# Neutrino mass hierarchy (ordering) determination. Promises and challenges.

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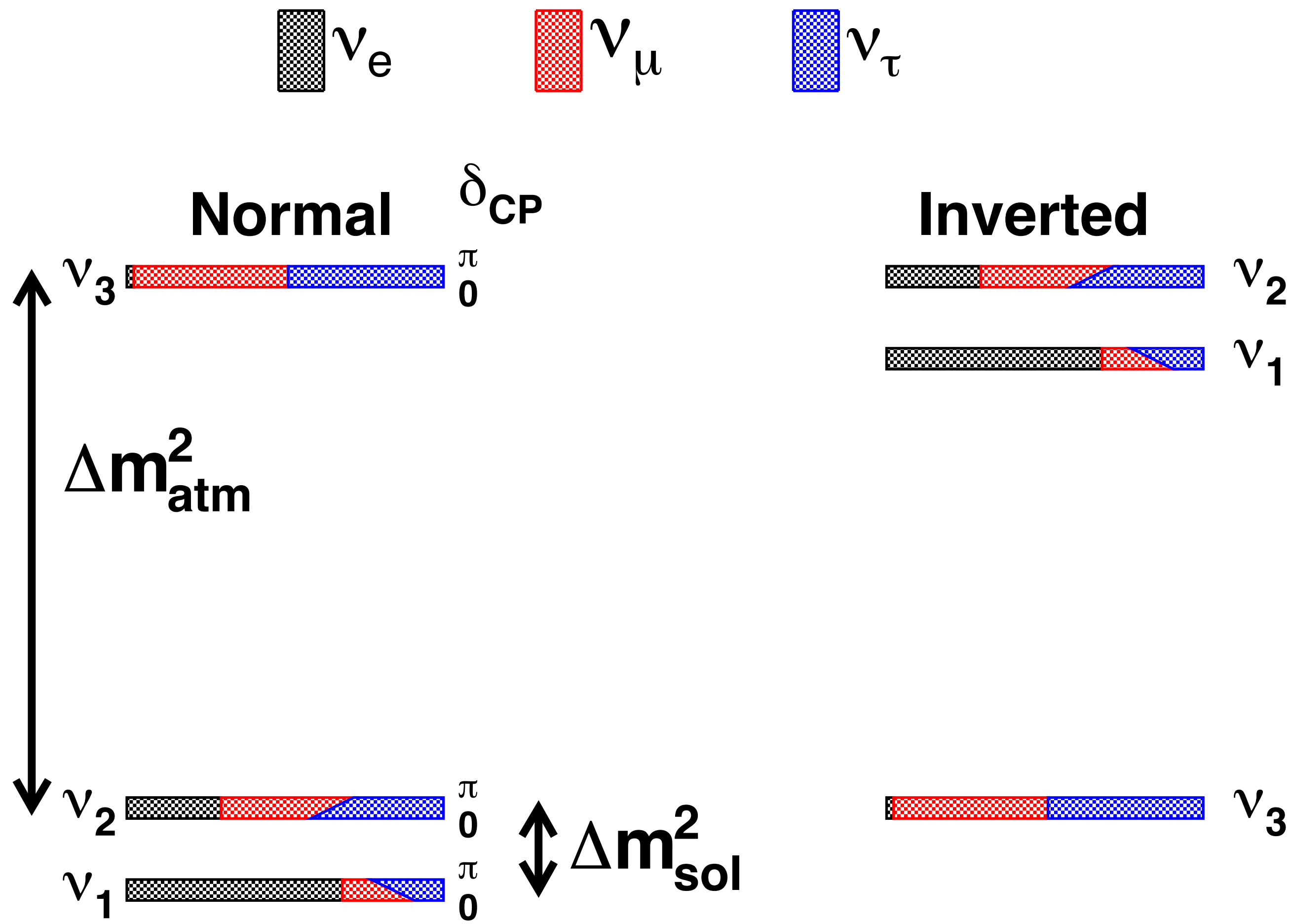
Erice, September 22, 2017

# Neutrino Mass Hierarchy




How can we find out which of these two actually exists in nature?

# Why is the hierarchy determination difficult?

In a reasonable approximation (0th order) the oscillation probabilities can be described in the two flavor picture with only one $\Delta m^2$ and only one mixing angle $\theta$

$$P(\nu_l \rightarrow \nu_{l'}) = \sin^2 2\theta \sin^2(1.27\, \Delta m^2\, L/E_\nu)$$

In this case, obviously, there is no effect when $\Delta m^2 \rightarrow -\Delta m^2$. Thus, to separate the hierarchies we must either consider three flavor oscillations and thus effects that are small due to the smallness of $\theta_{13}$ ($\sin^2\theta_{13} \sim 0.022$) and of $\Delta m^2_{21}/\Delta m^2_{31} \sim 1/30$, or go beyond the vacuum oscillation, i.e. use the matter effects that are sensitive to the sign of $\Delta m^2$.

Before considering them, there is yet another possibility. From oscillation results we know that the sum of the three neutrino masses, **$\Sigma = m_1 + m_2 + m_3$**, must be larger than **~0.06 eV** for NH and **~0.10 eV** for IH.

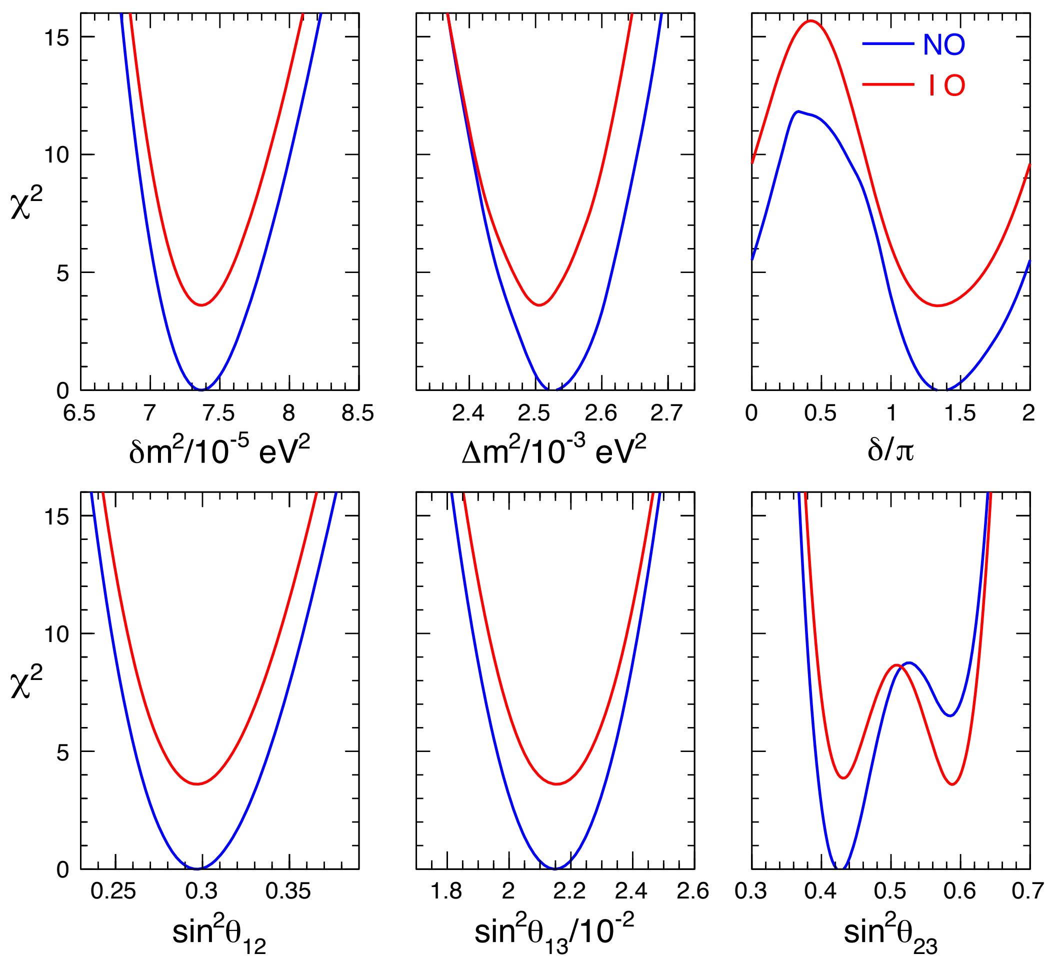
$\Sigma$ can be constrained and, perhaps, eventually determined, by cosmology in combination with various astrophysics data. Recent $\Sigma$ limits, at 95% CL, reach small values of 0.13 eV (Cuesta et al. 1511.05983) and 0.12 eV (Palanque-Delabrouille et al. 1506.05976). Based on that, Simpson et al. (1703.03425), use Bayesian analysis and claim a strong preference for NH (odds 42:1). This claim is based on using the logarithmic prior based on the so-called ``Bedford law" and is disputed (see Schwetz et al. 1703. 04585).

Nevertheless, if $\Sigma$ could be reliably restricted to values $\Sigma < 0.1$ eV, but still $\Sigma > 0.06$ eV, the NH would be obviously the only possibility.

One can also try to analyze all oscillation etc. data together to obtain

$$\Delta\chi^2_{IH-NH} = \chi^2_{min\ IH} - \chi^2_{min\ NH}$$

Capozzi et al. (1703.04471) obtain $\Delta\chi^2_{IH-NH} = 3.6$, i.e. about $2\sigma$ preference for NH.




They use the term ``ordering" instead of hierarchy. Thus NO means NH and IO means IH

Similar preference for NH $\Delta\chi^2_{IH-NH} = 2.7$ is found in the analogous global oscillation analysis by Salas et al. 1708.01186. See also the talk by M.C. Gonzales-Garcia on Sunday

## How can we approach the hierarchy determination using vacuum oscillations?

The basic idea is to use the small difference, $\Delta m^2_{21}$, between $\Delta m^2_{31}$ and $\Delta m^2_{32}$ in the three-flavor framework.

Of practical importance is the case of pure $\overline{\nu}_e$ beam from reactors.

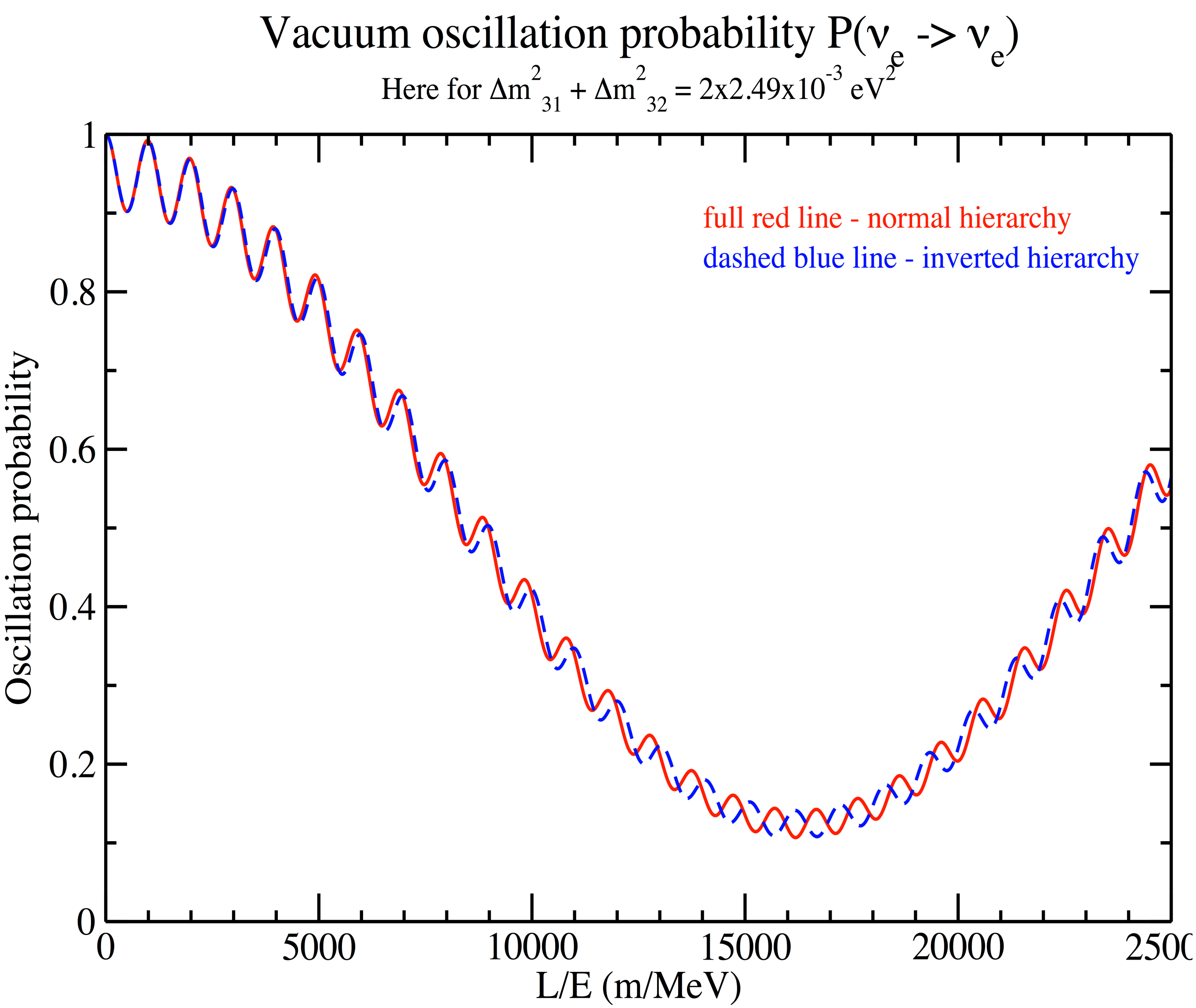
Survival probability for 3-neutrino mixing for the $\nu_e$ and its antineutrinos in vacuum is

$$P_{ee} = 1 - \{ \cos^4(\theta_{13}) \sin^2(2\theta_{12}) \sin^2(\Delta_{21}) + \cos^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta_{31}) + \sin^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta_{32}) \}$$

Where $\Delta_{ij}$ = **1.27 $|\delta m^2_{ji}$ (eV$^2$)$|$ L(m)/$E_\nu$(MeV).**
Since $\Delta_{31} \sim \Delta_{32} = \Delta_{atm}$ the $P_{ee}$ exhibits low frequency oscillations governed by $\Delta_{21}$ (dominant) and high frequency oscillations governed by $\Delta_{atm}$ (subdominant) with amplitude proportional to $\sin^2(2\theta_{13})$ . With the relatively large $\sin^2(\theta_{13})$ = 0.0215 +- 0.007 these subdominant oscillations are more easily visible.

Moreover, since for normal mass hierarchy (NH) $\Delta_{31} = \Delta_{32} + \Delta_{21}$ while for inverted mass hierarchy (IH) $\Delta_{31} = \Delta_{32} - \Delta_{21}$ , there is a phase shift between the two hierarchies proportional to L/$E_\nu$ .

(For proposals to use reactor neutrinos at intermediate distances see e.g. Choubey, Petcov, Piai (2003) or Schoenert, Lasserre, Oberauer (2003).)




There are two oscillation lengths in the problem.
The `solar'
$(L/E)_{osc}^{sol}$ ~ 32000 m/MeV
And the `atmospheric'
$(L/E)_{osc}^{atm}$ ~ 1000 m/MeV.

Depending on the hierarchy the atmospheric oscillation lengths are slightly different.

After several oscillations the phase difference between these two possibilities increases; this allows determination of the correct hierarchy.

For realistic input, and the typical reactor neutrino energy ~4 MeV the optimum distance is L ~ 40-60 km.

Survival probability formula in the previous slide can be rewritten as

$$P_{ee} = 1 - 2s^2_{13}c^2_{13} - 4c^4 13 s^2_{12}c^2_{12}\sin^2\Delta_{21} + 2s^2_{13}c^2_{13}\sqrt{1 - 4s^2_{12}c^2_{12}\sin^2\Delta_{21}}\cos(2\Delta_{32} \pm \phi_{ee}),$$

Where the relevant angle $\Phi_{ee}$ is defined as

$$\sin\phi_{ee} = \frac{c^2_{12}\sin 2\Delta_{21}}{\sqrt{1 - 4s^2_{12}c^2_{12}\sin^2\Delta_{21}}}, \qquad \cos\phi_{ee} = \frac{c^2_{12}\cos 2\Delta_{21} + s^2_{12}}{\sqrt{1 - 4s^2_{12}c^2_{12}\sin^2\Delta_{21}}}.$$

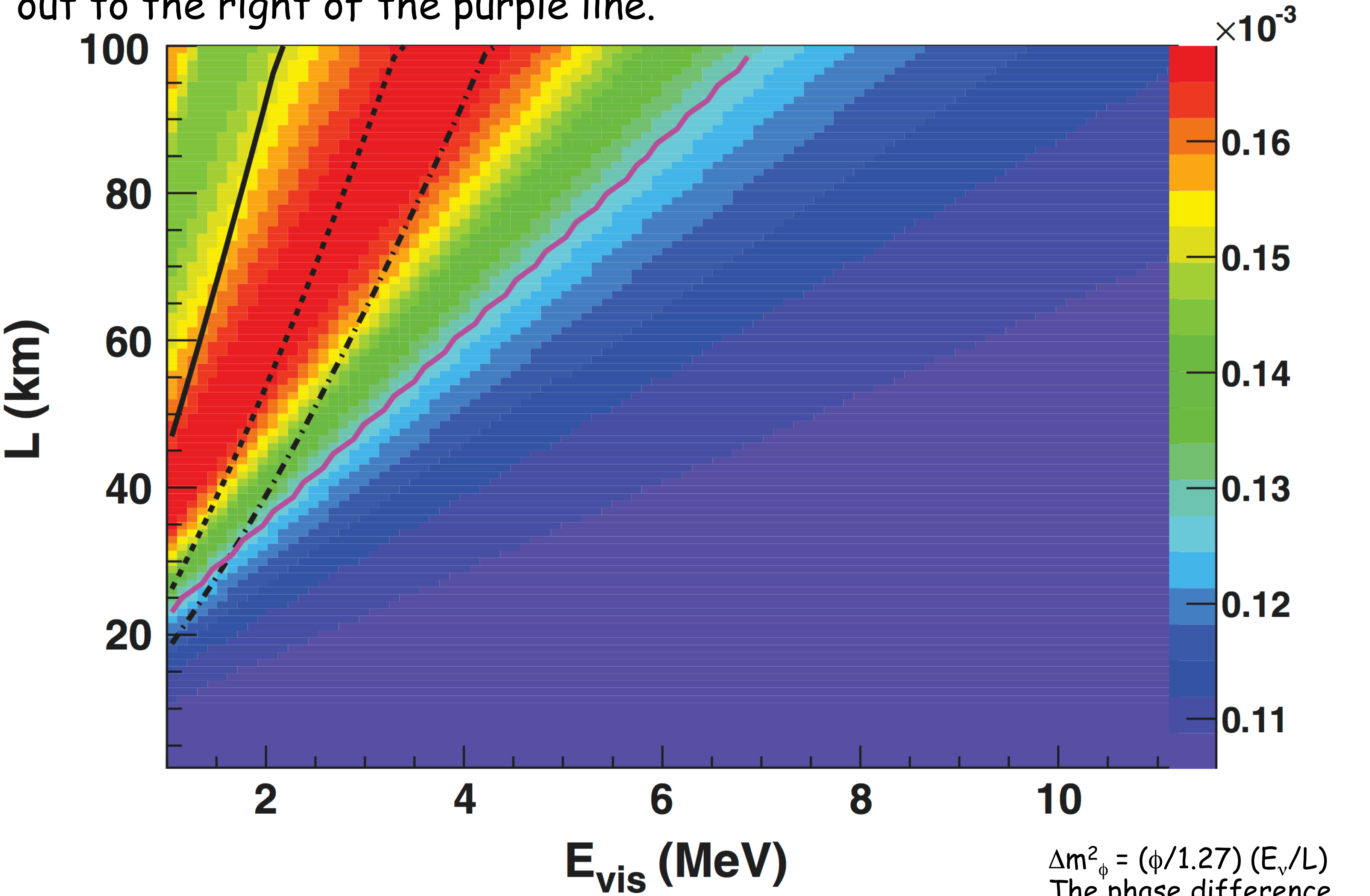
Lets define the effective mass square difference. In order to determine the MH we need to be able to separate $\Delta m^2_{32}$ from $\Delta m^2_{\Phi}$

$$\Delta m^2_{\phi} := 4 \cdot \phi_{ee} \cdot E_\nu / L.$$

Since, at the present time, the uncertainty in $\Delta m^2_{atm}$ is comparable to $\Delta m^2_{21}$ it means that for a fixed $E_\nu$ one cannot separate the NH and IH. However, the degeneracy could be, in principle, overcome, by considering a range of $L/E_\nu$ or, realistically, a range of $E_\nu$. When $\Delta m^2_{\phi}$ remains small and essentially unchanged with $E_{vis}$, it is impossible to determine the MH.

In any case, the energy resolution of the detector must be very good, since the oscillations corresponding to $\Delta m^2_{\Phi}$ vary fast. And the distance L must be properly chosen.

Plot of $\Delta m^2_\phi$ for the range of L and $E_{vis}$. The MH is smeared out to the right of the purple line.



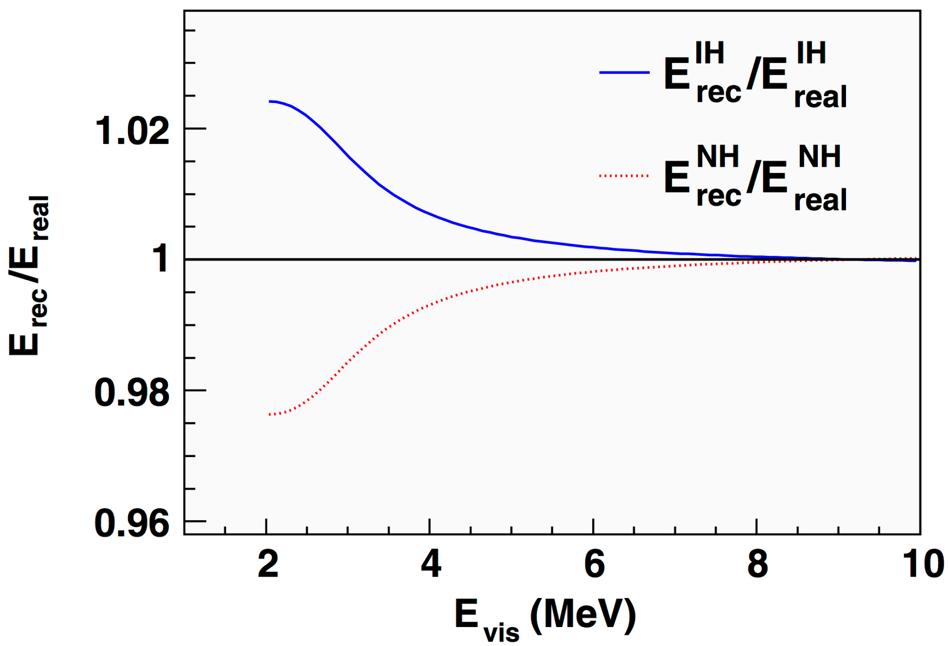

$\Delta m^2_\phi = (\phi/1.27)\,(E_\nu/L)$

The phase difference of NH and IH is $2\phi$

(figure from Qian, Dwyer, McKeown, Vogel, Wang and Zhang, (2013).)

# Additional challenge: Energy scale nonlinearity.

A small nonlinearity of the energy scale can lead to a substantial reduction of the hierarchy discovery potential (in particular in association with the $\Delta m^2_{32}$ uncertainty).
As an illustration, lets assume that the ratio $E_{reconstructed}/E_{real}$ is like in the




figure, for the case when the true hierarchy is IH (blue) or NH (red). In that case the spectrum analysis would lead to wrong MH.

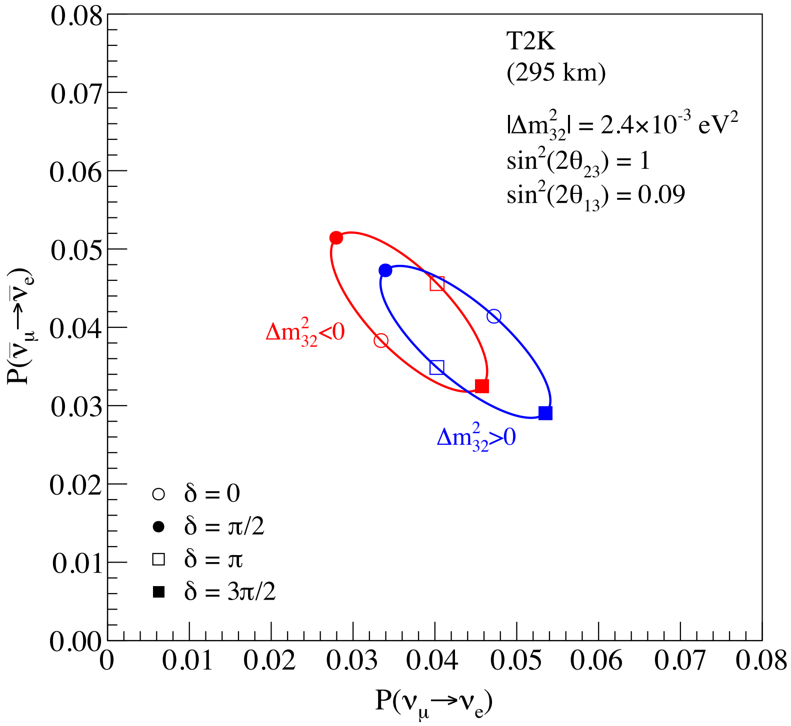
Thus, the nonlinearity of $E_{rec}/E_{real}$ need to be controlled to a fraction of 1% over a wide range of $E_{vis.}$.
Current state-of-the-art is ~1.9%.
Substantial improvement is required.

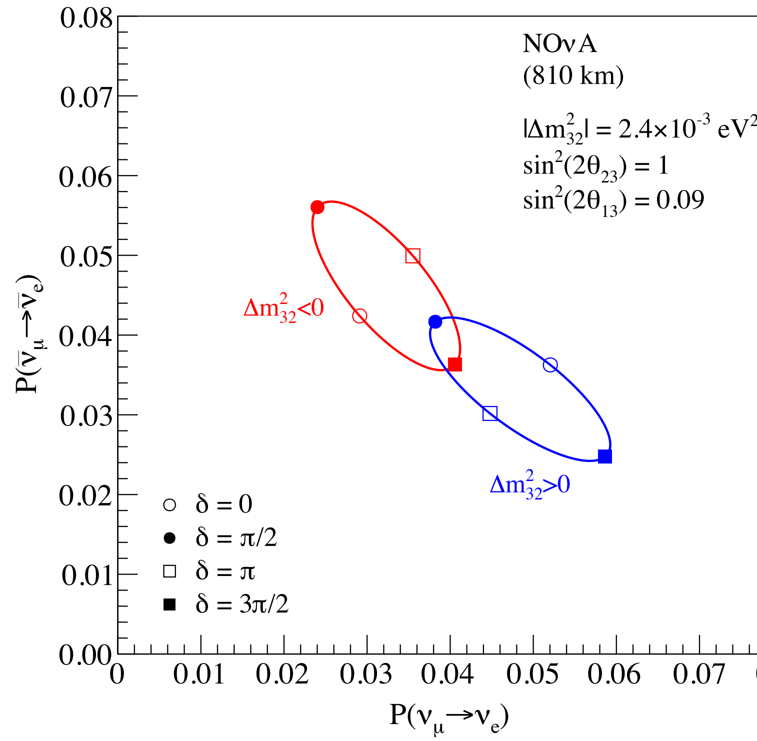
Nevertheless, the method is clean in the sense that the outcome is independent of other things, like matter effects, CP phase, octant, etc.

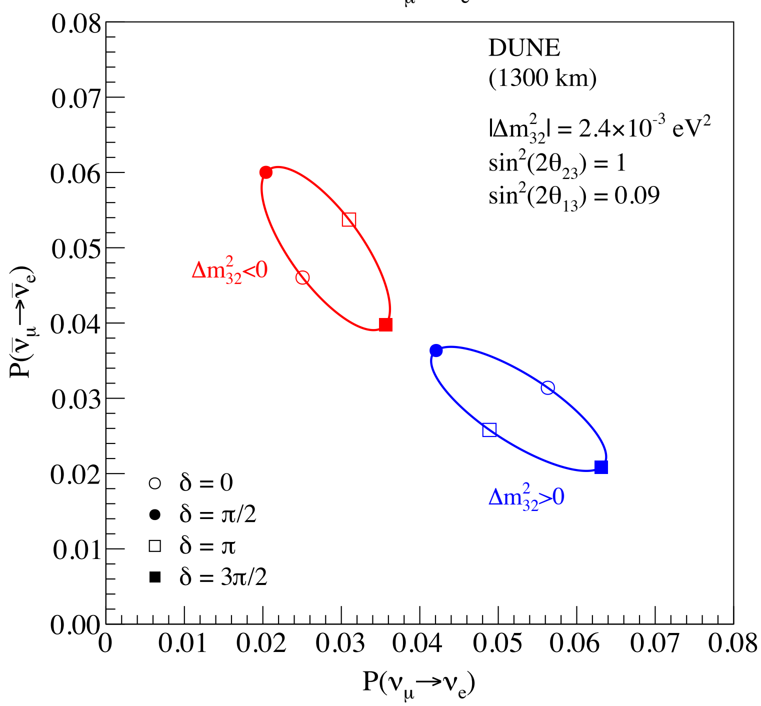
# Using the matter effects:

The effective masses of $\nu_e$ dominated $\nu_1$ and $\nu_2$ are increased by propagation in matter while they are decreased for antineutrinos. This modification of the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability, and its analog for the antineutrinos is the basis for the accelerator based hierarchy determination.

The effects of the so far unknown CP phase $\delta$ complicate the hierarchy determination. That complication decreases with the increasing distance L. However, it is important to use the distance and energy near the oscillation maximum, i.e. to have $\Delta m^2_{32} L/4E \sim \pi/2$.



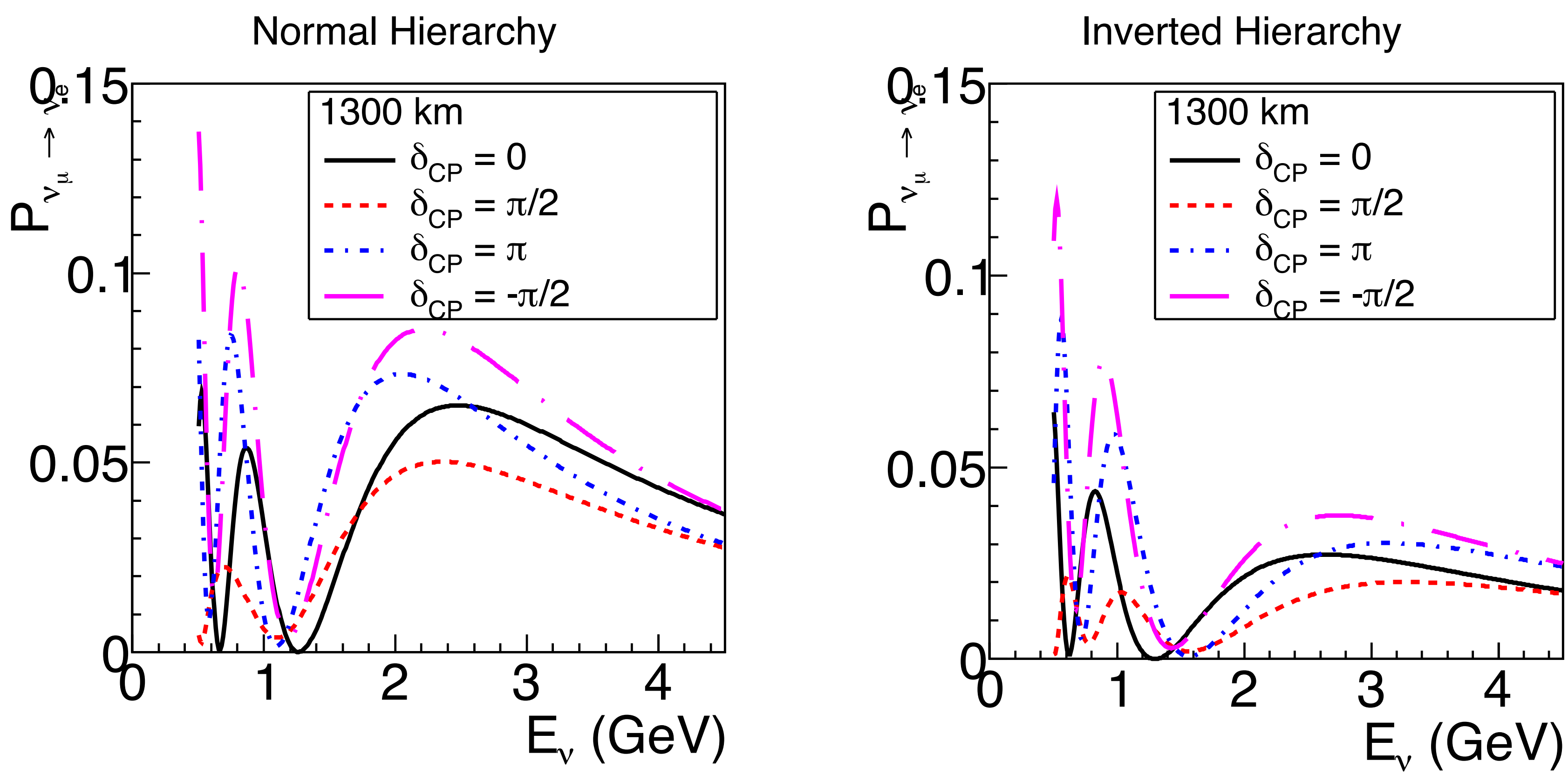




Oscillation probabilities for $\nu_\mu$ -> $\nu_e$ neutrinos and antineutrinos for different distances.

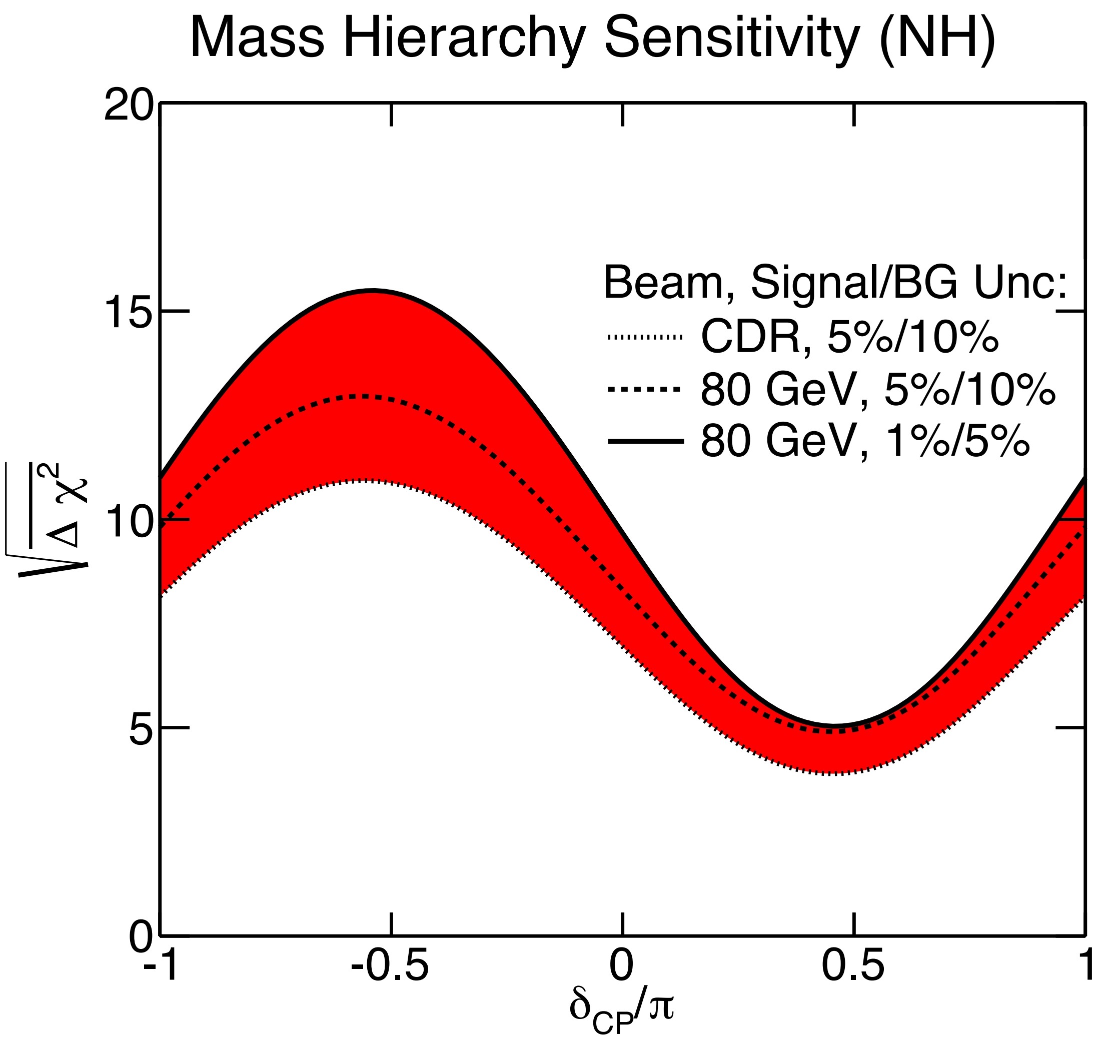
This is an illustration for a fixed L/E = 0.4 km/MeV. Other parameters are fixed. Statistical fluctuation are omitted.

Figure from R. B. Patterson, 1506.07917



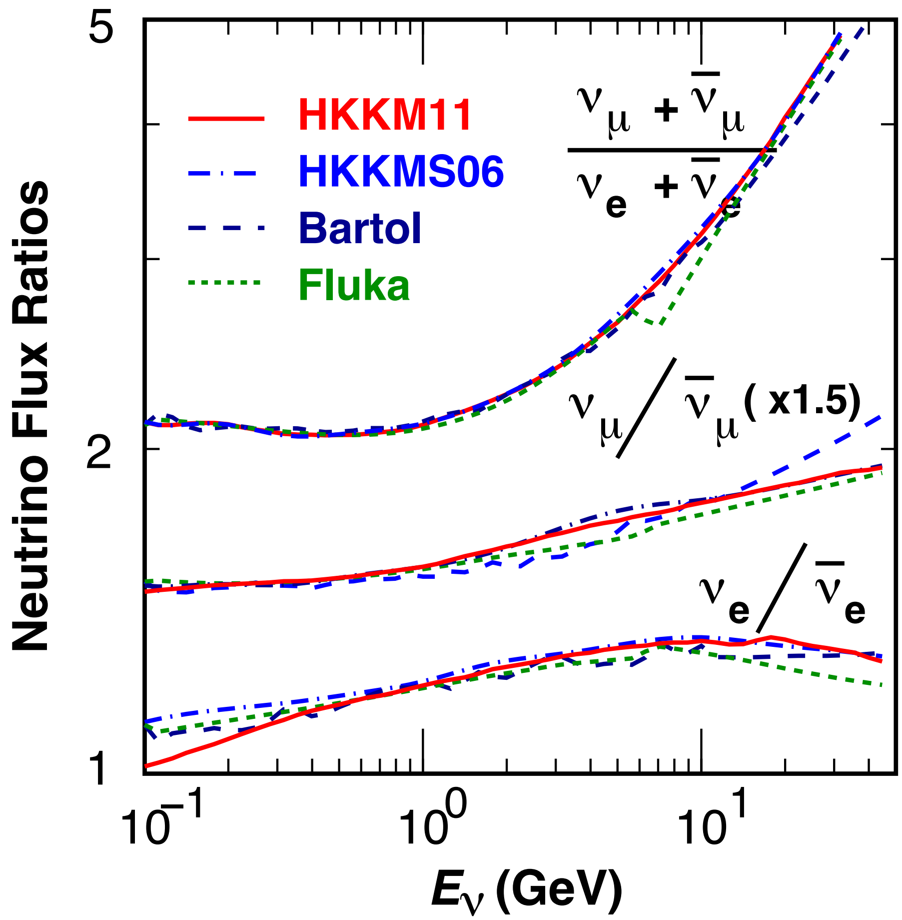

Oscillation probalities in DUNE as functions of the neutrino energy. At the oscillation maximum, $E_\nu \sim 2.5$ GeV the $\nu_e$ appearance is enhanced for NH (and suppressed for antineutrinos), while for IH the opposite is true. Additional information will be gained from the determination of the energy spectrum.

Hierarchy sensitivity of DUNE (34 kton, 10 yr, 1.2 MW Fermilab beam)
The difference of $\chi^2$ for the two hierarchies plotted against the CP phase $\delta$.



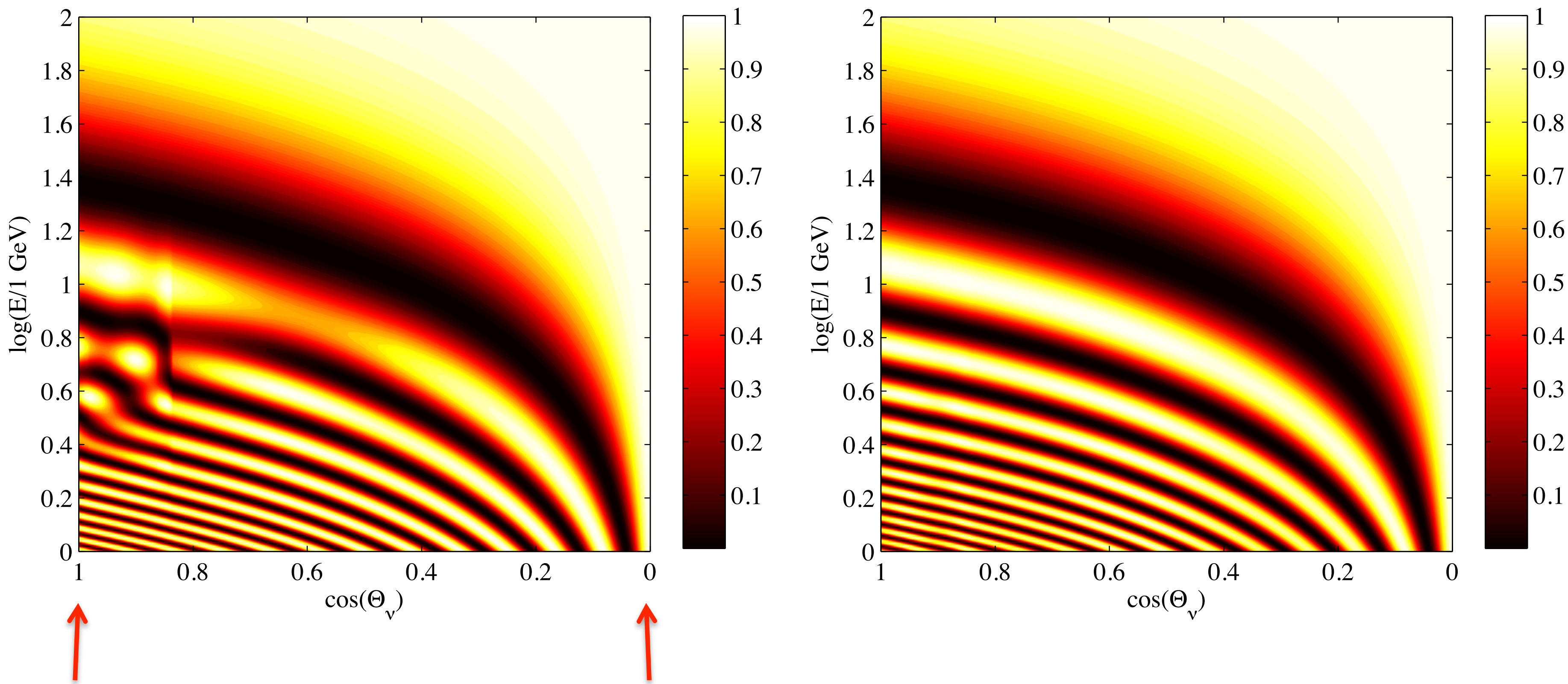

Dotted line: 5% uncertainty for signal, 10% for background
Full line: Aggressive design, 1% uncertainty for signal, 5% for background

Atmospheric neutrinos are produced by cosmic ray showers in the atmosphere. They contain a mix of $\nu_\mu$ and $\nu_e$ and their antineutrinos. Detectors can observe them from distances ranging from 13 000 Km to few km. Typically, only neutrino flavors can by separated from each other.




Atmospheric neutrino flux ratios. The ratio $\nu_\mu / \nu_e$ is ~ 2 increasing above ~ 2 GeV.
There is slight excess of neutrinos over antineutrinos.

Oscillograms, lines of equal flavor conversion probability. Left for $\nu_\mu \to \nu_\mu$ right for antineutrinos. Matter effects are for $\nu_\mu \to \nu_\mu$ and are absent for antineutrinos. This is for the NH, it is reversed for the IH.




Vertical direction
Throught the Earth

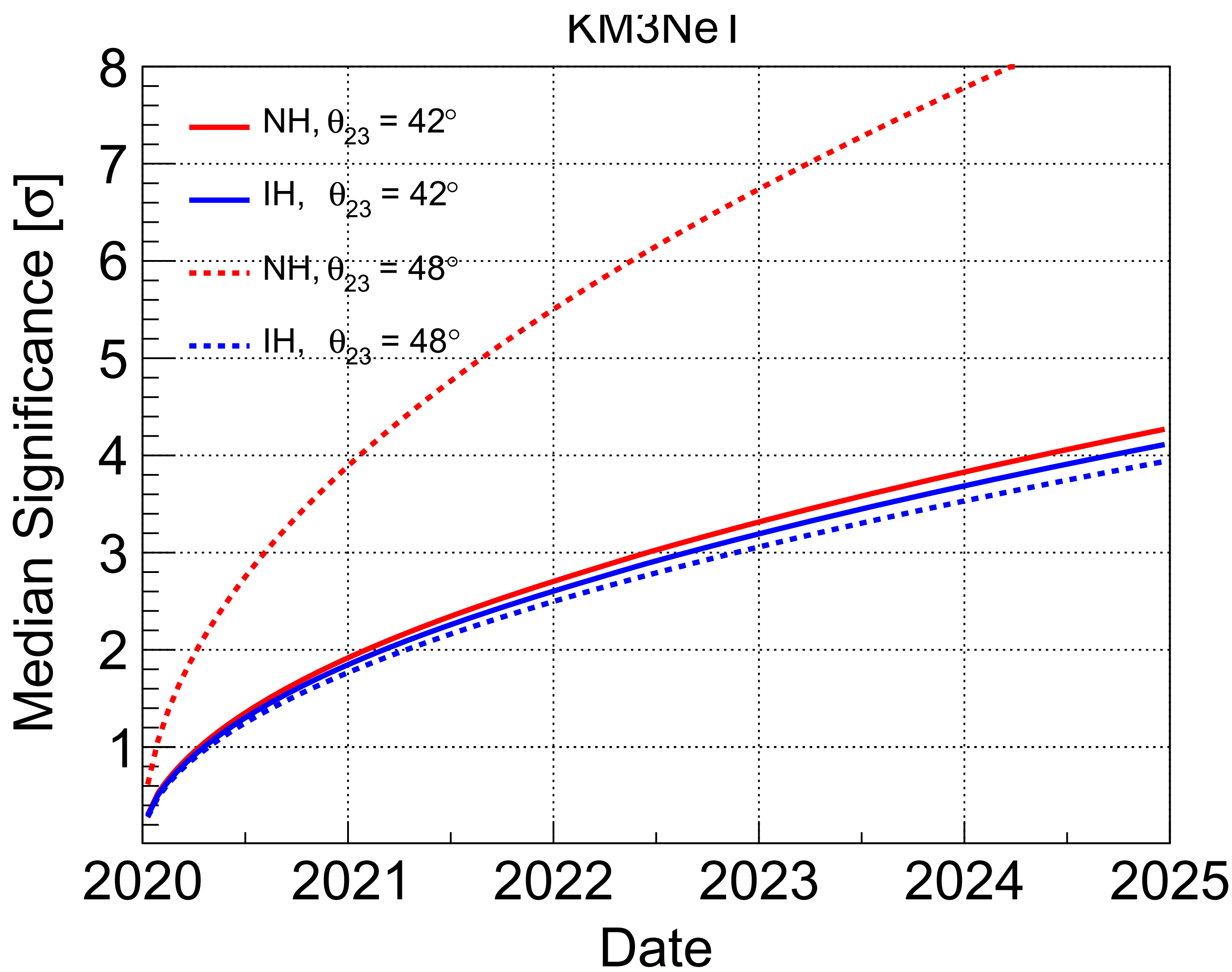
Horizontal
direction

From Blennow and Smirnov, 1306.2903

# PINGU and ORCA

The Precision IceCube Next Generation Upgrade (PINGU) is a proposed new multi megaton array for IceCube upgrade at the South Pole Station. Its primary purpose is the determination of neutrino mass hierarchy using the detection of atmospheric neutrinos. The design has a dense array of optical modules with the threshold for the energy reconstruction of 5 - 15 GeV, where the effect of MH is most evident.

ORCA (Oscillation Research with Cosmics in the Abyss) is aiming to determine the MH using the deep-sea neutrino telescope technology developed for the KM3NeT project (A multi-km$^3$ sized Neutrino Telescope) . The experimental principle of ORCA is similar to that of PINGU. Instead of deep ice in the south pole, ORCA will deploy large 3-dimensional arrays of photo-sensors to detect Cherenkov lights in the deep Mediterranean Sea.




Median projected sensitivity of the MH determination depends sensitively on the octant (whether $\theta_{23} < 45^0$ or $\theta_{23} > 45^0$ ). The figure (see J.Phys.*G* **43**, 084001 (2016). assumes that $\delta_{CP} = 0$. (T2K gives slight preference to the second octant, see the Talk by T. Nakaya on Tuesday.)

# Summary:

1) Currently, there is a slight, but consistent, ~2$\sigma$ preference for NH from global analyses.
2) Cosmology + astrophysics constrain $\Sigma$, the sum of neutrino masses. If that constraint, or actual determination, will reliably reach the conclusion that 0.06 eV < $\Sigma$ < 0.1 eV, then NH will be established.
3) Dedicated reactor experiments at L ~ 50 km are being developed that should be able to separate the two hierarchies.
4) Long baseline accelerator experiments will be able to do it as well, together with the determination of CP phase $\delta$. The T2K and NovA are running, DUNE might be the decisive experiment..
5) Determination of the MH with atmospheric neutrinos requires detection thresholds of 5-15 GeV. Dedicated modules like PINGU and ORCA are proposed and developed for that purpose.
6) Improvements, i.e. reduction of uncertainties in the determination of the CP phase $\delta$, of the $|\Delta m^2_{atm}$, and of the angle $\theta_{23}$ would make the hierarchy determination easier.

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## Conclusions for the case of the reactor MH determination

1) Determination of the MH in a reactor experiment at intermediate distance is obviously very challenging, but not unrealistic.
2) Besides the necessity of sufficient count rate (hence very large detector), it is necessary to have very good energy resolution, better than existing large detectors.
3) Improvement in the accuracy of the known oscillation parameters, in particular $\Delta m^2_{atm}$ would help.
4) The energy scale nonlinearity need to be improved as well.
5) One needs to be careful in determining the degree of confidence with which the MH was determined; the usual relation between the number of $\sigma$ and CL cannot be used.

Nevertheless, the method is clean in the sense that the outcome is independent of other things, like matter effects, CP phase etc.
It appears to be probably the best way to determine the MH.