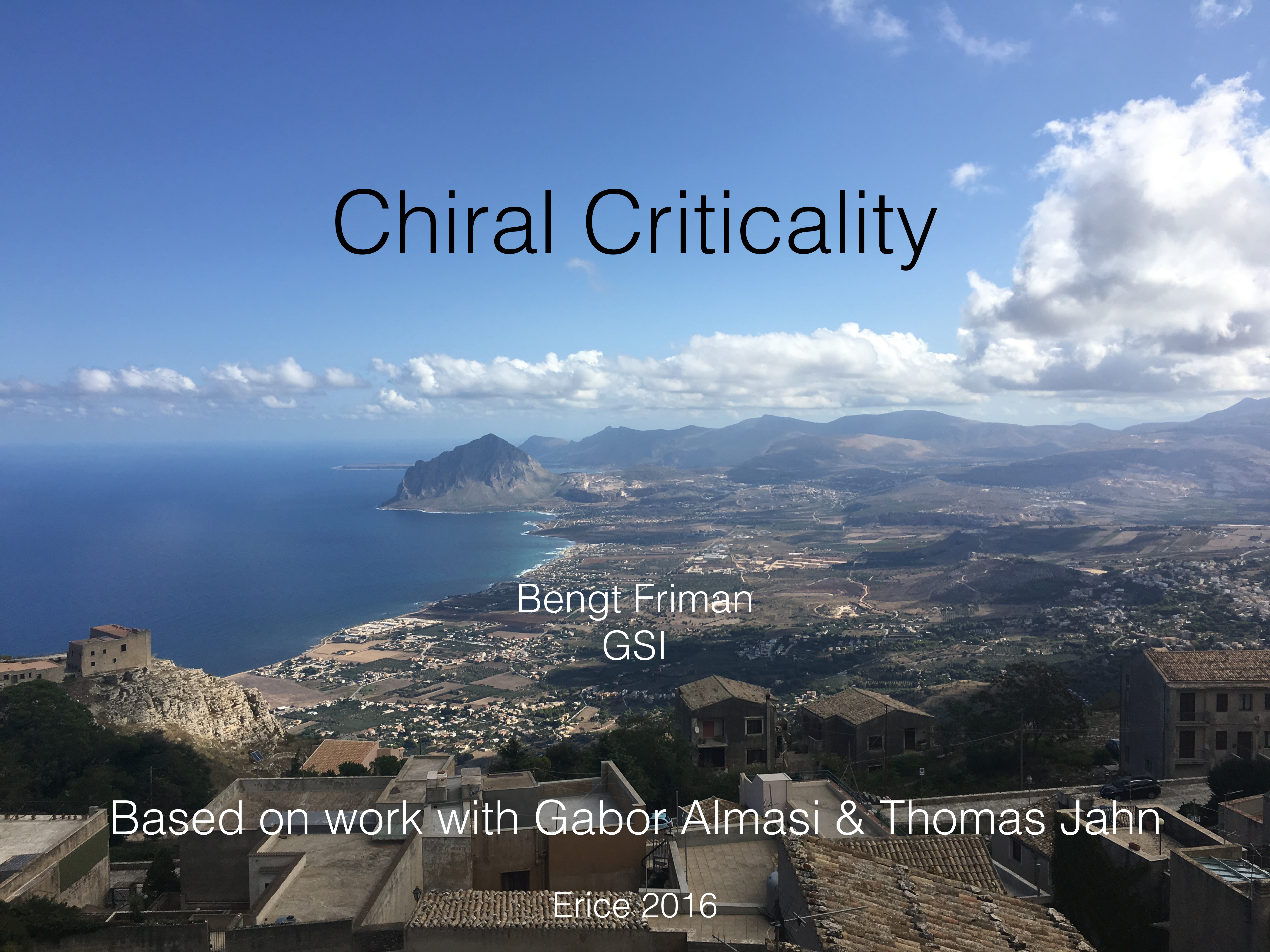


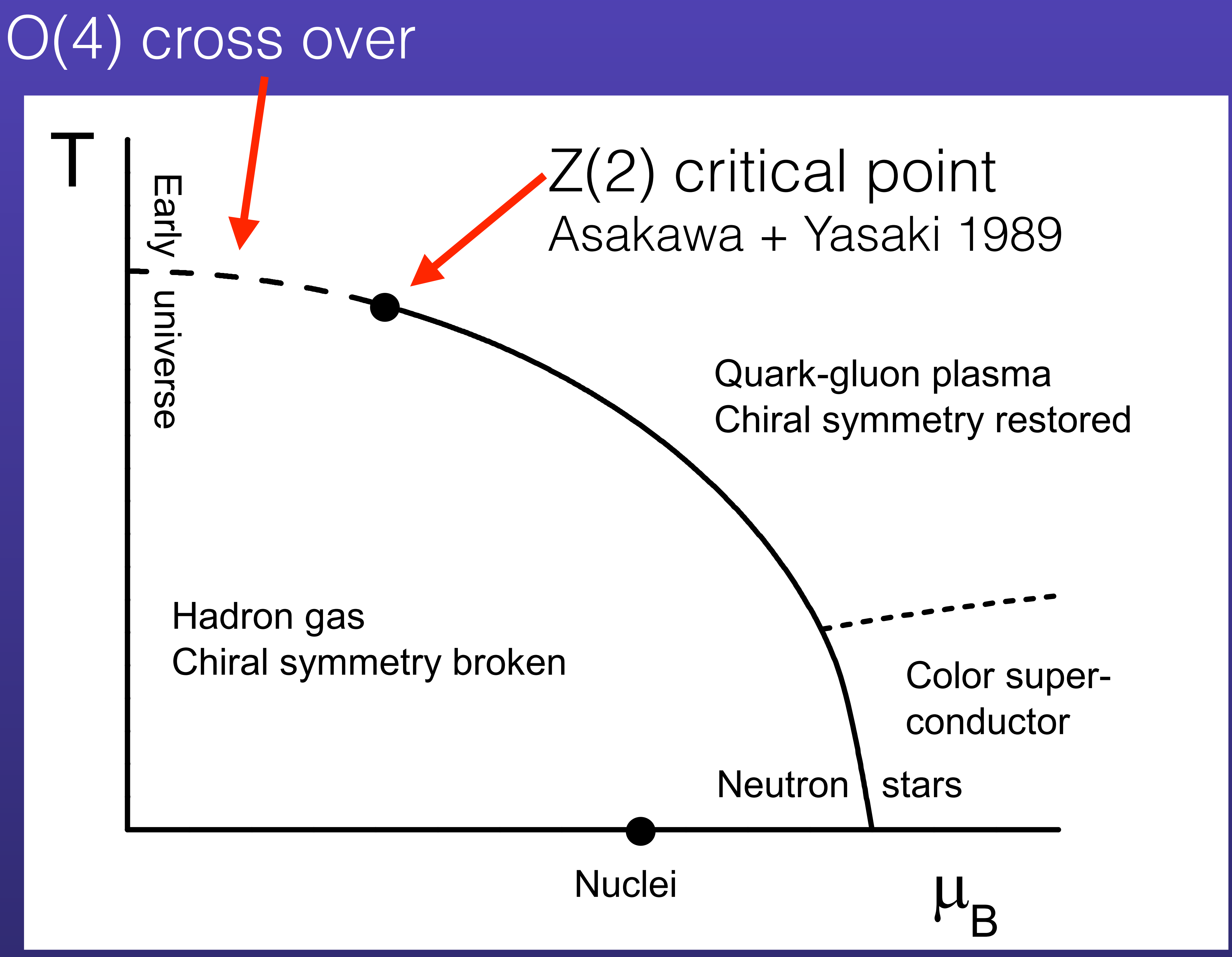
Chiral Criticality

Bengt Friman
GSI

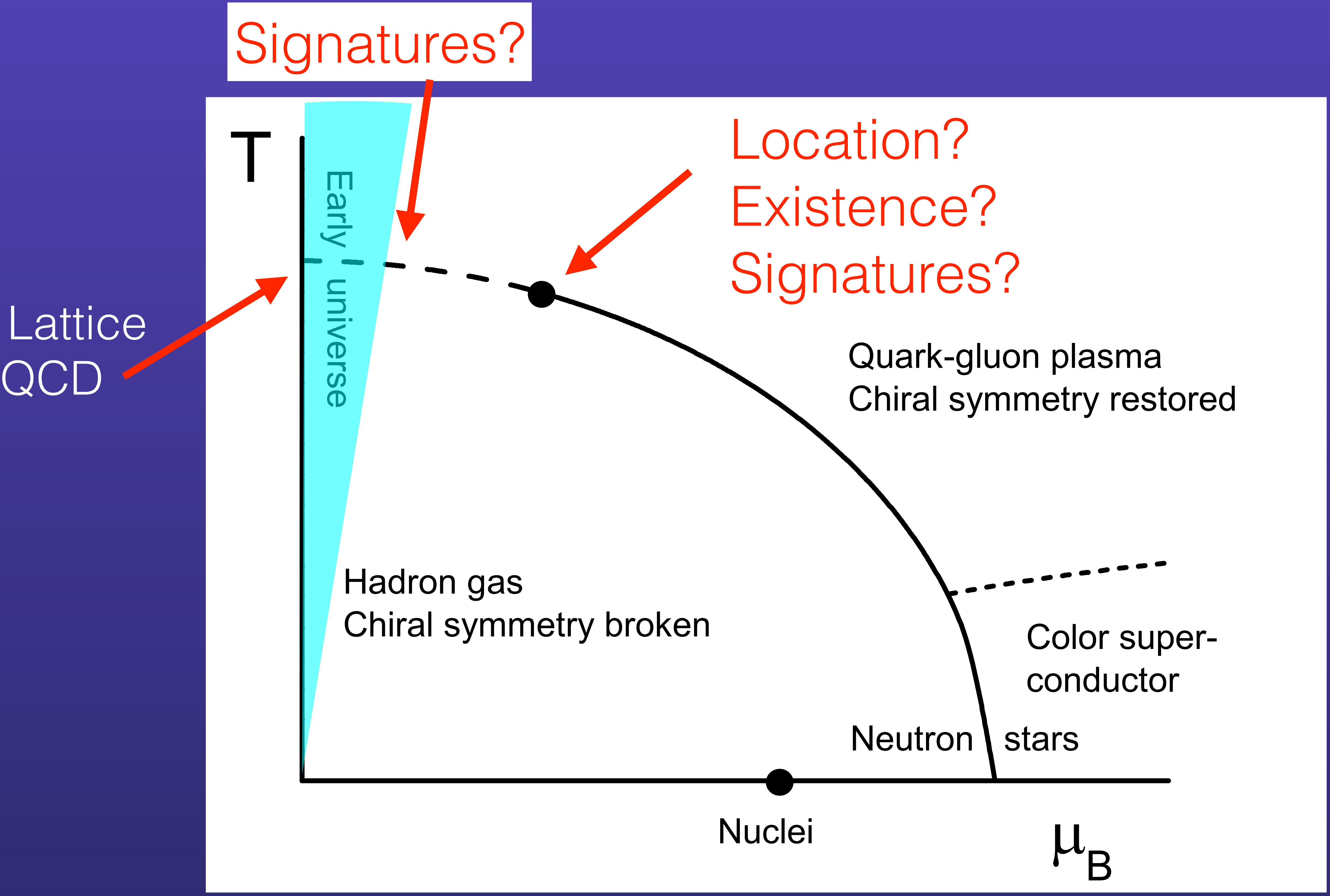
Based on work with Gabor Almasi & Thomas Jahn

Erice 2016

Conjectured phase diagram of QCD

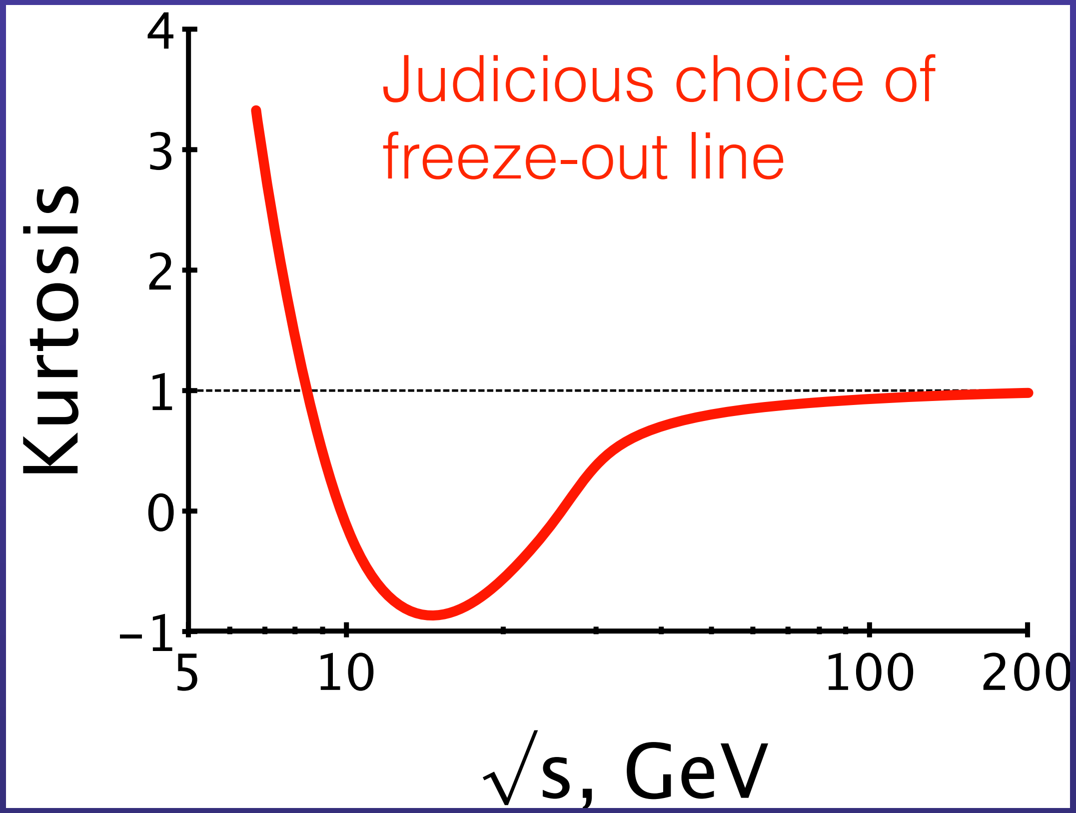


Knowns and unknowns

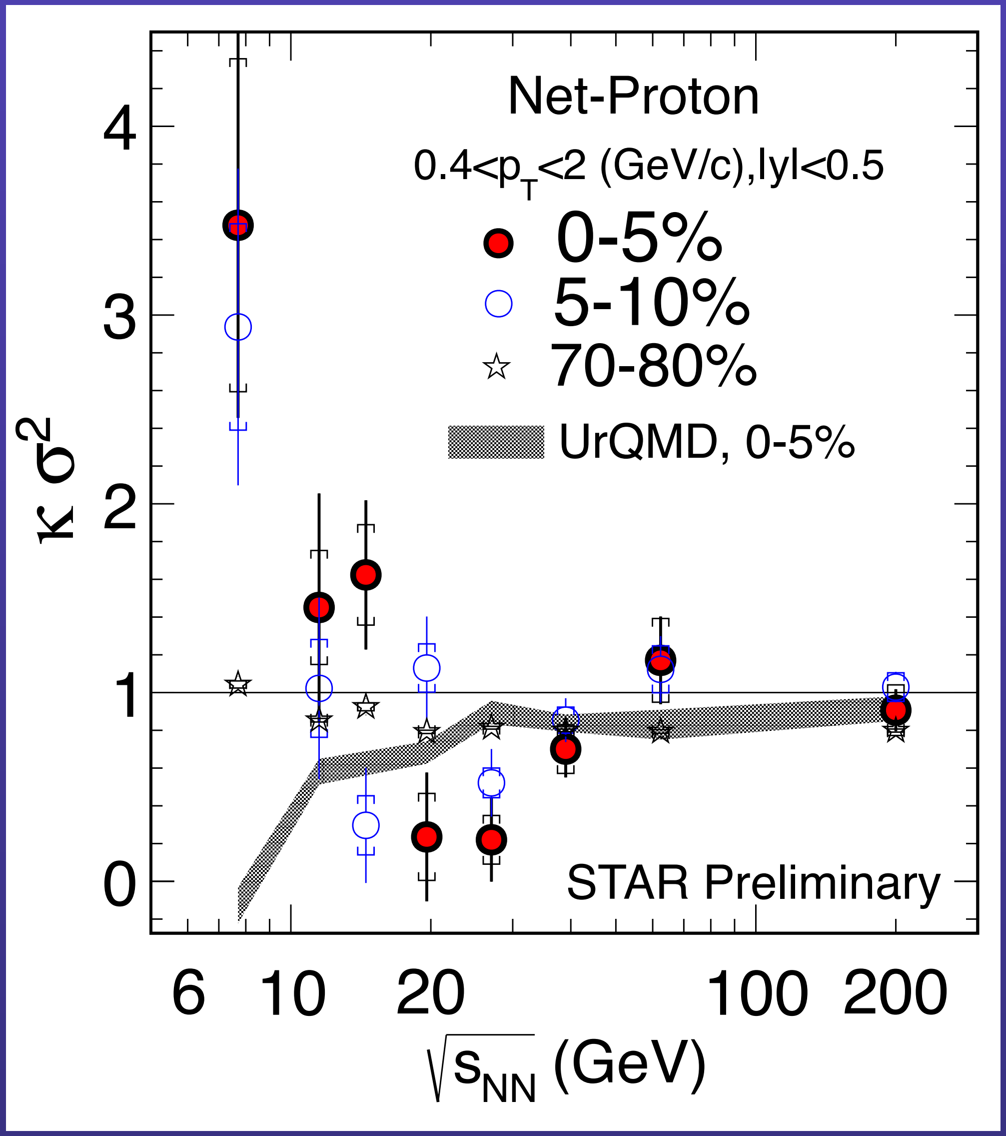


Fluctuations of conserved quantum numbers critical?

- STAR data on net-proton cumulants $(\kappa\sigma^2 = \chi_p^4/\chi_p^2)$
- Signature of Z(2) CP?

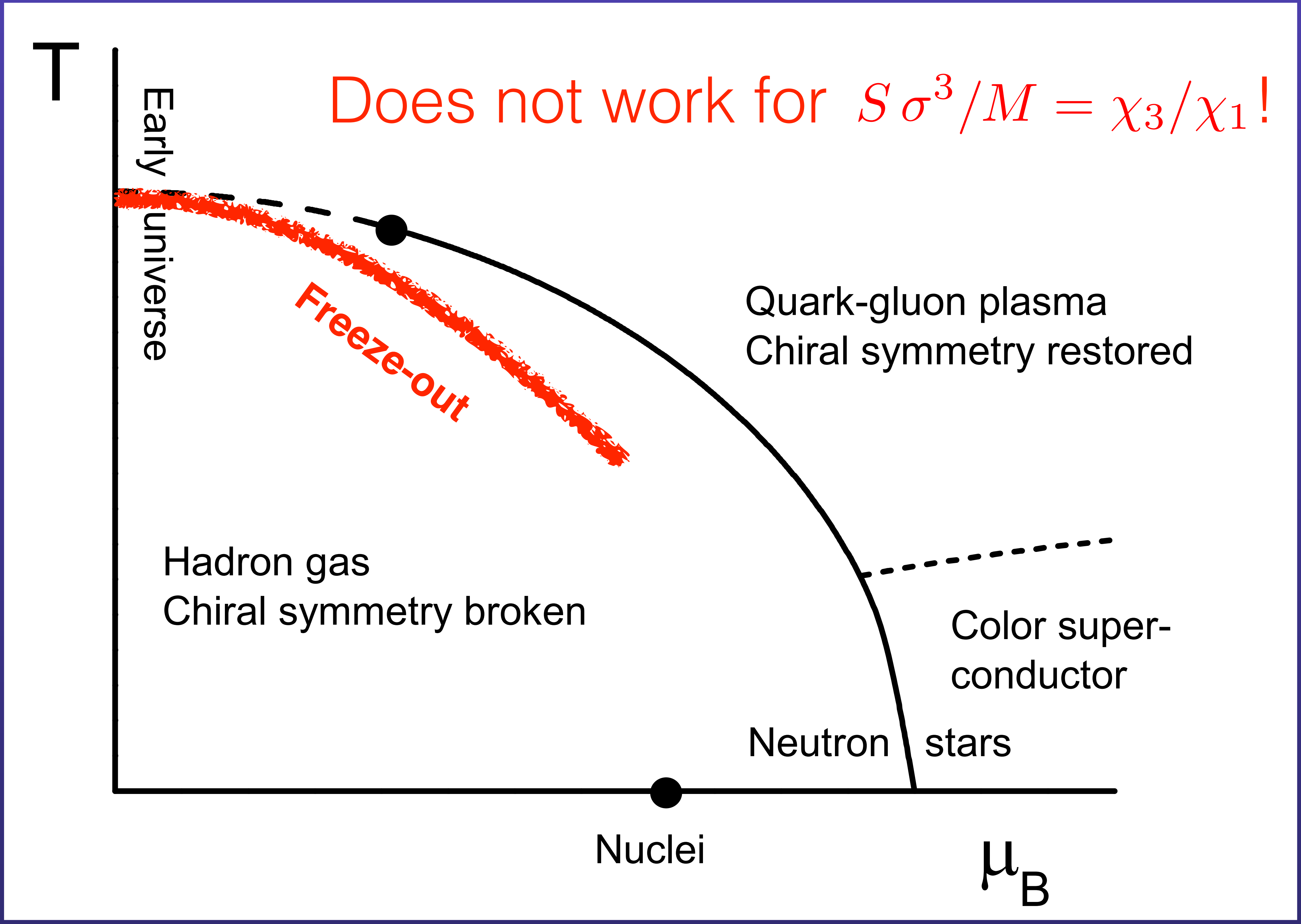


Stephanov, PRL ’11
Skokov, QM 2012



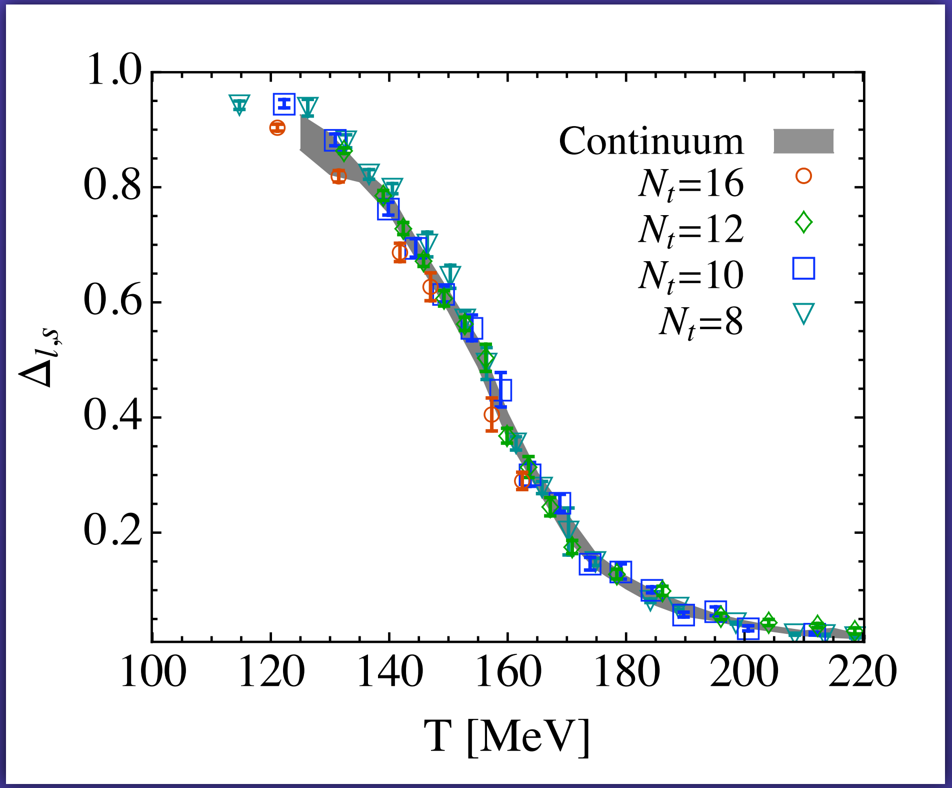
χ_p^4/χ_p^2 STAR data

Freeze-out line below CP



Lattice QCD

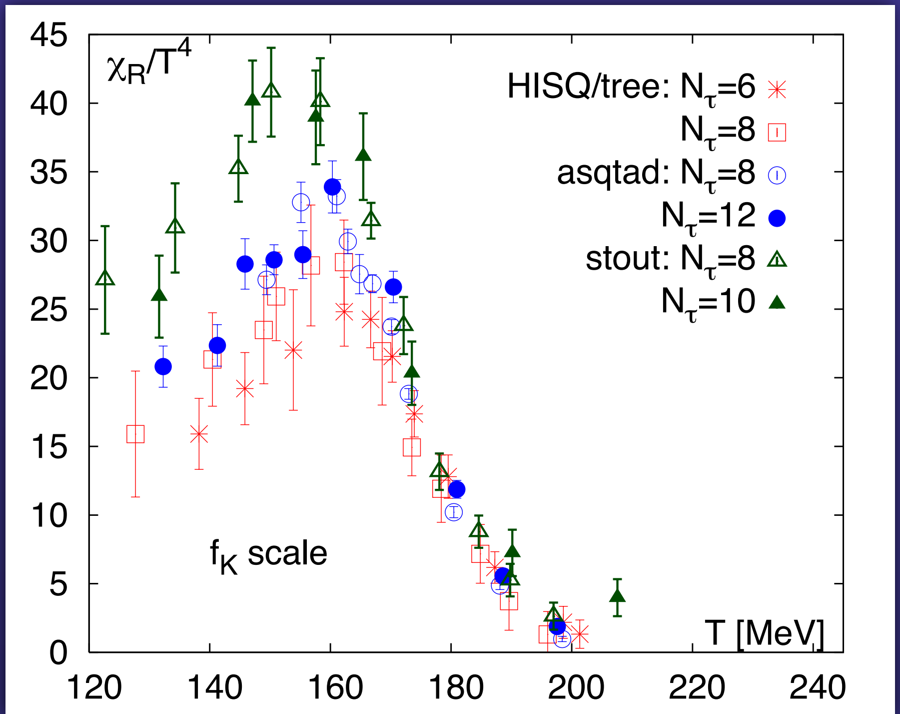
- Chiral symmetry broken in vacuum and restored at high temperatures.



Borsanyi et al. 2010

- Quark condensate $\rightarrow 0$ at high T (order parameter)

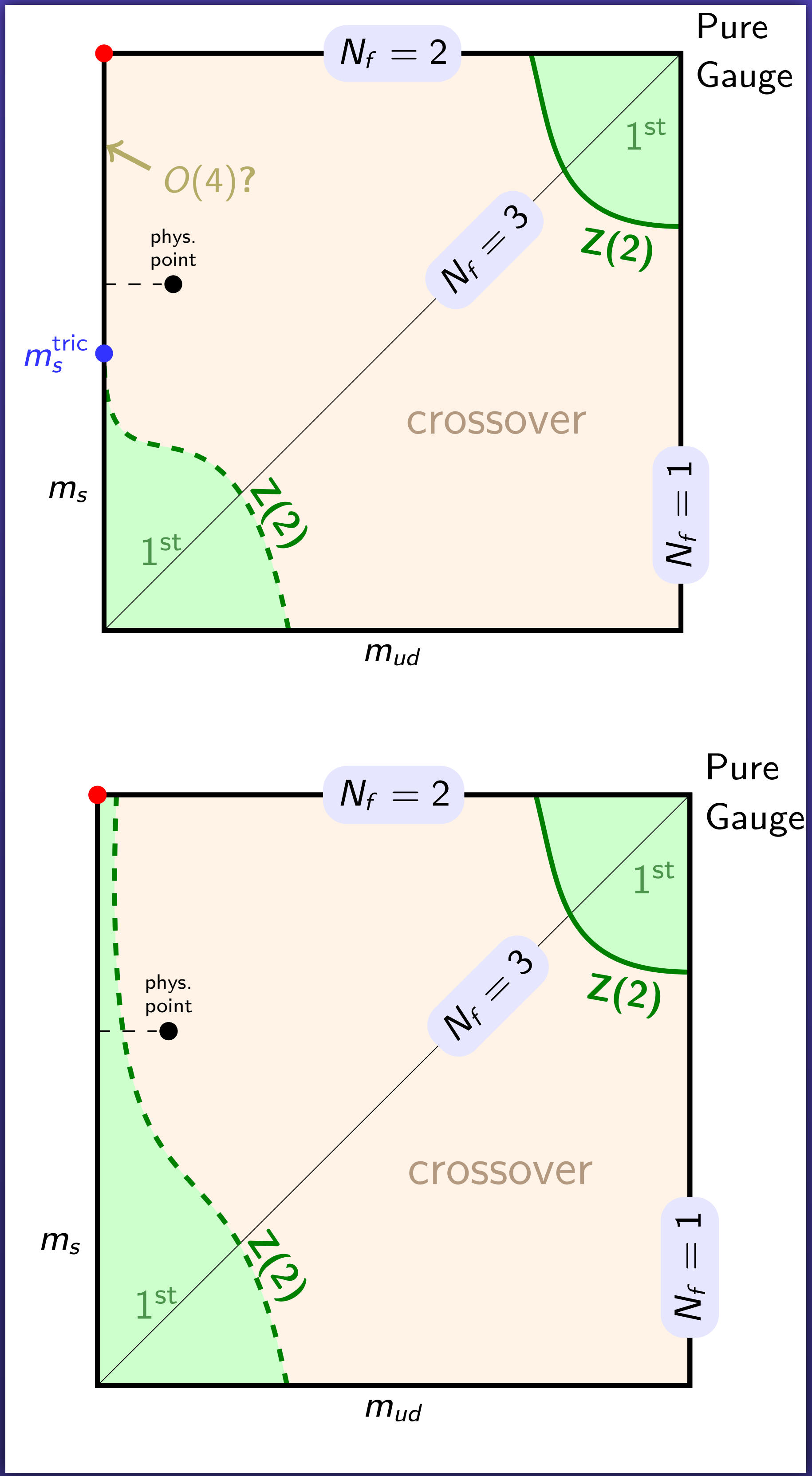
- Chiral susceptibility peaks at $T \simeq T_{pc}$ (fluctuations!)



Bazavov et al. 2012

Columbia plot ($\mu_B = 0$)

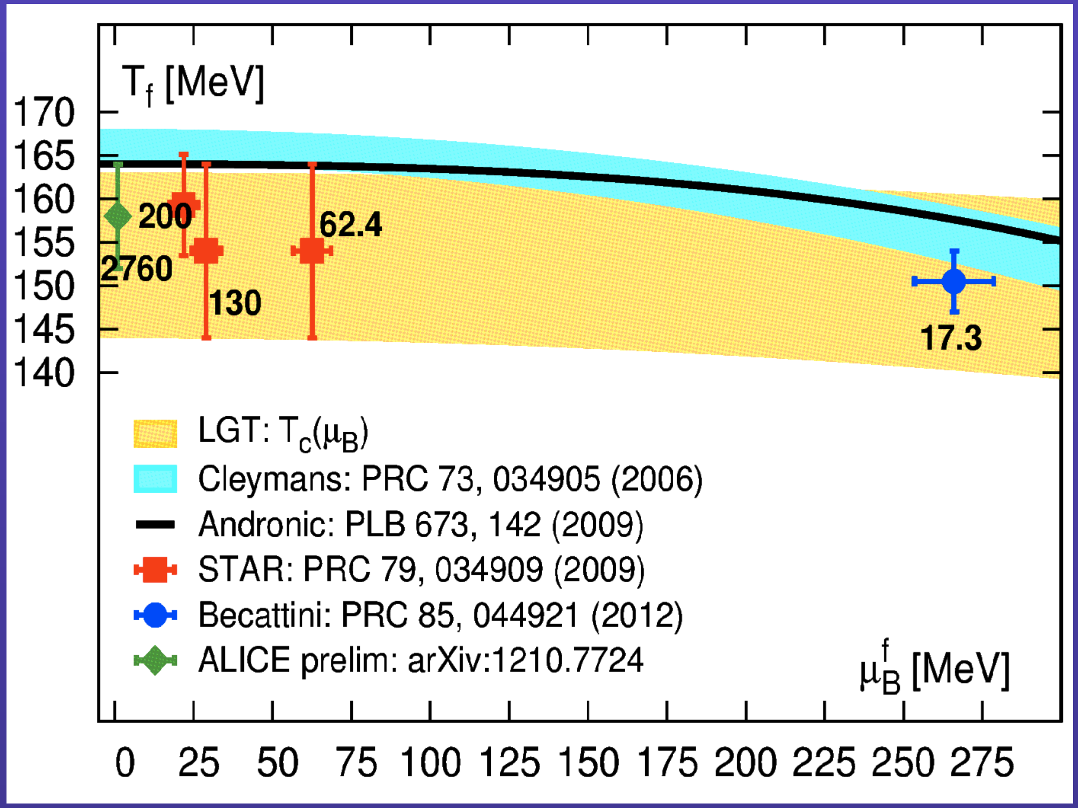
Philipsen & Pinke, 2016



- Dependence on quark masses
- Three massless flavors:
  1st order chiral transition
- Quark masses $\rightarrow \infty$
  1st order deconfinement trans.
- Order of transition in
  2-flavor chiral limit:
  - 2nd order $\rightarrow$ O(4) scaling
  - 1st order $\rightarrow$ Z(2) scaling

Lattice QCD @ small μ_B

- LQCD at $\mu_B \neq 0$ difficult due to sign problem



- $\rightarrow$ Taylor expansion about

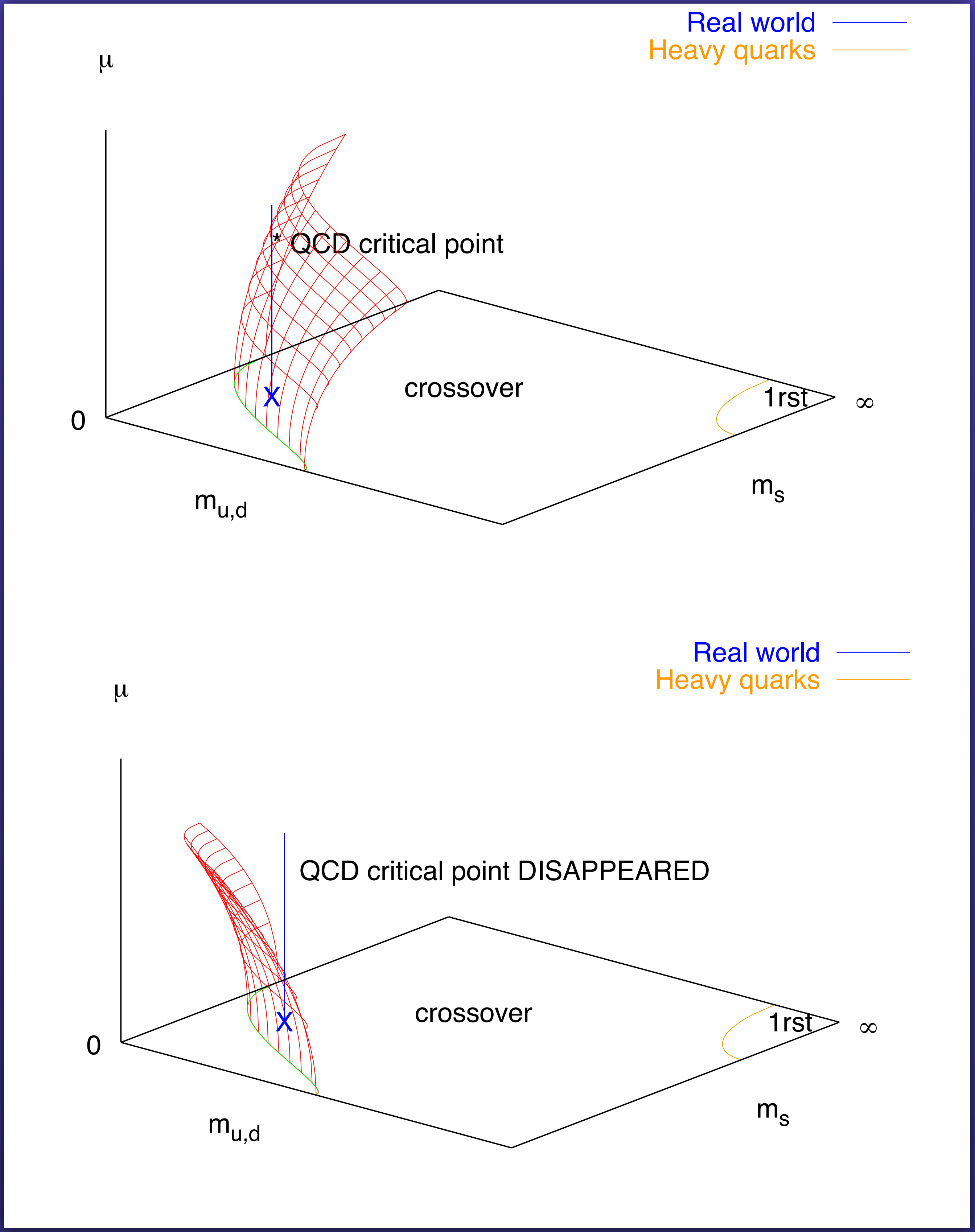
$$\mu_B = 0 \quad \rightarrow \mu_B \neq 0 \ (\mu_B/T \lesssim 1)$$

- Other schemes:
  - Im chemical potential + analytic continuation
  - Reweighting
  - Complex Langevin ➡ Gert Aarts talk

Columbia plot @ non-zero μ_B

deForcrand & Philipsen 2007

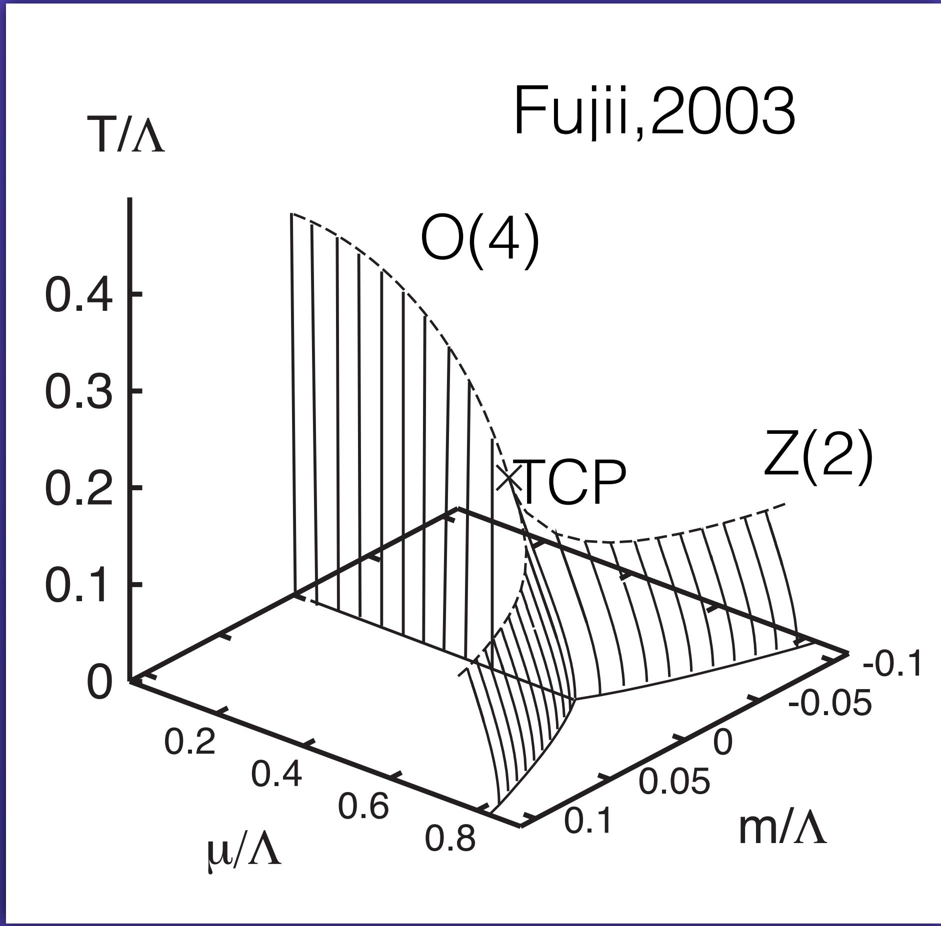
- Line of critical points $\rightarrow$ surface of cp's
- Physical point crosses surface $\rightarrow$ CP & 1st ord.

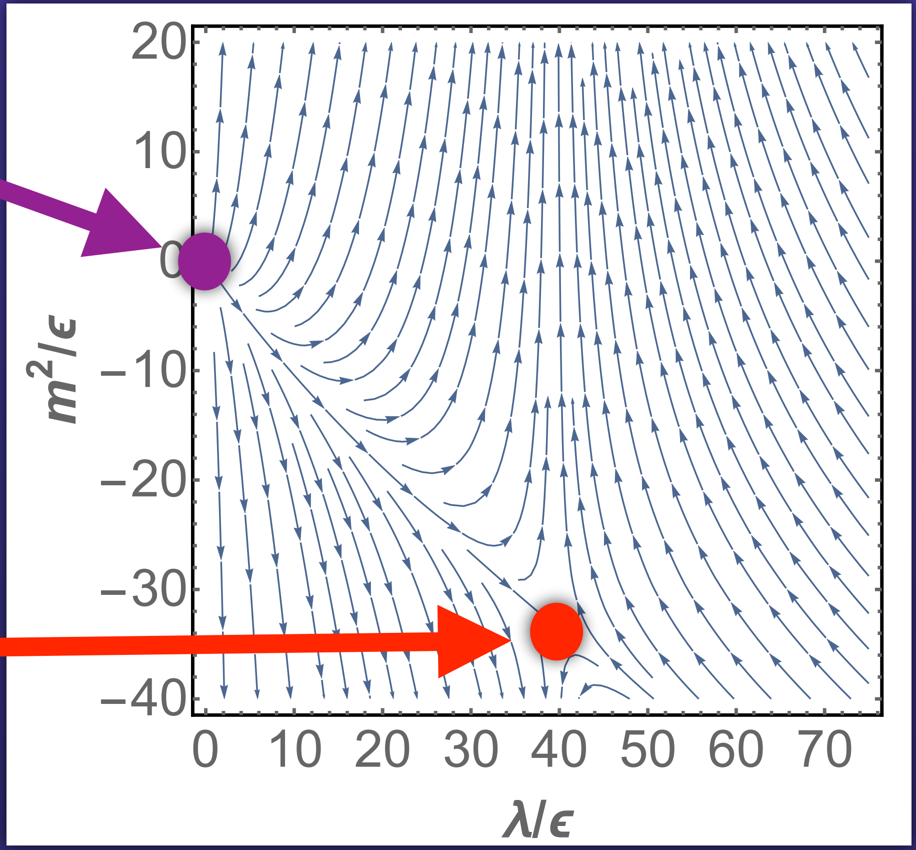


- No crossing $\rightarrow$ chiral transition remains of cross over type.
- Not settled due to strong cut-off effects

Wilson RG flow

- Assume O(4) at $\mu_B \simeq 0$ + critical point at larger μ_B
- TCP corresponds to Gaussian FP
- Crossover from O(4) to Gaussian to Z(2) fixed point with incr. μ_B? Interference between FP's?

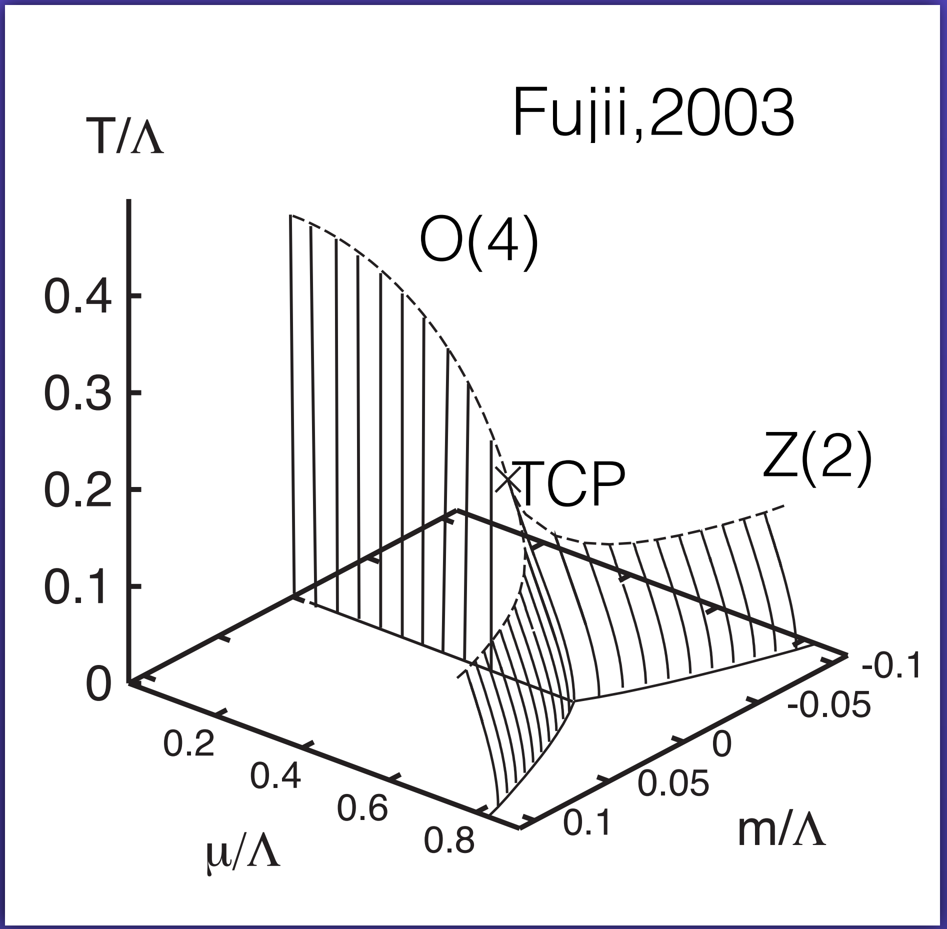




Wilson-Fisher O(4)

Wilson RG flow

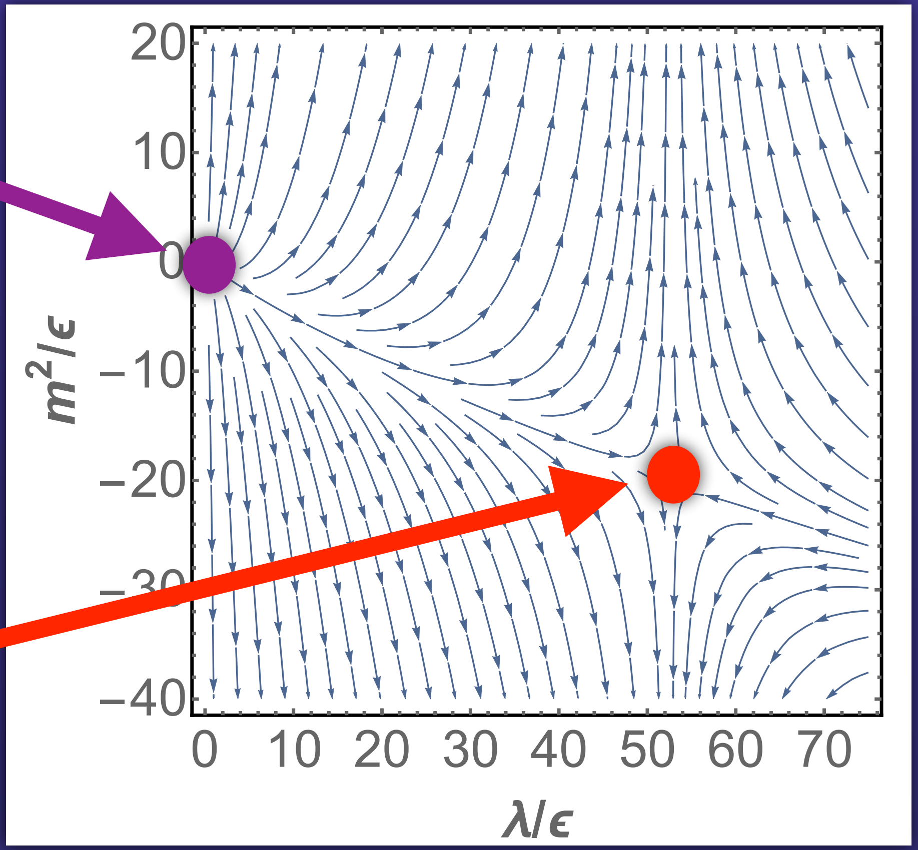
- Assume O(4) at $\mu_B \simeq 0$ + tricritical point at larger μ_B



- TCP corresponds to Gaussian FP

- Crossover from O(4) to Gaussian to Z(2) fixed point with incr. μ_B? Interference between FP's?

Wilson-Fisher O(1)=Z(2)



Outline

- Critical fluctuations and scaling
- Aside on focusing (on the cross over side)
- Magnetic equation of state and scaling window
- Scaling window near TCP
- Conclusions

Critical fluctuations

- Mixture of methanol and cyclohexane

Uniform mixture ⟷ Separated fluids

Light scatt. on critical fluctuations

Index of refraction $n_1 \neq n_2$

$T > T_c$

Uniform mixture

$T = T_c$

Critical opalescence

Criticality and scaling (heuristic)

- Close to a CP, $\xi\,(\to\infty)$ most important length scale; responsible for singular part of thermodynamics

- Partition function dimensionless & extensive

$$\log\mathcal{Z} = \left(\frac{L}{\xi}\right)^d \times g_s + \left(\frac{L}{a}\right)^d \times g_r$$

singular regular

g_s, g_r non-singular

a microsc. length

- Free energy density:

$$f(T,\mu,m) = f_s + f_r$$

$$f_s \sim \frac{\log\mathcal{Z}}{L^d} \sim \xi^{-d}$$

Widom scaling hypothesis

- Reduced temperature $\bar{t} = \frac{1}{t_0} \frac{T - T_c}{T_c}$

- Symmetry breaking field $\bar{h} = \frac{1}{h_0} \frac{H}{H_0}$

- Correlation length diverges @ $\bar{t} = \bar{h} = 0$

$$\xi \sim (\bar{t}\,)^{-1/y_t} \qquad \xi \sim (\bar{h}\,)^{-1/y_h}$$

- Scale invariance

$$\boxed{\xi \to \xi/\lambda} \quad \to \quad \boxed{\bar{t} \to \lambda^{y_t} \bar{t}, \quad \bar{h} \to \lambda^{y_h} \bar{h} \;\text{ or }\; f_s \to \lambda^d f_s}$$

$$f_s(\bar{t}, \bar{h}) = \lambda^{-d} f_s(\lambda^{y_t} \bar{t}, \lambda^{y_h} \bar{h})$$

generalized homogeneous fctn

Critical exponents

- $f_s(\bar{t},\bar{h}) = \lambda^{-d} f_s(\lambda^{y_t}\bar{t}, \lambda^{y_h}\bar{h})$ true for any λ

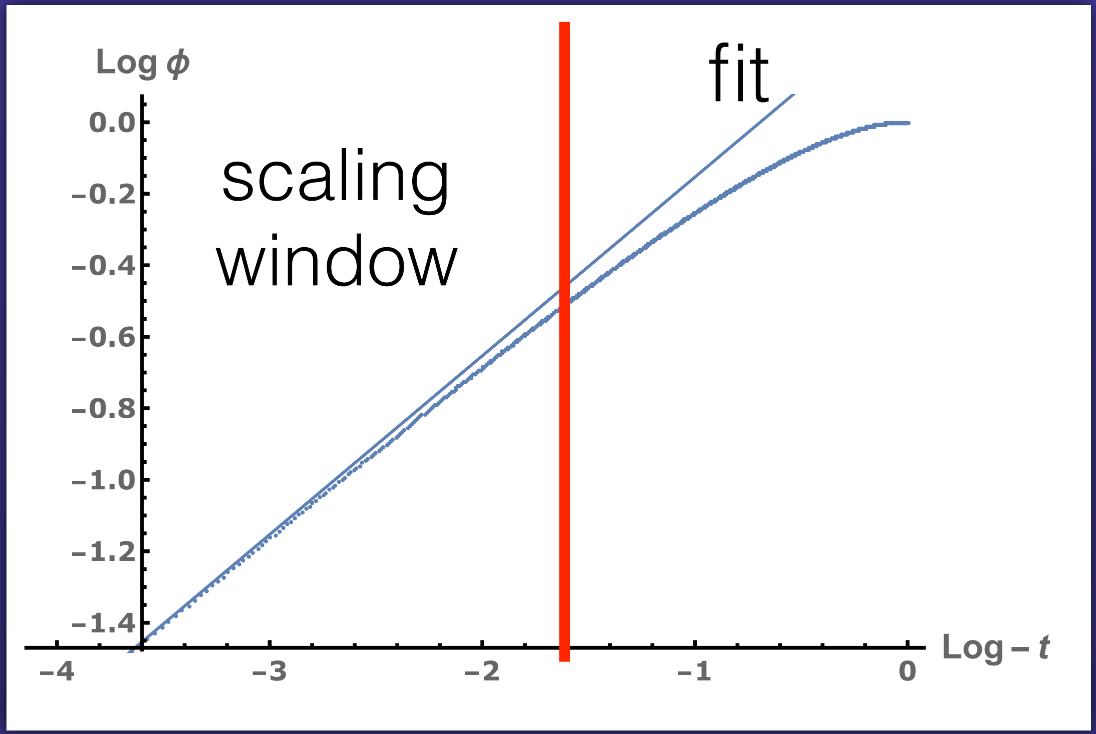
- Choose $\lambda^{y_h}\bar{h} = 1$ $\{y_t, y_h\} \to \{\beta, \delta\}$

$\rightarrow$ $f_s\left(\bar{t},\bar{h}\right) = \bar{h}^{\,1+1/\delta}\tilde{f}_s\left(z\right)$ $z = \bar{t}/\bar{h}^{1/\beta\delta}$

- Order parameter scaling

$\langle\phi\rangle = \partial f_s/\partial\bar{h}$ $\langle\phi\rangle = \bar{h}^{1/\delta}\, f_G(z)$

$\langle\phi\rangle(\bar{t}=0) \sim \bar{h}^{1/\delta}$ $\langle\phi\rangle(\bar{h}=0) \sim (-\bar{t})^{\beta}$



Critical region

- Scaling window: critical fluctuations dominate

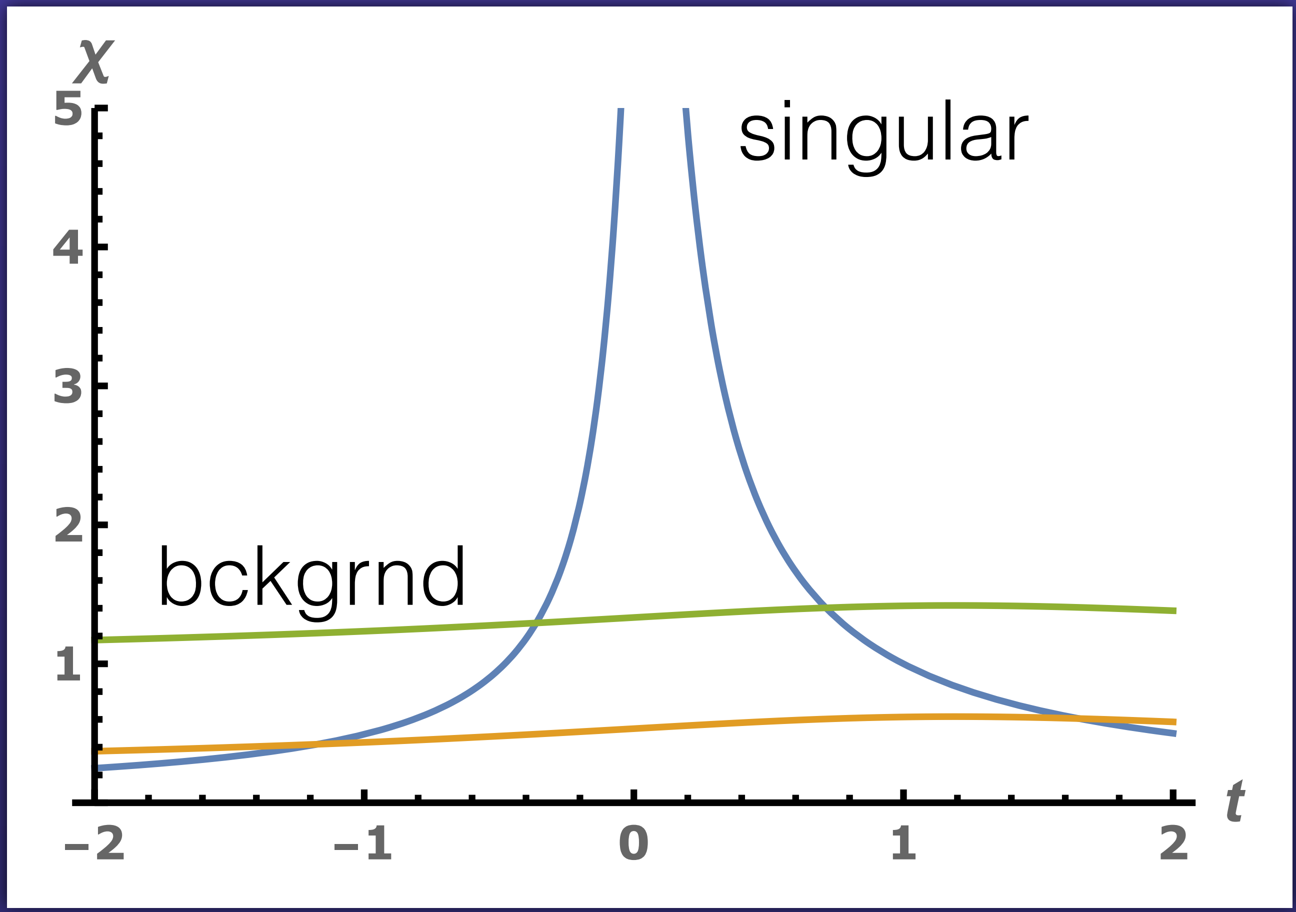
- More derivatives → stronger singularity

$$f(x) = x^{1/2} \qquad f(0) = 0$$

$$f'(x) = \frac{1}{2}x^{-1/2} \qquad f'(0) = \infty$$

$$f^{(n)}(x) \sim x^{1/2-n}$$

- Size of scaling window determ. by competition betw. sing. & reg. parts depends on observable



Aside on focusing

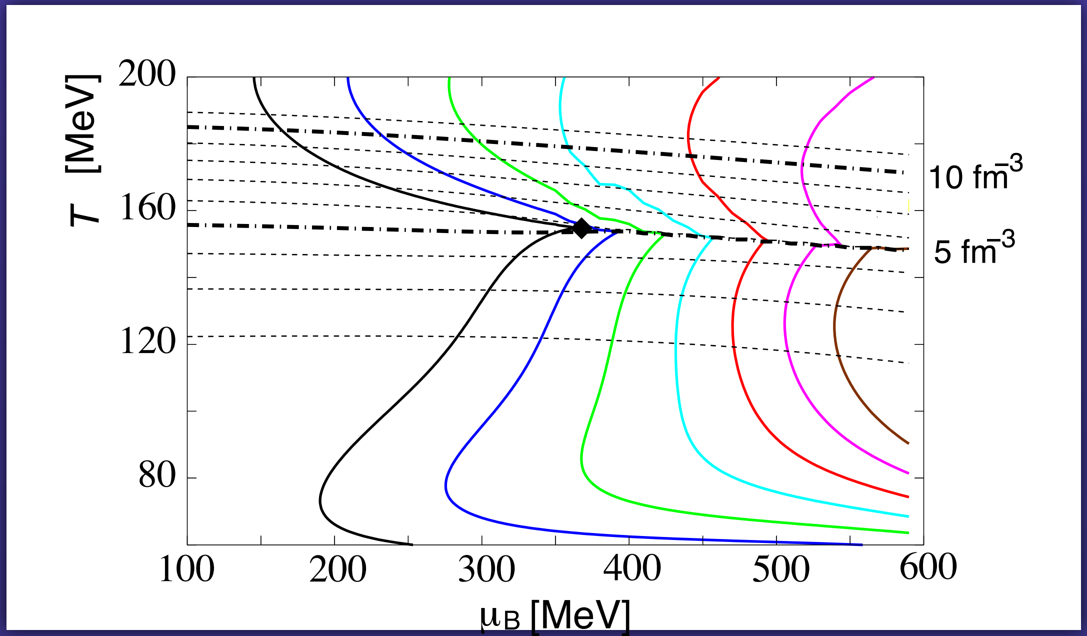
- Robust observable of criticality: Sing. part diverges!
- Singular free energy: $f_s(\bar{t}, \bar{h}) = \lambda^{-d} f_s(\lambda^{y_t}\bar{t}, \lambda^{y_h}\bar{h})$
- Choose $\lambda^{y_t}\bar{t} = 1$
- Yields:

$$f_s = |\bar{t}|^{2-\alpha}\tilde{f}_s\left(\bar{h}/\bar{t}^{y_h/y_t}\right)$$

$$S/N_B \sim -\partial f_s/\partial T \sim |\bar{t}|^{1-\alpha}$$

$1 - \alpha > 0$ No divergence!

Nonaka & Asakawa, 2005

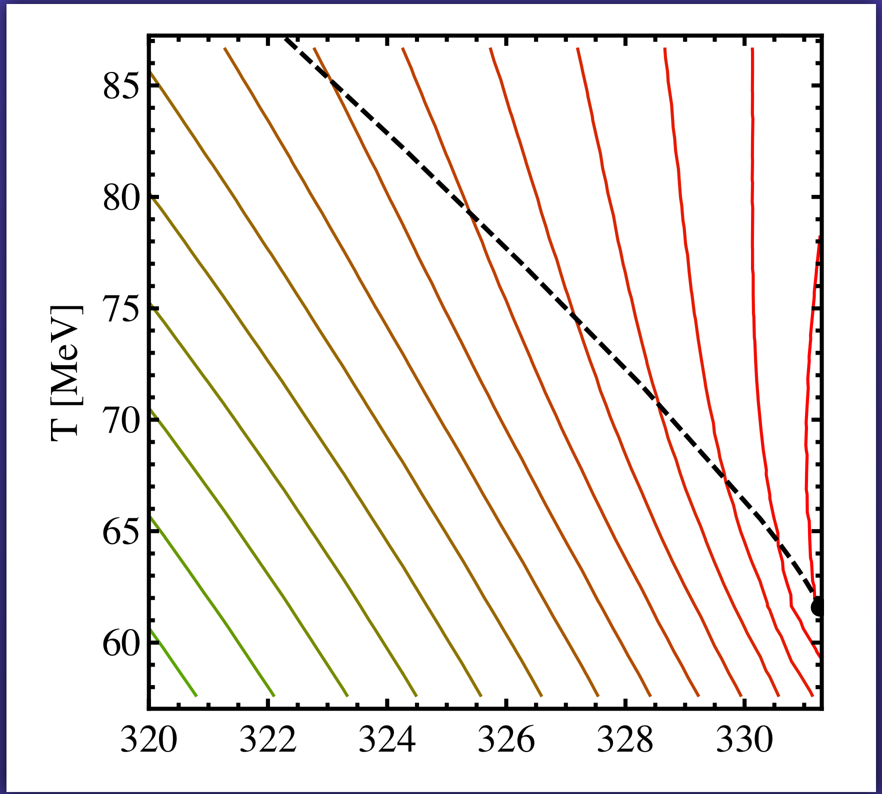


Strength of singularity tuned up

Aside on focusing

- Robust observable of criticality: Sing. part diverges!
- Singular free energy: $f_s(\bar{t},\bar{h}) = \lambda^{-d} f_s(\lambda^{y_t}\bar{t}, \lambda^{y_h}\bar{h})$
- Choose $\lambda^{y_t}\bar{t} = 1$

Nakano *et al., 2010*



PQM-FRG

- Yields:

$$f_s = |\bar{t}|^{2-\alpha} \tilde{f}_s\left(\bar{h}/\bar{t}^{y_h/y_t}\right)$$

$$S/N_B \sim -\partial f_s/\partial T \sim |\bar{t}|^{1-\alpha}$$

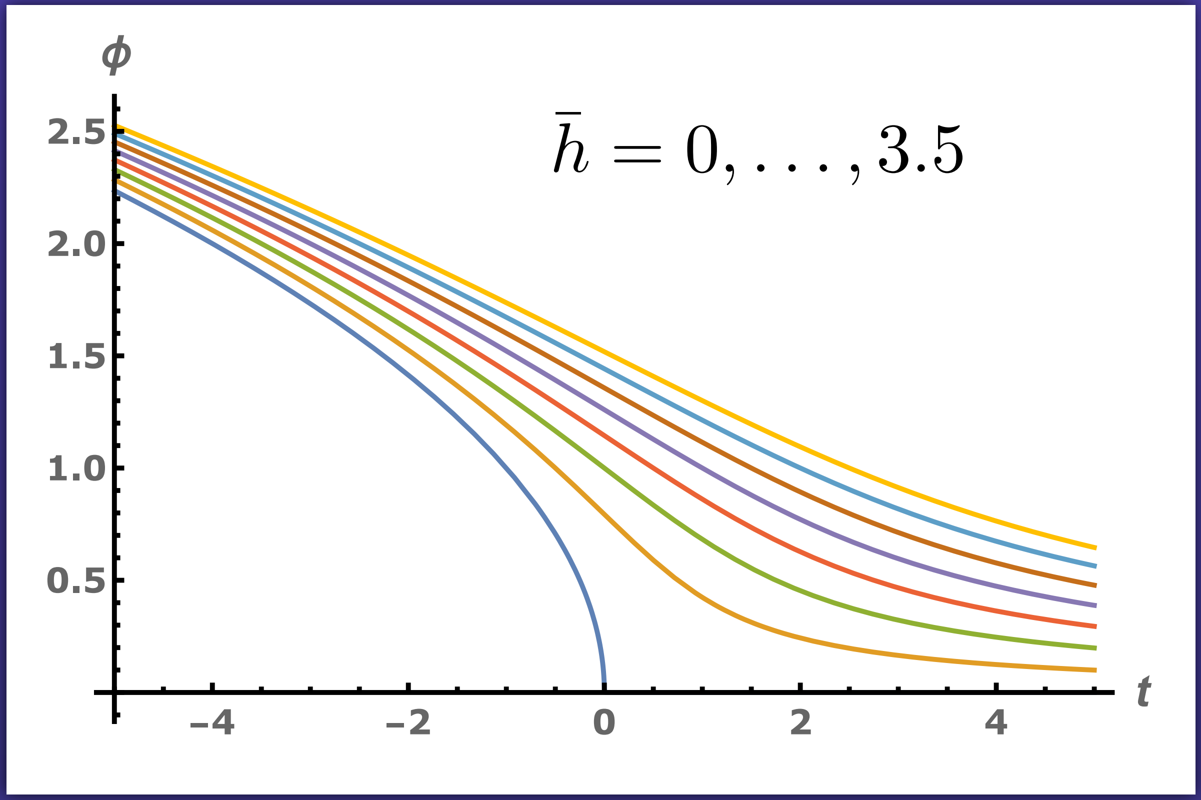
$1-\alpha > 0$ No divergence!

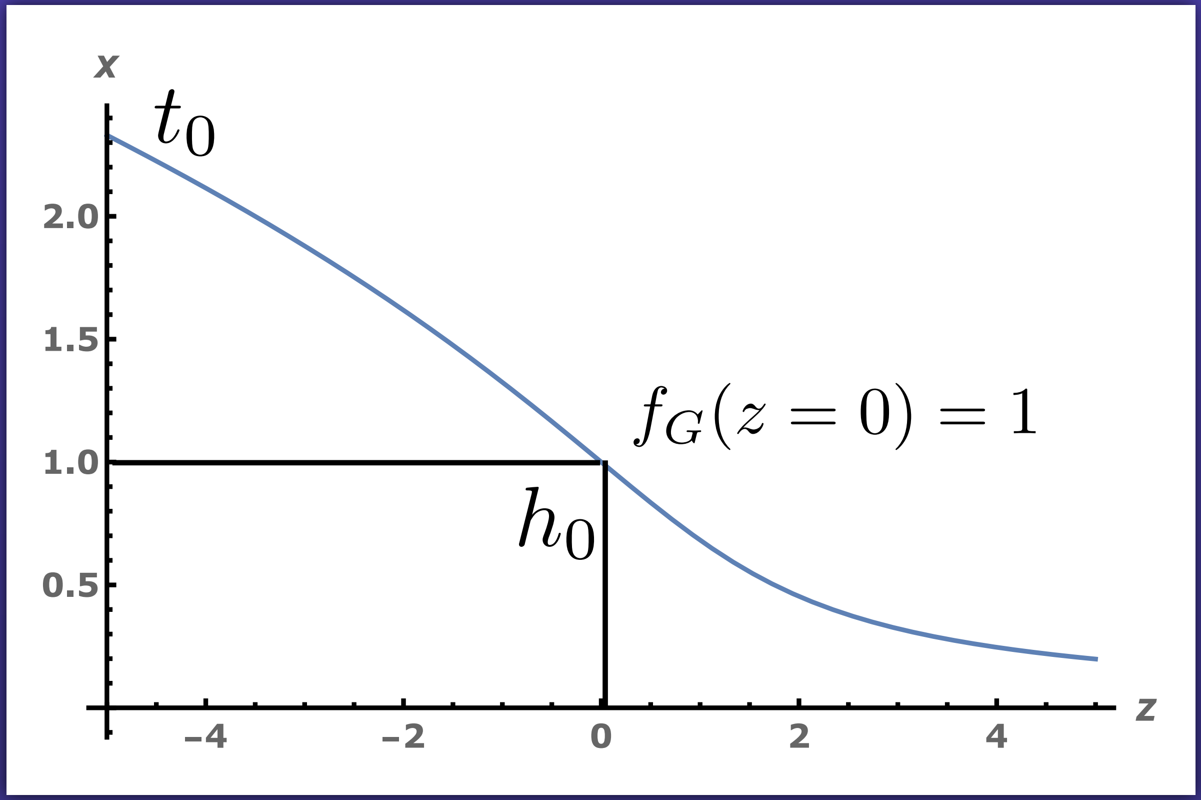
NA-focusing @ very unlikely!

Magnetic equation of state

- Dimensionless order parameter $(\phi = \sigma/\sigma_0)$

$$\langle\phi\rangle/\bar{h}^{1/\delta} \equiv x = f_G(z) \qquad z = \bar{t}/\bar{h}^{1/\beta\delta}$$





- Unique magnetic EOS for each universality class!

- Scaling violations → deviations from universal EOS

Landau Theory

- Landau effective free energy

$$\mathcal{L} = \frac{1}{2}a(T)\phi^2 + \frac{1}{4}b(T)\phi^4 + \frac{1}{6}c(T)\phi^6 - H\phi$$

- Temperature dependence $(a(T_c) = 0)$

$$a(T) = a_1\, t + a2\, t^2 \qquad b(T) = b_0 + b1\, t \qquad c(T) = c_0$$

- mEOS $x = \phi/\bar{h}^{1/\delta}$ $z = \bar{t}/\bar{h}^{1/\beta\delta}$

$$\big(x(x^2+z)-1\big) + \left(\frac{H}{b_0}\right)^{2/3}\left(\frac{c_0}{b_0}x^5 + \frac{b_1}{a_1}x^3 z + \frac{a_2 b_0}{a_1^2}x z^2\right)$$
$$+\mathcal{O}\left(\left(\frac{H}{b_0}\right)^{4/3}\right) = 0$$

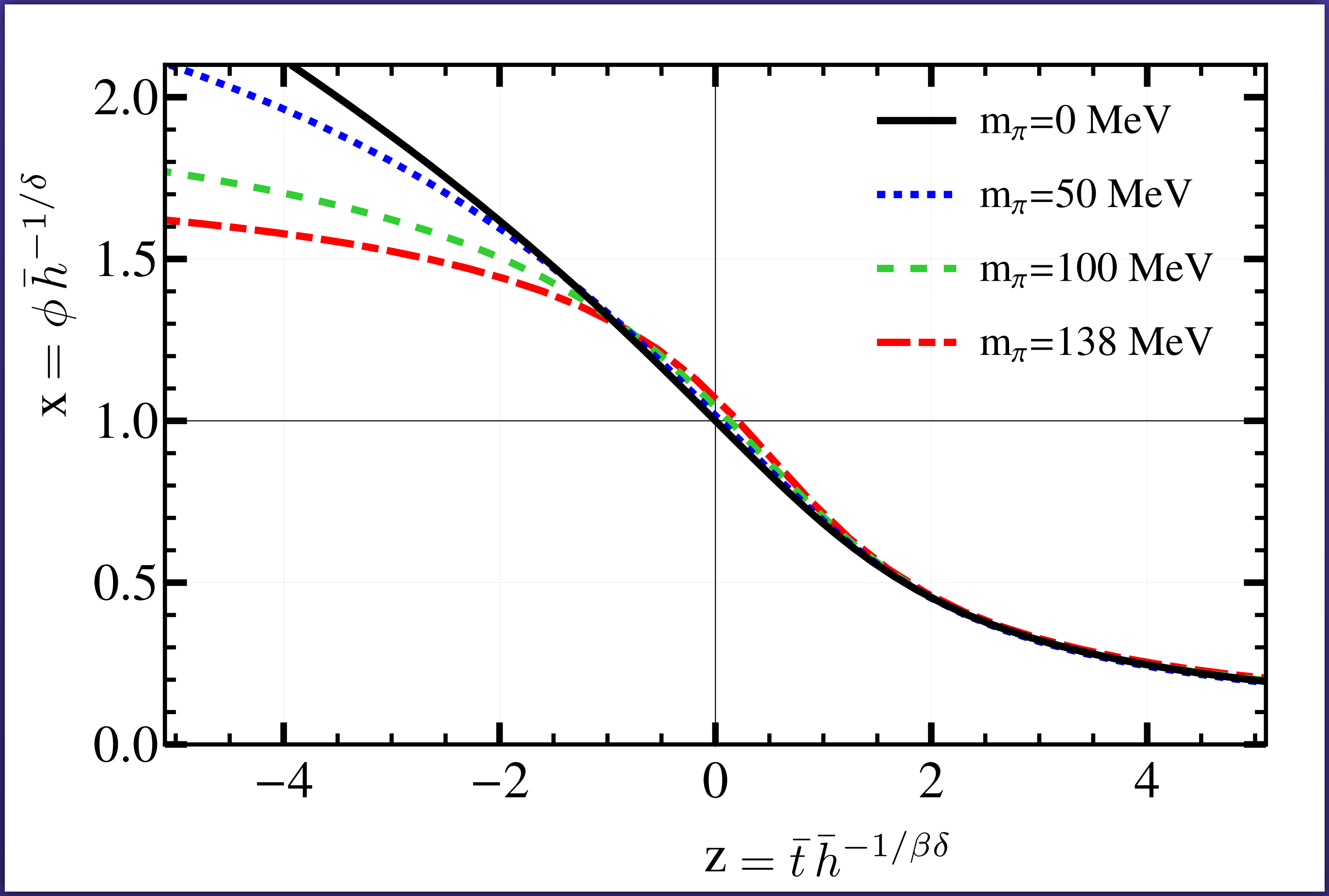
Scaling window

- Scaling violation in mEOS: $z = 0$ $x = 1 + \delta x$

$$\delta x = -\frac{1}{3}\bar{h}^{2/3}\,\sigma_0^2\,c_0/b_0$$

$$\boxed{\bar{h}_{sw} = \left(\frac{-3\,\delta x\,b_0}{\sigma_0^2\,c_0}\right)^{3/2}}$$

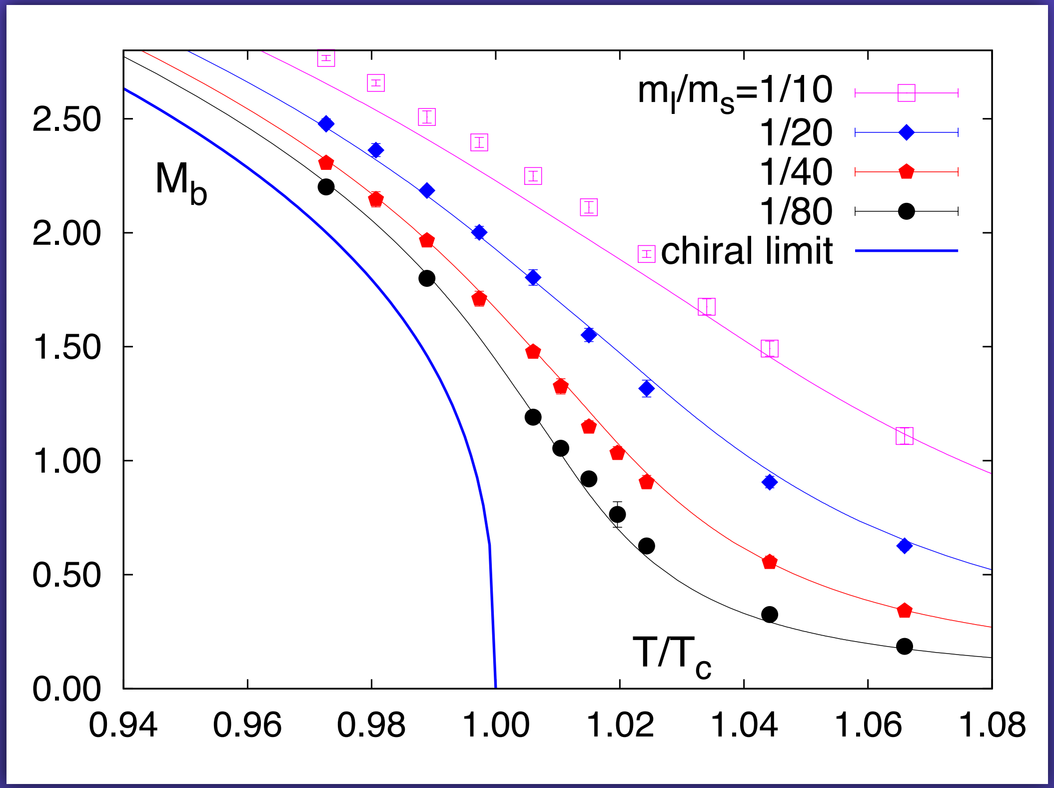
PQM-MF (Almasi *et al.* 2016)

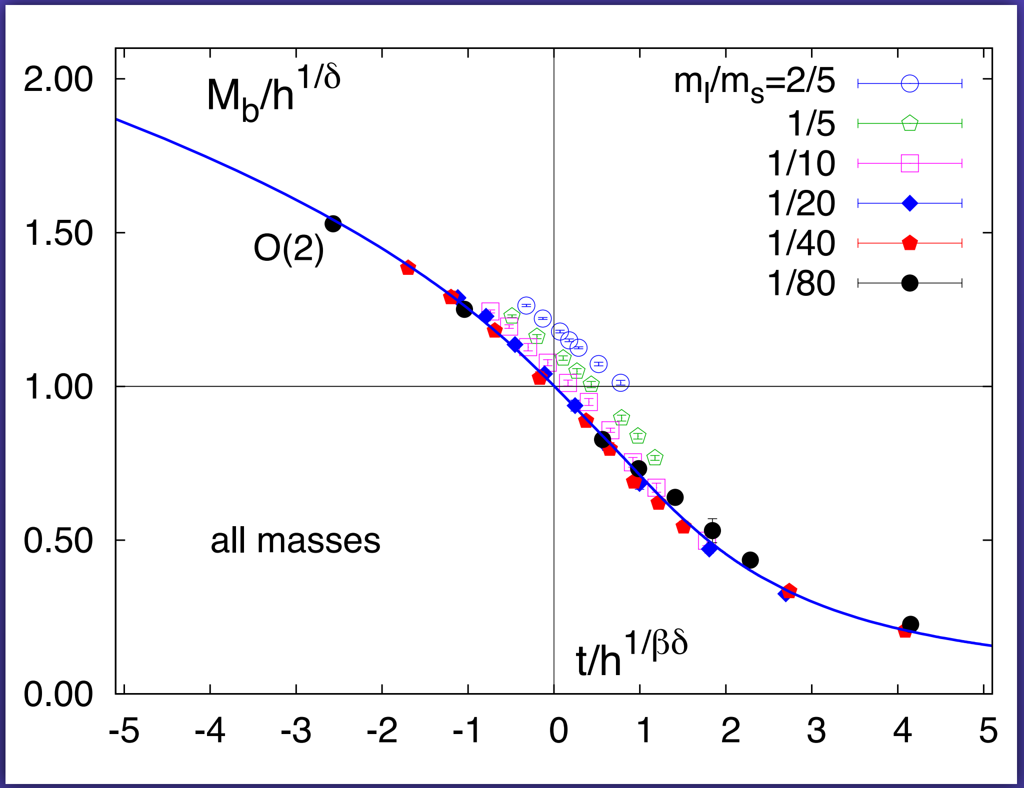


Scaling window

- Lattice QCD:

Ejiri *et al.* 2009



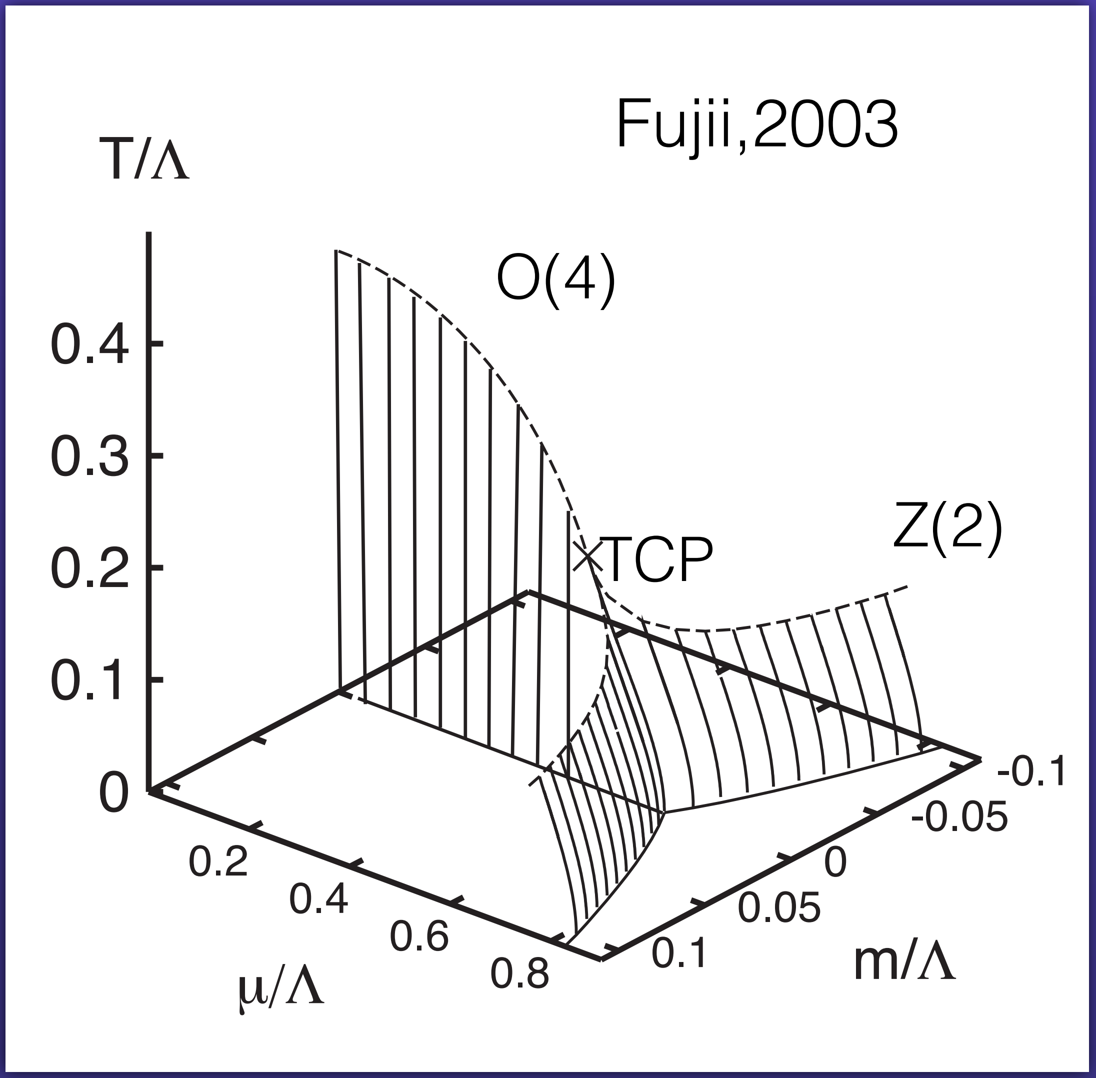


- Physical value $m_l/m_s = 1/27$
- Scaling window extends $\sim$ to physical $m_\pi \quad (\mu_B = 0)$
- Staggered fermions: one light pion → O(2)

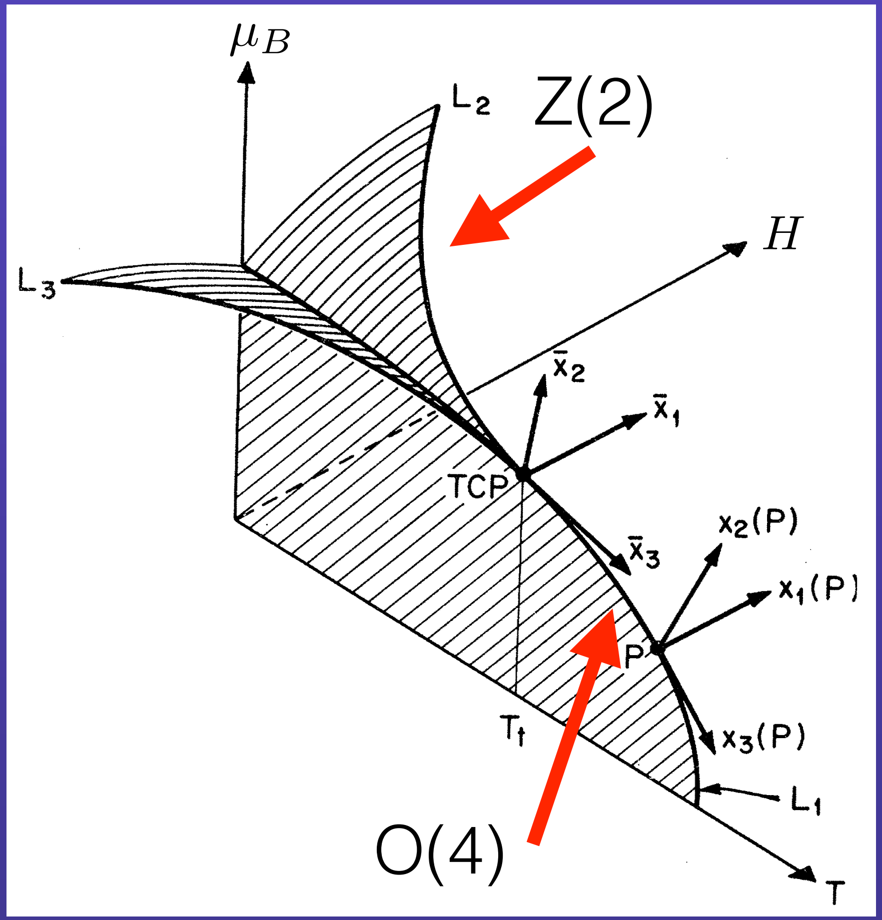
Scaling window at non-zero μ_B ?

Tricritical scaling

- Critical points mark the end of a first-order transition
- At a tricritical point three lines of critical points meet
- Advantageous to discuss scaling in three dimensions



Tricritical scaling



Hankey *et al, 1973*

- Strong direction: $x_1 \leftrightarrow m, H$
- Weak direction: $x_2 \leftrightarrow T\cos\theta + \mu\sin\theta$
- Independent direction: $x_3 \leftrightarrow -T\sin\theta + \mu\cos\theta$

At TCP all coord. systems coincide!

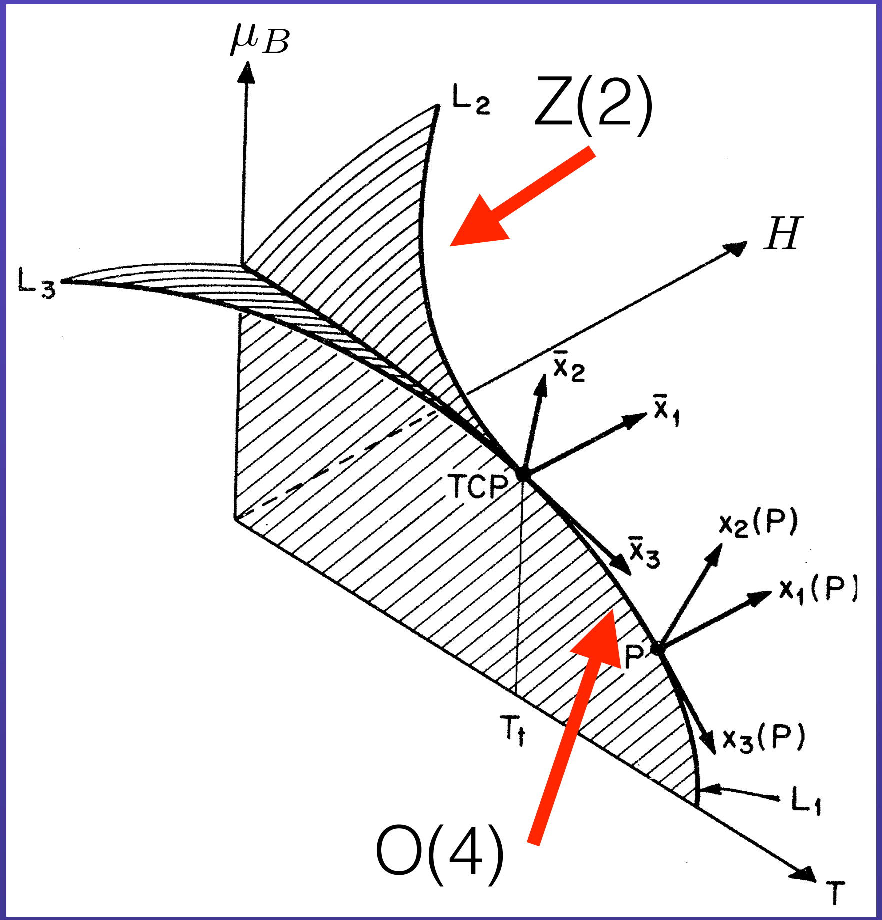
Tricritical scaling

Strongest divergence in χ

- Strong direction: $x_1 \leftrightarrow m, H$

- Weak direction: $x_2 \leftrightarrow T\cos\theta + \mu\sin\theta$

- Independent direction: $x_3 \leftrightarrow -T\sin\theta + \mu\cos\theta$

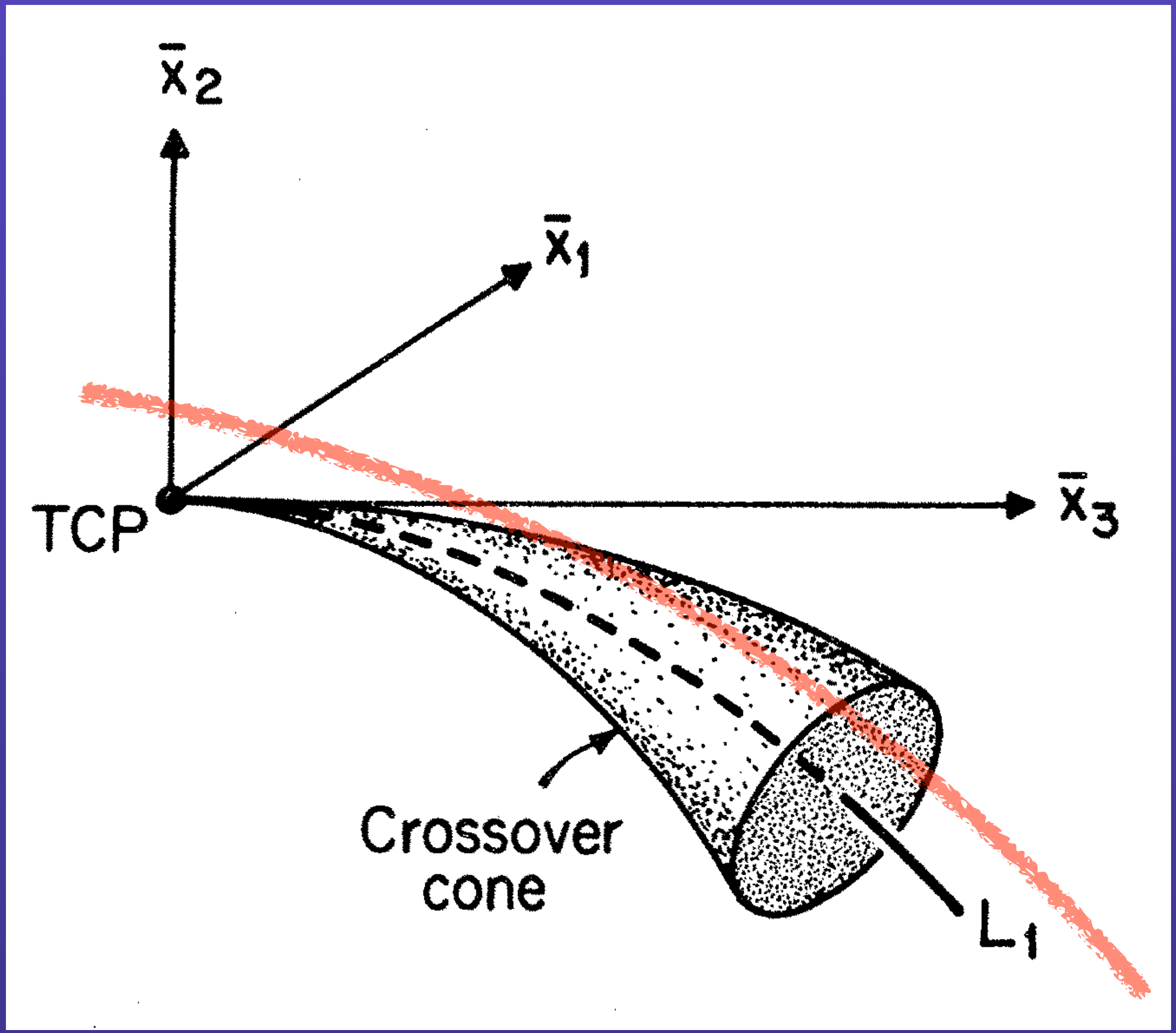


Hankey *et al, 1973*

At TCP all coord. systems coincide!

Scaling window near TCP

- Scaling arguments:
  - Scaling windows near TCP $\to 0$
- Expect O(4) scaling window to decrease with μ_B
- If physical m_q within scaling window @ $\mu_B = 0$ leave SW at some $\mu_B \neq 0$

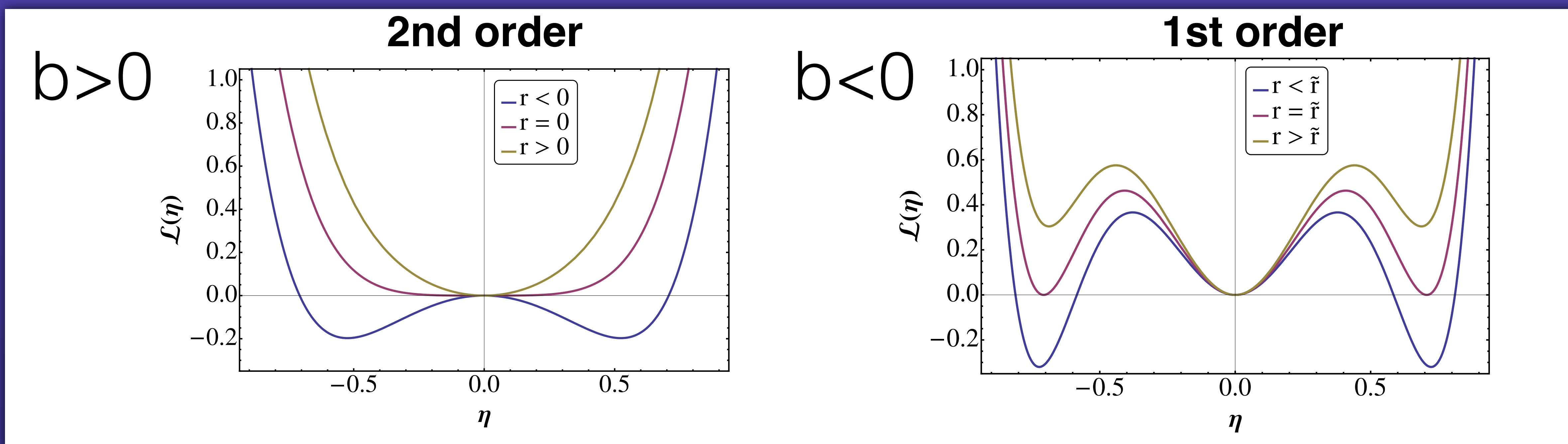


Chang *et al., 1973*

Landau Theory revisited

- Landau effective free energy

$$\mathcal{L} = \frac{1}{2}a(T,\mu)\phi^2 + \frac{1}{4}b(T,\mu)\phi^4 + \frac{1}{6}c(T,\mu)\phi^6 - H\phi$$



$$\bar{h}_{sw} = \left(\frac{-3\,\delta x\, b_0}{\sigma_0^2\, c_0}\right)^{3/2} \to 0 \quad \text{@ TCP}$$

If TCP expect scaling window to decrease with μ_B

Landau Theory revisited

- Landau effective free energy

$$\mathcal{L} = \frac{1}{2}a(T,\mu)\phi^2 + \frac{1}{4}b(T,\mu)\phi^4 + \frac{1}{6}c(T,\mu)\phi^6 - H\phi$$

- TCP:

$$a = b = 0 \rightarrow (T_{TCP}, \mu_{TCP})$$

- Scaling window:

$$\bar{h}_{sw} = \left(\frac{-3\,\delta x\, b_0}{\sigma_0^2\, c_0}\right)^{3/2} \rightarrow 0 \quad \text{@ TCP}$$

If TCP expect scaling window to decrease with μ_B

Critical scaling in QM-FRG

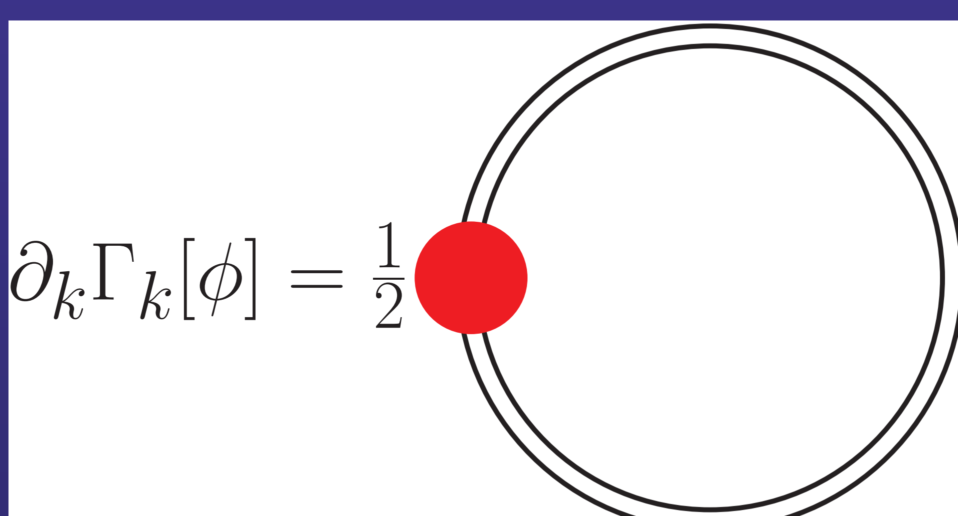
- Quark-meson model (O(4) universality class)

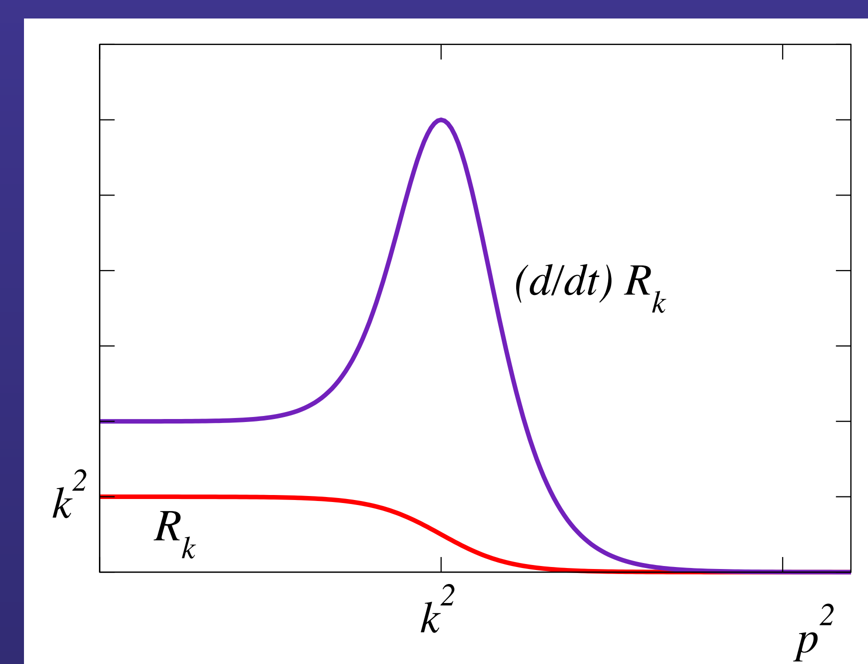
$$\mathcal{L} = \bar{q}\left[i\partial_\mu\gamma^\mu - g(\sigma + i\gamma_5\vec{\tau}\vec{\pi})\right]q + \frac{1}{2}\left[(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2\right] - U(\sigma,\vec{\pi})$$

$$U(\sigma,\vec{\pi}) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^{\,2} - v^2)^2 - H\sigma$$

- Critical fluctuations accounted for using FRG

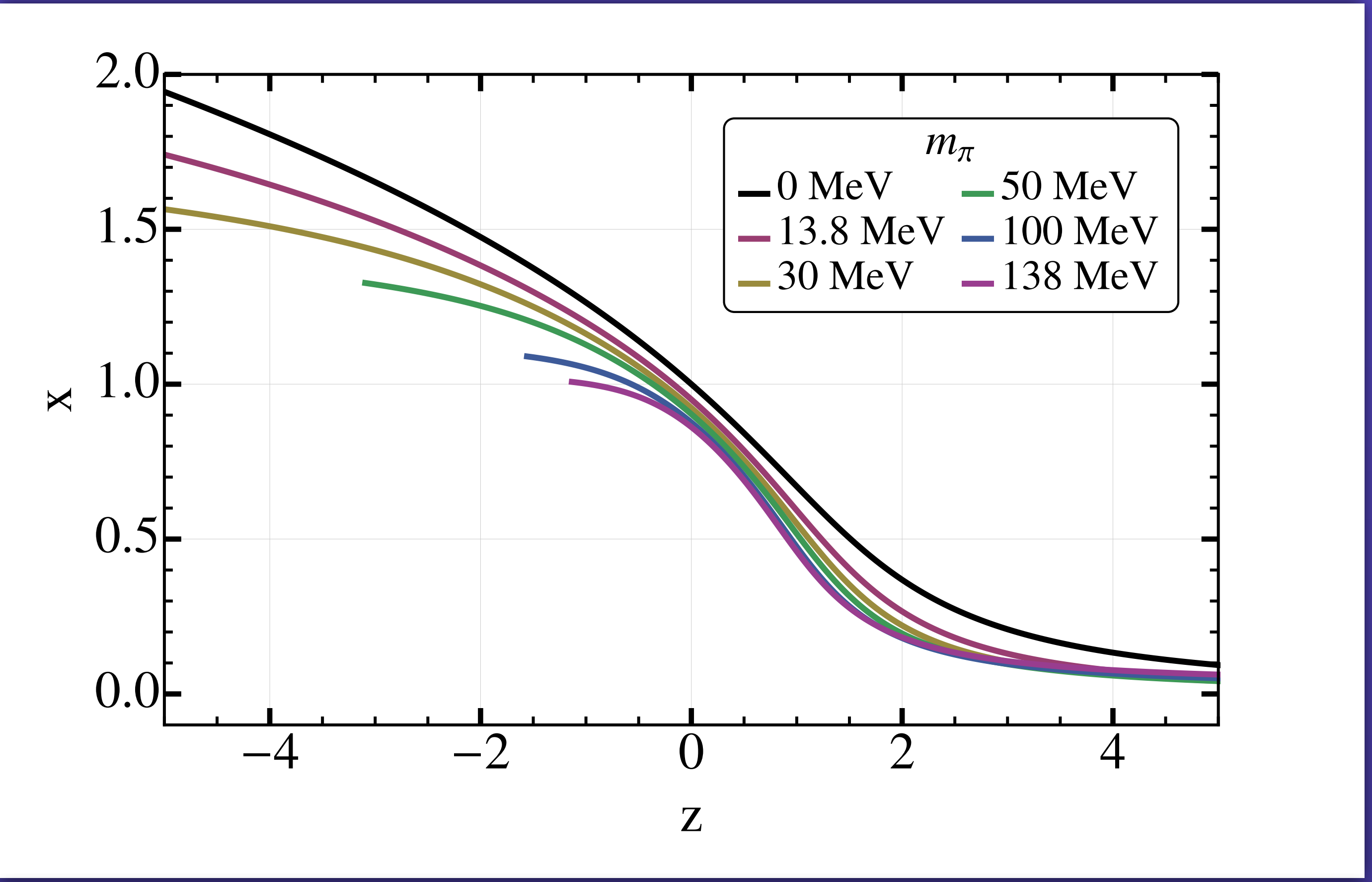
$$\partial_k\Gamma_k = \frac{1}{2}\mathrm{Tr}\left\{\partial_k R_k\left(\Gamma_k^{(2,0)} + R_k\right)^{-1}\right\}$$



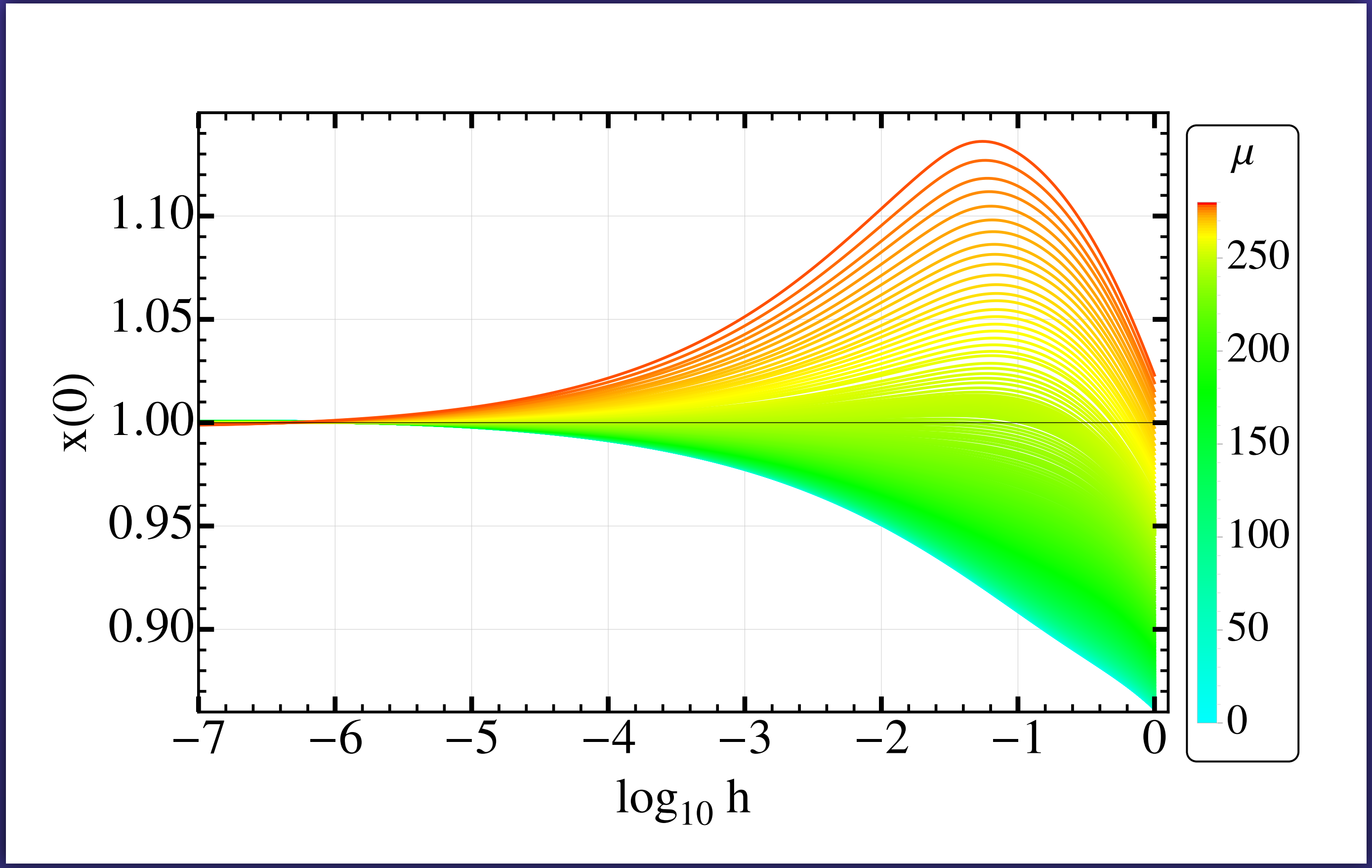


Scaling violation in mEOS

- Magnetic EOS
  @ $\mu_B = 0$

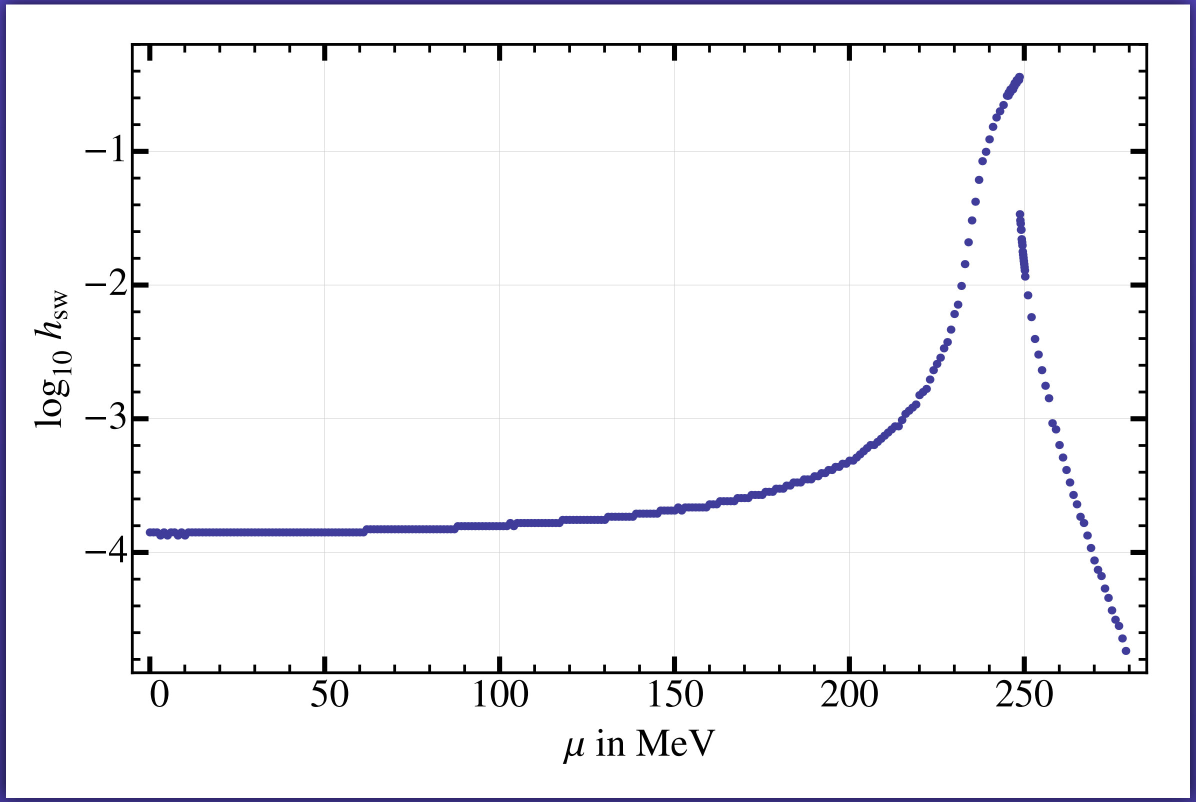


- Scaling violation
  @ $\bar{t} = 0$ along
  phase boundary

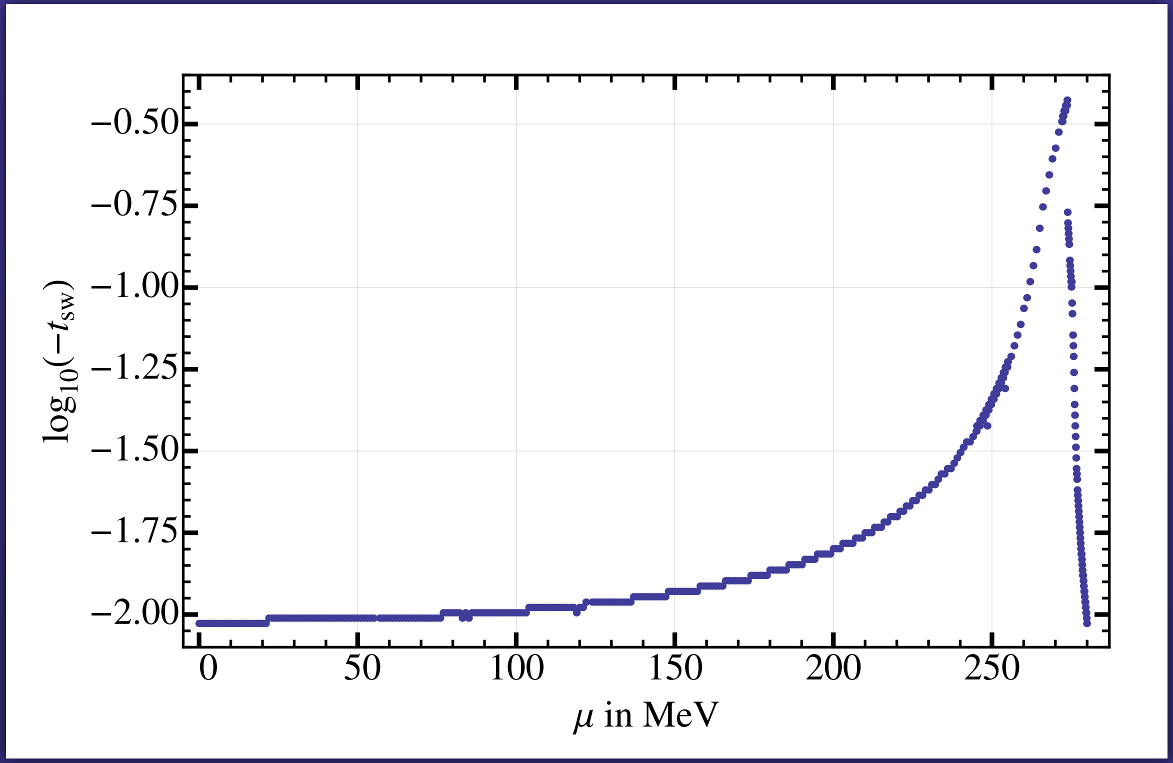


Scaling window

- 1% deviation from scaling



- Scaling region in t, for h=0



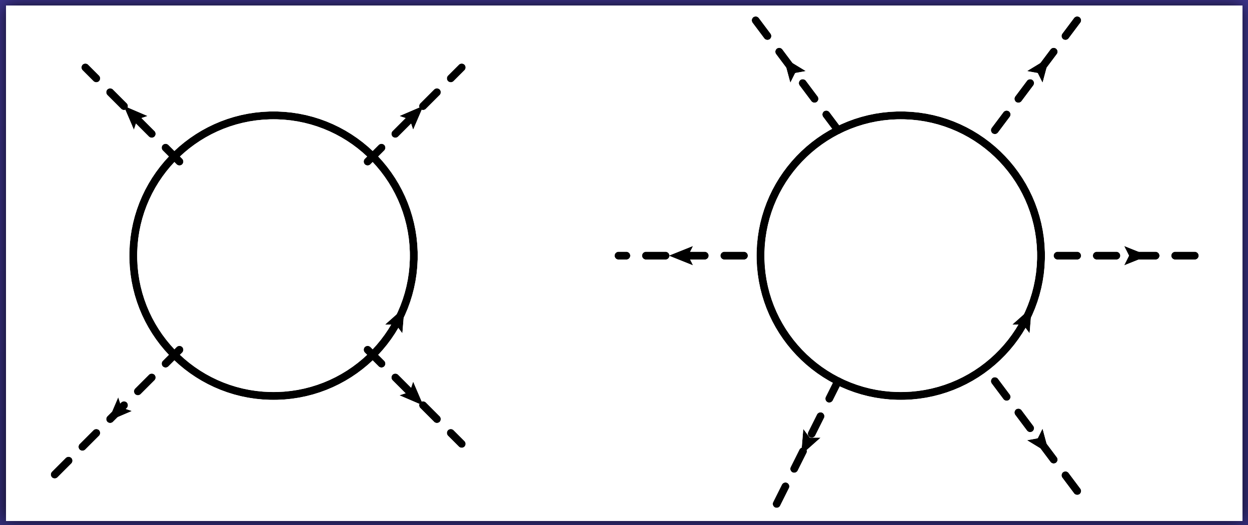
Interpretation?

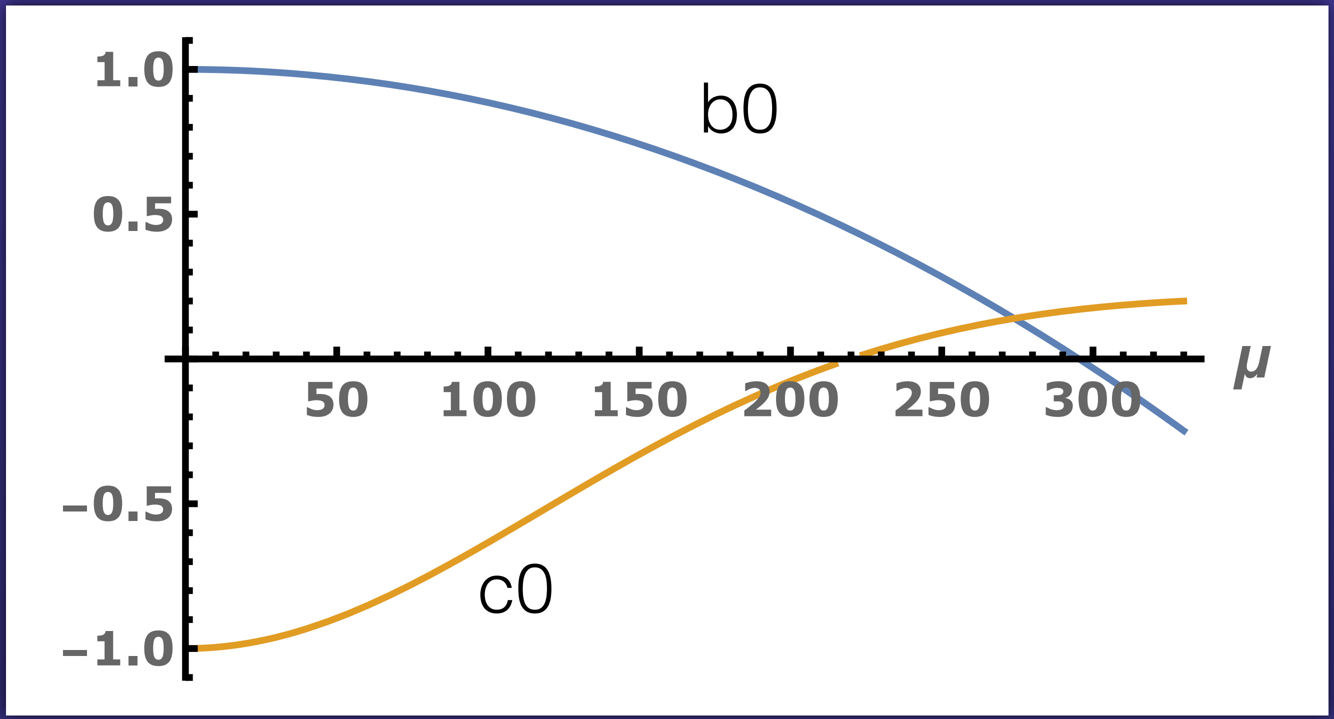
- QM-model in mean-field theory

$$\bar{h}_{sw} = \left(\frac{-3\,\delta x\, b_0}{\sigma_0^2\, c_0} \right)^{3/2} \quad \text{(leading order)}$$

- $b_0 \to 0$ @ TCP. What about c_0 ?

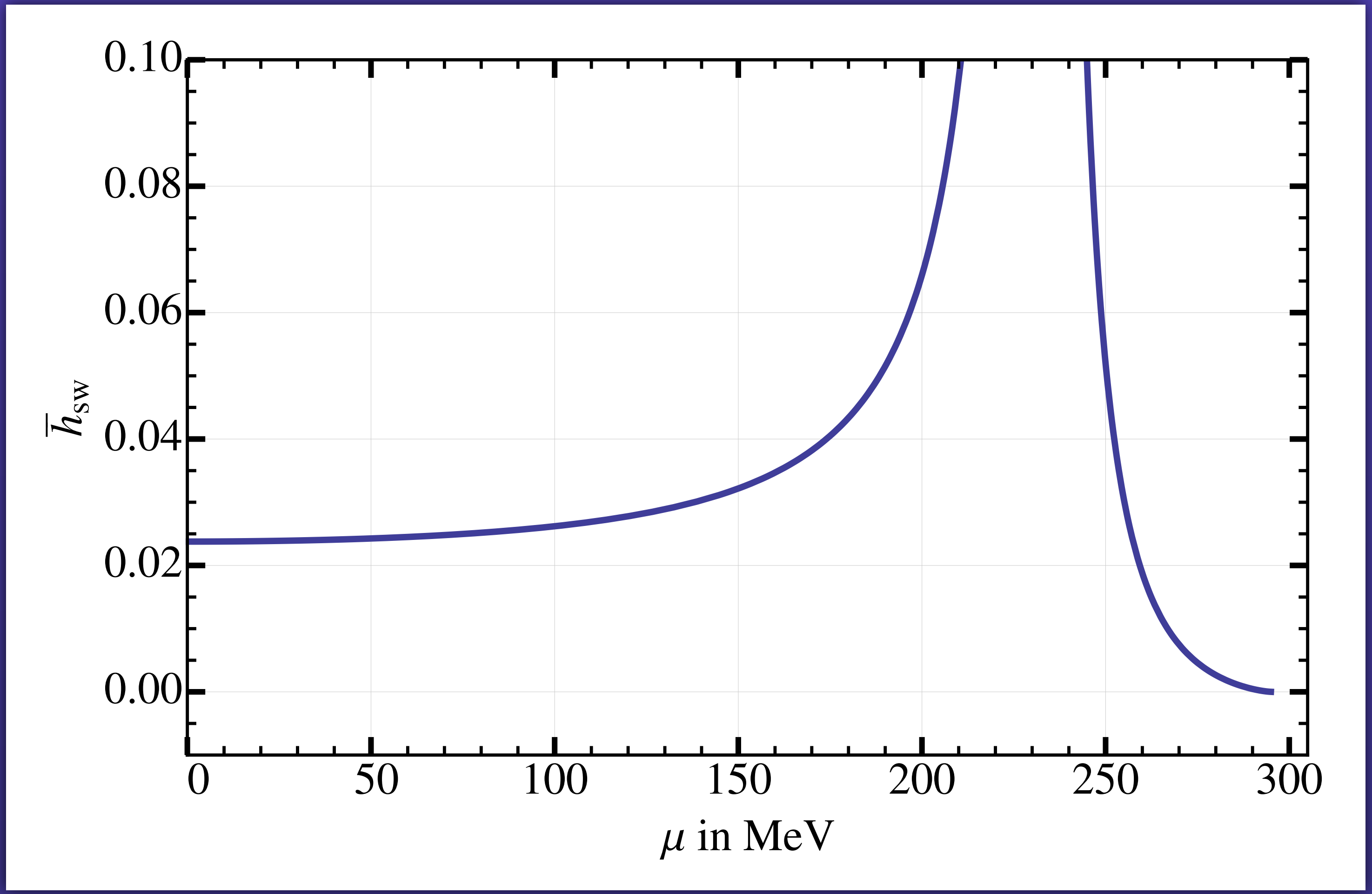
- High-temperature expansion of one-fermion-loop:





Scaling window MFT

- To leading order in scale-breaking field



- Maximum related to change in sign of scale breaking term

- μ_B dependence of one-fermion-loop

Summary and conclusions

- O(4) scaling window shows unexpected behavior in QM-FRG, maximal close to TCP
- Existence of TCP and scaling window non-universal; In MFT both traced back to one-fermion-loop; Enhanced scaling window survives critical fluct.
- Effect of confinement?
- Lattice QCD? (need to overcome sign problem)
- What happens along Z(2) line?
- Analogous effect in finite size scaling?