Hadron interactions from Lattice QCD

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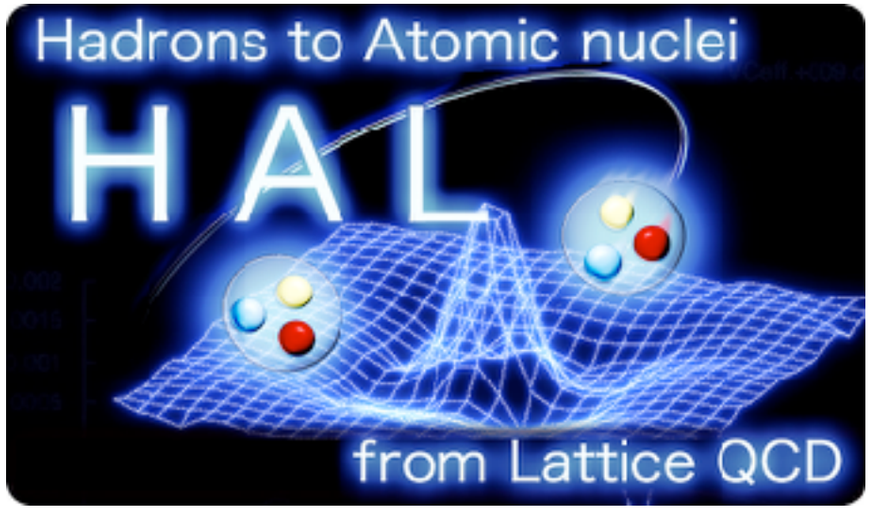


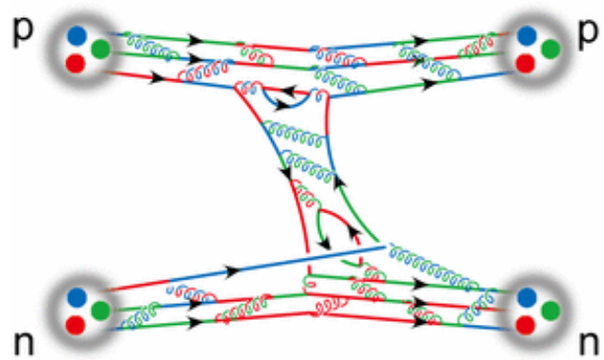
YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

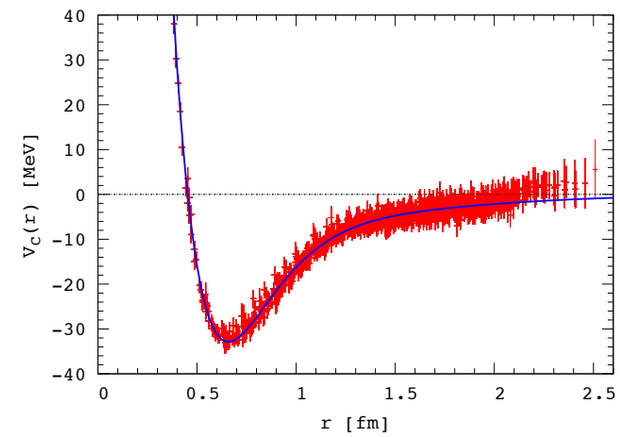
KYOTO UNIVERSITY FOUNDED 1897 KYOTO JAPAN

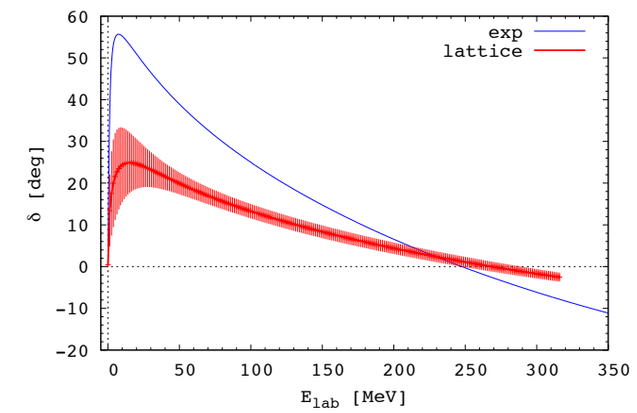
International School of Nuclear Physics 40th Source
"The Strong Interaction: From Quarks and Gluons to Nuclei and Stars"
Erice-Sicily, Italy, September 16-24, 2018
Blackett Institute (San Domenico)

HAL QCD approach to Nuclear/Astro physics

Potentials from lattice QCD

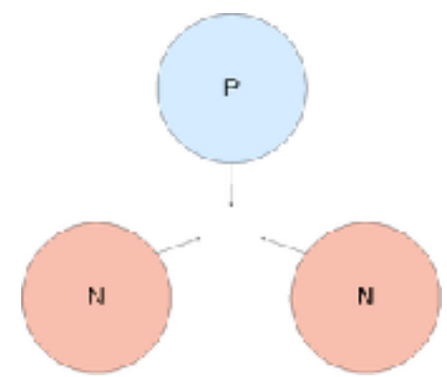






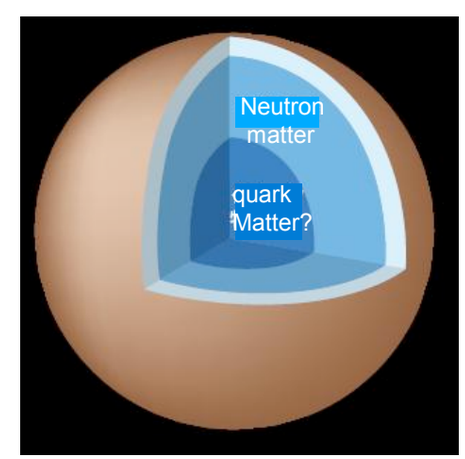
Nuclear Physics with these potentials

parameters of EFT



Neutron stars

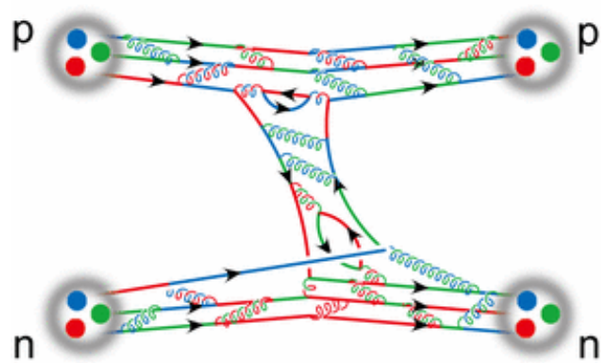
Supernova explosion

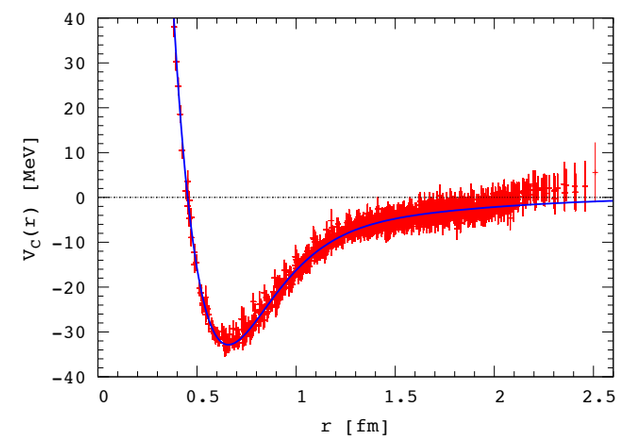


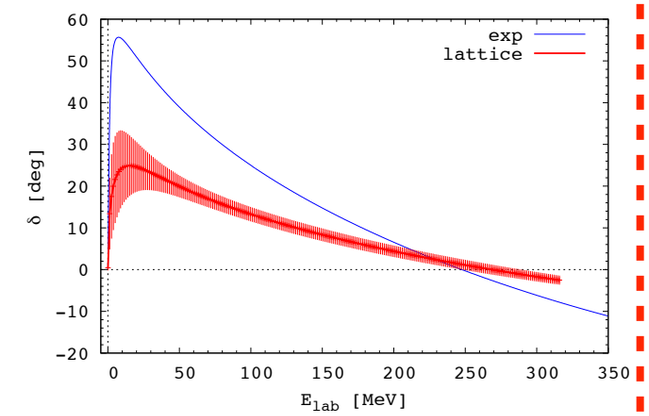
HAL QCD approach to Nuclear/Astro physics

Our task

Potentials from lattice QCD

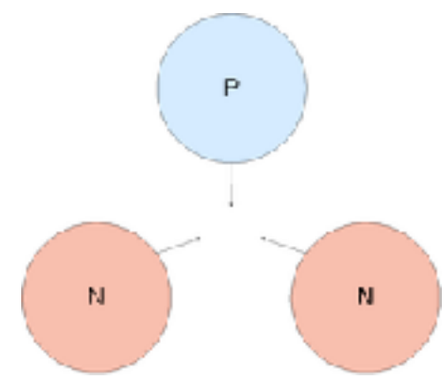






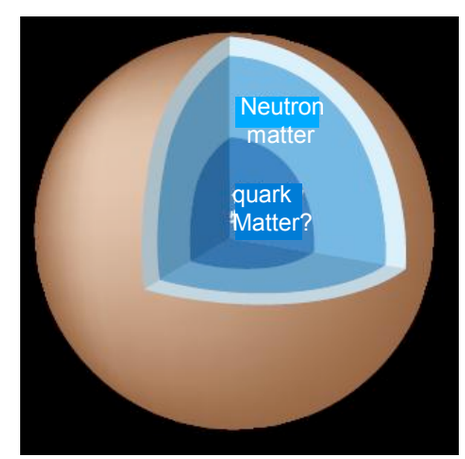
Nuclear Physics with these potentials

parameters of EFT



Neutron stars

Supernova explosion



Plan of my talk

I. HAL QCD potential method

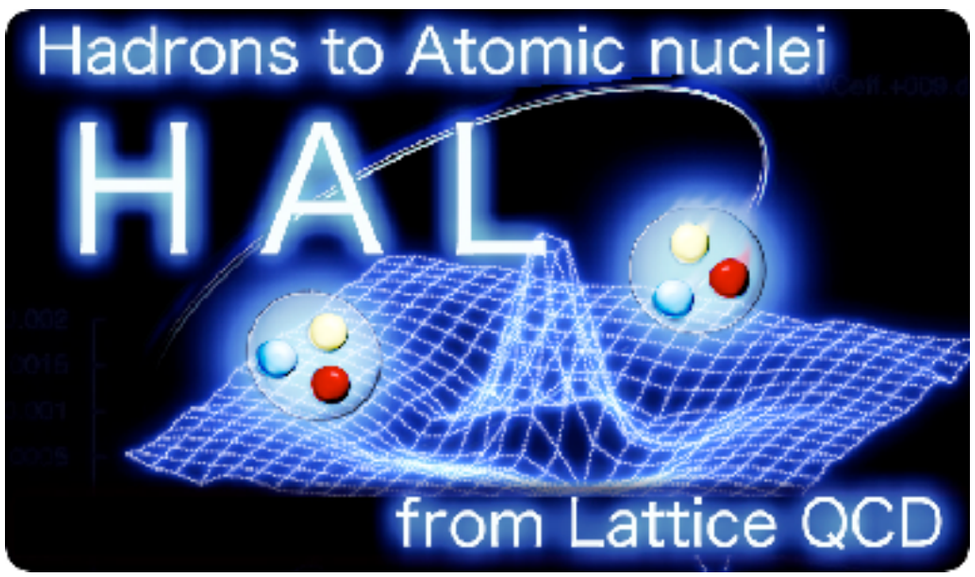
II. Recent results (selected)

1. Dibaryons

2. Exotic hadron Zc(3900)

III. Discussions

I. HAL QCD potential method



References

N. Ishii, S. Aoki, T. Hatsuda, Phys. Rev. Lett. 99 (2007) 022001,
"The Nuclear Force from Lattice QCD"

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theor. Phys. 123 (2010) 89-128,
" Theoretical Foundation of the Nuclear Force in QCD and its applications to Central and Tensor Forces in Quenched Lattice QCD Simulations "

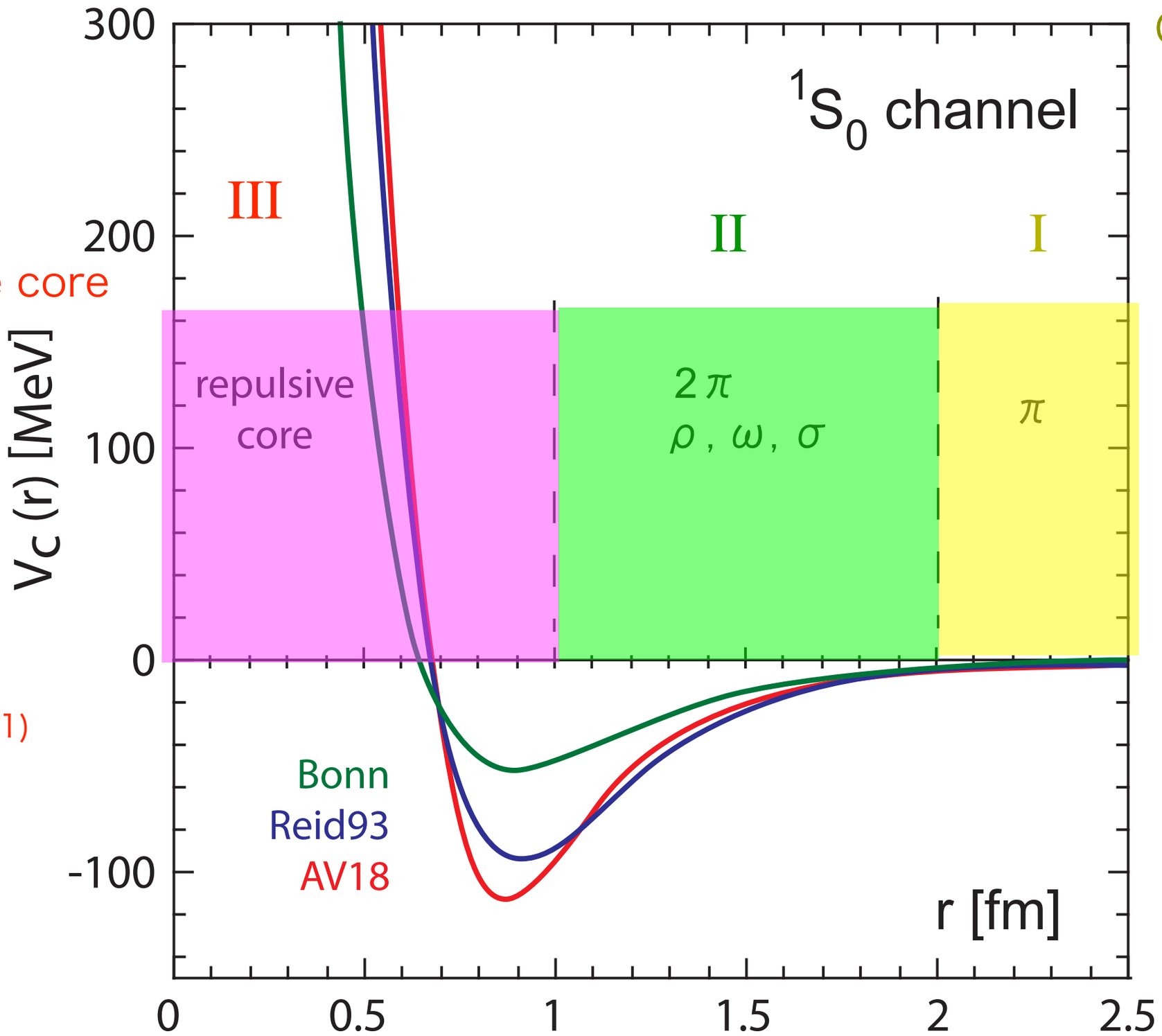
HAL QCD Collaboration (S. Aoki *et al.* ,), PTEP 2012 (2012) 01A105,
"Lattice QCD approach to Nuclear Physics"

Phenomenological NN potential

III

Repulsive core

Jastrow(1951)



I

One-pion exchange

Yukawa(1935)

II

Multi-pions

Taketani et al.(1951)

Problem

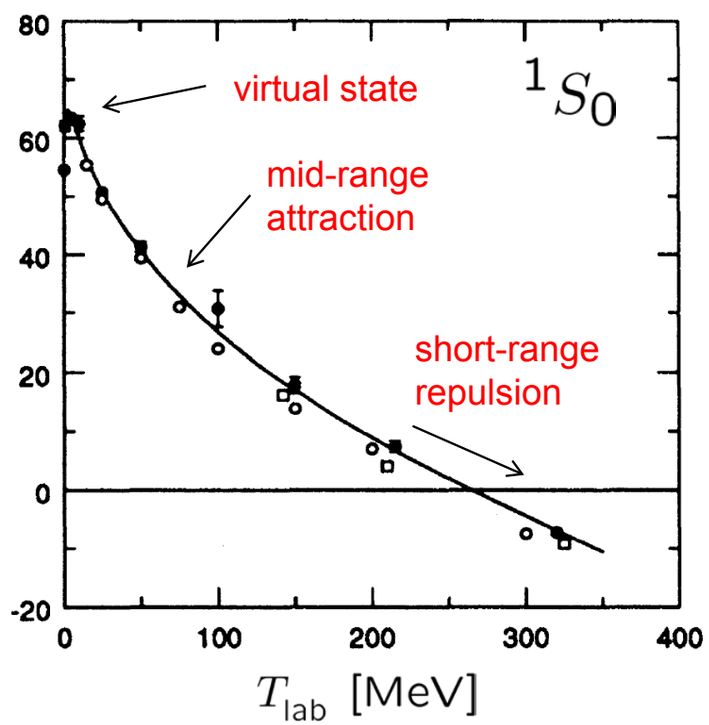
Potentials in QCD ?

What are “potentials” in quantum field theories such as QCD ?

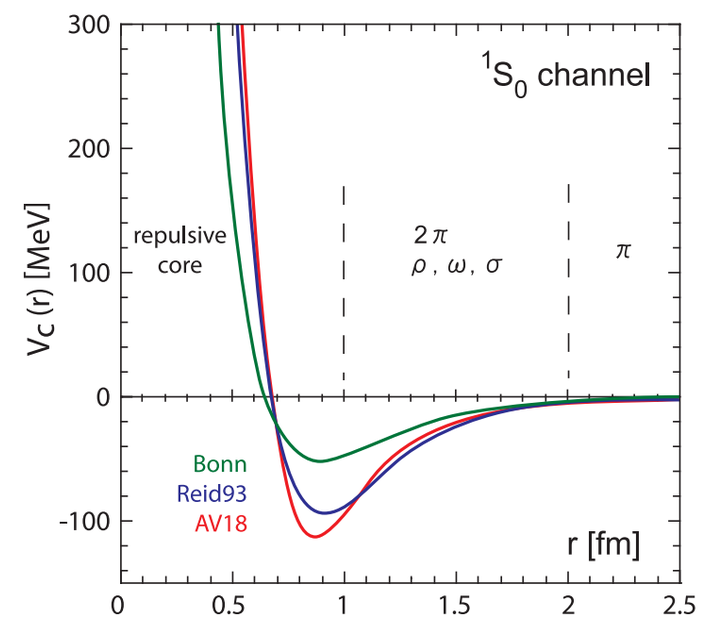
“Potentials” themselves can NOT be directly measured. analogy: running coupling

scheme dependent

experimental data of scattering phase shifts



potentials, but not unique



useful to “understand” physics

analogy: asymptotic freedom

“Potentials” are useful tools to extract scattering phase shift.

One may adopt a convenient definition for this purpose.

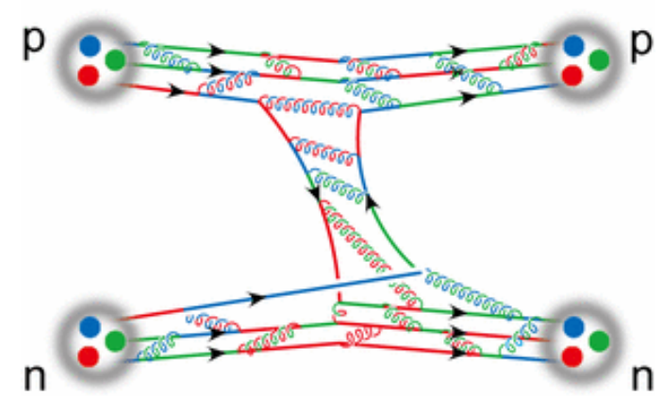
Our strategy in lattice QCD

Step 1 define (Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0|N(\mathbf{x}+\mathbf{r},0)N(\mathbf{x},0)|NN, W_k\rangle$$

QCD eigen-state

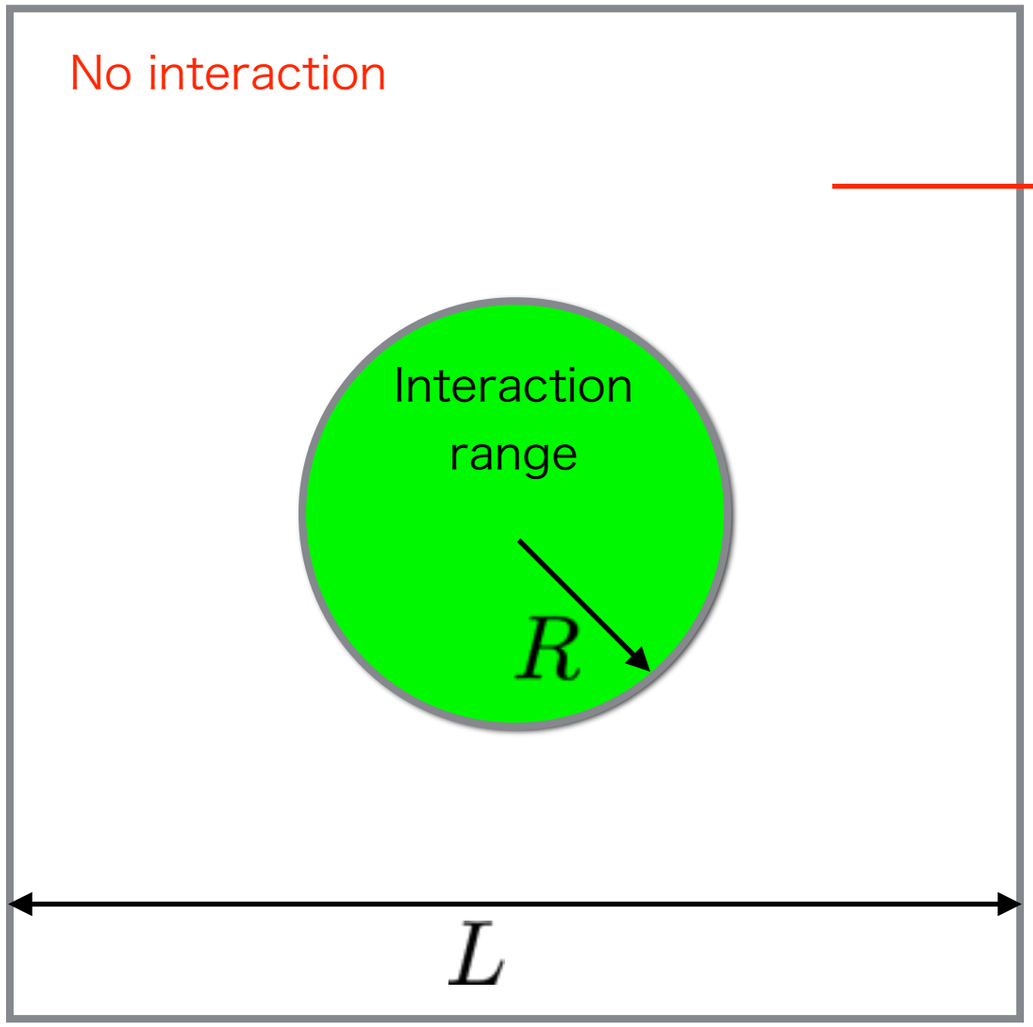
$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator "scheme"



energy $W_k = 2\sqrt{\mathbf{k}^2 + m_N^2} < W_{\rm th} = 2m_N + m_\pi$ Elastic threshold

$NN \to NN$ only elastic scattering

Main idea



Outside

$$\varphi_{\mathbf{k}}(\mathbf{r}) \simeq \sum_{l,m} C_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} Y_{lm}(\Omega_{\mathbf{r}})$$

scattering phase shift (phase of the S-matrix by unitarity) in QCD.

With PBC, the allowed momenta are restricted.

Free theory $(\delta_l(q) = 0)$ $\quad \vec{q} = \frac{2\pi}{L}\vec{n}$

Interacting theory $\quad \vec{q} - \frac{2\pi}{L}\vec{n} \neq 0$

$l = 0$ (S-wave)

$$q\cot(\delta_0(q)) = 4\pi \frac{1}{L^3} \sum_{\vec{p}\in\Gamma} \frac{1}{(\vec{p})^2 - q^2}, \qquad \Gamma = \left\{ \vec{p} = \frac{2\pi}{L}\vec{n} \right\}$$

CM frame

"Lüsher's finite volume formula"

We instead consider the inside region and extract information of interactions there.

Step 2 define the energy-independent "potential" with derivatives from the NBS wave function as

$$\boxed{\left[\epsilon_k - H_0\right]\varphi_{\mathbf{k}}(\mathbf{x}) = V(\mathbf{x},\nabla)\varphi_{\mathbf{k}}(\mathbf{x})}$$

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

For NN

$$V(\mathbf{x},\nabla) = \underbrace{V_0(r)}_{\text{LO}} + \underbrace{V_\sigma(r)(\sigma_\mathbf{1}\cdot\sigma_\mathbf{2})}_{\text{LO}} + \underbrace{V_T(r)S_{12}}_{\text{LO}} + \underbrace{V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S}}_{\text{NLO}} + \underbrace{O(\nabla^2)}_{\text{NNLO}}$$

tensor operator $S_{12} = \frac{3}{r^2}(\sigma_\mathbf{1}\cdot\mathbf{x})(\sigma_\mathbf{2}\cdot\mathbf{x}) - (\sigma_\mathbf{1}\cdot\sigma_\mathbf{2})$ spins

By construction

$$\boxed{\text{potential } V(\mathbf{x},\nabla) \text{ is faithful to QCD phase shift } \delta_l(k).}$$

Remark

Non-relativistic approximation is NOT used. We just take the specific (equal-time) frame.

Extraction of the potential (scalar without symmetry)

Define $$V(\mathbf{x}, \nabla, L) := \sum_{\mathbf{n}}{}' V_L^{(\mathbf{n})}(\mathbf{x}) \nabla^{(\mathbf{n})} \qquad \nabla^{(\mathbf{n})} := \nabla_x^{n_x} \nabla_y^{n_y} \nabla_z^{n_z}$$

A sum $\sum_{\mathbf{n}}{}'$ is restricted to $W_{\mathbf{k}} < W_{\mathrm{th}}$, where $\mathbf{k} = \frac{2\pi}{L}\mathbf{n}$.

Coefficient functions $V^{(\mathbf{n})}(\mathbf{x})$ can be determined from

$$\sum_{\mathbf{n}}{}' V_L^{(\mathbf{n})}(\mathbf{x}) \nabla^{(\mathbf{n})} \varphi_{\mathbf{k}}(\mathbf{x}) = [\epsilon_k - H_0]\, \varphi_{\mathbf{k}}(\mathbf{x}) \qquad W_{\mathbf{k}} \leq W_{\mathrm{th}}$$

a number of unknowns = a number of equations $\Rightarrow$ $V_L^{(\mathbf{n})}(\mathbf{x})$

$$\boxed{V(\mathbf{x}, \nabla) := \lim_{L\to\infty} V(\mathbf{x}, \nabla, L) = \lim_{L\to\infty} \sum_{\mathbf{n}}{}' V_L^{(\mathbf{n})}(\mathbf{x}) \nabla^{\mathbf{n}}}$$

If the volume is sufficiently large, $$V(\mathbf{x}, \nabla) \simeq \sum_{\mathbf{n}}{}' V_L^{(\mathbf{n})}(\mathbf{x}) \nabla^{\mathbf{n}}$$

Step 3 Determination of local terms

$$V_0^{\rm LO}(r) := V_0(r) + V_\sigma(r)(\sigma_1 \cdot \sigma_2) + V_T(r) S_{12}$$

One $\varphi_{\mathbf{k}}(\mathbf{x})$ → $V_0^{\rm LO}(r;\varphi_{\mathbf{k}}) = \dfrac{[\epsilon_{\mathbf{k}} - H_0]\,\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$ LO approximation

Another $\varphi_{\mathbf{q}}(\mathbf{x})$ → $V_0^{\rm LO}(r;\varphi_{\mathbf{q}}) = \dfrac{[\epsilon_{\mathbf{q}} - H_0]\,\varphi_{\mathbf{q}}(\mathbf{x})}{\varphi_{\mathbf{q}}(\mathbf{x})}$ LO approximation

If $V_0^{\rm LO}(r;\varphi_{\mathbf{q}}) \simeq V_0^{\rm LO}(r;\varphi_{\mathbf{k}})$ → **LO approximation is good at** $|\mathbf{k}| \leq |\mathbf{p}| \leq |\mathbf{q}|$

If $V_0^{\rm LO}(r;\varphi_{\mathbf{q}}) \neq V_0^{\rm LO}(r;\varphi_{\mathbf{k}})$ → **NLO term can be determined from**

$$[\epsilon_{\mathbf{k}} - H_0)]\,\varphi_{\mathbf{k}} = \left[V_0^{\rm NLO}(r) + V_1^{\rm NLO}(r)\mathbf{L}\cdot\mathbf{S}\right]\varphi_{\mathbf{k}}$$
$$[\epsilon_{\mathbf{q}} - H_0)]\,\varphi_{\mathbf{q}} = \left[V_0^{\rm NLO}(r) + V_1^{\rm NLO}(r)\mathbf{L}\cdot\mathbf{S}\right]\varphi_{\mathbf{q}}$$

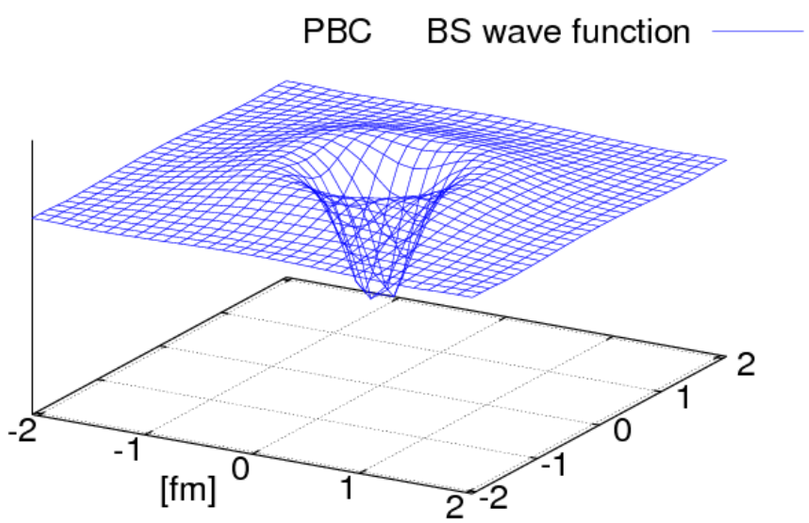
Example in quenched QCD

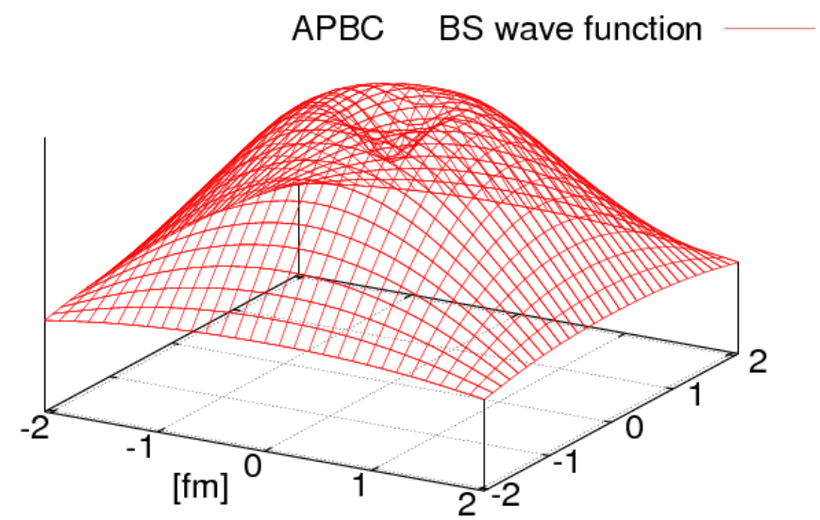
PTP 125 (2011)1225. K. Murano, N. Ishii, S. Aoki, T. Hatsuda

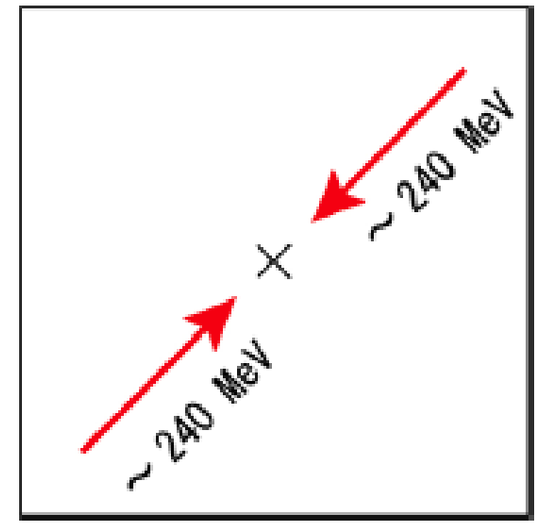
a=0.137fm, L=4.0 fm $m_\pi \simeq 0.53$ GeV

NBS wave functions

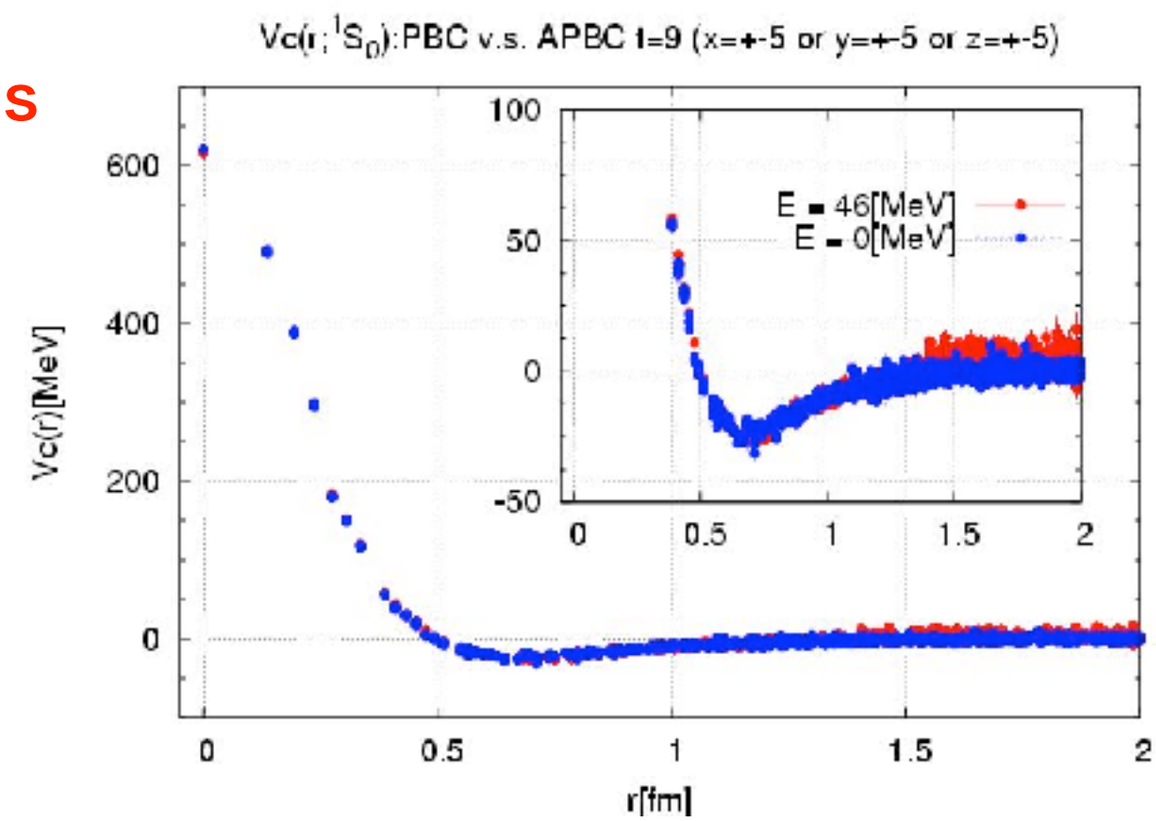
- PBC (E∼0 MeV)
- APBC (E∼46 MeV)







potentials

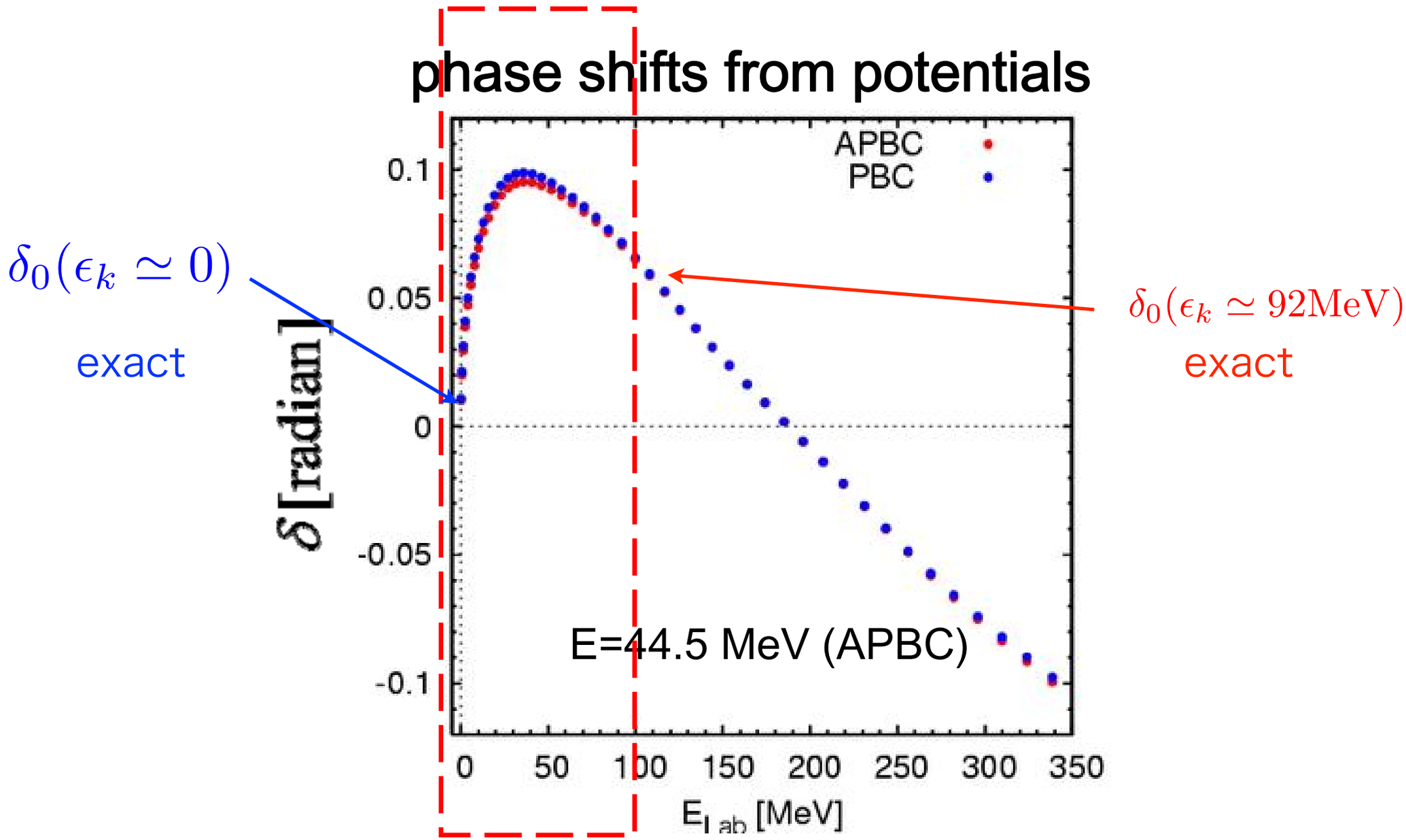


$$V_0^{\rm LO}(r;\varphi_{\mathbf{q}}) \simeq V_0^{\rm LO}(r;\varphi_{\mathbf{k}})$$

Higher order terms turn out to be very small at low energy in our scheme.

Step 4 solve the Schroedinger Eq. in the infinite volume with the potential.

phase shifts and binding energy below inelastic threshold



$\delta_0(\epsilon_{\mathbf{k}})$ is reliable at $\epsilon_{\mathbf{k}} < 92$ MeV.

Advantage of potential method

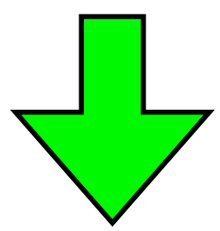
Direct method

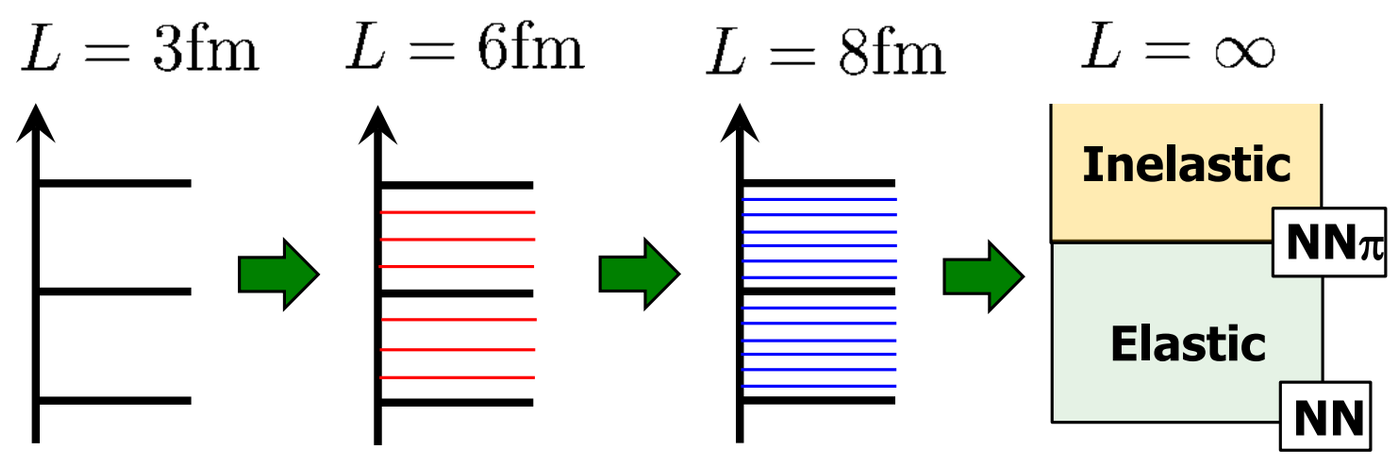
$$G_{NN}(t) = \langle N(t)N(t)\bar{N}(0)\bar{N}(0)\rangle = Z_0 e^{-E_0 t} + Z_1 e^{-E_1 t} + \cdots \to Z_0 e^{-E_0 t}, \quad t \to \infty$$

Excitation energy

finite volume energy for scattering state $\simeq \dfrac{1}{m_N}\dfrac{(2\pi)^2}{L^2}$

$E_1 - E_0 \simeq 50$ MeV at $L = 4$ fm





$t \gg 1/(E_1 - E_0) \simeq 4$ fm is needed to suppress excited states.

Large time separation is very difficult due to the bad signal-to-noise ratio.

$$\frac{S}{N} := \frac{\langle N^2(t)\bar{N}^2(t)\rangle}{\sqrt{\langle |N^2(t)\bar{N}^2(t)|^2\rangle}} \sim \exp[-(2m_N - 3m_\pi)t]$$

Improved extraction of potentials

4-pt Correlation function

$$F(\mathbf{r}, t-t_0) = \langle 0|T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\}\underline{\overline{\mathcal{J}}(t_0)}|0\rangle \quad = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-W_n(t-t_0)} + \cdots$$

source for NN

Normalized 4-pt function

$$R(\mathbf{r},t) \equiv F(\mathbf{r},t)/G_N(t)^2 = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-\Delta W_n t}$$

a sum of many NBS wave functions

$$V(\mathbf{x},\nabla)\varphi^{W_0}(\mathbf{x}) = (E_{W_0} - H_0)\varphi^{W_0}(\mathbf{x})$$

$$V(\mathbf{x},\nabla)\varphi^{W_1}(\mathbf{x}) = (E_{W_1} - H_0)\varphi^{W_1}(\mathbf{x})$$

$$\cdots$$

controlled by the same V

$$\left[\frac{\mathbf{k}_n^2}{m_N} - H_0\right]\varphi^{W_n} = V \cdot \varphi^{W_n}$$

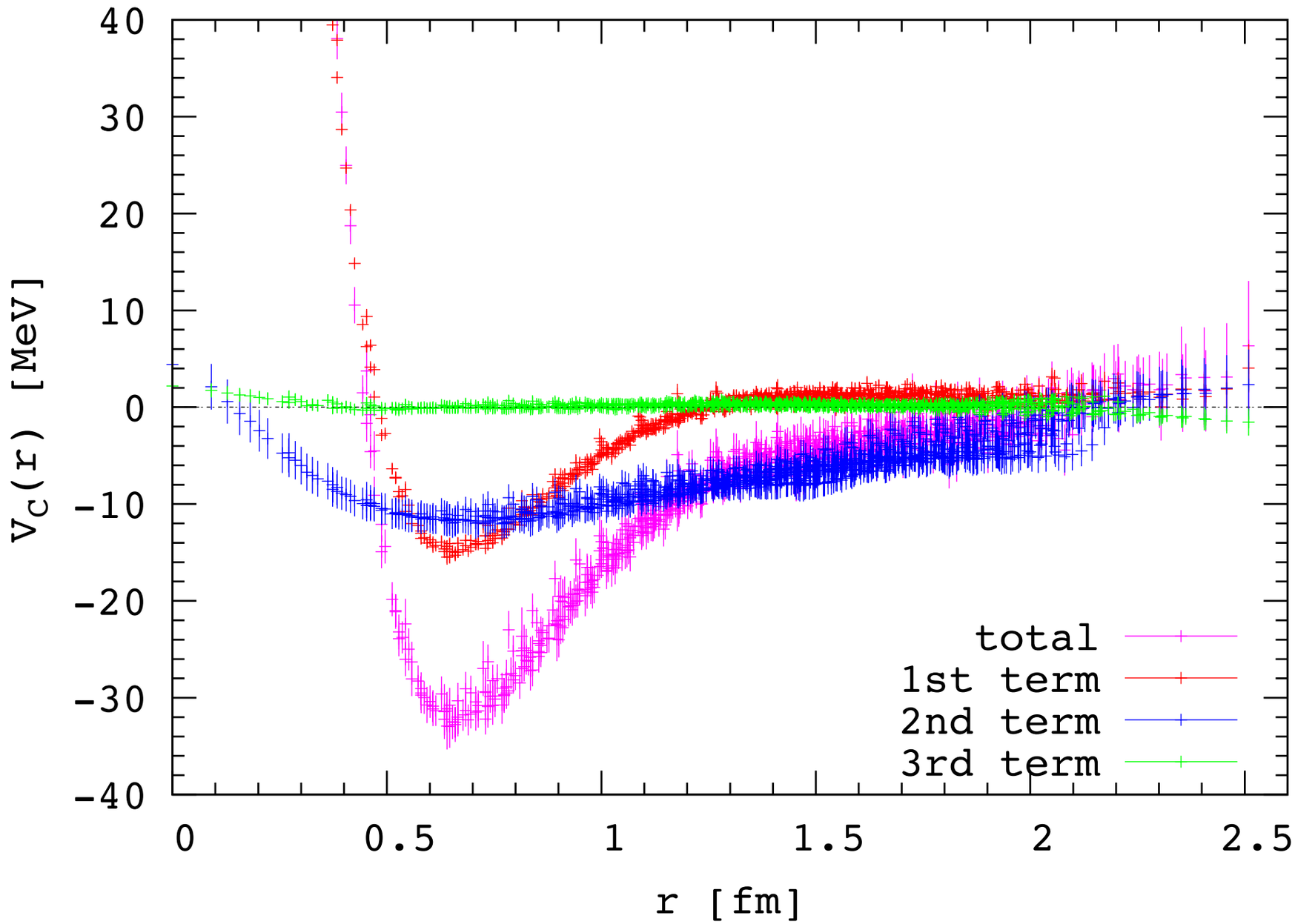
$$\Delta W_n = W_n - 2m_N = \frac{\mathbf{k}_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N} \quad \longrightarrow \quad -\frac{\partial}{\partial t}R(\mathbf{r},t) = \left\{H_0 + V - \frac{1}{4m_N^2}\frac{\partial^2}{\partial t^2}\right\}R(\mathbf{r},t)$$

$$W_n = 2\sqrt{m_N^2 + \mathbf{k}_n^2}$$

$$\left\{-H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N^2}\frac{\partial^2}{\partial t^2}\right\} R(\mathbf{r},t) = V(\mathbf{r},\nabla)R(\mathbf{r},t) = V_C(\mathbf{r})R(\mathbf{r},t) + \cdots$$

1st 2nd 3rd

Potential



3rd term(relativistic correction) is negligible.

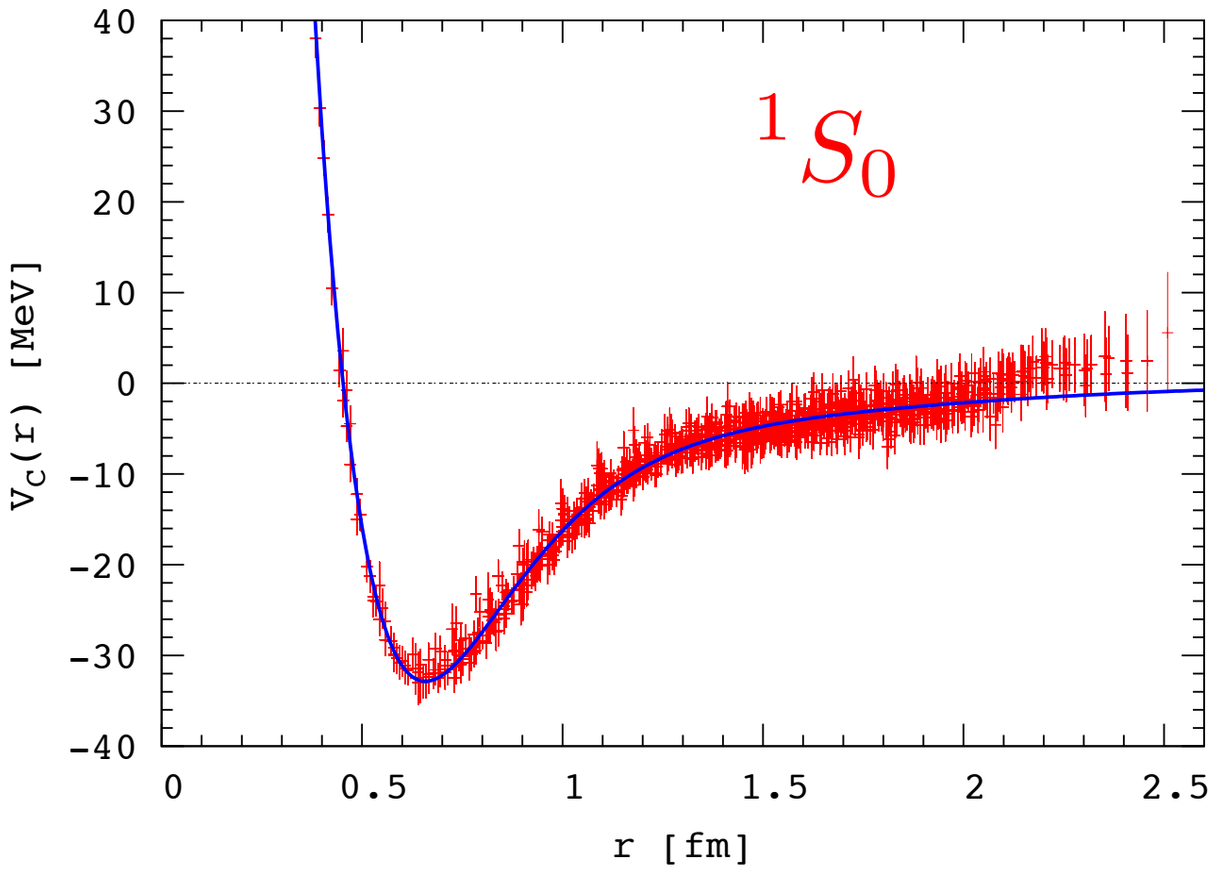
This time-dependent method overcomes the difficulty of the direct method.

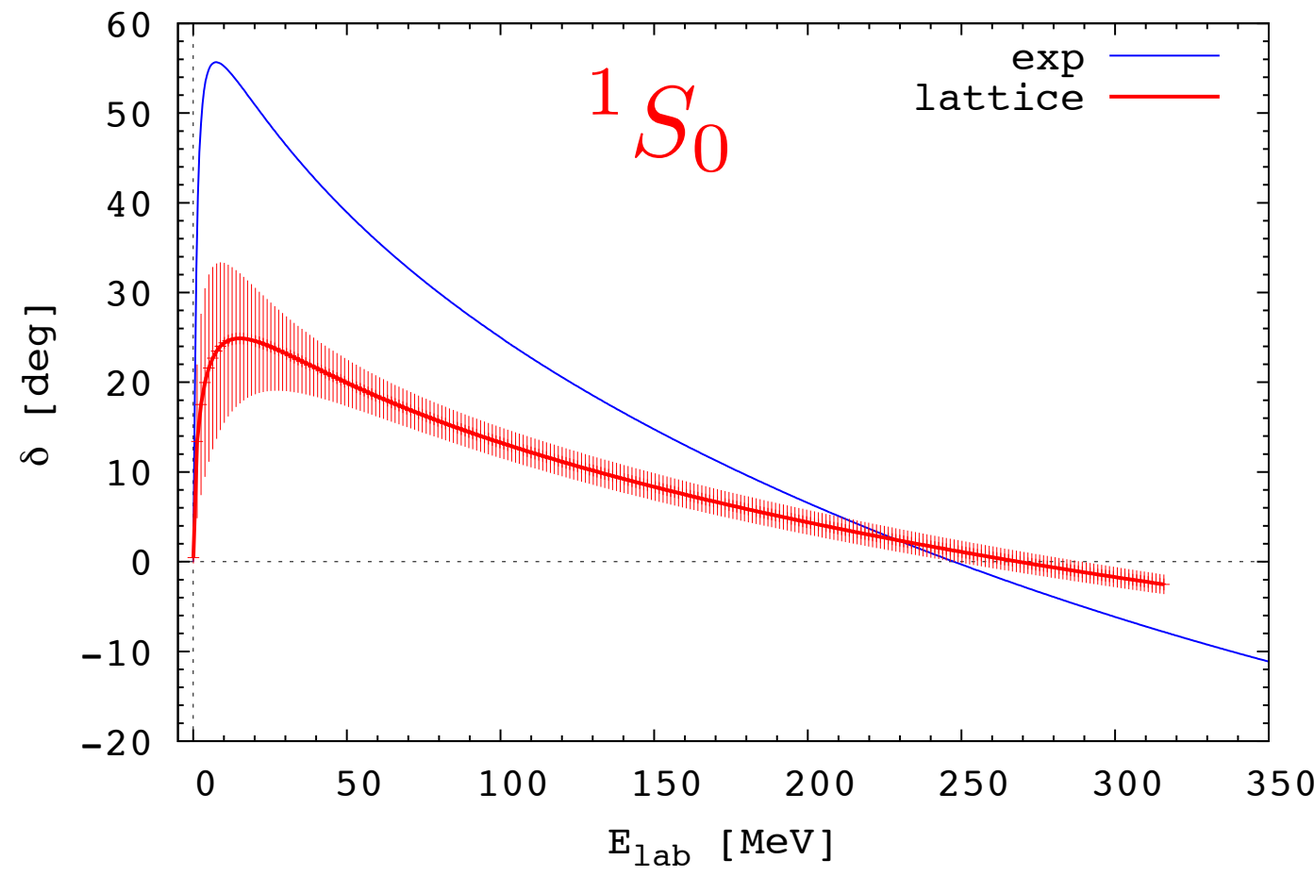
2+1 flavor QCD a=0.09fm, L=2.9fm

Ishii et al. (HALQCD), PLB712(2012) 437.

$m_\pi \simeq 700$ MeV

NN potential → phase shift





Qualitative features of NN potential are reproduced. → It has a reasonable shape. The strength is weaker due to the heavier quark mass.

No dineutron at heavier pion mass.

Systematics: derivative expansion

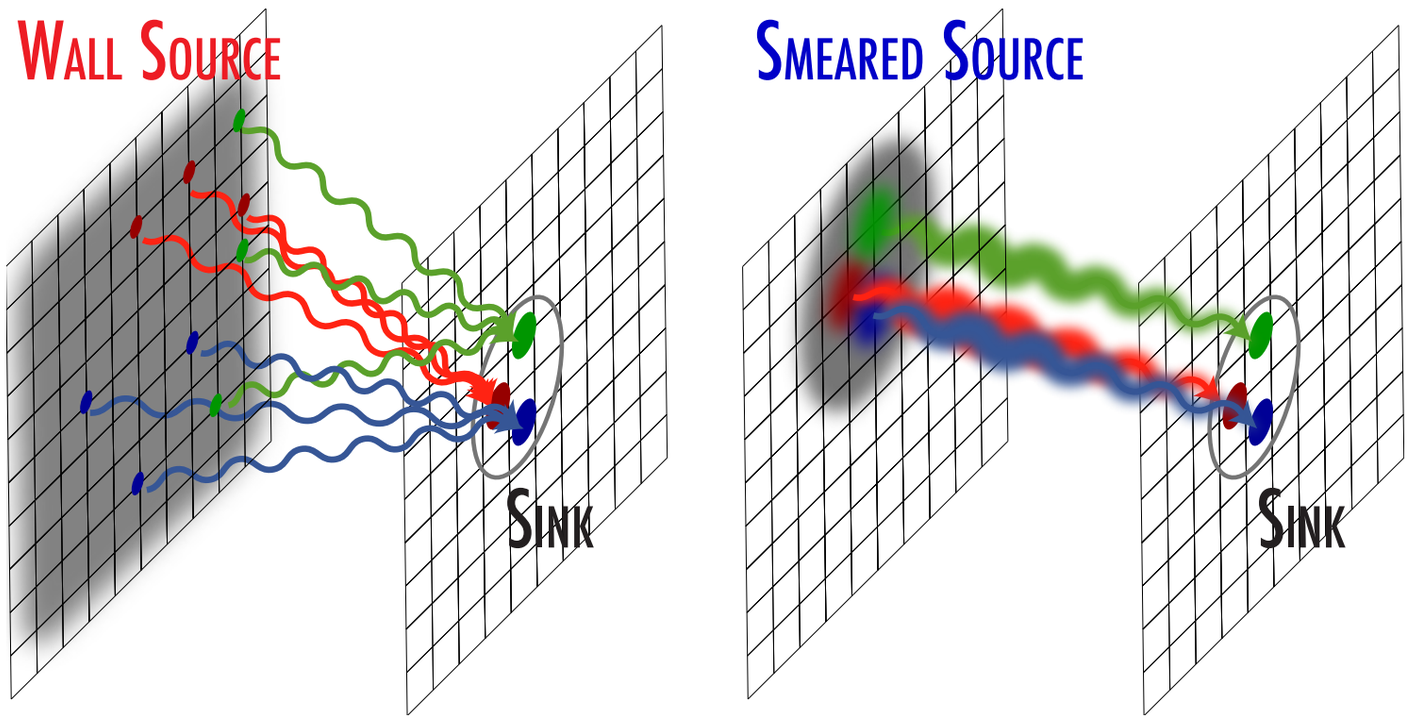
T. Iritani, Talk at Lat2016, arXiv1610.09779[hep-lat].

T. Iritani, Talk at Lat2017

T. Iritani et al., arXiv:1805.02365[hep-lat].

Two source operators

quark wall source vs quark smeared source



$$\sum_{\mathbf{y}} q(\mathbf{y}, t_0) \qquad \sum_{\mathbf{y}} e^{-B|\mathbf{x}_0 - \mathbf{y}|} q(\mathbf{y}, t_0)$$

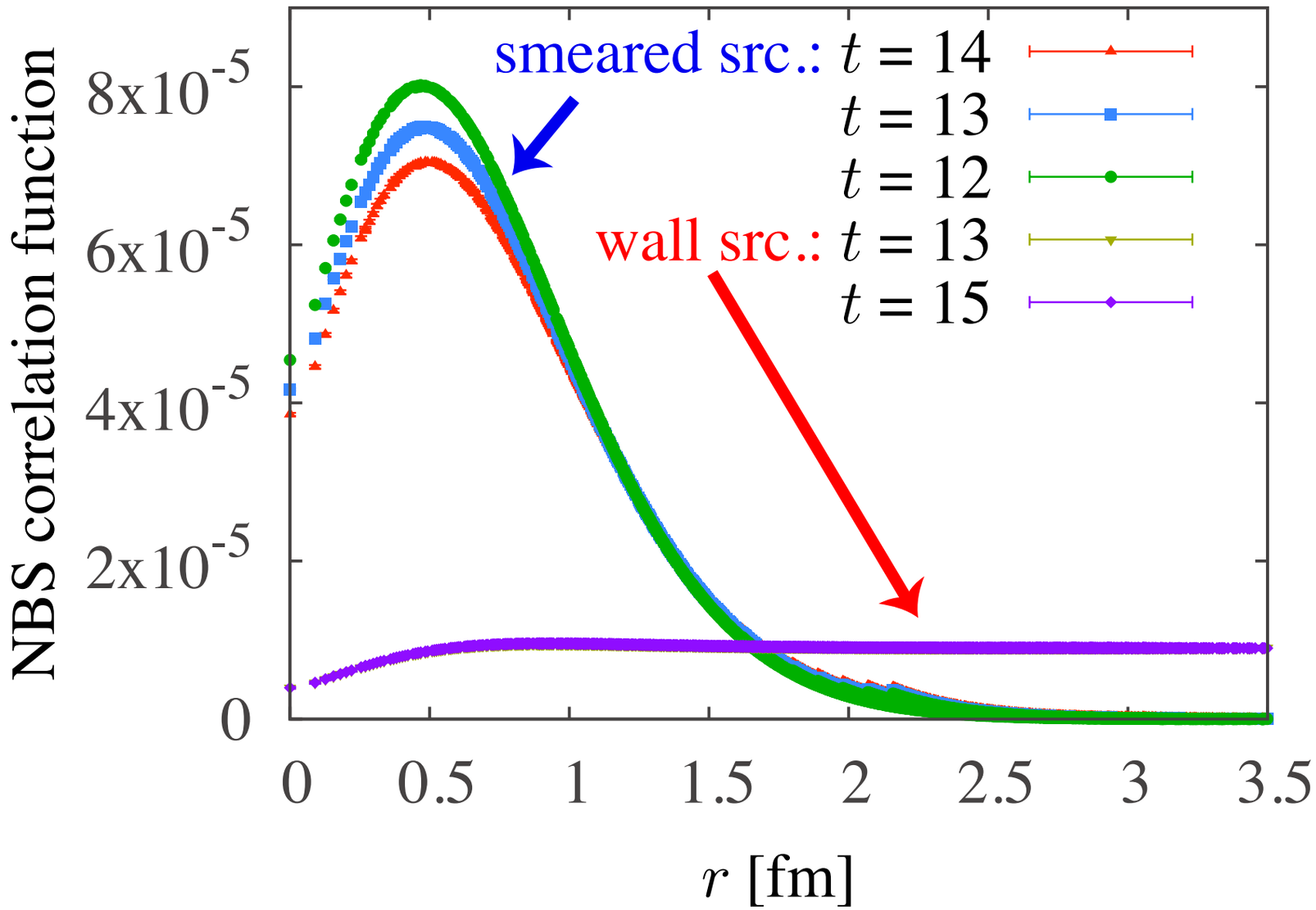
Lattice setup 2+1 flavor QCD

$$a = 0.09 \text{ fm } (a^{-1} = 2.2 \text{ GeV})$$

$$m_\pi = 0.51 \text{ GeV}, \; m_N = 1.32 \text{ GeV}, \; m_K = 0.62 \text{ GeV}, \; m_\Xi = 1.46 \text{ GeV}$$

NBS wave function

$\Xi\Xi(^1S_0)$



wall source shape weakly depends on t

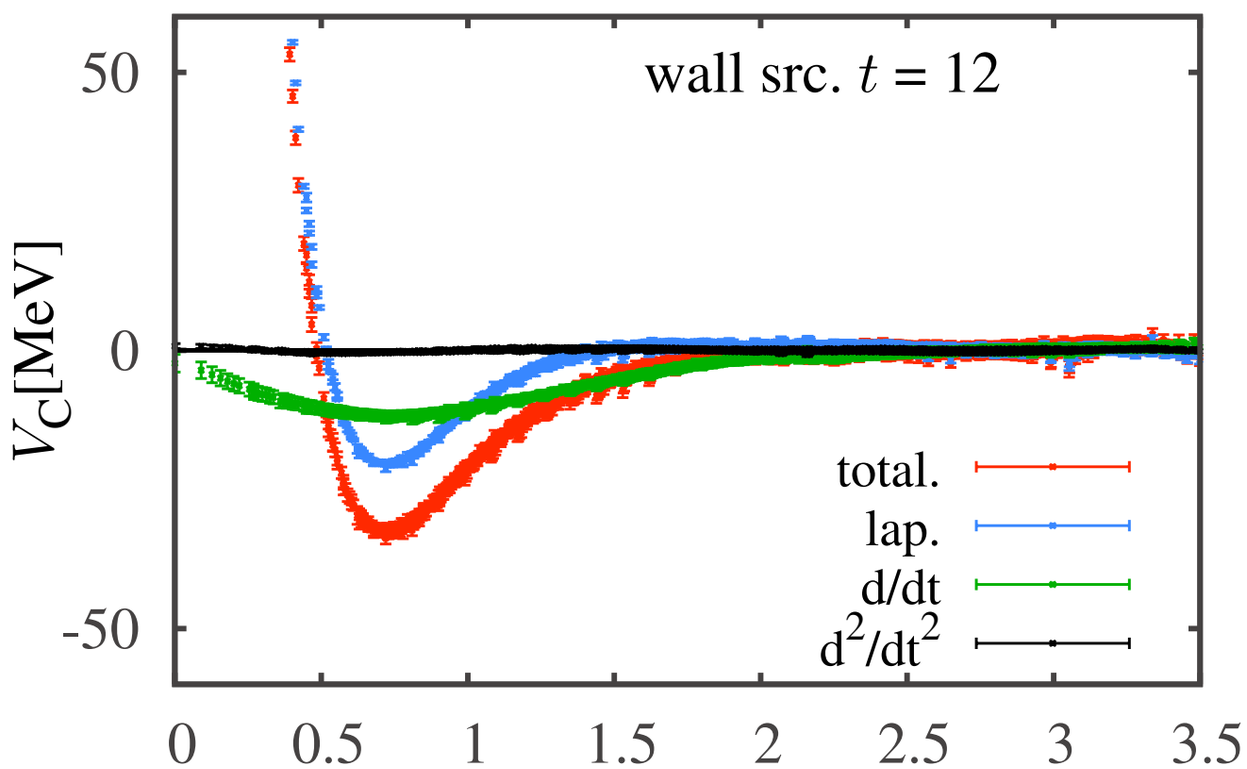
smeared source shape strongly depend on t → contributions from excited states

Both shapes must agree in very large t.

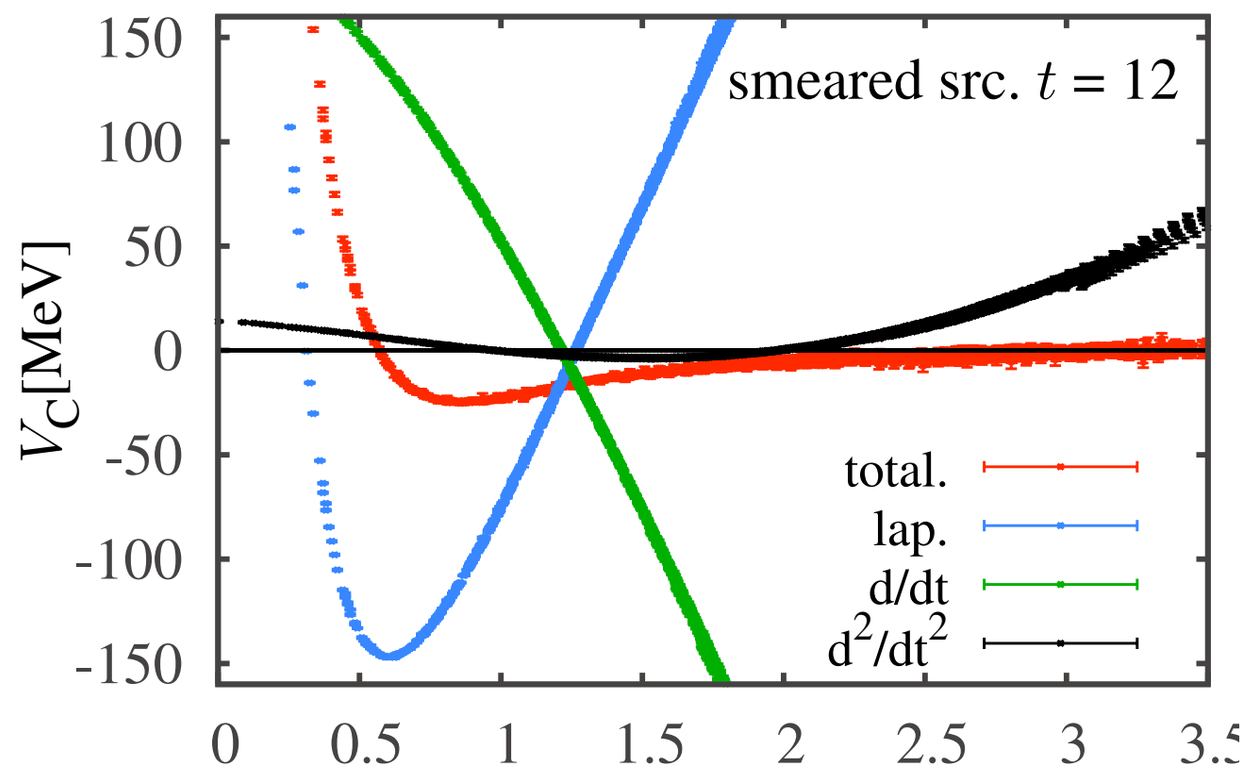
LO Potential

$$V_c(\mathbf{r}) = -\frac{H_0 R}{R} - \frac{(\partial/\partial t)R}{R} + \frac{(\partial/\partial t)^2 R}{4mR}$$

wall

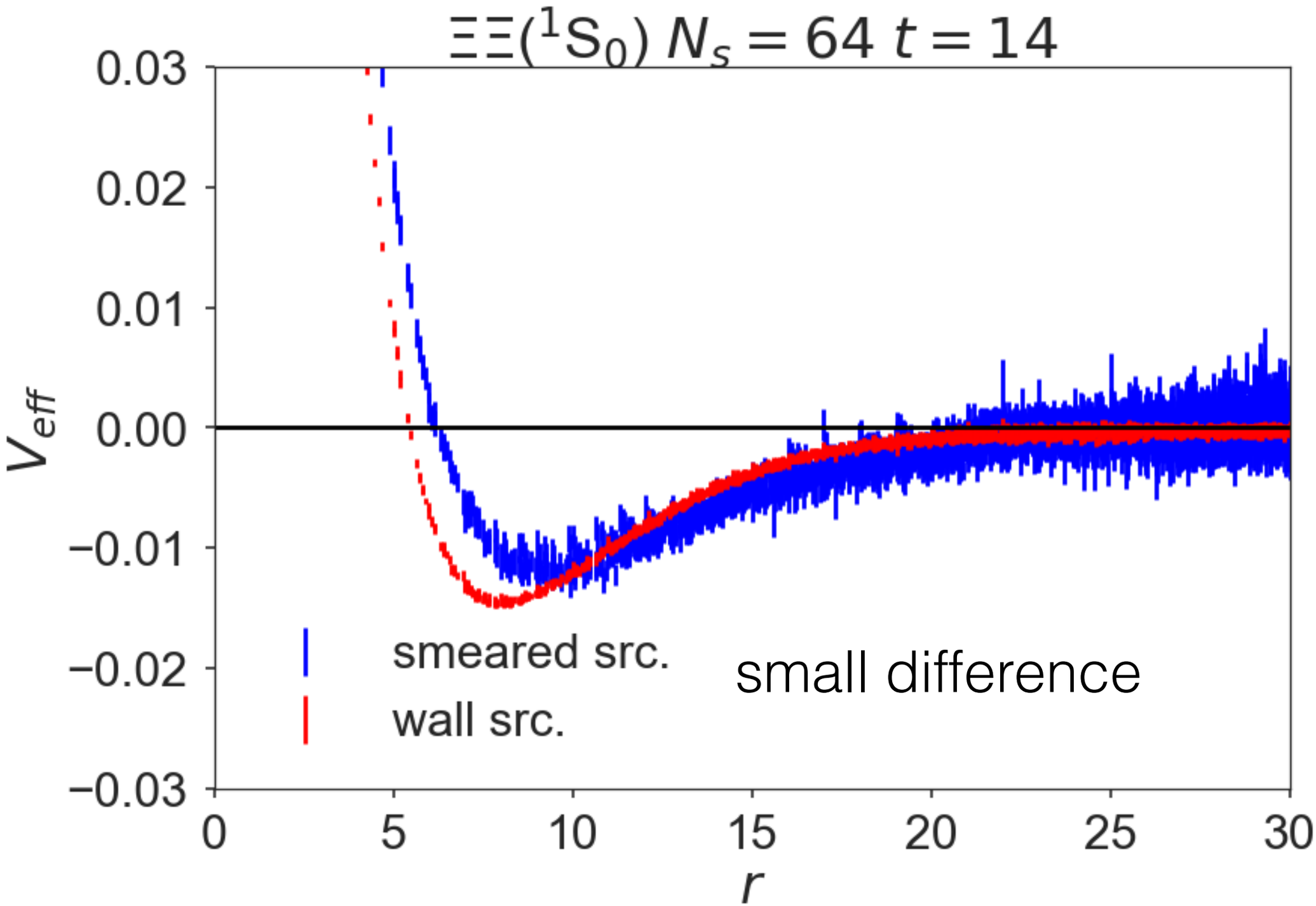


smeared



O(100) MeV cancellation

time-dependent HAL method works well



Small difference → **potentials at NNLO ?**

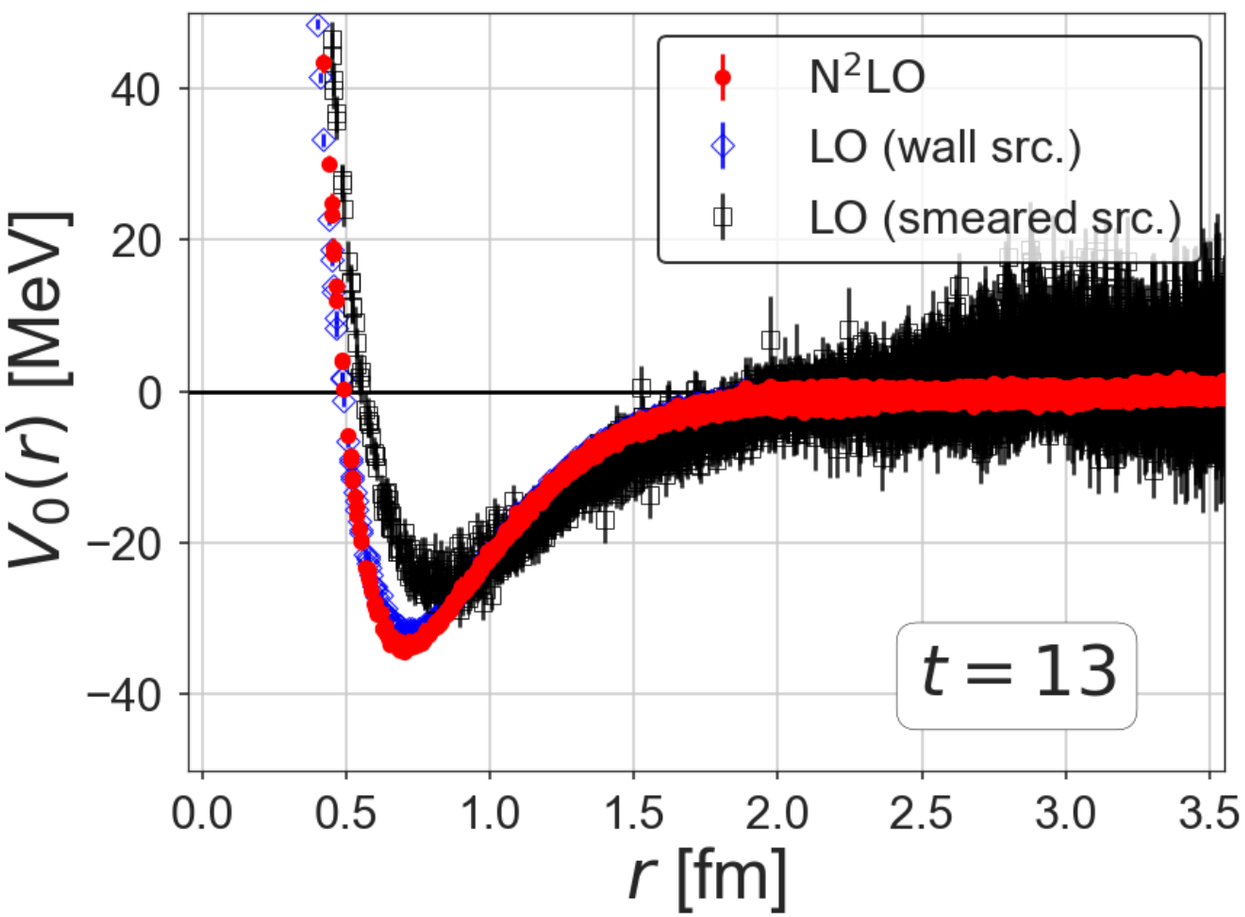
NNLO potentials

$$V_X^{\mathrm{LO}}(r,t) = \frac{1}{4m}\frac{\frac{\partial^2}{\partial t^2}R^X(r,t)}{R^X(r,t)} - \frac{\frac{\partial}{\partial t}R^X(r,t)}{R^X(r,t)} - \frac{H_0 R^X(r,t)}{R^X(r,t)}$$

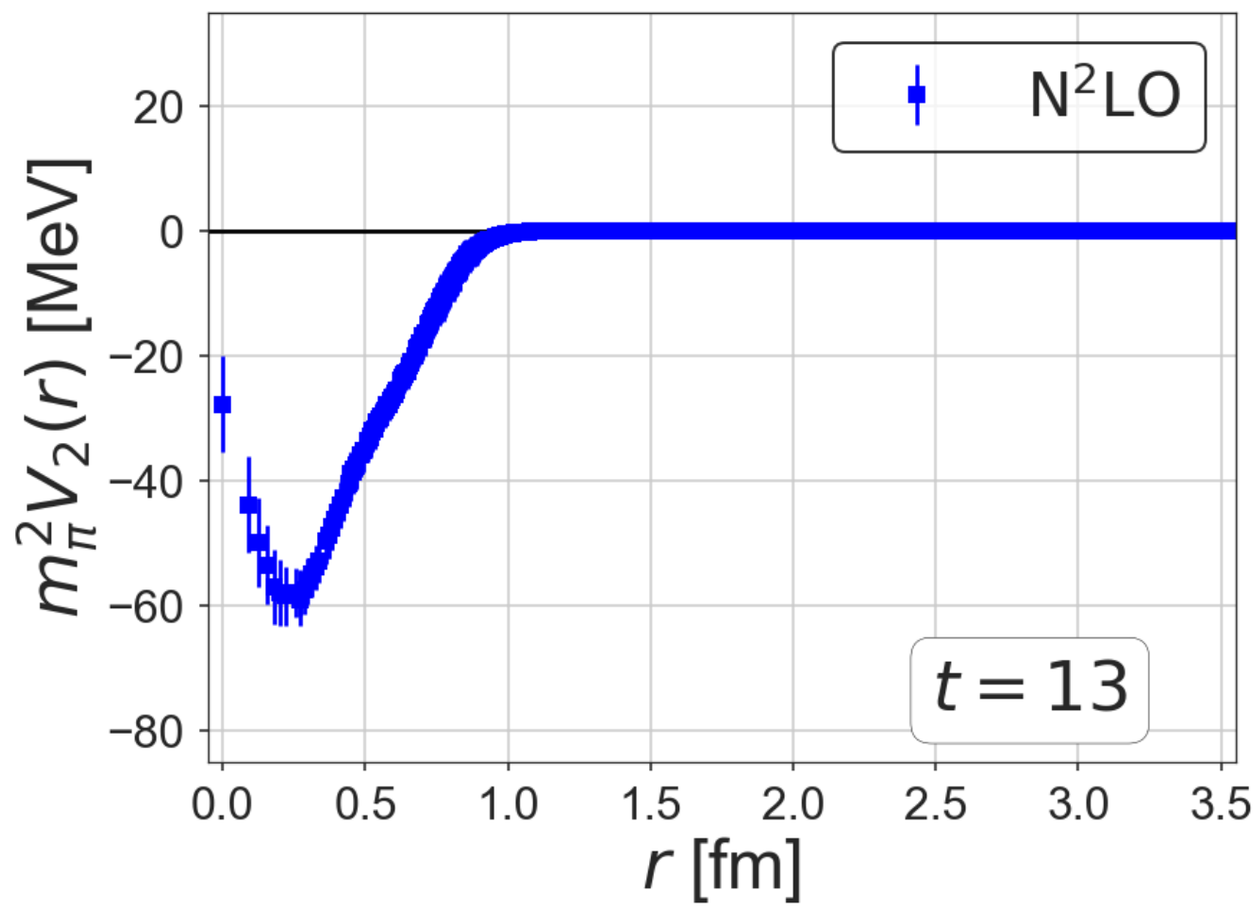
$$= \underline{V_0^{\mathrm{N^2LO}}(r)} + \underline{V_2^{\mathrm{N^2LO}}}\frac{\nabla^2 R^X(r,t)}{R^X(r,t)}$$

$X = \mathrm{Wall},\ \mathrm{Smear}$

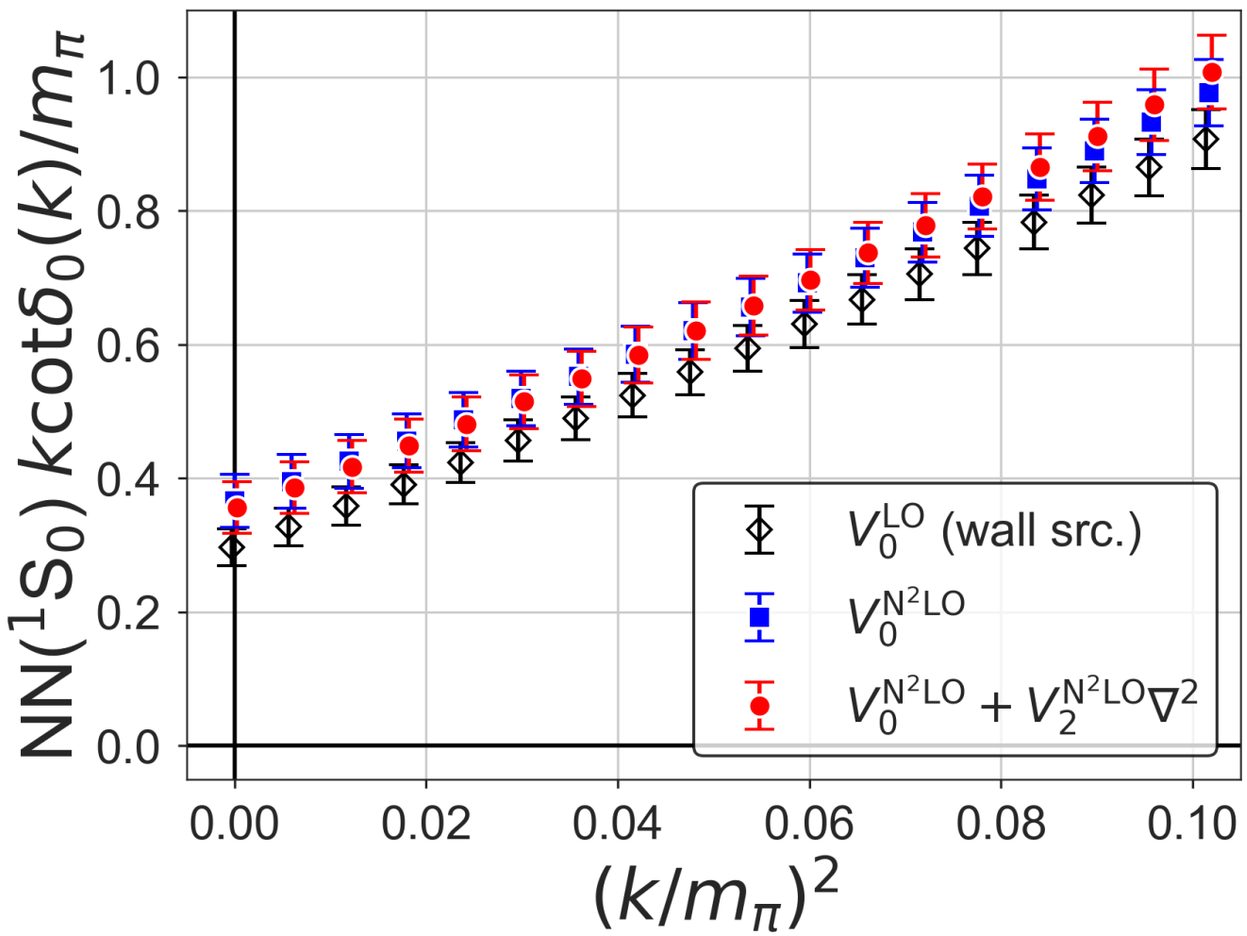
$V_0^{\mathrm{N^2LO}}(r)$

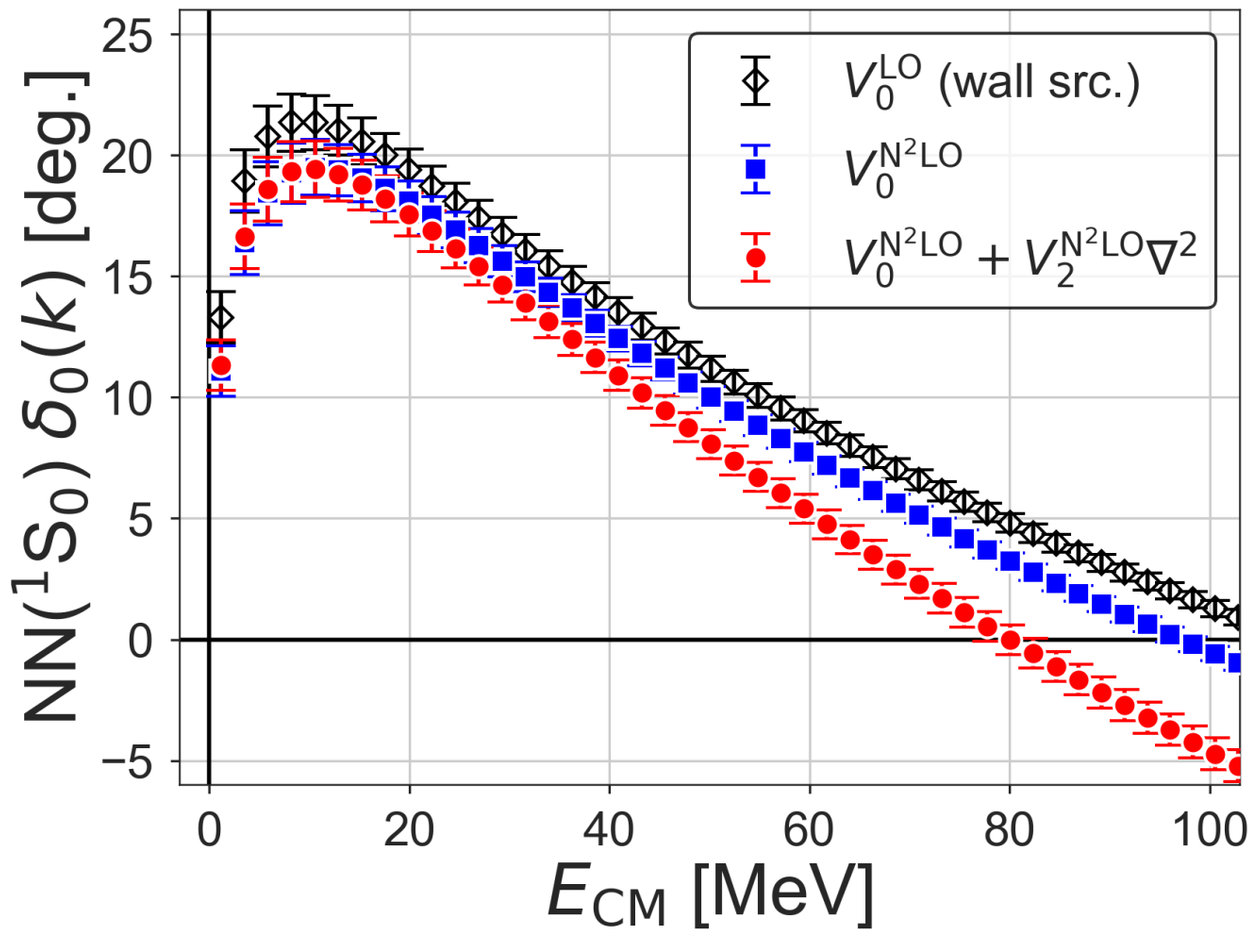


$m_\pi^2 V_2^{\mathrm{N^2LO}}(r)$



Phase shift at NNLO analysis





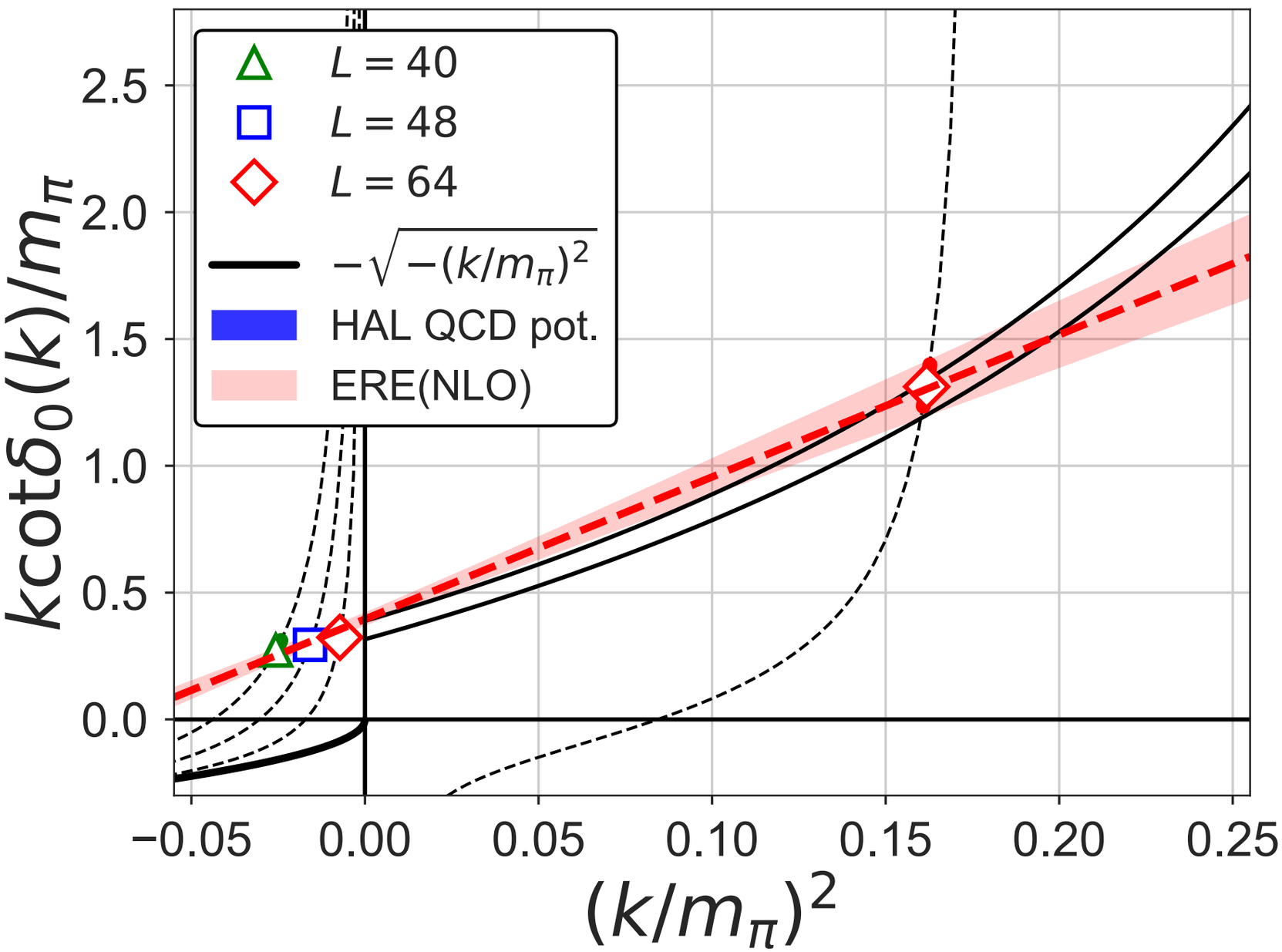
NNLO correction appears at high momentum.

Derivative expansion works well at NNLO.
LO approximation from the wall source also works rather well.

Phase shift from finite volume spectra

$$H = H_0 + V_0^{\mathrm{N^2LO}} + V_2^{\mathrm{N^2LO}}$$

finite volume spectra → phase shift



II. Recent results (selected)

1. Dibaryons

Dibaryon = a bound state with baryon number B=2

Only deuteron is the dibaryon observed in Nature so far.

Other candidates.

H-dibaryon (uuddss)

Omega-Omega (ssssss)

N-Omega

Delta-Delta

Some of them are investigated in the potential method.

ΩΩ

(ssssss)

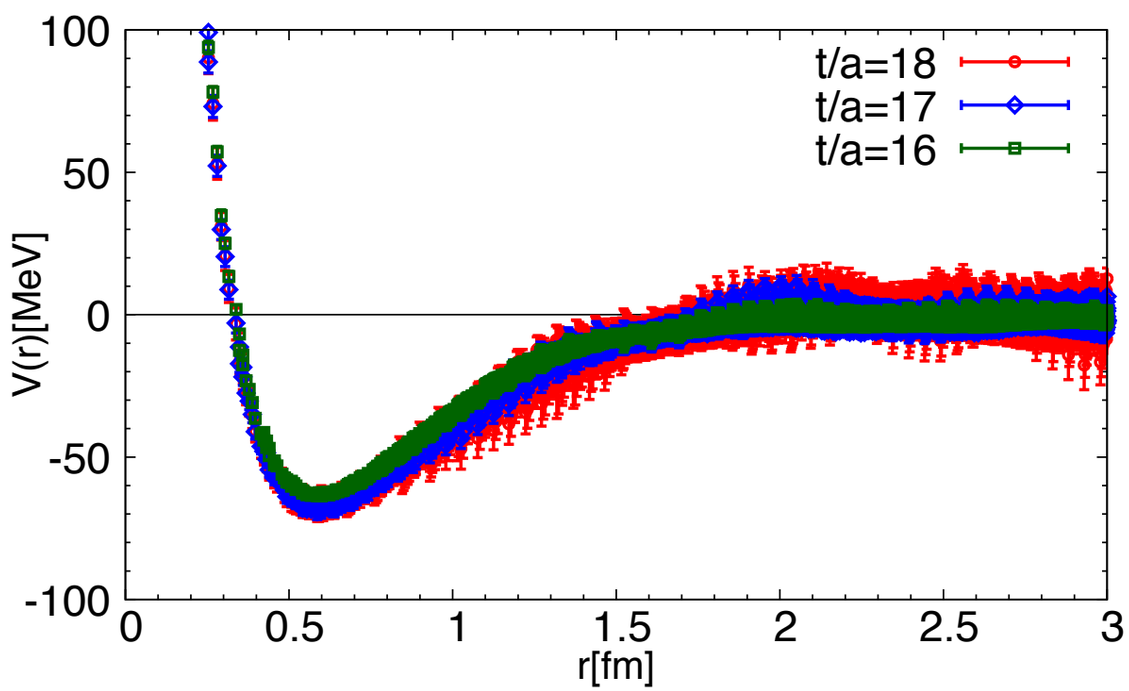
S. Gongyo et al., Phys. Rev. Lett. 120 (2018) 212001.

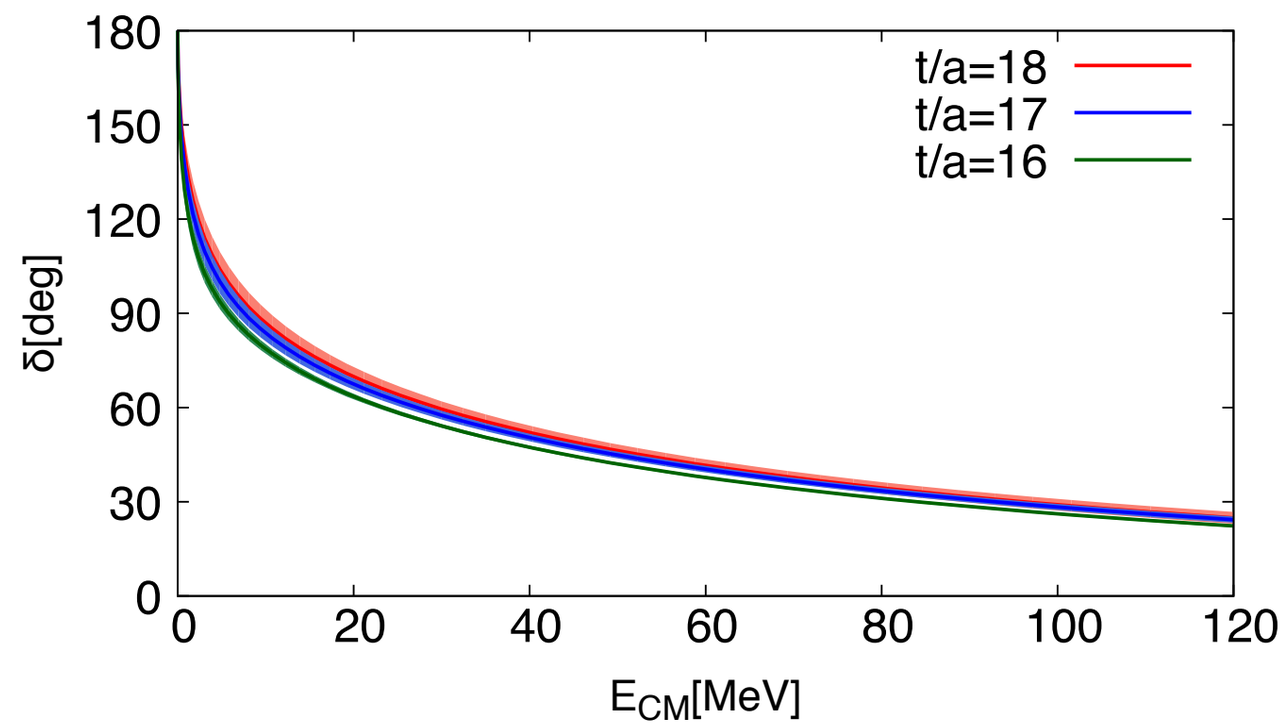
Lattice QCD at (almost) physical pion mass

2+1 flavor QCD, $m_\pi \simeq 145$ MeV, $a \simeq 0.085$ fm, $L \simeq 8$ fm

K-computer [10PFlops]

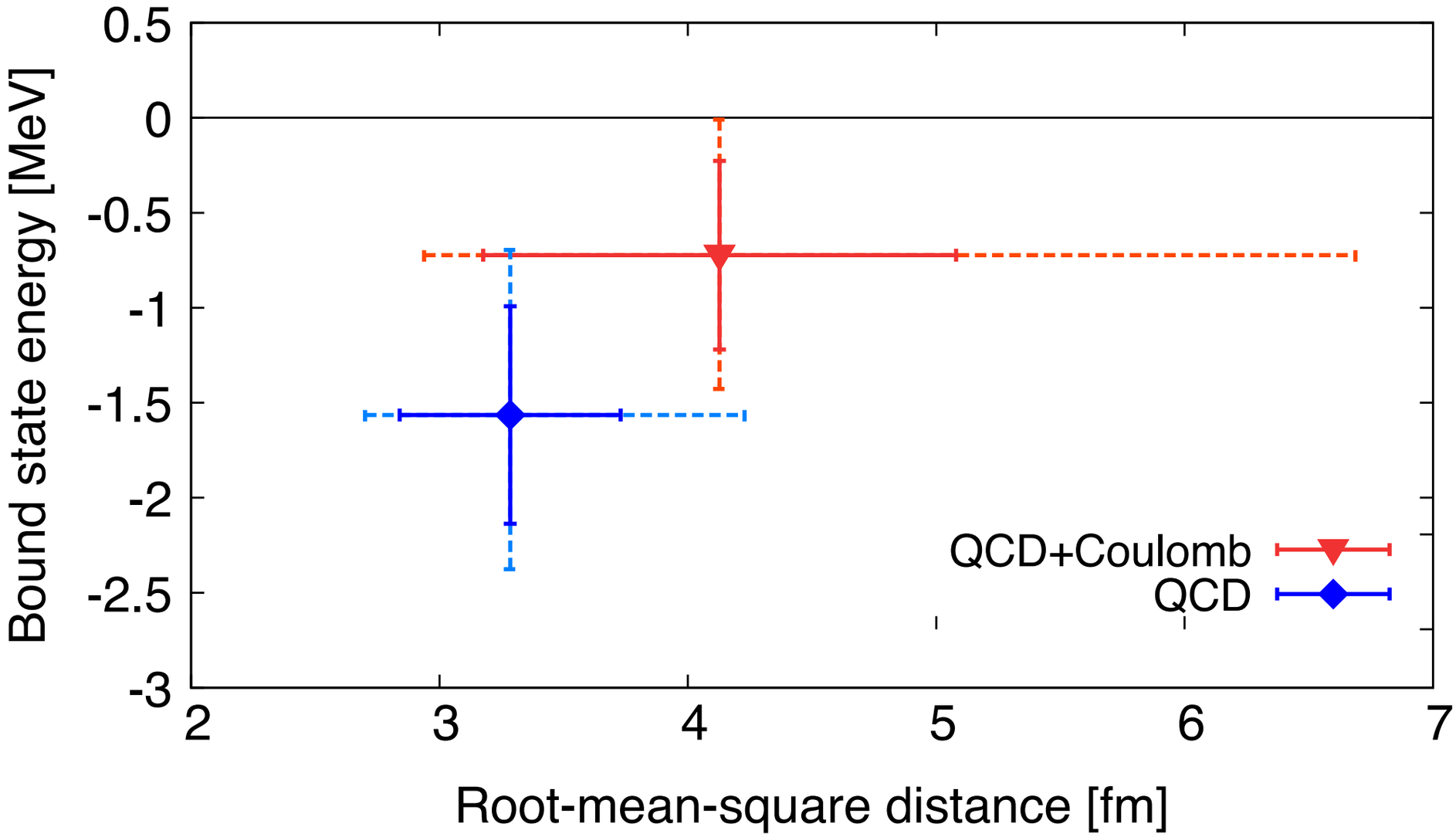
$\Omega\Omega$ potential → phase shift





Strong attraction → **Vicinity of bound/unbound (~ unitary limit)**

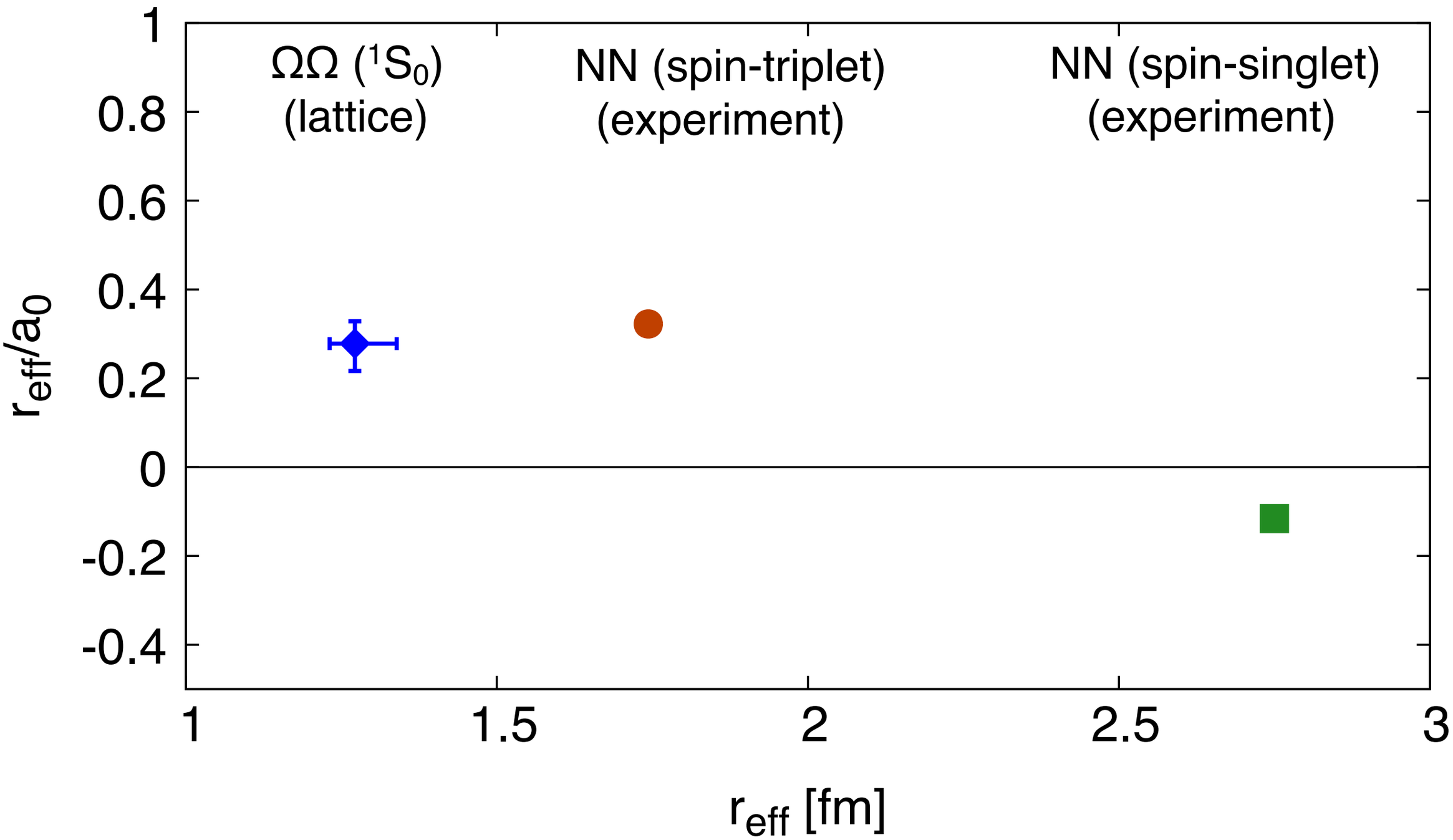
Binding energy



The most strange (sss sss) dibaryon ?

Comparison

$$\frac{r_{\rm eff}}{a_0} \text{ vs } r_{\rm eff}$$



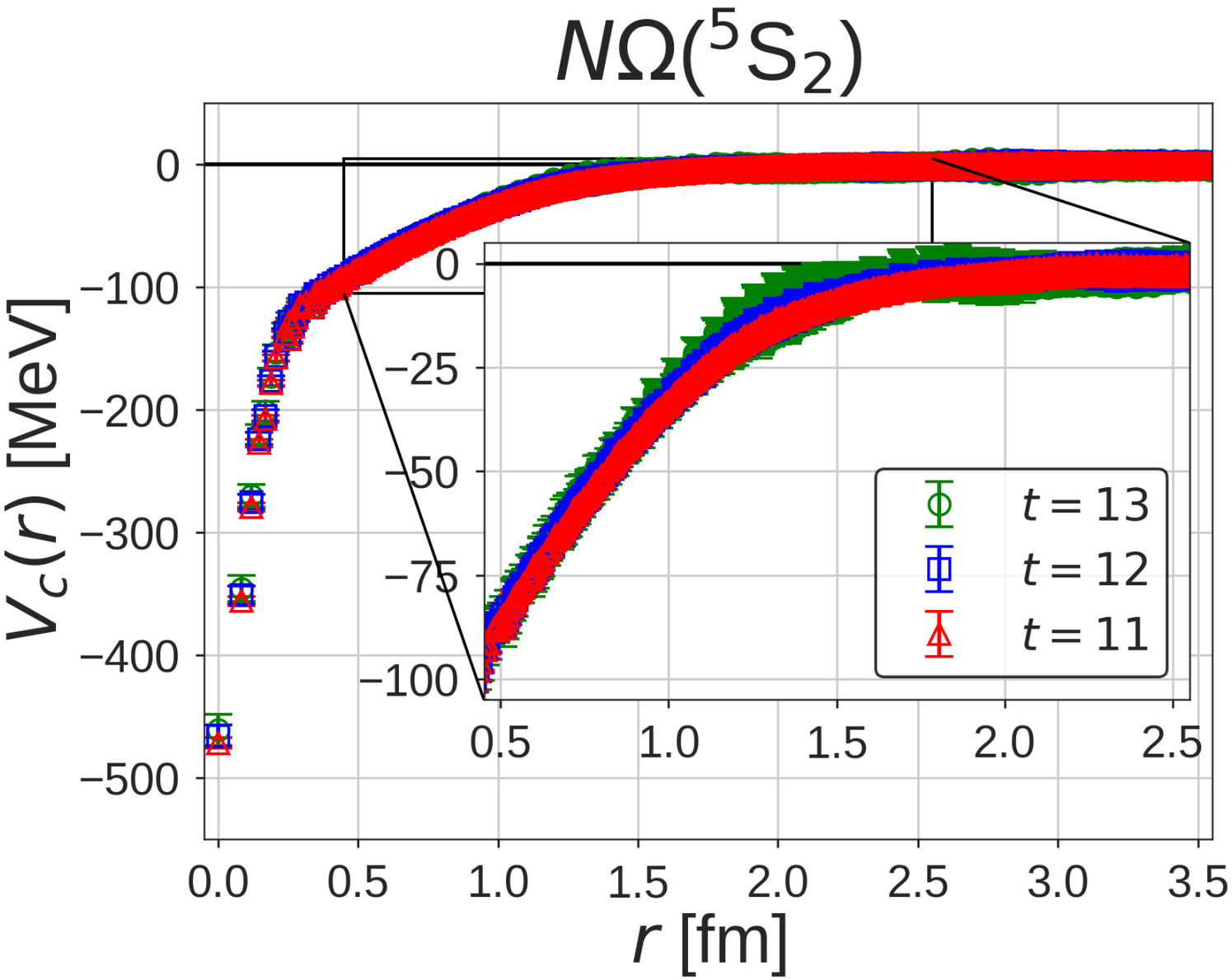
$N\Omega$

T. Iritani, Lat2018

T. Iritani et al., in preparation.

$N\Omega$ potential in $^5S_2$ channel

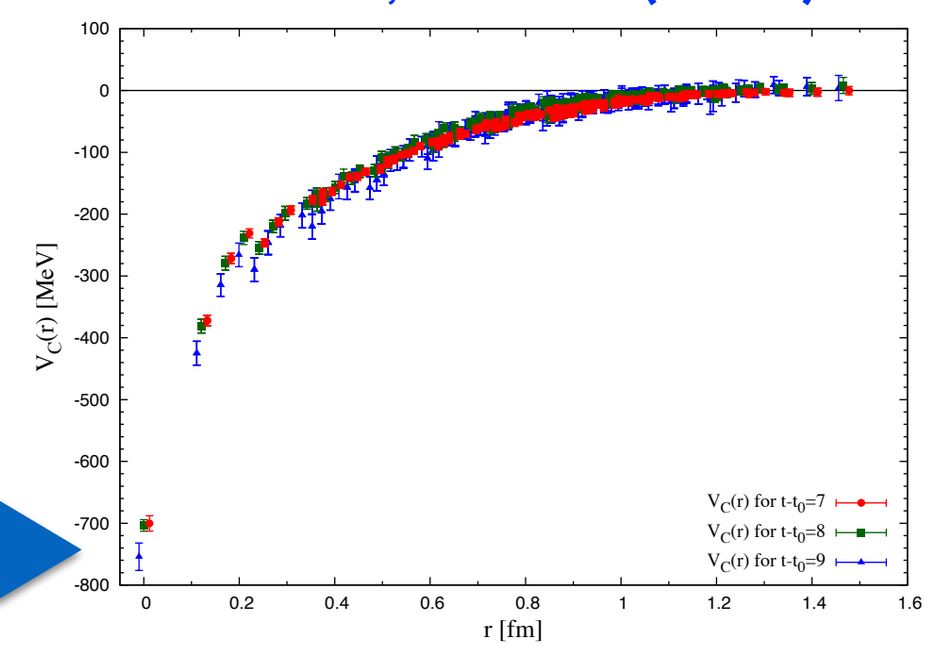
at physical pion mass



* attractive potential without repulsive core

* long range attraction

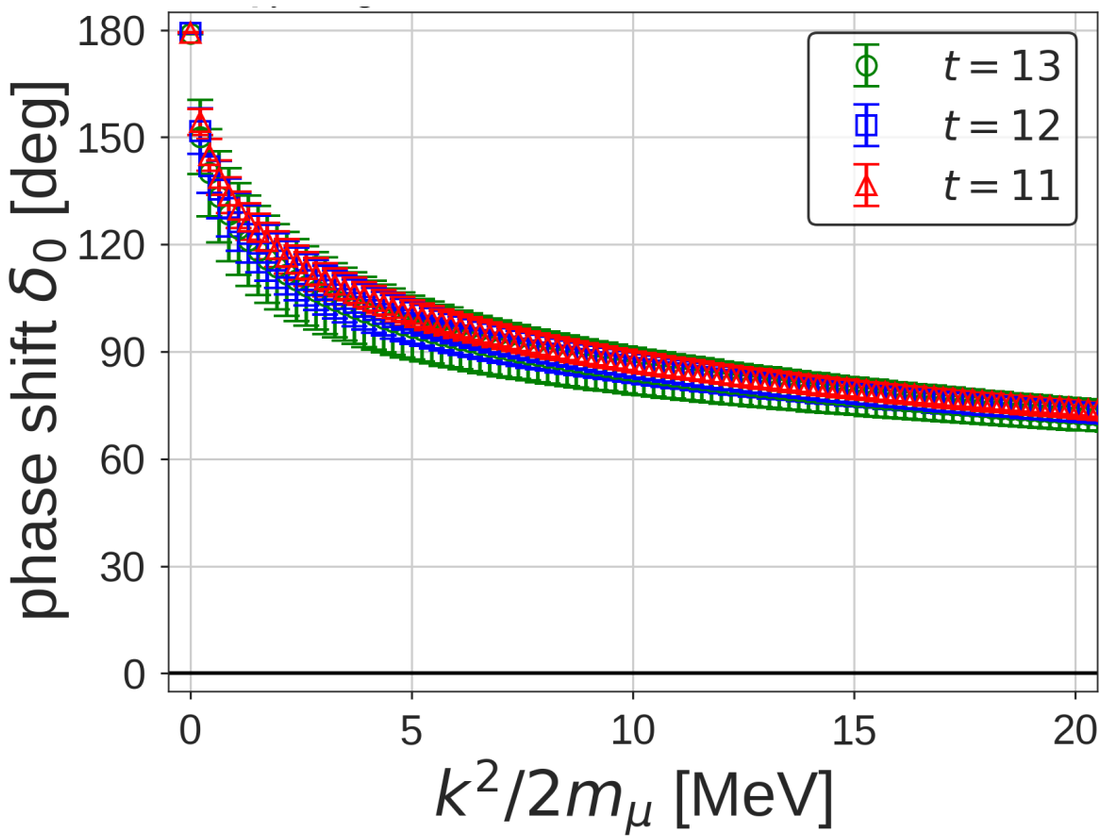
Etminan et al., NPA928(2014)89



B.E. = 18.9(5.0)(+12.1)(-1.8) MeV

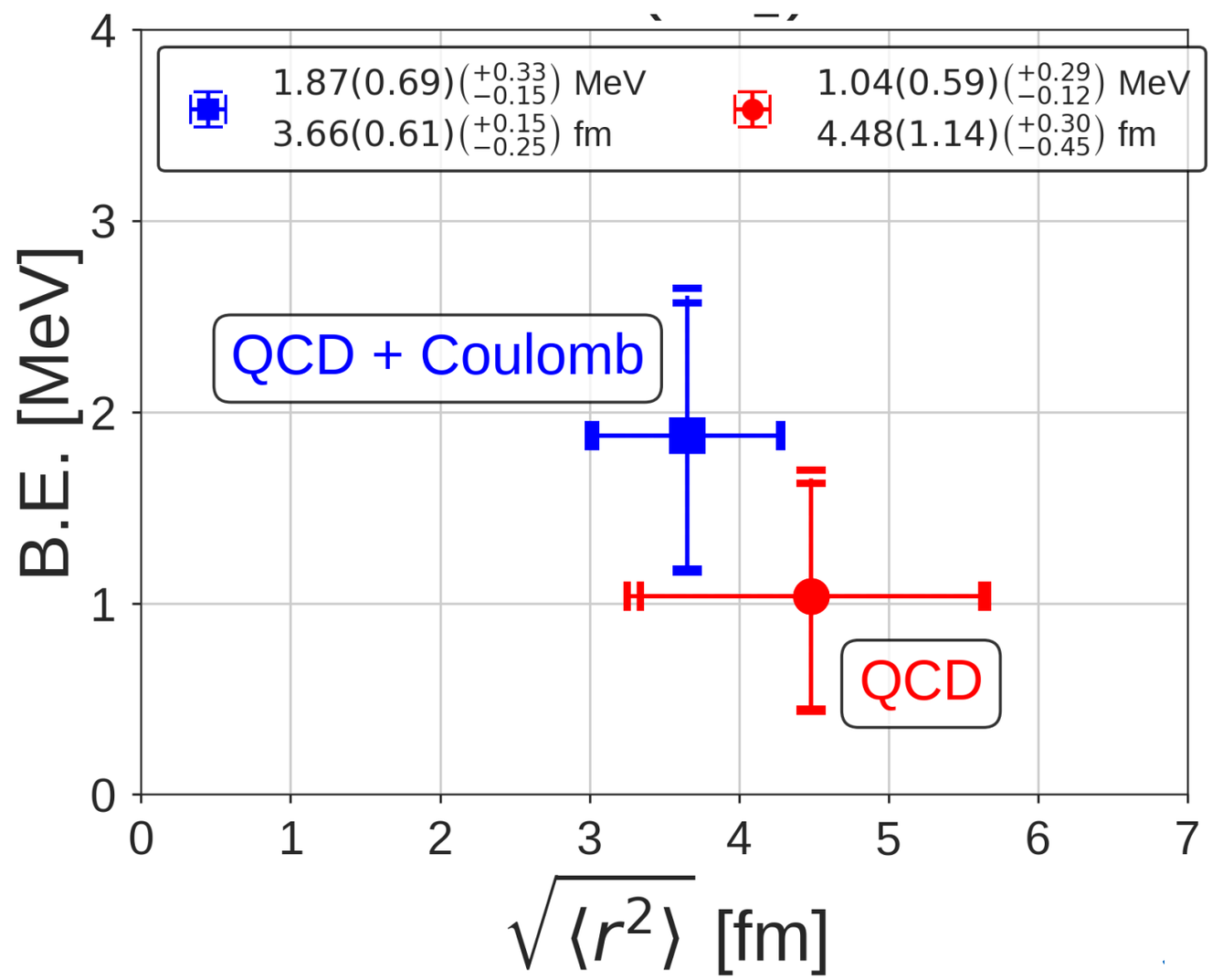
qualitatively the same at $m_\pi \simeq 875$ MeV

Phase shift and binding energy



$$\frac{r_{\rm eff}}{a_0} \simeq 0.2$$

Preliminary !

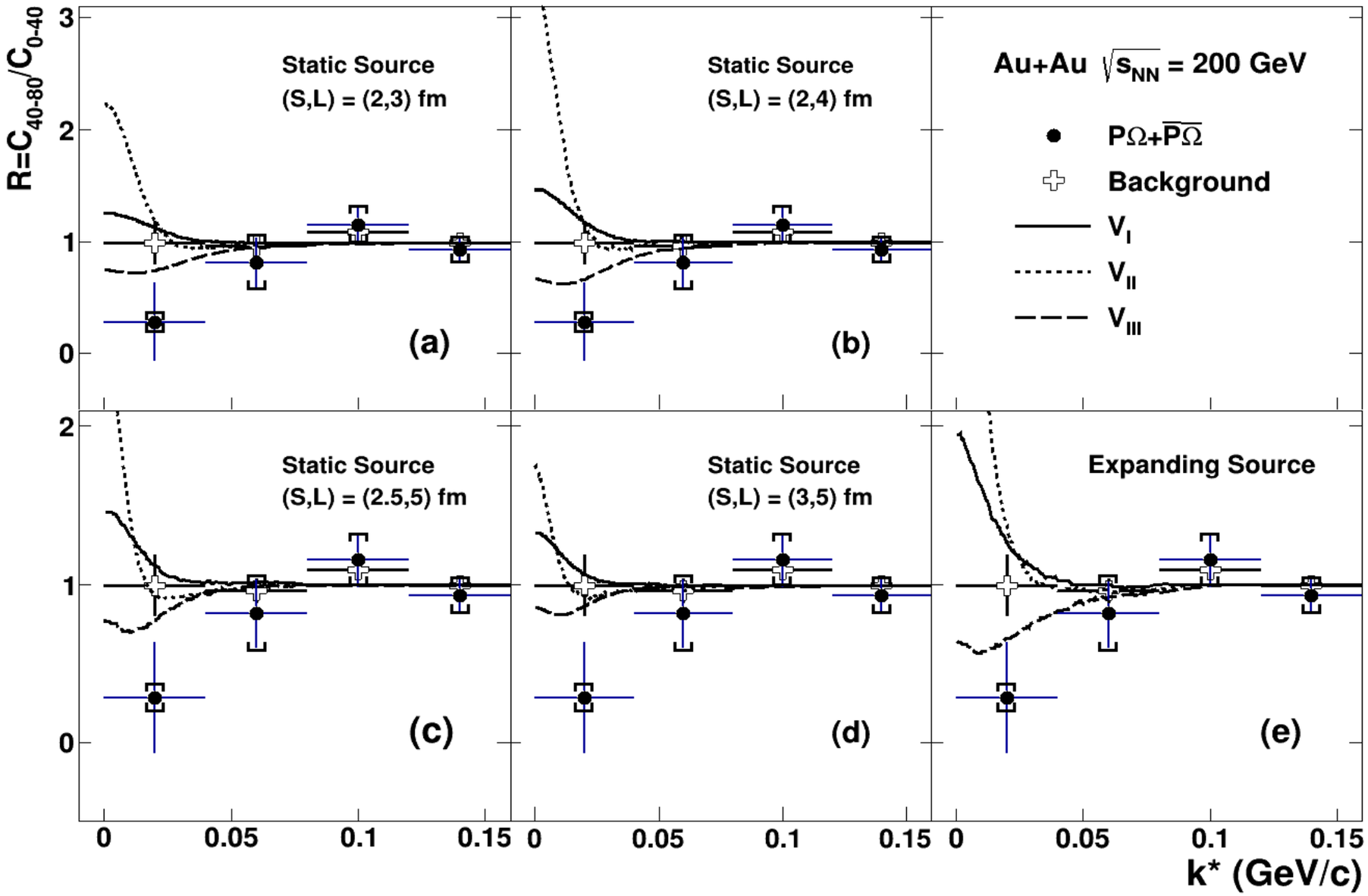


New dibaryon ?

Proton-Ω correlation in RHIC

STAR collaboration, arXiv:1808.0251[hep-ex]

ratio of small to large systems



centrality

40-80% (small)

0-40% (large)

Morita et al., PRC94(2016)031901

V_I : unbound

V_{II} : $E_B = 6.3$ MeV

V_{III} : $E_B = 26.9$ MeV

potential at $m_\pi = 875$ MeV

Data at $k^* < 40$ MeV favor V_{III}.

H-dibaryon

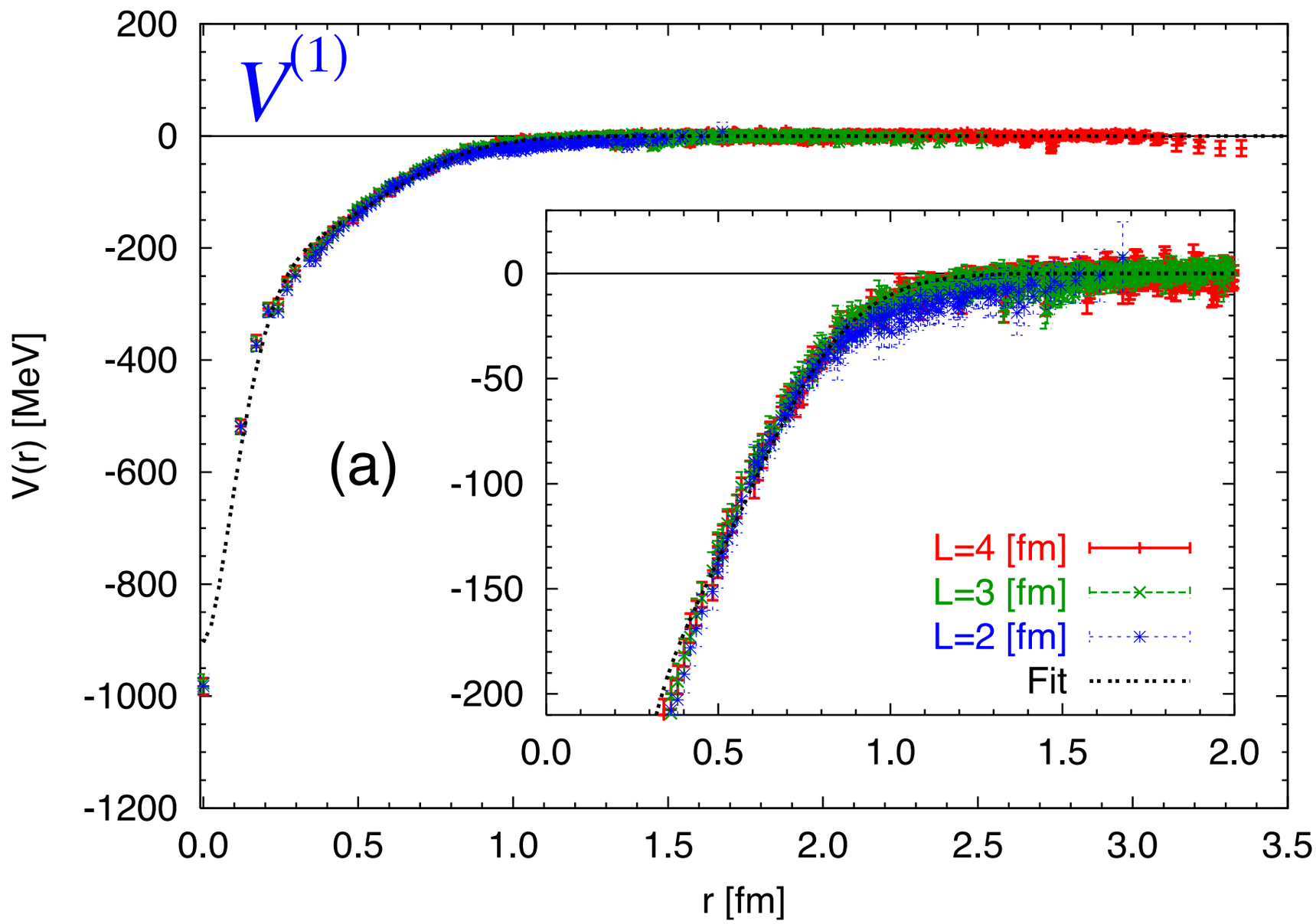
(uuddss)

flavor SU(3) singlet potential

Lattice QCD in the flavor SU(3) limit

$m_u = m_d = m_s$

Inoue *et al.* (HAL QCD Coll.), Progress of Theoretical Physics 124(2010)591



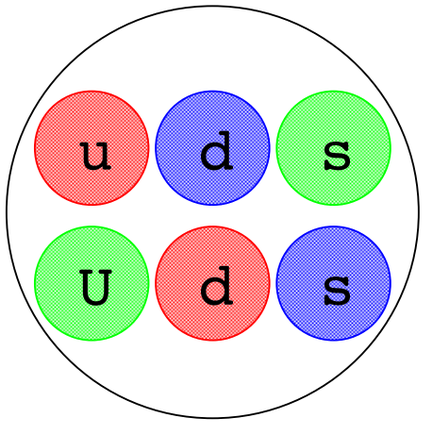
$a \simeq 0.12$ fm

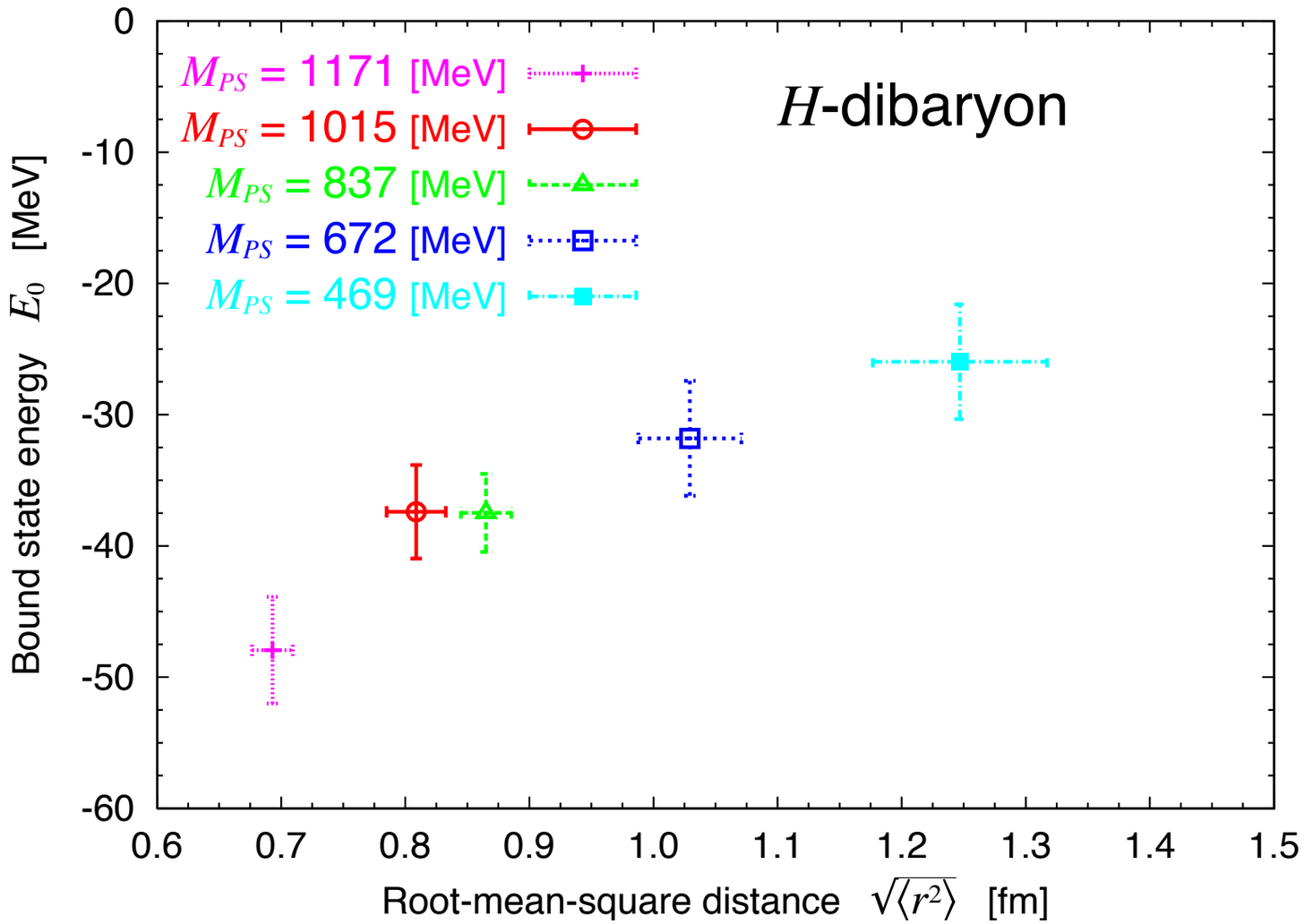
$m_\pi \simeq 470$ MeV

Force is attractive at all distances. Bound state ?

possibility of a bound state (H-dibaryon) $\Lambda\Lambda - N\Xi - \Sigma\Sigma$

Inoue et al. (HAL QCD Coll.), PRL106(2011)162002





An H-dibaryon exists in the flavor SU(3) limit.

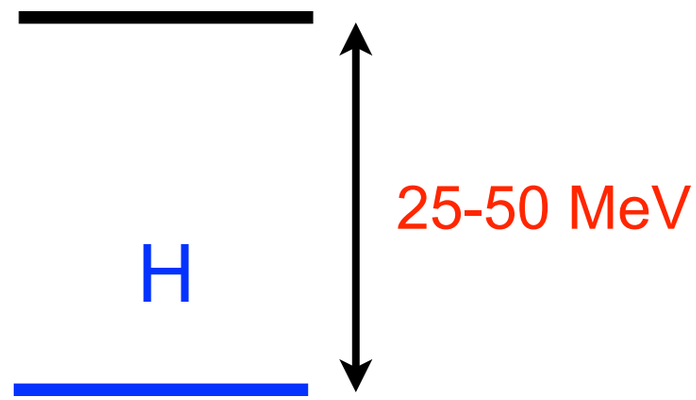
Binding energy = 25-50 MeV at this range of quark mass. Real world ?

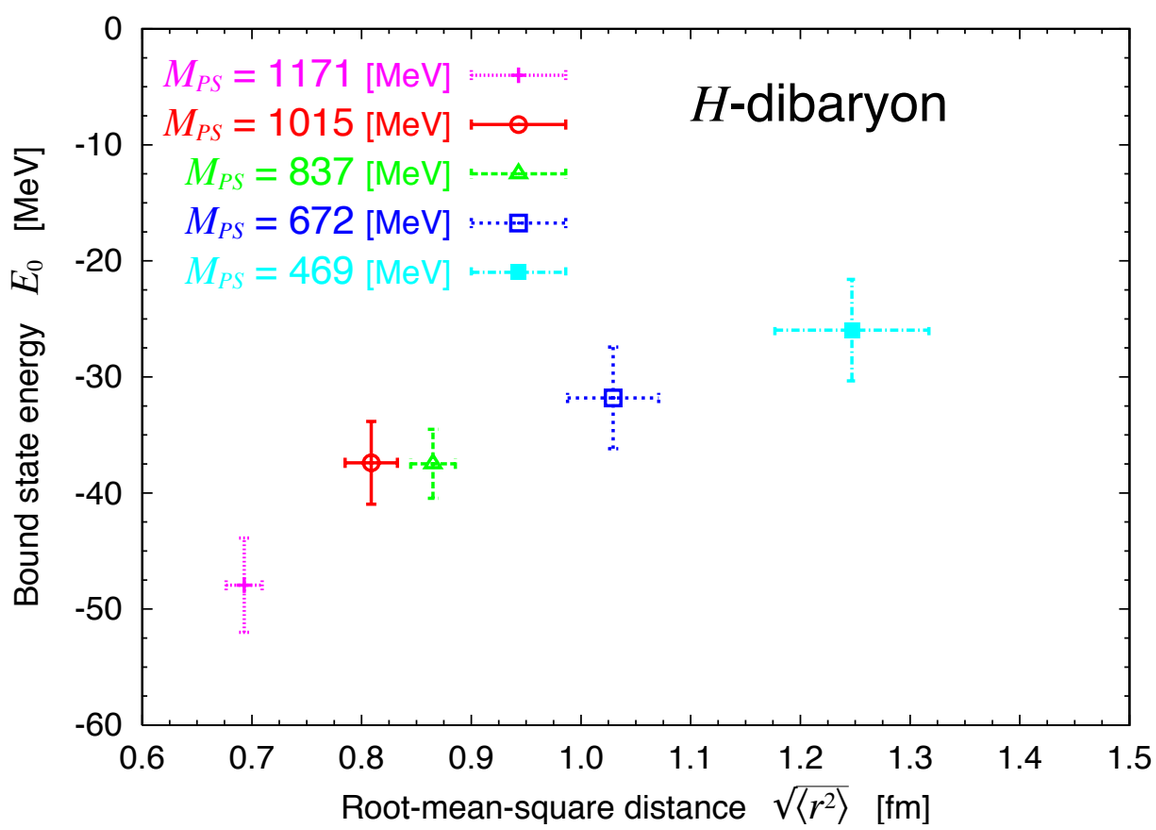
A mild quark mass dependence.

H-dibaryon with the flavor SU(3) breaking

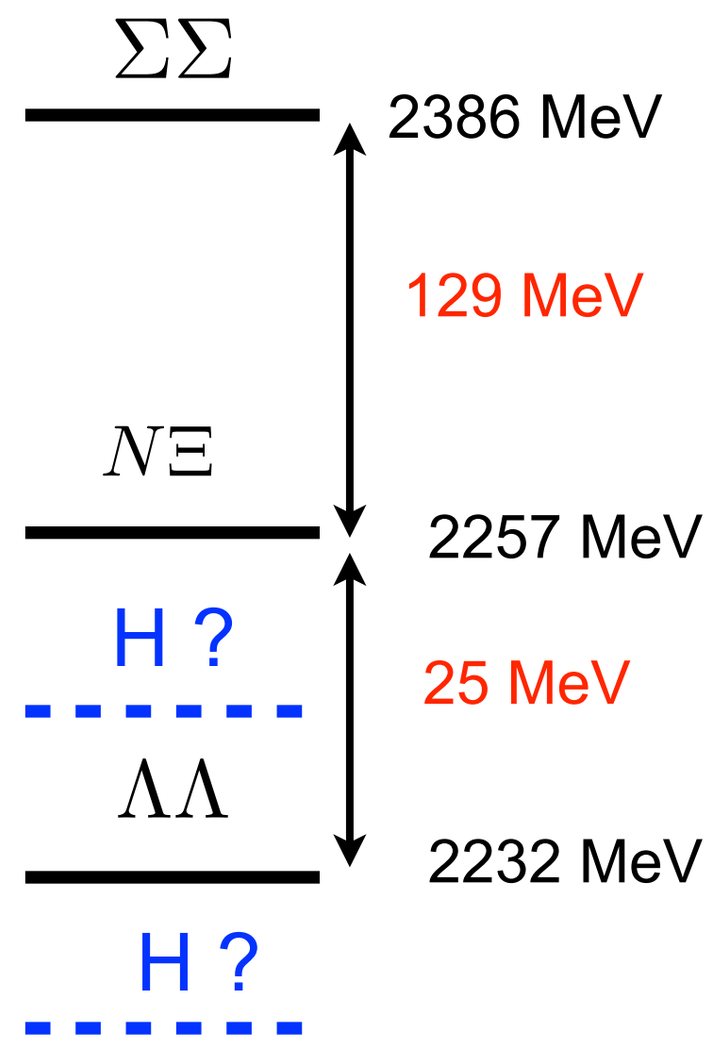
SU(3) limit

$\Lambda\Lambda - N\Xi - \Sigma\Sigma$





Real world $m_u = m_d \neq m_s$



Preliminary results from HAL QCD Collaboration

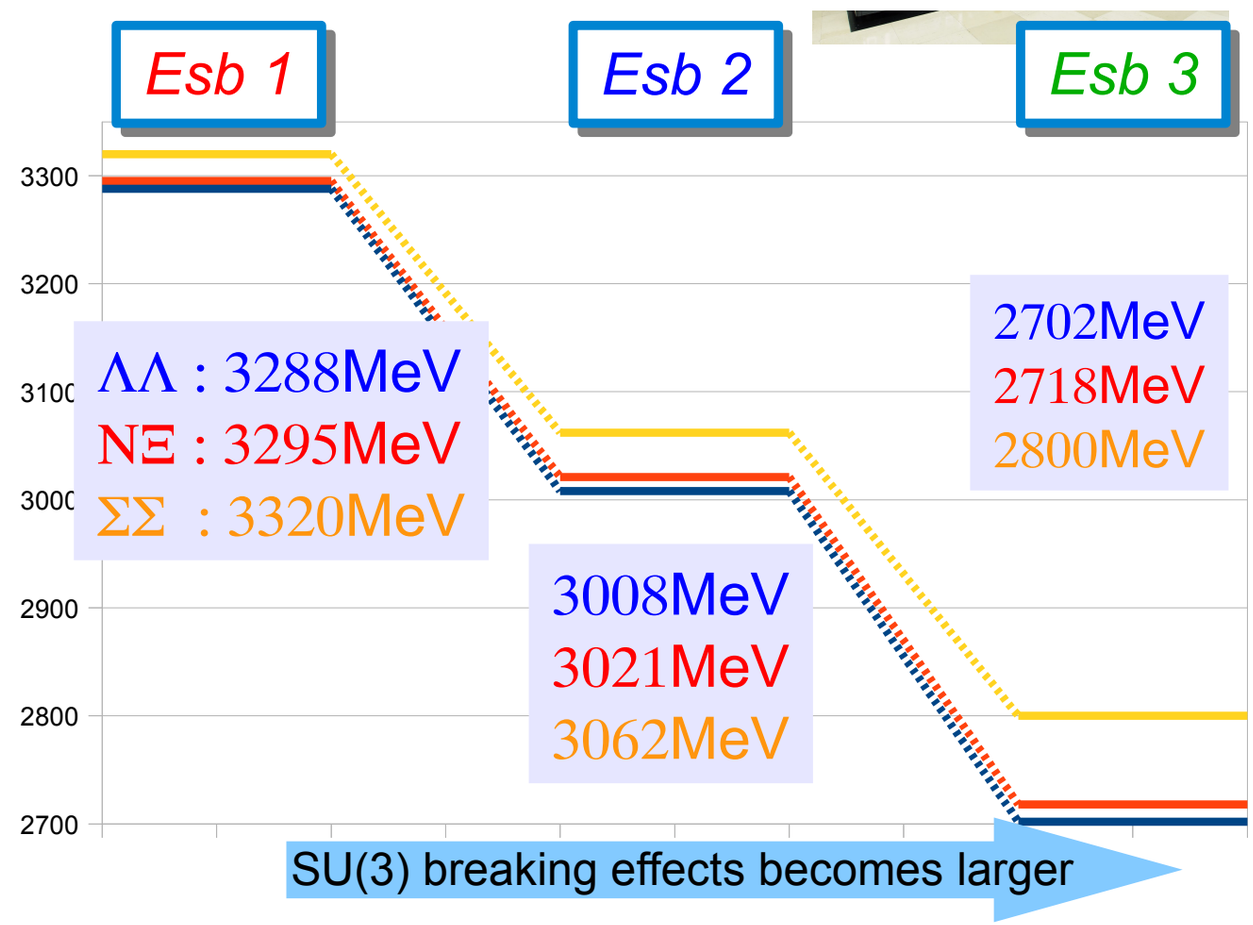
Sasaki for HAL QCD Collaboration

Gauge ensembles

In unit of MeV	Esb 1	Esb 2	Esb 3
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

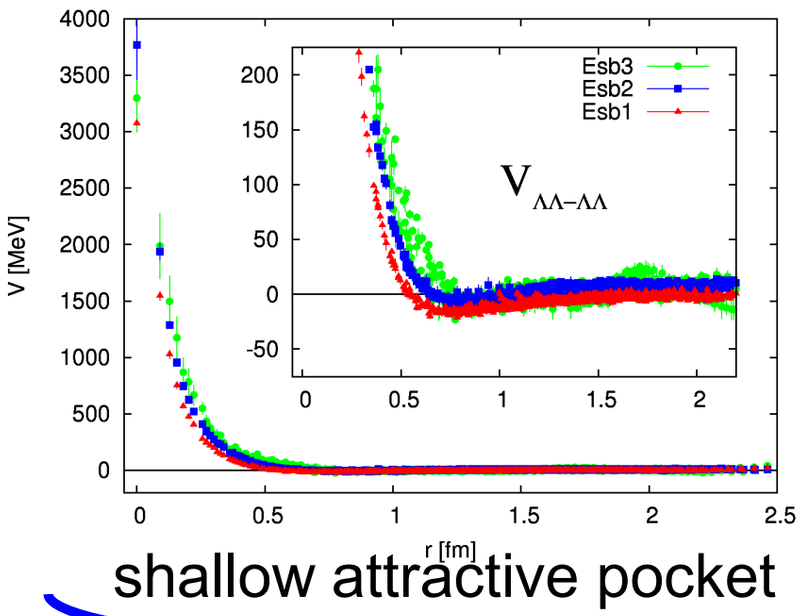
u,d quark masses lighter

thresholds

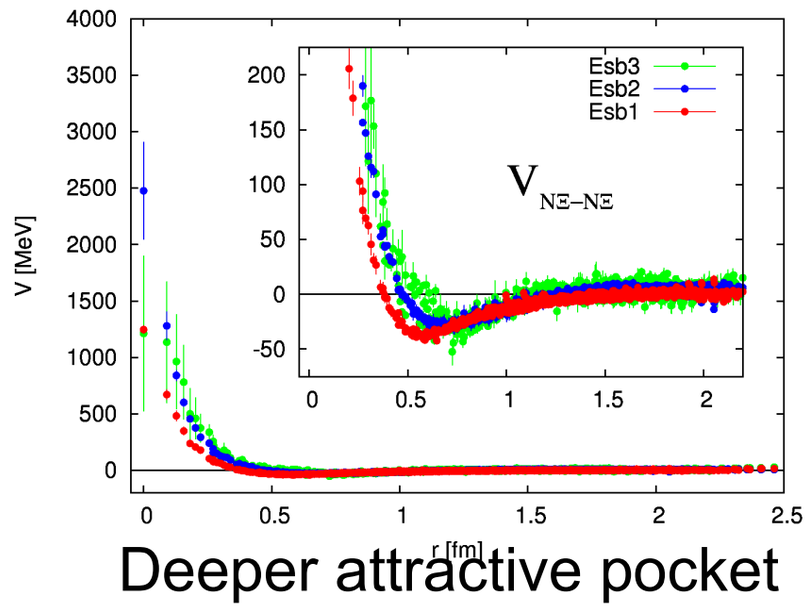


coupled channel 3x3 potentials

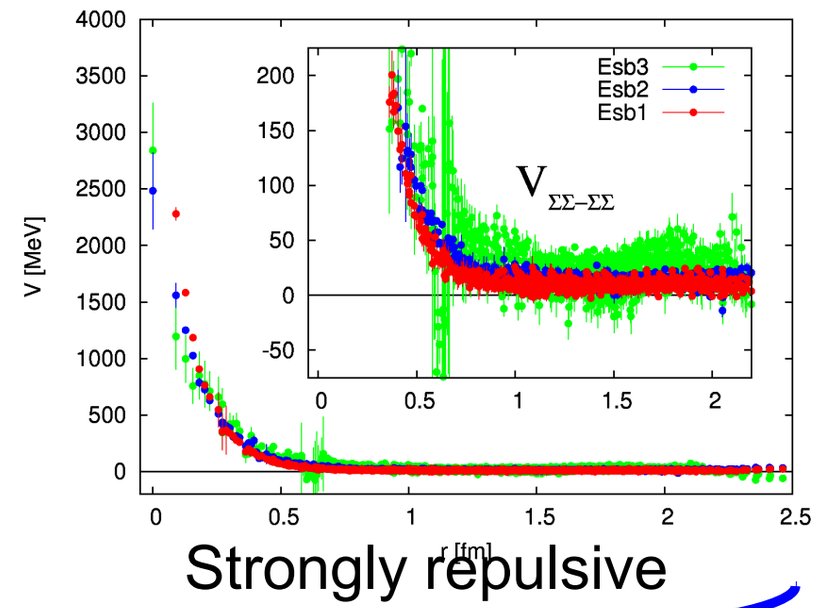
Diagonal elements



shallow attractive pocket



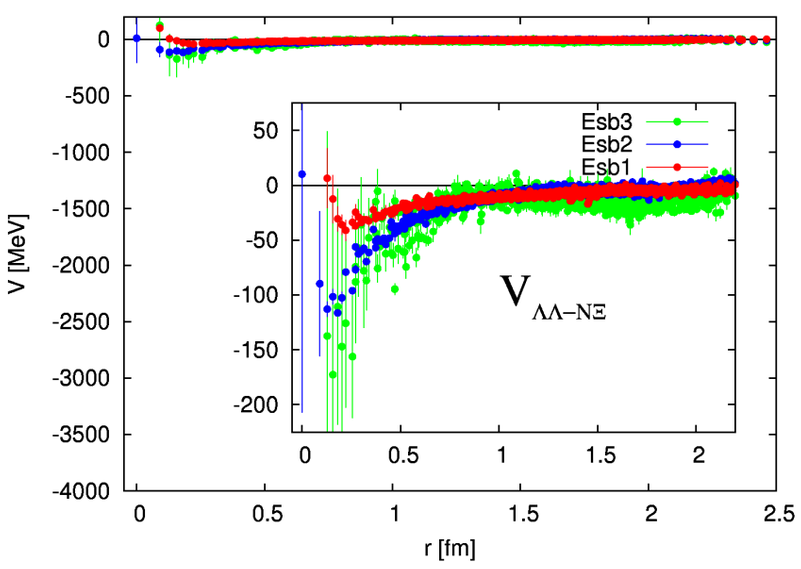
Deeper attractive pocket

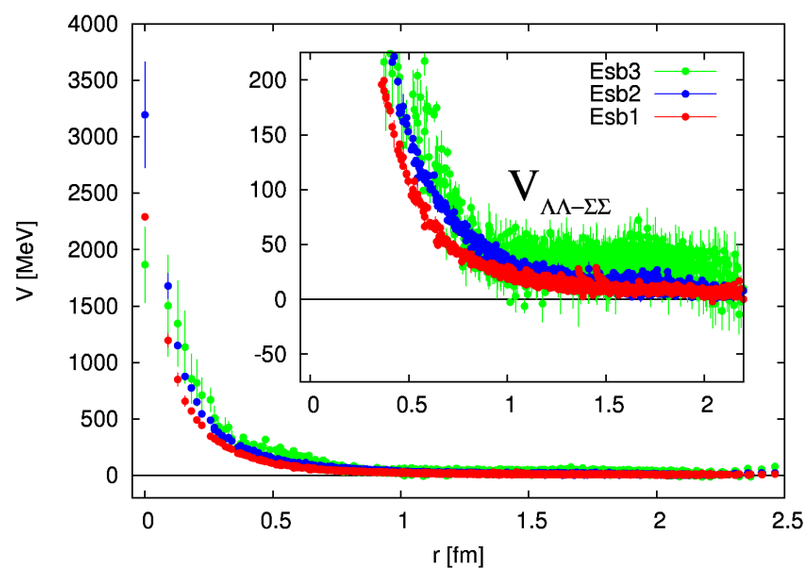


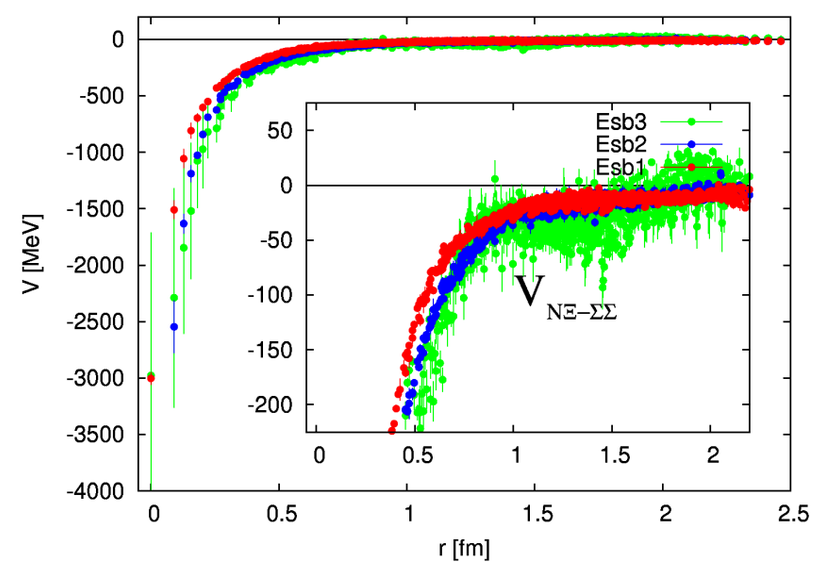
Strongly repulsive

All channels have repulsive core

Off-diagonal elements



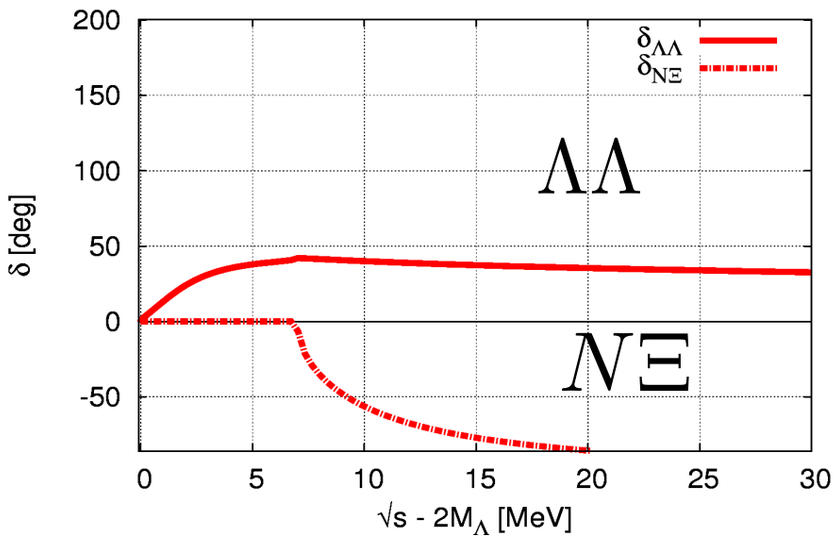




$\Lambda\Lambda$ and $N\Xi$ phase shift

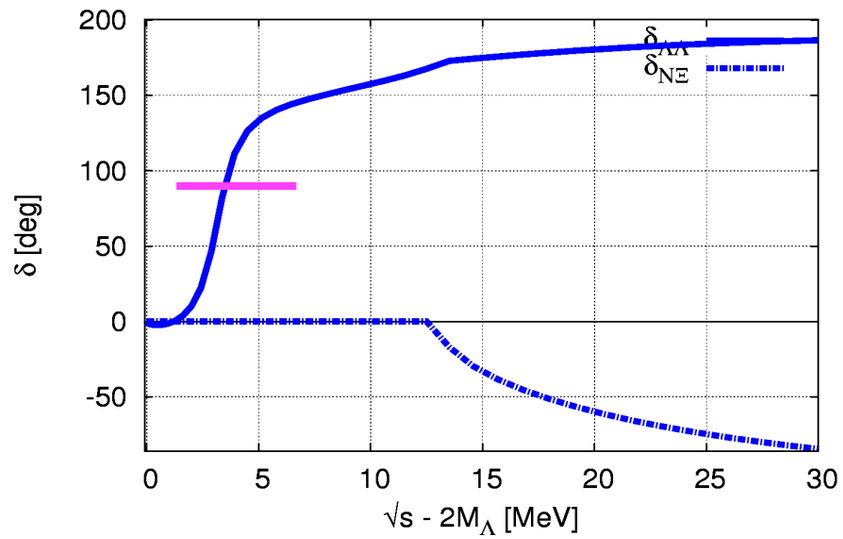
Preliminary !

Esb1 : mπ= 701 MeV



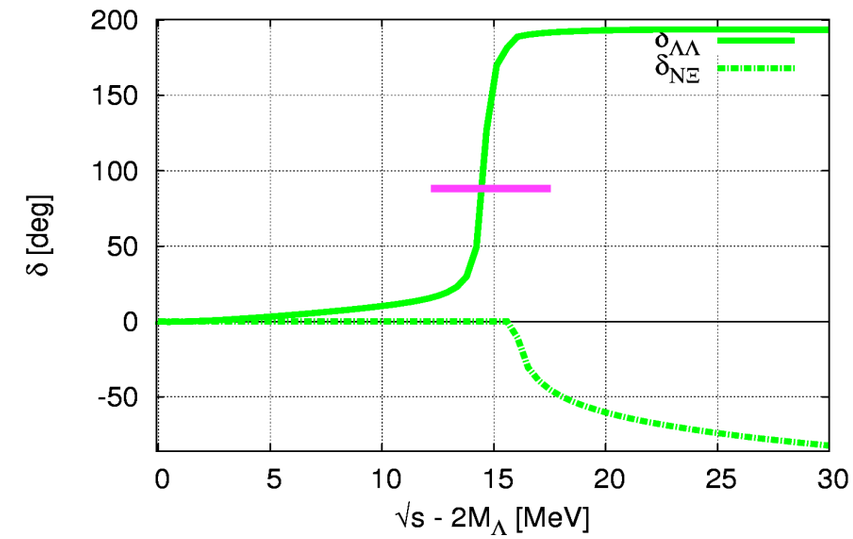
Bound H-dibaryon
coupled to NΞ

Esb2 : mπ= 570 MeV



H as ΛΛ resonance
H as bound NΞ

Esb3 : mπ= 411 MeV



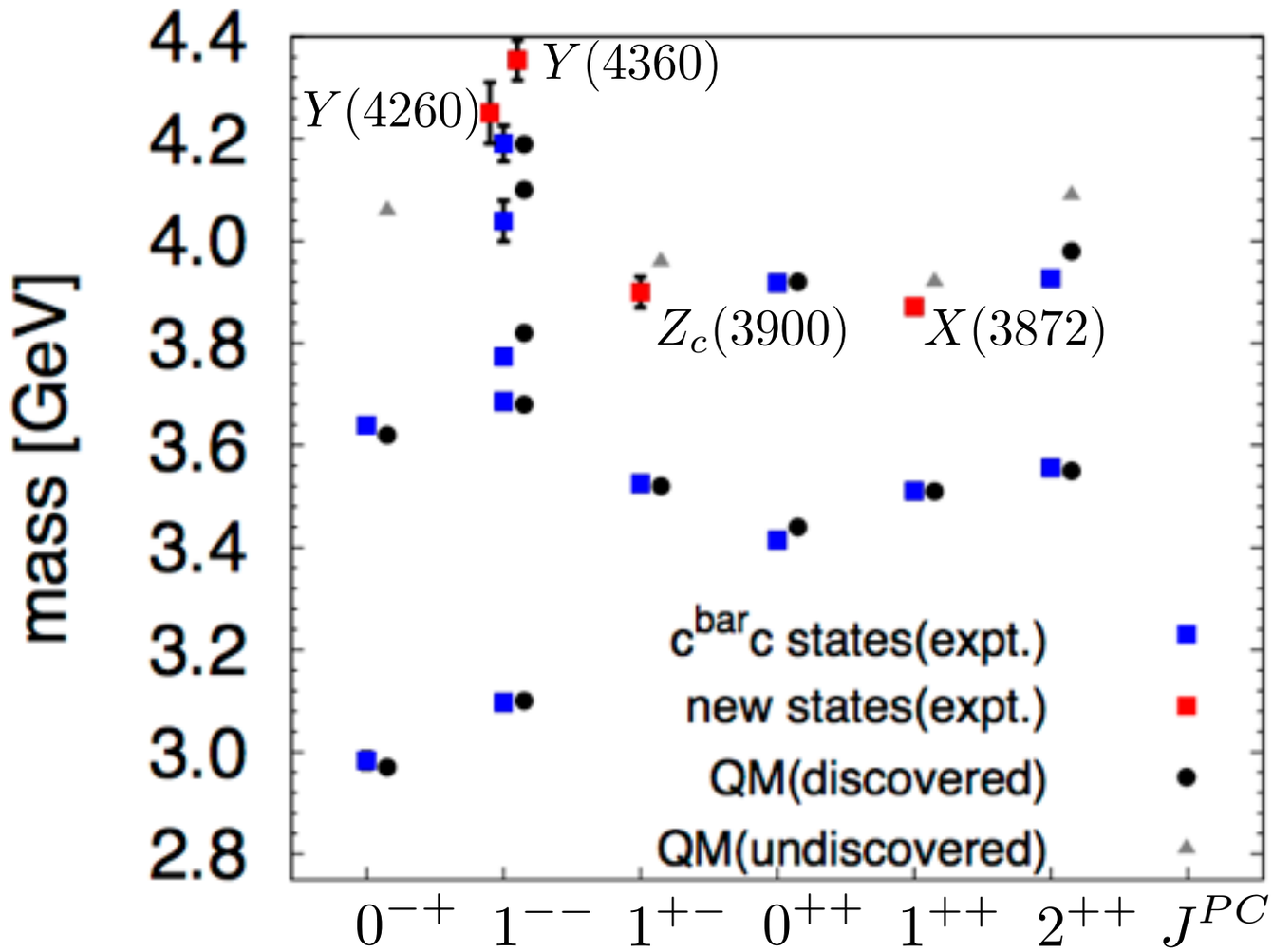
H as ΛΛ resonance
H as bound NΞ

This suggests that H-dibaryon becomes resonance at physical point.
Below or above NΞ ?
=> Simulation at physical point on K-computer

Physically, it is essential that H-dibaryon is a bound state in the flavor SU(3) limit.

2. Exotic hadron Zc(3900)

Charmonium(-like) spectrum



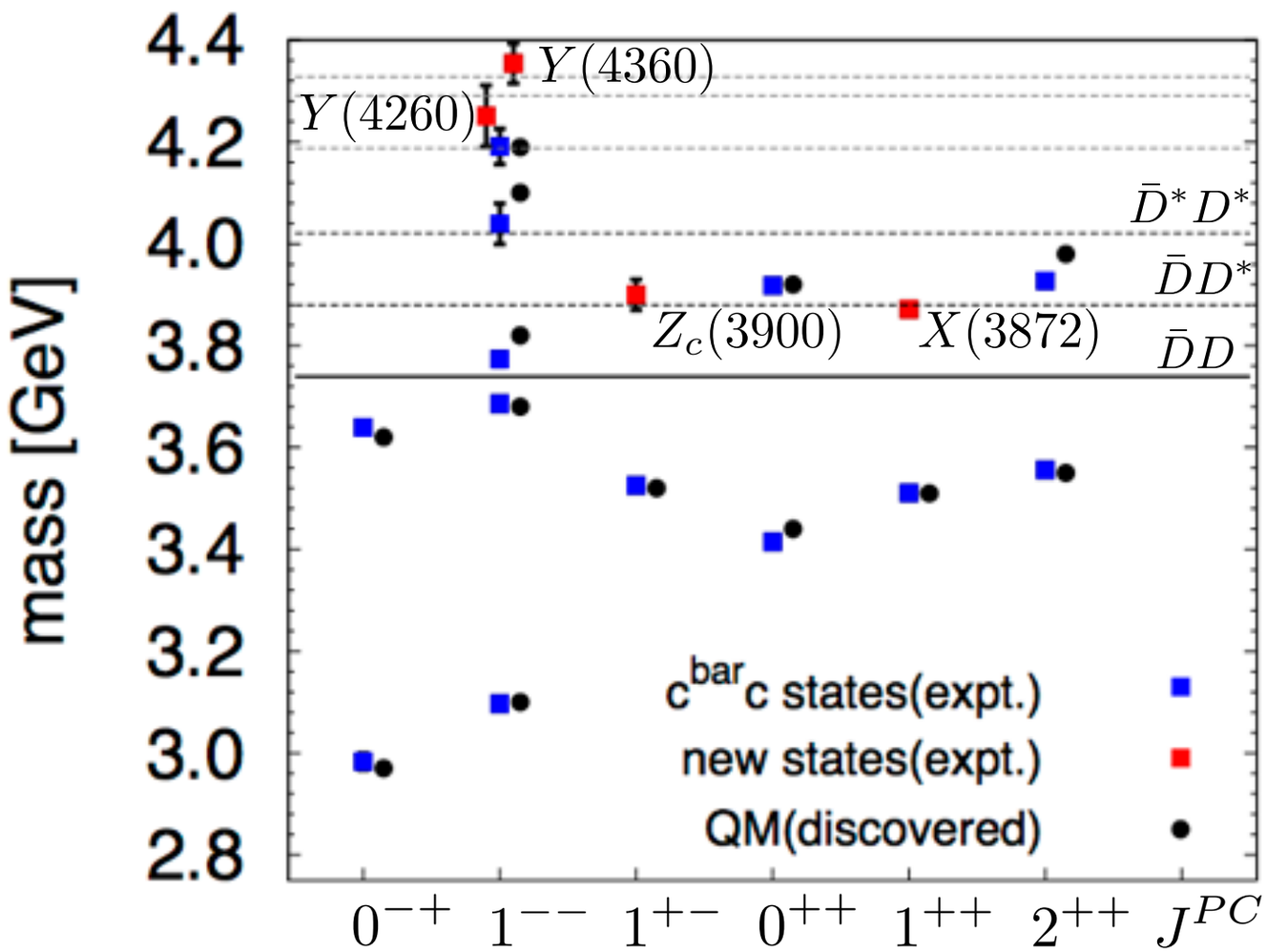
Quark model well describes observed mass spectra below 3.8 GeV.

Several states above 3.8 GeV are not discovered.

New (X,Y,Z) states, not predicted by QM, are experimentally observed.

Exotic ?

Charmonium(-like) spectrum



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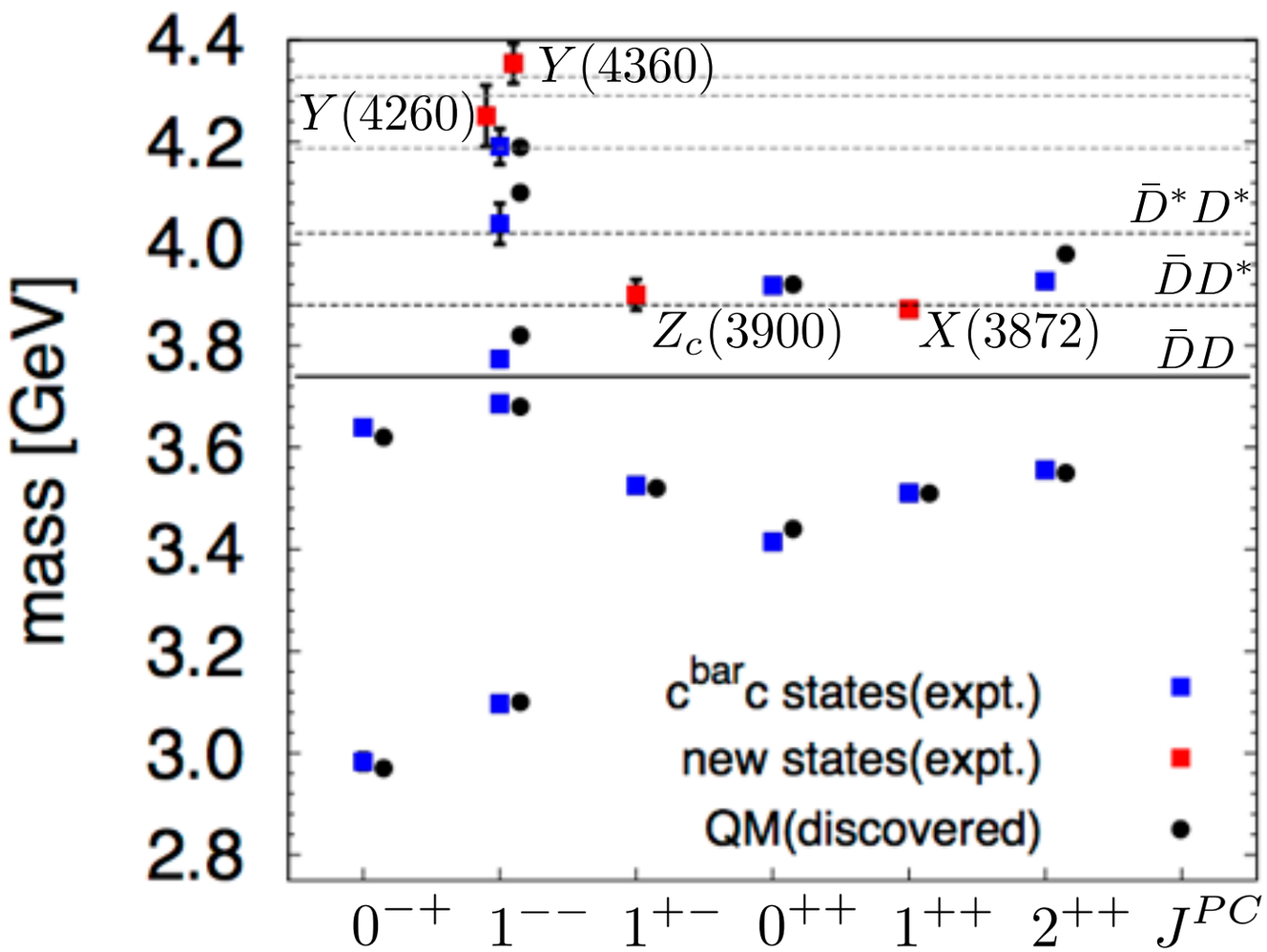
Exotic ?

Charmonium(-like) spectrum



Quark model well describes observed mass spectra below 3.8 GeV.

Several states above 3.8 GeV are not discovered.

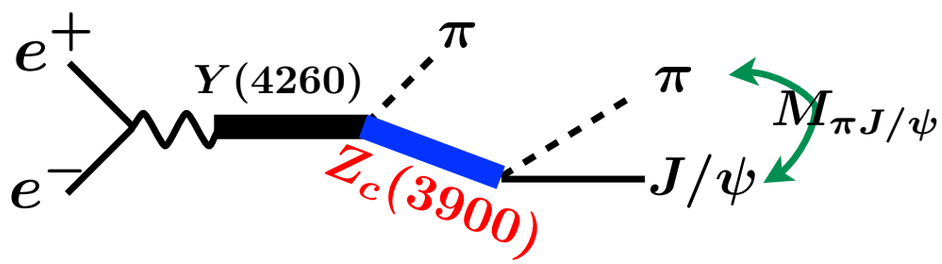
New (X,Y,Z) states, not predicted by QM, are experimentally observed.

Exotic ?

☆ All new states are found above 3.8 GeV

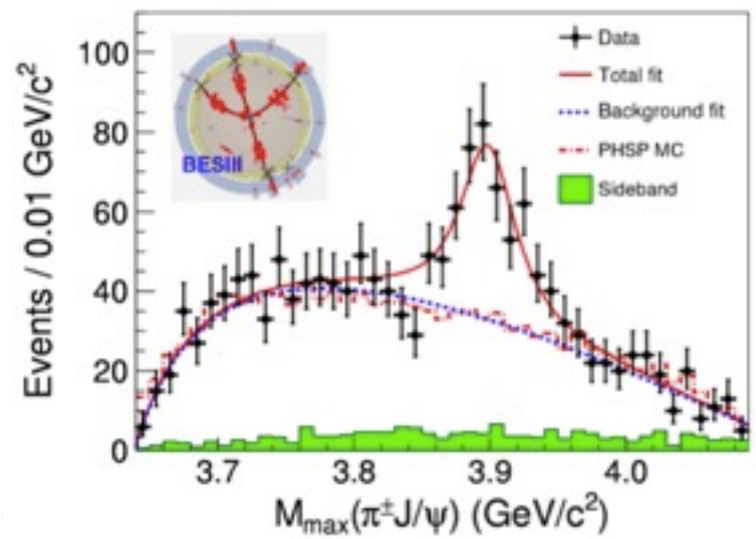
- Lowest open charm threshold ($D^{bar}D$) is 3.75 GeV
- All new states embedded in two-meson continuum($D^{bar}D$, $D^{bar}D^*$, $D^{bar*}D^*$,⋯)
- Channel coupling is a key to investigate X, Y, Z states

Tetra quark candidate $Z_c(3900)$

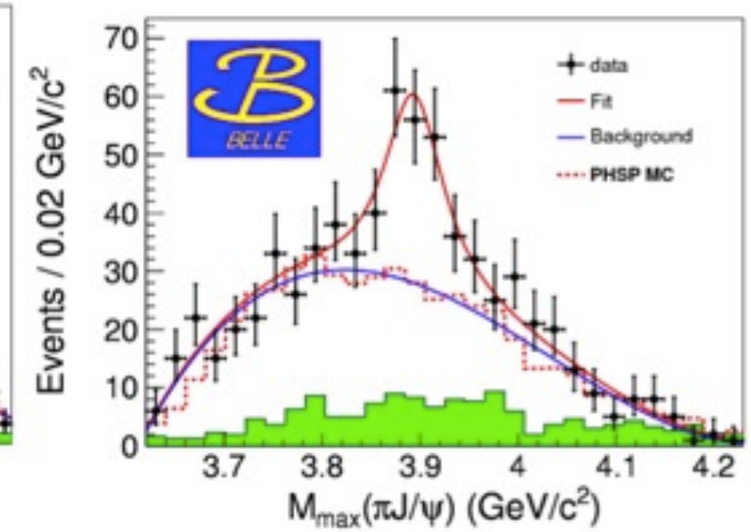


$$e^+ + e^- \to Y(4260) \to \pi + Z_c(3900)$$

BESIII Coll., PRL110 (2013).



Belle Coll., PRL110 (2013).



peak in $\pi^{\pm}J/\psi$ invariant mass (minimal quark content $c\bar{c}u\bar{d}$) terra quark candidate

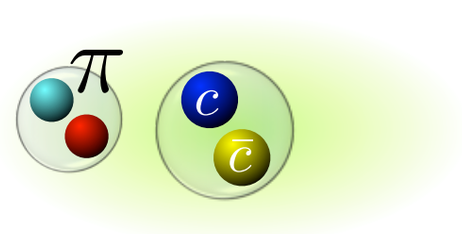
$M + i\Gamma \sim 3900 + i60$ MeV (Breit-Wigner) just above D^bar^D* threshold

interpretations

tetraquark

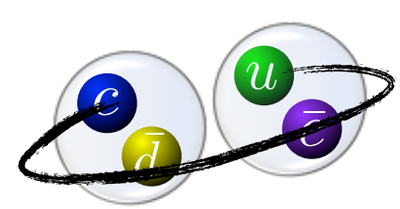
Maiani et al.('13)

J/ψ + π atom



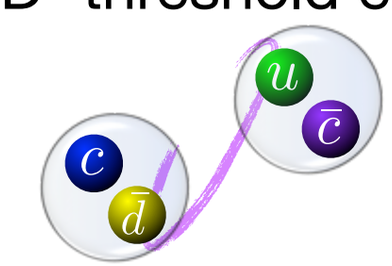
Voloshin('08)

D^bar^D* molecule



Nieves et al.('11) + many others

D^bar^D* threshold effect



Chen et al.('14), Swanson('15)

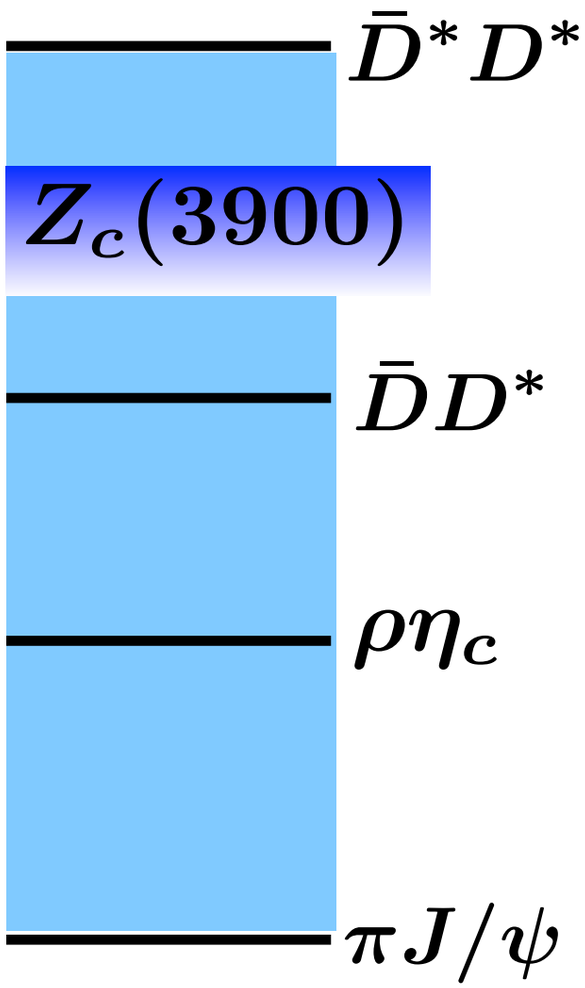
genuine state

kinematical origin

coupled channel potential and $Z_c(3900)$

Y. Ikeda et al. [HAL QCD], PRL117, 24001 (2016)

coupled channel potential $V_{AB}(\vec{r})$ $A, B = \pi J/\psi, \rho\eta_c, \bar{D}D^*$



$a \simeq 0.09$ fm, $L \simeq 2.9$ fm

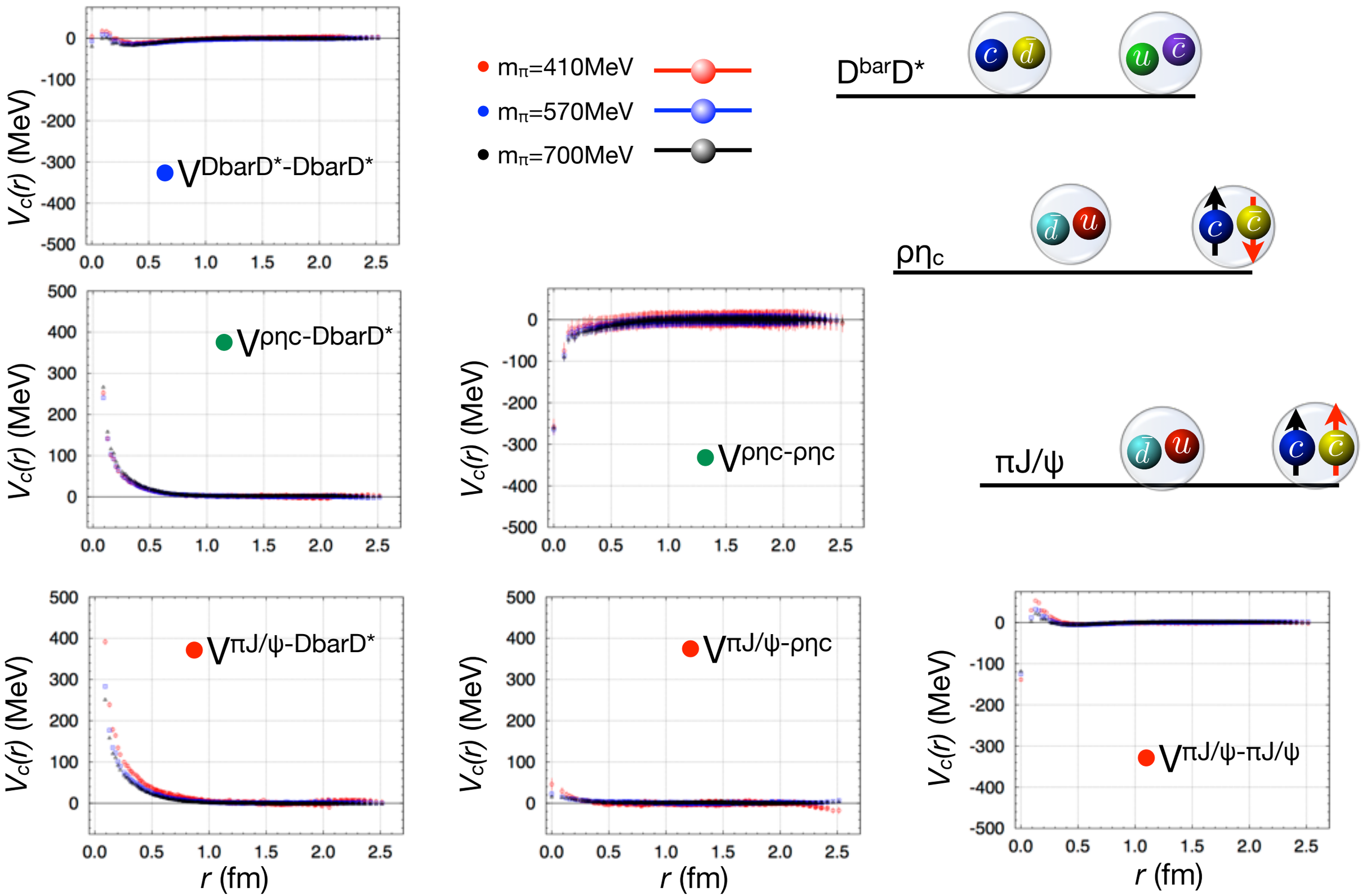
$m_\pi = 411, 572, 701$ MeV

$\bar{D} = \bar{c}u$ (spin 0) $D^* = \bar{d}c$ (spin 1)

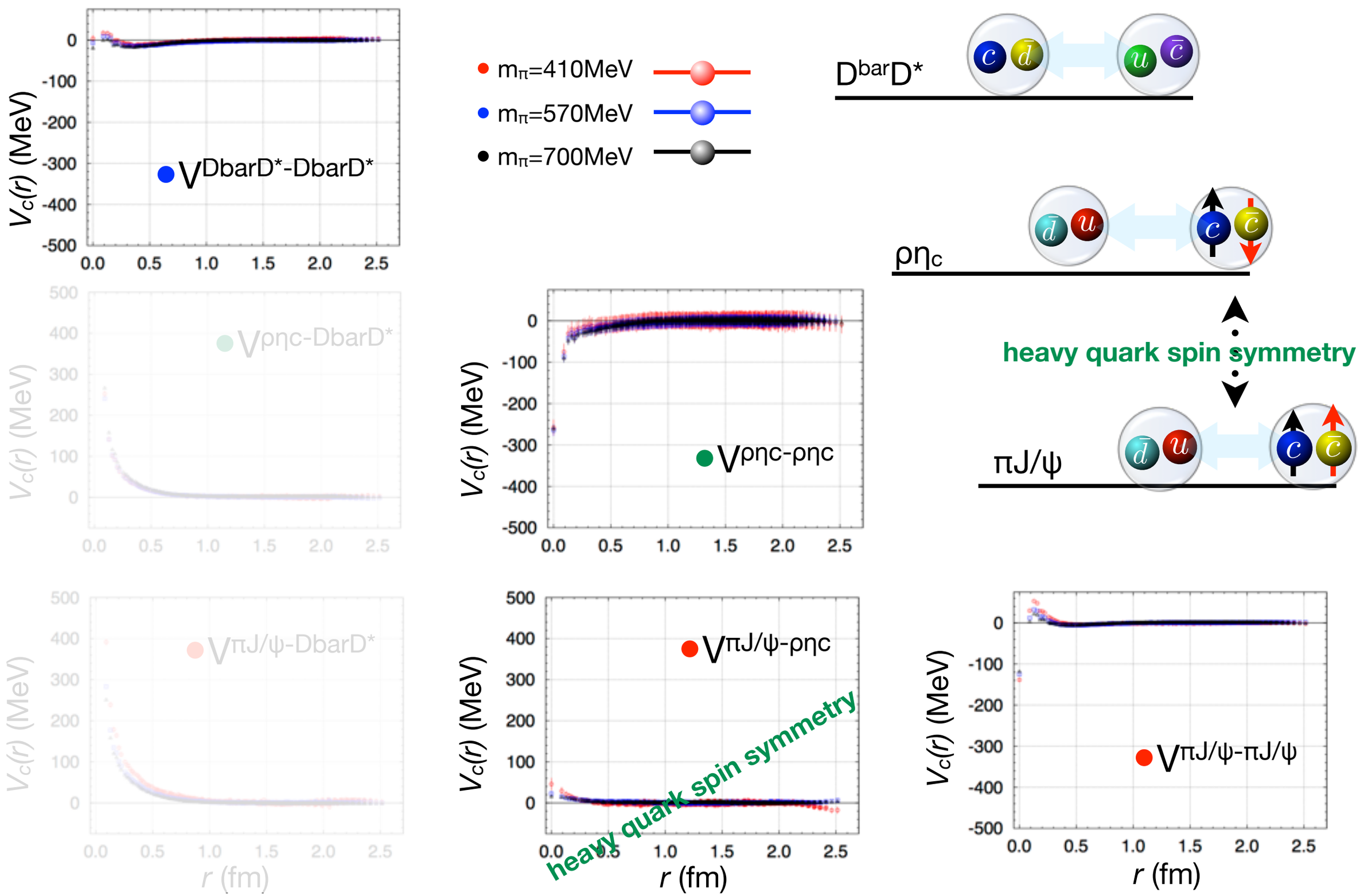
$\eta_c = \bar{c}c$ (spin 0) $\rho = \bar{d}u$ (spin 1)

$\pi = \bar{d}u$ (spin 0) $J/\psi = \bar{c}c$ (spin 1)

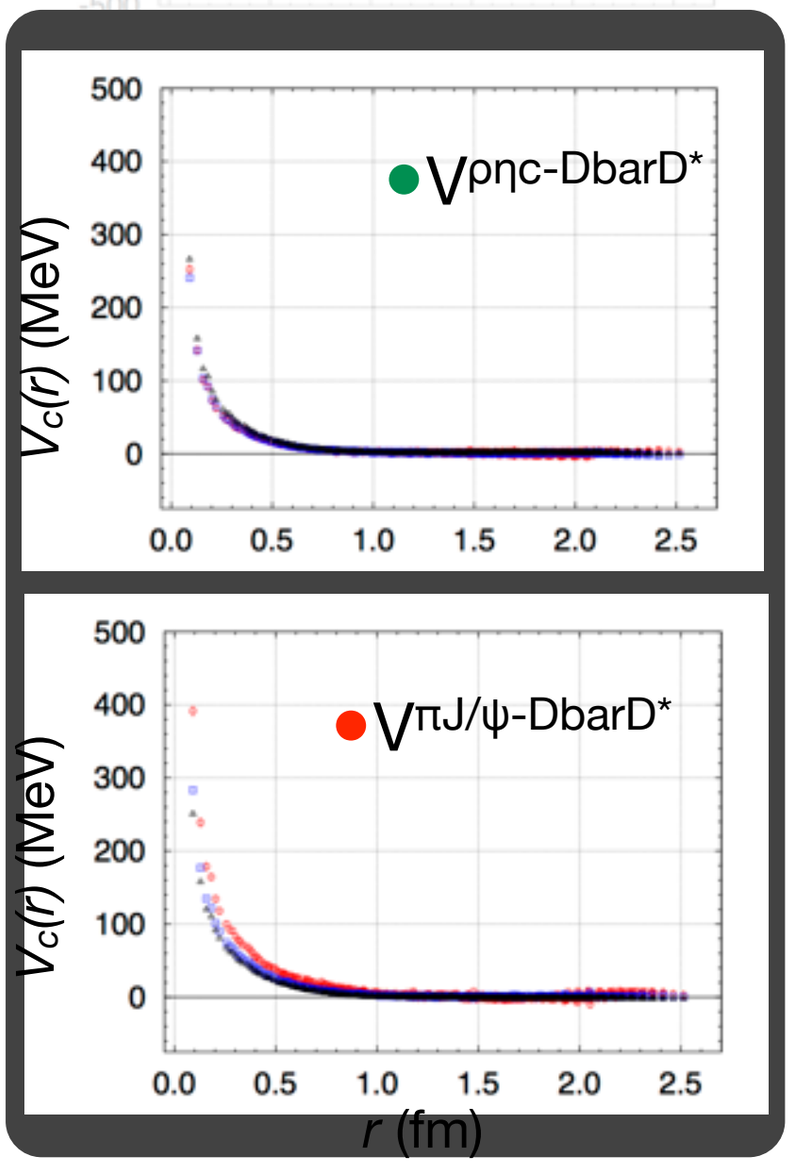
3×3 potential matrix $(\pi J/\psi - \rho\eta_c - \bar{D}D^*)$

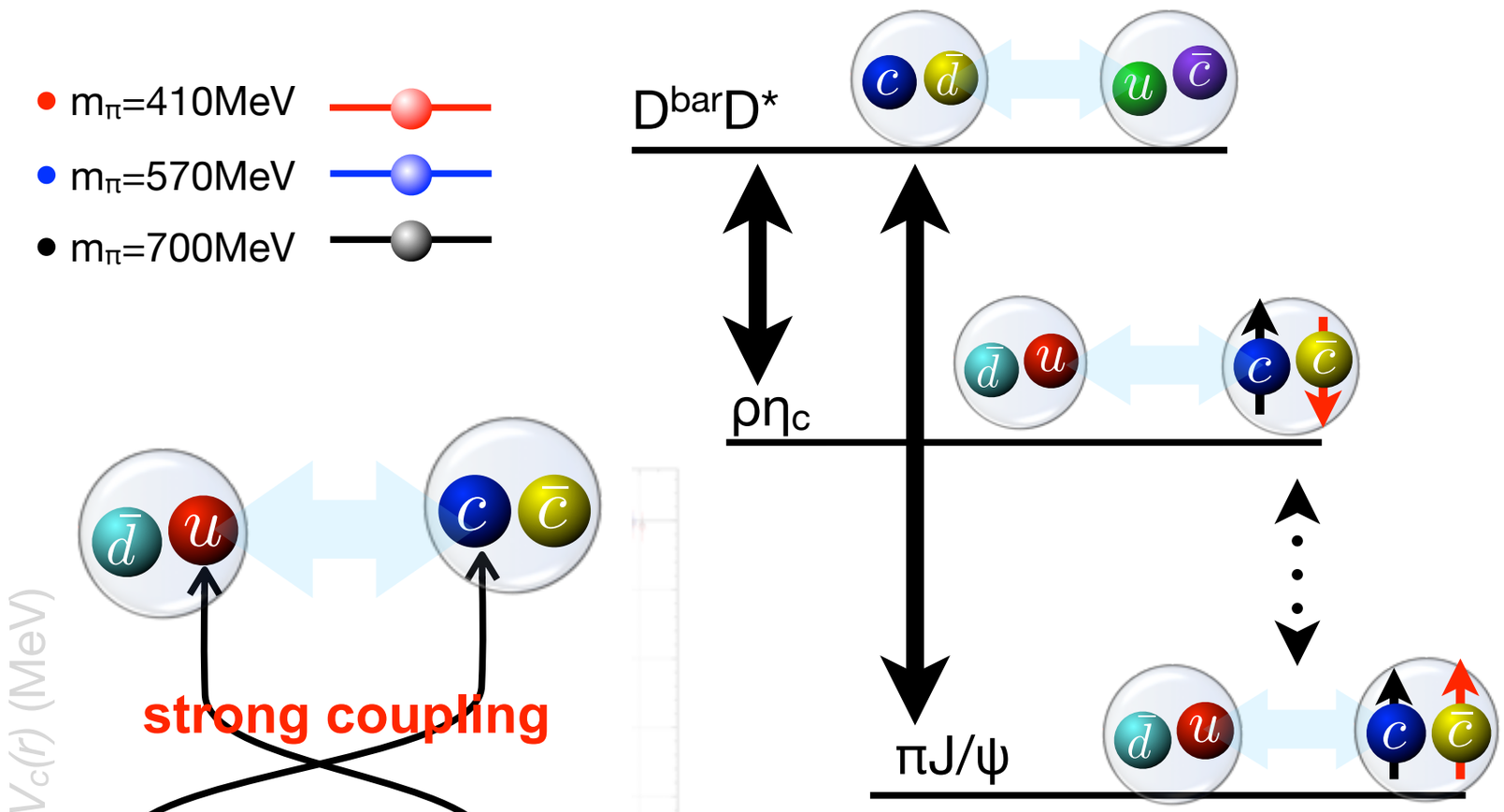


3×3 potential matrix $(\pi J/\psi - \rho\eta_c - \bar{D}D^*)$



3×3 potential matrix $(\pi J/\psi - \rho\eta_c - \bar{D}D^*)$





- strong $V^{\pi J/\psi,DbarD^*}$ & $V^{\rho\eta c,DbarD^*}$

➡ charm quark exchange process

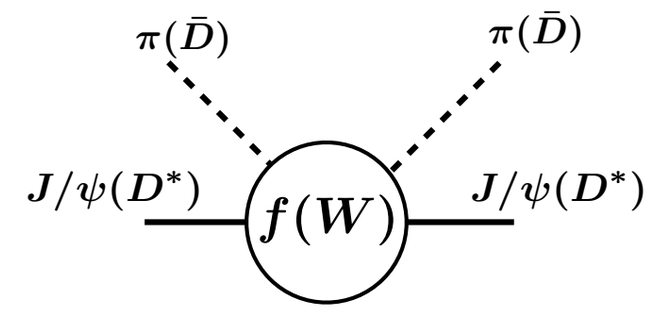
Structure of $Z_c(3900)$

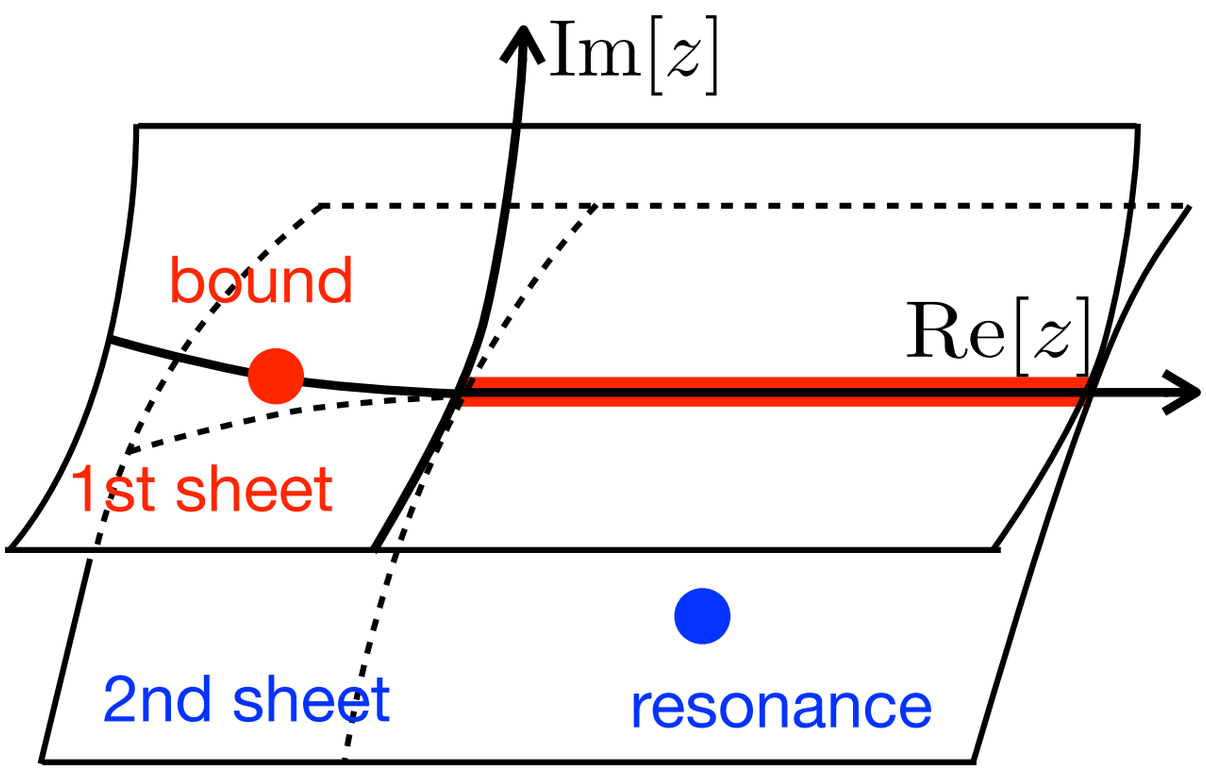
S-wave $\pi J/\psi - \rho\eta_c - \bar{D}D^*$ coupled channel scattering

1. invariant mass distribution of 2-body scattering

$$N_{\rm sc} \propto ({\rm flax}) \cdot \sigma(W) \propto {\rm Im} f(W)$$

2. pole position of S-matrix





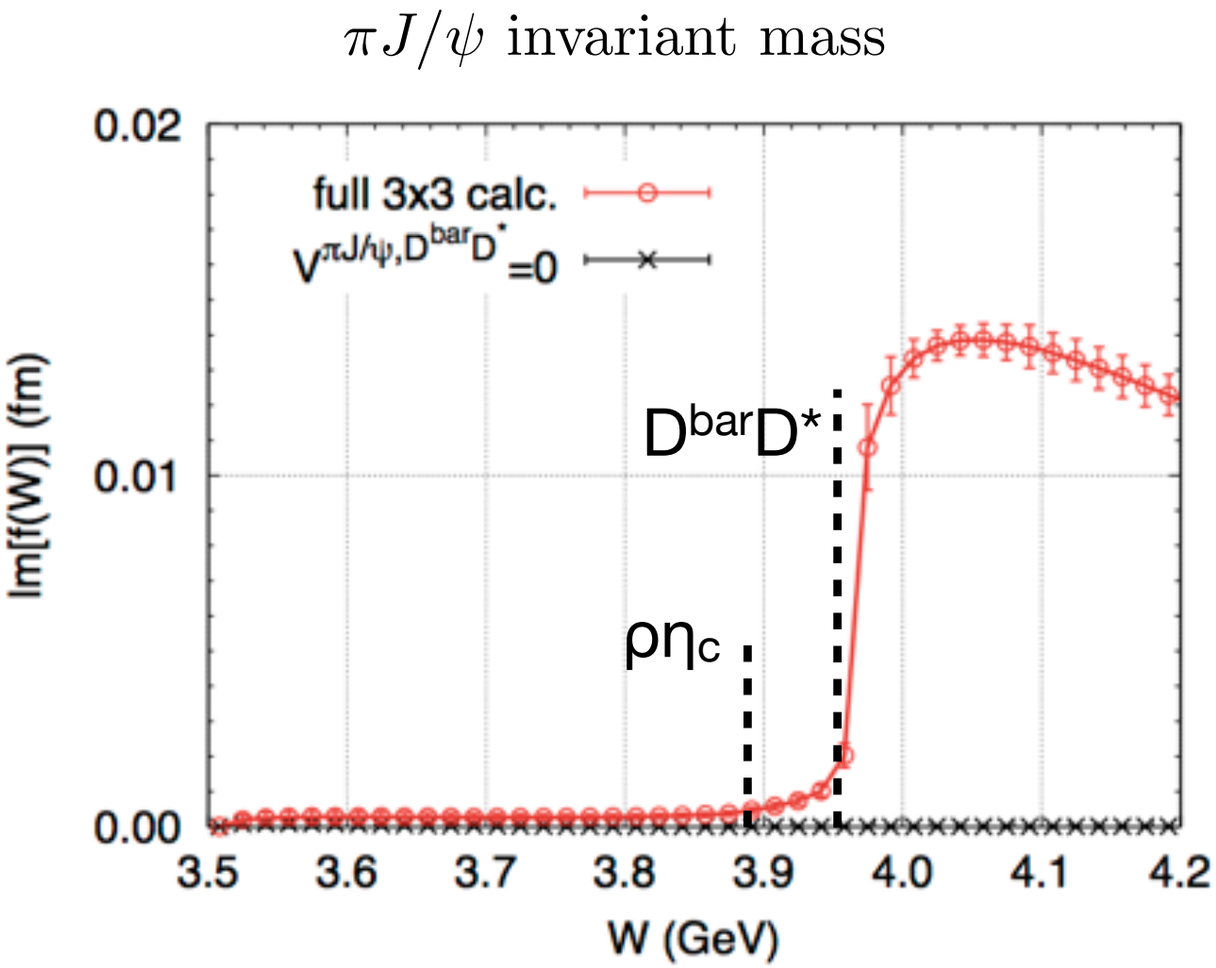
bound state (1st sheet)

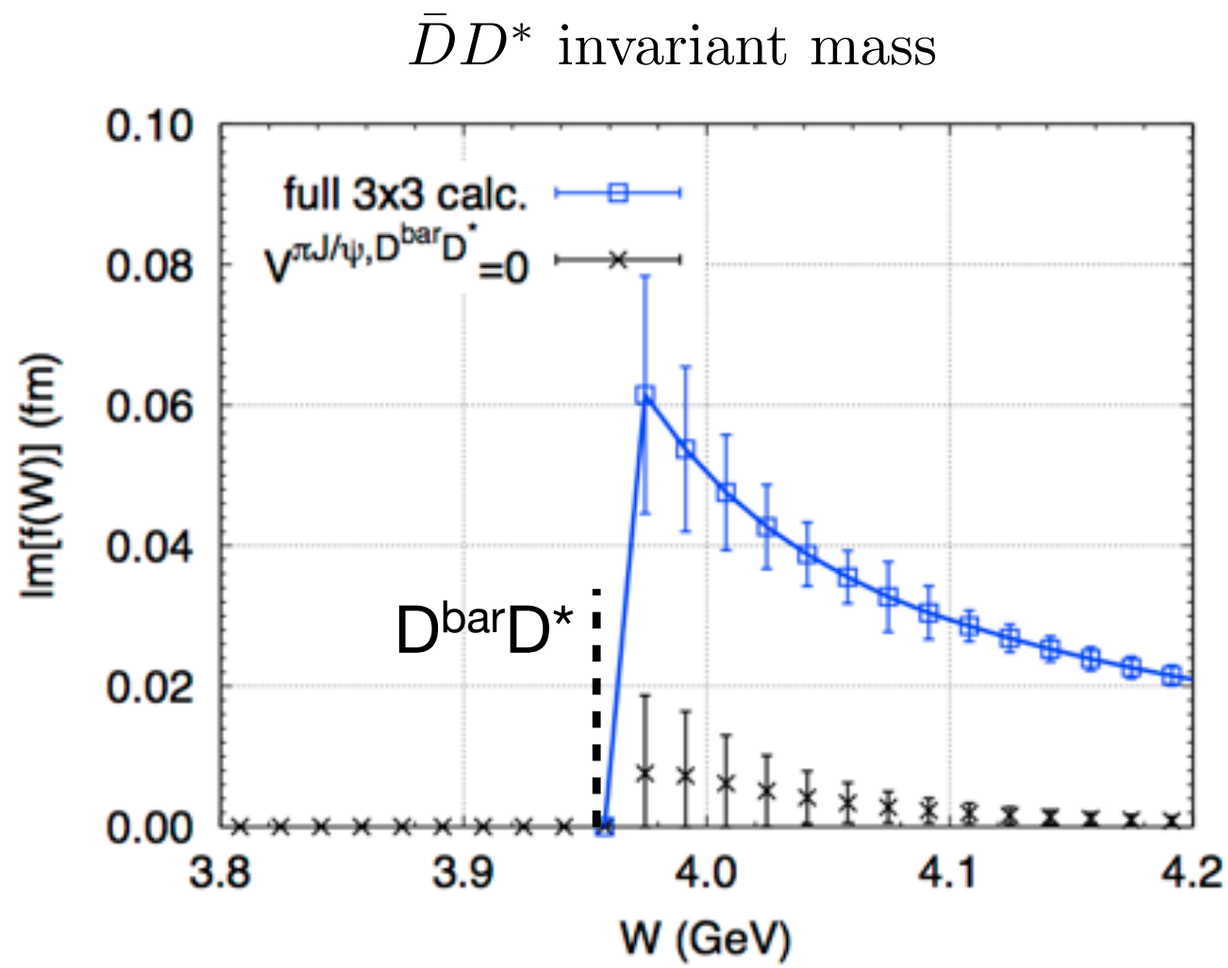
- pole position --> binding energy
- residue --> coupling to scattering state

resonance (2nd sheet)

- analytic continuation onto 2nd sheet
- pole position --> resonance energy
- residue --> coupling to scat. state, partial decay

Invariant mass distribution





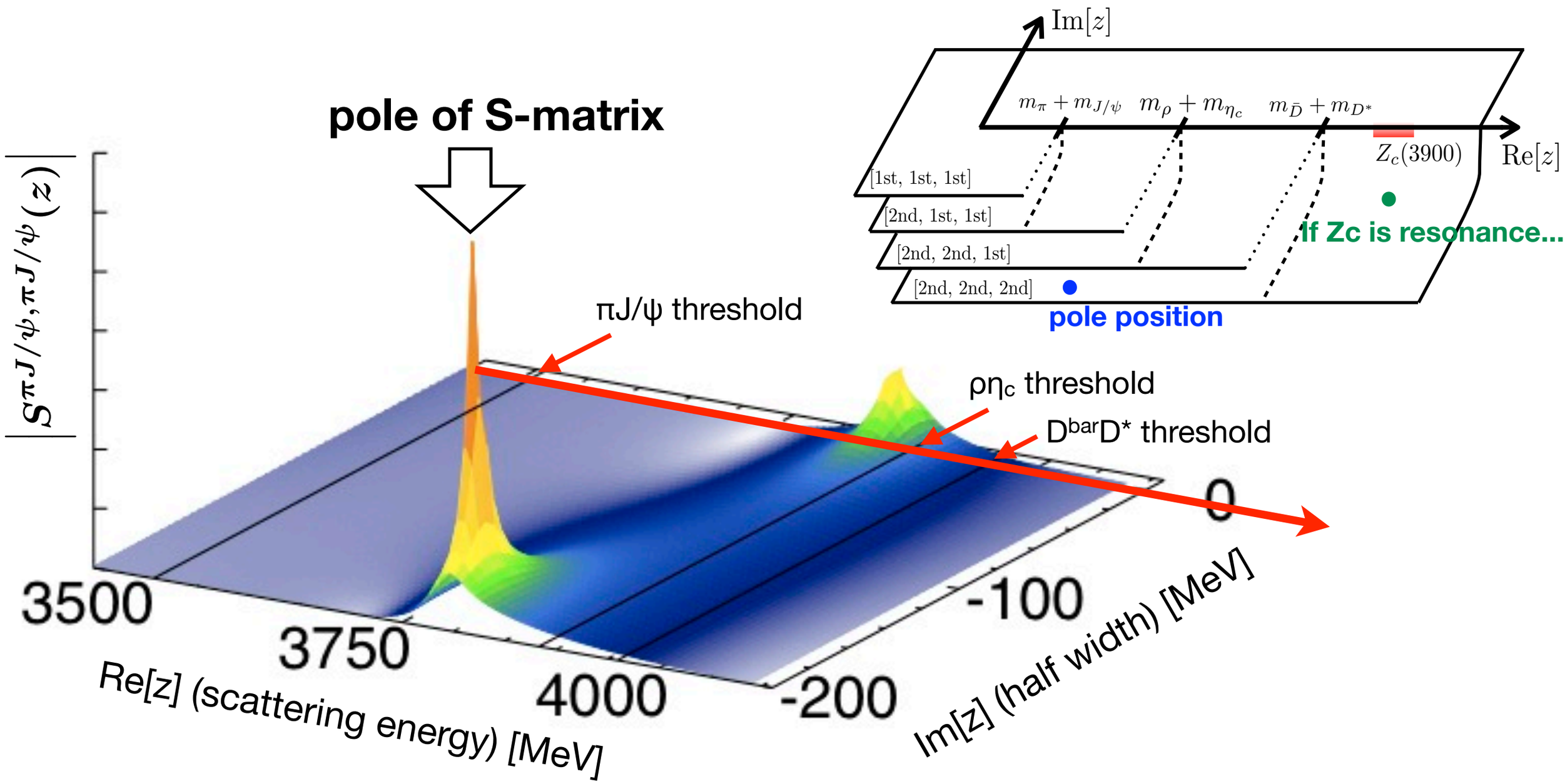
☆ Enhancement just above $D^{bar}D^*$ threshold in both amplitudes

- effect of strong off-diagonal parts (black—>off-diagonal=0)
- peak is not the Breit-Wigner shape

✱ Is $Z_c(3900)$ conventional resonance？ —> pole of S-matrix

Pole of S-matrix

$(\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $\bar{D}D^*$:2nd)



☆Pole corresponds to "virtual state"

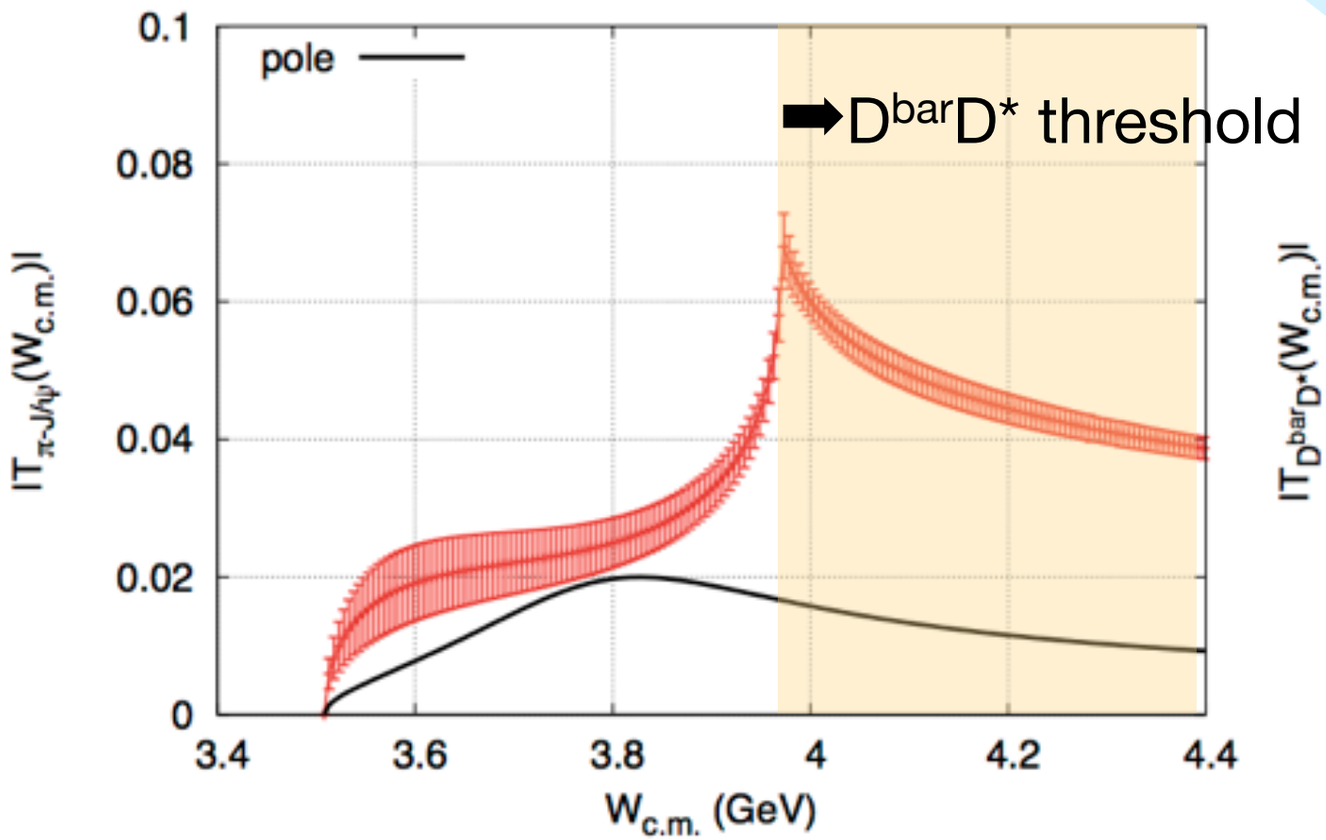
☆ Pole contribution to scat. observable is small (far from scat. axis)

$Z_c(3900)$ is not a resonance but "threshold cusp" induced by strong channel couplings

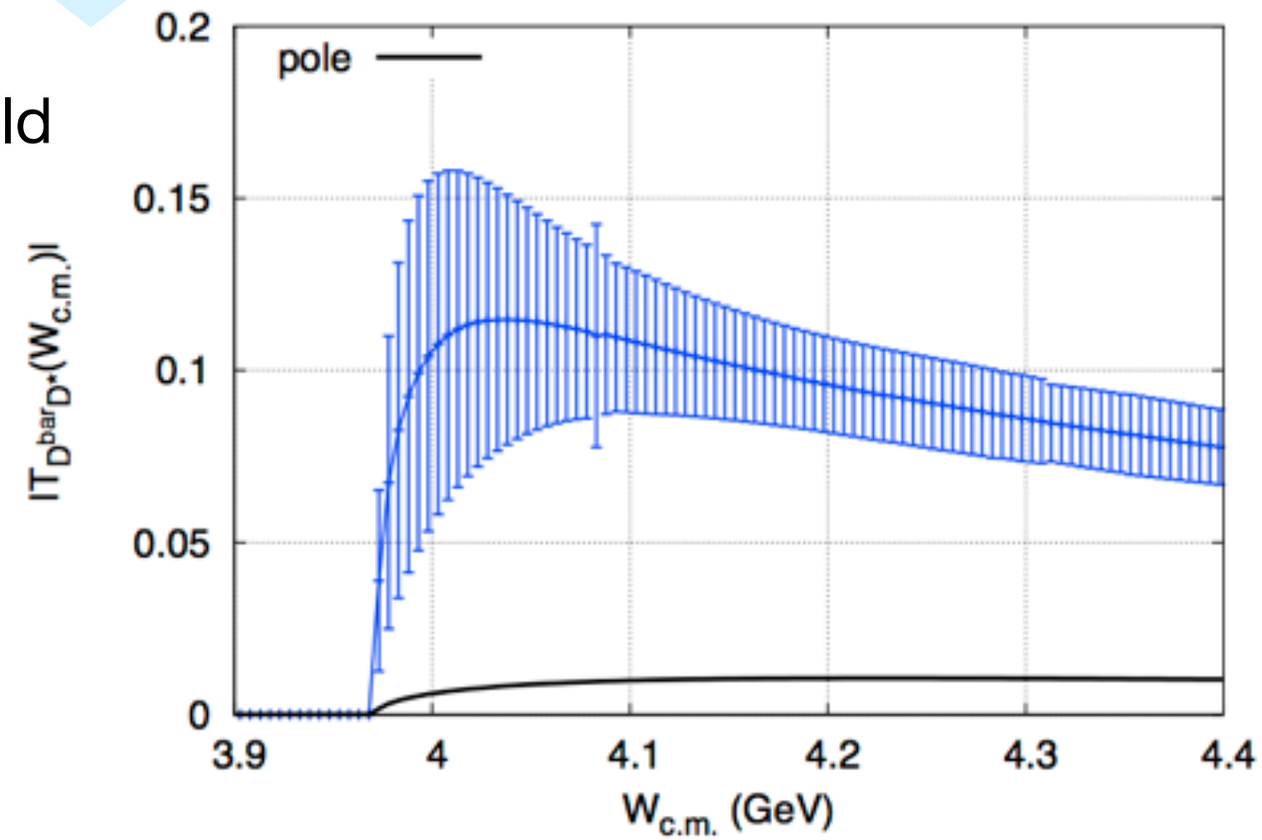
Contributions from the pole to T-matrices

$$S = 1 + iT$$

$\pi J/\psi - \pi J/\psi$



$\bar{D}D^\star - \bar{D}D^\star$

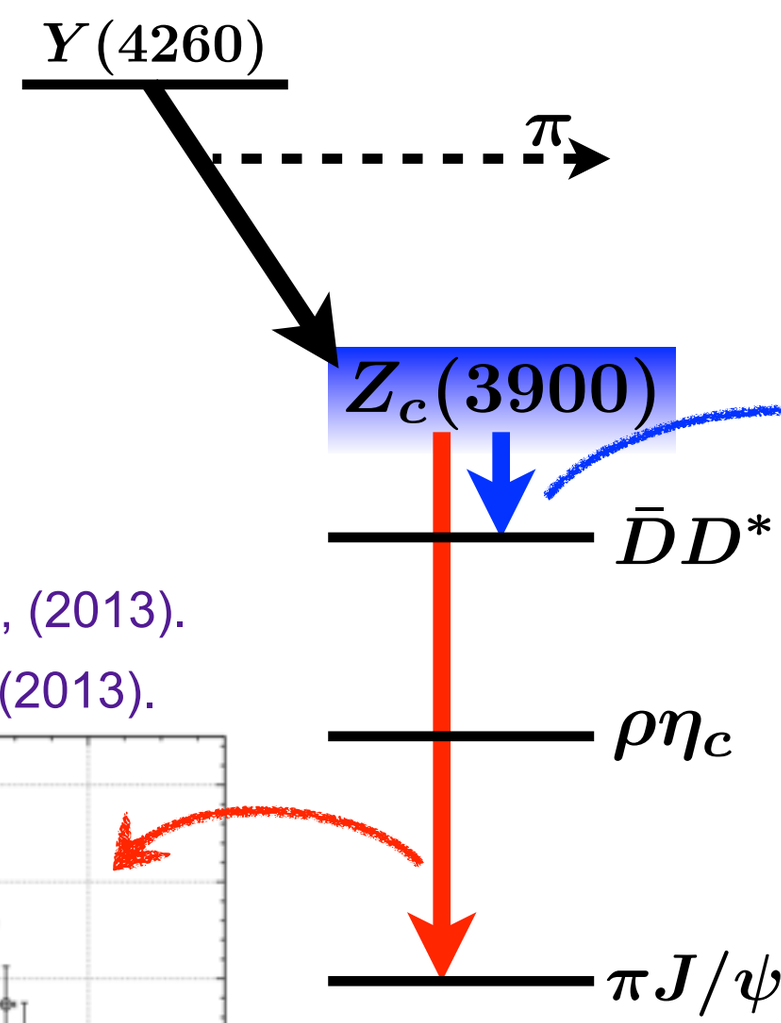


☆ contribution from virtual pole to T-matrix is small

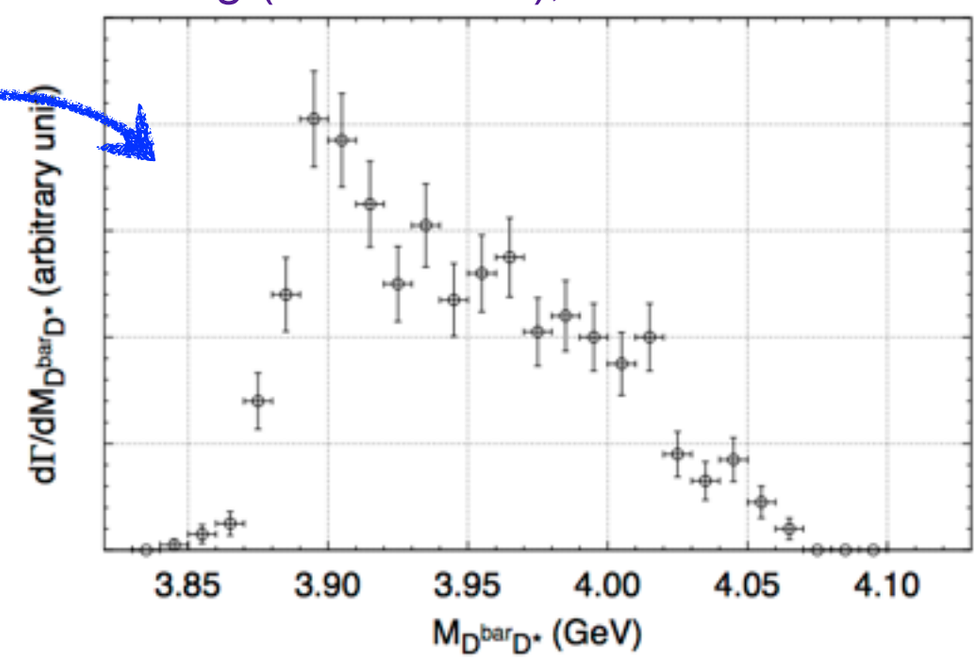
☆ $Z_c(3900)$ is “threshold cusp” induced by strong channel couplings

Comparison with experimental data

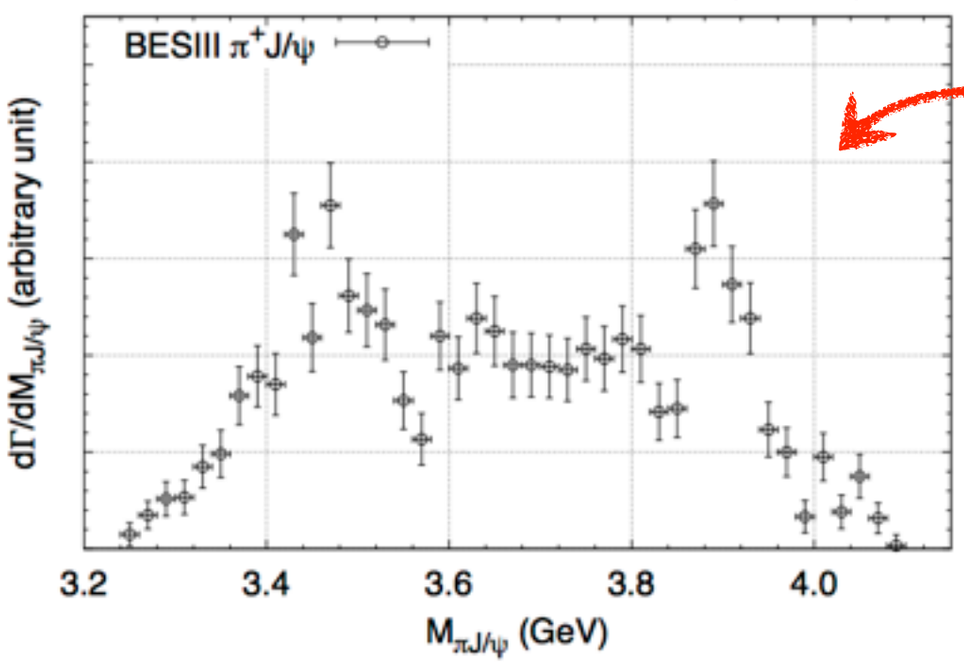
spectrum of Y(4260) 3-body decay

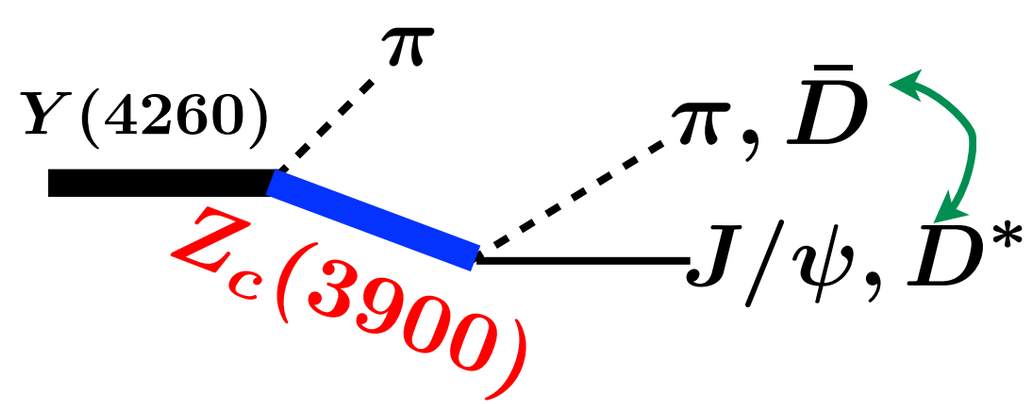


BESIII Coll., PRL112, 022001, (2014).
Wang (BESIII Coll.), MENU2016 talk



BESIII Coll., PRL110, 252001, (2013).
Belle Coll., PRL110, 252002, (2013).



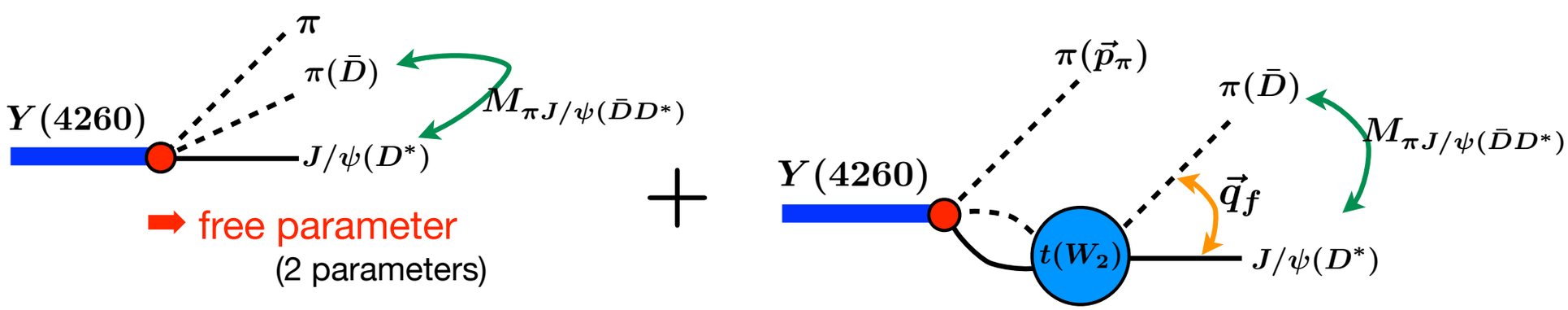


$$Y(4260) \to \pi\pi J/\psi, \pi\bar{D}D^*$$

$$d\Gamma_{Y\to\pi+f} = (2\pi)^4\delta(W_3 - E_\pi(\vec{p}_\pi) - E_f(\vec{q}_f))d^3p_\pi d^3q_f|T_{Y\to\pi+f}(\vec{p}_\pi, \vec{q}_f; W_3)|^2$$

3-body T-matrix

$$T_{Y\to\pi+f}(\vec{p}_\pi, \vec{q}_f; W_3) = \sum_{n=\pi J/\pi, \bar{D}D^*} C^{Y\to\pi+n}\left[\delta_{nf} + \int d^3q' \frac{t_{nf}(\vec{q}', \vec{q}_f, \vec{p}_\pi; W_3)}{W_3 - E_\pi(\vec{p}_\pi) - E_n(\vec{q}', \vec{p}_n) + i\epsilon}\right]$$

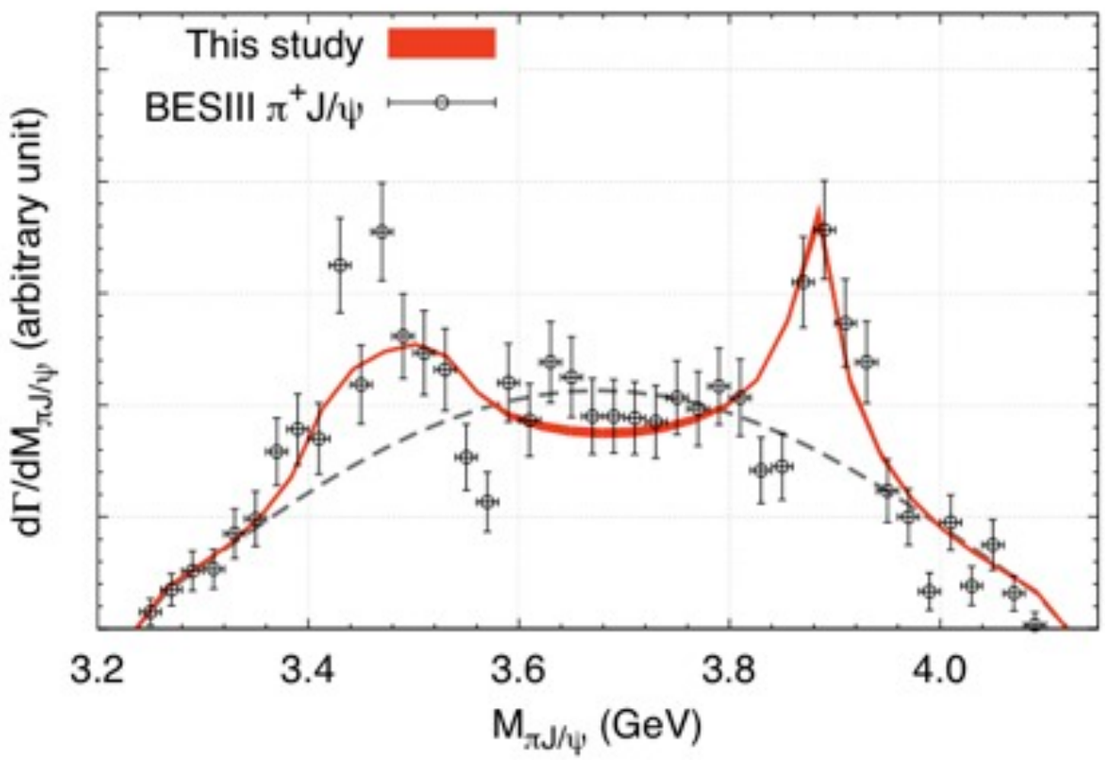


t-matrix from potential matrix

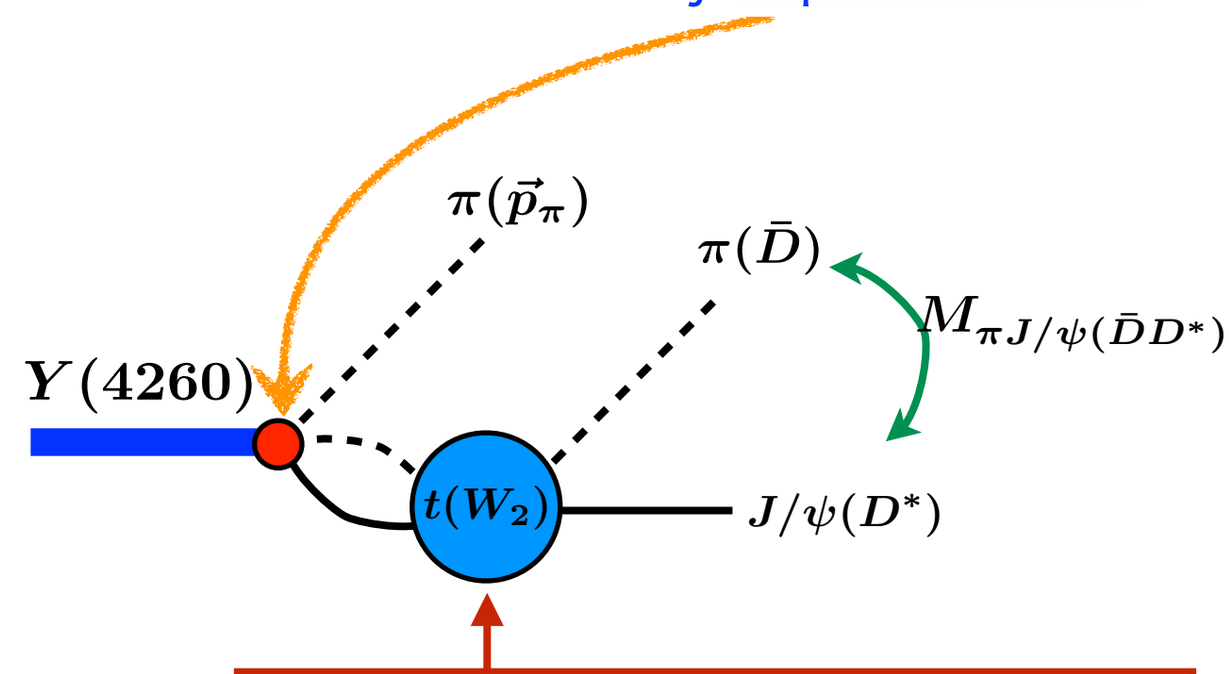
employ physical hadron masses for a comparison with experimental data

- potential matrix is used to calculate t-matrix in subsystem
- fix free parameters by fitting Y(4260) —>$\pi\ \pi$ J/ψ experimental data

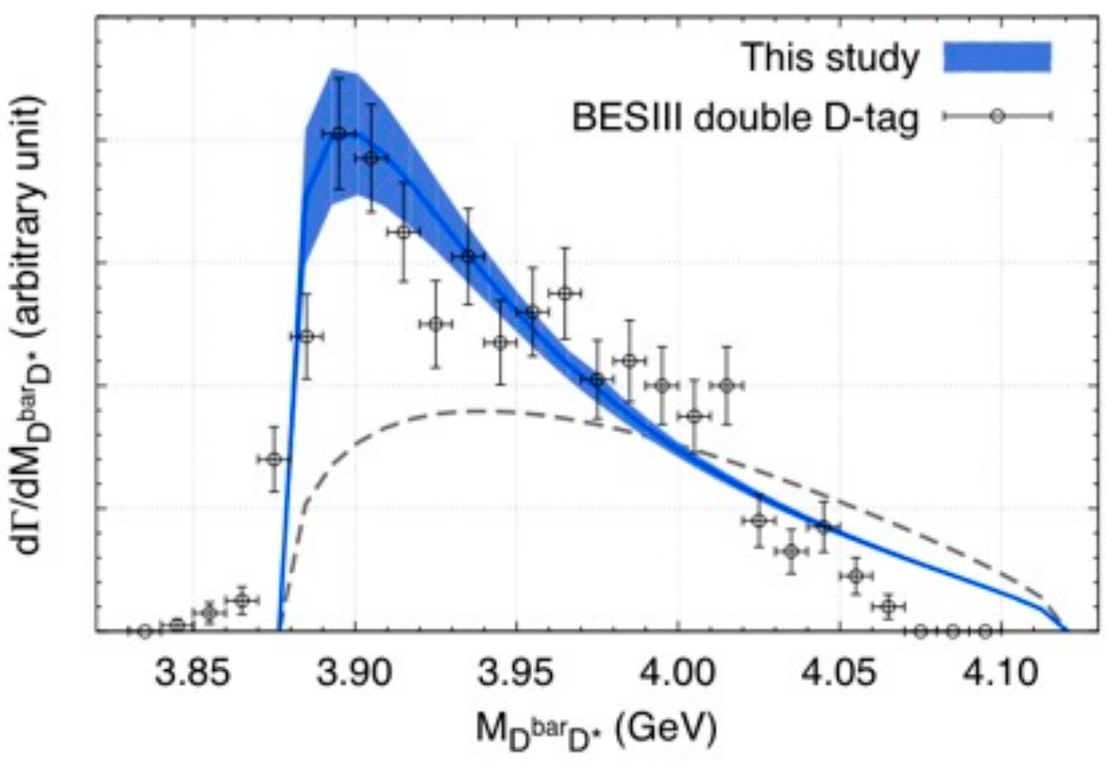
Invariant mass of 3-body decay



Data are well described by 2 parameters



Our coupled channel potential



Without off-diagonal parts (dashed curves), peak structures are not reproduced.

$Z_c(3900)$ is threshold cusp caused by strong channel coupling

III. Discussion

Pros and Cons of HAL QVD potential method

Pros

- phase shift as a function of momentum and S-matrix in complex momentum (with approximation)
- no ground state saturation is required with t-dep. method
- extension to coupled channel is easy
- less finite volume effect, can be checked, partial wave mixing does not matter
- more NBS wave functions give more higher order terms in the derivative expansion

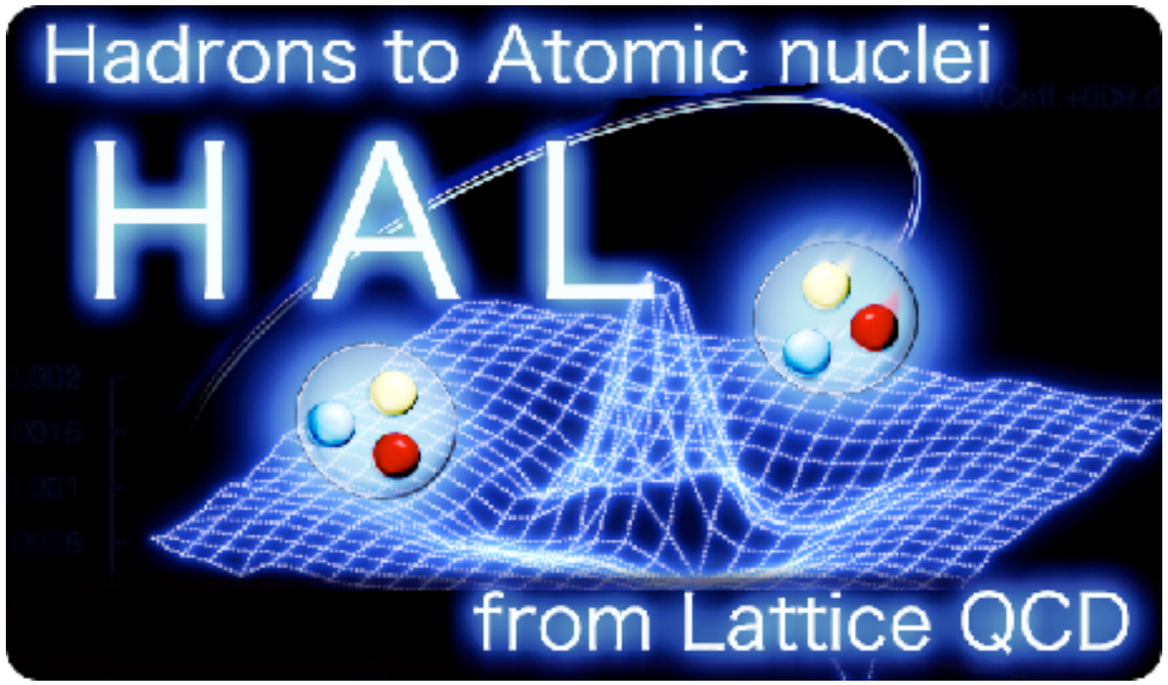
Cons

- potential is scheme-dependent
- some errors from the truncation of the derivative expansion.
- approximated phase shift with t-dep. method
- more numerical costs

Summary

- Currently the HAL QCD Potential method is the only way to investigate baryon interactions reliably in lattice QCD.
- Nuclear potentials, Hyperon potentials and more
- Omega-Omega : shallow bound state at physical pion mass
- N-Omega: bound state at physical pion mass ?
- H-dibaryon: bound state at SU(3) limit, resonance with SU(3) breaking
  - physical point simulation is on-going with K-computer.
- Application to exotic hadron: Zc(3900) is the threshold effect
- Other applications (rho & sigma resonances, heavy baryons, Tetra quark, Penta quark, 3 body forces and more)

HAL QCD Collaboration



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Riken: Takumi Doi, Takahiro Doi, Sinya Gongyo, Tetsuo Hatsuda, Takumi Iritani

RCNP, Osaka: Yoichi Ikeda, Noriyoshi Ishii, Keiko Murano, Hidekatsu Nemura

Nihon: Takashi Inoue

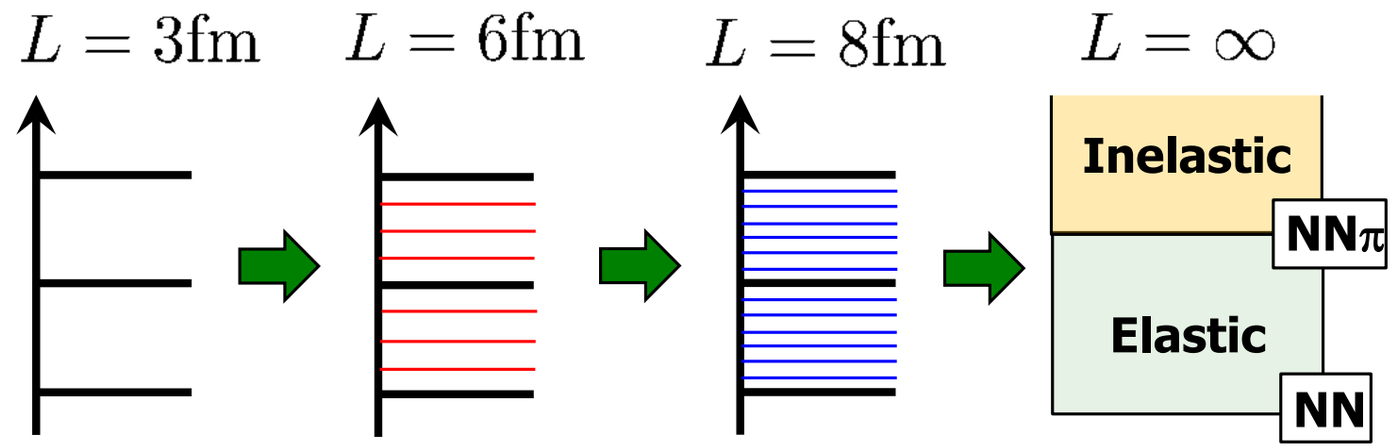
Birjand, Iran: Faisal Etminan

Backup slides

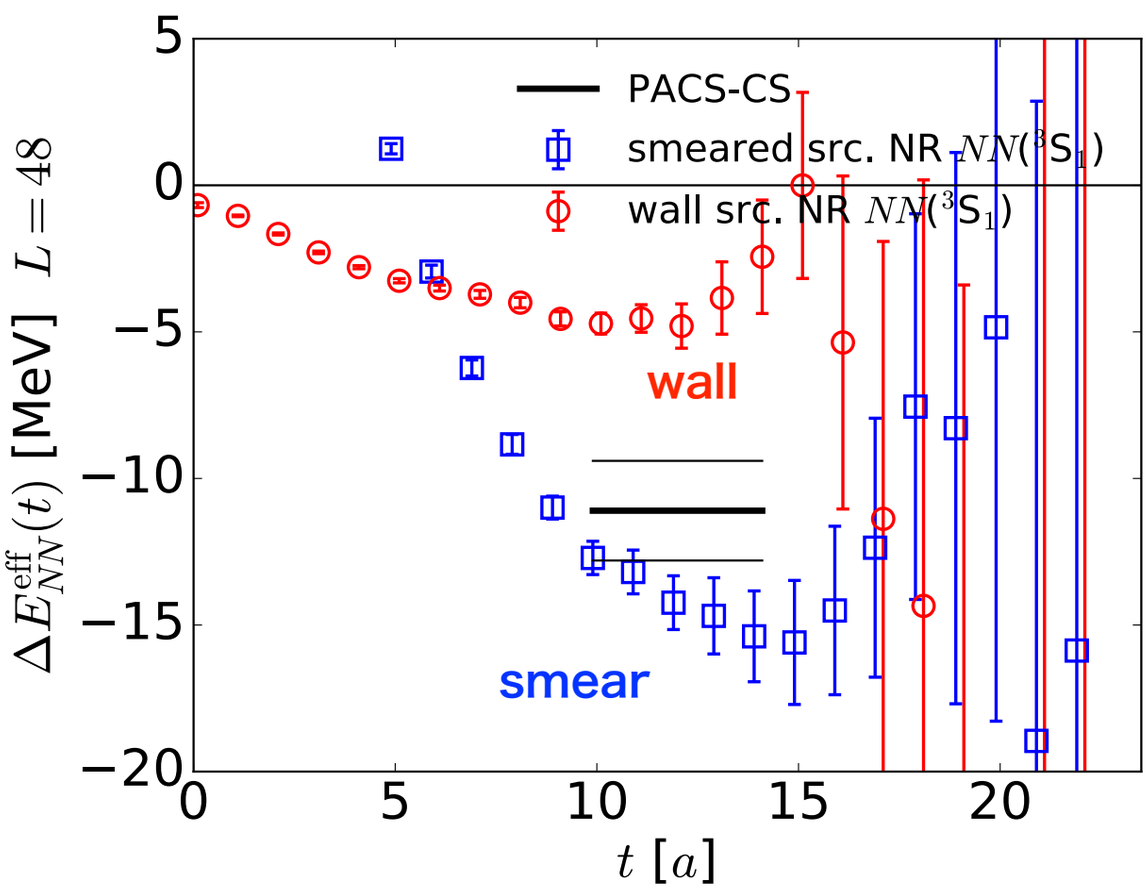
Failure of the direct method for baryons

Excited state contaminations

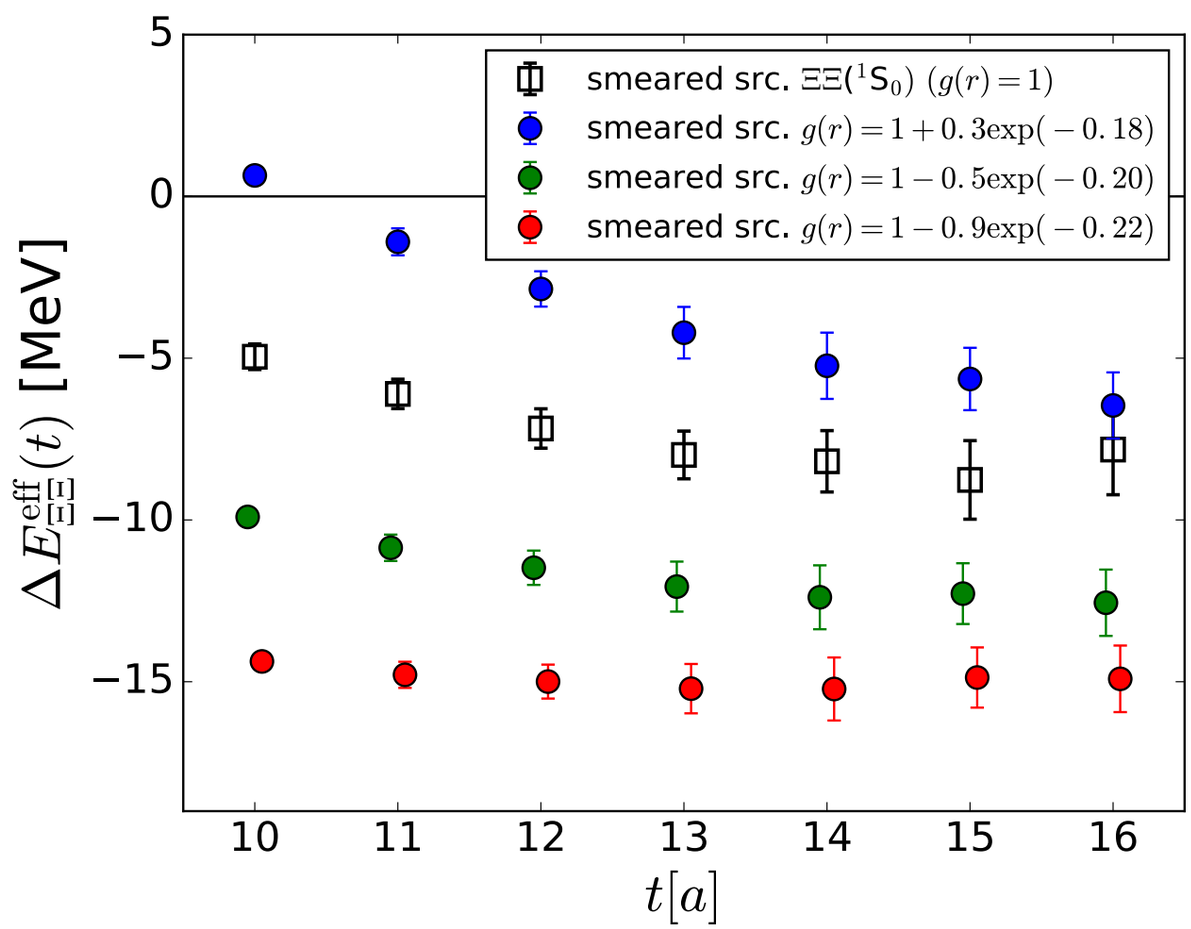
$\Delta E = E - 2m_B$



$NN(^3S_1)$



$\Xi\Xi(^1S_0)$ Smeared source



strong source operator dependence

strong sink operator dependence

Fake plateau problem

Mock-up data @ $m_\pi = 0.5$ GeV, $L = 4$ fm (setup of YIKU2012)

$$R(t) = e^{-\Delta E t}\left(1 + b\, e^{-\delta E_{\rm el} t} + c\, e^{-\delta E_{\rm inel} t}\right)$$

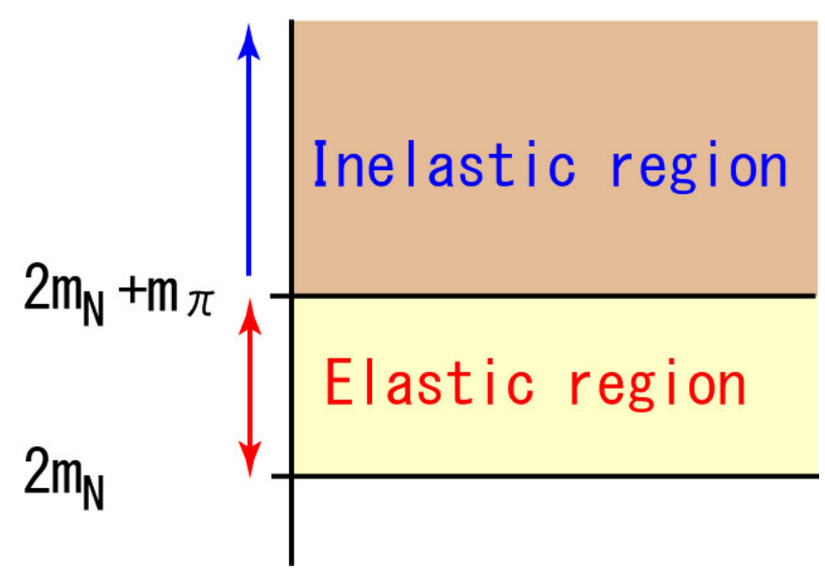
$\delta E_{\rm el} \propto \dfrac{1}{L^2}$ the lowest excitation energy of elastic scattering state

$\delta E_{\rm el} = 50$ MeV at $L \simeq 4$ fm

$b = \pm 0.1$ 10 % contamination $b = 0$ for a comparison

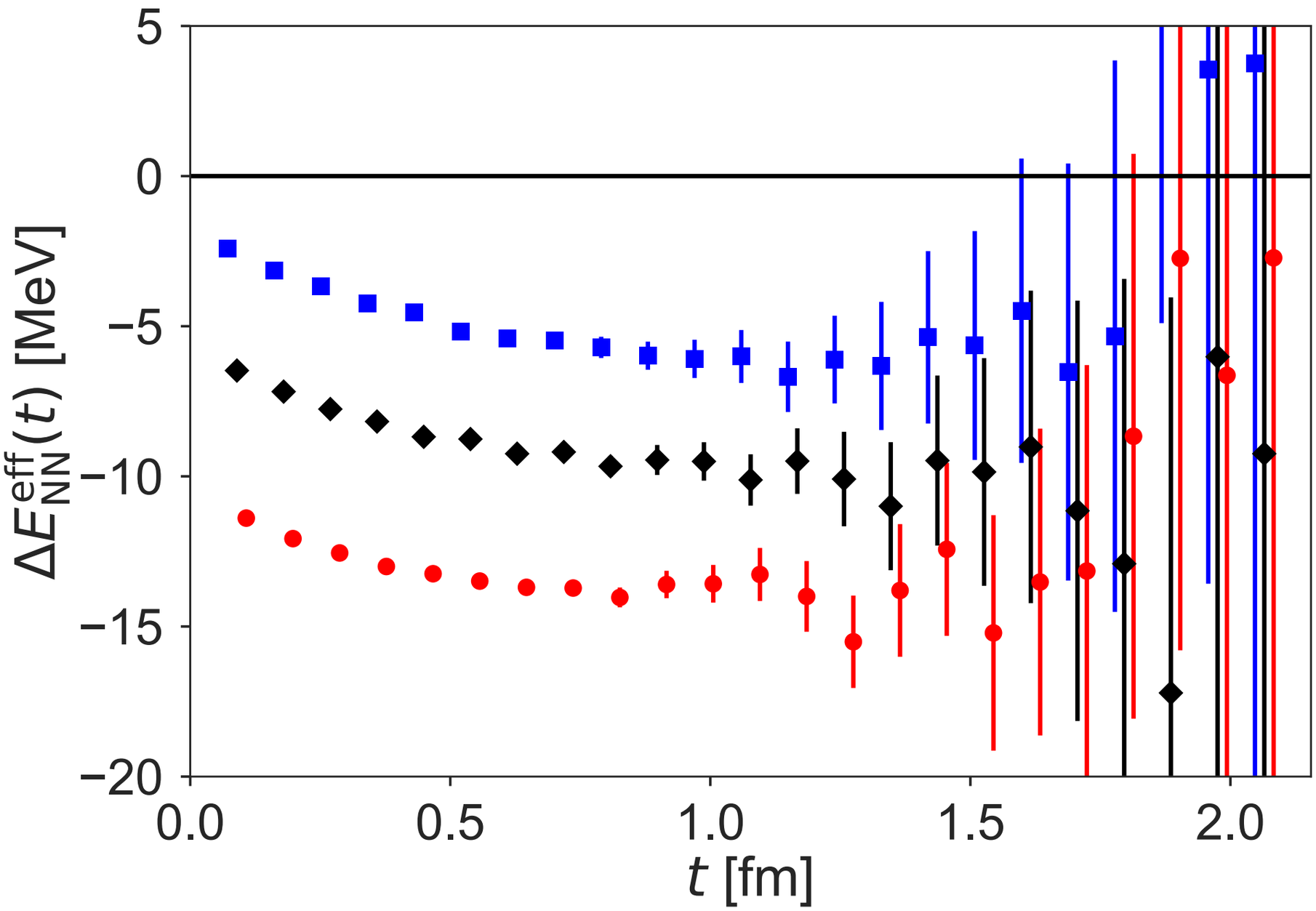
$\delta E_{\rm inel} = 500$ MeV the inelastic energy from heavy pions

$c = 0.01$ 1% contamination



increasing errors and fluctuations in time

$$\Delta E^{\rm eff}(t) = -\frac{1}{a}\log\frac{R(t+a)}{R(t)}$$



"Plateaux" at t ~ 1 fm but some are fake.

One can not tell which is correct by its plateau behavior at small t.

Frequently Asked Questions

[Q1] Scheme/Operator dependence of the potential

- The potential depends on the definition of the wave function, in particular, on the choice of the nucleon operator N(x). (Scheme-dependence)
  - local operator -> manifest causality
  - a similar example: running coupling is scheme-dependent
- Moreover, the potential itself is NOT a physical observable. Therefore it is NOT unique and is naturally scheme-dependent.
  - Observables: scattering phase shift of NN, binding energy of deuteron

QM: (wave function, potential) -> observables
QFT: (asymptotic field, vertex) -> observables
EFT: (choice of field, vertex) -> observables

- Is the scheme-dependent potential useful ? Yes !
  - useful to understand/describe physics
  - cf. running coupling: it is useful to understand the deep inelastic scattering data (asymptotic freedom)
- “good” scheme ?
  - good convergence of the perturbative expansion for the running coupling
  - good convergence of the velocity expansion for the potential ?
    - local operator is found to be “good” (see later)
    - completely local and energy-independent one is the best (Inverse scattering method)