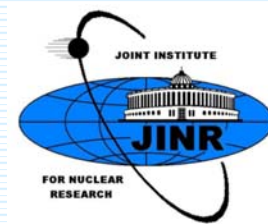


Neutrinos in Cosmology, in Astro-, Particle- and Nuclear Physics
Erice-Sicily: September 16-24, 2017

Different modes of double beta decay

Fedor Šimkovic



OUTLINE

- *Introduction*
 ν -oscillations and ν -masses
- *The $0\nu\beta\beta$ -decay scenarios due neutrinos exchange*
(simplest, sterile ν , LR-symmetric model)
- *Quenching of g_A and $2\nu\beta\beta$ -decay*
(nuclear structure issues)
- *Double beta-decay with emission of single electron*
- *Conclusion*

Acknowledgements: **A. Faesler** (Tuebingen), **P. Vogel** (Caltech), **S. Kovalenko** (Valparaiso U.), **M. Krivoruchenko** (ITEP Moscow), **D. Štefánik**, **R. Dvornický** (Comenius U.), **A. Babič**, **A. Smetana**, **J. Terasaki** (IEAP CTU Prague), ...



MESONIUM AND ANTIMESONIUM

B. PONTECORVO

Joint Institute for Nuclear Research

Submitted to JETP editor May 23, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 549-551 (August, 1957)

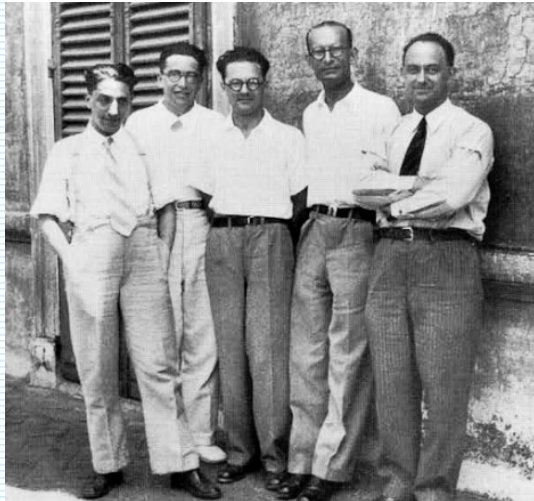
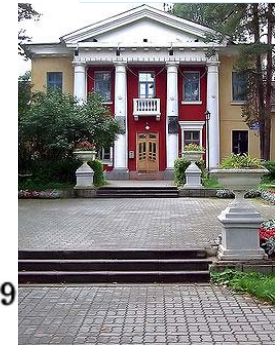
INVERSE BETA PROCESSES AND NONCONSERVATION OF LEPTON CHARGE

B. PONTECORVO

Joint Institute for Nuclear Research

Submitted to JETP editor October 19, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 247-249
(January, 1958)



I ragazzi di via Panisperna

It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are “mixed” particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity.⁵



1968 Gribov, Pontecorvo [PLB 28(1969) 493]
oscillations of neutrinos - a solution
of deficit of solar neutrinos in Homestake exp.

Observation of ν -oscillations = the first prove of the BSM physics

mass-squared differences: $\Delta m^2_{\text{SUN}} \cong 7.5 \cdot 10^{-5} \text{ eV}^2$, $\Delta m^2_{\text{ATM}} \cong 2.4 \cdot 10^{-3} \text{ eV}^2$

The observed **small neutrino masses** (limits from tritium β -decay, cosmology) have profound implications for our understanding of the Universe and are now a major focus in astro, particle and nuclear physics and in cosmology.

PMNS
unitary
mixing
matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

large off-diagonal values

$$\begin{pmatrix} 0.82 & 0.54 & -0.15 \\ -0.35 & 0.70 & 0.62 \\ 0.44 & -0.45 & 0.77 \end{pmatrix}$$

3 angles: $\theta_{12}=33.36^\circ$ (solar), $\theta_{13}=8.66^\circ$ (reactor), $\theta_{23}=40.0^\circ$ or 50.4° (atmospheric)

$$U^{PMNS} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

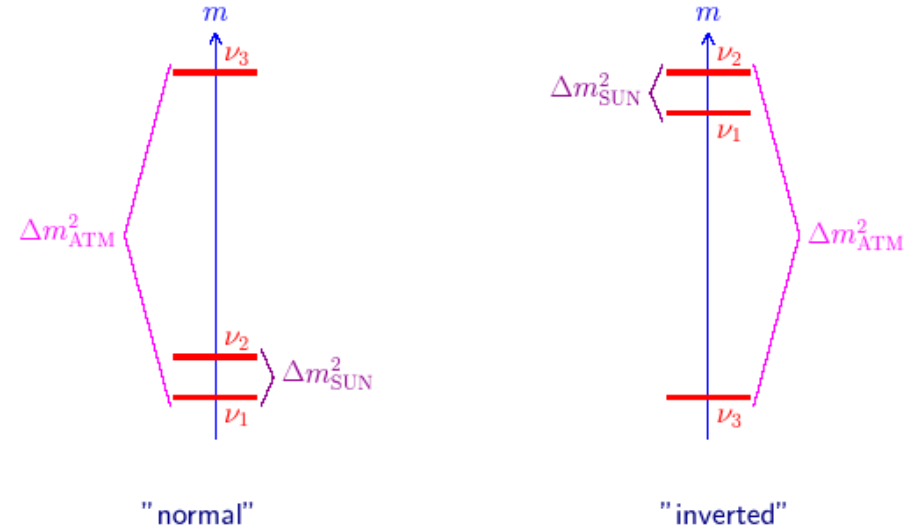
unknown (CP violating) phases: δ , α_1 , α_2

Neutrinos mass spectrum

$0\nu\beta\beta$ Measurements

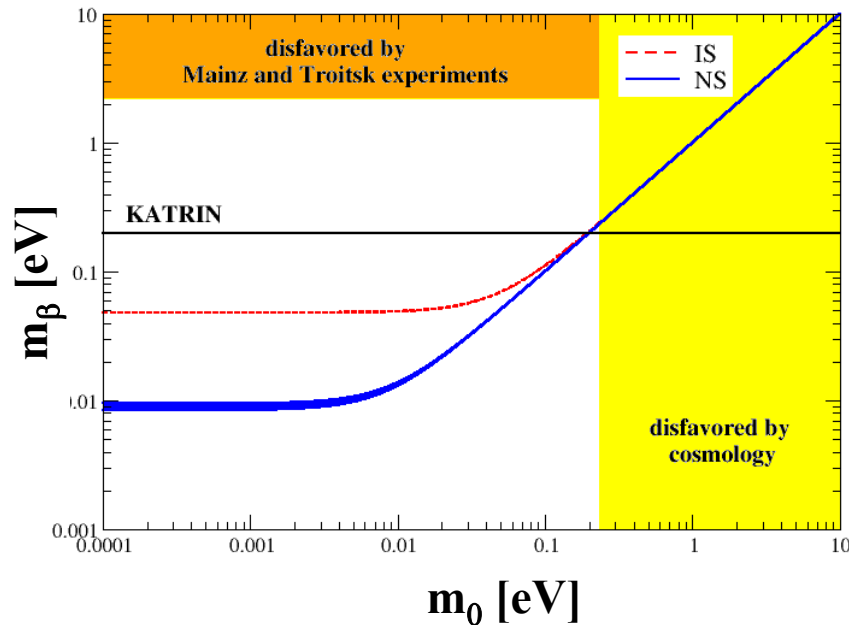
$$m_{\beta\beta} =$$

$$\left| c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3 \right|$$



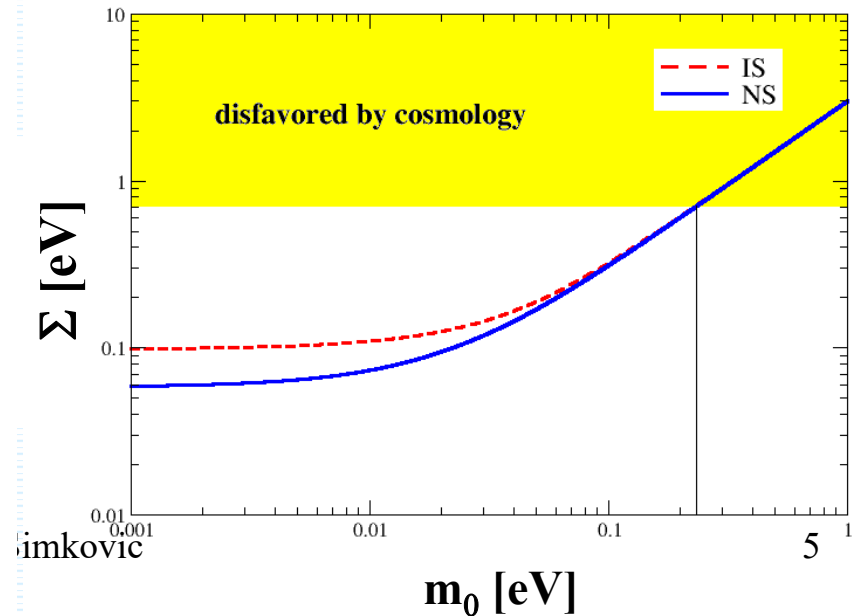
Beta Decay Measurements

$$m_{\beta} = \sqrt{c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2}$$



Cosmological Measurements

$$\Sigma = m_1 + m_2 + m_3$$



The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

What is the nature of neutrinos?



ν



GUT's



Symmetric Theory of Electron and Positron
Nuovo Cim. 14 (1937) 171

Only the $0\nu\beta\beta$ -decay can answer this fundamental question

Analogy with
kaons: K_0 and \bar{K}_0

Fedor Simkovic

Analogy with
 π_0

Majorana fermion



https://en.wikipedia.org/wiki/File:Ettore_Majorana.jpg

Celebrating
80
years
1937 - 2017

9/23/2017

Fedor S

TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

Nota di ETTORE MAJORANA

Symmetric Theory of Electron and Positron Nuovo Cim. 14 (1937) 171

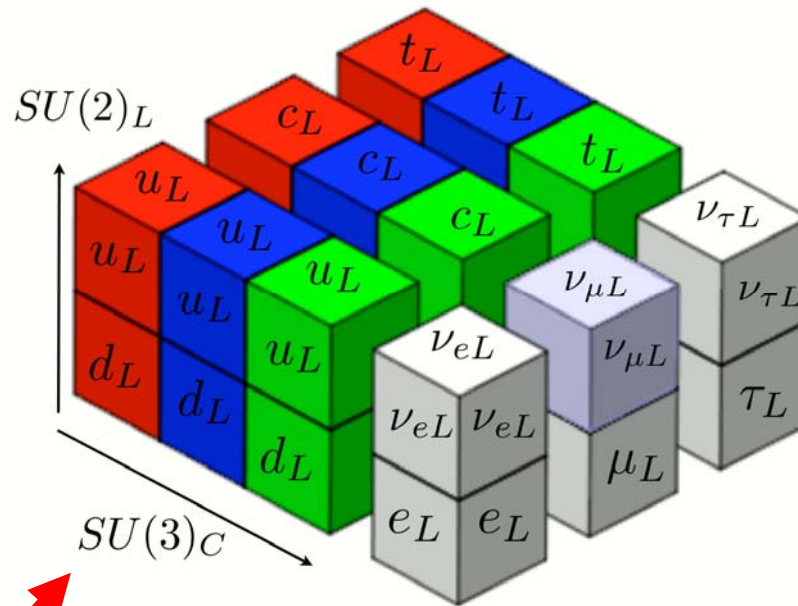
Sunto. - *Si dimostra la possibilità di pervenire a una piena simmetrizzazione formale della teoria quantistica dell'elettrone e del positrone facendo uso di un nuovo processo di quantizzazione. Il significato delle equazioni di DIRAC ne risulta alquanto modificato e non vi è più luogo a parlare di stati di energia negativa; nè a presumere per ogni altro tipo di particelle, particolarmente neutre, l'esistenza di « antiparticelle » corrispondenti ai « vuoti » di energia negativa.*

L'interpretazione dei cosiddetti « stati di energia negativa » proposta da DIRAC ⁽¹⁾ conduce, come è ben noto, a una descrizione sostanzialmente simmetrica degli elettroni e dei positroni. La sostanziale simmetria del formalismo consiste precisamente in questo, che fin dove è possibile applicare la teoria girando le difficoltà di convergenza, essa fornisce realmente risultati del tutto simmetrici. Tuttavia gli artifici suggeriti per dare alla teoria una forma simmetrica che si accordi con il suo contenuto, non sono del tutto soddisfacenti; sia perchè si parte sempre da una impostazione asimmetrica, sia perchè la simmetrizzazione viene in seguito ottenuta mediante tali procedimenti (come la cancellazione di costanti infinite) che possibilmente dovrebbero evitarsi. Perciò abbiamo tentato una nuova via che conduce più direttamente alla meta.

Per quanto riguarda gli elettroni e i positroni, da essa si può veramente attendere soltanto un progresso formale; ma ci sembra importante, per le possibili estensioni analogiche, che venga a cadere la nozione stessa di stato di energia negativa. Vedremo infatti che è perfettamente possibile costruire, nella maniera più naturale, una teoria delle particelle neutre elementari senza stati negativi.

⁽¹⁾ P. A. M. DIRAC, « Proc. Camb. Phil. Soc. », **30**, 150, 1924. V. anche W. HEISENBERG, « ZS. f. Phys. », **90**, 209, 1934.

Beyond the Standard model physics (*EFT scenario*)



$$\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_i c_i^{(5)} \mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} \mathcal{O}_i^{(6)} + O\left(\frac{1}{\Lambda^3}\right)$$

Minimal SM + EFT

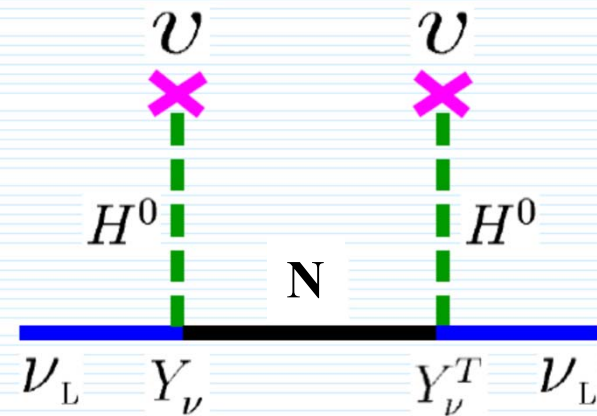
S.M. Bilenky,
Phys.Part.Nucl.Lett. 12 (2015) 453-461

The **absence of the right-handed neutrino fields** in the Standard Model is the simplest, most economical possibility. In such a scenario **Majorana mass term** is the only possibility for neutrinos to be massive and mixed. This mass term is generated by the **lepton number violating Weinberg effective Lagrangian**.

$$\mathcal{L}_5^{eff} = -\frac{1}{\Lambda} \sum_{l_1 l_2} \left(\bar{\Psi}_{l_1 L}^{lep} \tilde{\Phi} \right) Y_{l_1 l_2} \left(\tilde{\Phi}^T (\Psi_{l_2 L}^{lep})^c \right)$$

$$m_i = \frac{v}{\Lambda} (y_i v), \quad i = 1, 2, 3 \quad \Lambda \geq 10^{15} \text{ GeV}$$

Heavy Majorana leptons N_i ($N_i = N_i^c$)
singlet of $SU(2)_L \times U(1)_Y$ group
Yukawa lepton number violating int.

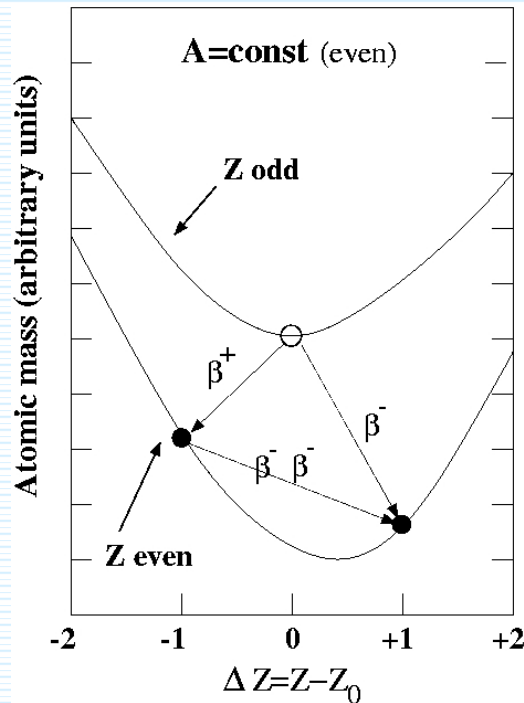


The three Majorana neutrino masses are suppressed by the ratio of the electroweak scale and a scale of a lepton-number violating physics.

The discovery of the $\beta\beta$ -decay and absence of transitions of flavor neutrinos into sterile states would be evidence in favor of this minimal scenario.

I. The simplest $0\nu\beta\beta$ -decay scenario (SM + EFT scenario)

$$\left(T_{1/2}^{0\nu}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 |M_\nu^{0\nu}|^2 G^{0\nu}$$



transition	$G^{01}(E_0, Z)$ $\times 10^{14}y$	$Q_{\beta\beta}$ [MeV]	Abund. (%)	$ M^{0\nu} ^2$
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	26.9	3.667	6	?
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	8.04	4.271	0.2	?
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	7.37	3.350	3	?
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	6.24	2.802	7	?
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	5.92	2.479	9	?
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	5.74	3.034	10	?
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	5.55	2.533	34	?
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	3.53	2.995	9	?
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	0.79	2.040	8	?

The NMEs for $0\nu\beta\beta$ -decay must be evaluated using tools of nuclear theory

Effective mass of Majorana neutrinos (in vacuum)

$$|m_{\beta\beta}| = \left| c_{12}^2 c_{13}^2 e^{i\alpha_1} m_1 + s_{12}^2 c_{13}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3 \right|$$

$m_1, m_2, m_3, \theta_{12}, \theta_{13}, \alpha_1, \alpha_2$
(3 unknown parameters)

Measured quantity

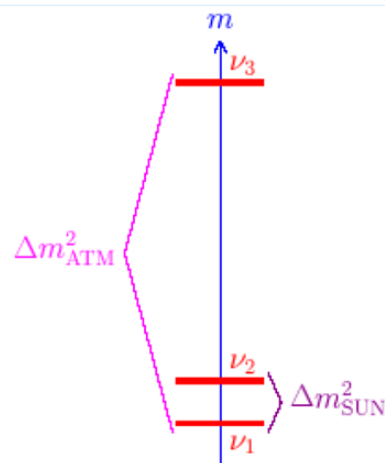
$$|m_{\beta\beta}|^2 = c_{12}^4 c_{13}^4 m_1^2 + s_{12}^4 c_{13}^4 m_2^2 + s_{13}^4 m_3^2 + 2c_{12}^2 s_{12}^2 c_{13}^4 m_1 m_2 \cos(\alpha_1 - \alpha_2) + 2c_{12}^2 c_{13}^2 s_{13}^2 m_1 m_3 \cos \alpha_1 + 2s_{12}^2 c_{13}^2 s_{13}^2 m_2 m_3 \cos \alpha_2.$$

Limiting cases

Normal hierarchy

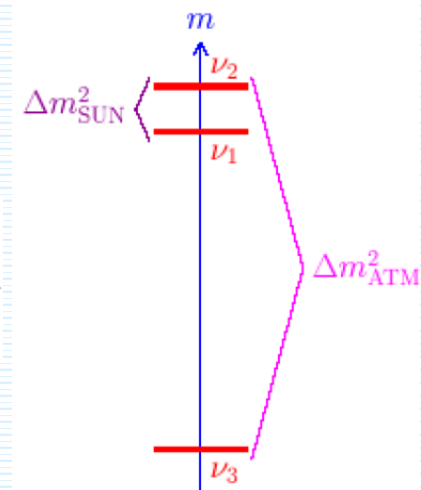
$$\begin{aligned} m_1 &\ll \sqrt{\Delta m_{\text{SUN}}^2} \\ m_2 &\simeq \sqrt{\Delta m_{\text{SUN}}^2} \\ m_3 &\simeq \sqrt{\Delta m_{\text{ATM}}^2} \end{aligned}$$

9/23/2017

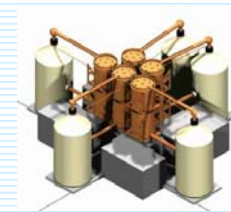


Inverted hierarchy

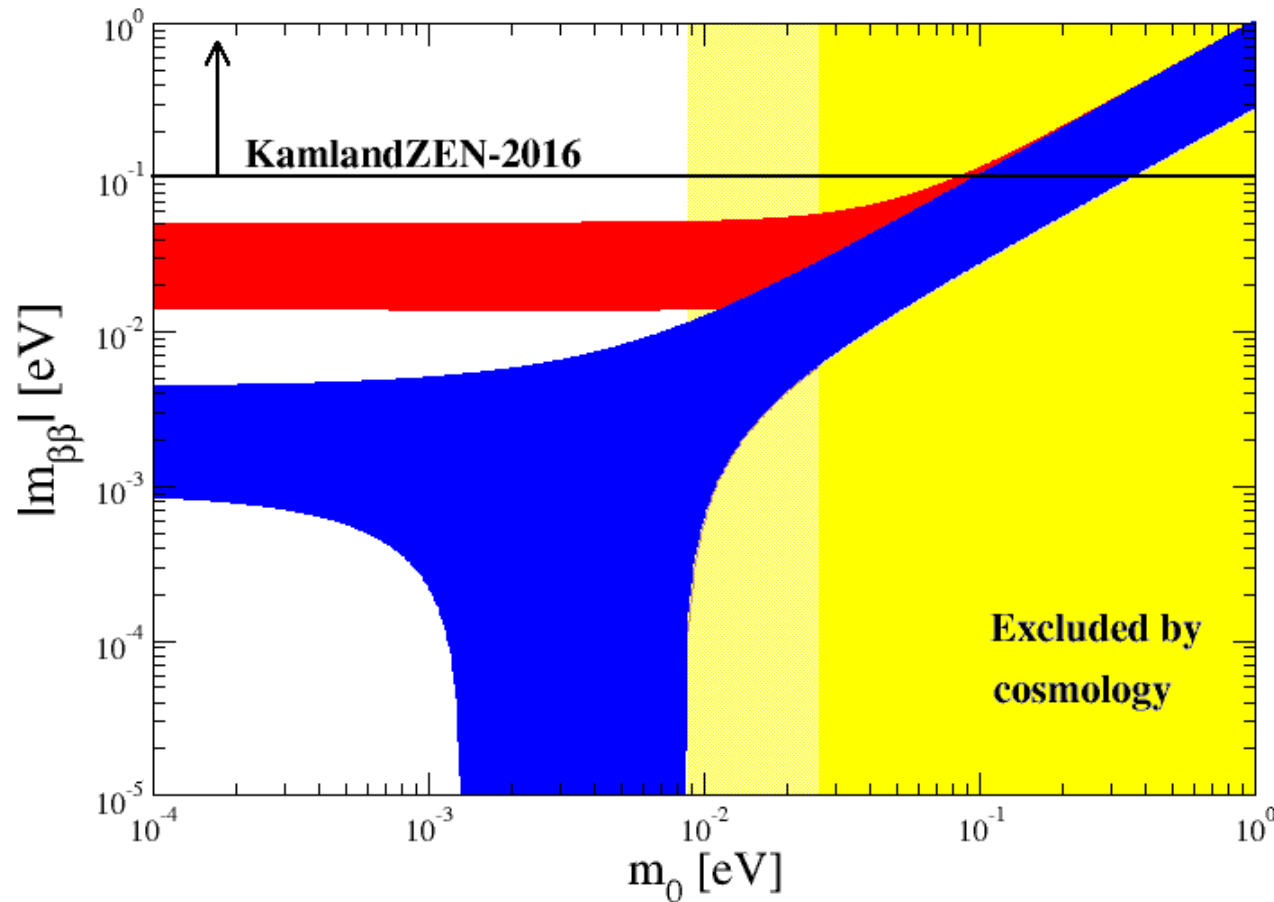
$$\begin{aligned} m_3 &\ll \sqrt{\Delta m_{\text{ATM}}^2} \\ m_1 &\simeq m_2 \simeq \sqrt{\Delta m_{\text{ATM}}^2} \end{aligned}$$



Andor Simkovic



Issue: Lightest neutrino mass m_0



Complementarity of $0\nu\beta\beta$ -decay, β -decay and cosmology

β -decay (Mainz, Troitsk)

$$m_\beta^2 = \sum_i |U_{ei}^L|^2 m_i^2 \leq (2.2 \text{ eV})^2$$

KATRIN: $(0.2 \text{ eV})^2$

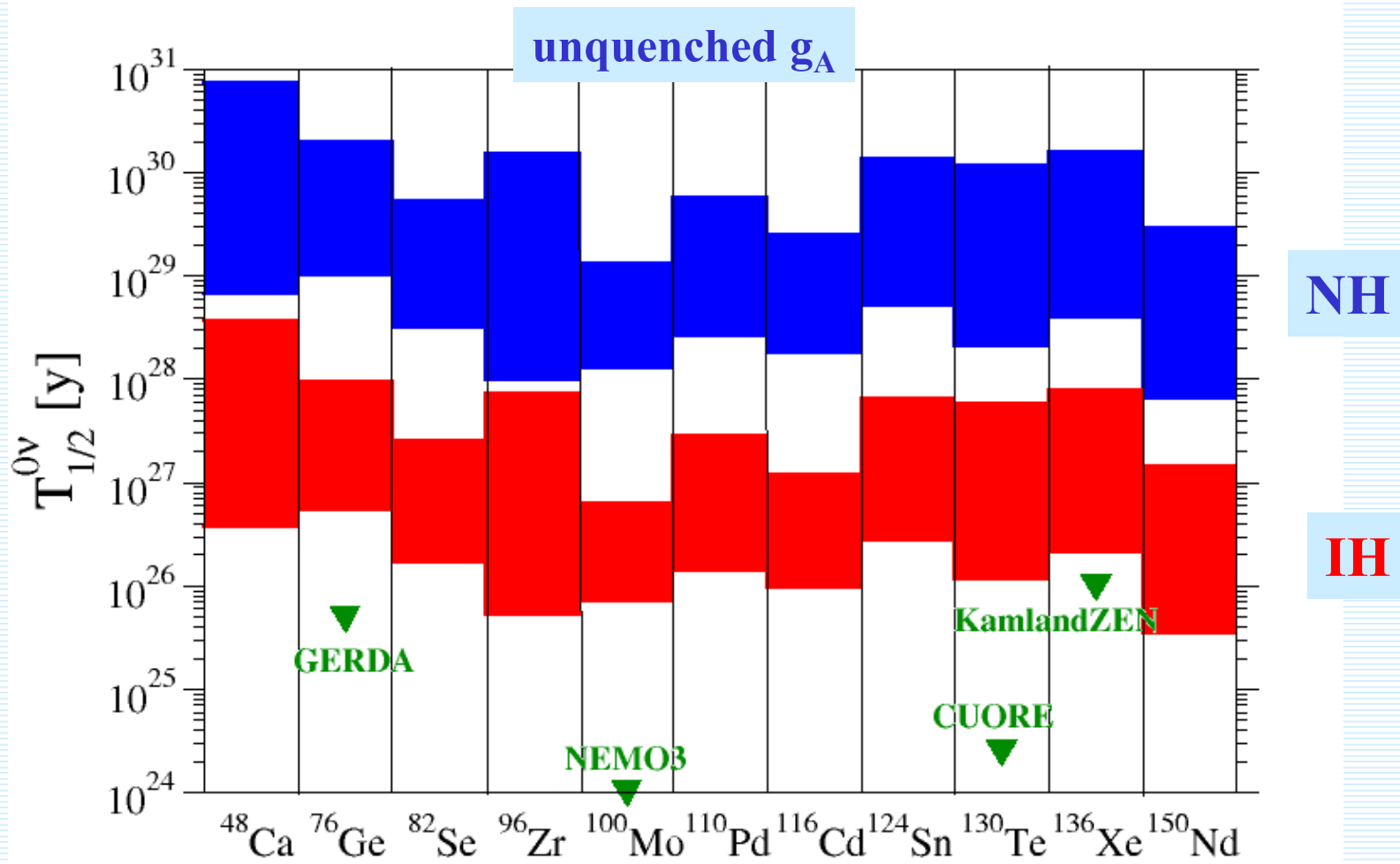
Cosmology (Planck)

$$\Sigma < 110 \text{ meV}$$

$$m_0 > 26 \text{ meV (NS)}$$

$$87 \text{ meV (IS)}$$

0νββ –half lives for NH and IH with included uncertainties in NMEe



NH: $m_1 \ll m_2 \ll m_3$ $m_3 \simeq \sqrt{\Delta m^2}$

IH: $m_3 \ll m_1 < m_2$ $m_1 \simeq m_2 \simeq \sqrt{\Delta m^2}$

$m_1 \ll \sqrt{\delta m^2}$, $m_2 \simeq \sqrt{\delta m^2}$

$m_3 \ll \sqrt{\Delta m^2}$

13

$1.4 \text{ meV} \leq m_{\beta\beta} \leq 3.6 \text{ meV}$

Lightest ν -mass equal to zero

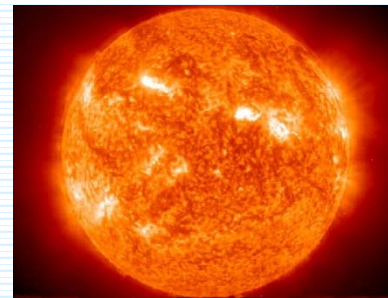
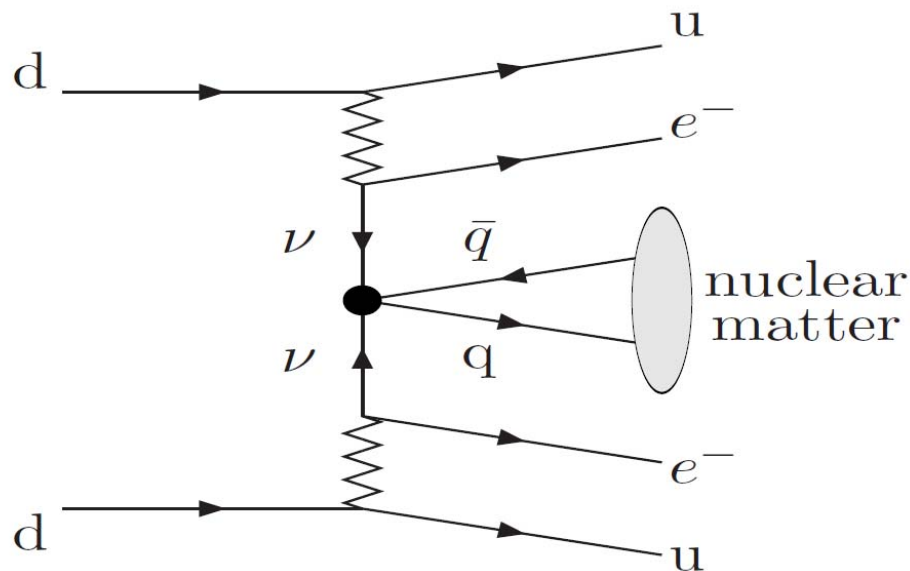
$20 \text{ meV} \leq m_{\beta\beta} \leq 49 \text{ meV}$

Nuclear medium effect on the light neutrino mass exchange mechanism of the $0\nu\beta\beta$ -decay

S.G. Kovalenko, M.I. Krivoruchenko, F. Š., Phys. Rev. Lett. 112 (2014) 142503

A novel effect in $0\nu\beta\beta$ decay related with the fact, that its underlying mechanisms take place in the nuclear matter environment:

- + Low energy 4-fermion $\Delta L \neq 0$ Lagrangian**
- + In-medium Majorana mass of neutrino**
- + $0\nu\beta\beta$ constraints on the universal scalar couplings**

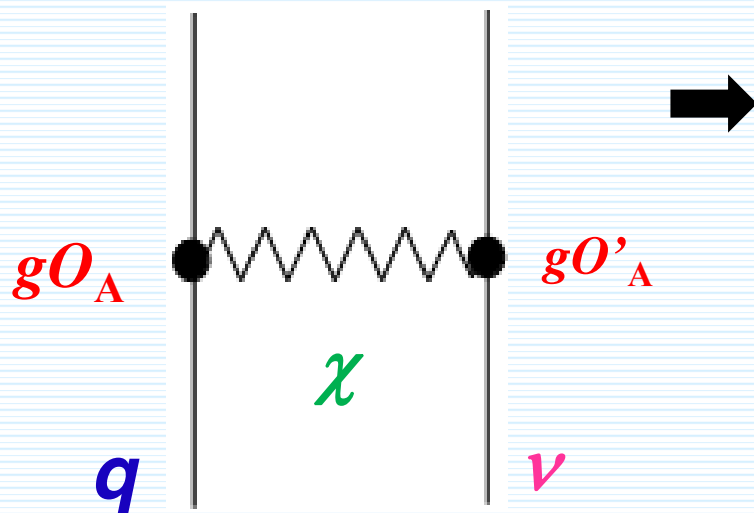


**Non-standard
 ν -int. discussed
e.g., in the context
of ν -osc. at Sun**

imkovic

$$\begin{aligned} \rho_{\text{Sun}} &= 1.4 \text{ g/cm}^3 \\ \rho_{\text{Earth}} &= 5.5 \text{ g/cm}^3 \\ \rho_{\text{nucleus}} &= 2.3 \cdot 10^{14} \text{ g/cm}^3 \end{aligned}$$

Non-standard interactions might be easily detected in nucleus rather than in vacuum



oscillation experiments
tritium β -decay, cosmology

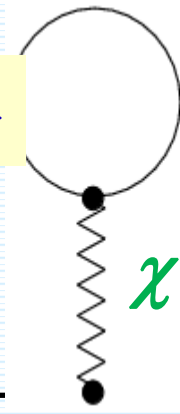
$$\sum_{\nu}^{\text{vac}} = -\times-$$

Low energy 4-fermion
 $\Delta L \neq 0$ **Lagrangian**

$$L_{\text{eff}} = \frac{g^2}{m_{\chi}^2} \sum_A (\bar{q} O_A q) (\bar{\nu} O'_A \nu),$$

$$m_{\chi} \sim M_W.$$

$0\nu\beta\beta$ -decay

density \rightarrow 

$$\sum_{\nu}^{\text{medium}} = -\times- + \text{diagram}$$

Mean field:

$$\bar{q}q \rightarrow \langle \bar{q}q \rangle$$

and

$$\langle \bar{q}q \rangle \approx 0.5 \langle q^\dagger q \rangle \approx 0.25 \text{ fm}^{-3}$$

The effect depends on

$$\langle \chi \rangle = -\frac{g_\chi}{m_\chi^2} \langle \bar{q}q \rangle$$

A comparison with G_F :

$$\frac{g_\chi g_{ij}^a}{m_\chi^2} = \frac{G_F}{\sqrt{2}} \varepsilon_{ij}^a$$

Typical scale:

$$\langle \chi \rangle g_{ij}^a = -\frac{G_F}{\sqrt{2}} \langle \bar{q}q \rangle \varepsilon_{ij}^a \approx -25 \varepsilon_{ij}^a \text{ eV}$$

We expect:

$$25 \varepsilon_{ij}^a < 1 \rightarrow m_\chi^2 > 25 \frac{g_\chi g_{ij}^a \sqrt{2}}{G_F} \sim 1 \text{ TeV}^2$$

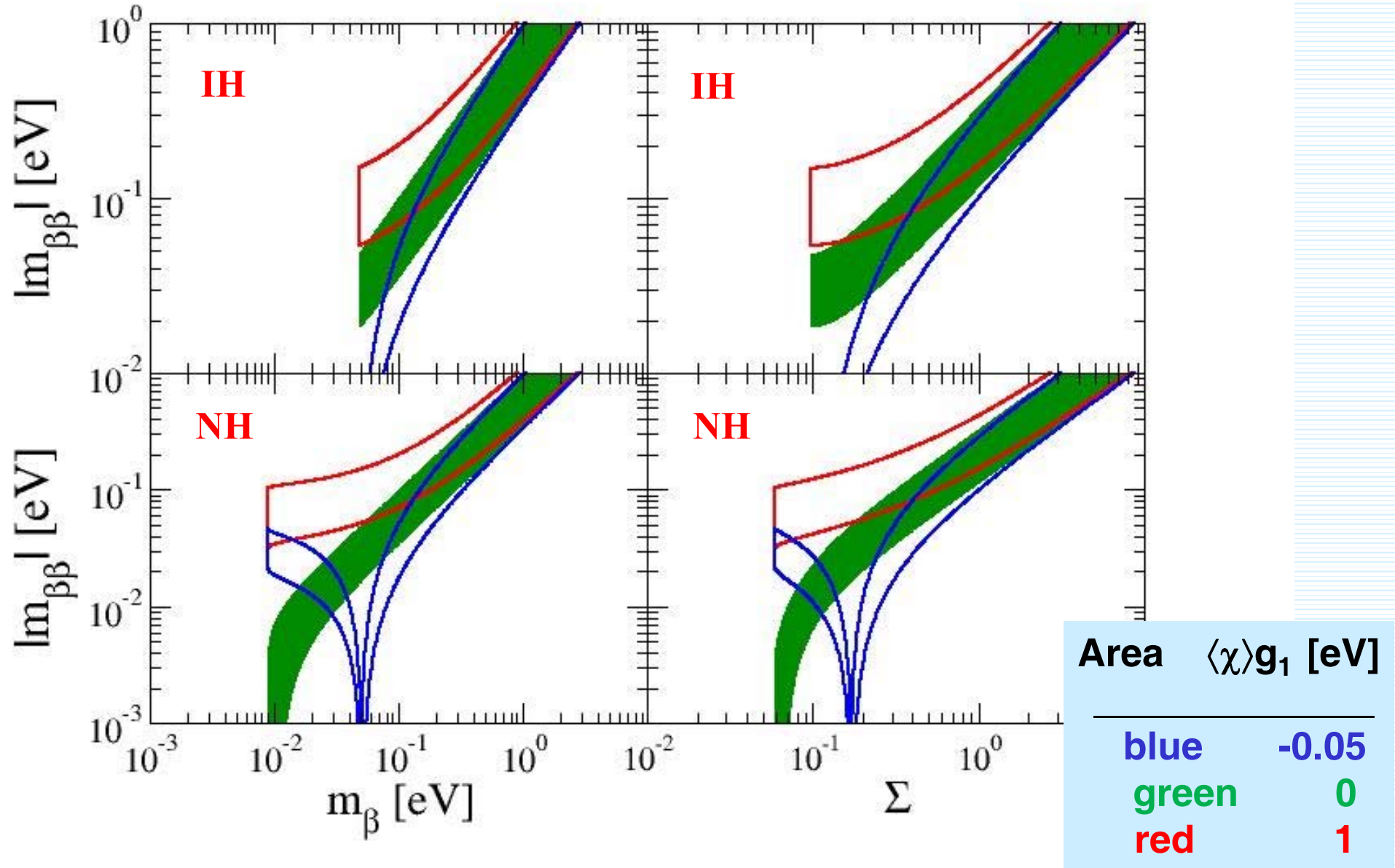
Universal scalar interaction

$$g_{ij}^a = \delta_{ij} g_a \quad \varepsilon_{ij}^a = \delta_{ij} \varepsilon_a$$

In medium
effective
Majorana ν mass

$$m_{\beta\beta} = \sum_{i=1}^n U_{ei}^2 \xi_i \frac{\sqrt{(m_i + \langle \chi \rangle g_1)^2 + (\langle \chi \rangle g_2)^2}}{(1 - \langle \chi \rangle g_4)^2}$$

Complementarity between β -decay, $0\nu\beta\beta$ -decay and cosmological measurements might be spoiled



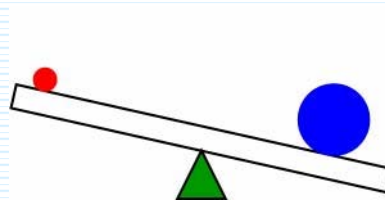
II. *The sterile ν mechanism of the $0\nu\beta\beta$ -decay* (*D-M mass term, V-A SM int.*)

$$N = \sum_{\alpha=s,e,\mu,\tau} U_{N\alpha} \nu_{\alpha}$$

Mixing of
active-sterile
neutrinos

Dirac-Majorana
mass term

$$\begin{pmatrix} 0 & m_D \\ m_D & m_{LNV} \end{pmatrix}$$

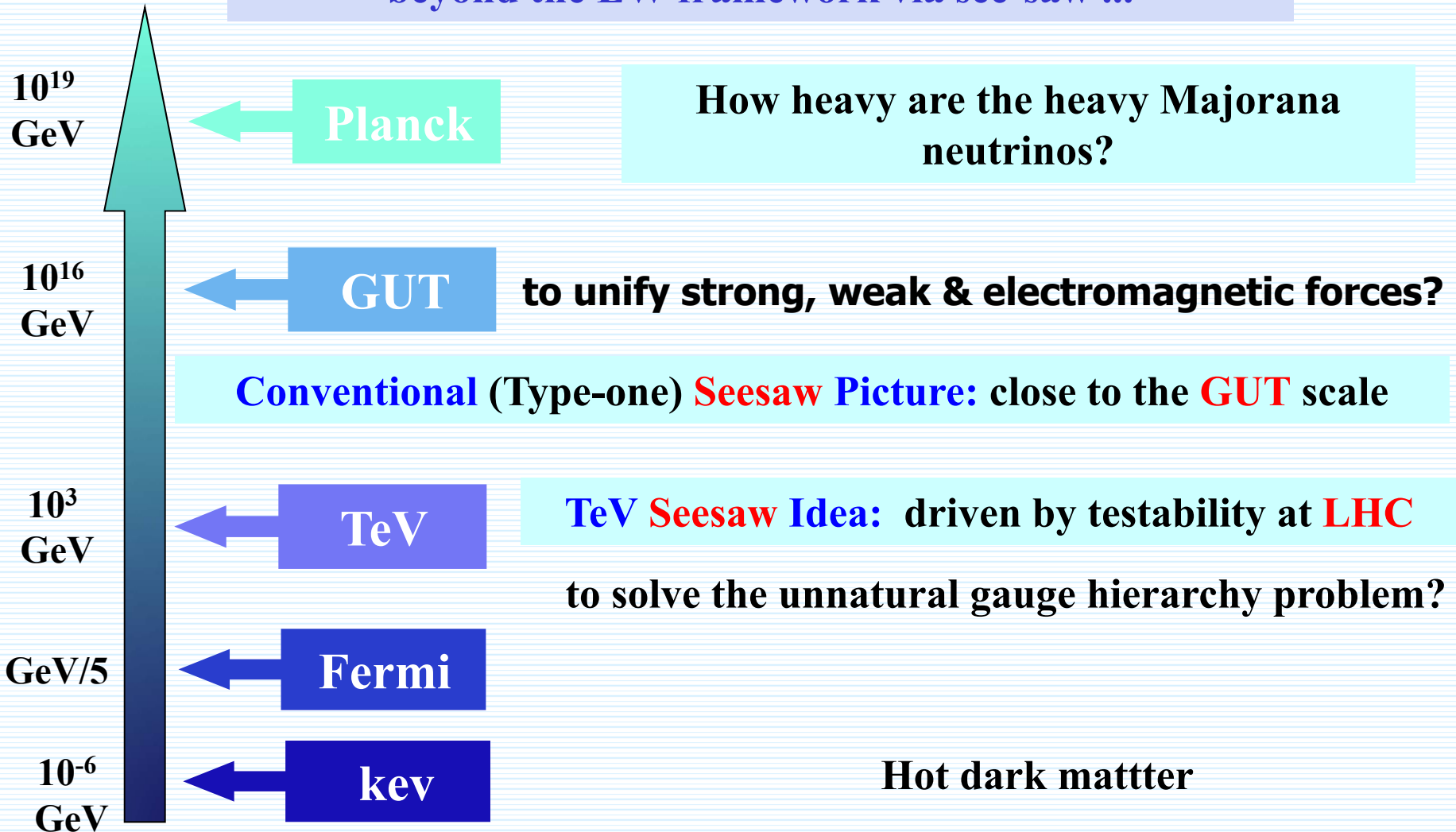


Light ν mass $\approx (m_D/m_{LNV}) m_D$
Heavy ν mass $\approx m_{LNV}$

small ν masses due to see-saw
mechanism

Possible lepton number violating scale - m_{LNV}

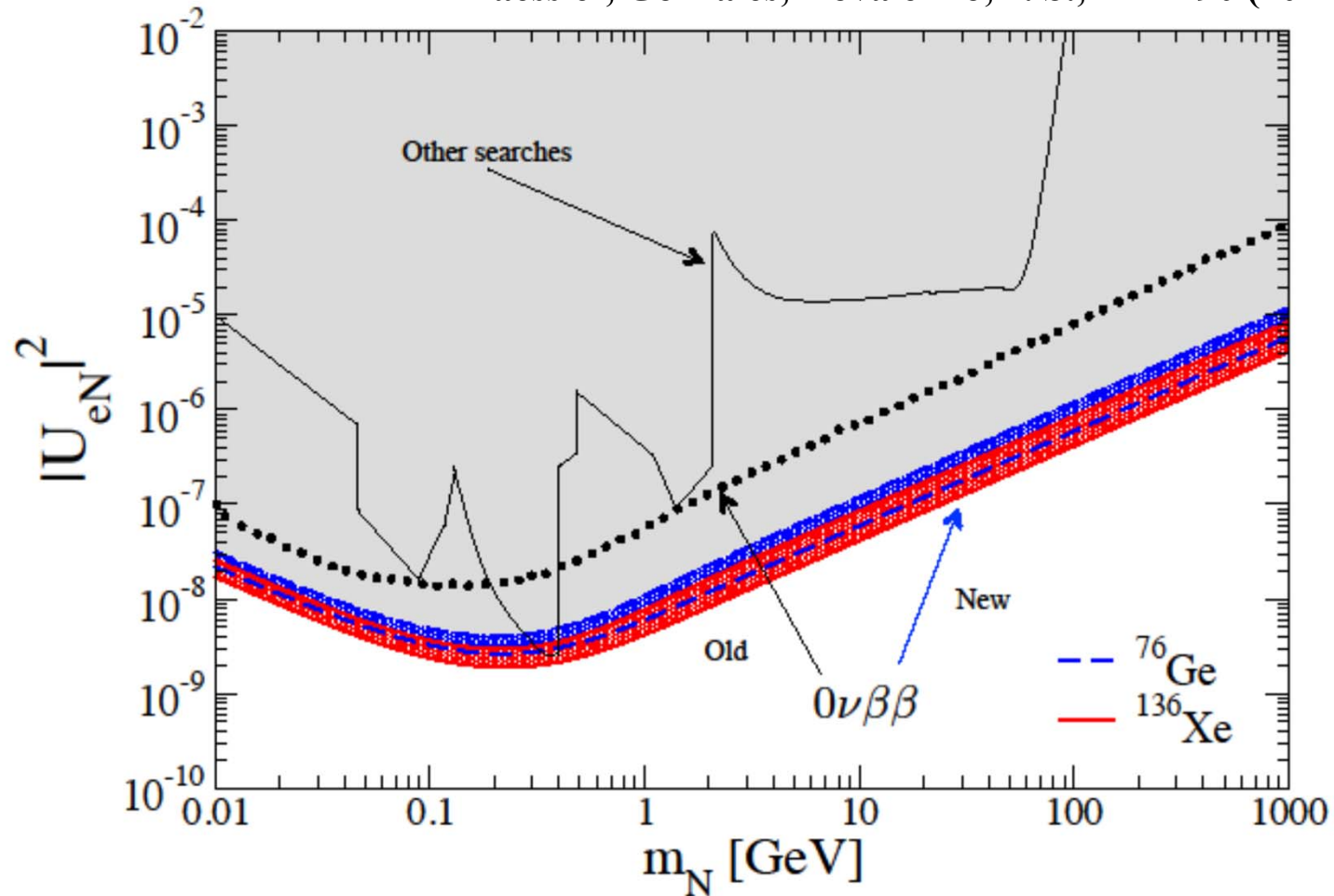
Neutrinos masses may offer a great opportunity to jump beyond the EW framework via see-saw ...



**Exclusion plot
in $|U_{eN}|^2 - m_N$ plane**

$$T_{1/2}^{0\nu}({}^{76}\text{Ge}) \geq 3.0 \cdot 10^{25} \text{ yr}$$
$$T_{1/2}^{0\nu}({}^{136}\text{Xe}) \geq 3.4 \cdot 10^{25} \text{ yr}$$

Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010]



Improvements: i) QRPA (constrained Hamiltonian by $2\nu\beta\beta$ half-life, self-consistent treatment of src, restoration of isospin symmetry ...),
ii) More stringent limits on the $0\nu\beta\beta$ half-life

III. *The $0\nu\beta\beta$ -decay within L-R symmetric theories* (D-M mass term, see-saw, V-A and V+A int., exchange of light neutrinos)

Effective β -decay Hamiltonian

$$H^\beta = \frac{G_\beta}{\sqrt{2}} \left[j_L^\rho J_{L\rho} + \chi j_L^\rho J_{R\rho} + \eta j_R^\rho J_{L\rho} + \lambda j_R^\rho J_{R\rho} + h.c. \right].$$

left- and right-handed lept. currents

$$j_L^\rho = \bar{e}\gamma^\rho(1 - \gamma_5)\nu_{eL}$$

$$j_R^\rho = \bar{e}\gamma^\rho(1 + \gamma_5)\nu_{eR}$$

Mixing of vector bosons W_L and W_R

$$\begin{pmatrix} W_L^- \\ W_R^- \end{pmatrix} = \begin{pmatrix} \cos \zeta & \sin \zeta \\ -\sin \zeta & \cos \zeta \end{pmatrix} \begin{pmatrix} W_1^- \\ W_2^- \end{pmatrix}$$

$$\eta = -\tan \zeta, \quad \chi = \eta,$$

$$\lambda = (M_{W_1}/M_{W_2})^2$$

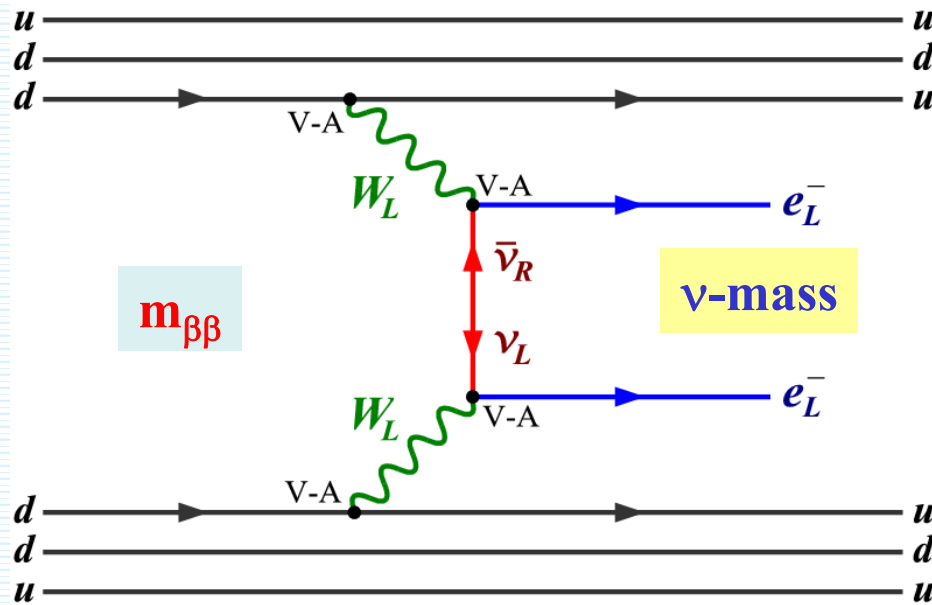
The $0\nu\beta\beta$ -decay half-life

$$\begin{aligned} [T_{1/2}^{0\nu}]^{-1} &= \frac{\Gamma^{0\nu}}{\ln 2} = g_A^4 |M_{GT}|^2 \left\{ C_{mm} \frac{|m_{\beta\beta}|^2}{m_e} \right. \\ &+ C_{m\lambda} \frac{|m_{\beta\beta}|}{m_e} \langle \lambda \rangle \cos \psi_1 + C_{m\eta} \frac{|m_{\beta\beta}|}{m_e} \langle \eta \rangle \cos \psi_2 \\ &\left. + C_{\lambda\lambda} \langle \lambda \rangle^2 + C_{\eta\eta} \langle \eta \rangle^2 + C_{\lambda\eta} \langle \lambda \rangle \langle \eta \rangle \cos(\psi_1 - \psi_2) \right\} \end{aligned}$$

$\langle \lambda \rangle$ - W_L - W_R exch.

$\langle \eta \rangle$ - W_L - W_R mixing

Left-right symmetric models $SO(10)$



Mixing of light and heavy neutrinos

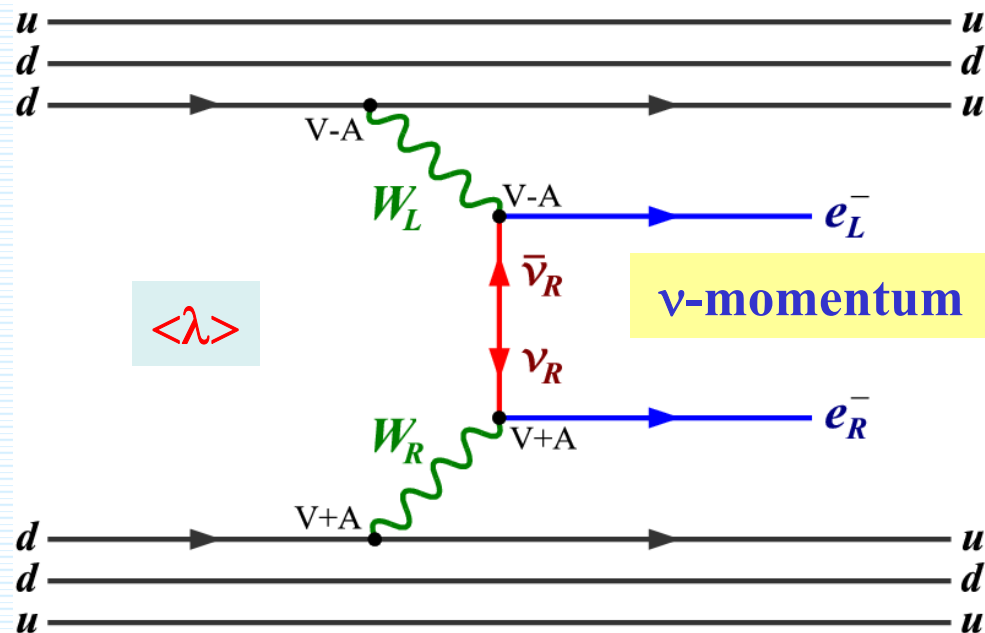
$$\nu_{eL} = \sum_{j=1} \left(U_{ej} \nu_{jL} + S_{ej} (N_{jR})^C \right),$$

$$\nu_{eR} = \sum_{j=1}^3 \left(T_{ej}^* (\nu_{jL})^C + V_{ej}^* N_{jR} \right)$$

Effective LNV parameters due to RHC

$$\langle \lambda \rangle = \lambda \left| \sum_{j=1}^3 U_{ej} T_{ej}^* \right|$$

$$\langle \eta \rangle = \eta \left| \sum_{j=1}^3 U_{ej} T_{ej}^* \right|$$



Mixing and masses of vector bosons

$$\eta = -\tan \zeta, \quad \chi = \eta,$$

$$\lambda = (M_{W_1}/M_{W_2})^2$$

3x3 block matrices

U, S, T, V are generalization of PMNS matrix

Zhi-zhong Xing, Phys. Rev. D 85, 013008 (2012)

6x6 neutrino mass matrix

Basis

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

$$(\nu_L, (N_R)^c)^T$$

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix}$$

15 angles, 10+5 phases

Decomposition

$$\mathcal{U} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

The see-saw structure and neglecting mixing between different generations

Approximation

$$A \approx \mathbf{1}, \quad B \approx \mathbf{1}, \quad R \approx \frac{m_D}{m_{LNV}} \mathbf{1}, \quad S \approx -\frac{m_D}{m_{LNV}} \mathbf{1}$$

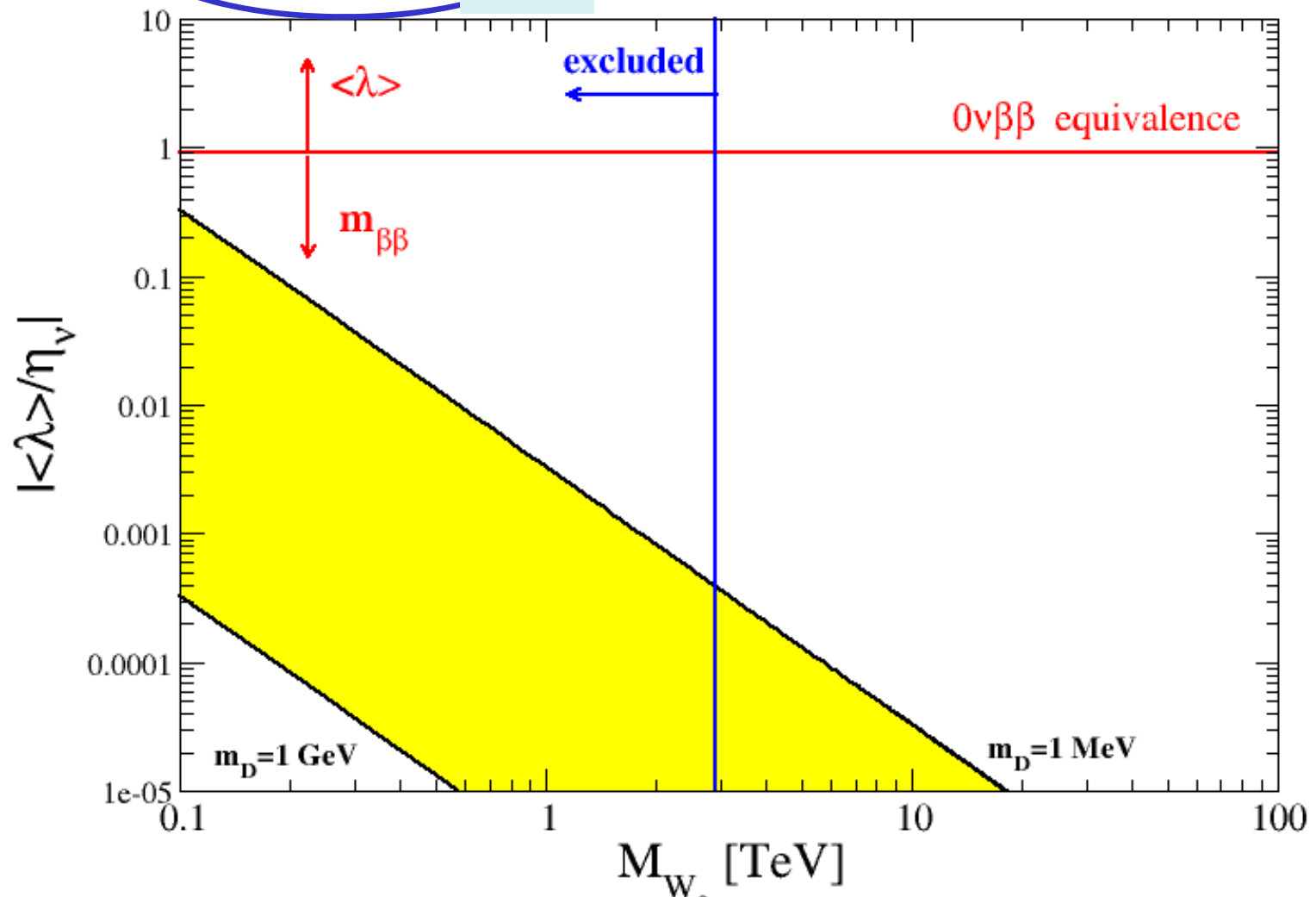
$$U_0 \simeq V_0$$

LNV parameters

$$|\langle \lambda \rangle| \approx \frac{m_D}{m_{LNV}} \left(\frac{M_{W_1}}{M_{W_2}} \right)^2 |\xi| \quad |\langle \eta \rangle| \approx \frac{m_D}{m_{LNV}} \tan(\zeta) |\xi| \quad |\xi| \simeq 0.82$$

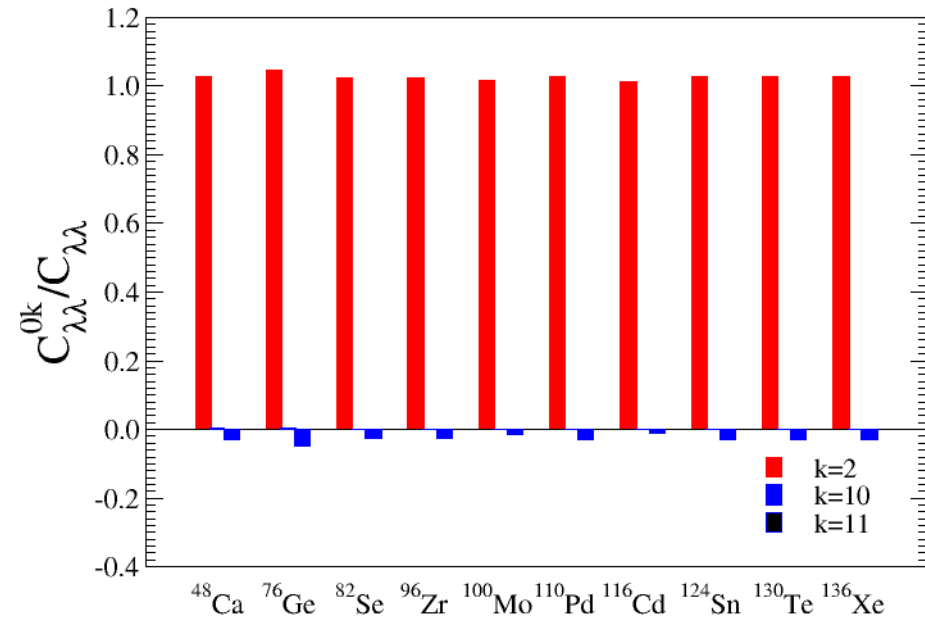
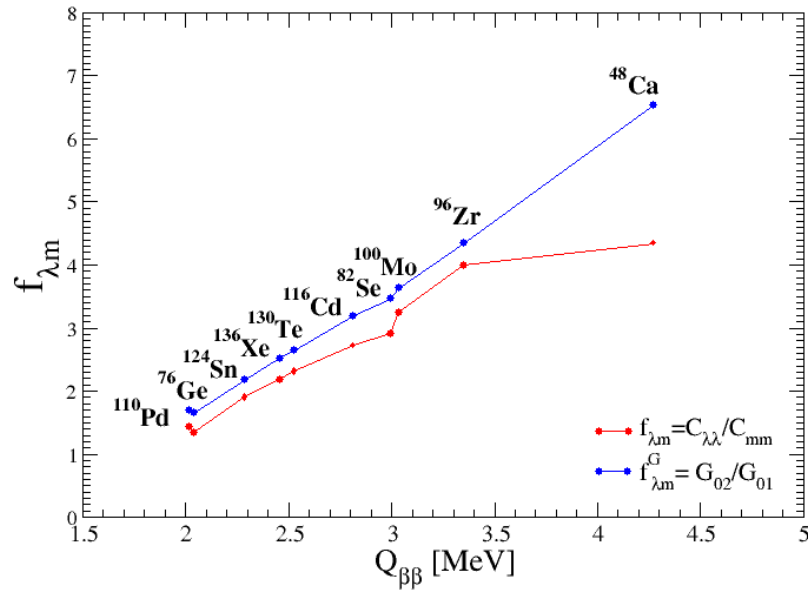
$$\eta_\nu = \frac{m_{\beta\beta}}{m_e} = \sum_i ((U_0)_{ei})^2 \frac{m_i}{m_e}$$

$$\approx \frac{m_D}{m_{LNV}} \frac{m_D}{m_e} \sum_i (U_0)_{ei}^2 \frac{m_i m_{LNV}}{m_D^2} \quad \text{if } \approx 1 \quad |\langle \lambda \rangle| \approx \frac{m_D}{m_{LNV}} \left(\frac{M_{W_1}}{M_{W_2}} \right)^2 |\xi|$$



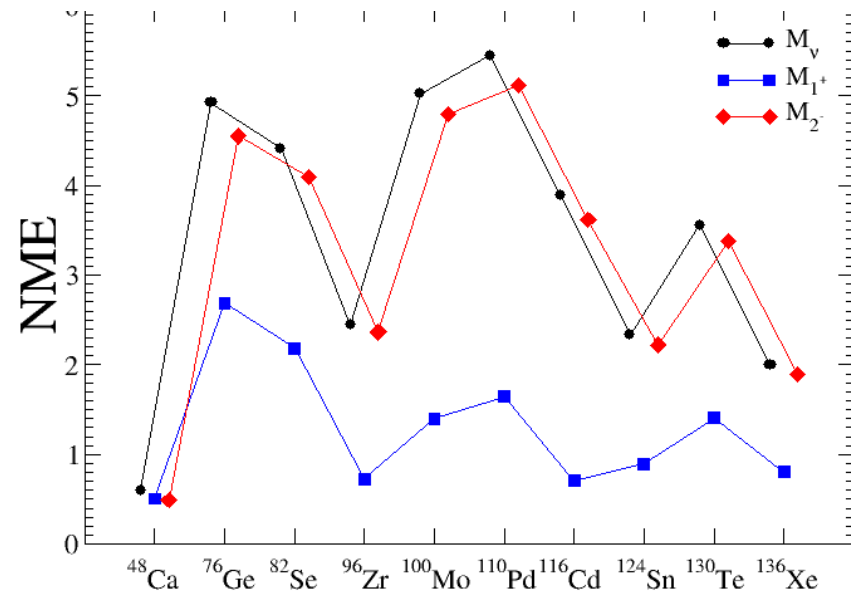
Clear dominance of $m_{\beta\beta}$ over $\langle \lambda \rangle$ mechanism by current constraint on mass of heavy vector boson

$m_{\beta\beta}$ and λ mechanisms



$$\begin{aligned} [T_{1/2}^{0\nu}]^{-1} &= (\eta_\nu^2 + \eta_\lambda^2 f_{\lambda m}) C_{mm} \\ &\simeq (\eta_\nu^2 + \eta_\lambda^2 f_{\lambda m}^G) g_A^4 M_\nu^2 G_{01} \end{aligned}$$

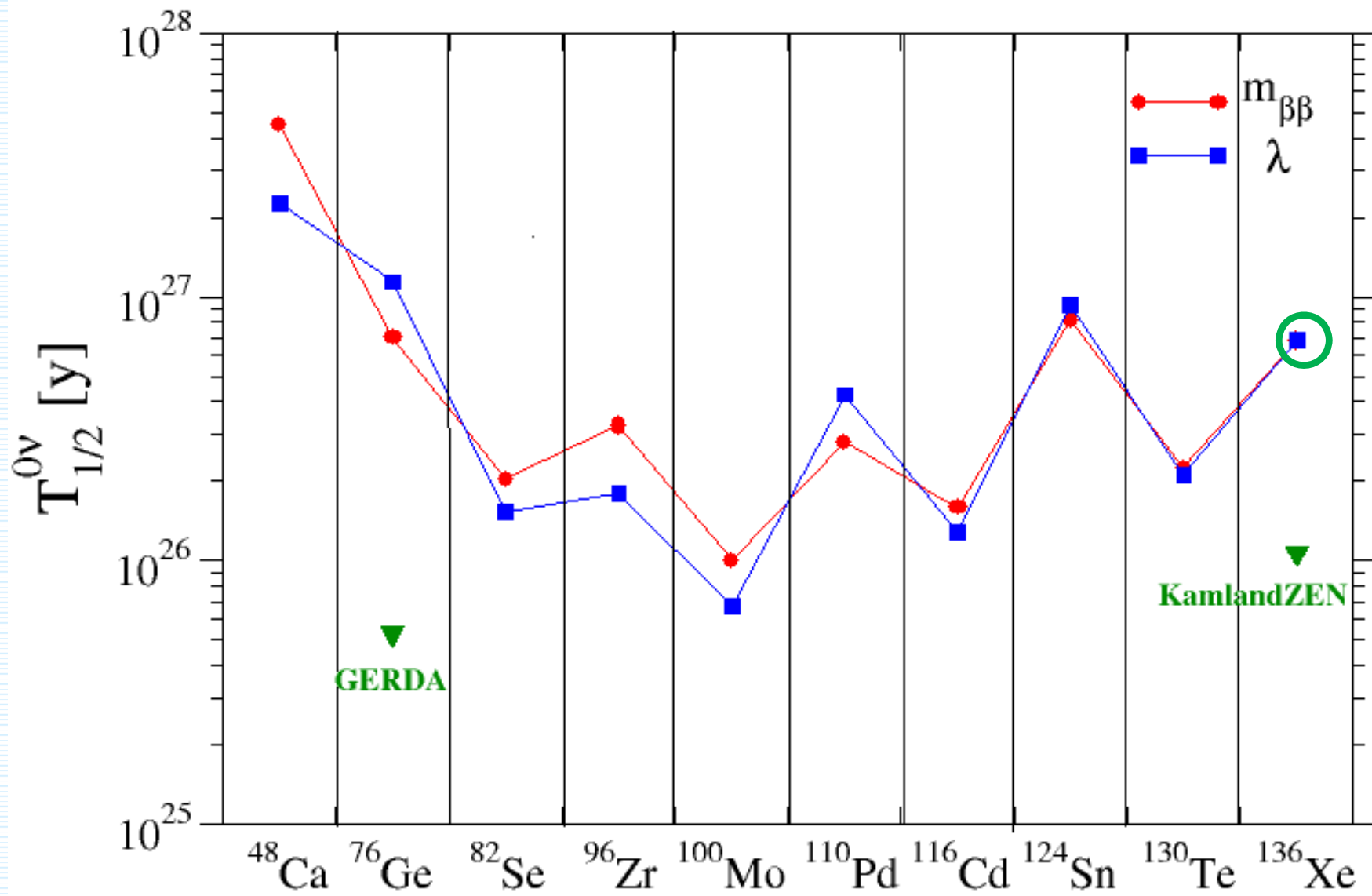
$$\begin{aligned} f_{\lambda m} &= \frac{C_{\lambda\lambda}}{C_{mm}} \\ &\simeq f_{\lambda m}^G = \frac{G_{02}}{G_{01}} \end{aligned}$$



9/23/2017

Fedc

$m_{\beta\beta} = 50 \text{ meV}$ (^{136}Xe), $g_A = 1.269$, QRPA NMEs



IV. The $0\nu\beta\beta$ -decay within L-R symmetric theories (D-M mass term, see-saw, V-A and V+A int., exchange of heavy neutrinos)

J.D.Vergados, H. Ejiri, , F.Š., Int. J. Mod. Phys. E25, 1630007(2016)

$$\left(T_{1/2}^{0\nu} G^{0\nu} g_A^2\right)^{-1} = \left|\eta_\nu M_\nu^{0\nu} + \eta_N^L M_N^{0\nu}\right|^2 + \left|\eta_N^R M_N^{0\nu}\right|^2$$

$$\eta_\nu = \frac{m_{\beta\beta}}{m_e} = \sum_i ((U_0)_{ei})^2 \frac{m_i}{m_e} \quad \eta_N^L = \frac{m_p}{m_{LNV}} \sum_i (U_{ei}^{(12)})^2 \frac{m_{LNV}}{M_i} \quad \eta_\nu \gg \eta_N^L$$

$$\approx \frac{m_p}{m_{LNV}} \frac{m_D^2}{m_e m_p} \sum_i (U_0)_{ei}^2 \frac{m_i m_{LNV}}{m_D^2} \sim 1 \quad \approx \frac{m_p}{m_{LNV}} \left(\frac{m_D}{m_{LNV}}\right)^2 \sum_i \frac{m_{LNV}}{M_i} \sim 1$$

$$\eta_N^R = \frac{m_p}{m_{LNV}} \left(\frac{M_{W_1}}{M_{W_2}}\right)^2 \sum_i (U_{ei}^{22})^2 \frac{m_{LNV}}{M_i}$$

$$\approx \frac{m_p}{m_{LNV}} \left(\frac{M_{W_1}}{M_{W_2}}\right)^2 \sum_i (V_0)_{ei}^2 \frac{m_{LNV}}{M_i} \sim 1$$

**Sensitivity of 0.1 eV scale to LNV
comparable
to sensitivity of TeV scale to LNV**

$$\sum_i (U_0)_{ei}^2 \frac{m_i m_{LNV}}{m_D^2} \simeq \sum_i (V_0)_{ei}^2 \frac{m_{LNV}}{M_i}$$

$$\frac{m_D^2}{m_e m_p} M_\nu^{0\nu} \simeq \left(\frac{M_{W_1}}{M_{W_2}}\right)^2 M_N^{0\nu}$$

**η_ν and η_N^R might
be comparable, if e.g.**

9/23/2017

$m_D \approx m_e$

~ 5

$< 10^{-4}$

~ 200

27

Method	g_A	src	$M_\nu^{0\nu}$					
			^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{110}Pd
ISM-StMa	1.25	UCOM	0.85	2.81	2.64			
ISM-CMU	1.27	Argonne	0.80	3.37	3.19			
		CD-Bonn	0.88	3.57	3.39			
IBM	1.27	Argonne	1.75	4.68	3.73	2.83	4.22	4.05
QRPA-TBC	1.27	Argonne	0.54	5.16	4.64	2.72	5.40	5.76
		CD-Bonn	0.59	5.57	5.02	2.96	5.85	6.26
QRPA-Jy	1.26	CD-Bonn		5.26	3.73	3.14	3.90	6.52
dQRPA-NC	1.25	without		5.09				
PHFB	1.25	Argonne				2.84	5.82	7.12
		CD-Bonn				2.98	6.07	7.42
NREDF	1.25	UCOM	2.37	4.60	4.22	5.65	5.08	
REDF	1.25	without	2.94	6.13	5.40	6.47	6.58	
Mean value			1.34	4.55	4.02	3.78	5.57	6.12
variance			0.81	1.20	0.91	2.49	0.58	1.78

Method	g_A	src	$M_\nu^{0\nu}$					
			^{116}Cd	^{124}Sn	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd
ISM-StMa	1.25	UCOM		2.62		2.65	2.19	
ISM-CMU	1.27	Argonne		2.00		1.79	1.63	
		CD-Bonn		2.15		1.93	1.76	
IBM	1.27	Argonne	3.10	3.19	4.10	3.70	3.05	2.67
QRPA-TBC	1.27	Argonne	4.04	2.56	4.56	3.89	2.18	
		CD-Bonn	4.34	2.91	5.08	4.37	2.46	3.37
QRPA-Jy	1.26	CD-Bonn	4.26	5.30	4.92	4.00	2.91	
dQRPA-NC	1.25	without				1.37	1.55	2.71
PHFB	1.27	Argonne			3.90	3.81		2.58
		CD-Bonn			4.08	3.98		2.68
NREDF	1.25	UCOM	4.72	4.81	4.11	5.13	4.20	1.71
REDF	1.25	without	5.52	4.33		4.98	4.32	5.60
Mean value			4.34	3.07	4.34	3.42	2.59	3.01
variance			0.79	1.01	0.23	1.67	1.10	1.34

**NMEs for
unquenched value
of g_A**

**Mean field approaches
(PHFB, NREDF, REDF)
⇒ Large NMEs**

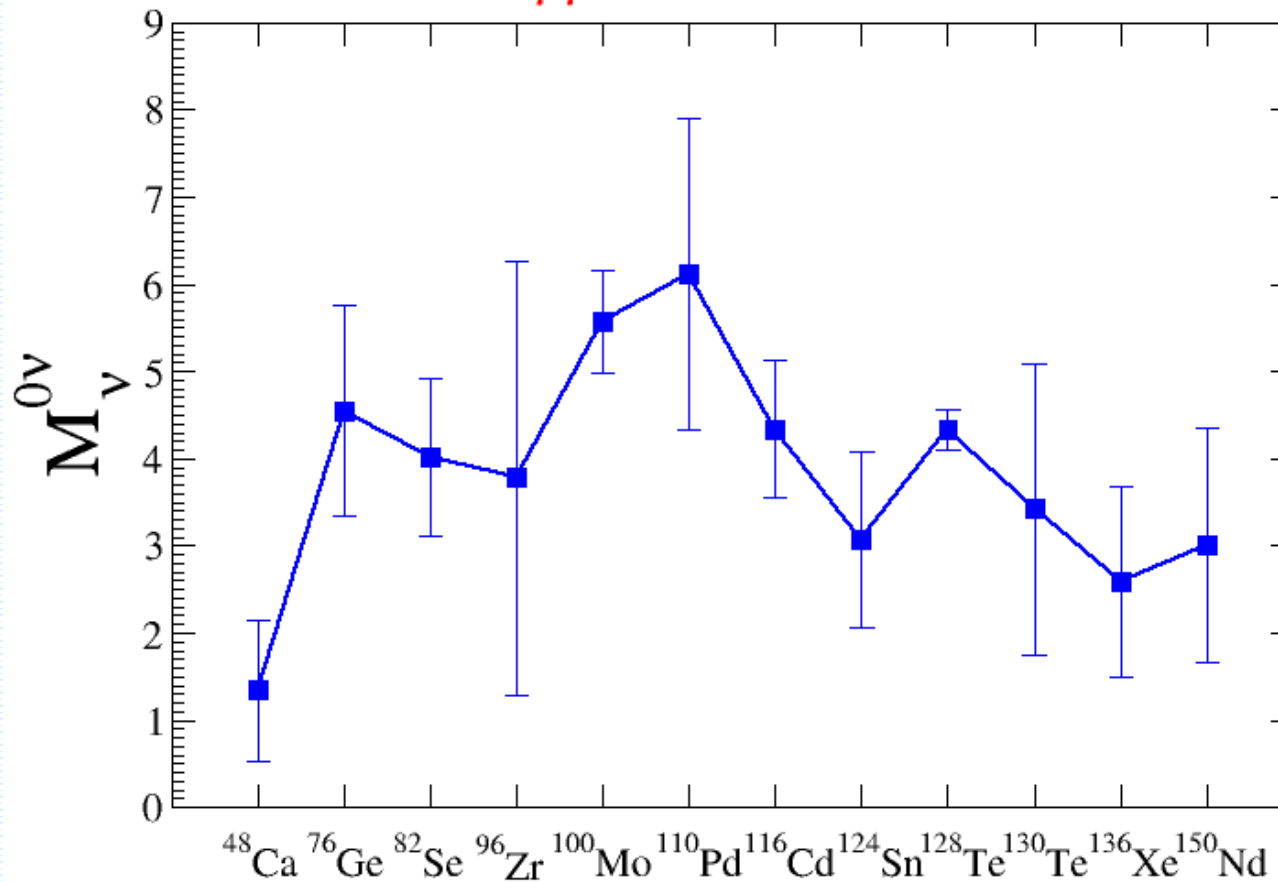
**Interacting Shell Model
(ISM-StMa, ISM-CMU)
⇒ small NMEs**

**Quasiparticle Random
Phase Approximation
(QRPA-TBC, QRPA-Jy,
dQRPA-NC)
⇒ Intermediate NMEs**

**Interacting Boson Model
(IBM)
⇒ Close to QRPA results**

unquenched g_A

$0\nu\beta\beta$ NMEs -status 2017



J.D.Vergados, H. Ejiri, F.Š., Int. J. Mod. Phys. E25, 1630007(2016)

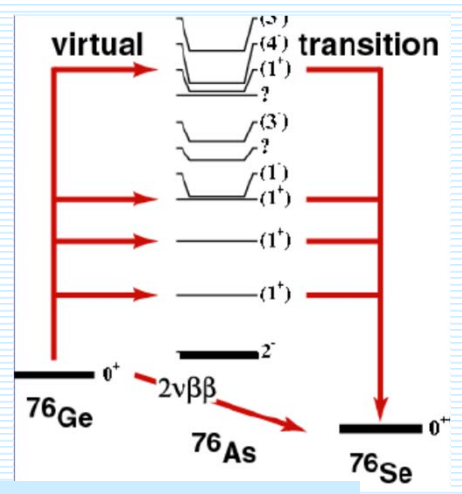
	mean field meth.	ISM	IBM	QRPA
Large model space	yes	no	yes	yes
Constr. Interm. States	no	yes	no	yes
Nucl. Correlations	limited	all	restricted	restricted

$g_A^4 = (1.269)^4 = 2.6$ **Quenching of g_A** (from exp.: $T_{1/2}^{0\nu}$ up 2.5 x larger)

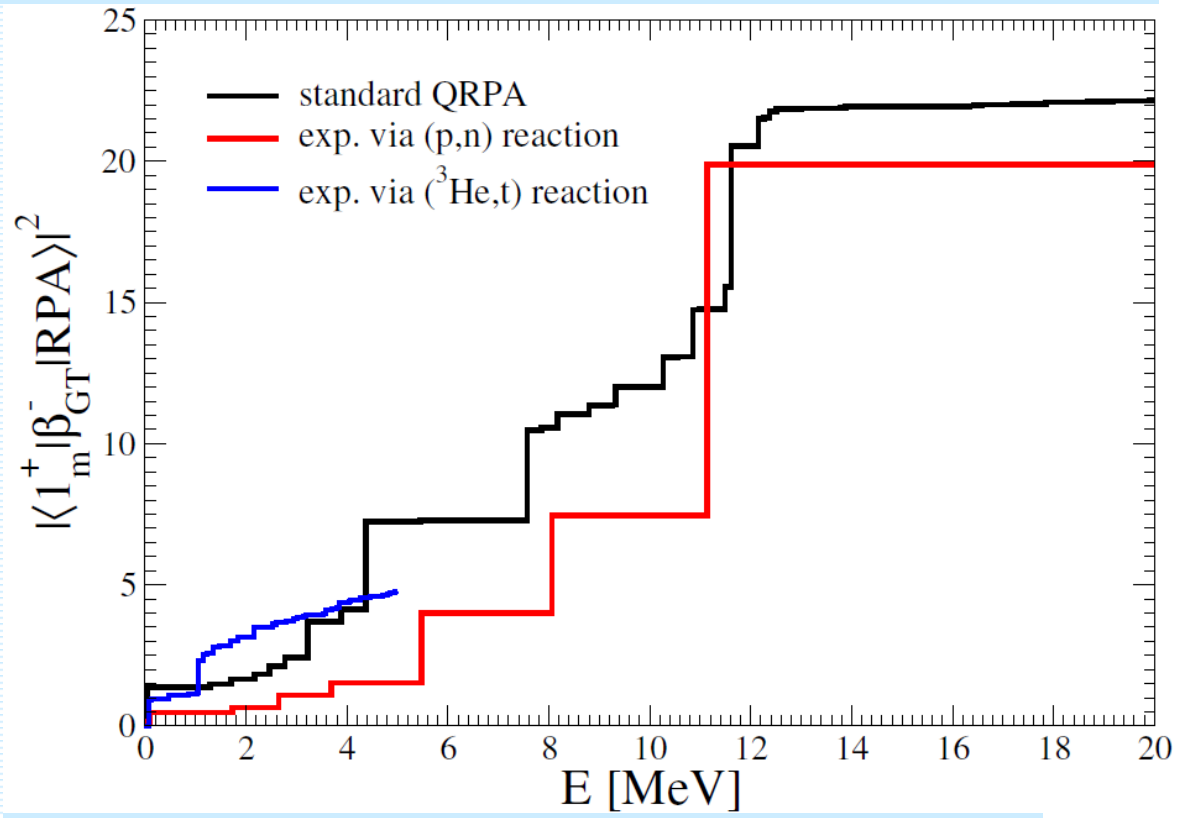
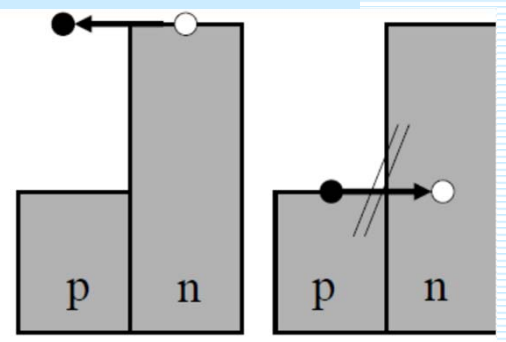
$(g_A^{\text{eff}})^4 = 1.0$

Strength of GT trans. (approx. given by Ikeda sum rule = $3(N-Z)$) has to be quenched to reproduce experiment

${}^{76}_{32}\text{Ge}_{44} \Rightarrow$
 $S_{\beta^-} - S_{\beta^+} = 3(N-Z) = 36$



Pauli blocking



Cross-section for charge exchange reaction:

$$\left[\frac{d\sigma}{d\Omega} \right] = \left[\frac{\mu}{\pi\hbar} \right]^2 \frac{k_f}{k_i} N_d |v_{\sigma\tau}|^2 |\langle f | \sigma\tau | i \rangle|^2$$

$q = 0!!$

largest at 100 - 200 MeV/A

Quenching of g_A (from theory: $T_{1/2}^{0\nu}$ up 50 x larger)

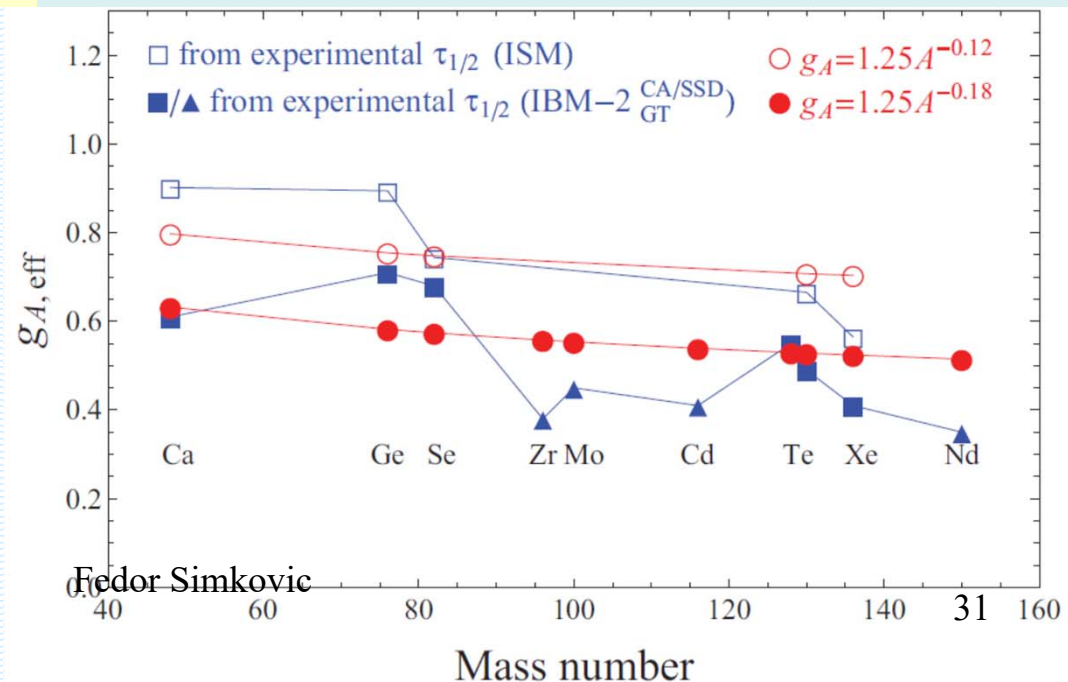
$(g_A^{\text{eff}})^4 \simeq 0.66$ (^{48}Ca), 0.66 (^{76}Ge), 0.30 (^{76}Se), 0.20 (^{130}Te) and 0.11 (^{136}Xe)

The Interacting Shell Model (ISM), which describes qualitatively well energy spectra, does reproduce experimental values of $M^{2\nu}$ only by consideration of significant quenching of the Gamow-Teller operator, typically by **0.45 to 70%**.

$(g_A^{\text{eff}})^4 \simeq (1.269 A^{-0.18})^4 = 0.063$ (**The Interacting Boson Model**). This is an incredible result. The quenching of the axial-vector coupling within the IBM-2 is more like **60%**.

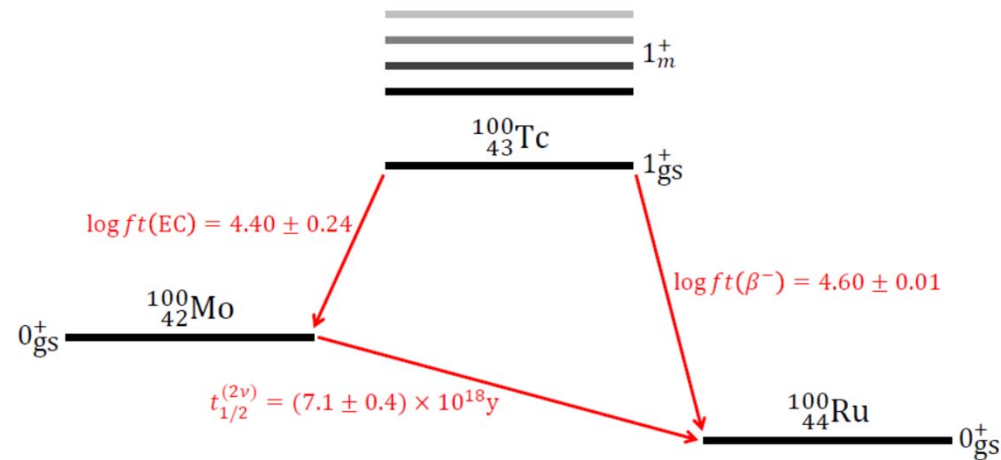
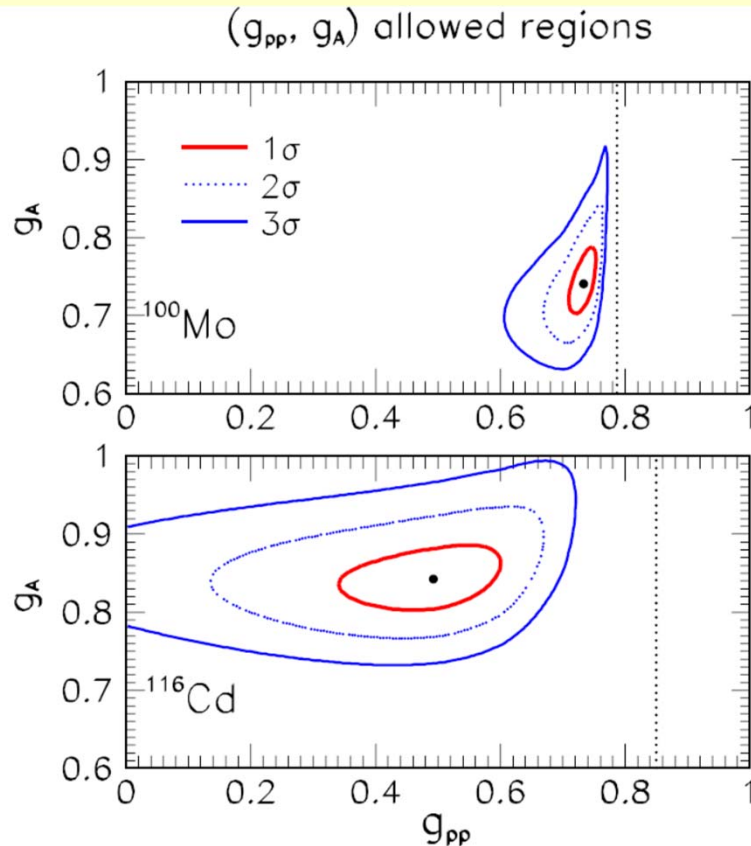
J. Barea, J. Kotila, F. Iachello, PRC 87, 014315 (2013).

It has been determined by theoretical prediction for the $2\nu\beta\beta$ -decay half-lives, which were based on within **closure approximation** calculated corresponding NMEs, with the measured half-lives.



Faessler, Fogli, Lisi, Rodin, Rotunno, F. Š, J. Phys. G 35, 075104 (2008).

$(g_A^{\text{eff}})^4 = 0.30$ and 0.50 for ^{100}Mo and ^{116}Cd , respectively (**The QRPA prediction**). g_A^{eff} was treated as a completely free parameter alongside g_{pp} (used to renormalize particle-particle interaction) by performing calculations within the QRPA and RQRPA. It was found that a least-squares fit of g_A^{eff} and g_{pp} , where possible, to the **β -decay rate** and **β +/**EC rate**** of the $J = 1^+$ ground state in the intermediate nuclei involved in double-beta decay in addition to the **$2\nu\beta\beta$ rates** of the initial nuclei, leads to an effective g_A^{eff} of about **0.7** or **0.8**.



Extended calculation also for neighbor isotopes performed by

F.F. Depisch and J. Suhonen, PRC 94, 055501 (2016)

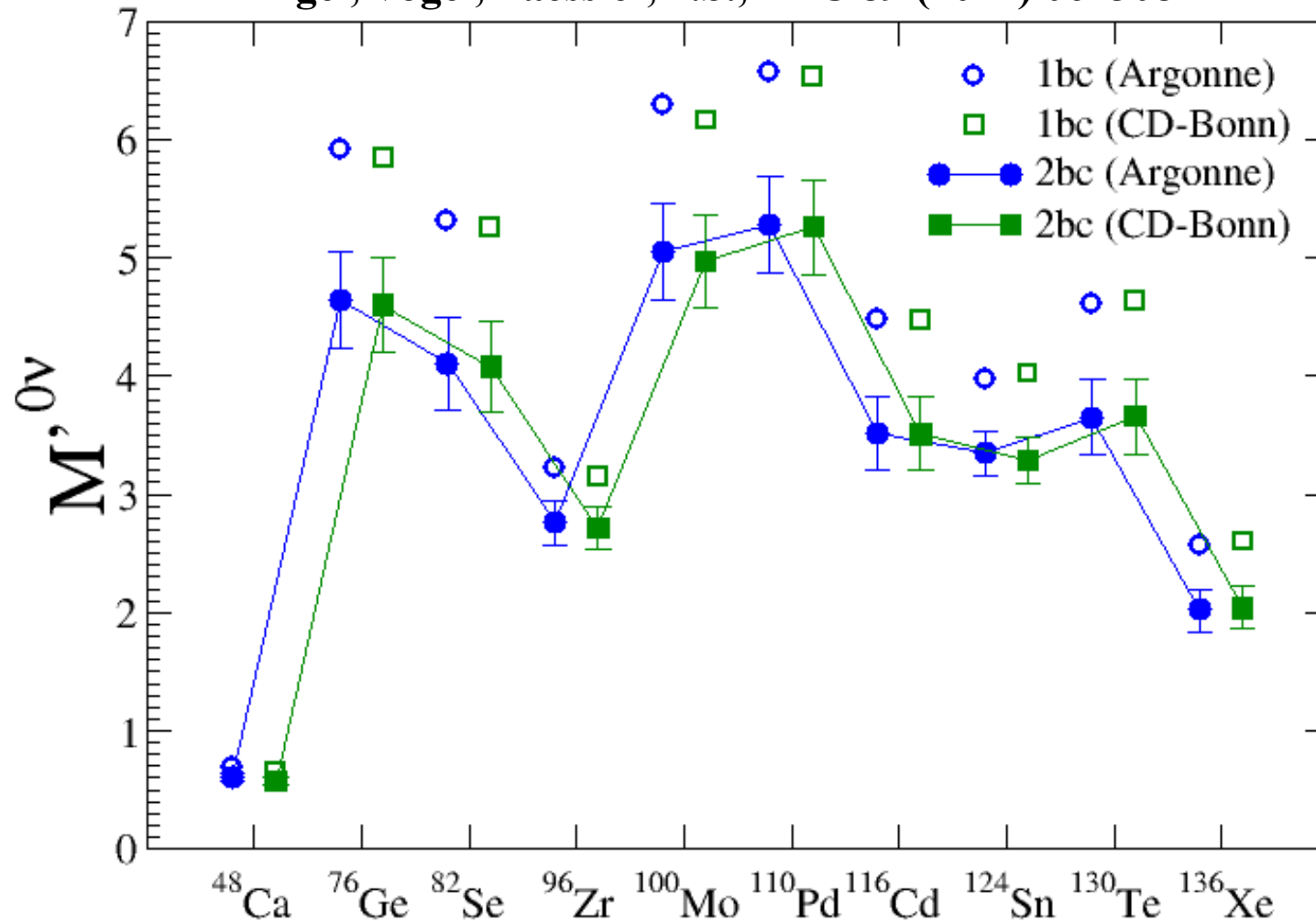
r Simkovic

Dependence of g_A^{eff} on A was not established.

Quenching of g_A , two-body currents and QRPA

(Suppression of only about 20% due to momentum dependence of 2bc)

Engel, Vogel, Faessler, F.Š., PRC 89 (2014) 064308



The $0\nu\beta\beta$ operator calculated within effective field theory. Corrections appear as 2-body current predicted by EFT. The 2-body current contributions are related to the quenching of Gamow-Teller transitions found in nuclear structure calc.

Improved description of the $0\nu\beta\beta$ -decay rate

F. Š., R. Dvornický, D. Štefánik, A. Faessler, in preparation

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \epsilon_{K,L}^2}$$

Let perform Taylor expansion

$$\frac{\epsilon_{K,L}}{E_n - (E_i + E_f)/2} \quad \epsilon_{K,L} \in \left(-\frac{Q}{2}, \frac{Q}{2}\right)$$

We get

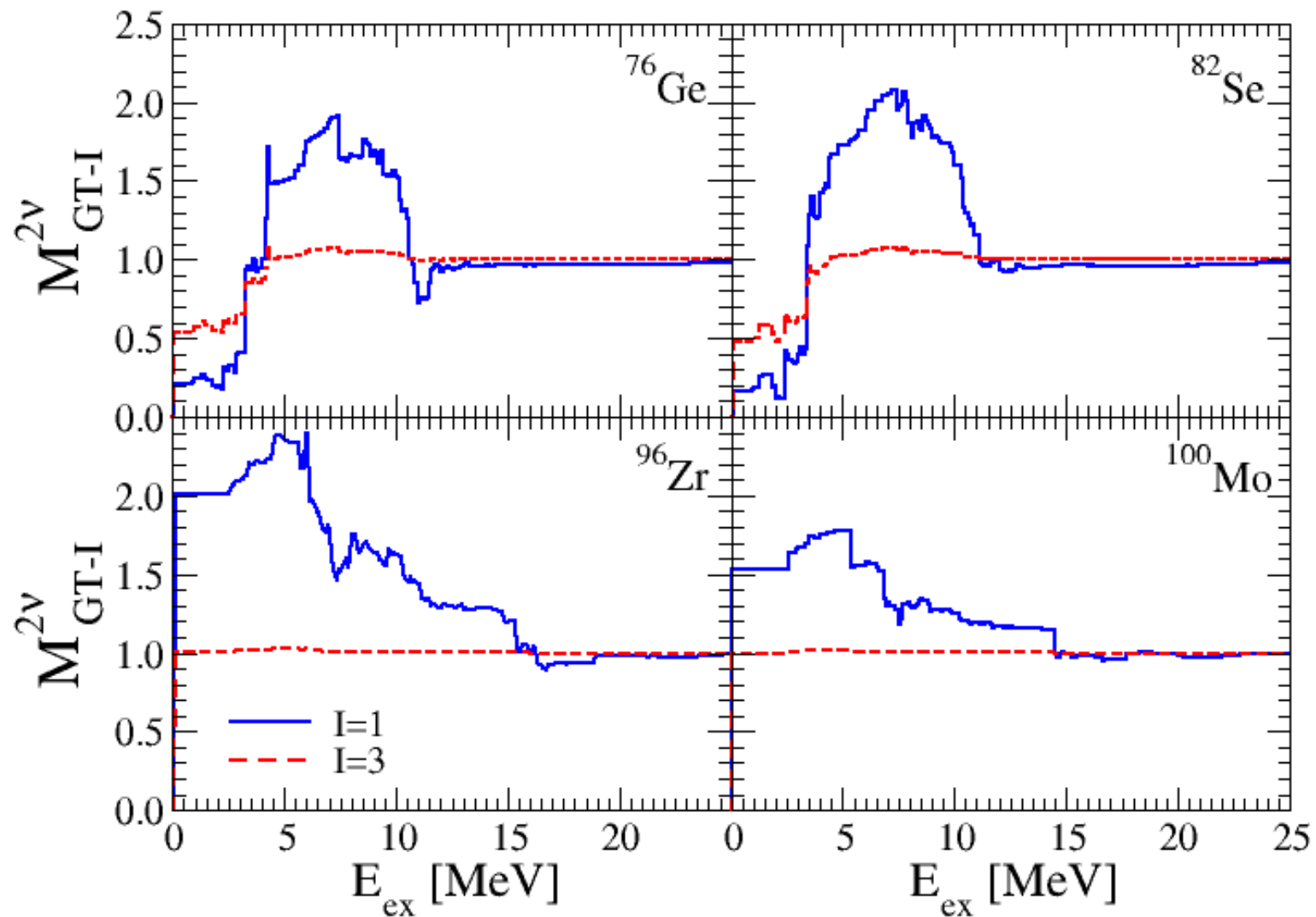
$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \simeq \left(g_A^{\text{eff}}\right)^4 \left|M_{GT-3}^{2\nu}\right|^2 \frac{1}{|\xi_{13}^{2\nu}|^2} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu}\right)$$

$$M_{GT-1}^{2\nu} \equiv M_{GT}^{2\nu}$$

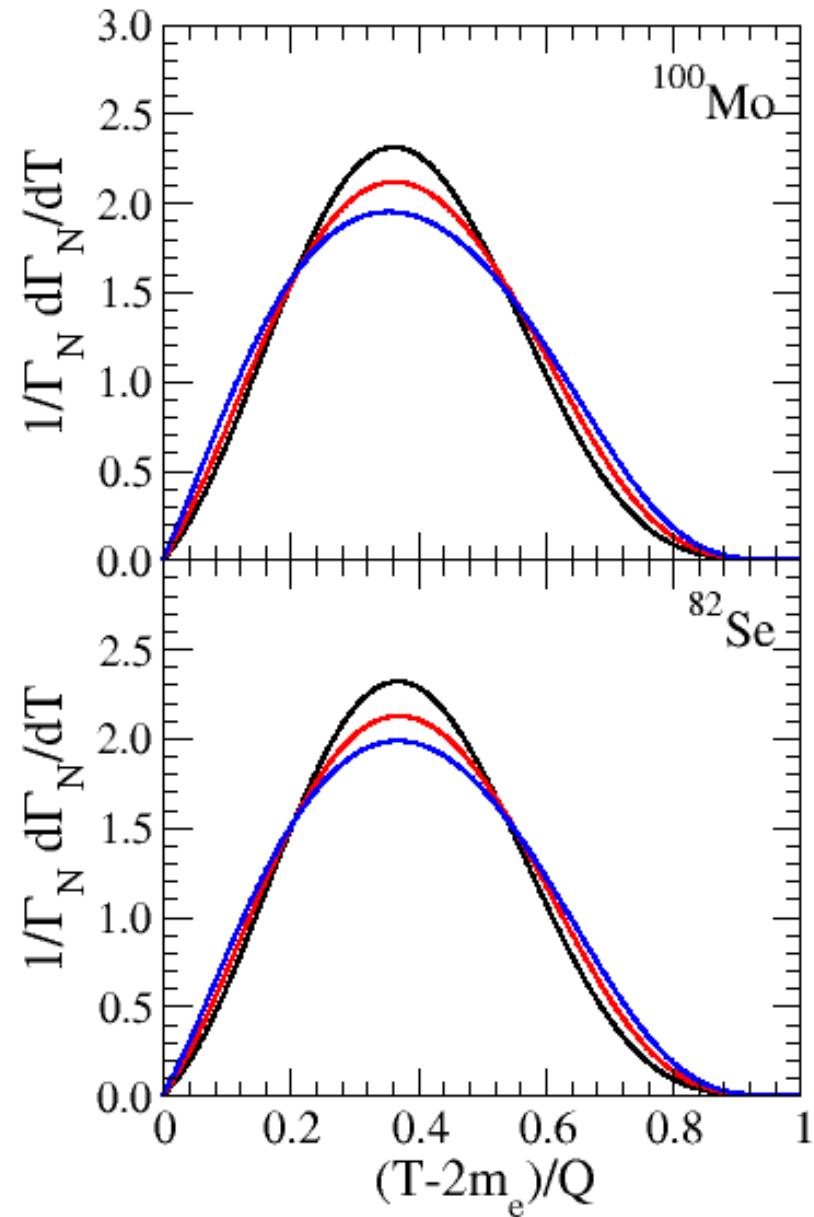
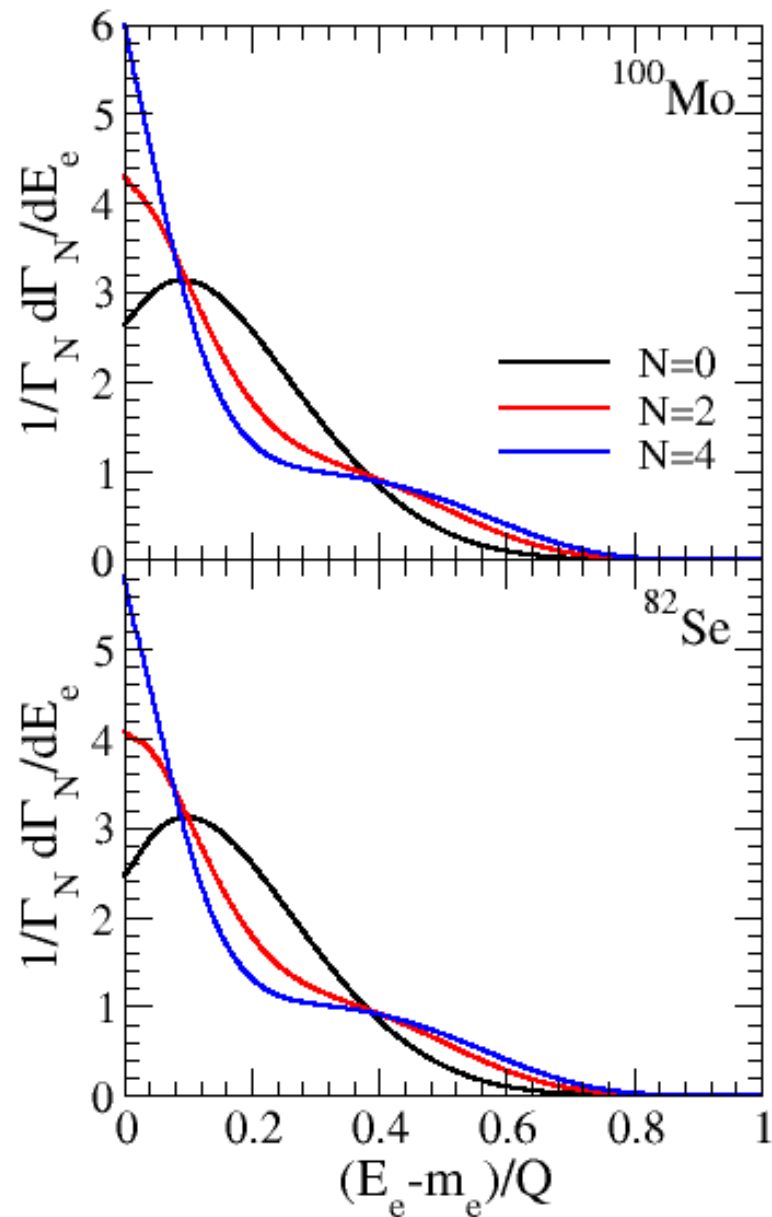
$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4 m_e^3}{(E_n - (E_i + E_f)/2)^3} \quad \xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

The g_A^{eff} can be determined with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM?)

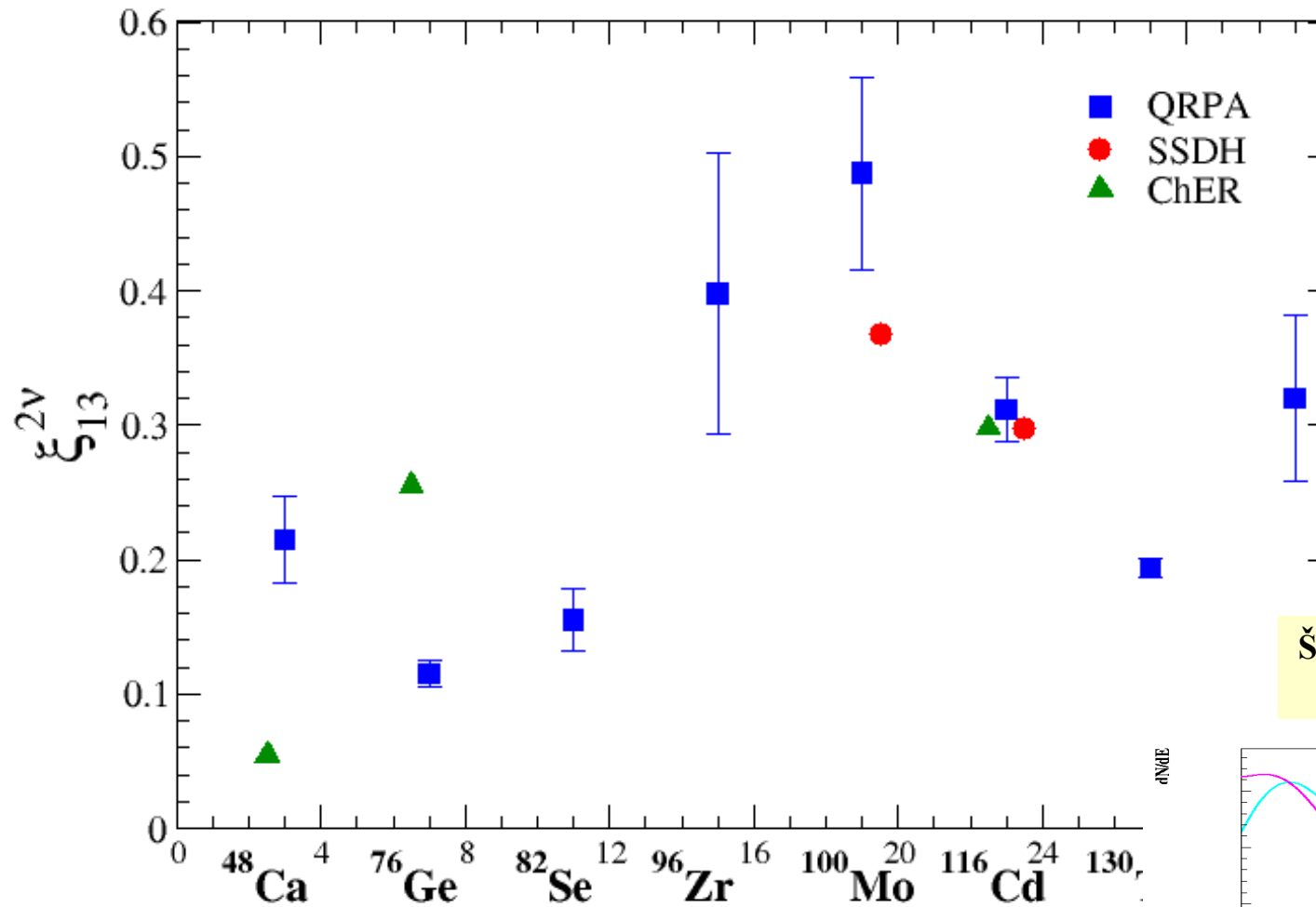
The running sum of the $2\nu\beta\beta$ -decay NMEs



Normalized to unity different partial energy distributions



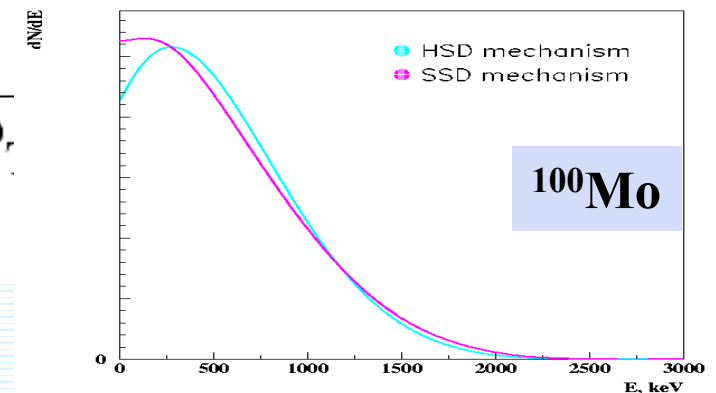
ξ_{13}^r tell us about importance of higher lying states of int. nucl.



HSD: $\xi_{13}=0$

Šimkovic, Šmotlák, Semenov
J. Phys. G, 27, 2233, 2001

ξ_{13}^r can be determined phenomenologically from the shape of energy distributions of emitted electrons

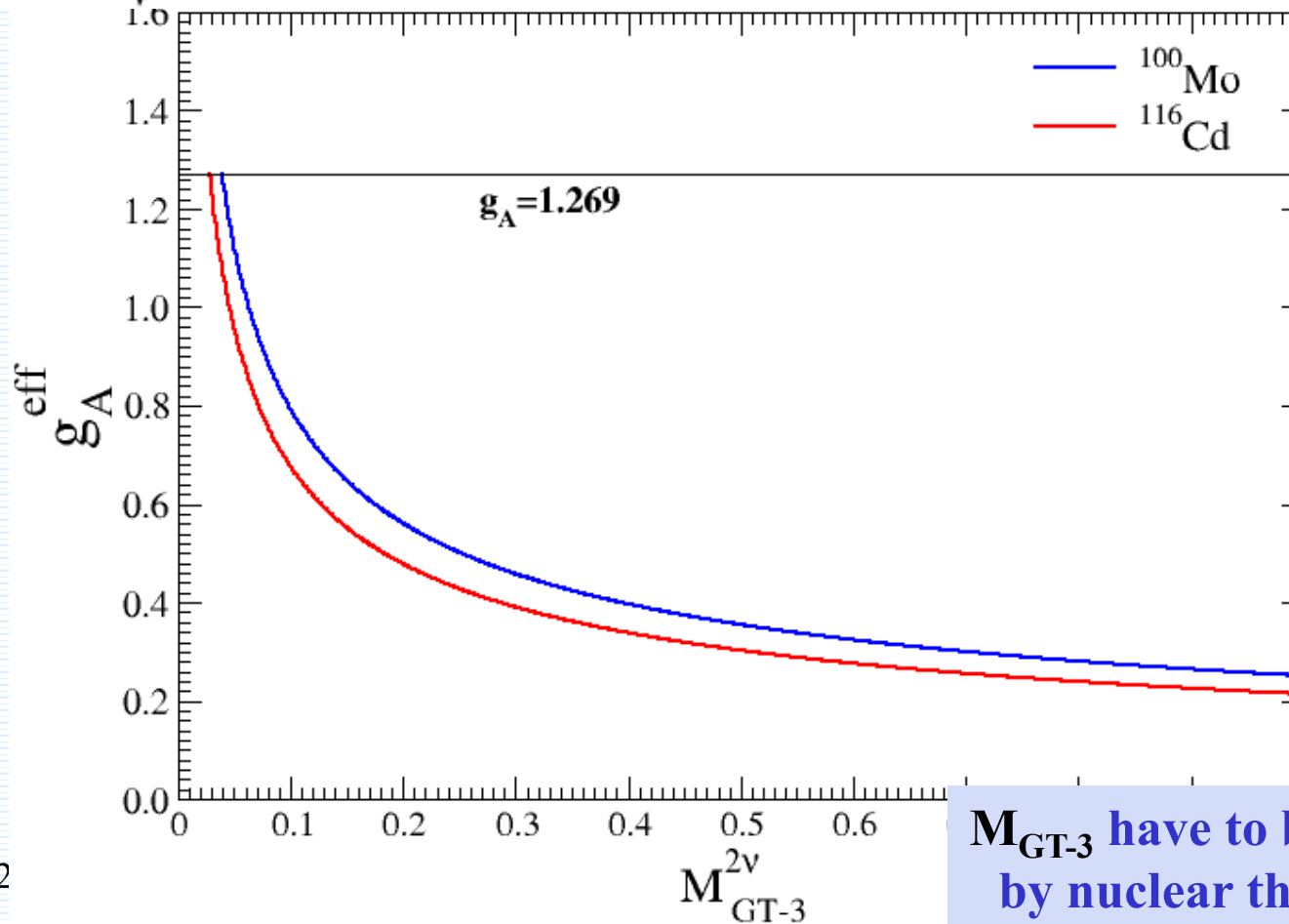


Solution: NEMO3/Supernemo measurement of ξ and calculation of M_{GT-3}

$$(g_A^{\text{eff}})^2 = \frac{1}{|M_{GT-3}^{2\nu}|} \frac{|\xi_{S13}^{2\nu}|}{\sqrt{T_{1/2}^{2\nu-\text{exp}} (G_0^{2\nu} + \xi_{S13}^{2\nu} G_2^{2\nu})}}$$

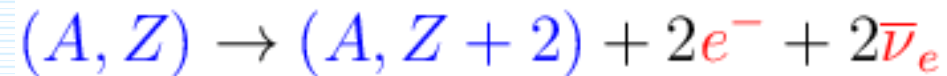
$$g_A^{\text{eff}}(^{100}\text{Mo}) = \frac{0.251}{\sqrt{M_{GT-3}^{2\nu}}}$$

$$g_A^{\text{eff}}(^{100}\text{Cd}) = \frac{0.214}{\sqrt{M_{GT-3}^{2\nu}}}$$



M_{GT-3} have to be calculated by nuclear theory - ISM

Understanding of the $2\nu\beta\beta$ -decay NMEs is of crucial importance for correct evaluation of the $2\nu\beta\beta$ -decay NMEs



*Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states.
Both change two neutrons into two protons.*

Explaining $2\nu\beta\beta$ -decay is necessary but not sufficient

There is no reliable calculation of the $2\nu\beta\beta$ -decay NMEs

**Calculation via intermediate nuclear states: QRPA (sensitivity to pp-int.)
ISM (quenching, truncation of model space, spin-orbit partners)**

Calculation via closure NME: IBM, PHFB

No calculation: EDF

The DBD Nuclear Matrix Elements and the SU(4) symmetry

D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

Suppression of the Two Neutrino Double Beta Decay by Nuclear Structure Effects

P. Vogel, M.R. Zirnbauer, PRL (1986) 3148

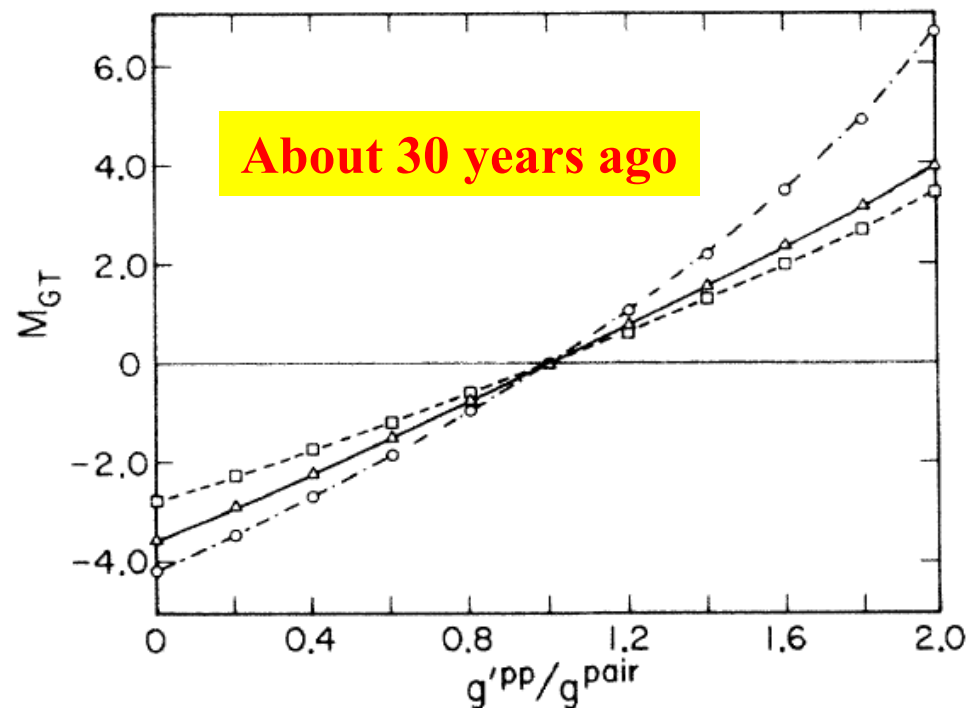
O. Civitarese, A. Faessler, T. Tomoda,
PLB 194 (1987) 11

E. Bender, K. Muto, H.V. Klapdor,
PLB 208 (1988) 53

...

The isospin is known to be a
good approximation in nuclei

In heavy nuclei the SU(4) symmetry
is strongly broken
by the spin-orbit splitting.



What is beyond this behavior? Is it an artifact of the QRPA?

s.p. mean-field

Conserves SU(4) symmetry

$$H = \underbrace{e_n N_n + e_p N_p - g_{pair} \left(\sum_{M_T=-1,0,1} A_{0,1}^\dagger(0, M_T) A_{0,1}(0, M_T) + \sum_{M_S=-1,0,1} A_{1,0}^\dagger(M_S, 0) A_{1,0}(M_S, 0) \right)}_{H_0} + g_{ph} \sum_{a,b} E_{a,b}^\dagger E_{a,b}$$

$$+ \underbrace{(g_{pair} - g_{pp}^{T=0}) \sum_{M_S=-1,0,1} A_{1,0}^\dagger(M_S, 0) A_{1,0}(M_S, 0) + (g_{pair} - g_{pp}^{T=1}) A_{0,1}^\dagger(0, 0) A_{0,1}(0, 0)}_{H_I}.$$

H_I violates SU(4) symmetry

g_{pair} - strength of isovector like nucleon pairing (L=0, S=0, T=1, M_T=±1)

$g_{pp}^{T=1}$ - strength of isovector spin-0 pairing (L=0, S=0, T=1, M_T=0)

$g_{pp}^{T=0}$ - strength of isoscalar spin-1 pairing (L=0, S=1, T=0)

g_{ph} - strength of particle-hole force

M_F and M_{GT} do not depend on the mean-field part of **H** and are governed by a weak violation of the **SU(4)** symmetry by the particle-particle interaction of **H**

$$M_F^{2\nu} = - \frac{48 \sqrt{\frac{33}{5}} (g_{pair} - g_{pp}^{T=1})}{(5g_{pair} + 3g_{ph})(10g_{pair} + 6g_{ph})}$$

$$M_{GT}^{2\nu} = \frac{144 \sqrt{\frac{33}{5}}}{5g_{pair} + 9g_{ph}} \left\{ \frac{(g_{pair} - g_{pp}^{T=0})}{(10g_{pair} + 20g_{ph})} + \frac{2g_{ph}(g_{pair} - g_{pp}^{T=1})}{(10g_{pair} + 20g_{ph})(10g_{pair} + 6g_{ph})} \right\}$$

Reproduction of exact solutions of Lipkin model by nonlinear higher random-phase approximation

J. Terasaki, A. Smetana, F. Š., M.I. Krivoruchenko, arXiv:1701.08368 [nucl-th]

Useful for test of theory often used.

H.J. Lipkin et al., N.P. **62**, 188 (1965)

Hamiltonian

$$H = \varepsilon J_z + \frac{V}{2} (J_+^2 + J_-^2)$$

The nonlinear phonon operator

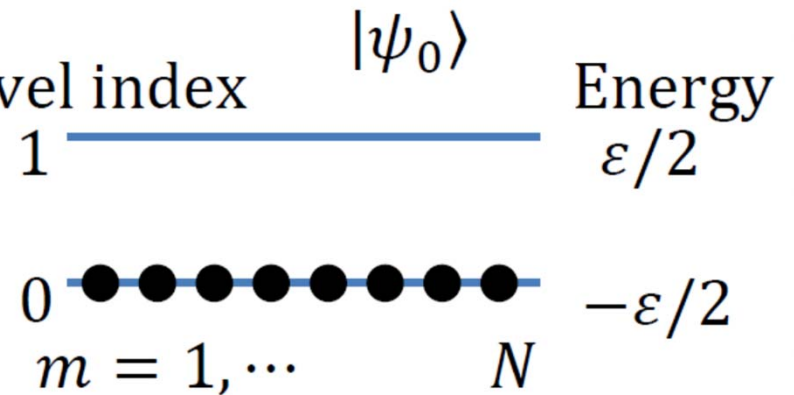
$$Q_k^{ot} = \sum_{l=1}^n (X_{2l-1}^k \mathcal{J}_+^{2l-1} + Y_{2l-1}^k \mathcal{J}_-^{2l-1}),$$

(odd-order subspace)

$$Q_k^{et} = c_k + \sum_{l=1}^n (X_{2l}^k \mathcal{J}_+^{2l} + Y_{2l}^k \mathcal{J}_-^{2l}),$$

(even-order subspace)

Lipkin model



Algebra

$$\begin{aligned} [J_z, J_+] &= J_+ \\ [J_z, J_-] &= -J_- \\ [J_+, J_-] &= 2J_z \end{aligned}$$

RPA ground state

$$Q_k |\Psi_0\rangle = 0 \quad 42$$

Eigen states, wave functions, total energies, excitation energies and phonon-creation operators obtained for $N=2$ by **the nonlinear higher RPA.**

Eigenstate	Wave function	Total energy
Ground	$ \Psi_0\rangle = \frac{V}{\sqrt{2E_{10}^o(E_{10}^o-\varepsilon)}} \left(1 - \frac{E_{10}^o-\varepsilon}{2V} J_+^2\right) \psi_0\rangle$	$-E_{10}^o$
Odd-order excited	$Q_1^{o\dagger} \Psi_0\rangle = \frac{1}{\sqrt{2}} J_+ \psi_0\rangle$	0
Even-order excited	$Q_1^{e\dagger} \Psi_0\rangle = \frac{V}{\sqrt{2E_{10}^o(E_{10}^o+\varepsilon)}} \left(1 + \frac{E_{10}^o+\varepsilon}{2V} J_+^2\right) \psi_0\rangle$	E_{10}^o

Eigenstate	Excitation energy	Phonon-creation operator
Ground	0	
Odd-order excited	$E_{10}^o = \sqrt{\varepsilon^2 + V^2}$	$Q_1^{o\dagger} = \frac{\sqrt{E_{10}^o}}{2\varepsilon} \left(\frac{V}{ V } \sqrt{E_{10}^o + \varepsilon} J_+ + \sqrt{E_{10}^o - \varepsilon} J_- \right)$
Even-order excited	$E_{10}^e = 2E_{10}^o$	$Q_1^{e\dagger} = \frac{V}{ V } \left(\frac{V}{2\varepsilon} + \frac{E_{10}^o+\varepsilon}{4\varepsilon} J_+^2 + \frac{E_{10}^o-\varepsilon}{4\varepsilon} J_-^2 \right)$

RPA
equation

$$\begin{pmatrix} A^o & B^o \\ B^o & A^o \end{pmatrix} \begin{pmatrix} X_k^o \\ Y_k^o \end{pmatrix} = E_{k0}^o \begin{pmatrix} U^o & O \\ O & -U^o \end{pmatrix} \begin{pmatrix} X_k^o \\ Y_k^o \end{pmatrix}$$

$$A_{ij}^o = \langle \Psi_0 | [\mathcal{J}_-^{2i-1}, H, \mathcal{J}_+^{2j-1}] | \Psi_0 \rangle$$

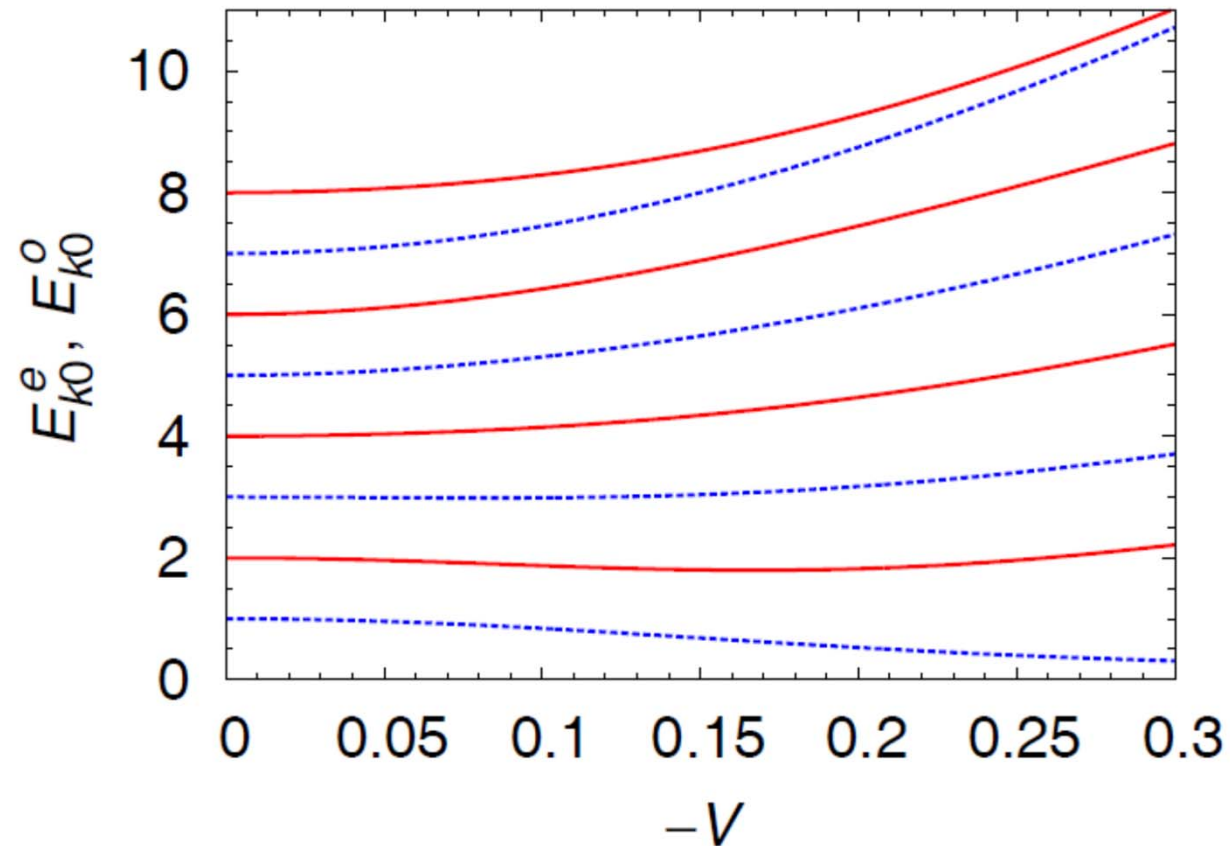
$$B_{ij}^o = \langle \Psi_0 | [\mathcal{J}_-^{2i-1}, H, \mathcal{J}_-^{2j-1}] | \Psi_0 \rangle$$

$$U_{ij}^o = \langle \Psi_0 | [\mathcal{J}_-^{2i-1}, \mathcal{J}_+^{2j-1}] | \Psi_0 \rangle$$

$$[A, B, C] = (1/2)[[A, B], C] + (1/2)[A, [B, C]]$$

N=8, $\epsilon=1$
Breaking point
of RPA
is $V=-0.143$

Exact
agreement
of RPA results
with those
obtained by
diagonalization
of H

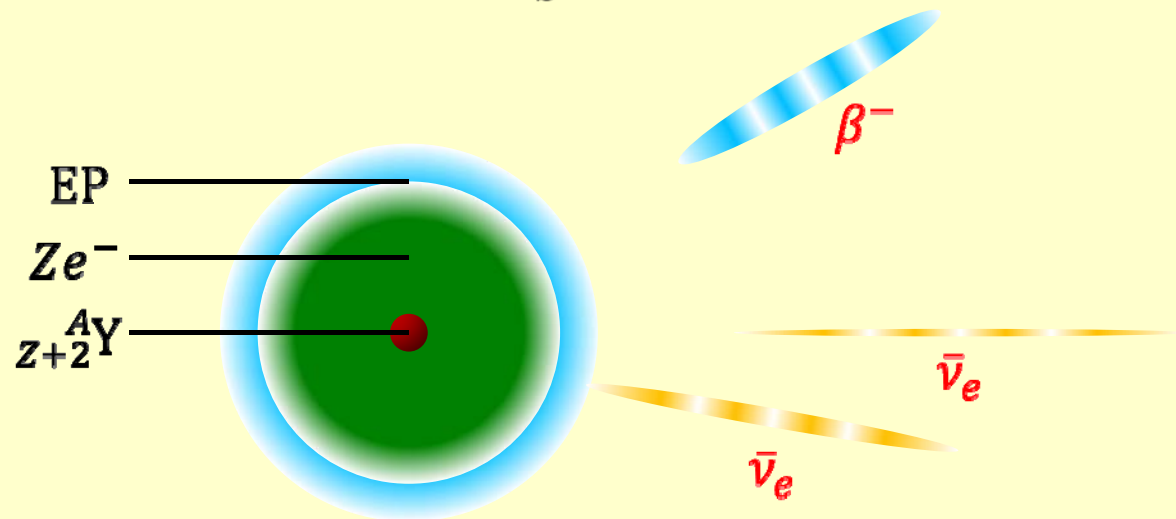
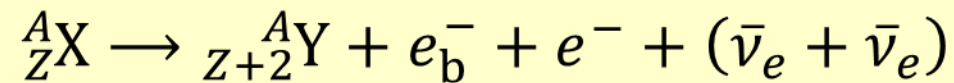


Double Beta Decay with emission of a single electron

A. Babič, M.I. Krivoruchenko, F.Š., to be submitted to PRC

[Jung *et al.* (GSI), 1992] observed beta decay of $^{163}_{66}\text{Dy}^{66+}$ ions with Electron Production (EP) in K or L shells: $T_{1/2}^{\text{EP}} = 47$ d

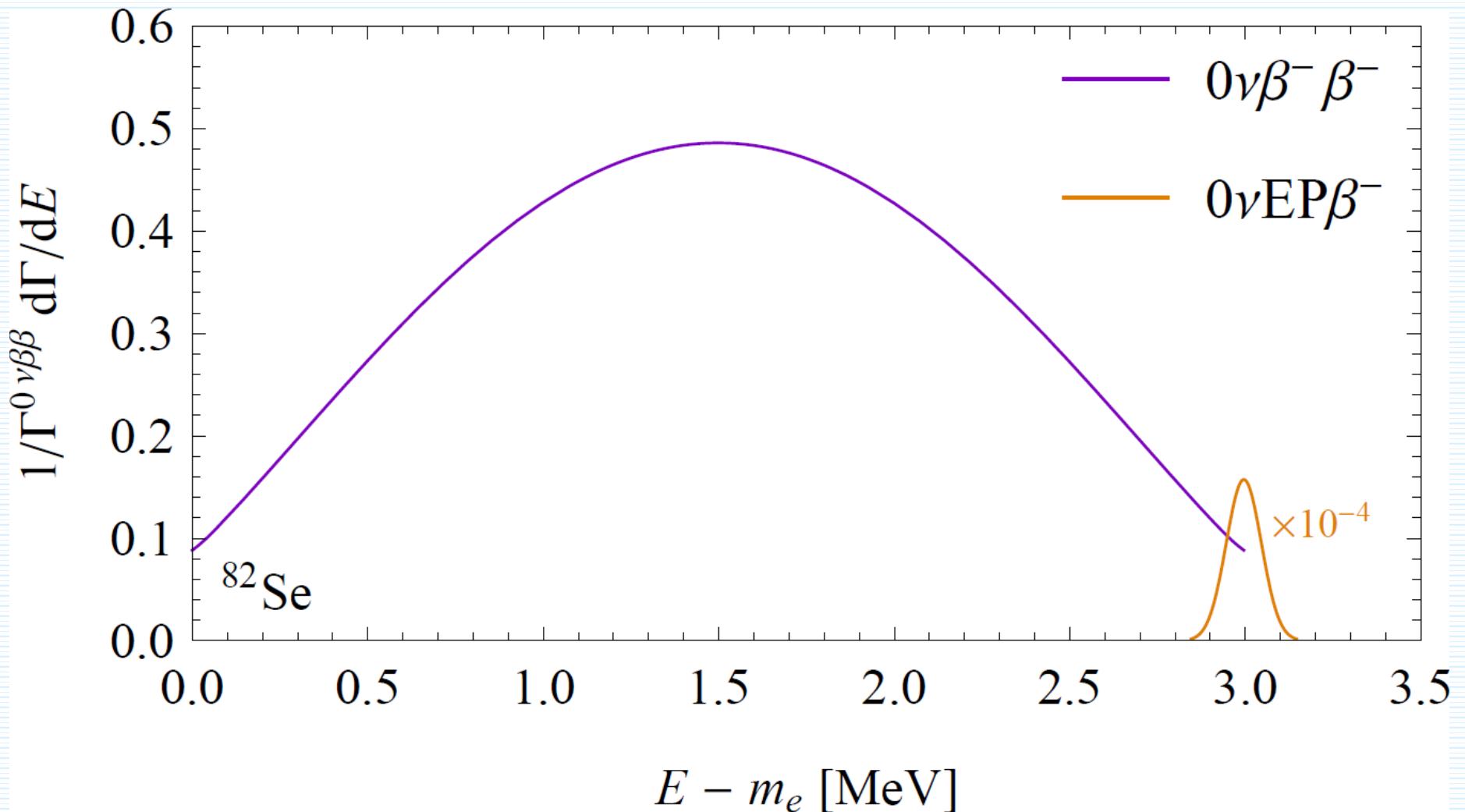
Bound-state double-beta decay $0\nu\text{EP}\beta^-$ ($2\nu\text{EP}\beta^-$) with EP in available $s_{1/2}$ or $p_{1/2}$ subshell of daughter $2+$ ion:



Search for possible manifestation in single-electron spectra...

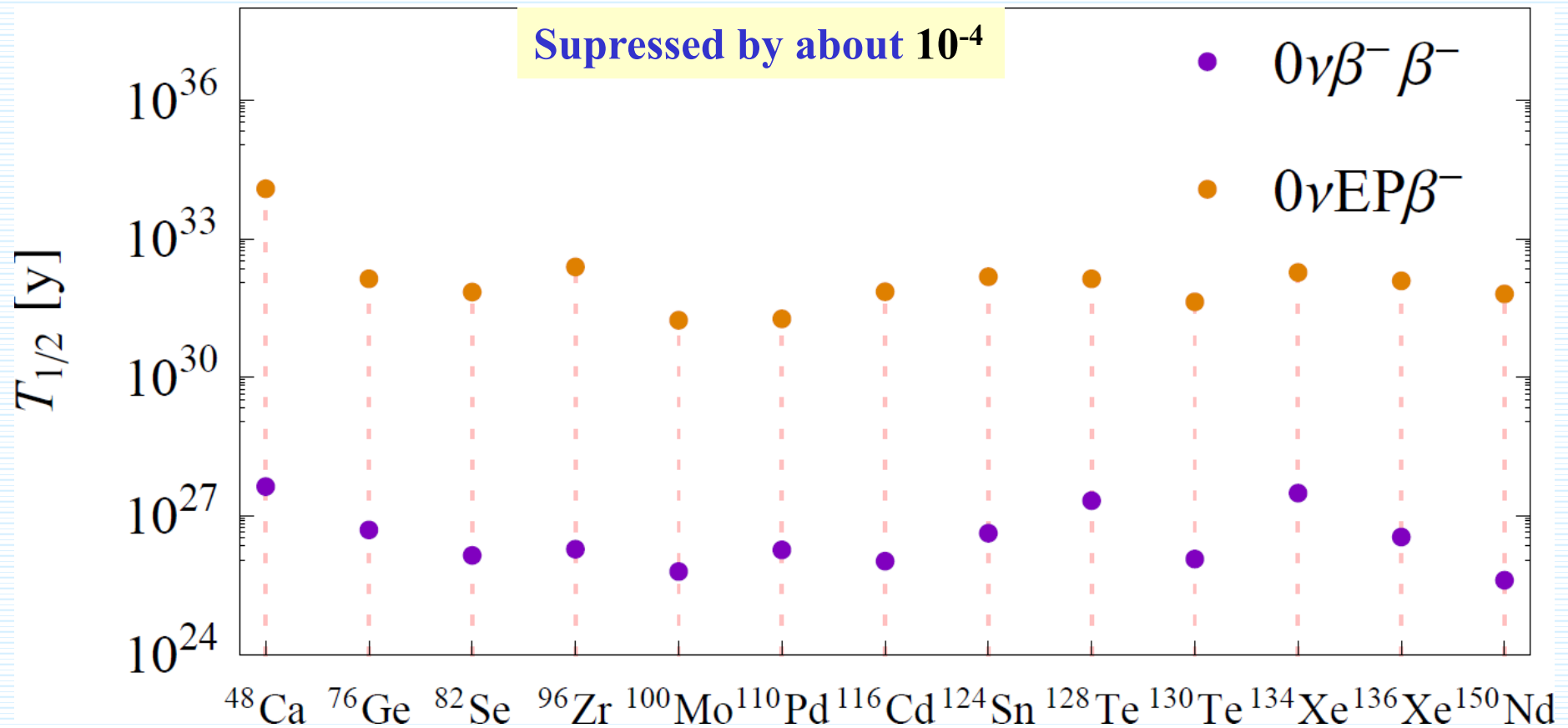
$0\nu\text{EP}\beta^-$ Single-Electron Spectrum (^{82}Se)

$0\nu\beta^-\beta^-$ and $0\nu\text{EP}\beta^-$ single-electron spectra $1/\Gamma^{0\nu\beta\beta} d\Gamma/dE$ vs. electron kinetic energy $E - m_e$ for ^{82}Se ($Q = 2.996$ MeV)



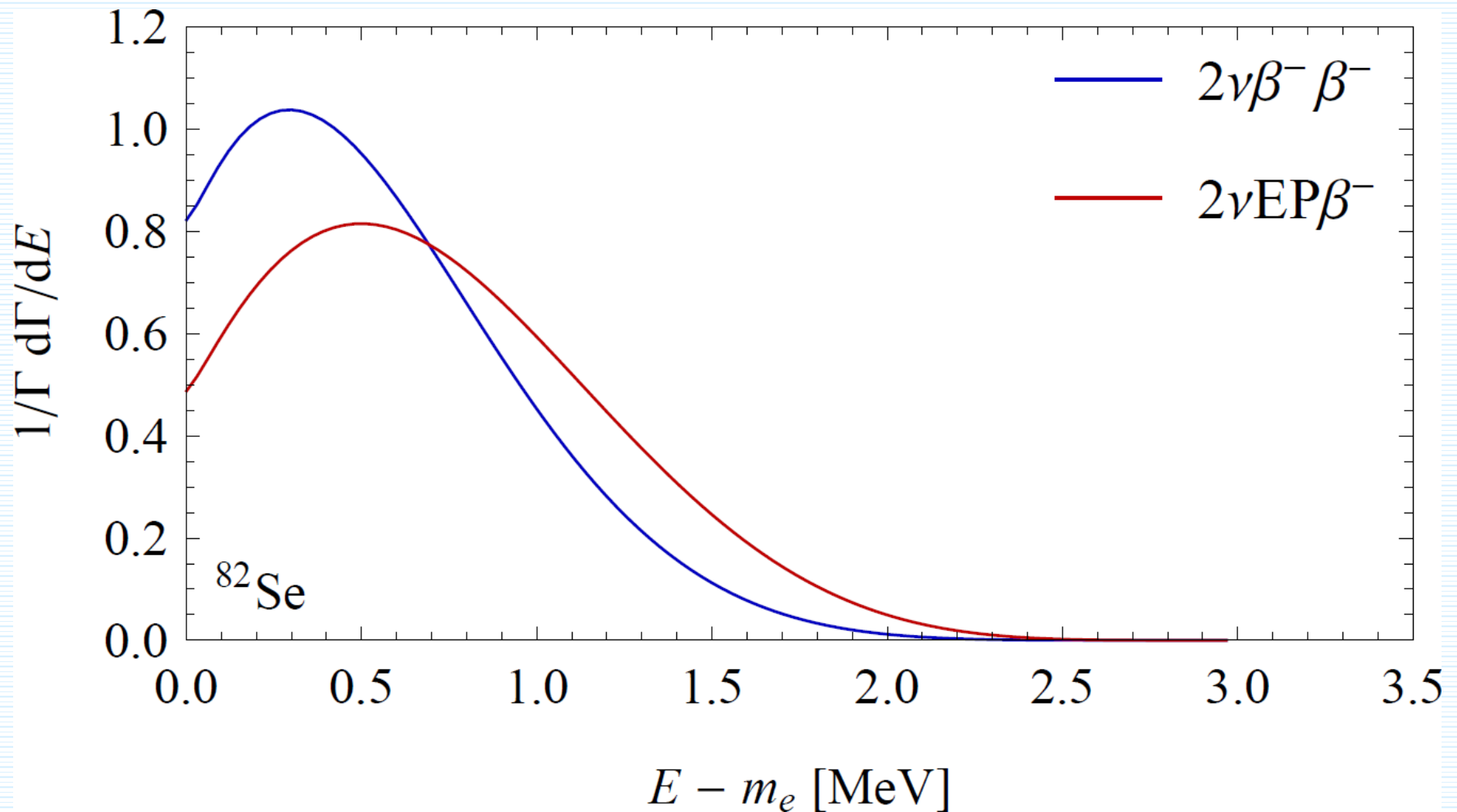
$0\nu\text{EP}\beta^-$ Half-Lives

$0\nu\beta^-\beta^-$ and $0\nu\text{EP}\beta^-$ half-lives $T_{1/2}^{0\nu\beta\beta}$ and $T_{1/2}^{0\nu\text{EP}\beta}$ estimated for $\beta^-\beta^-$ isotopes with known NME $|M^{0\nu\beta\beta}|$, assuming unquenched $g_A = 1.269$ and $|m_{\beta\beta}| = 50$ meV



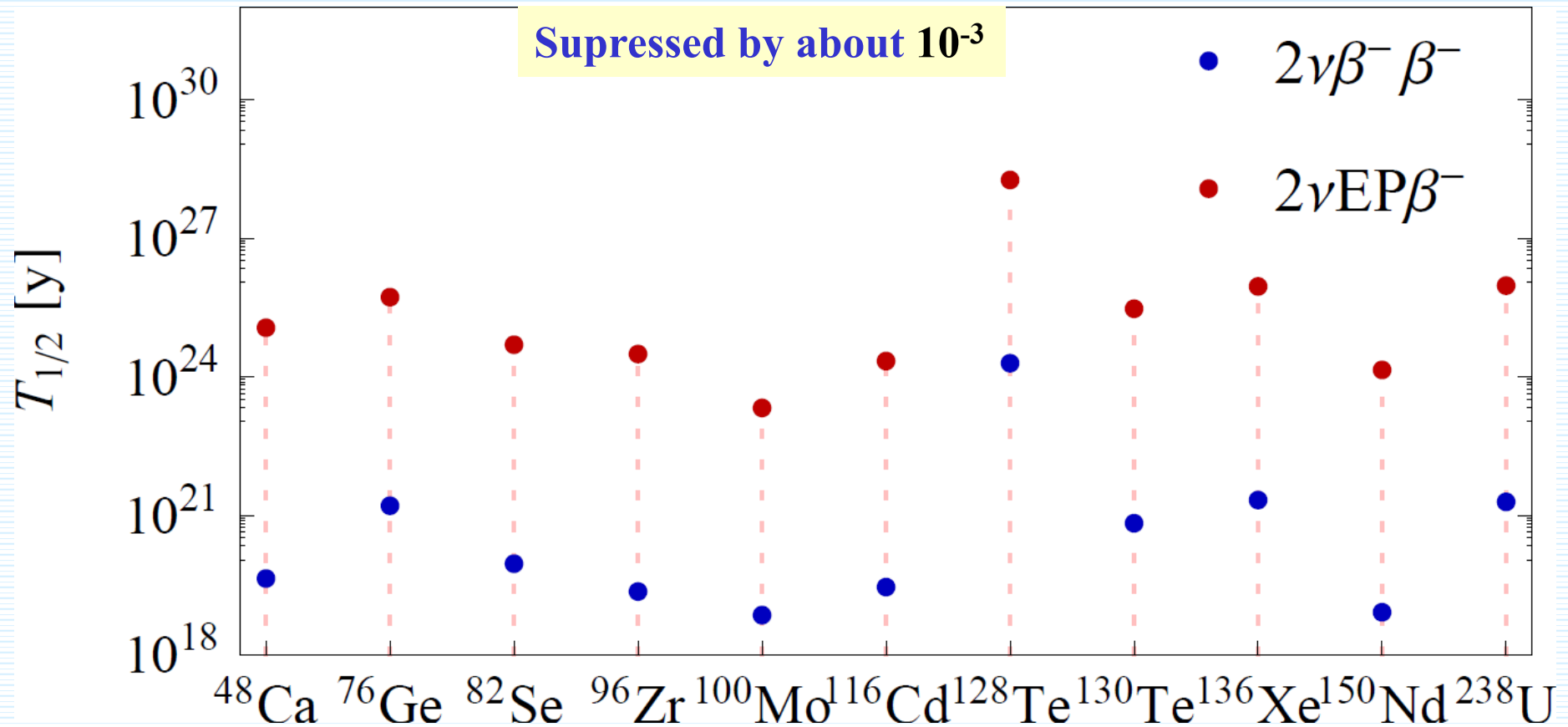
$2\nu\text{EP}\beta^-$ Single-Electron Spectrum (^{82}Se)

$2\nu\beta^-\beta^-$ and $2\nu\text{EP}\beta^-$ single-electron spectra $1/\Gamma d\Gamma/dE$ vs. electron kinetic energy $E - m_e$ for ^{82}Se ($Q = 2.996$ MeV)



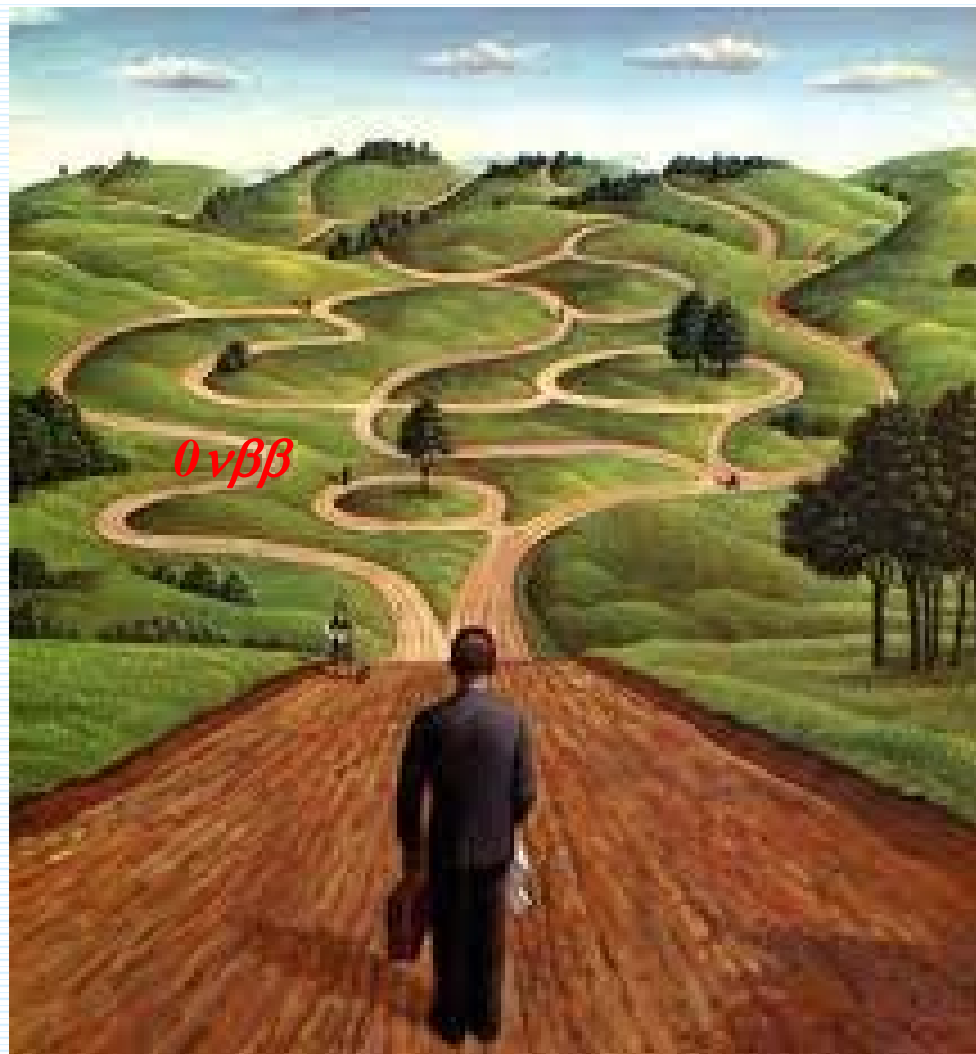
$2\nu\text{EP}\beta^-$ Half-Lives predictions (independent on g_A and value of NME)

$2\nu\beta^-\beta^-$ and $2\nu\text{EP}\beta^-$ half-lives $T_{1/2}^{2\nu\beta\beta}$ and $T_{1/2}^{2\nu\text{EP}\beta}$ calculated for $\beta^-\beta^-$ isotopes observed experimentally, assuming unquenched $g_A = 1.269$



Instead of Conclusions

Progress
in
nuclear
structure
calculations
is
highly
required



We are at the beginning of the **BSM** Road...

9/23/2017