Electroweak precision tests & constraints on neutrino models

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1. **Overview of electroweak precision tests**

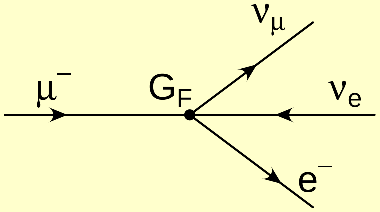
2. **Constraints on neutrino models**

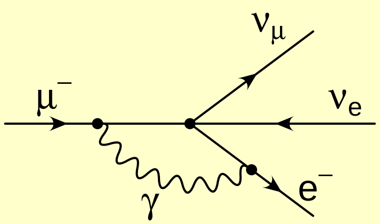
3. **Future $\mathrm{e}^+\mathrm{e}^-$ colliders**

Overview of electroweak precision tests

W mass

μ decay in Fermi Model



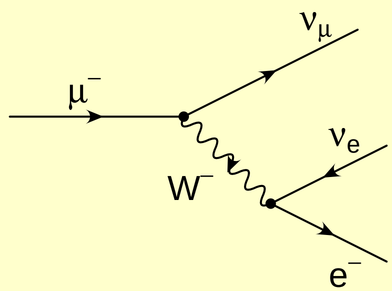


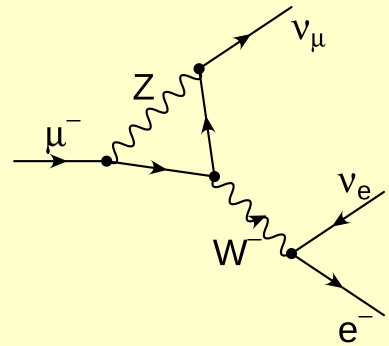
← QED corr. (2-loop)

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)(1 + \Delta q)$$

Ritbergen, Stuart '98
Pak, Czarnecki '08

μ decay in Standard Model





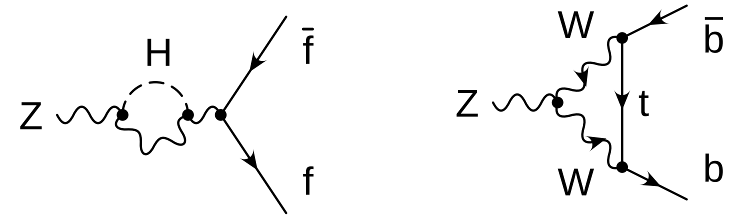
$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8 s_{\rm W}^2 M_{\rm W}^2}(1 + \Delta r)$$

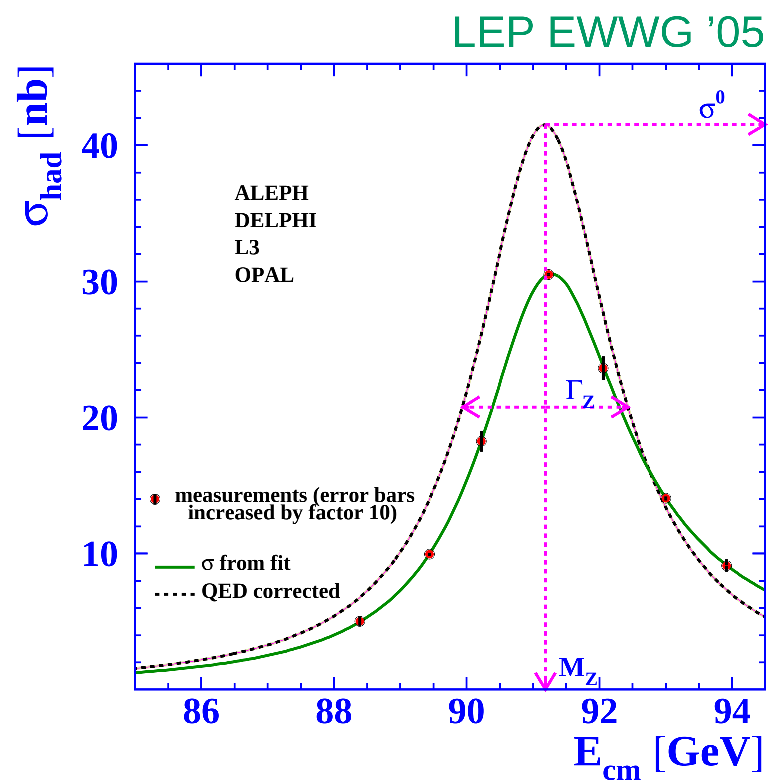
electroweak corrections

Z-pole observables

Indirect sensitivity to **top**, **Higgs**, and **new physics** through quantum corrections

- Z-pole cross-section
  $\sigma^0[e^+e^- \to (Z) \to f\bar{f}]$
- Z-boson width Γ_Z
- Partial widths $\overline{\Gamma}_f = \Gamma[Z \to f\bar{f}]_{s=\overline{M}_Z^2}$
- Branching ratios:
  $R_q = \Gamma_q/\Gamma_{\text{had}} \qquad (q = b, c)$
  $R_\ell = \Gamma_{\text{had}}/\Gamma_\ell \qquad (\ell = e, \mu, \tau)$





Z-pole observables

Z-pole asymmetries:

$$A_{\mathrm{FB}}^{f} \equiv \frac{\sigma(\theta < \frac{\pi}{2}) - \sigma(\theta > \frac{\pi}{2})}{\sigma(\theta < \frac{\pi}{2}) + \sigma(\theta > \frac{\pi}{2})} = \frac{3}{4}\mathcal{A}_e \mathcal{A}_f$$

$$A_{\mathrm{LR}} \equiv \frac{\sigma(\mathcal{P}_e > 0) - \sigma(\mathcal{P}_e < 0)}{\sigma(\mathcal{P}_e > 0) + \sigma(\mathcal{P}_e < 0)} = \mathcal{A}_e$$

$$\mathcal{A}_f = 2\frac{g_{Vf}/g_{Af}}{1+(g_{Vf}/g_{Af})^2} = \frac{1-4|Q_f|\sin^2\theta_{\mathrm{eff}}^{f}}{1-4|Q_f|\sin^2\theta_{\mathrm{eff}}^{f} + 8(|Q_f|\sin^2\theta_{\mathrm{eff}}^{f})^2}$$

$\sin^2\theta_{\mathrm{eff}}^{f}$ = “effective weak mixing angle”

$\sin^2\theta_{\mathrm{eff}}^{f} = s_{\mathrm{W}} \equiv 1 - M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2$ at tree-level

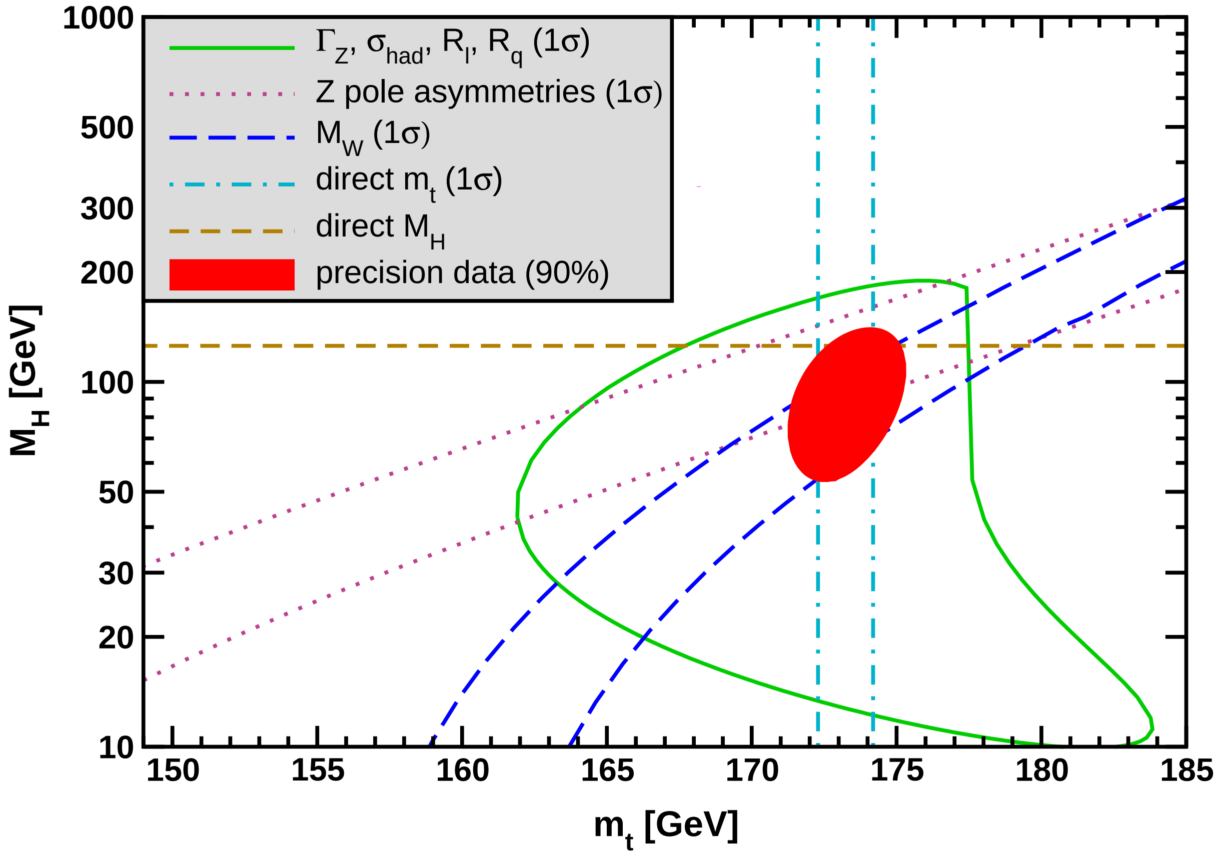
Most precisely measured for $f = \ell$

Current status of electroweak precision tests

Standard Model after Higgs discovery:

- Good agreement between measured mass and indirect prediction
- Very good agreement over large number of observables

Erler '13



Direct measurements:

$M_H = 125.6 \pm 0.4$ GeV

$m_t = 173.24 \pm 0.95$ GeV

Indirect prediction:

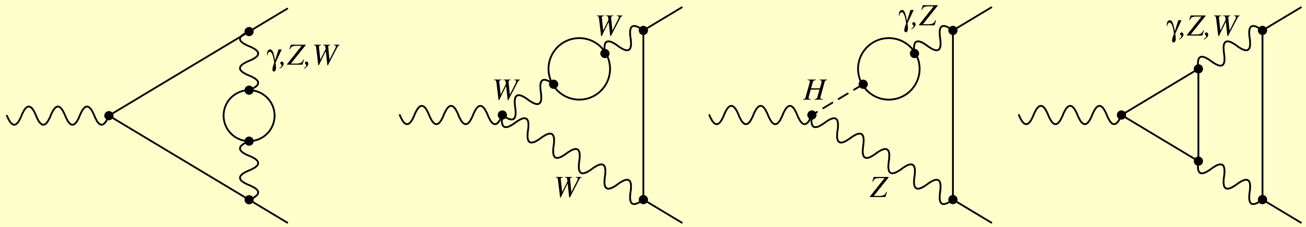
$M_H = 123.7 \pm 2.3$ GeV (with LHC BRs)

$M_H = 89^{+22}_{-18}$ GeV (w/o LHC data)

$m_t = 177.0 \pm 2.1$ GeV

Current status of SM loop results

Known corrections to Δr, $\sin^2\theta^f_{\text{eff}}$, g_{Vf}, g_{Af}:



- Complete NNLO corrections (Δr, $\sin^2\theta^\ell_{\text{eff}}$) Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02; Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14
- "Fermionic" NNLO corrections (g_{Vf}, g_{Af}) Czarnecki, Kühn '96
  Harlander, Seidensticker, Steinhauser '98
  Freitas '13,14
- Partial 3/4-loop corrections to ρ/T-parameter
  $\mathcal{O}(\alpha_t\alpha_s^2)$, $\mathcal{O}(\alpha_t^2\alpha_s)$, $\mathcal{O}(\alpha_t\alpha_s^3)$ Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05; Chetyrkin et al. '06
  Boughezal, Czakon '06

  $(\alpha_t \equiv \frac{y_t^2}{4\pi})$

Current uncertainties

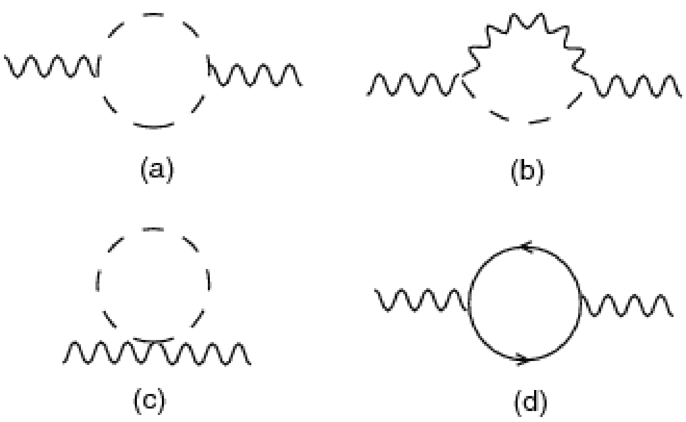
	Experiment	Theory error	Main source
M_W	80.385 ± 0.015 MeV	4 MeV	α^3, $\alpha^2\alpha_\mathrm{s}$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	α^2_bos, α^3, $\alpha^2\alpha_\mathrm{s}$, $\alpha\alpha_\mathrm{s}^2$
σ^0_had	41540 ± 37 pb	6 pb	α^2_bos, α^3, $\alpha^2\alpha_\mathrm{s}$
$R_b \equiv \Gamma^b_\mathrm{Z}/\Gamma^\mathrm{had}_\mathrm{Z}$	0.21629 ± 0.00066	0.00015	α^2_bos, α^3, $\alpha^2\alpha_\mathrm{s}$
$\sin^2\theta^\ell_\mathrm{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_\mathrm{s}$

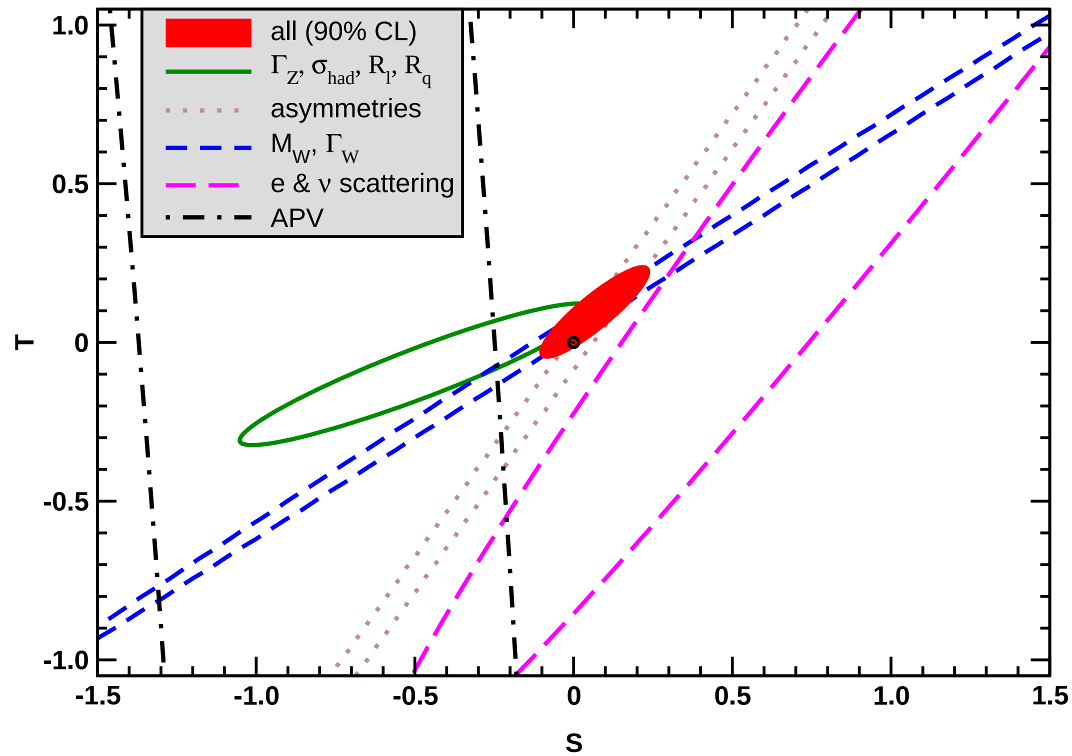
- Theory error estimate is not well defined, ideally $\Delta_\mathrm{th} \ll \Delta_\mathrm{exp}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence
- Also parametric error from external inputs (m_t, m_b, α_s, $\Delta\alpha_\mathrm{had}$, ...)

Oblique parameters:

$$\alpha T = \frac{\Sigma_{WW}(0)}{M_W} - \frac{\Sigma_{ZZ}(0)}{M_Z}$$

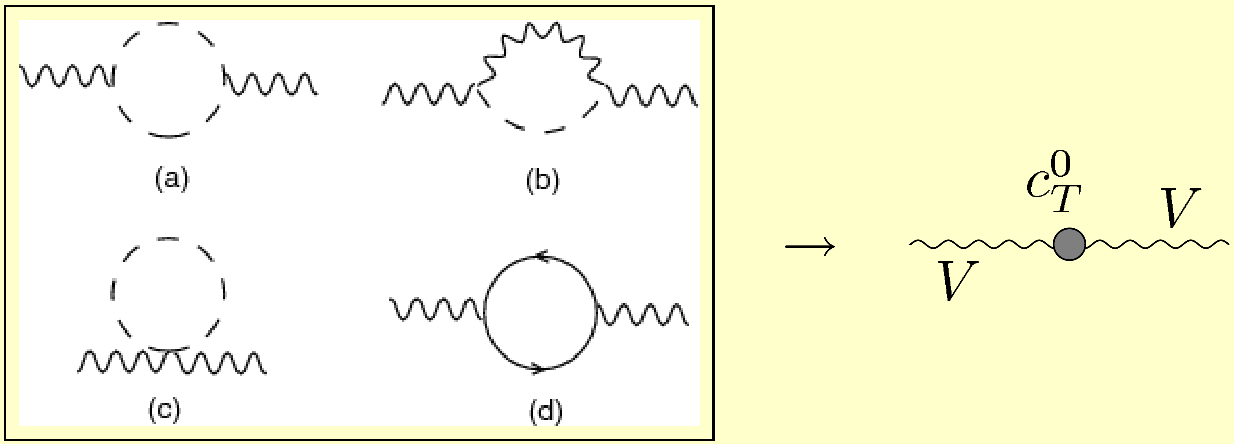
$$\frac{\alpha}{4s^2c^2} S = \frac{\Sigma_{ZZ}(M_Z^2) - \Sigma_{ZZ}(0)}{M_Z} + \frac{s^2 - c^2}{sc}\frac{\Sigma_{Z\gamma}(M_Z^2)}{M_Z} - \frac{\Sigma_{\gamma\gamma}(M_Z^2)}{M_Z}$$





Erler '16

General parametrization of new physics



- "Integrate out" heavy praticles with mass $m \sim \Lambda$
  (expand full result in M_Z/Λ)

- Generate higher-dimensional operators, leading dimension 6
  $$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3}) \qquad (\Lambda \gg M_Z)$$

- Possible operators restricted by SM field content and gauge invariance

Buchmüller, Wyler '86

Grzadkowski, Iskrzynski, Misiak, J. Rosiek '10

General parametrization of new physics

Effective field theory: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3}) \qquad (\Lambda \gg M_\mathrm{Z})$

$$\mathcal{O}_\mathrm{T} = (D_\mu\Phi)^\dagger\Phi\,\Phi^\dagger(D^\mu\Phi) \qquad \alpha\Delta T = -\frac{v^2}{2}\frac{c_\mathrm{T}}{\Lambda^2}$$

$$\mathcal{O}_\mathrm{BW} = \Phi^\dagger B_{\mu\nu}W^{\mu\nu}\Phi \qquad \alpha\Delta S = -e^2v^2\frac{c_\mathrm{BW}}{\Lambda^2}$$

$$O_\mathrm{LL}^{(3)e} = (\bar{L}_\mathrm{L}^e\sigma^a\gamma_\mu L_\mathrm{L}^e)(\bar{L}_\mathrm{L}^e\sigma^a\gamma^\mu L_\mathrm{L}^e) \qquad \Delta G_F = -\sqrt{2}\frac{c_\mathrm{LL}^{(3)e}}{\Lambda^2}$$

$$O_\mathrm{R}^f = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{f}_\mathrm{R}\gamma^\mu f_\mathrm{R}) \qquad f = e, \mu\,\tau, b, lq$$

$$O_\mathrm{L}^F = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{F}_\mathrm{L}\gamma^\mu F_\mathrm{L}) \qquad F = \binom{\nu_e}{e}, \binom{\nu_\mu}{\mu}, \binom{\nu_\tau}{\tau}, \binom{u, c}{d, s}, \binom{t}{b}$$

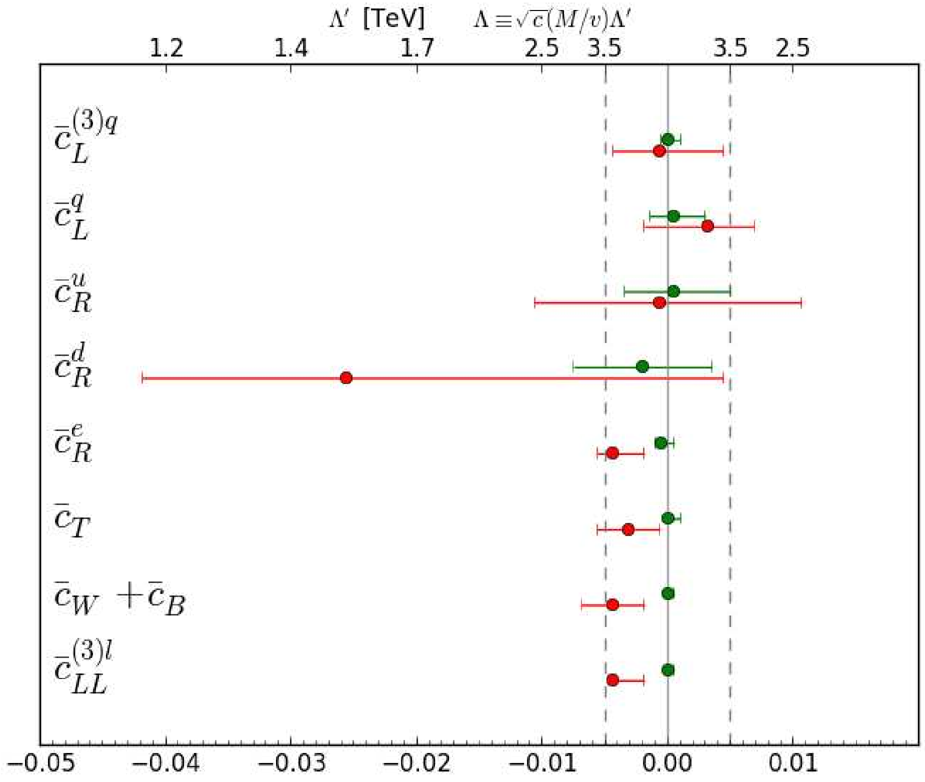
$$O_\mathrm{L}^{(3)F} = i(\Phi^\dagger \overleftrightarrow{D}_\mu^a \Phi)(\bar{F}_\mathrm{L}\sigma_a\gamma^\mu F_\mathrm{L})$$

In general more operators than EWPOs

$\longrightarrow$ Some can be constrained by $W \to \ell\nu$, had., $e^+e^- \to W^+W^-$

Current constraints on some dim-6 operators

Assuming flavor universality: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3})$ $(\Lambda \gg M_Z)$



$$\mathcal{O}_\mathrm{T} = (D_\mu\Phi)^\dagger\Phi\,\Phi^\dagger(D^\mu\Phi)$$

$$\mathcal{O}_\mathrm{BW} = \Phi^\dagger B_{\mu\nu}W^{\mu\nu}\Phi$$

$$O_\mathrm{LL}^{(3)e} = (\bar{L}_\mathrm{L}^e\sigma^a\gamma_\mu L_\mathrm{L}^e)(\bar{L}_\mathrm{L}^e\sigma^a\gamma^\mu L_\mathrm{L}^e)$$

$$O_\mathrm{R}^f = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{f}_\mathrm{R}\gamma^\mu f_\mathrm{R})$$

$$O_\mathrm{L}^F = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{F}_\mathrm{L}\gamma^\mu F_\mathrm{L})$$

$$O_\mathrm{L}^{(3)F} = i(\Phi^\dagger \overleftrightarrow{D}_\mu^a \Phi)(\bar{F}_\mathrm{L}\sigma_a\gamma^\mu F_\mathrm{L})$$

Pomaral, Riva '13
Ellis, Sanz, You '14

Constraints on neutrino models

Heavy sterile neutrinos

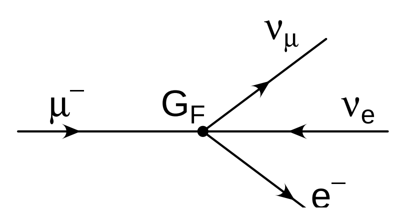
Extend SM with one (several) sterile neutrinos

Loinaz, Okamura, Rayyan, Takeuchi, Wijewardhana '02,04
Akhmedov, Kartavtsev, Lindner, Michaels, Smirnov '13

Mass matrix in $(\nu_e, \nu_\mu, \nu_\tau, \nu_s)$ basis:
$$\begin{pmatrix} 0 & 0 & 0 & y_{es} \\ 0 & 0 & 0 & y_{\mu s} \\ 0 & 0 & 0 & y_{\tau s} \\ y^*_{es} & y^*_{\mu s} & y^*_{\tau s} & M \end{pmatrix}, \qquad M \gg M_Z$$

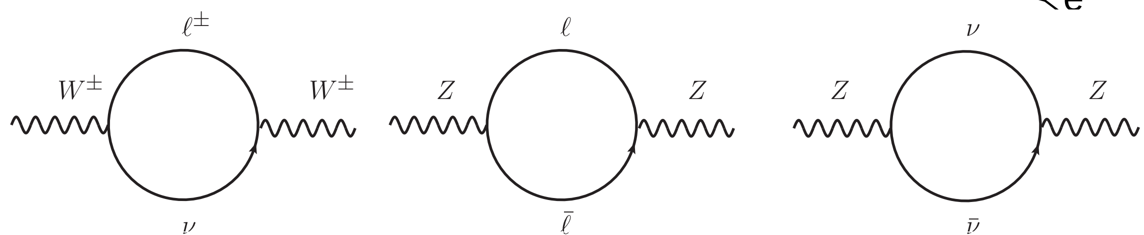
$\longrightarrow$ Heavy neutrino mass eigenstate ν_4 with $m_4 \approx M$,
small mixing with SM leptons $\epsilon_i = |U_{i4}|^2, \quad (i = e, \mu, \tau)$

- Impact on $Z \to \nu\bar{\nu}$: $\Gamma_\nu / \Gamma_\nu^{\mathrm{SM}} = 1 - \frac{1}{3}(\epsilon_e + \epsilon_\mu + \epsilon_\tau)$



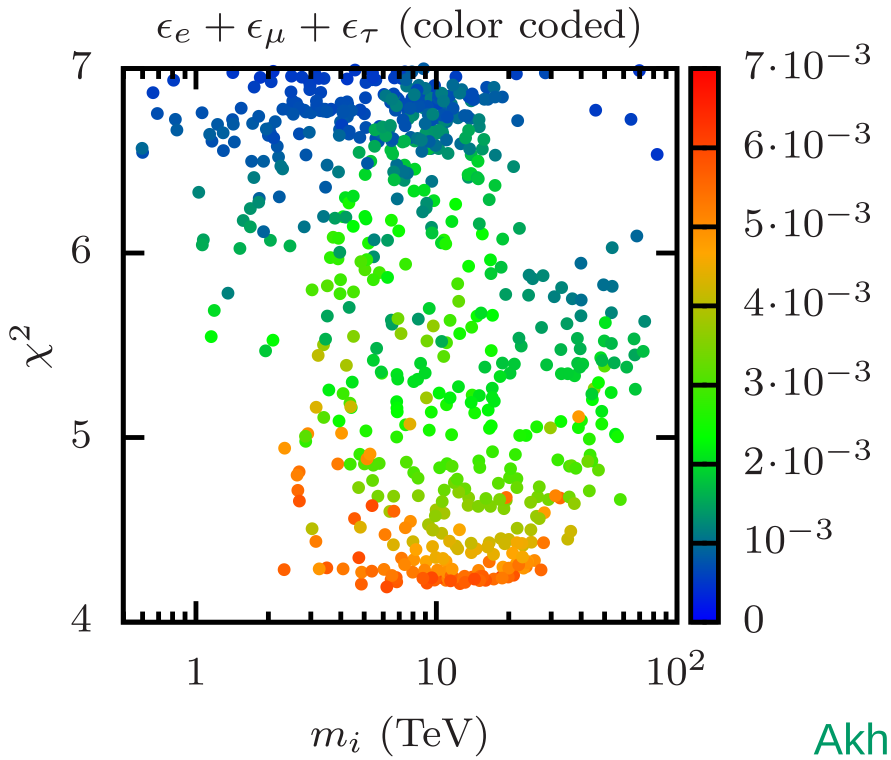
- Impact on Fermi constant: $G_F^2 \to G_F^2(1 - \epsilon_e)(1 - \epsilon_\mu)$

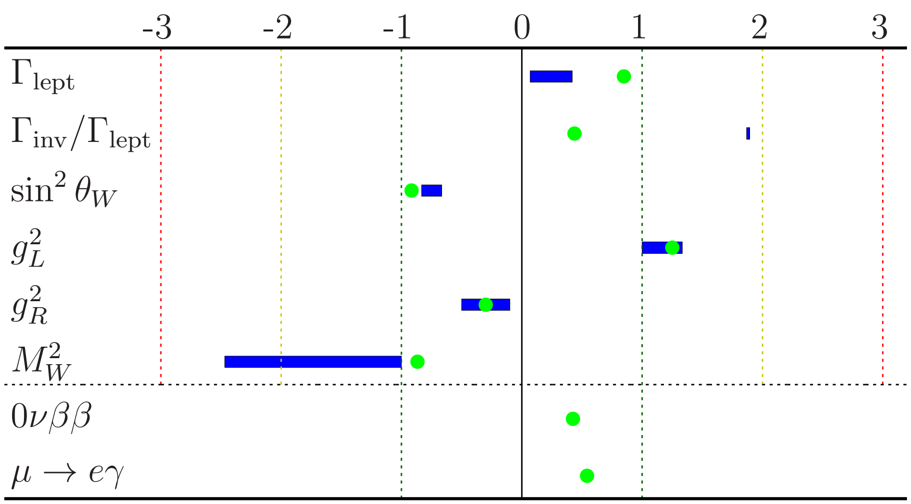
- Impact on S/T parameter



Cancellation between tree-level and loop corrections possible

$\longrightarrow$ Relatively large mixing angles allowed by EWPOs





Akhmedov, Kartavtsev, Lindner, Michaels, Smirnov ’13

Heavy sterile neutrinos

Additonal constraint: lepton number universality

de Gouvea, Kobach '15

Observable	SM	Observed	$\lvert g_\ell/g_{\ell'}\rvert^2$
$\Gamma(\tau\to\mu\nu\overline{\nu})/\Gamma(\tau\to e\nu\overline{\nu})$	0.9726	0.9764 ± 0.0030	$\lvert g_\mu/g_e\rvert^2=1.0040\pm0.0031$
$\Gamma(\pi\to e\nu)/\Gamma(\pi\to\mu\nu)$	1.235×10^{-4} [18]	$(1.230\pm0.004)\times10^{-4}$	$\lvert g_e/g_\mu\rvert^2=0.9958\pm0.0032$
$\Gamma(K\to e\nu)/\Gamma(K\to\mu\nu)$	2.477×10^{-5} [18]	$(2.488\pm0.010)\times10^{-5}$	$\lvert g_e/g_\mu\rvert^2=1.0044\pm0.0040$
$\Gamma(K\to\pi\mu\nu)/\Gamma(K\to\pi e\nu)$	0.6591 ± 0.0031 [19]	0.6608 ± 0.0030	$\lvert g_\mu/g_e\rvert^2=1.0026\pm0.0065$
$\Gamma(K_L\to\pi\mu\nu)/\Gamma(K_L\to\pi e\nu)$	0.6657 ± 0.0031 [19]	0.6669 ± 0.0027	$\lvert g_\mu/g_e\rvert^2=1.0018\pm0.0062$
$\Gamma(W\to\mu\nu)/\Gamma(W\to e\nu)$	1.000 [25]	0.993 ± 0.019	$\lvert g_\mu/g_e\rvert^2=0.993\pm0.020$
$\Gamma(\tau\to e\nu\overline{\nu})/\Gamma(\mu\to e\nu\overline{\nu})$	1.345×10^{6}	$(1.349\pm0.004)\times10^{6}$	$\lvert g_\tau/g_\mu\rvert^2=1.003\pm0.003$
$\Gamma(\tau\to\pi\nu)/\Gamma(\pi\to\mu\nu)$	9771 ± 14 [26]	9704 ± 56	$\lvert g_\tau/g_\mu\rvert^2=0.993\pm0.006$
$\Gamma(\tau\to K\nu)/\Gamma(K\to\mu\nu)$	480 ± 1 [26]	469 ± 7	$\lvert g_\tau/g_\mu\rvert^2=0.977\pm0.015$
$\Gamma(D_s\to\tau\nu)/\Gamma(D_s\to\mu\nu)$	9.76 [17]	10.0 ± 0.6	$\lvert g_\tau/g_\mu\rvert^2=1.02\pm0.06$
$\Gamma(\bar{B}\to D^*\tau\nu)/\Gamma(\bar{B}\to D^*\mu\nu)$	0.252 ± 0.003 [24]	0.336 ± 0.040 [20]	$\lvert g_\tau/g_\mu\rvert^2=1.333\pm0.159$
$\Gamma(\tau\to\pi\nu)/\Gamma(\pi\to e\nu)$	$(7.91\pm0.01)\times10^{7}$ [18, 26]	$(7.89\pm0.05)\times10^{7}$	$\lvert g_\tau/g_e\rvert^2=1.000\pm0.007$
$\Gamma(\tau\to K\nu)/\Gamma(K\to e\nu)$	$(1.940\pm0.004)\times10^{7}$ [18, 26]	$(1.89\pm0.03)\times10^{7}$	$\lvert g_\tau/g_e\rvert^2=0.974\pm0.015$
$\Gamma(W\to\tau\nu)/\Gamma(W\to e\nu)$	0.999 [25]	1.063 ± 0.027	$\lvert g_\tau/g_e\rvert^2=1.063\pm0.027$
$\Gamma(\bar{B}\to D^*\tau\nu)/\Gamma(\bar{B}\to D^*\ell\nu)$	0.252 ± 0.003 [24]	0.318 ± 0.024 [21, 22]	$2\lvert g_\tau\rvert^2/(\lvert g_e\rvert^2+\lvert g_\mu\rvert^2)=1.262\pm0.096$
$\Gamma(\bar{B}\to D\tau\nu)/\Gamma(\bar{B}\to D\ell\nu)$	0.299 ± 0.011 [23]	0.406 ± 0.050 [21, 22]	$2\lvert g_\tau\rvert^2/(\lvert g_e\rvert^2+\lvert g_\mu\rvert^2)=1.359\pm0.171$

Lighter sterile neutrinos

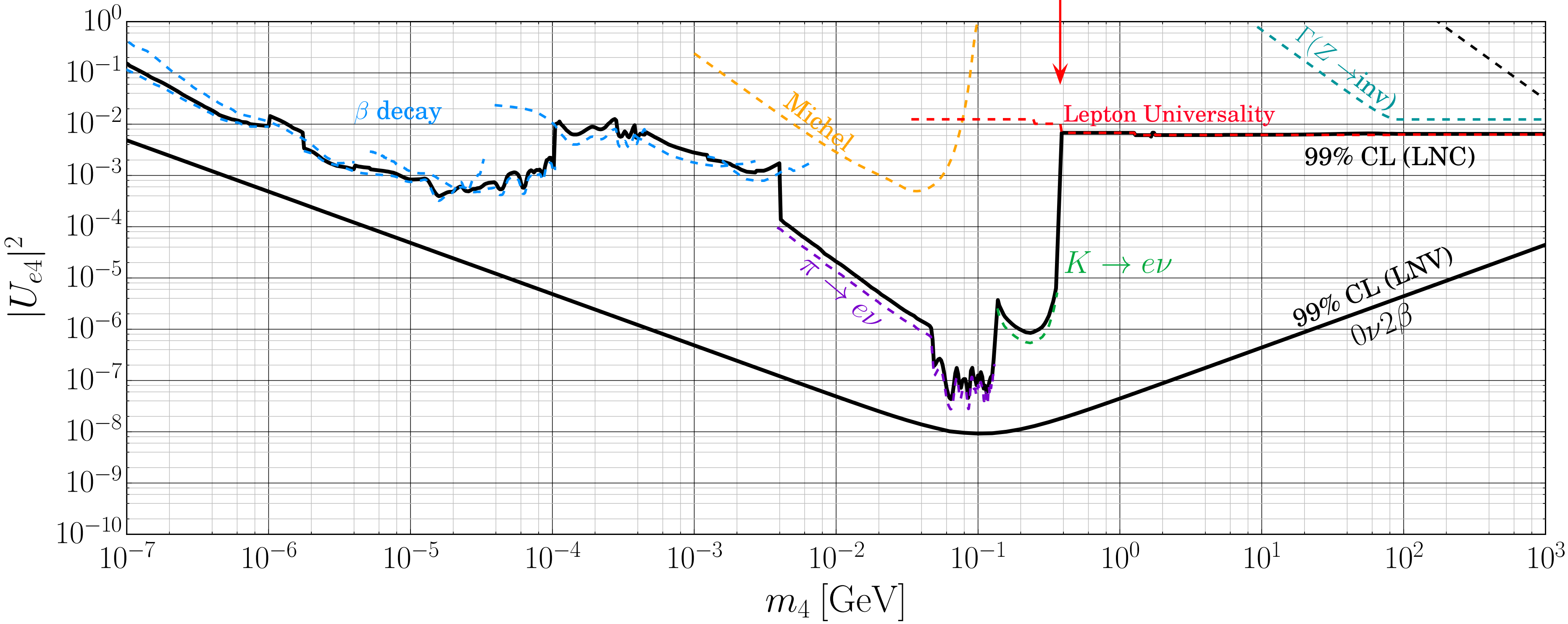
For $m_4 < M_Z$: Decay $Z \to \nu_i \nu_4$ possible

If ν_4 does not decay in detector:

$\longrightarrow$ Lesser suppression of Γ_ν

only for $U_{\mu 4} = U_{\tau 4} = 0$

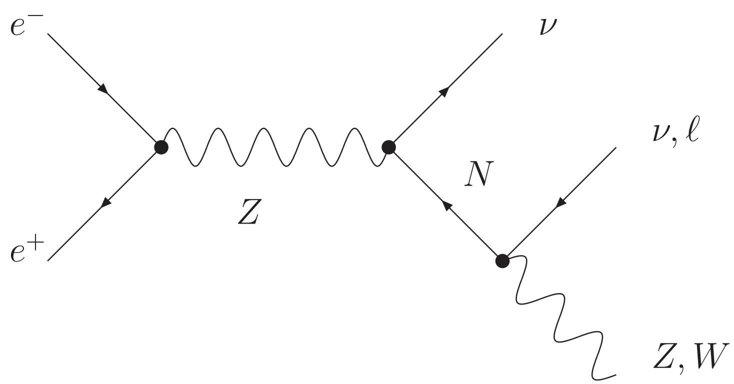
de Gouvea, Kobach '15



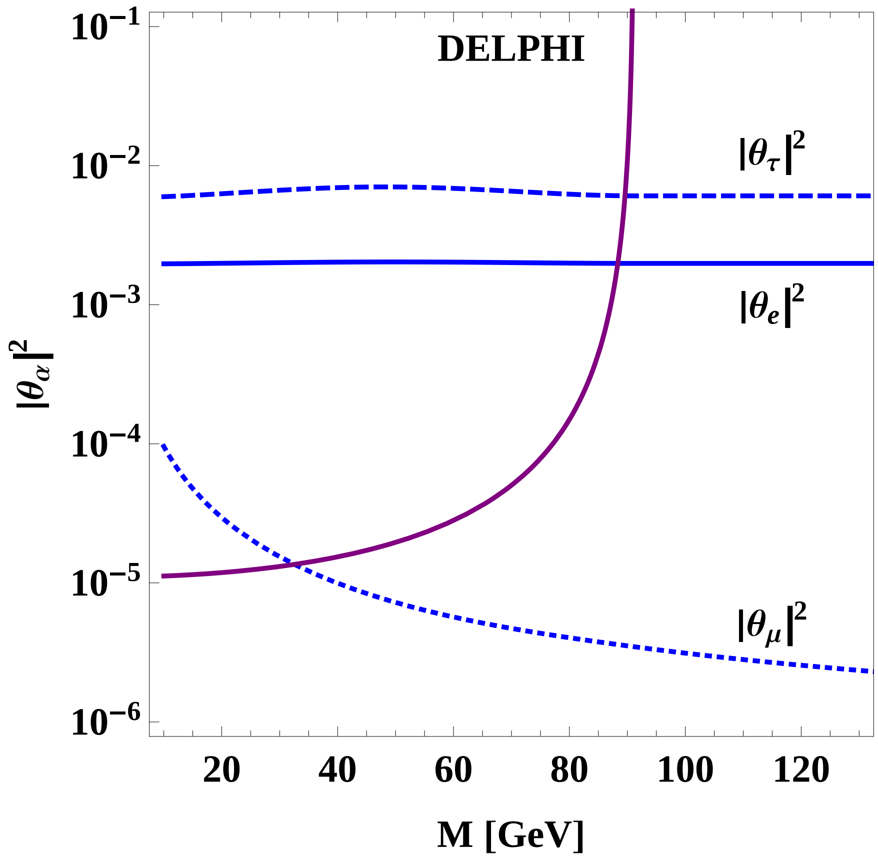
Lighter sterile neutrinos

For $m_4 < M_Z$: Decay $Z \to \nu_i \nu_4$ possible

If ν_4 **does** decay in detector:
$\longrightarrow$ Direct search limits

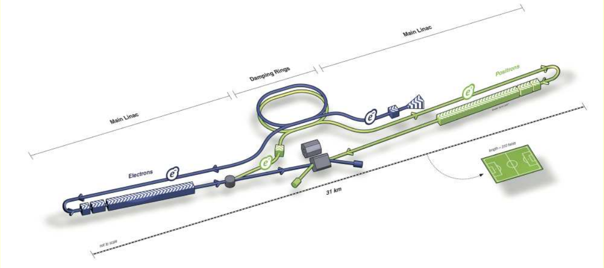


Antusch, Fischer '15



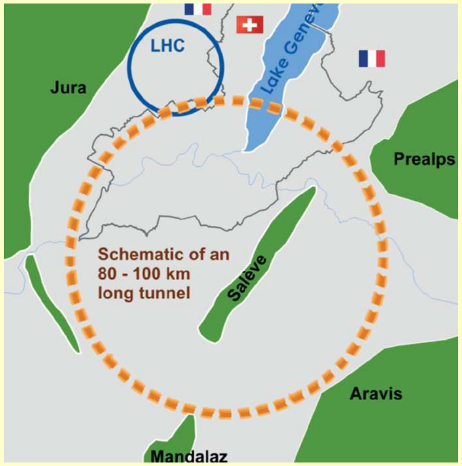
Future e^+e^- colliders

- International Linear Collider (ILC)
  Int. lumi at $\sqrt{s} \sim M_Z$: 50–100 fb^{-1}



- Circular Electron-Positron Collider (CEPC)
  Int. lumi at $\sqrt{s} \sim M_Z$: 2×150 fb^{-1}

- Future Circular Collider (FCC-ee)
  Int. lumi at $\sqrt{s} \sim M_Z$: $> 2 \times 30$ ab^{-1}



Future projections

	Measurement error				Intrinsic theory	
	Current	ILC	CEPC	FCC-ee	Current	Future†
M_W [MeV]	15	3–4	3	1	4	1
Γ_Z [MeV]	2.3	0.8	0.5	0.1	0.5	0.2
R_b $[10^{-5}]$	66	14	17	6	15	7
$\sin^2\theta^\ell_\mathrm{eff}$ $[10^{-5}]$	16	1	2.3	0.6	4.5	1.5

$\longrightarrow$ Existing theoretical calculations adequate for LEP/SLC/LHC, but not ILC/CEPC/FCC-ee!

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_\mathrm{s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_\mathrm{s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_\mathrm{s})$

($N_f^n =$ at least n closed fermion loops)

Future projections

	Measurement		Intrinsic theory		Parametric	
	ILC	FCC-ee	Current	Future	ILC	FCC-ee
M_{W} [MeV]	3–4	1	4	1	2.6	0.6–1
Γ_Z [MeV]	0.8	0.1	0.5	0.2	0.5	0.1
R_b [10^{-5}]	14	6	15	7	< 1	< 1
$\sin^2\theta^{\ell}_{\mathrm{eff}}$ [10^{-5}]	1	2.3	4.5	1.5	2	1–2

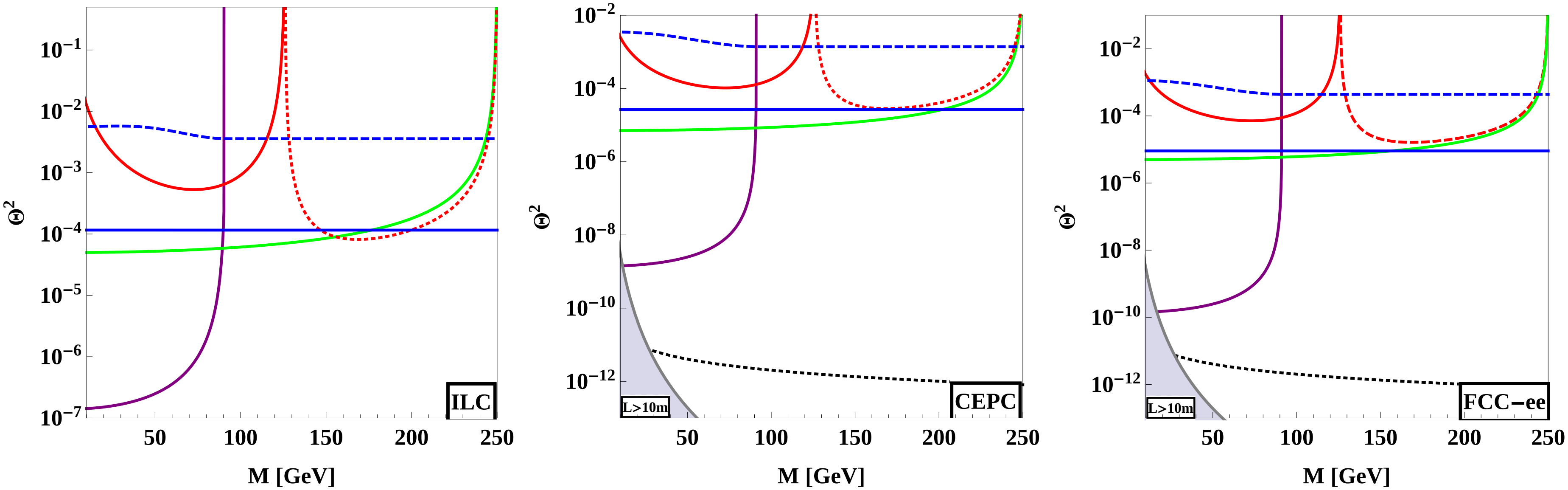
Projected parameter measurements:

	δm_t	$\delta\alpha_{\mathrm{s}}$	δM_{Z}	$\delta(\Delta\alpha)$
ILC:	50 MeV	0.001	2.1 MeV	5×10^{-5}
FCC-ee:	50 MeV	0.0002	0.1 MeV	$3\text{–}5 \times 10^{-5}$

Projected bounds on neutrino models

Improved bounds both from EWPOs and direct searches

Antusch, Fischer ’15



Direct searches

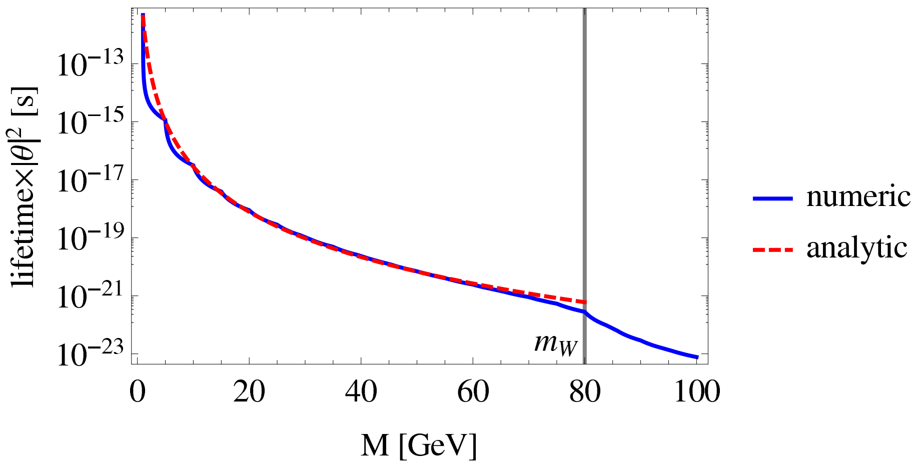
- Z pole search @2σ: $|y|=\sqrt{\sum_\alpha |y_{\nu_\alpha}|^2}$, $\Theta^2=\sum_\alpha|\theta_\alpha|^2$
- Higgs $\to$ WW @1σ: $|y|=\sqrt{\sum_\alpha |y_{\nu_\alpha}|^2}$, $\Theta^2=\sum_\alpha|\theta_\alpha|^2$
- $e^+e^-\to h + ME_{(T)}$ @1σ: $|y|=|y_{\nu_e}|$, $\Theta^2=|\theta_e|^2$
- $e^+e^-\to l\nu l\nu^*$ @1σ: $|y|=|y_{\nu_e}|$, $\Theta^2=|\theta_e|^2$

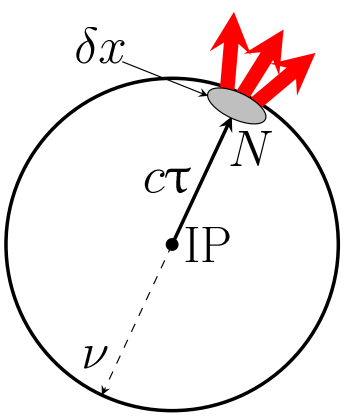
Other

- Precision constraints: $|y|=\sqrt{|y_{\nu_e}|^2+|y_{\nu_\mu}|^2}$, $\Theta^2=|\theta_e|^2+|\theta_\mu|^2$
- Precision constraints: $|y|=|y_{\nu_\tau}|$, $\Theta^2=|\theta_\tau|^2$
- ”Unprotected” type−I seesaw

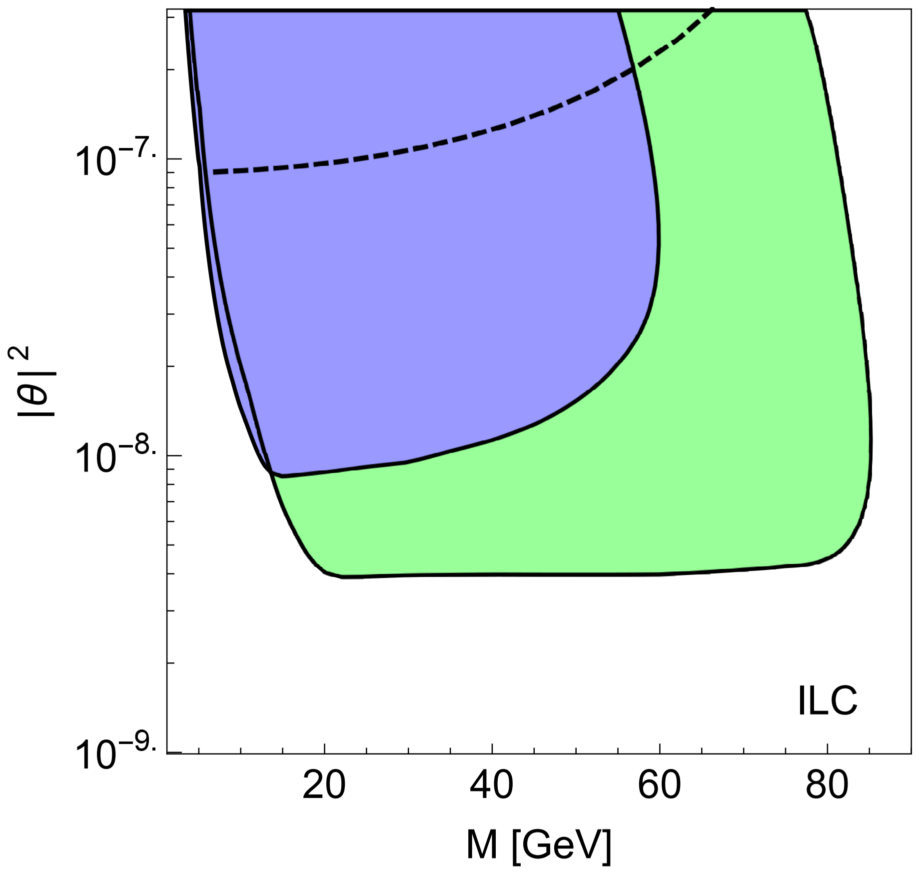
Displaced vertex search

For small mixing $|\theta|^2 = \sum_i \epsilon_i$ and small m_4,
ν_4 propagates in detector before decaying





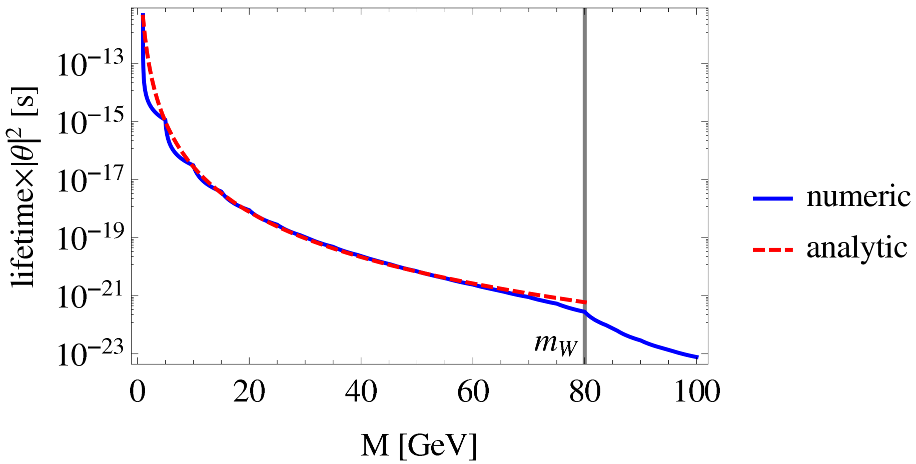
Antusch, Cazzato, Fischer ’16

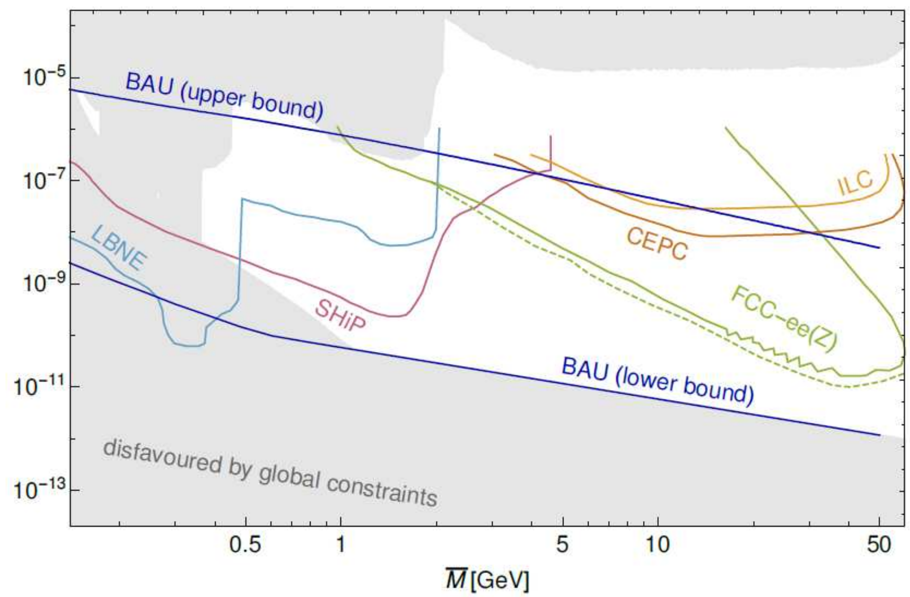


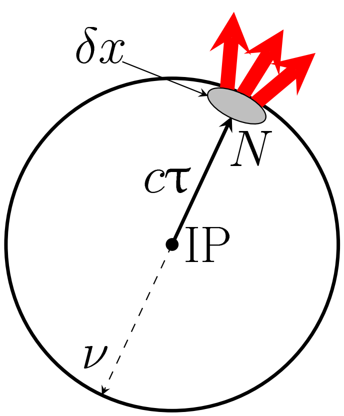
Displaced vertex search

For small mixing $|\theta|^2 = \sum_i \epsilon_i$ and small m_4, ν_4 propagates in detector before decaying

Drewes, Garbrecht, Gueter, Klaric '16







$\longrightarrow$ see talk by J. Klaric

Summary

- W/Z precision data provides trong constraints on multi-TeV new physics
  $\longrightarrow$ Non-trivial test for low-scale seesaw / left-right symmetric models, etc.

- For some neutrino models and regions of parameter space, EWPOs are the most important constraints

- Future e^+e^- colliders will probe new, theoretically motivated parameter space

Backup slides

Theory challenges

Full SM corrections at $\geq$2-loop:

- Large number of diagrams and tensor integrals, $\mathcal{O}(100) - \mathcal{O}(10000)$
- Many different scales (masses and ext. momenta)

Computer algebra methods:

- Generation of diagrams with *FeynArts*, *QGraf*, ...

  Küblbeck, Eck, Mertig '92, Hahn '01

  Nogueira '93
- Dirac/Lorentz algebra with *Form*, *FeynCalc*, ...

  Vermaseren '89,00

  Mertig '93

Evaluation of loop integrals:

- In general not possible analytically
- Numerical methods must be automizable, stable, fastly converging
- Need procedure for isolating divergent pieces