Neutrino Oscillation & Other Quantum Oscillations:

A collection of various oscillations & mixings

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Motivation of this talk:
Physics education based on the oscillation mechanism

The origin of the neutrino oscillation is the transitions between different flavor neutrinos, such as,

$$\nu_e \Leftrightarrow \nu_\mu$$

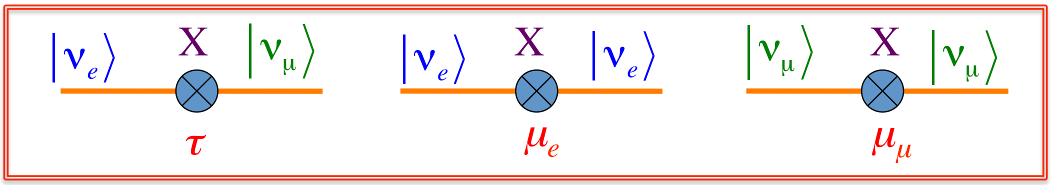
In fact, many kinds of transitions take place in various physics phenomena; many of them bear important physics effects.
Such important physics can be understood as the same way as neutrino oscillation mechanism.
In some cases, abstract concepts, such as parity, can be understood by a concrete idea of oscillation and mixing.
It should be useful to teach various physics to students using such the unified and concrete point of view.
In this talk, a few examples of transitions, oscillations, mixings and their phenomena are collected.

A Review of the Neutrino Oscillation Mechanism

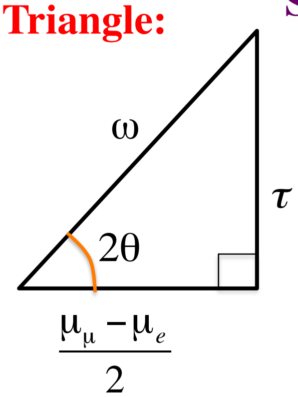
For two flavor (ν_e, ν_μ) case:

General neutrino state: $\Psi_\nu[t] = C_e[t]|\nu_e\rangle + C_\mu[t]|\nu_\mu\rangle$

Something (X) changes ν_μ to ν_e, and causes self-transition



Mixing Triangle:



State equation:

$$i\frac{d}{dt}\begin{pmatrix} C_e[t] \\ C_\mu[t] \end{pmatrix} = \begin{pmatrix} \mu_e & \tau \\ \tau & \mu_\mu \end{pmatrix}\begin{pmatrix} C_e[t] \\ C_\mu[t] \end{pmatrix}$$

Mass eigenstate:

$$\begin{cases} \nu_1 = (\cos\theta|\nu_e\rangle - \sin\theta|\nu_\mu\rangle)\exp[-im_1 t] \\ \nu_2 = (\sin\theta|\nu_e\rangle + \cos\theta|\nu_\mu\rangle)\exp[-im_2 t] \end{cases}$$

Masses: $m_{1,2} = \bar{\mu} \mp \omega$

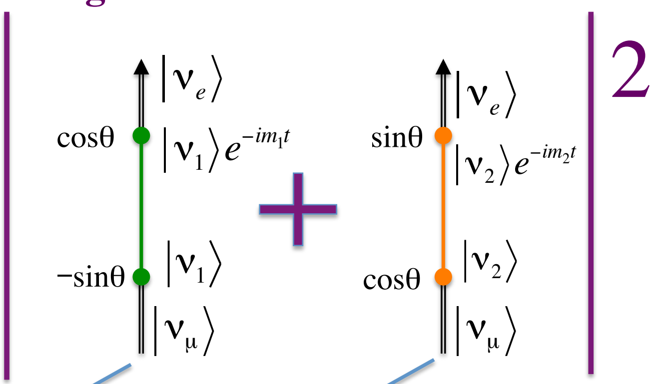
$$\bar{\mu} = \frac{\mu_e + \mu_\mu}{2}, \quad \omega = \frac{1}{2}\sqrt{(\mu_\mu - \mu_e)^2 + 4\tau^2}$$

Mixing between mass and flavor eigenstates

$$\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

To calculate the probability that ν_μ changes ν_e at time t, we take the absolute-square of the sum of diagrams of different intermediate mass eigenstates.

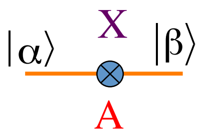
$P(\nu_\mu \rightarrow \nu_e) =$



$$= \left| -e^{-im_1 t}\cos\theta\sin\theta + e^{-im_2 t}\cos\theta\sin\theta \right|^2 = \sin^2 2\theta \sin^2 \omega t$$

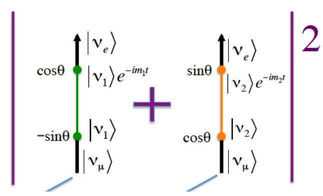
General Process of the Derivation of the Oscillation

$$\Psi[t] = C_\alpha[t]|\alpha\rangle + C_\beta[t]|\beta\rangle$$



$$i\frac{d}{dt}\begin{pmatrix} C_\alpha \\ C_\beta \end{pmatrix} = \begin{pmatrix} * & * \\ * & * \end{pmatrix}\begin{pmatrix} C_\alpha \\ C_\beta \end{pmatrix}$$

$$\psi[t] = (\cos\theta|\alpha\rangle + \sin\theta|\beta\rangle)\exp[-iMt]$$



Define basis sates

Look for transition origin and amplitude

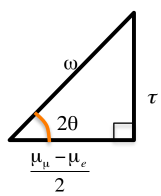
Form state equation

Obtain mass eigenstate

Choose the initial & final states

Oscillation

Mass



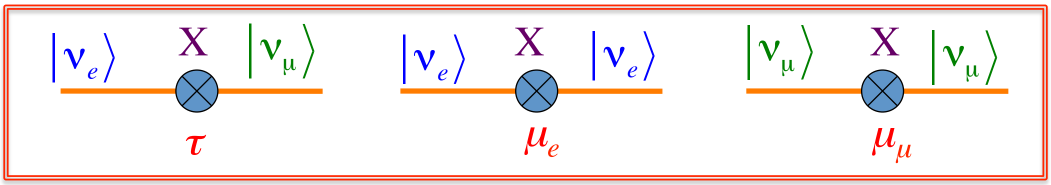
Mixing angle

Cabbibo angle θ_C

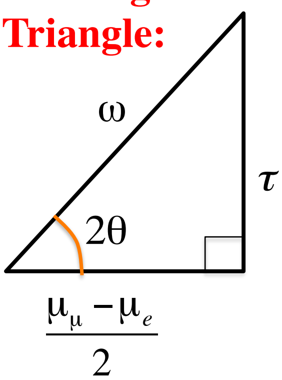
Neutrino Oscillation case

General neutrino state: $\Psi_\nu[t] = C_e[t]|\nu_e\rangle + C_\mu[t]|\nu_\mu\rangle$

Something (X) changes ν_μ to ν_e and gives self-transition



Mixing Triangle:



State equation:

$$i\frac{d}{dt}\begin{pmatrix} C_e[t] \\ C_\mu[t] \end{pmatrix} = \begin{pmatrix} \mu_e & \tau \\ \tau & \mu_\mu \end{pmatrix}\begin{pmatrix} C_e[t] \\ C_\mu[t] \end{pmatrix}$$

Mass eigenstate:

$$\begin{cases} \nu_1 = \left(\cos\theta|\nu_e\rangle - \sin\theta|\nu_\mu\rangle\right)\exp[-im_1 t] \\ \nu_2 = \left(\sin\theta|\nu_e\rangle + \cos\theta|\nu_\mu\rangle\right)\exp[-im_2 t] \end{cases}$$

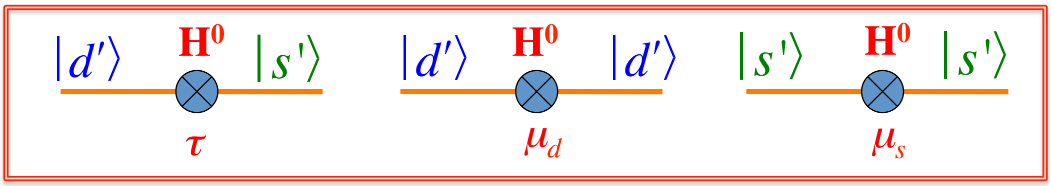
Masses: $m_{1,2} = \bar{\mu} \mp \omega$

$$\bar{\mu} = \frac{\mu_e + \mu_\mu}{2}, \quad \omega = \frac{1}{2}\sqrt{(\mu_\mu - \mu_e)^2 + 4\tau^2}$$

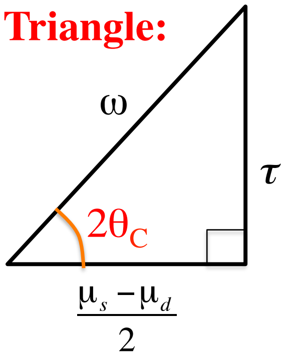
Cabbibo Angle case ($\nu_e, \nu_\mu \rightarrow d', s'$)

General neutrino state: $\Psi_q[t] = C_d[t]|d'\rangle + C_s[t]|s'\rangle$

Higgs potential (H^0) changes d' to s' and gives self-transition



Mixing Triangle:



State equation:

$$i\frac{d}{dt}\begin{pmatrix} C_d[t] \\ C_s[t] \end{pmatrix} = \begin{pmatrix} \mu_d & \tau \\ \tau & \mu_s \end{pmatrix}\begin{pmatrix} C_d[t] \\ C_s[t] \end{pmatrix}$$

Mass eigenstate:

$$\begin{cases} d = \left(\cos\theta_C |d'\rangle - \sin\theta_C |s'\rangle\right)\exp[-im_d t] \\ s = \left(\sin\theta_C |d'\rangle + \cos\theta_C |s'\rangle\right)\exp[-im_s t] \end{cases}$$

Masses: $m_{d,s} = \bar{\mu} \mp \omega$

$$\bar{\mu} = \frac{\mu_s + \mu_d}{2}, \quad \omega = \frac{1}{2}\sqrt{(\mu_s - \mu_d)^2 + 4\tau^2}$$

Important Difference

ν and quark oscillations are two extreme cases of the uncertainty principle.

$$P[t] = \sin^2 2\theta \sin^2 \frac{\Delta m}{2} t$$

This is the oscillation of particles at rest.
Usually elementary particle travels with relativistic speed.
The time delation changes;

$$\frac{\Delta m}{2} t \xrightarrow{\text{Lorentz Boost}} \frac{\Delta m}{2}\left(\frac{t}{\gamma}\right) \sim \frac{\Delta m^2}{4E} L \quad \begin{cases} \lambda_\nu \sim 10^6 \, m \cdot GeV \\ \lambda_q \sim 10^{-14} \, m \cdot GeV \end{cases}$$

ν: Measurement of the flavor evolution possible but it is impossible to distinguish ν_1 and ν_2 by their masses

q: It is possible to distinguish d and s by their masses but it is impossible to measure the evolution of d' nor s' (always superposition)

Chirality

A problem:

μ_R^- can be produced by EM interaction.
~2μs after, it decays weakly.

$$e^+ + e^- \rightarrow \gamma^* \rightarrow \mu_L^+ + \mu_R^-$$

EM

$$\hookrightarrow e^- + \nu_\mu + \bar{\nu}_e$$

Weak

But why μ_R^- can decay weakly?

Chirality Oscillation & Muon Decay

Definition of Chirality State:

$$\begin{cases} \psi_R = \dfrac{1+\gamma^5}{2}\begin{pmatrix} u \\ v \end{pmatrix} = \dfrac{u+v}{\sqrt{2}}\dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \equiv \dfrac{u+v}{\sqrt{2}}|R\rangle, \\ \psi_L = \dfrac{1-\gamma^5}{2}\begin{pmatrix} u \\ v \end{pmatrix} = \dfrac{u-v}{\sqrt{2}}\dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \dfrac{u-v}{\sqrt{2}}|L\rangle \end{cases}$$

Muon satisfies the Dirac equation

$$i\frac{d}{dt}\psi_\mu = m_\mu \gamma_0 \psi_\mu$$

Wave function can be expressed in chirality basis

$$\frac{d}{dt}\psi_\mu = \begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \frac{\dot{u}+\dot{v}}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{\dot{u}-\dot{v}}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \dot{C}_R|\mu_R\rangle + \dot{C}_L|\mu_L\rangle$$

The right-hand side of the Dirac equation is expressed as

$$\gamma_0\psi_\mu = \begin{pmatrix} u \\ -v \end{pmatrix} = \frac{u-v}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{u+v}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = C_L|\mu_R\rangle + C_R|\mu_L\rangle$$

Therefore, the Dirac equation can be expressed as

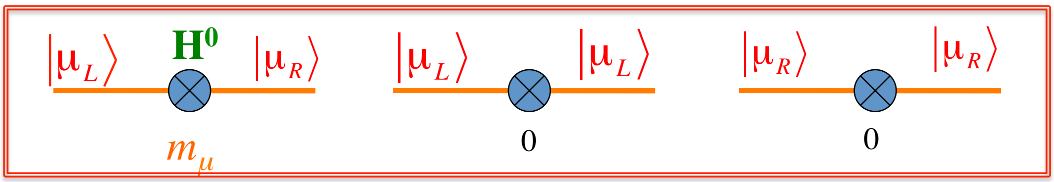
$$\Rightarrow \quad i\left(\dot{C}_L|\mu_L\rangle + \dot{C}_R|\mu_R\rangle\right) = m_\mu\left(C_R|\mu_L\rangle + C_L|\mu_R\rangle\right)$$

or,

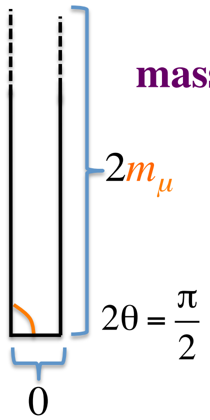
$$i\frac{d}{dt}\begin{pmatrix} C_L \\ C_R \end{pmatrix} = \begin{pmatrix} 0 & m_\mu \\ m_\mu & 0 \end{pmatrix}\begin{pmatrix} C_L \\ C_R \end{pmatrix}$$

← Dirac equation is actually chirality swapping equation

State equation = Dirac equation: $i\frac{d}{dt}\begin{pmatrix} C_L \\ C_R \end{pmatrix} = \begin{pmatrix} 0 & m_\mu \\ m_\mu & 0 \end{pmatrix}\begin{pmatrix} C_L \\ C_R \end{pmatrix}$



Mixing "Triangle"



mass eigenstates:

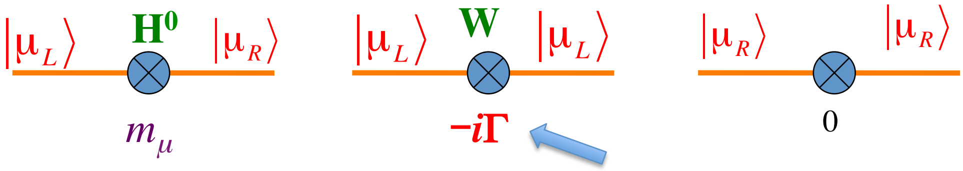
$$\begin{cases} \mu[E>0] = \frac{1}{\sqrt{2}}\left(|\mu_L\rangle + |\mu_R\rangle\right)\exp\left[-im_\mu t\right] \\ \mu[E<0] = \frac{1}{\sqrt{2}}\left(|\mu_L\rangle - |\mu_R\rangle\right)\exp\left[+im_\mu t\right] \end{cases}$$

$\mu_R \Leftrightarrow \mu_L$ Oscillation is taking place

$$P\left[\mu_R \Leftrightarrow \mu_L\right] = \sin^2 m_\mu t$$

muon decays weakly while it is in μ_L state

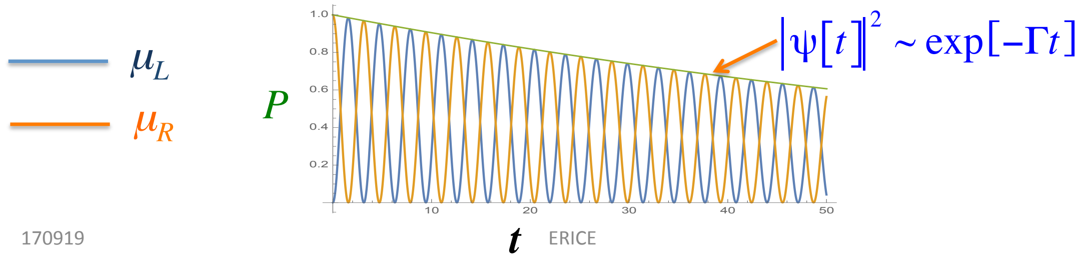
The weak decay effect can be included by putting imaginary amplitude to the μ_L self transition



State equation: $i\begin{pmatrix}\dot{C}_L \\ \dot{C}_R\end{pmatrix} = \begin{pmatrix}-i\Gamma & m_\mu \\ m_\mu & 0\end{pmatrix}\begin{pmatrix}C_L \\ C_R\end{pmatrix}$ $\quad \Gamma = \dfrac{1}{\tau_\mu} = 3\times10^{-10}\, eV \ll m_\mu$

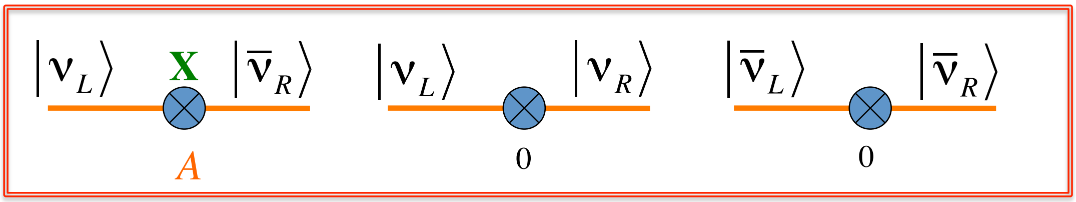
The solution for the condition $\psi[0] = |\mu_R\rangle$ **is,**

$$\psi[t] \sim \left(\cos[m_\mu t]|\mu_R\rangle - i\sin[m_\mu t]|\mu_L\rangle\right)\exp\left[-\frac{\Gamma}{2}t\right]$$



Majorana Neutrino

If something (X) transforms $|\nu_L\rangle \leftrightarrow |\bar{\nu}_R\rangle$



Dirac Mass = 0

$$i\frac{d}{dt}\begin{pmatrix} C_\nu \\ C_{\bar{\nu}} \end{pmatrix} = \begin{pmatrix} 0 & A \\ A & 0 \end{pmatrix}\begin{pmatrix} C_\nu \\ C_{\bar{\nu}} \end{pmatrix}$$

Mass eigensatate →

$$\begin{cases} \nu_+ = \dfrac{1}{\sqrt{2}}\left(|\nu_L\rangle + |\bar{\nu}_R\rangle\right)\exp[-iAt] \\ \nu_- = \dfrac{1}{\sqrt{2}}\left(|\nu_L\rangle - |\bar{\nu}_R\rangle\right)\exp[+iAt] \end{cases}$$

$$CP|\nu_\pm\rangle = \frac{1}{\sqrt{2}}\left(|\bar{\nu}_R\rangle \pm |\nu_L\rangle\right) = \pm|\nu_\pm\rangle$$

$\nu_\pm$ are regarded as Majorana neutrinos

Weinberg Angle θ_W

The Lagrangian for the interaction of the gauge boson and Higgs fields is written as,

U(1) G SU(3) G Higgs

$$\mathcal{L}_{\Phi G_{\smile}} = \frac{1}{4}\left|(g'B^\mu + g(\vec{W}^\mu\cdot\vec{\sigma}))\Phi\right|^2 \xrightarrow{\text{SSB}}$$

$$\frac{(v_0+h)^2}{8}(g^2(|W^+|^2+|W^-|^2) + \underbrace{(g^2W_3^2 + g'^2B^2 - gg'(W_3B+BW_3))}_{\text{Neutral component}})$$

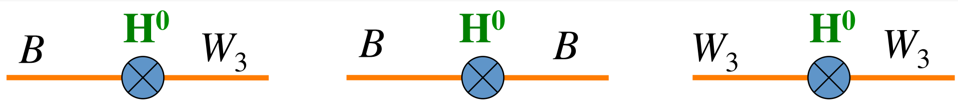
Euler-Lagrange equation
→State equation,

$$\frac{d^2}{dt^2}\begin{pmatrix}B\\W_3\end{pmatrix} = \frac{v_0^2}{4}\begin{pmatrix}-g'^2 & gg'\\ gg' & -g^2\end{pmatrix}\begin{pmatrix}B\\W_3\end{pmatrix}$$

This 2^{nd} differential equation can be rewritten by 2 times of the 1^{st} differential equation:

$$i\frac{d}{dt}\begin{pmatrix}B\\W_3\end{pmatrix} = \frac{v_0}{2\sqrt{g^2+g'^2}}\begin{pmatrix}g'^2 & -gg'\\ -gg' & g^2\end{pmatrix}\begin{pmatrix}B\\W_3\end{pmatrix}$$

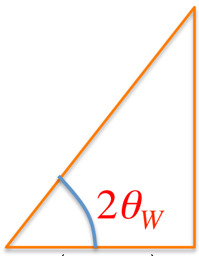
Oscillation View



$-gg'\times\frac{v_0}{2\sqrt{g^2+g'^2}}$ $g'^2\times\frac{v_0}{2\sqrt{g^2+g'^2}}$ $g^2\times\frac{v_0}{2\sqrt{g^2+g'^2}}$

geometrical average of g^2 and g'^2

Mass eigenstate
$$\begin{cases}\psi_1 = (B\cos\theta_W - W_3\sin\theta_W)\exp[-i\times 0\times t] & \Rightarrow A\\ \psi_2 = (B\sin\theta_W + W_3\cos\theta_W)\exp[-iM_Zt] & \Rightarrow Z^0\end{cases}$$



$\frac{v_0gg'}{2\sqrt{g^2+g'^2}}$

$\frac{v_0(g^2-g'^2)}{4\sqrt{g^2+g'^2}}$

$$M_Z = \frac{v_0}{2}\sqrt{g^2+g'^2},\quad M_A = 0$$

$$\tan 2\theta_W = \frac{2gg'}{g^2-g'^2} \text{ or } \tan\theta_W = \frac{g'}{g}$$

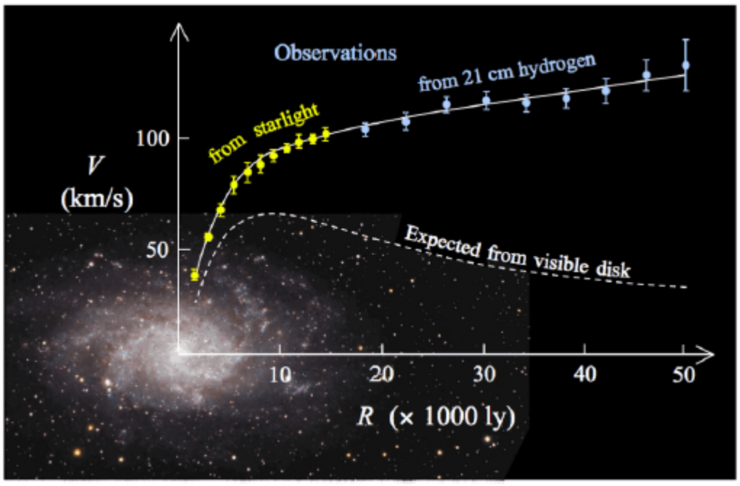
In A and Z^0, B and W_3 are oscillating very quickly.

$$P[B\Leftrightarrow W_3] = \sin^2 2\theta_W \sin^2\left[\frac{v_0\sqrt{g^2+g'^2}}{4}t\right]$$

0.71 **ω=1/1.5x10^{-26} s**

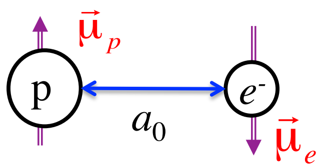
Hydrogen 21cm line

➔ **This is important for us too, because the motivation of the dark matter came from the Doppler shift of the 21cm line**



https://www.forbes.com/sites/startswithabang/2017/06/29/dark-matter-theory-triumphs-in-sweeping-new-study/#75e906452d81

Electron has spin and magnetic dipole moment.
There is dipole-dipole interaction.



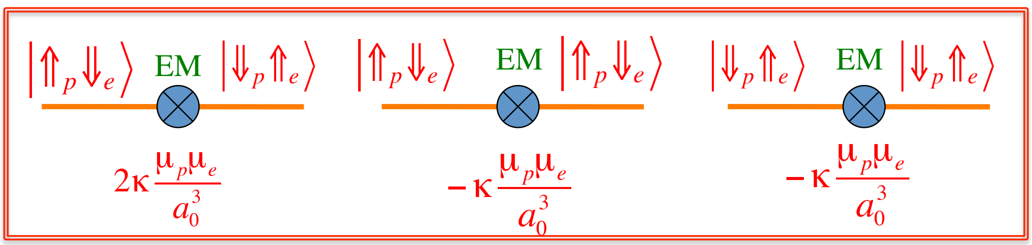
Classical Electrodynamics

$$\Delta E = \kappa \frac{1}{a_0^3}(\vec{\mu}_p \cdot \vec{\mu}_e)$$

Quantum Mechanics

$$\Rightarrow \quad i\frac{d}{dt}\psi = \kappa \frac{\mu_p \mu_e}{a_0^3}(\vec{\sigma}_p \cdot \vec{\sigma}_e)\psi$$

$$(\vec{\sigma}_p \cdot \vec{\sigma}_e)|\Uparrow_p \Downarrow_e\rangle = 2|\Downarrow_p \Uparrow_e\rangle - |\Uparrow_p \Downarrow_e\rangle, \quad \cdots$$



State equation:

$$i\frac{d}{dt}\begin{pmatrix} C_{\uparrow\downarrow} \\ C_{\downarrow\uparrow} \end{pmatrix} = \kappa\frac{\mu_p\mu_e}{a_0^3}\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} C_{\uparrow\downarrow} \\ C_{\downarrow\uparrow} \end{pmatrix}$$

Energy Eigenstate:

$$\begin{cases} |H(S=0)\rangle = \frac{1}{\sqrt{2}}\left(|\Uparrow\Downarrow\rangle - |\Downarrow\Uparrow\rangle\right)\exp\left[-i\left(M_0 - 3\kappa\frac{\mu_p\mu_e}{a_0^3}\right)t\right] \\ |H(S=1)\rangle = \frac{1}{\sqrt{2}}\left(|\Uparrow\Downarrow\rangle + |\Downarrow\Uparrow\rangle\right)\exp\left[-i\left(M_0 + \kappa\frac{\mu_p\mu_e}{a_0^3}\right)t\right] \end{cases}$$

Oscillation

$$P\left[|\Uparrow\Downarrow\rangle \leftrightarrow |\Downarrow\Uparrow\rangle\right] = \sin^2\left[2\kappa\frac{\mu_p\mu_e}{a_0^3}t\right]$$

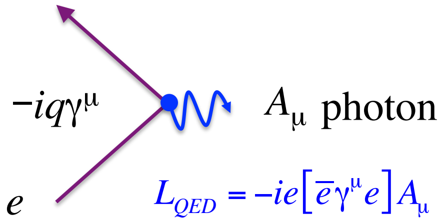
$\omega/2 = 29\mu\text{eV}$ ➔ $\lambda = 21\text{cm}$

Hadron Mass Pattern

(From analogy of hydrogen 21cm line)

Mass difference between ρ⁺(770MeV) and π⁺(140MeV)

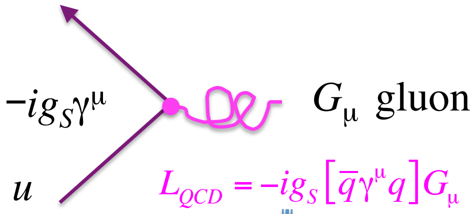
QED:



$$L_{QED} = -ie\left[\bar{e}\gamma^{\mu}e\right]A_{\mu}$$

$$\vec{\mu}_{EM} = \frac{e}{2m}\vec{\sigma}$$

QCD:



$$L_{QCD} = -ig_S\left[\bar{q}\gamma^{\mu}q\right]G_{\mu}$$

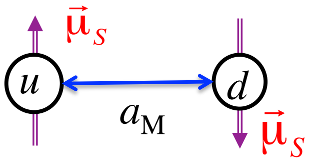
$$\vec{\mu}_S = \frac{g_S}{2m}\vec{\sigma}$$

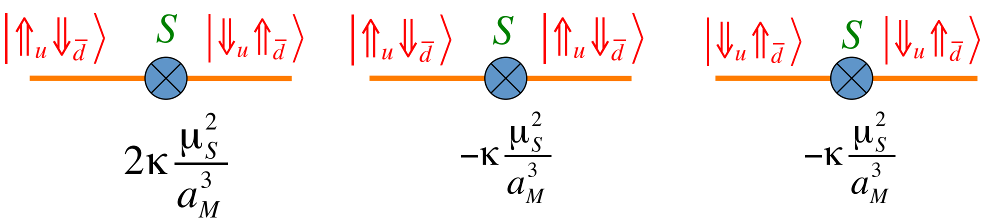
The structure of the fermion coupling is same for QED and QCD .
There is "Strong dipole moment" .

$$\mu_S = \frac{g_S}{2m_q}$$

Then we can follow the exactly same way as the Hydrogen 21cm line

Strong dipole-dipole interaction.





$$\begin{cases} |\pi^+\rangle = \frac{1}{\sqrt{2}}|u\bar{d}\rangle\left(|\Uparrow\Downarrow\rangle - |\Downarrow\Uparrow\rangle\right)\exp\left[-i\left(M_0 - 3\frac{\kappa\mu_S^2}{a_M^3}\right)t\right] \\ |\rho^+\rangle = \frac{1}{\sqrt{2}}|u\bar{d}\rangle\left(|\Uparrow\Downarrow\rangle + |\Downarrow\Uparrow\rangle\right)\exp\left[-i\left(M_0 + \frac{\kappa\mu_S^2}{a_M^3}\right)t\right] \end{cases}$$

$$\Delta m_{\rho-\pi} = \frac{4\pi\kappa\alpha_S}{m_u m_d a_M^3} = 630\,MeV$$

$$M_0 = 610\,MeV \sim 2m_q$$

$\Delta m_{\rho-\pi}$ can be estimated from hydrogen $\Delta E_{21\text{cm}}$

$$\Delta m_{\rho-\pi} \sim \Delta E_{21cm} \times \frac{1}{g_p}\left(\frac{\alpha_S}{\alpha}\right)\left(\frac{m_p m_e}{m_u m_d}\right)\left(\frac{a_0}{a_M}\right)^3$$

$$= 58[\mu eV] \times \frac{1}{2.7} \times (137 \times 0.2) \times \left(\frac{940[MeV] \times 0.5[MeV]}{300[MeV] \times 300[MeV]}\right) \times \left(\frac{5 \times 10^{-11}[m]}{10^{-15}[m]}\right)^3$$

$$\sim O[GeV]$$

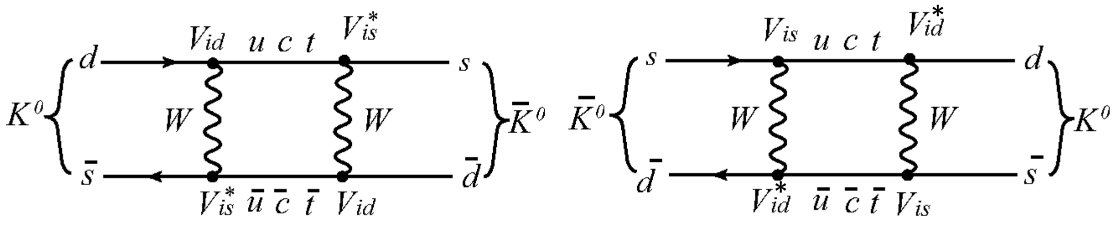
The large mass difference (630MeV) can be explained mainly by the small meson size (a_M)

-- Other hadron mass patterns can be understood similarly.

$K^0 \Leftrightarrow \bar{K}^0$ Oscillation and CP non-conservation:

A case of imaginary cross transition amplitude

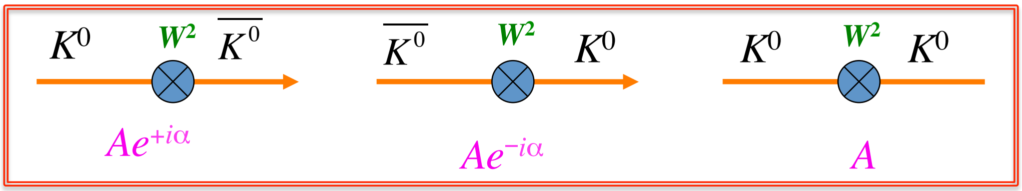
K^0, $\bar{K}^0$ system



(a) $K^0 \to \bar{K}^0$ (b) $\bar{K}^0 \to K^0$

Transition amplitudes includes imaginary number

$$\begin{cases} M\left(K^0 \to \overline{K^0}\right) = g_W^4 \left(\sum_{i=u,c,t} V_{id} V_{is}^* \Pi_K(m_i) \right)^2 = A\exp[+i\alpha] \\ M\left(\overline{K^0} \to K^0\right) = g_W^4 \left(\sum_{i=u,c,t} V_{id}^* V_{is} \Pi_K(m_i) \right)^2 = A\exp[-i\alpha] \\ M\left(K^0 \to K^0\right) = g_W^4 \left| \sum_{i=u,c,t} V_{id} V_{is}^* \Pi_K(m_i) \right|^2 = A \end{cases}$$



State equation: $$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix} = A\begin{pmatrix} 1 & e^{-i\alpha} \\ e^{i\alpha} & 1 \end{pmatrix}\begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix}$$

On the other hand, CP eigenstates are

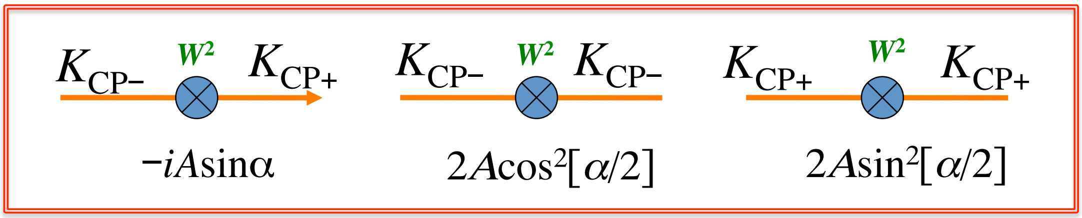
$$\begin{pmatrix} K_{CP+} \\ K_{CP-} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix}$$

$$\left(\mathrm{CP}K_{CP+} = -\frac{\overline{K^0} - K^0}{\sqrt{2}} = K_{CP+}, \quad \mathrm{CP}K_{CP-} = -\frac{\overline{K^0} + K^0}{\sqrt{2}} = -K_{CP-} \right)$$

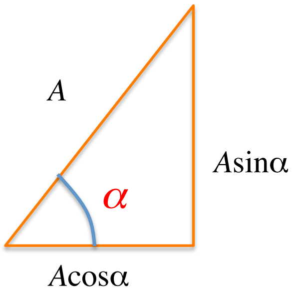
State equation of CP basis:

$$i\frac{d}{dt}\begin{pmatrix} K_{CP+} \\ K_{CP-} \end{pmatrix} = A\begin{pmatrix} 2\sin^2[\alpha/2] & -i\sin\alpha \\ i\sin\alpha & 2\cos^2[\alpha/2] \end{pmatrix}\begin{pmatrix} K_{CP+} \\ K_{CP-} \end{pmatrix}$$

Transitions between CP eigenstates



Mass eigenstate
$$\begin{cases}\psi_1 = \left(\cos[\alpha/2]K_{CP+} - i\sin[\alpha/2]K_{CP-}\right)\exp[-iM_K t] \\ \psi_2 = \left(-i\sin[\alpha/2]K_{CP+} + \cos[\alpha/2]K_{CP-}\right)\exp[-i(M_K + 2A)t]\end{cases}$$



$K_{CP+} \Leftrightarrow K_{CP-}$ **oscillation takes place**

$$P\left[K_{CP+} \Leftrightarrow K_{CP-}\right] = \sin^2\alpha \sin^2 At$$

$$\omega = 2A \sim 1/10ns$$

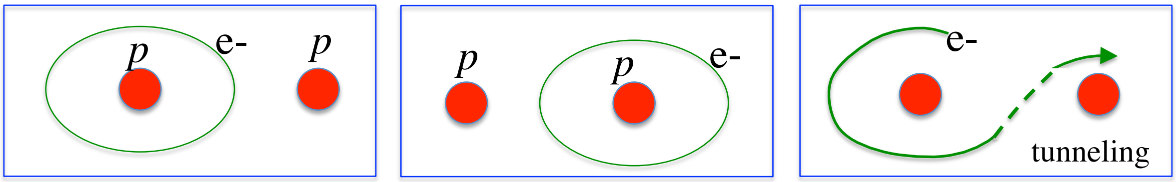
➔ **CP does not conserve**

The oscillation and mixing view is useful to understand abstract properties concretely

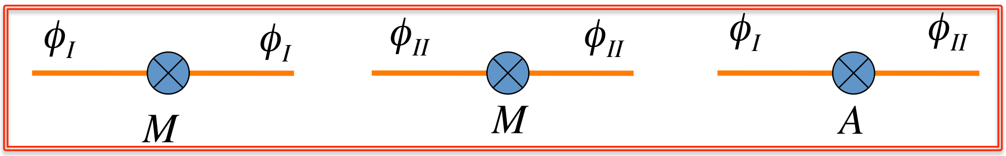
* Parity
* C-Parity
* Isospin

Parity (Feynman's explanation)

An Hydrogen ion H_2^+ has two basis states: ϕ_I and ϕ_{II}.

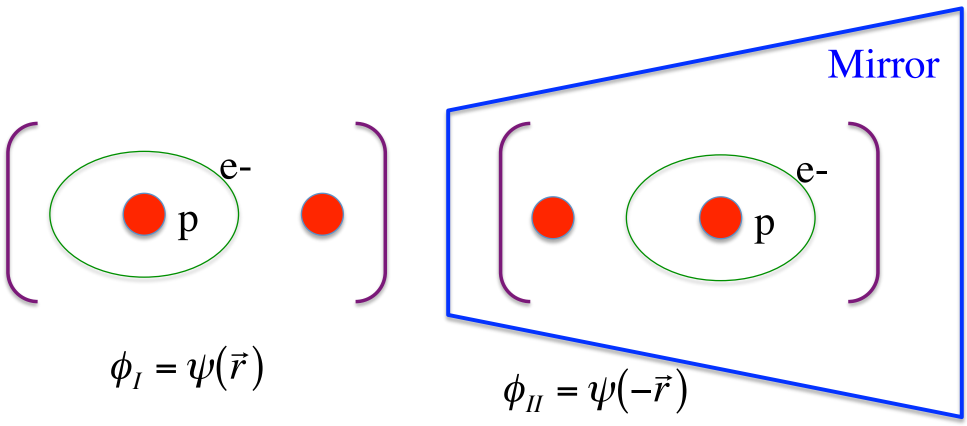


ϕ_I ϕ_{II} $\phi_I \rightarrow \phi_{II}$



➔ The energy eigenstates

$$\begin{cases} \Phi_+ = \dfrac{1}{\sqrt{2}}(\phi_I + \phi_{II})e^{-i(M+A)t} \\ \Phi_- = \dfrac{1}{\sqrt{2}}(\phi_I - \phi_{II})e^{-i(M-A)t} \end{cases}$$

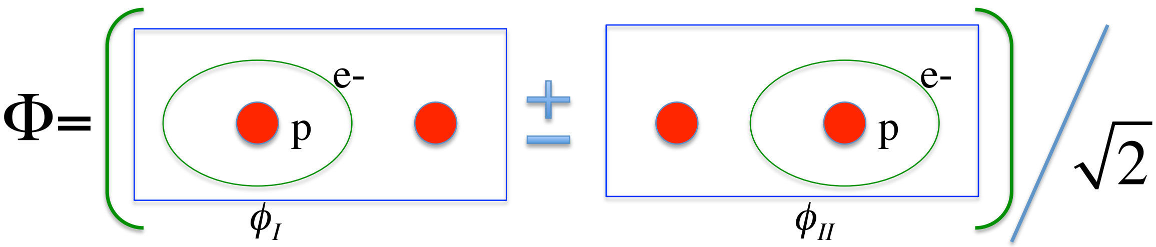


$\phi_I = \psi(\vec{r})$ $\phi_{II} = \psi(-\vec{r})$

Actually, ϕ_{II} is a mirror image of ϕ_I

Therefore,
mass eigenstates are

$$\begin{cases} \Phi_+(\vec{r}) = \dfrac{1}{\sqrt{2}}(\phi_I(\vec{r}) + \phi_I(-\vec{r}))\exp[-i(M+A)t] \\ \Phi_-(\vec{r}) = \dfrac{1}{\sqrt{2}}(\phi_I(\vec{r}) - \phi_I(-\vec{r}))\exp[-i(M-A)t] \end{cases}$$



If parity of the mass eigenstates is reversed

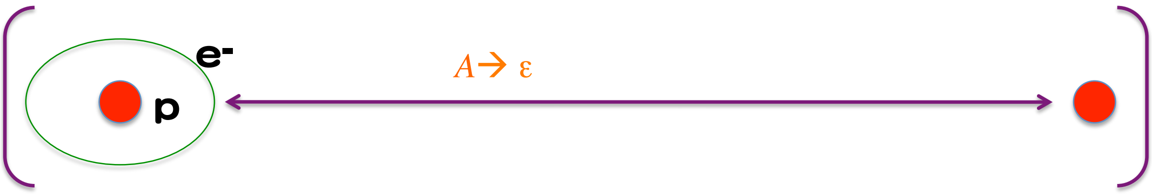
Parity = + structure

$$\begin{cases} \Phi_+(-\vec{r}) = \frac{1}{\sqrt{2}}\left(\phi_I(-\vec{r}) + \phi_I(\vec{r})\right)\exp\left[-i(M+A)t\right] = +\Phi_+(\vec{r}) \\ \Phi_-(-\vec{r}) = \frac{1}{\sqrt{2}}\left(\phi_I(-\vec{r}) - \phi_I(\vec{r})\right)\exp\left[-i(M-A)t\right] = -\Phi_-(\vec{r}) \end{cases}$$

Parity =- structure

=> Energy eigenstates have fixed parities.

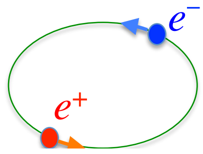
What if the protons are separate apart

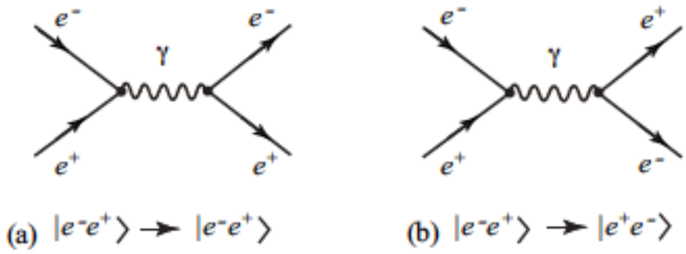


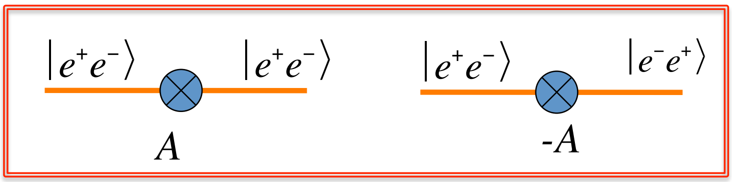
If the distance between the protons is large and the transition amplitude *A* is too small for us to distinguish the two states (energy degeneracy case), we can not see pure Φ_+ nor Φ_- state anymore.
Therefore, if we measure the parity of the system, it will not show fixed parity.

C- parity

Example: Positronium

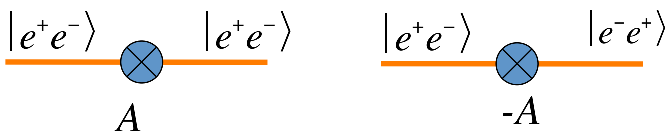


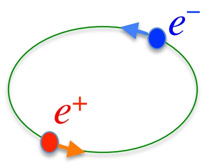




$$i\frac{d}{dt}\begin{pmatrix} C_{+-} \\ C_{-+} \end{pmatrix} = \begin{pmatrix} M+A & -A \\ -A & M+A \end{pmatrix}\begin{pmatrix} C_{+-} \\ C_{-+} \end{pmatrix}$$

C- parity





$$\begin{cases} \Psi_1 = \frac{1}{\sqrt{2}}\left(|e^+e^-\rangle + |e^-e^+\rangle\right)|S\rangle \exp[-iMt] \\ \Psi_2 = \frac{1}{\sqrt{2}}\left(|e^+e^-\rangle - |e^-e^+\rangle\right)|S\rangle \exp[-i(M+2A)t] \end{cases}$$

$$\begin{cases} C\Psi_1 = \frac{1}{\sqrt{2}}\left(|e^-e^+\rangle + |e^+e^-\rangle\right)|S\rangle \exp[-iMt] = +\Psi_1 \\ C\Psi_2 = \frac{1}{\sqrt{2}}\left(|e^-e^+\rangle - |e^+e^-\rangle\right)|S\rangle \exp[-i(M+2A)t] = -\Psi_2 \end{cases}$$

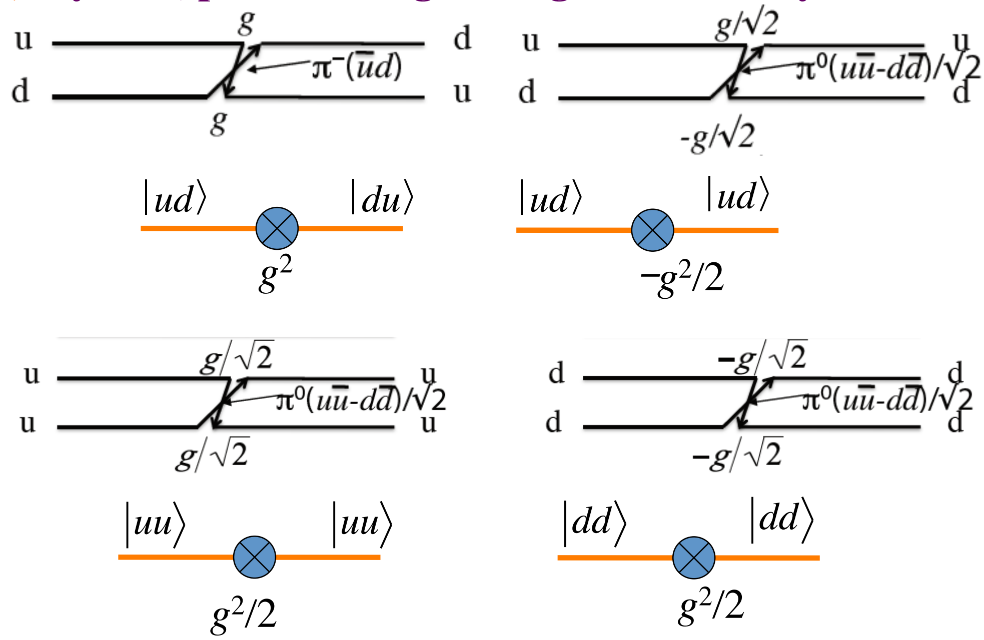
Structure for C=+1

Structure for C=-1

Instead of saying "This is C= − state.",
we may say "This is ($|e^+e^-\rangle - |e^-e^+\rangle$) state.", as well.

Isospin

For *u,d* system, pion exchange changes the basis system



State equation

$$i\frac{d}{dt}\begin{pmatrix} C_{uu} \\ C_{ud} \\ C_{du} \\ C_{dd} \end{pmatrix} = \begin{pmatrix} g^2/2 & 0 & 0 & 0 \\ 0 & -g^2/2 & g^2 & 0 \\ 0 & g^2 & -g^2/2 & 0 \\ 0 & 0 & 0 & g^2/2 \end{pmatrix}\begin{pmatrix} C_{uu} \\ C_{ud} \\ C_{du} \\ C_{dd} \end{pmatrix}$$

←This is the same form as spin dipole moment interaction (cf. 21cm line)

Mass eigenstates

$$\begin{cases} \psi_U = |uu\rangle \exp\left[-i\left(M_0 + \frac{1}{2}g^2\right)t\right] \\ \psi_D = |dd\rangle \exp\left[-i\left(M_0 + \frac{1}{2}g^2\right)t\right] \\ \psi_+ = \frac{|ud\rangle + |du\rangle}{\sqrt{2}} \exp\left[-i\left(M_0 + \frac{1}{2}g^2\right)t\right] \\ \psi_- = \frac{|ud\rangle - |du\rangle}{\sqrt{2}} \exp\left[-i\left(M_0 - \frac{3}{2}g^2\right)t\right] \end{cases}$$

Analogy to the Spin combination

3 same mass state: I=1 state

$$|\Sigma^+\rangle = |uu\rangle|s\rangle$$

$$|\Sigma^-\rangle = |dd\rangle|s\rangle$$

$$|\Sigma^0\rangle = \frac{|ud\rangle + |du\rangle}{2}|s\rangle$$

singlet state: I=0 → $|\Lambda\rangle = \frac{|ud\rangle - |du\rangle}{2}|s\rangle$

There are a lot more interesting oscillations and mixings . . .

Name	Origin	Transition	Energy eigenstate
Neutrino Oscillation	X	$\lvert\nu_e\rangle \Leftrightarrow \lvert\nu_\mu\rangle \Leftrightarrow \lvert\nu_\tau\rangle$	$\nu_1,\ \nu_2,\ \nu_3$
Cabbibo Angle	Higgs	$\lvert d'\rangle \Leftrightarrow \lvert s'\rangle$	$d,\ s$
Chirality Osc.	Higgs	$\lvert L\rangle \Leftrightarrow \lvert R\rangle$	$\lvert R\rangle \pm \lvert L\rangle$
Majorana Neutrino	X	$\lvert\nu_L\rangle \Leftrightarrow \lvert\bar\nu_R\rangle$	$\lvert\nu_L\rangle \pm \lvert\bar\nu_R\rangle$
Seesaw Mechanism	X	$\lvert\nu_R\rangle \Leftrightarrow \lvert\bar\nu_L\rangle,\ \lvert\nu_L\rangle \Leftrightarrow \lvert\nu_R\rangle$	$\nu = \lvert\nu_L\rangle - \lvert\bar\nu_R\rangle,\ N = \lvert\nu_R\rangle + \lvert\bar\nu_L\rangle$
Weinberg angle	Higgs	$W_3 \Leftrightarrow B$	$\gamma,\ Z^0$
Hydrogen 21 cm line	$\vec\mu_p \cdot \vec\mu_e$	$\lvert p(\Uparrow)e(\Downarrow)\rangle \Leftrightarrow \lvert p(\Downarrow)e(\Uparrow)\rangle$	$\lvert\Uparrow\Downarrow\rangle \pm \lvert\Downarrow\Uparrow\rangle$
$\pi^+ - \rho^+$ mass difference	Strong	$\lvert\Uparrow\Downarrow\rangle_S \Leftrightarrow \lvert\Downarrow\Uparrow\rangle_S$	$\pi^+,\ \rho^+$
CPV	Weak	$K_{CP+} \Leftrightarrow K_{CP-}$	$K_1,\ K_2$
Hydrogen Ion (H_2^+)	tunneling	$\lvert(pe^-)p\rangle \Leftrightarrow \lvert p(e^-p)\rangle$	$\lvert(pe^-)p\rangle \pm \lvert p(e^-p)\rangle$
Positronium	EM	$\lvert e^+e^-\rangle \Leftrightarrow \lvert e^-e^+\rangle$	o-Ps, p-Ps
Isospin	S	$\lvert ud\rangle \Leftrightarrow \lvert du\rangle,\ \lvert u\bar u\rangle \Leftrightarrow \lvert d\bar d\rangle$	$(\Lambda,\ \Sigma),\ (\rho,\omega)$
Baryon Color	Strong	$\lvert RGB\rangle \Leftrightarrow \lvert GRB\rangle$	$\lvert RGB\rangle - \lvert RBG\rangle + \lvert BRG\rangle - \cdots$
ρ^0, ω, ϕ structure	Strong	$\lvert u\bar u\rangle \Leftrightarrow \lvert d\bar d\rangle \Leftrightarrow \lvert s\bar s\rangle$	$\rho^0,\ \omega,\ \phi$
Spin precession in $\vec B$	$\vec\mu \vec B$	$\lvert\Uparrow\rangle \Leftrightarrow \lvert\Downarrow\rangle$	$\lvert\Uparrow\rangle_\theta$
Deuteron	S	$\lvert pn\rangle \Leftrightarrow \lvert np\rangle$	$(\lvert pn\rangle - \lvert np\rangle)\lvert\Uparrow\Uparrow\rangle$
sp^3 hybrid orbit	EM	$\psi_{2S} \Leftrightarrow \psi_{2P_i}$	$\psi_{2S} \pm \psi_{2P_x} \pm \psi_{2P_y} \pm \psi_{2P_z}$
⋮	⋮	⋮	⋮

Conclusion

*** The same mechanism as neutrino oscillation is working in various other places and is playing important physics roles.**

*** Many important physics can be understood by analogy of neutrino oscillation mechanism (or vice versa).**

*** Abstract properties, such as parity, etc. can be attributed to the structure of the mixings.**

*** It should be educative to teach such ideas to students.**

Back-up

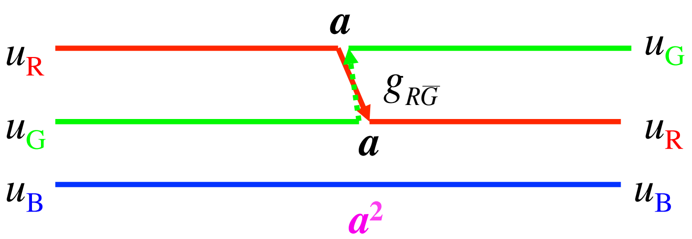
More oscillations and mixings

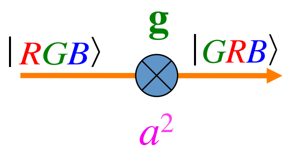
Baryon Color State

$$\Delta^{++} \rightarrow |uuu\rangle |\Uparrow\Uparrow\Uparrow\rangle |RGB\rangle$$

flavor spin color

Gluon carries color and anti-color and changes quark color

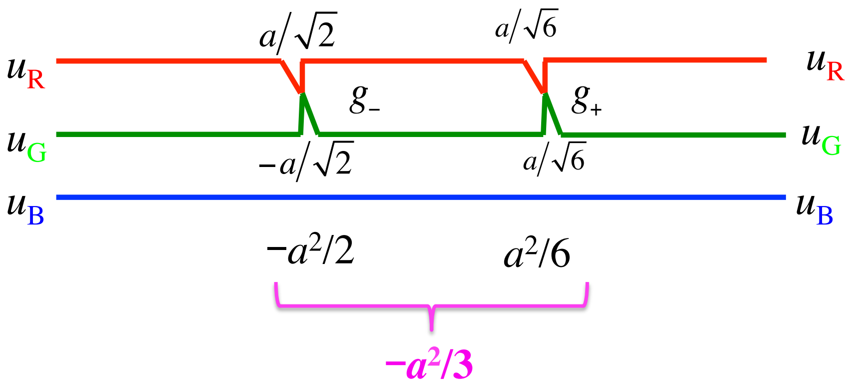


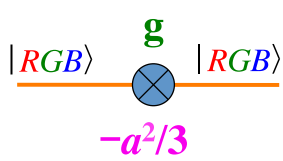


Color states are oscillating $|RGB\rangle \Leftrightarrow |GRB\rangle$

Self transition

$$|RGB\rangle \Leftrightarrow |RGB\rangle$$





State Equation:

$$i\frac{d}{dt}\begin{pmatrix} C_{RGB} \\ C_{GRB} \\ C_{GBG} \\ C_{BRG} \\ C_{GBR} \\ C_{BGR} \end{pmatrix} = \frac{a^2}{3}\begin{pmatrix} -1 & 3 & 3 & 0 & 0 & 3 \\ 3 & -1 & 0 & 3 & 3 & 0 \\ 3 & 0 & -1 & 3 & 3 & 0 \\ 0 & 3 & 3 & -1 & 0 & 3 \\ 0 & 3 & 3 & 0 & -1 & 3 \\ 3 & 0 & 0 & 3 & 3 & -1 \end{pmatrix}\begin{pmatrix} C_{RGB} \\ C_{GRB} \\ C_{GBG} \\ C_{BRG} \\ C_{GBR} \\ C_{BGR} \end{pmatrix}$$

6 Energy eigenstates

V<0; attractive

$$\begin{cases} \Psi_A = (|RGB\rangle - |GRB\rangle + |GBR\rangle - |BGR\rangle + |BRG\rangle - |RBG\rangle)/\sqrt{6}\exp\left[+i(10a^2/3)t\right] \\ \Psi_S = (|RGB\rangle + |GRB\rangle + |GBR\rangle + |BGR\rangle + |BRG\rangle + |RBG\rangle)/\sqrt{6}\exp\left[-i(8a^2/3)t\right] \\ \Psi_3 = (|GRB\rangle - |RBG\rangle)/\sqrt{2}\exp\left[+i(a^2/3)t\right] \\ \vdots \end{cases}$$

V>0; repulsive

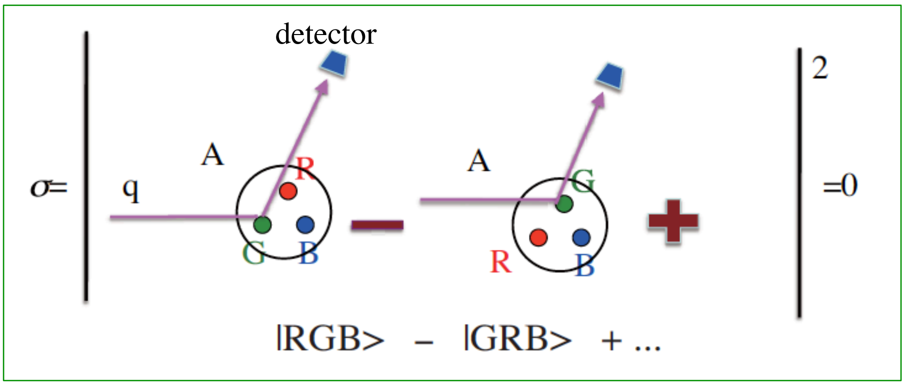
& total antisymmetricity ➔

$$|\Delta^{++}\rangle = |uuu\rangle|\Uparrow\Uparrow\Uparrow\rangle\frac{|RGB\rangle - |GRB\rangle + |GBR\rangle - |BGR\rangle + |BRG\rangle - |RBG\rangle}{\sqrt{6}}$$

Why baryon is colorless although there is no anti-color?

$$|\text{Color}\rangle = (|RGB\rangle - |GRB\rangle + |GBR\rangle - |BGR\rangle + |BRG\rangle - |RBG\rangle)/\sqrt{6}$$

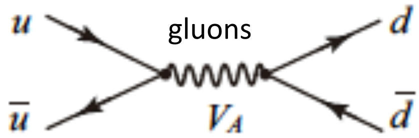
Color state oscillates in baryon; $|RGB\rangle \Leftrightarrow |GRB\rangle$ **with relative phase π.**
When we measure quark scattering, we can not know in which color state the quark was scattered. The scattering amplitude from $|RGB\rangle$ **and** $|GRB\rangle$ **cancels and there is no net scattering.**
➔ definition of the colorless.



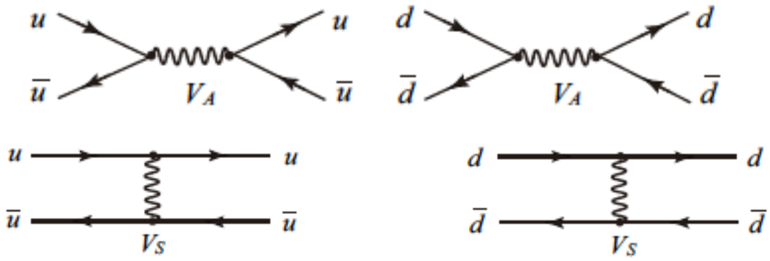
ρ^0(775MeV), ω(783MeV) mass difference

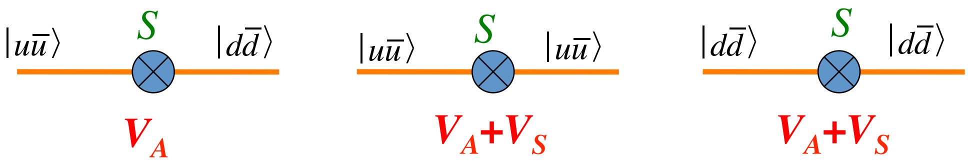
ρ^0 & ω are made of $|u\bar{u}\rangle$ and $|d\bar{d}\rangle$ states.
The 2 system transforms to each other by;

annihilation:

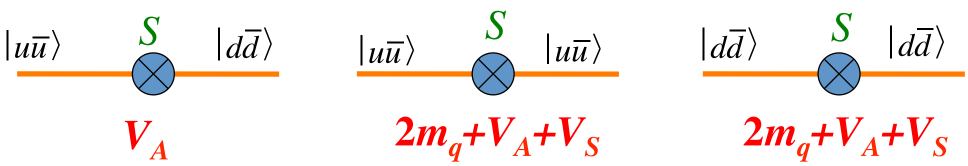


There are also self transitions





ρ^0, ω mass difference

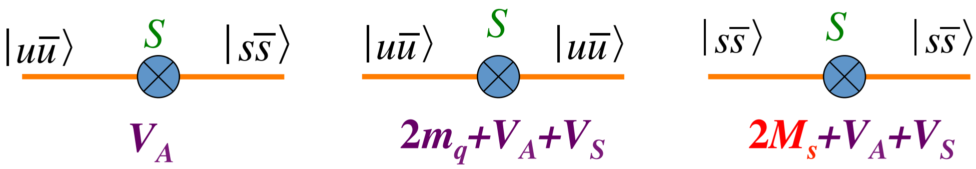


$$i\frac{d}{dt}\begin{pmatrix} C_U \\ C_D \end{pmatrix} = \begin{pmatrix} 2m_q + V_A + V_S & V_A \\ V_A & 2m_q + V_A + V_S \end{pmatrix}\begin{pmatrix} C_U \\ C_D \end{pmatrix}$$

$$\begin{cases} \rho^0 = \dfrac{|u\bar{u}\rangle - |d\bar{d}\rangle}{\sqrt{2}} \exp\left[-i\left(2m_q + V_S\right)t\right] \\ \omega = \dfrac{|u\bar{u}\rangle + |d\bar{d}\rangle}{\sqrt{2}} \exp\left[-i\left(2m_q + V_S + 2V_A\right)t\right] \end{cases}$$

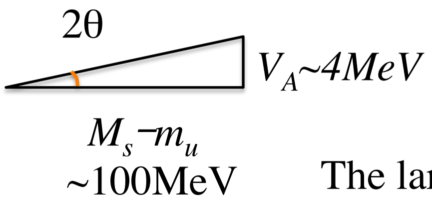
$$V_A = \frac{m_\omega - m_{\rho^0}}{2} \sim 4\,MeV$$

$|s\bar{s}\rangle \leftrightarrow |u\bar{u}\rangle$ Mixing



$$i\frac{d}{dt}\begin{pmatrix} C_U \\ C_S \end{pmatrix} = \begin{pmatrix} 2m_q+V_A+V_S & V_A \\ V_A & 2M_s+V_A+V_S \end{pmatrix}\begin{pmatrix} C_U \\ C_S \end{pmatrix}$$

$$\begin{cases} \psi_1 = \left(\cos\theta|u\bar{u}\rangle - \sin\theta|d\bar{d}\rangle\right)\exp[-iM_1 t] \\ \psi_2 = \left(\sin\theta|u\bar{u}\rangle + \cos\theta|s\bar{s}\rangle\right)\exp[-iM_2 t] \end{cases} \qquad \sin\theta \sim \frac{V_A}{2(M_S - m_q)} \sim 0.02$$

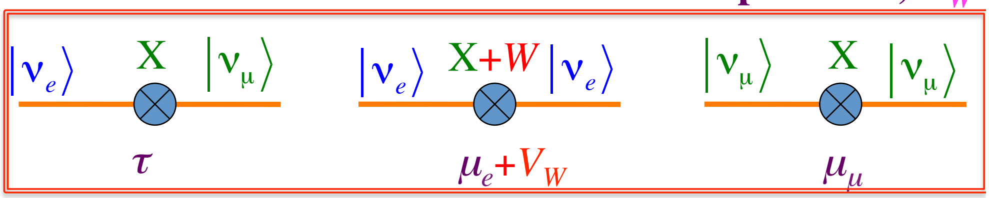


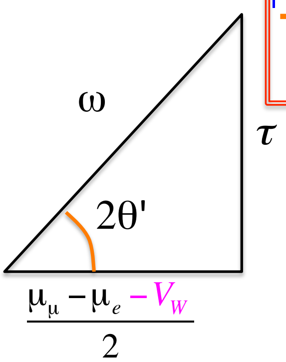
$|s\bar{s}\rangle$ component in ρ^0, ω and $|d\bar{d}\rangle$, $|u\bar{u}\rangle$ component in $|s\bar{s}\rangle$ (ϕ) is only 0.04%

The large mass of s quark enable that pure $|s\bar{s}\rangle$ can be physical meson.

Matter Effect (solar neutrino case)

Only ν_e perform this and it can be treated as additional potential; V_W





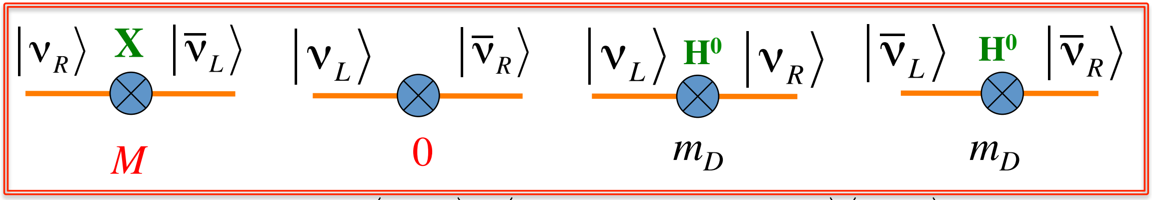
The matter effect to oscillation can be treated by just replacing $\mu_e \rightarrow \mu_e + V_W$

Oscillation:

$$\sin^2 2\theta' = \frac{4\tau^2}{\left(\mu_\mu - \mu_e - V_W\right)^2 + 4\tau^2}$$

If $\mu_e+V_W=\mu_\mu$ (resonance), $\sin^2 2\theta' = 1$:

Seesaw mechanism



$M >> m_D$

State equation:

$$i\frac{d}{dt}\begin{pmatrix}\nu_L\\ \nu_R\\ \bar{\nu}_L\\ \bar{\nu}_R\end{pmatrix} = \begin{pmatrix}0 & m_D & 0 & 0\\ m_D & 0 & M & 0\\ 0 & M & 0 & m_D\\ 0 & 0 & m_D & 0\end{pmatrix}\begin{pmatrix}\nu_L\\ \nu_R\\ \bar{\nu}_L\\ \bar{\nu}_R\end{pmatrix}$$

Mass eigenstate:

$$\psi_I = \frac{1}{\sqrt{2}}\left(|\nu_L\rangle - |\bar{\nu}_R\rangle\right)\exp\left[-i\left(m_D^2/M\right)t\right]$$ ← Very light LH Neutrino (our neutrino)

$$\psi_{II} = \frac{1}{\sqrt{2}}\left(|\nu_R\rangle + |\bar{\nu}_L\rangle\right)\exp[-iMt]$$ ← Very heavy RH Neutrino (unknown neutrino)

$$\psi_{III} = \frac{1}{\sqrt{2}}\left(|\nu_R\rangle - |\bar{\nu}_L\rangle\right)\exp[iMt]$$

$$\psi_{IV} = \frac{1}{\sqrt{2}}\left(|\nu_L\rangle + |\bar{\nu}_R\rangle\right)\exp\left[i\left(m_D^2/M\right)t\right]$$

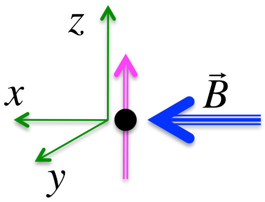
}Negative energy states

Spin precession under magnetic field

(Very basic model of oscillations)

Pauli Equation

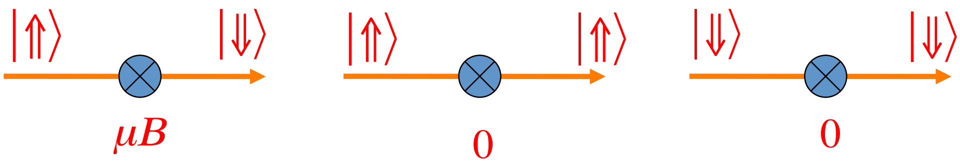
$$i\frac{d\Psi}{dt} = \mu\vec{B}\vec{\sigma}\Psi$$

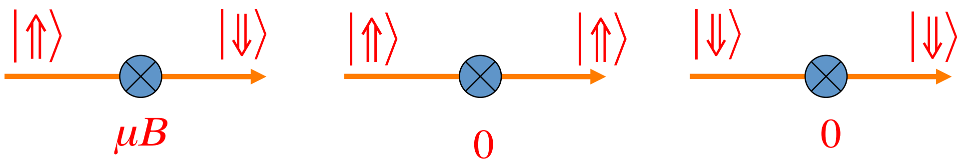


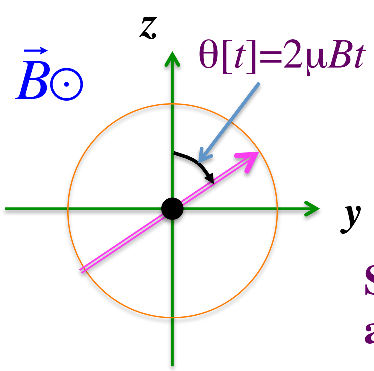
$$\Psi[t] = \alpha[t]|\Uparrow\rangle + \beta[t]|\Downarrow\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\vec{B} = B(1,0,0)$$

$$i\frac{d}{dt}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \mu B\sigma_x \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \mu B \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$







$$\begin{cases} \psi_1 = \frac{1}{\sqrt{2}}(|\Uparrow\rangle - |\Downarrow\rangle)\exp[+i\mu Bt] \\ \psi_2 = \frac{1}{\sqrt{2}}(|\Uparrow\rangle + |\Downarrow\rangle)\exp[-i\mu Bt] \end{cases}$$

$$\Psi[0] = |\Uparrow\rangle \quad \rightarrow \quad \Psi[t] = \cos[\mu Bt]|\Uparrow\rangle - i\sin[\mu Bt]|\Downarrow\rangle$$

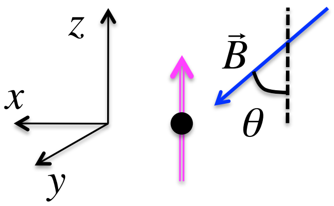
Spin precession happens within *y-z* plane with angular velocity $\omega=2\mu B$.

$$P[|\Uparrow\rangle \rightarrow |\Downarrow\rangle] = \sin^2[\mu Bt]$$

➔ ν oscillation correspond to be *a* precession.

Pauli Equation

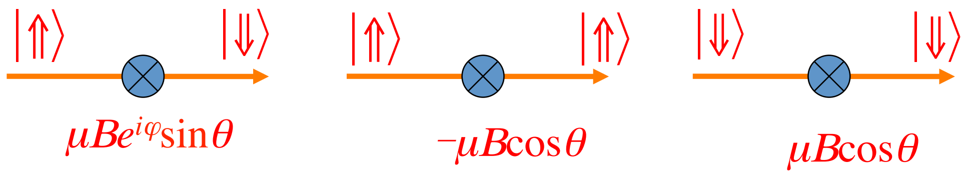
$$i\frac{d\Psi}{dt} = \mu\vec{B}\vec{\sigma}\Psi$$

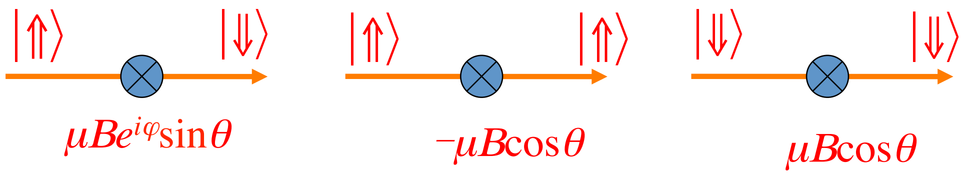


$$\Psi[t] = \alpha[t]|\Uparrow\rangle + \beta[t]|\Downarrow\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

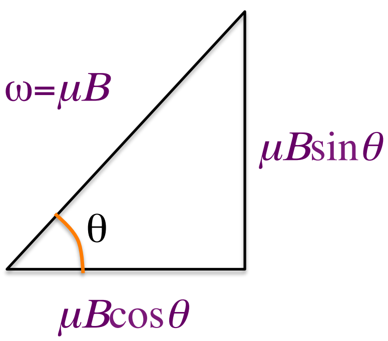
$$\vec{B} = B(\sin\theta\cos\varphi, \sin\theta\sin\varphi, -\cos\theta)$$

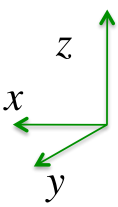
$$i\frac{d}{dt}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \mu B \begin{pmatrix} -\cos\theta & e^{-i\varphi}\sin\theta \\ e^{i\varphi}\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

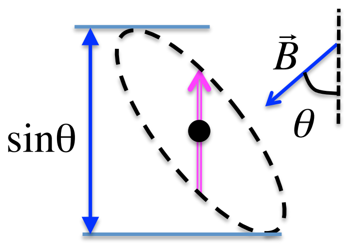




$$\begin{cases} \psi_1 = \left(|\Uparrow\rangle\cos(\theta/2) - |\Downarrow\rangle e^{i\varphi}\sin(\theta/2)\right)\exp[+i\mu Bt] \\ \psi_2 = \left(|\Uparrow\rangle e^{-i\varphi}\sin(\theta/2) + |\Downarrow\rangle\cos(\theta/2)\right)\exp[-i\mu Bt] \end{cases}$$







$$P[|\Uparrow\rangle \to |\Downarrow\rangle] = \sin^2\theta\sin^2[\mu Bt]$$