Modelling Nueutron Star Matter in the Age of Gravitational Waves

Omar Benhar

INFN and Department of Physics, “Sapienza” University
I-00185 Roma, Italy

International School of Nuclear Physics
Erice, September 17-23,2018

A Little History

- ⋆ AD 1998: the seminal paper Andersson and Kokkotas heralds the advent of GW astereoseismology, declaring that "The day of the first undeniable detection of gravitational waves should not be far away"

Mon. Not. R. Astron. Soc. **299,** 1059–1068 (1998)

Towards gravitational wave asteroseismology

Nils Andersson[1,2] and Kostas D. Kokkotas[3,4]
[1]*Department of Physics, Washington University, St Louis, MO 63130, USA*
[2]*Institut für Theoretische Astrophysik, Universität Tübingen, D-72076 Tübingen, Germany*
[3]*Department of Physics, Aristotle University of Thessaloniki, Thessaloniki 54006, Greece*
[4]*Max-Planck-Institut für Gravitationsphysik, Schlaatzweg 1, 14473 Potsdam, Germany*

Accepted 1998 May 19. Received 1997 December 4

- ⋆ AD 2004: OB, Ferrari & Gualtieri argue that GW astereoseismology is promising to the extent to which neutron stars are described within a realistic model

PHYSICAL REVIEW D **70,** 124015 (2004)

Gravitational wave asteroseismology reexamined

Omar Benhar,[2,1] Valeria Ferrari,[1,2] and Leonardo Gualtieri[1,2]
[1]*Dipartimento di Fisica "G. Marconi", Universitá degli Studi di Roma, "La Sapienza", P.le A. Moro 2, 00185 Roma, Italy*
[2]*INFN, Sezione Roma 1, P.le A. Moro 2, 00185 Roma, Italy*
(Received 26 July 2004; published 10 December 2004)

Continuous GW from Neutron Stars

ALBERT EINSTEIN INSTITUTE HANNOVER
Max Planck Institute for Gravitational Physics and
Leibniz Universität Hannover

About us | Research | Publications | People | Career | Services & Resources | Outreach & Med

Outreach & Media ‣ Research News ‣ AEI Hannover ‣ Searching for continuous gravitational waves

Research News

- Recent News
- AEI Hannover
- AEI Potsdam

Videos, Pictures and Texts

Einstein Online

Science for the public

New independent research group

Searching for continuous gravitational waves

Permanent Max Planck Independent Research Group established at AEI Hannover

April 10, 2018

A permanent Max Planck Independent Research Group under the leadership of Dr. M. Alessandra Papa has been established at the Max Planck Institute for Gravitational Physics (Albert Einstein Institute; AEI) in Hannover. The primary goal of the research group "Searching for continuous gravitational waves" is to make the first direct detection of gravitational waves from rapidly rotating neutron stars. It is the largest group worldwide dedicated to this topic and conducts the most sensitive searches for this kind of gravitational wave with the globally distributed volunteer computing project Einstein@Home. In addition to its permanent funding, the group will receive additional funds from the Max Planck Society for the first five years.

NEUTRON STARS OSCILLATION MODES

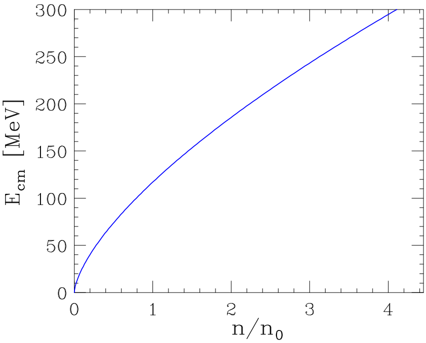
- Neutron stars have a rich spectrum of oscillation modes, whose frequencies reflect different features of the star structure and dynamics
  - **f-mode**: the most effective mechanism of GW emission. Depends on the average density
  - **p-modes**: acoustic modes, driven by pressure
  - **g-modes**: driven by thermal or composition gradients. The main restoring force is buoyancy
  - **w-modes**: pure space-time modes
  - **r-modes**: inertial mode of rotating stars, restored by the Coriolis force. Driven unstable by GW emission (CFS instability)!
- The onset of the CFS instability depends on a variety of properties of neutron star matter, ranging from the shear and bulk viscosity to the superfluid and superconducting gap. The development of a theoretical framework providing a *realistic* and *consistent* description of these is needed

Preamble: Collision Energy in Degenerate Matter

- In degenerate matter, the center-of-mass energy of nucleon-nucleon collisions, E_{cm}, is simply related to the particle density, n

- Head-on collisions in pure neutron matter at density n

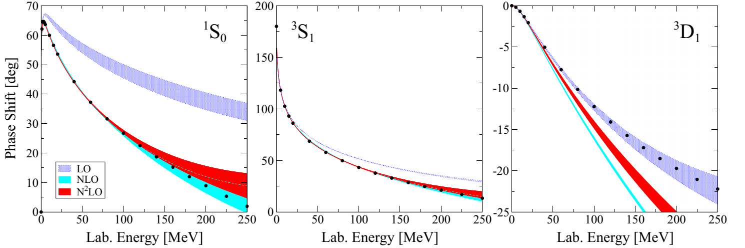
$$E_{\mathrm{cm}} = \frac{1}{m}(3\pi^2 n)^{2/3}$$



- Potential models used to predict the properties of dense nuclear mater must be capable to describe nucleon-nucleon collisions at high energies

Potentials from Chiral Effective Field Theory

- ⋆ Chiral Effective Field Theory (χEFT) provides a powerful framework—based on the symmetries of the fundamental theory of strong interactions—to derive two- and many-nucleon potentials in a fully consistent fashion
- ⋆ Being based on a low-momentum expansion, however, χEFT is inherently limited, when it comes to describing nuclear interactions in high-density nuclear matter

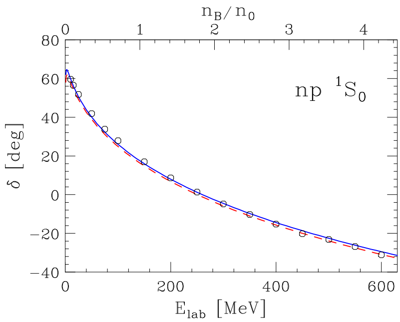


- ⋆ Phase shifts obtained from the local potential of A. Gezerlis et al, PRC 90 054323 (2014). Recall: $E_{\mathrm{LAB}} = 250$ MeV corresponds to $n \approx 1.1 n_0$ in neutron matter

PHENOMENOLOGICAL POTENTIALS

- ⋆ Purely phenomenological potentials, such as those developed at ANL, reproduce the nucleon-nucleon scattering phase shift up to the highest available energies

- ⋆ Phase shifts obtained from the full AV18 (full line) and the truncated AV6p model (dashes). Data from gwdac.phys.gwu.edu.

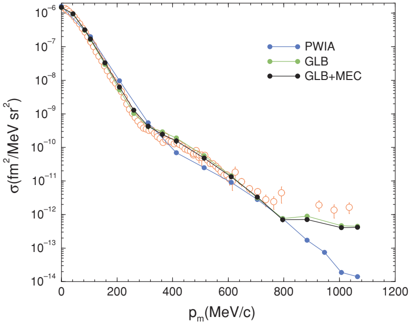


- ⋆ Phenomenological Hamiltonians also include three-nucleon potentials, such as the UIX model, *designed* to explain the properties of the three-nucleon system and the saturation density of isospin-symmetric nuclear matter

Validation of the Phenomenological Approach

- ⋆ The validity of the phenomenological approach in the high energy regime relevant to dense matter has been extensively tested, exploiting the availability of independent data

- ⋆ Cross section of the $^3\mathrm{He}(e, e'p)$ cross section measured at Jefferson Lab, as a function of the initial momentum of the knocked-out nucleon

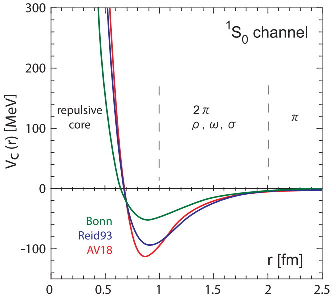


- ⋆ Monte Carlo calculations carried out using the phenomenological AV18+UIX Hamiltonian provide an excellent description of the data for proton momenta up to ~ 700 MeV. No adjustable parameters involved!

Perturbation Theory with Strongly Repulsive Forces

- ⋆ A prominent feature of the nucleon-nucleon potential is the presence of a strong repulsive core

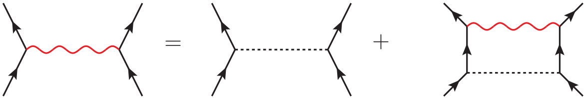
- ⋆ NN potential in the $^1S_0$ channel



- ⋆ Perturbative calculations of nuclear matter properties can only be performed using *effective* interactions, obtained from *renormalisation* of the bare potential

Renormalisation of the Nucleon-Nucleon Potential

- In the early days of nuclear matter theory, renormalisation was based on the replacement of the bare interaction, v. with the G-matrix describing nucleon-nucleon scattering in the nuclear medium



$$G = v + v \frac{Q}{H_0 - W} G$$

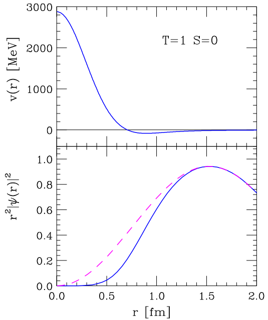
- The G-matrix approach has been extensively employed in conjunction with phenomenological potentials
- More recently, soft of the nucleon-nucleon interactions have been obtained from renormalisation group evolution of potentials derived within χEFT

Screening of the Repulsive Core

- ⋆ Renormalisation group evolution essentially amounts to screening the repulsive core of the potential through the action of a cutoff, Λ, in momentum space
- ⋆ Screening can also be implemented in *coordinate space*, through a transformation of the basis of eigenstates of the non interacting system

- ⋆ Transformation of the two-nucleons wave function in nuclear matter

$$\phi_{ij}(r) \to \psi_{ij}(r) = f_{ij}(r)\phi_{ij}(r)$$



- ⋆ The role of the momentum cutoff Λ is played by the correlation range, d, such that $f_{ij}(r \geq d) = 1$, which depends on density

The CBF Effective Interaction

- The Correlated Basis Function (CBF) formalism is based on the transformation from Fermi gas (FG) states to correlated states

$$|n_{FG}\rangle \to |n\rangle = F|n_{FG}\rangle$$

- The definition of the CBF effective interaction follows from the requirement (note: H include both the two- and three-nucleon potentials)

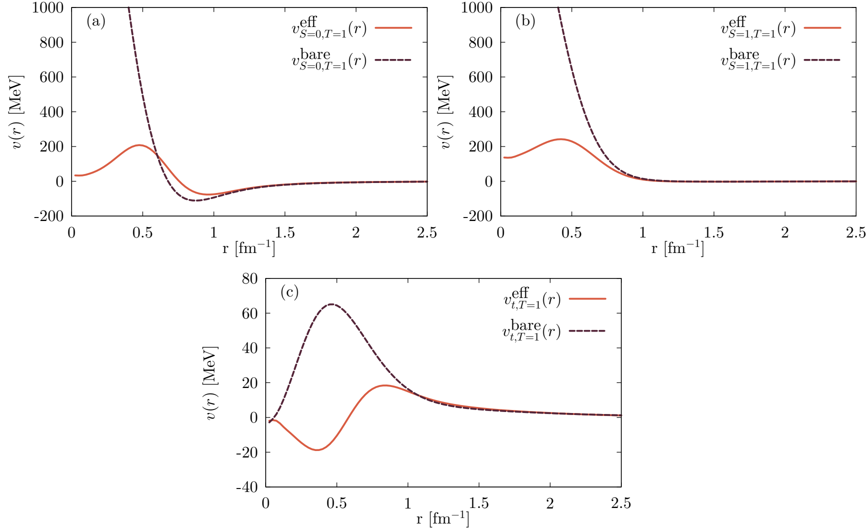
$$\langle H\rangle = \langle 0|H|0\rangle = \frac{3}{5}\frac{k_F^2}{2m} + \langle 0_{FG}|V_{\rm eff}|0_{FG}\rangle$$

implying

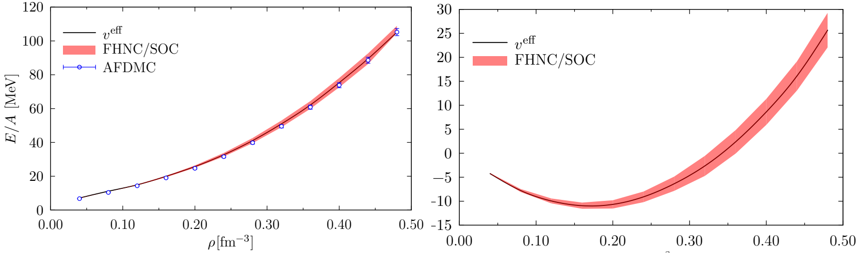
$$H_{\rm eff} = H_0 + V_{\rm eff} = F^\dagger H F$$

- For any given density, the operator F is determined in such a way as to reproduce the value of $\langle H\rangle$ obtained from accurate many-body calculations (Quantum Monte Carlo, or Variational FHNC/SOC)

- ⋆ CBF effective interaction in the $T = 1$ channel at nuclear matter equilibrium density, obtained from the Argonne $v_6' + UIX$ nuclear Hamiltonian

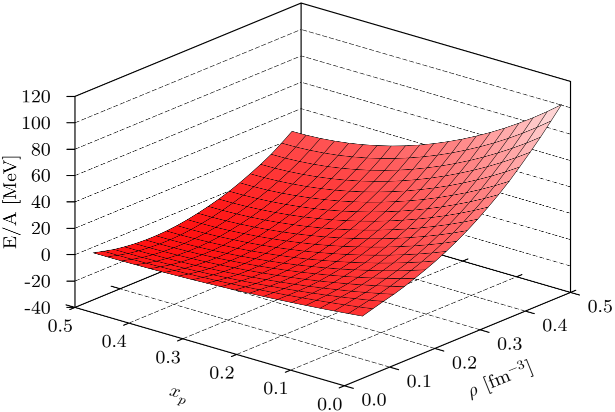


- Density dependence of the ground state energy per nucleon of unpolarized pure neutron matter (PNM) and isopspin-symmetric nuclear matter (SNM) obtained from the Argonne $v_6' + UIX$ nuclear Hamiltonian



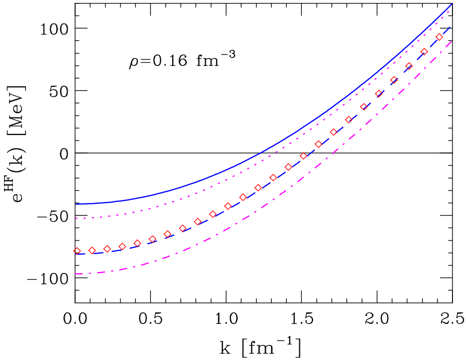
- Note that the $v_6' + UIX$ Hamiltonian, while yielding saturation at $\rho \approx \rho_0 = 0.16 \text{ fm}^{-3}$, underestimates the equilibrium energy of SNM by ~ 5 MeV, corresponding to a $\sim 15\%$ underestimate of the interaction energy

- Energy of unpolarized nuclear matter as a function of baryon density and proton fraction $0 \leq x_p \leq 0.5$. The generalization to spin-polarized matter is straightforward.



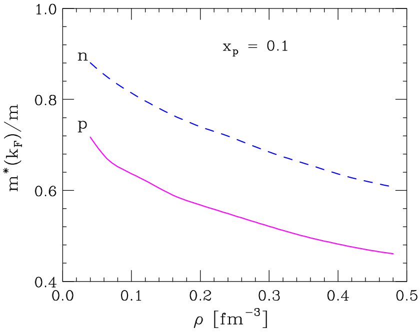
Single-nucleon spectrum

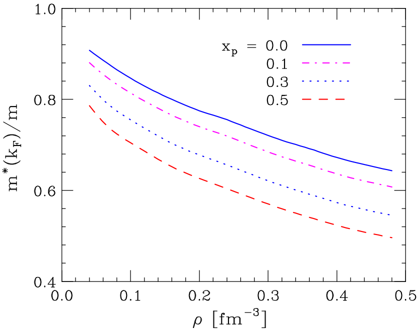
- Momentum dependence of proton and neutron spectra at nuclear matter equilibrium density and different proton fraction



EFFECTIVE MASS (HARTEE-FOCK)

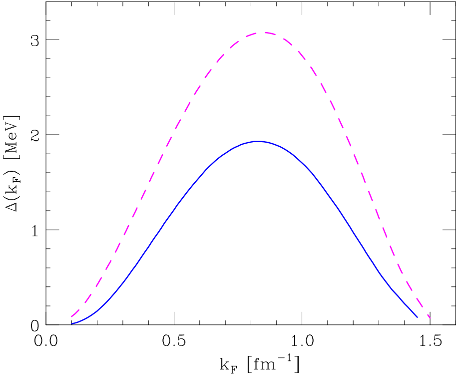
★ Density dependence of $m^\star(k_F)/m$





DENSITY DEPENDENCE OF Δ_F IN PURE NEUTRON MATTER

- Gap function obtained using the bare v_6' potential (dashed line) with kinetic energy spectrum (dashed line) and the CBF effective interaction with Hartee-Fock spectrum (solid line)

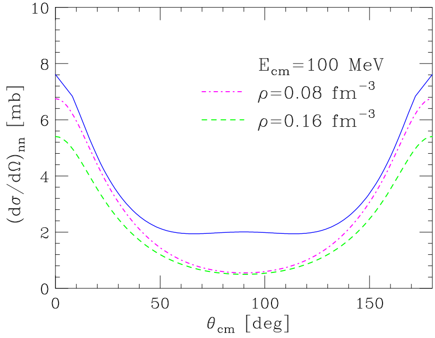


IN-MEDIUM CROSS SECTION

- Neutron-Neutron Channel

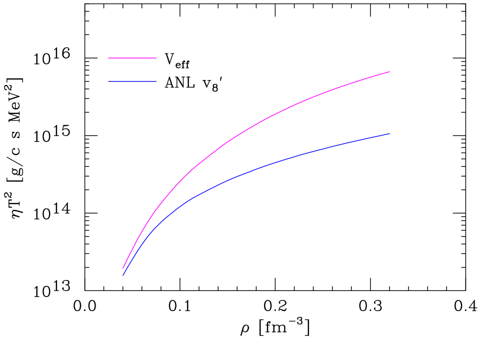
$$W(\mathbf{p},\mathbf{p}') = 2\pi\left|V_{\text{eff}}(\mathbf{p}-\mathbf{p}')\right|^2 \rho(\mathbf{p}')$$

$$\frac{d\sigma}{d\Omega_{\mathbf{p}'}} = \frac{m^{\star 2}}{16\pi^2}\left|V_{\text{eff}}(\mathbf{p}-\mathbf{p}')\right|^2$$



Shear Viscosity of PNM

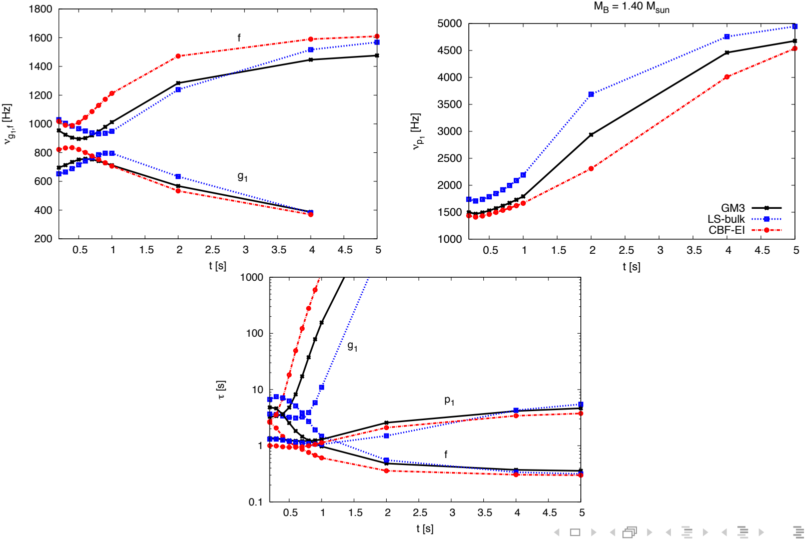
- ⋆ Density dependence of ηT^2 of PNM



- ⋆ Medium modifications of the scattering cross section increase ηT^2 by a factor $\sim 3 - 7$ @ $\rho/\rho_0 \sim 1 - 2$

FREQUENCIES OF QUASI NORMAL MODES OF PNS

G. Camelio, A, Lovato, L. Gualtieri, OB, J. Pons, and V. Ferrari
PRD 96, 043015 (2017)



Summary & Outlook

- ⋆ The detection of continuous gravitational waves will provide an unprecedented opportunity and a challenge for the understanding of the properties of neutron stars
- ⋆ Nuclear Hamiltonians suitable for the description of dense matter can be obtained from phenomenology, exploiting the availability of a wealth of experimental information on short-range nuclear dynamics
- ⋆ The formalism based on Correlated Basis Function provides a framework to obtain effective interactions—as well as the associated effective operators—allowing the calculation of nuclear matter properties in low-order perturbation theory
- ⋆ The existing results suggest that a fully consistent treatment of processes such as the onset of the CFS instability will be feasible in the not too far future

Credits

People involved in the development of the project over the past two decades

⋆ **Gravitational Waves**

- Emanuele Berti
- Giovanni Camelio
- Valeria Ferrari
- Leonardo Gualtieri

⋆ **Nuclear Theory**

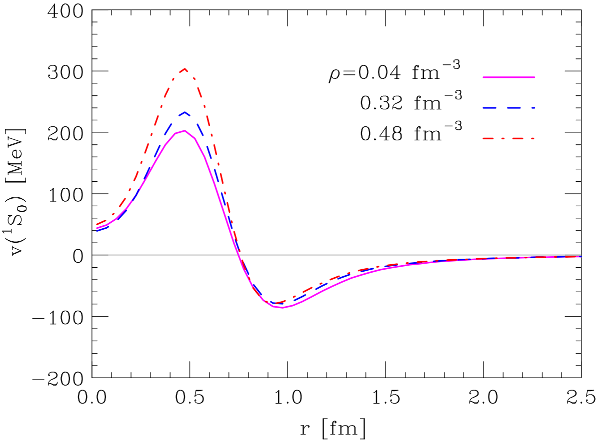
- Marco Valli
- Arianna Carbone
- Andrea Cipollone
- Andrea Loreti
- Cristina Losa
- Alessandro Lovato

Thank you!

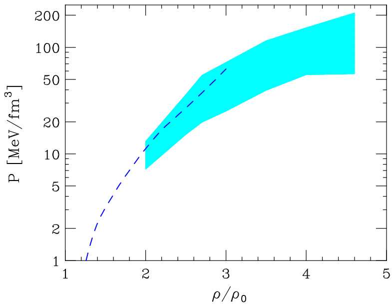
Backup slides

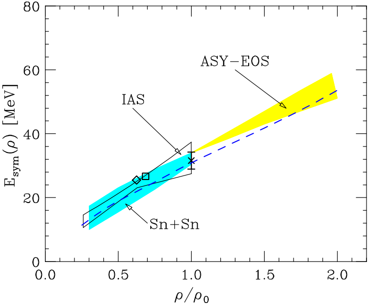
Density dependence of the effective interaction

⋆ $^1S_0$ channel



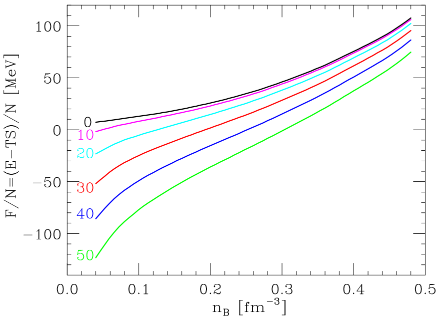
PRESSURE OF SNM AND SYMMETRY ENERGY

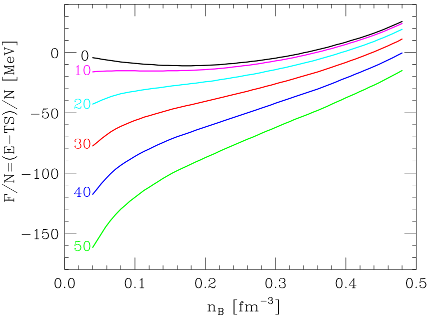




EXTENSION TO $T > 0$

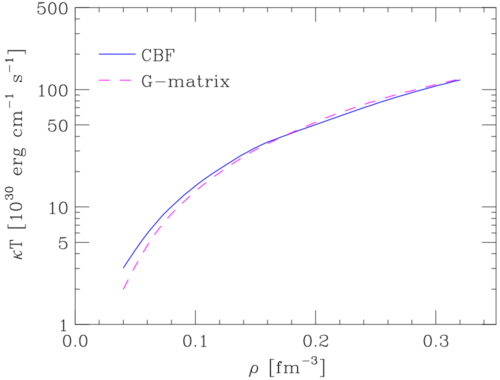
- ⋆ Assuminhg that thermal effect do not significantly affect the dynamics of strong interactions, the effective interaciotns can be used to obtain the properties of nuclear matter at $T > 0$
- ⋆ Replace $\theta(k_F - k) \to \{1 + \exp[e(k) - \mu]/T\}^{-1}$





THERMAL CONDUCTIVITY OF PNM

- Results from PRC 81, 024305 (2009). Three-nucleon interactions not taken into account.



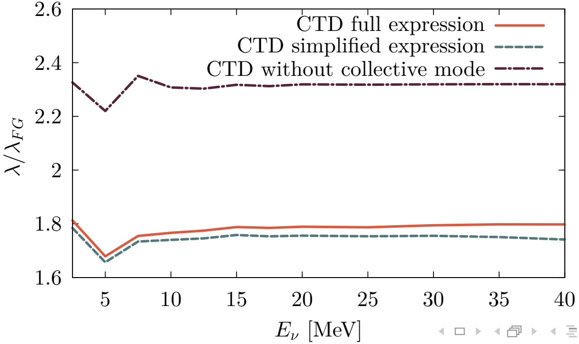
- The transport coefficients computed using the CBF effective interaction is remarkably close to the result obtained within the G-matrix approach using the same bare NN potential.

Neutrino Mean Free Path in Cold Neutron Matter

- The mean free path of non degenerate neutrinos at zero temperature is obtained from

$$\frac{1}{\lambda} = \frac{G_F^2}{4}\rho \int \frac{d^3q}{(2\pi)^3}\left[(1+\cos\theta)S(\mathbf{q},\omega) + \mathbf{C_A^2}(\mathbf{3} - \cos\theta)\mathcal{S}(\mathbf{q},\omega)\right]$$

where S and $\mathcal{S}$ are the density (Fermi) and spin (Gamow Teller) response, respectively [A. Lovato et al, NPA 89, 025804 (2013); PRC 89, 025804 (2013)]



Neutrino Luninosity of Proto Neutron Stars (PNS)