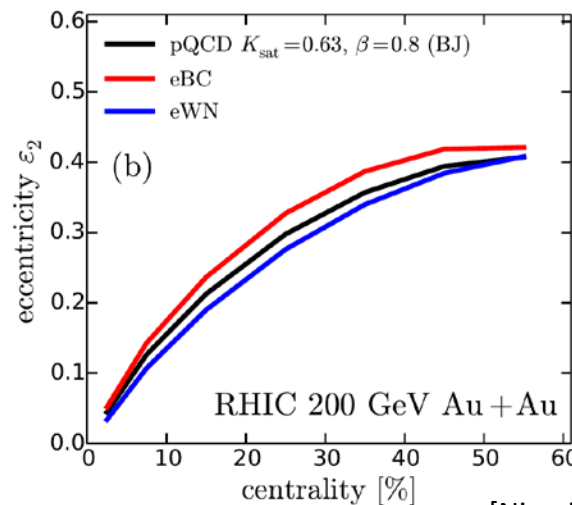
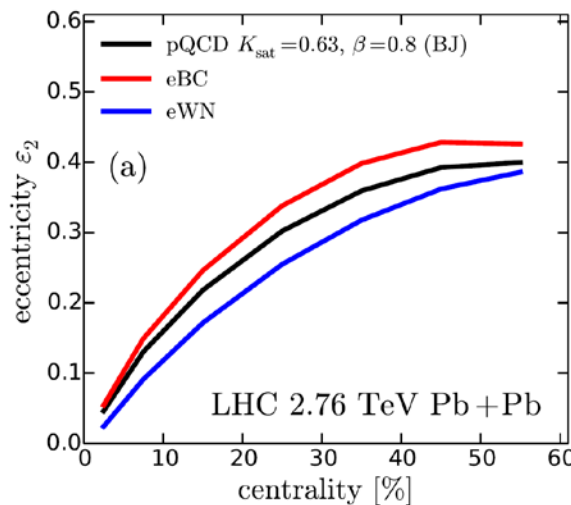
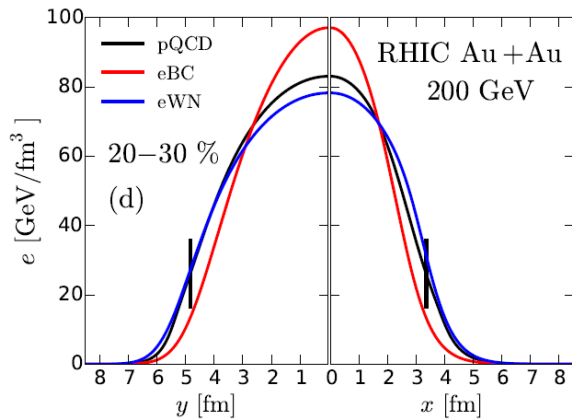
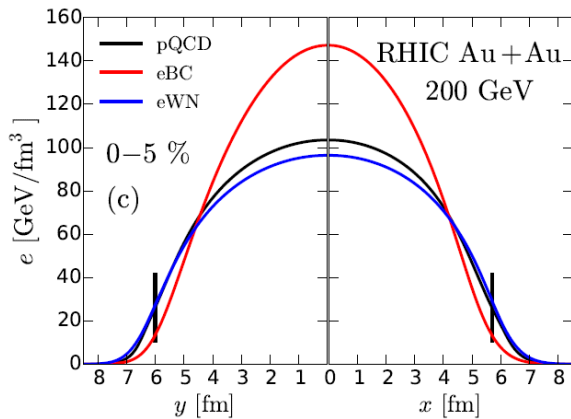
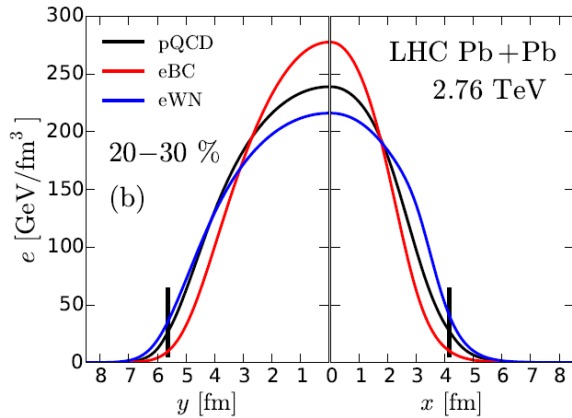
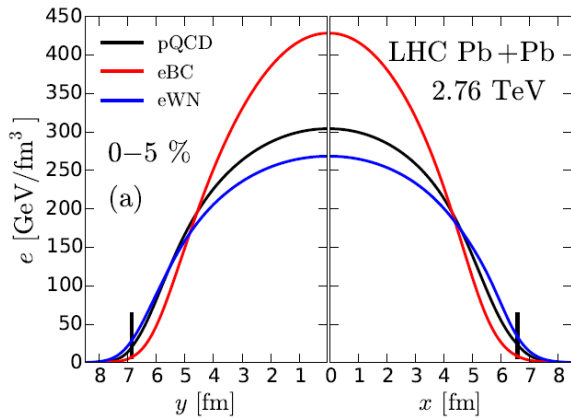
Back up slides



bulk $\langle p_T \rangle \sim \text{OK}$



bulk $dN/dy \sim \text{OK}$



Average energy densities

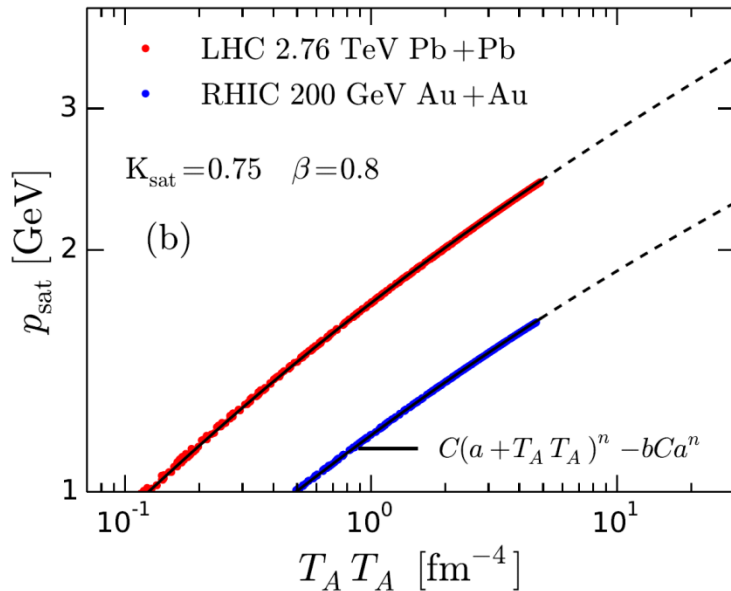
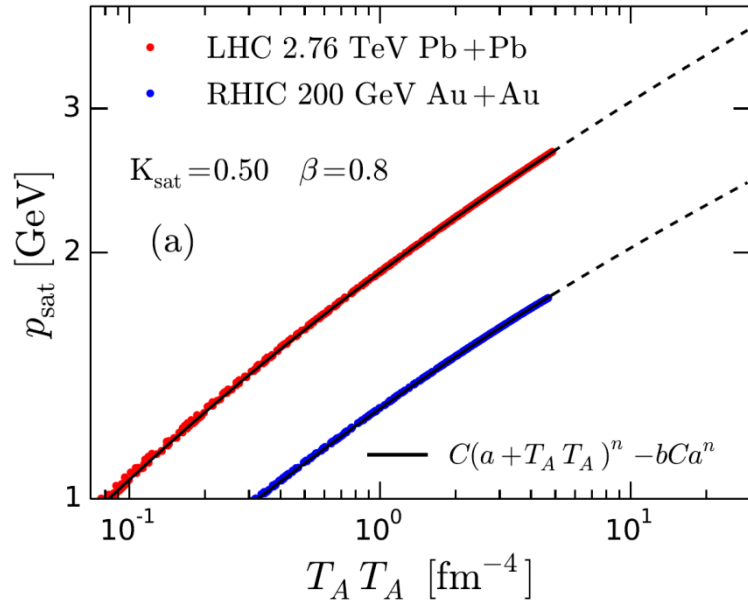
$$K_{\text{sat}} = 0.63 \text{ and } \beta = 0.8$$

$$\tau_0 = 0.20 \text{ fm}$$

$$\Leftrightarrow \text{eta/s} = 0.2$$

The vertical lines show where $p_{\text{sat}} < 1 \text{ GeV}$ and where matching to BC profile is made

Average initial eccentricities are btw eBC and eWN



Parametrization of $p_{\text{sat}}(K_{\text{sat}}, \beta)$ available:

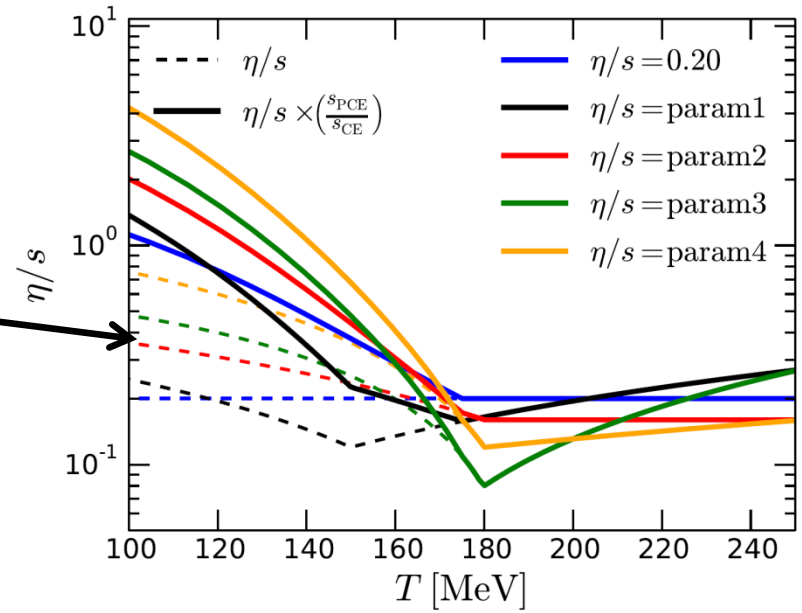
$$p_{\text{sat}}(\rho_{AA}) = C [a + \rho_{AA}]^n - b C a^n$$

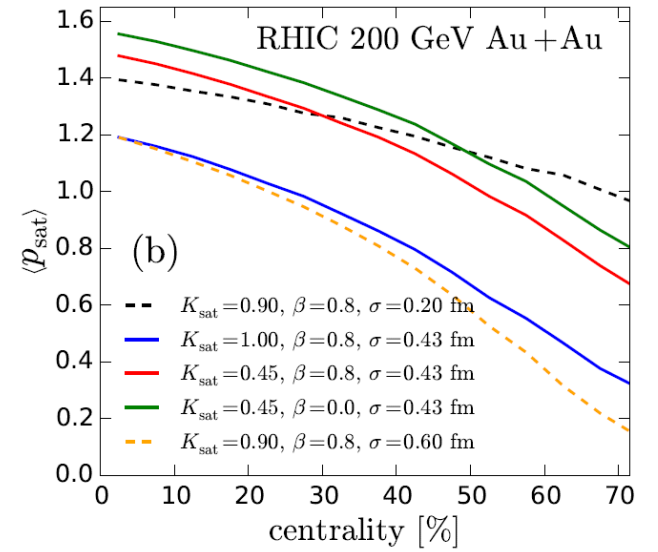
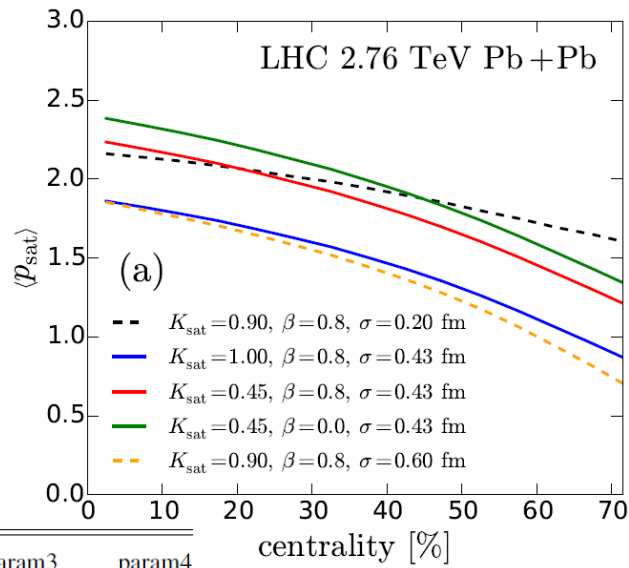
$$P_i(K_{\text{sat}}, \beta) = a_{i0} + a_{i1} K_{\text{sat}} + a_{i2} \beta + a_{i3} K_{\text{sat}} \beta + a_{i4} \beta^2 + a_{i5} K_{\text{sat}}^2$$

TABLE II. The parametrization of $p_{\text{sat}}(K_{\text{sat}}, \beta)$ for $\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb collisions for $K_{\text{sat}} \in [0.4, 2.0]$ and $\beta < 0.9$

$P_i \rightarrow$	C	n	a	b
a_{i0}	3.9027590	0.1312476	-0.0044020	0.8537670
a_{i1}	-0.6277216	-0.0157637	0.0220154	-0.0580163
a_{i2}	1.0703962	-0.0362980	-0.0005974	0.0957157
a_{i3}	0.0692793	-0.0022506	0.0125320	-0.0016413
a_{i4}	-1.9808449	0.0615129	-0.0032844	-0.1788390
a_{i5}	0.1106879	0.0052116	-0.0033841	0.0220187

Interestingly,
 the modest T-dependence of the
 hadronic viscosity obtained here in **PCE**
 is in fact **not** inconsistent with
 microscopic calculations in **CE** [e.g.
 Cernai, Kapusta, McLerran, PRL 97
 (2006) 152303]





η/s	0.20	param1	param2	param3	param4
K_{sat}	0.63	0.50	0.75	0.45	0.64

Average p_{sat} as a function of centrality in $\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb collisions at the LHC (a), and in $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions at RHIC (b) with different values of K_{sat} , β and σ .

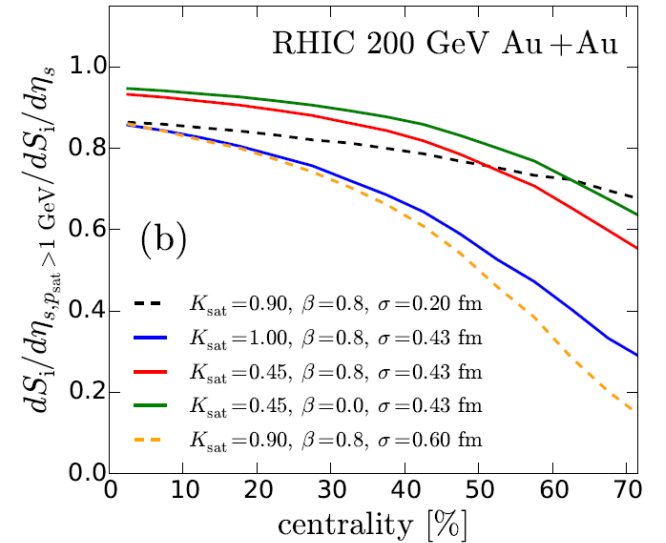
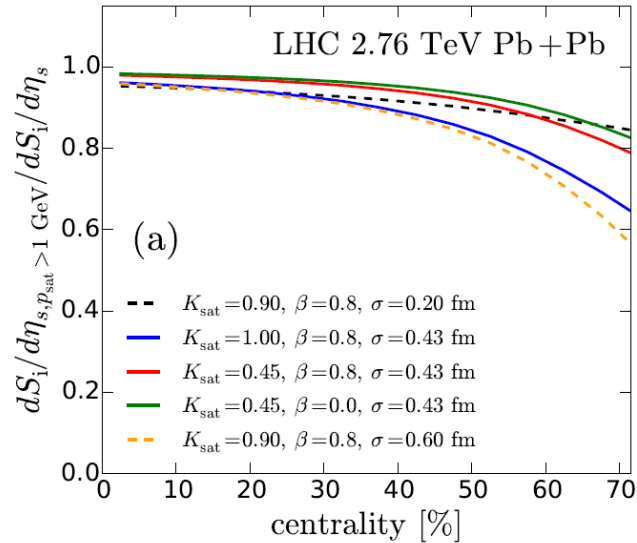
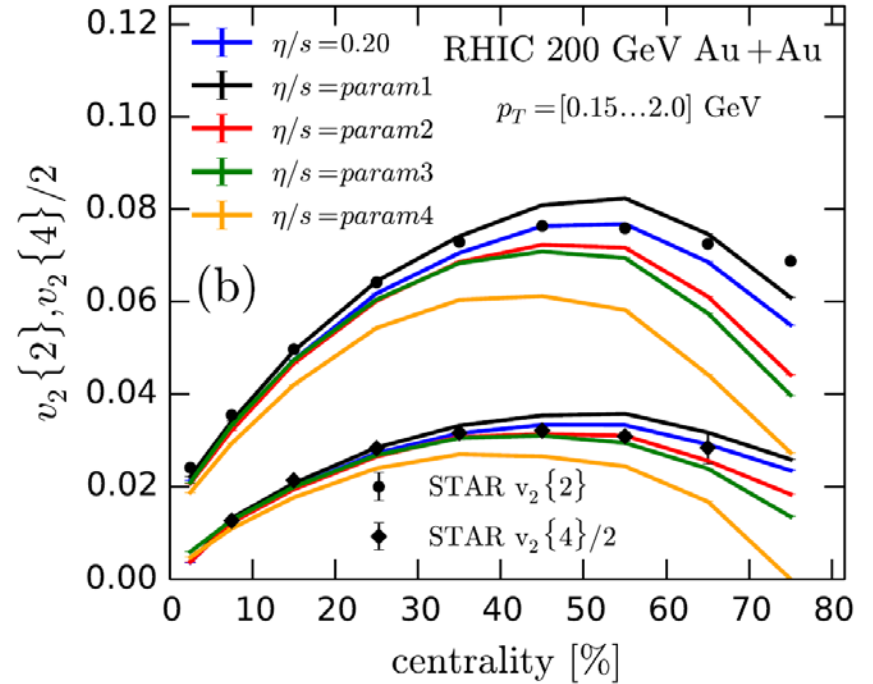
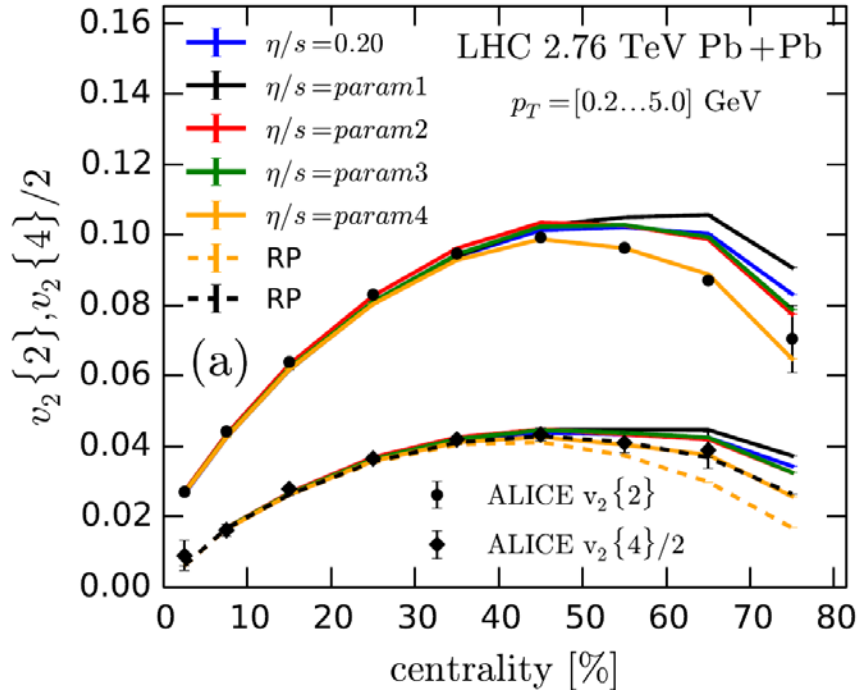


FIG. 9. (Color online) Fraction of dS_i/dn_s from the region $p_{\text{sat}} \geq 1$ GeV as a function of centrality in $\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb collisions at the LHC (a), and in $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions at RHIC (b).

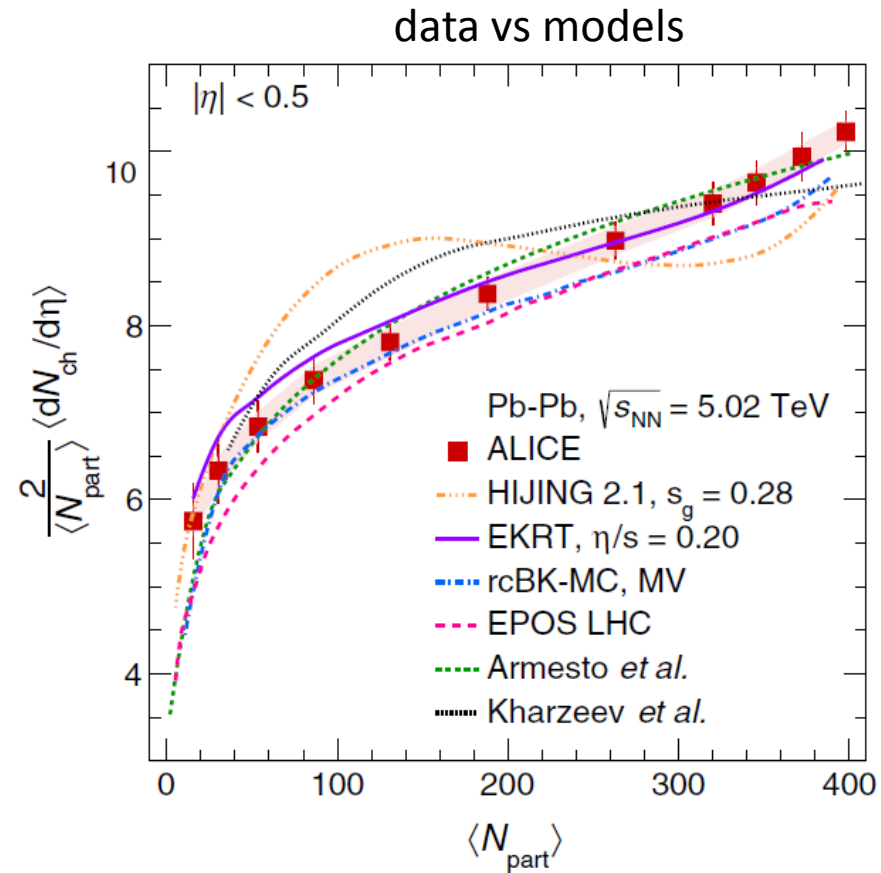
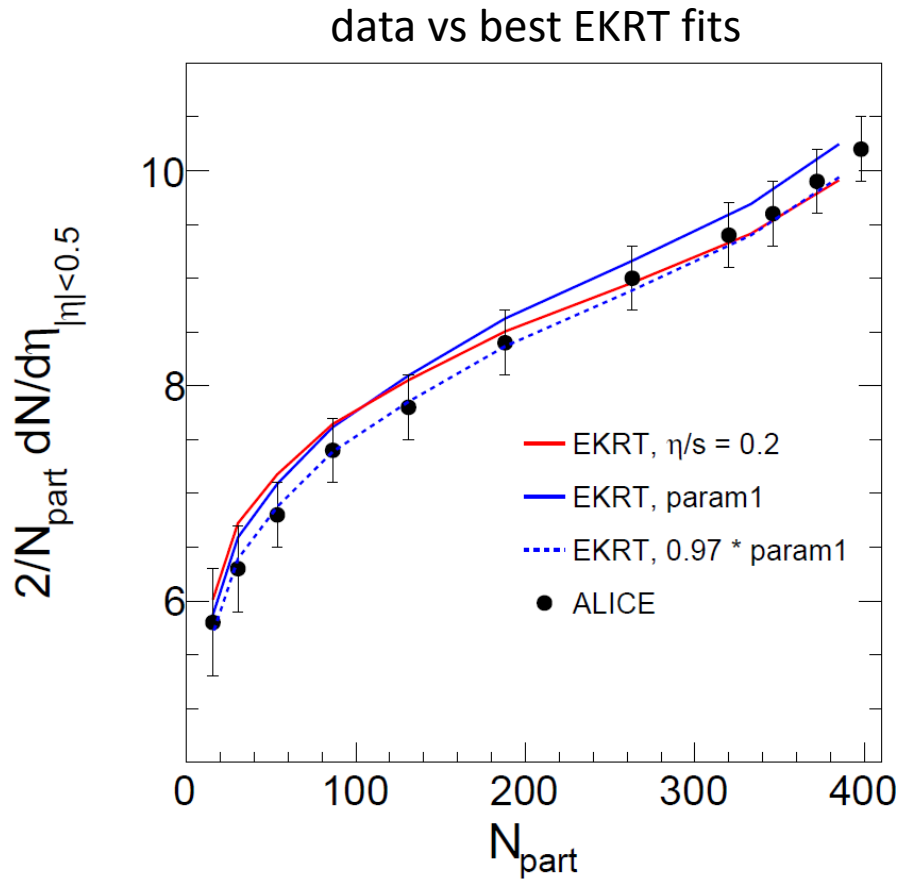
Centrality dependence of 2,4-particle cumulant flow coefficients v_n



EKRT prediction [Phys.Rev. C93 (2016) 014912, arXiv:1511.04296 [hep-ph]]

vs. 5.02 TeV ALICE data [Phys.Rev.Lett. 116 (2016) 222302, arXiv:1512.06104 [nucl-ex]] ,

using ALICE's Npart



[ALICE, PRL116 (2016) 222302]

F-components for each event: $v_n(y)e^{in\Psi_n(y)} = \langle e^{in\phi} \rangle_{\phi, p_T}$

event plane angle $\Psi_n = \text{atan2}(\langle \sin n\phi \rangle / \langle \cos n\phi \rangle) / n$

$$v_n = \langle \cos(n(\phi - \Psi_n)) \rangle \quad \langle \dots \rangle_{\phi, p_T} = \left(\frac{dN}{dy} \right)^{-1} \int d\phi dp_T^2 \frac{dN}{dy dp_T^2 d\phi} (\dots)$$

Exp's:

$$v_n\{\text{EP}\}(p_T) = \langle \cos[n(\phi - \Psi_n\{\text{EP}\})] \rangle_{\phi} \Big|_{\text{ev}} \rightarrow \langle v_n \rangle_{\text{ev}}, \langle v_n^2 \rangle_{\text{ev}}^{1/2}$$

high / low resolution

$$\Psi_n\{\text{EP}\} = \frac{1}{n} \text{atan2}(\langle w \cos(n\phi) \rangle_{\phi, p_T}, \langle w \sin(n\phi) \rangle_{\phi, p_T})$$

2-particle cumulant, $v_n\{2\}^2 = \langle e^{in(\phi_1 - \phi_2)} \rangle_{\phi} \equiv \frac{1}{N_2} \int d\phi_1 d\phi_2 \frac{dN_2}{d\phi_1 d\phi_2} e^{in(\phi_1 - \phi_2)}$
 not sensitive to $\Psi_n[\text{EP}]$

$$\frac{dN_2}{d\phi_1 d\phi_2} = \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} + \delta_2(\phi_1, \phi_2)$$

$$v_n\{2\} = \langle v_n^2 + \delta_2 \rangle_{\text{ev}}^{1/2} \stackrel{\text{flow}}{=} \langle v_n^2 \rangle_{\text{ev}}^{1/2} \quad (\text{no non-flow in our results})$$

3,4-particle cumulants $v_4\{3\} \equiv \frac{\langle v_2^2 v_4 \cos(4[\Psi_2 - \Psi_4]) \rangle_{\text{ev}}}{\langle v_2^2 \rangle_{\text{ev}}}$ $v_n\{4\} \equiv (2\langle v_n^2 \rangle_{\text{ev}}^2 - \langle v_n^4 \rangle_{\text{ev}})^{1/4}$

$$\varepsilon_{m,n} e^{in\Psi_{m,n}} = -\{r^m e^{in\phi}\} / \{r^m\} \quad \{\dots\} = \int dx dy e(x, y, \tau_0) (\dots)$$

$$\Psi_{m,n} = \frac{1}{n} \text{atan2}(\{r^m \cos(n\phi)\}, \{r^m \sin(n\phi)\}) + \frac{\pi}{n} \quad \text{Participant plane angle}$$

EoM for shear-stress tensor from 14-moment approx to UR gas

$$\begin{aligned} \tau_\pi \frac{d}{d\tau} \pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} + c_1\pi^{\mu\nu}\nabla^\alpha u_\alpha + c_2\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} \\ &+ c_3\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} + c_4\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha}, \end{aligned} \quad (4)$$

$$\begin{aligned} c_1 &= -(4/3)\tau_\pi \\ c_2 &= -(10/7)\tau_\pi \\ c_3 &= 2\tau_\pi \\ c_4 &= 9/(70P_0) \end{aligned}$$

$$\tau_\pi = \frac{5\eta}{e + P_0}$$

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 Denicol, Niemi, Molnár, Rischke, Phys. Rev. D85, 114047 (2012)
 Molnár, Niemi, Denicol, Rischke, Phys. Rev. D89, 074010 (2014)

$$\begin{aligned} \sigma^{\mu\nu} &= \dot{\nabla}^{\langle\mu}u^{\nu\rangle} & \omega^{\mu\nu} &= \frac{1}{2}(\nabla^\mu u^\nu - \nabla^\nu u^\mu) & \nabla^\mu &= \Delta^{\mu\nu}\partial_\nu \\ & & & & \Delta^{\mu\nu} &= g^{\mu\nu} - u^\mu u^\nu \end{aligned}$$

Landau frame $eu^\mu = T^{\mu\nu}u_\nu$

$$T^{\mu\nu} = eu^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$N_i^\mu = n_i u^\mu + n_i^\mu,$$

$$e = T^{\mu\nu}u_\mu u_\nu \quad P = P_0 + \Pi \quad n_i = N_i^\mu u_\mu \quad n_i^\mu = N_i^{\langle\mu}$$

$$\pi^{\mu\nu} = T^{\langle\mu\nu\rangle}$$

$$A^{\langle\mu\rangle} = \Delta^{\mu\nu}A_\nu$$

$$A^{\langle\mu\nu\rangle} = \frac{1}{2} \left[\Delta_\alpha^\mu \Delta_\beta^\nu + \Delta_\beta^\mu \Delta_\alpha^\nu - \frac{2}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta} \right] A^{\alpha\beta}$$