Erice International School on Nuclear Physics - 40th course

Exploring few-nucleon systems and hadronic matter with effective interactions

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Outline

- Motivation: Nuclear Force
- Framework: Effective Nuclear Forces
- Two-Nucleon System: Pions vs Contacts
- Pairing Gap: Chiral N4LO vs Pionless
- Neutron matter: Unitary Limit
- Final Remarks

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T. Frederico (ITA)

E. F. Batista (UESB)

S. Szpigel (UPM)

L. Tomio (UNESP)

Special thanks to Prof. Machleidt

for the chiral forces

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FAPESP

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CNPq

Nuclear Forces

Phenomenological forces (Argonne, Nijmegen, …)

High precision fits to scattering data, but too many parameters and no relation to QCD

Boson Exchange forces (Bonn, Paris, …)

Phenomenological short range + meson exchanges, hybrid approach

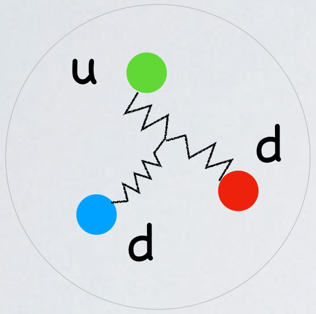
Chiral forces (LO, NLO, N2LO, N3LO, N4LO, N4LO+, …)

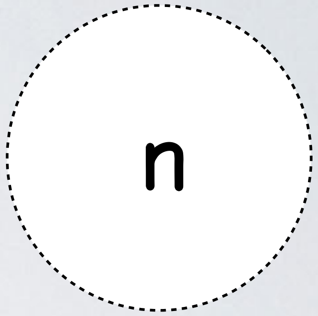
Chiral expansion, systematic improvement, QCD inspired, Quantum Field Theory

Effective theory principle

Physics at low energy (large distance) scales is insensitive to the details of the physics at high energy (small distance) scales

$$S \longrightarrow S'$$





Effective Interactions Timeline

1991	1994	1998	2003	2005	2013	2015	2017
EFT formulation Weinberg	ED N2LO van Kolck et al.	EI N2LO Epelbaum et al.	N3LO Entem Machleidt	N3LO (SFR) Epelbaum et al.	optimized N2LO Ekström et al.	N4LO Idaho, Salamanca Bochum, Bonn	N4LO+ Idaho, Salamanca

Many other important works by:

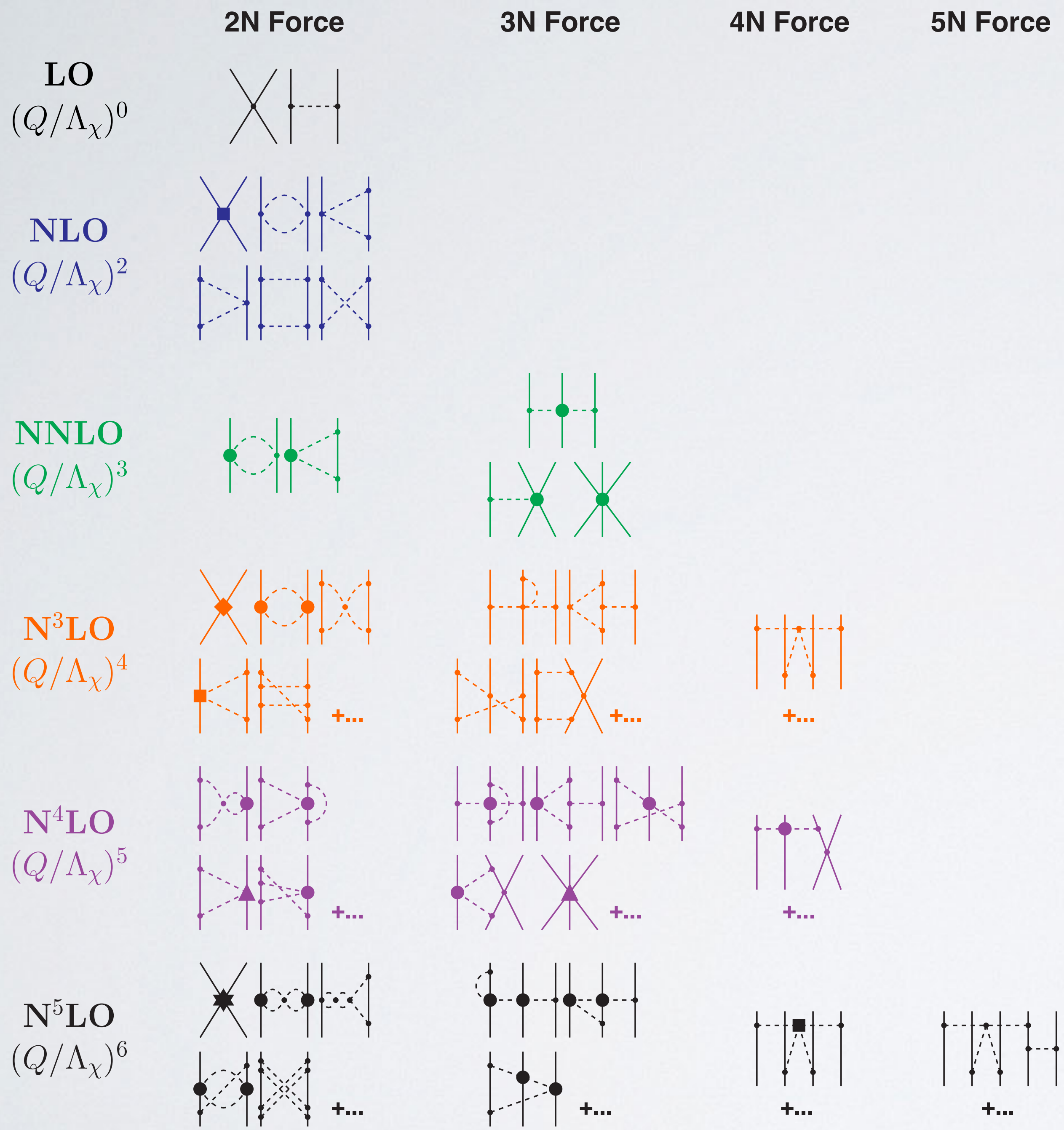
Kaiser, Robilotta, Ruiz Arriola, Frederico, Friar,

Birse, Kaplan, Savage, Wise, Bedaque, Beane, ...

Chiral Forces with pions & nucleons as fundamental d.o.f.

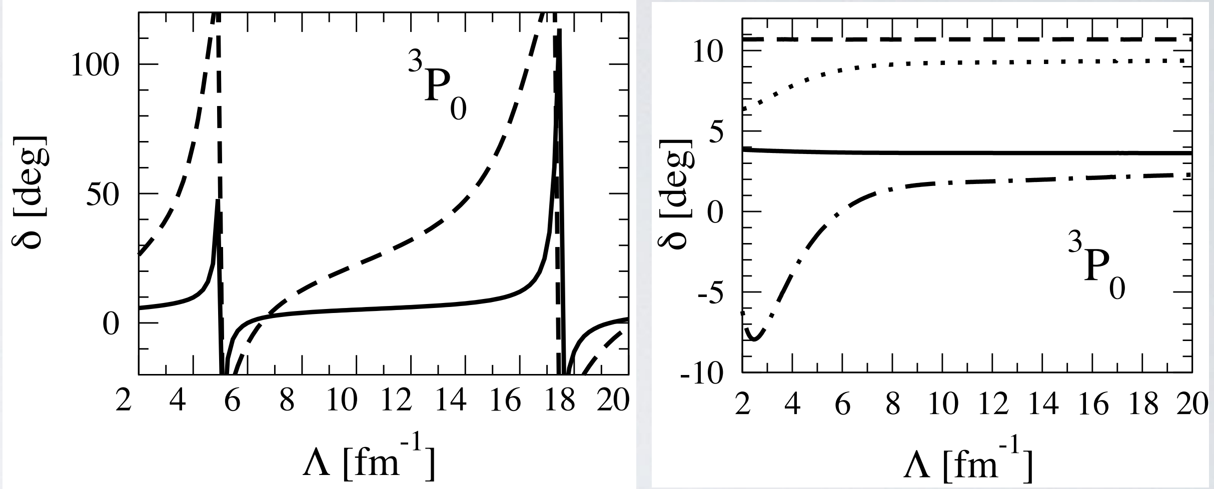
Chiral expansion

see works from Machleidt et al. and Epelbaum et al.



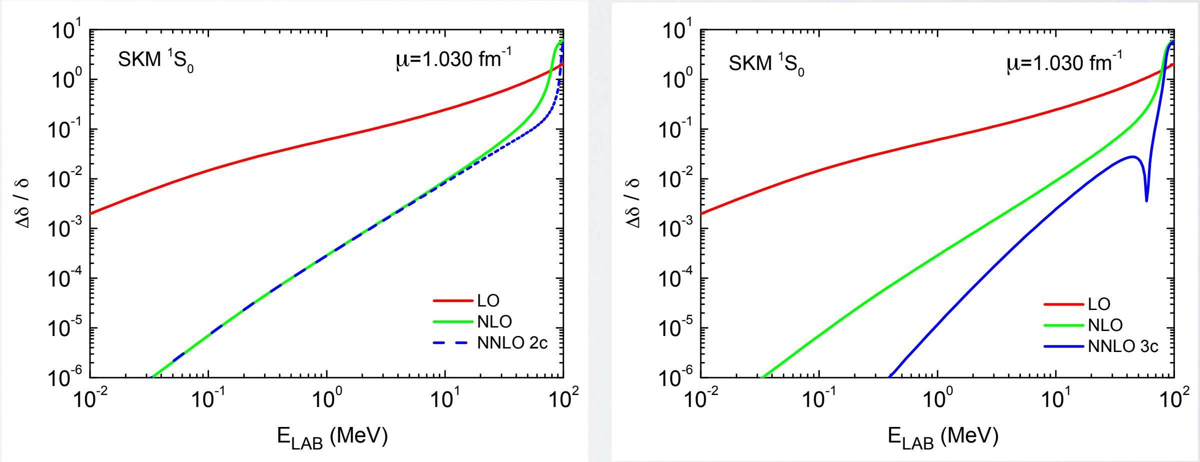
Power counting

Nogga, Timmermans, van Kolck, Phys Rev C 72 (2005) 054006



LO

Szpigel & VST, J Phys G 39 (2012) 105102



N2LO

Similarity Renormalization Group (SRG)

$$\mathcal{H}\ |\psi\rangle \ = \ E\ |\psi\rangle$$

- Pre-diagonalization
- Reduces off-shellness
- Improves convergence in many-body calculations
- Nuclei and Nuclear matter

S. Glazek
K. Wilson

R. Furnstahl
R. Perry
S. Bogner
E. Jurgenson

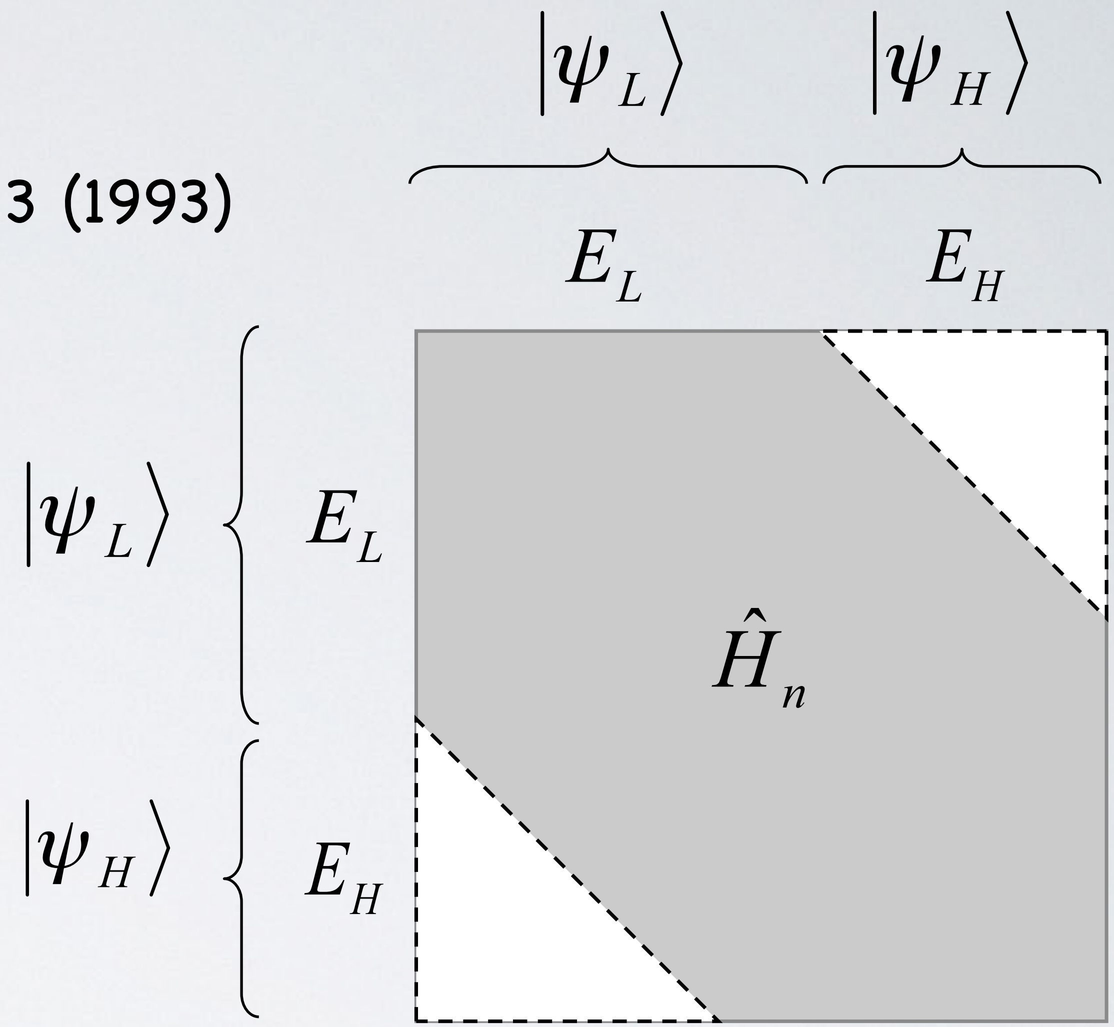
R. Roth
A. Schwenk
P. Navratil
J. Vary

H. Hammer
K. Hebeler
A. Calci
S. Binder

Similarity Renormalization Group

Similarity Transformation:

S. D. Glazek and K. G. Wilson, Phys. Rev. D 48, 5863 (1993)



$$H_n\left[\Lambda_n\right] = T^{(n)}_{Sim.}\left\{H_0\left[\Lambda_0\right]\right\}$$

Doesn't remove degrees of freedom

But suppresses states with large energy difference (off-diagonal elements):

$$\langle\psi_L|H|\psi_H\rangle \rightarrow \Lambda_n \leq (E_H - E_L) \leq \Lambda_0$$

off-shellness

Unitary transformation: doesn't affect the spectrum

$$T^{\dagger}T = 1$$

Similarity Renormalization Group

Wegner's formulation:

F. Wegner, Annalen der Physik (Berlin) 3, 77 (1994)

Flow equation:

$$H_s = U(s)\ H\ U^\dagger(s) = T + V_s$$

$$\frac{d}{ds} H_s = [H_s, \eta_s]$$

Flow parameter: $s = \frac{1}{\lambda^4}$ $(0 \leq s \leq \infty)$

similarity cutoff λ: dimension of momentum

Boundary condition: $\lim_{s \to s_0} H_s = H_{s_0}$

Generators for the similarity transformation

Free hamiltonian (kinetic energy): $\eta_s = [H_s, T]$

Diagonal part of the running hamiltonian: $\eta_s = [H_s, H_D]$

SRG - Wilson Generator

(two-nucleon system)

$$\eta_s = [H_s, T] \longrightarrow \frac{d}{ds} H_s = [H_s, [H_s, T]]$$

$$\boxed{\frac{d}{ds} V_s(p,p') = -(p^2 - p'^2)\ V_s(p,p') + \frac{2}{\pi} \int dq\ q^2\ (p^2 + p'^2 - 2q^2)\ V_s(p,q)\ V_s(q,p')}$$

S. Szpigel and R. J. Perry, in "Quantum Field Theory, A 20th Century Profile", ed. A. N. Mitra, Hindustan Publishing, New Delhi (2000)

S.K. Bogner, R.J. Furnstahl, and R.J. Perry, Phys. Rev. C **75**, 061001(R) (2007)

S.K. Bogner, R.J. Furnstahl, R.J. Perry, and A. Schwenk, Phys. Lett. B **649**, 488 (2007)

E.D. Jurgenson, P. Navratil, R.J. Furnstahl, Phys. Rev. Lett. **103** (2009) 082501

$V_{s=0} \longrightarrow$ regular or regularised

SRG - Wegner Generator

(two-nucleon system)

$$\lambda = \frac{1}{\sqrt[4]{s}}$$

$$\eta_s = [\ H_s\ ,\ \mathrm{diag}(H_s)\ ]$$

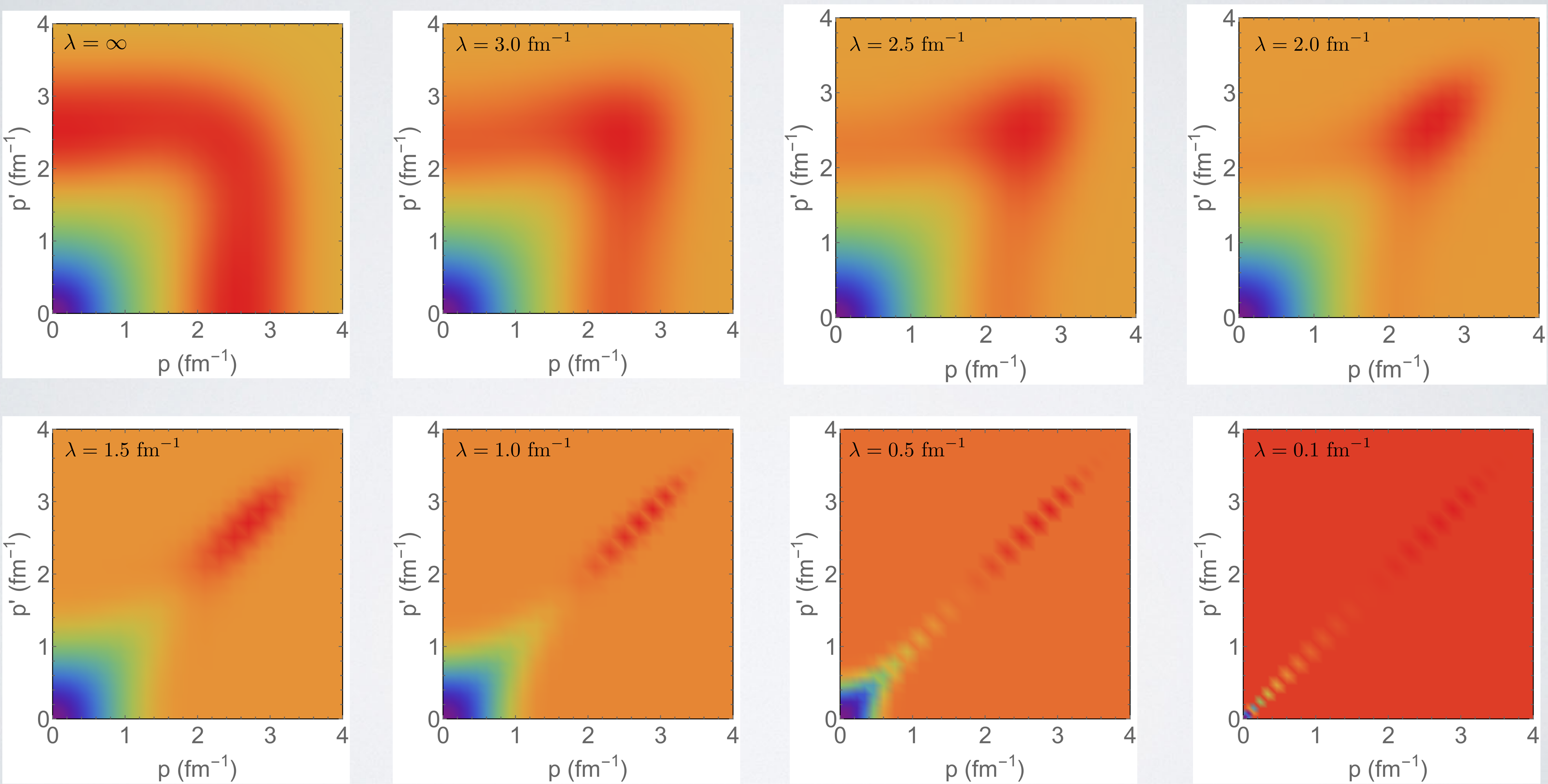
$$\frac{d}{ds} H_s = [\ H_s\ , [\ H_S\ , \mathrm{diag}(H_s)\ ]\ ]$$

$$T\ |p\rangle = p^2\ |p\rangle \qquad [\mathrm{diag}(H_s)] |p\rangle = \epsilon_p |p\rangle$$

$$\boxed{\frac{d}{ds} V_s(p,p') = \frac{2}{\pi} \int_0^\infty dq\ q^2\ (\epsilon_p + \epsilon_{p'} - 2\epsilon_q)\ H_s(p,q)\ H_s(q,p')}$$

SRG evolution (Wilson Gen.) - Chiral N3LO - 1S0

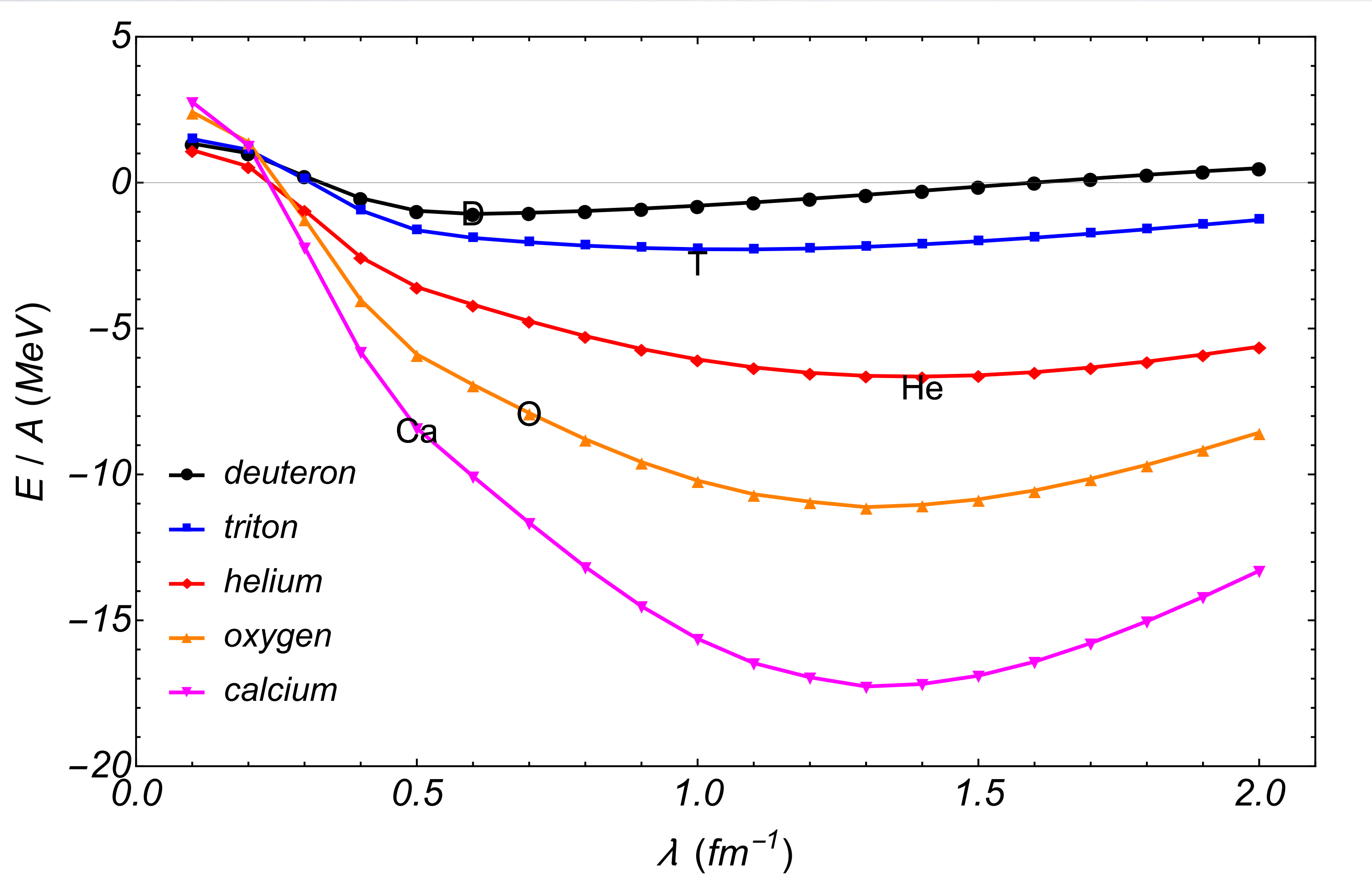
for a review on applications of SRG to nuclear physics see
Furnstahl & Hebeler, Rept Prog Phys 76 (2013) 126301



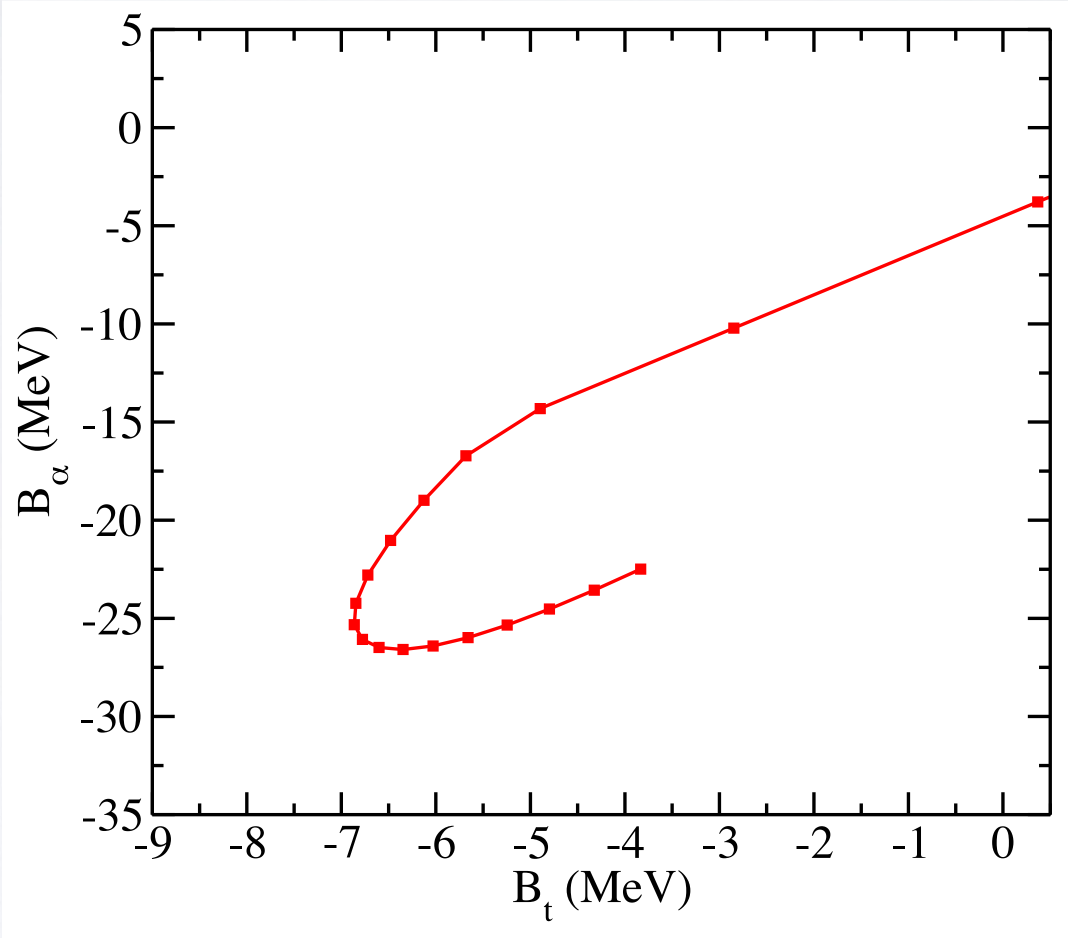
Finite Nuclei: SRG flow

No three-body force

Binding energies



Tjon line



No universal value for the SRG cutoff

Quantifying offshellness

The Frobenius norm:

$$\phi = ||V_\lambda|| = \sqrt{\mathrm{Tr}\ V_\lambda^2}$$

$$V_\lambda^2 = \frac{2}{\pi} \int_0^\infty dq\ q^2\ V_\lambda(p,q)\ V_\lambda(q,p')$$

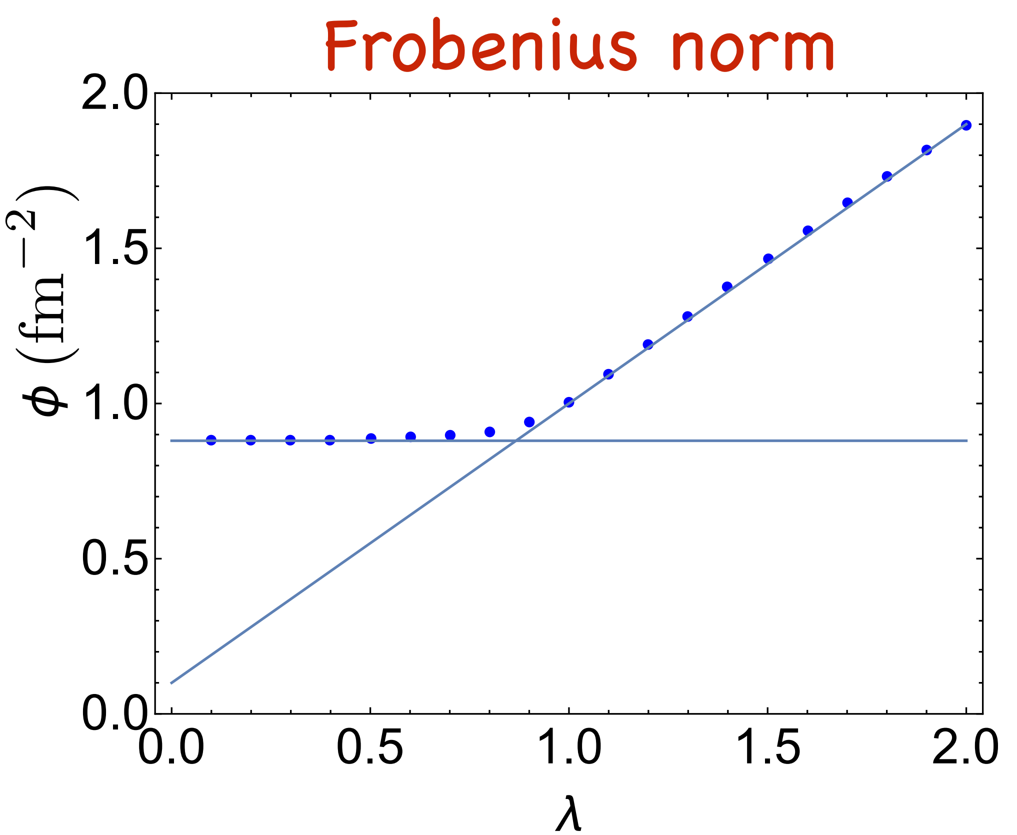
Order parameter:

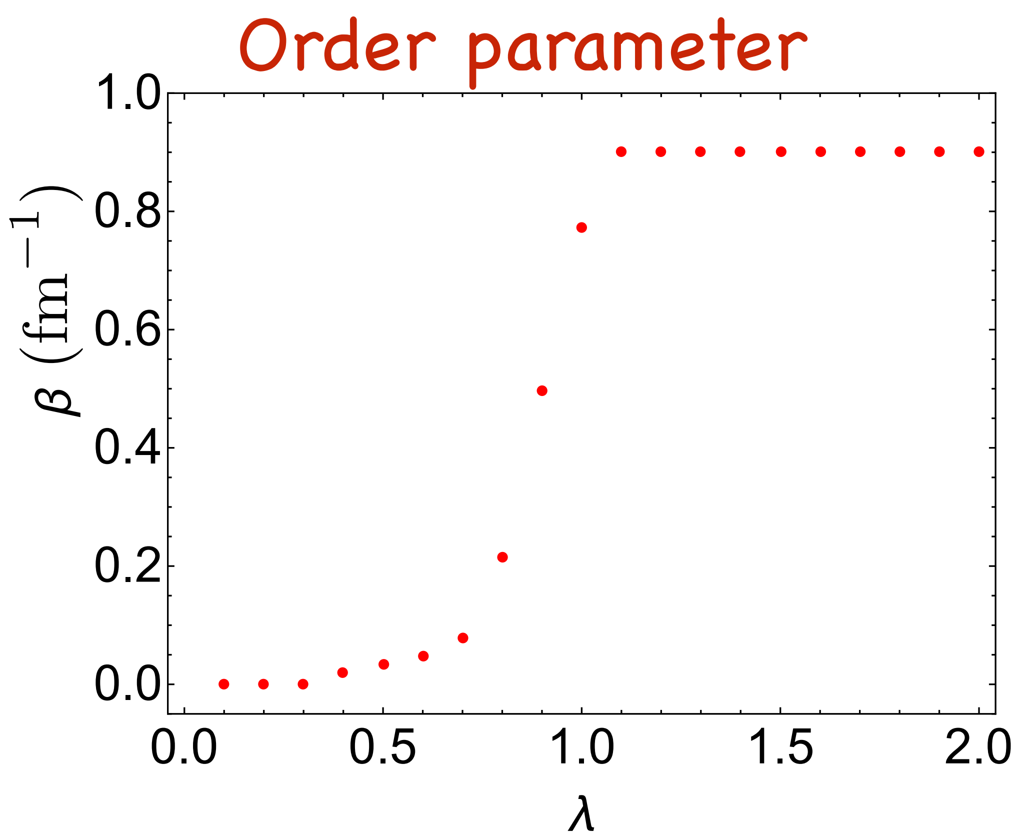
$$\beta = \frac{d\phi}{d\lambda}$$

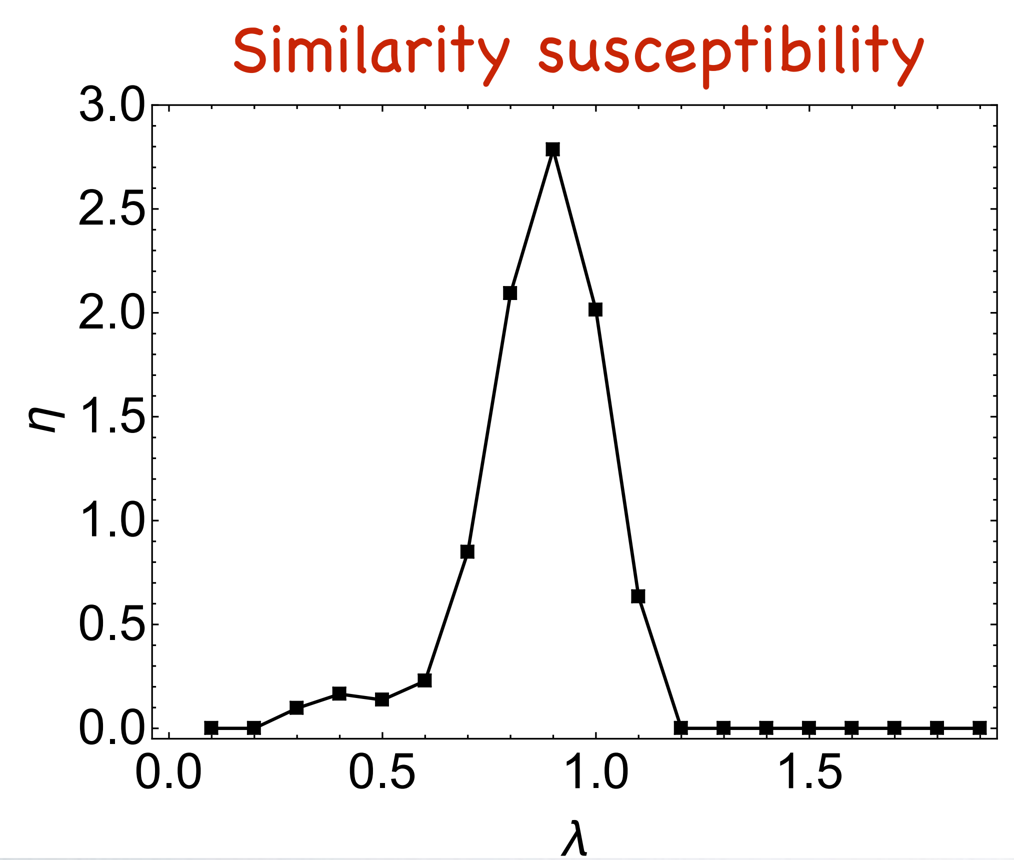
Similarity susceptibility:

$$\eta = \frac{d\beta}{d\lambda} = \frac{d^2\phi}{d\lambda}$$

The on-shell transition - N3LO







Critical λ

$$\lambda_c = 0.9\ \mathrm{fm}^{-1}$$

Szpigel, Ruiz Arriola, VST

Few-Body Syst (2014)

Physics Letters B 728 (2014) 596

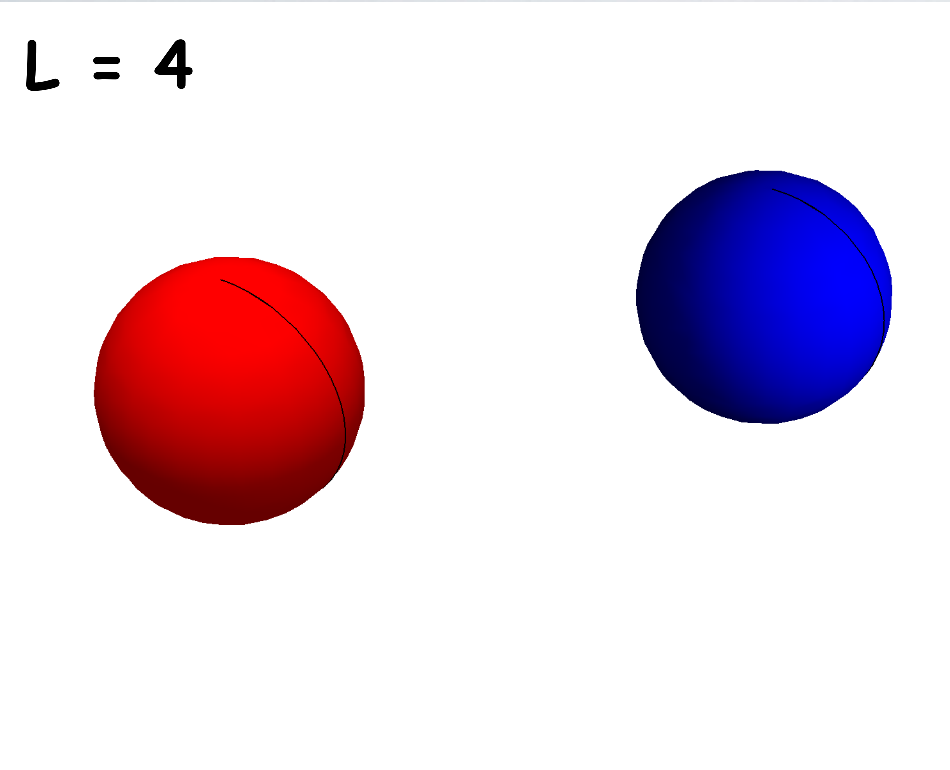
Physics Letters B 735 (2014) 149

Annals of Physics 353 (2015) 129

Annals of Physics 371 (2016) 398

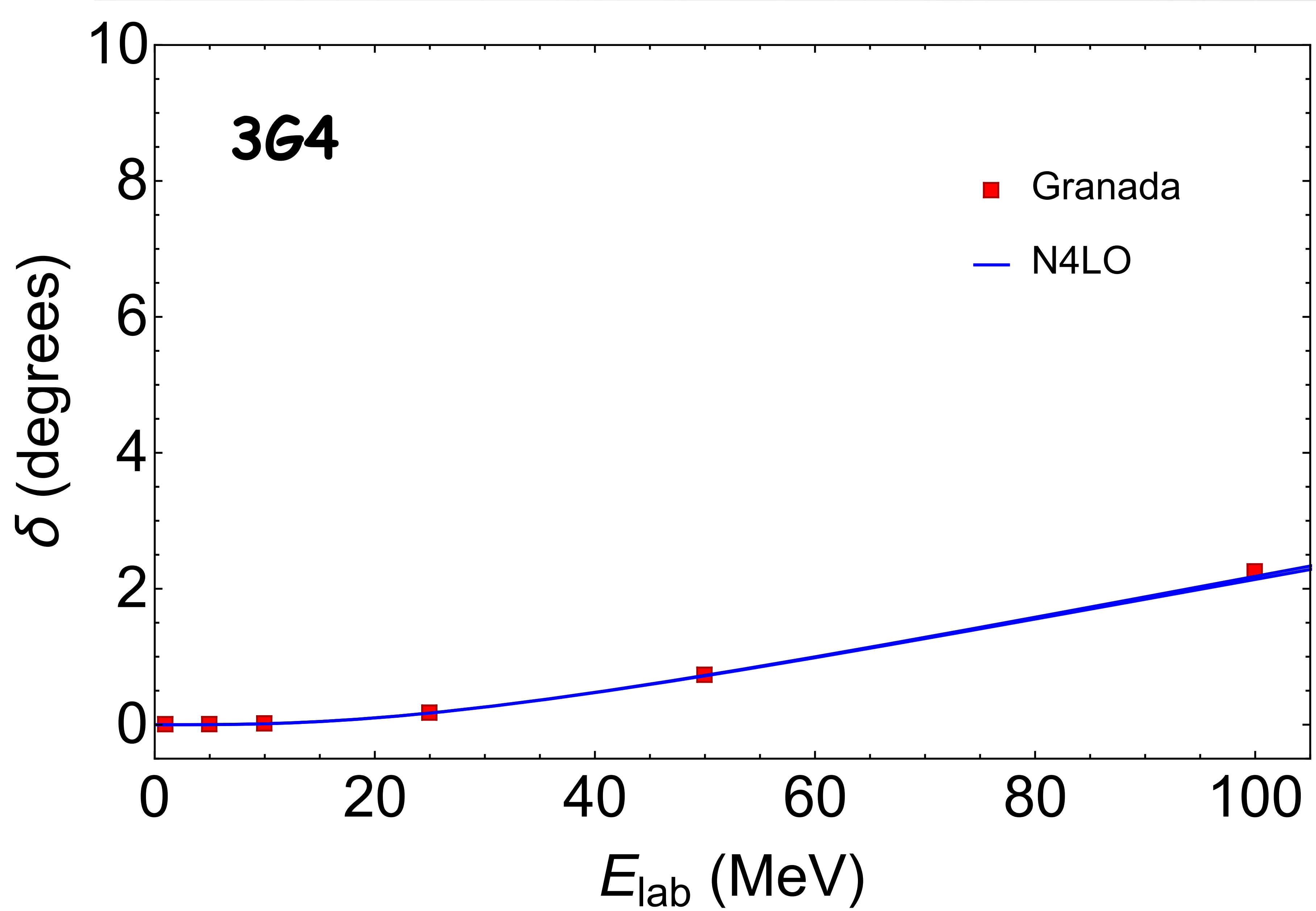
Few-Body Syst (2017) 58:62

Peripheral waves: pions



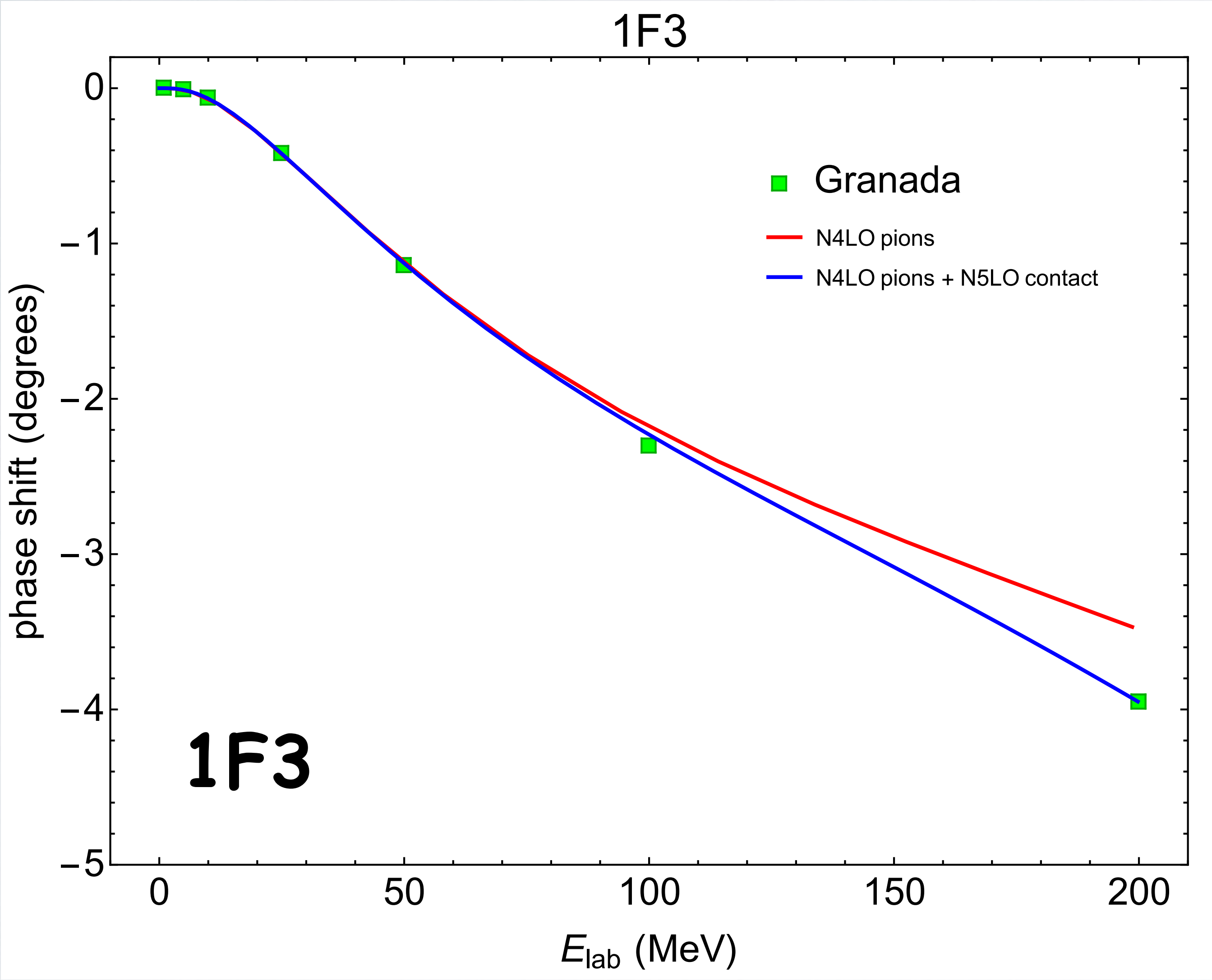
$$V_{N4LO}^{3G4} = 1\pi E + 2\pi E + 3\pi E$$

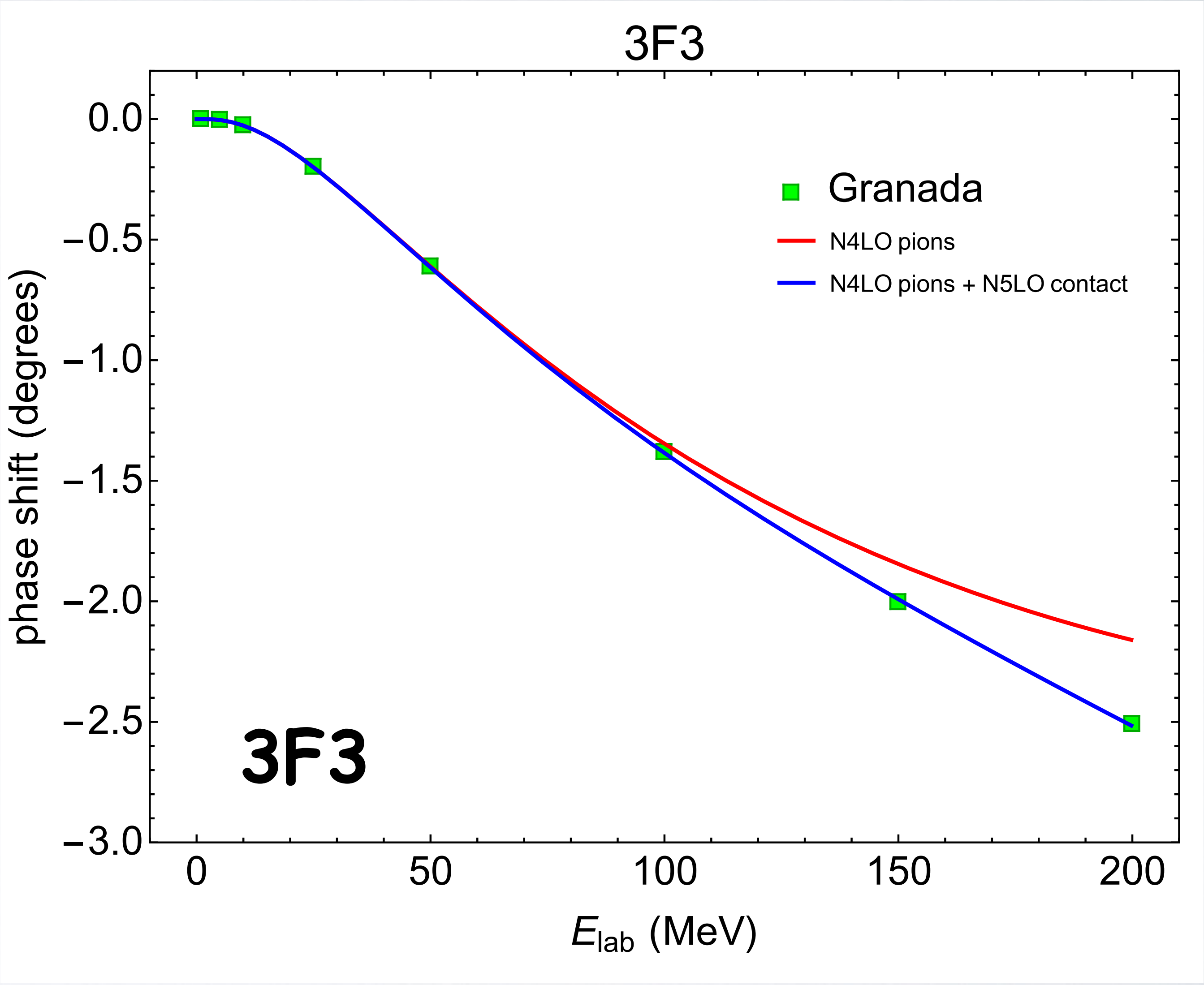
No contacts !!!



Pions+Contacts vs Granada PWA

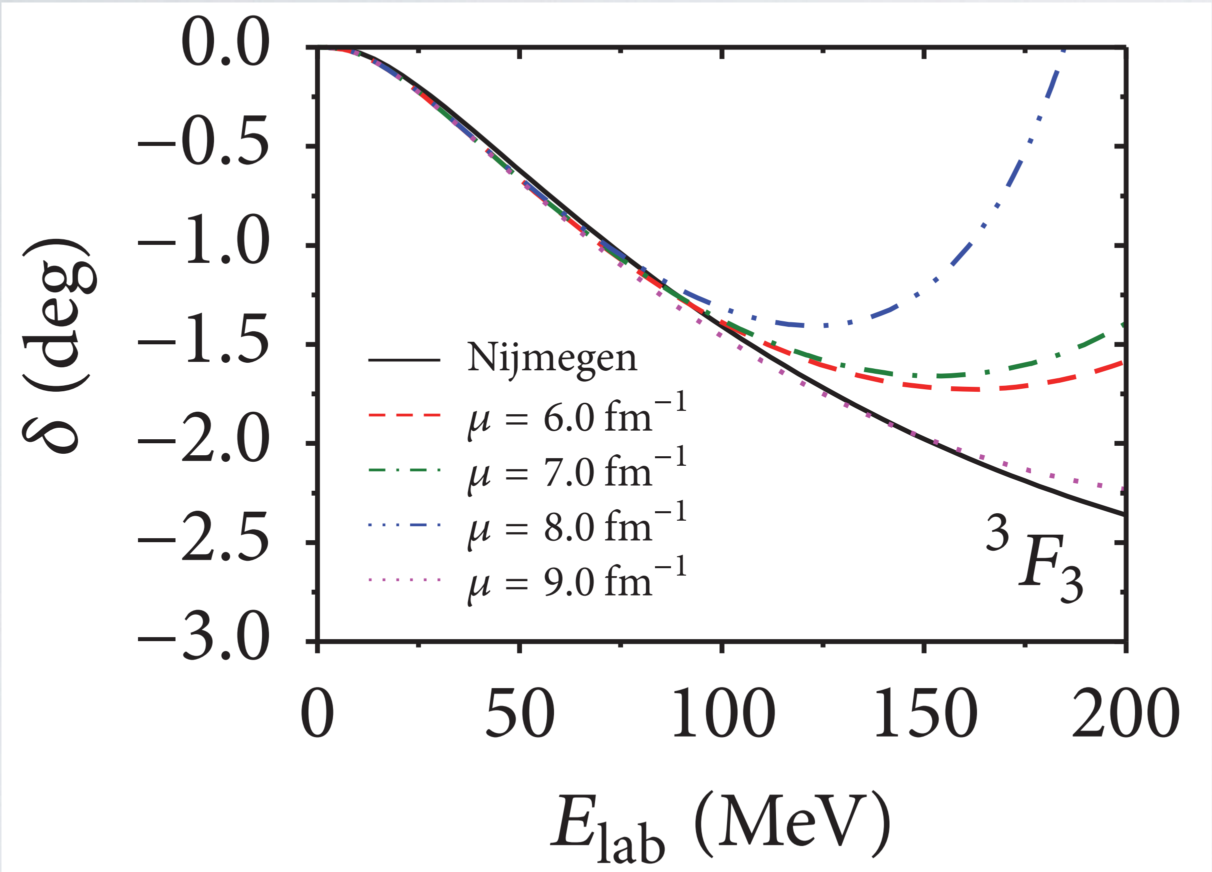
Peripheral, but sensitive to contacts





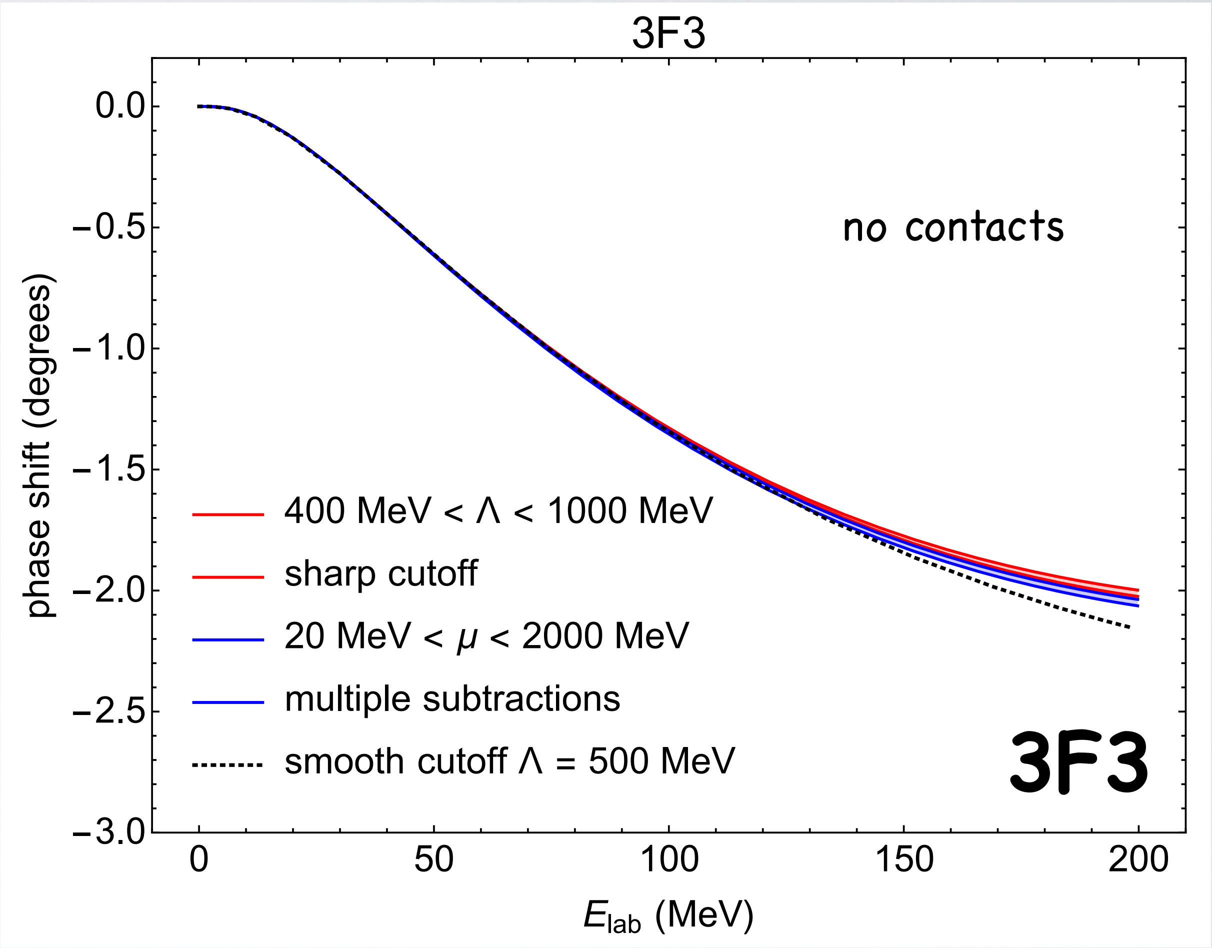
Scale dependence

N3LO



E. F. Batista, S. Szpigel and VST, AHEP (2017) 2316247

N4LO

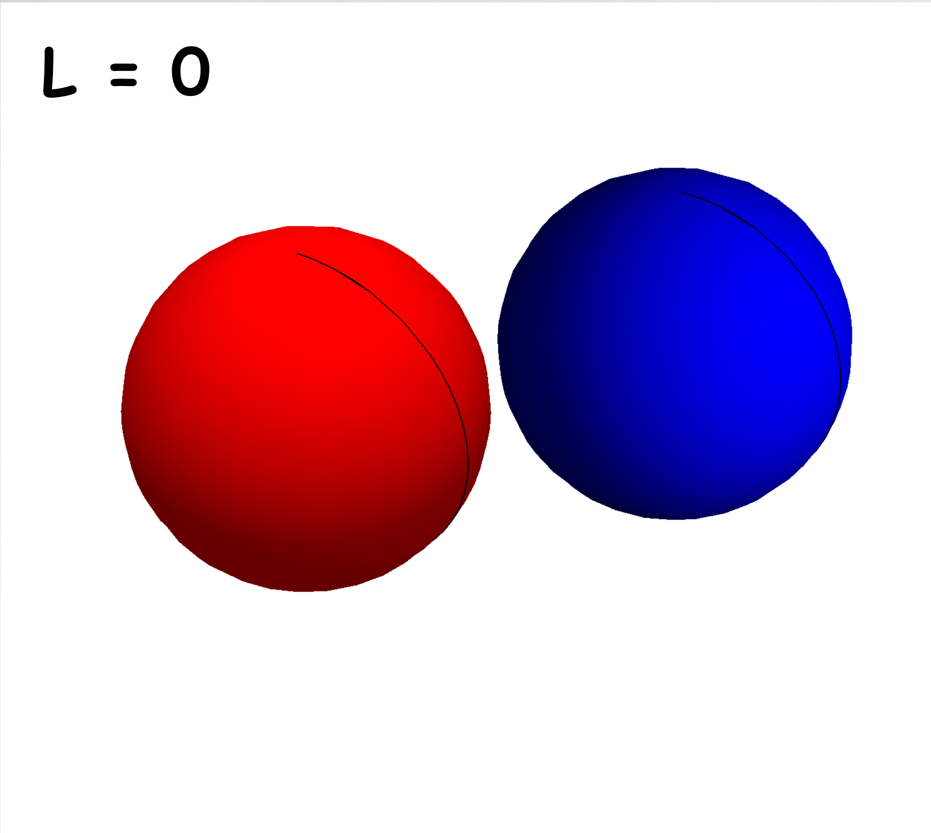


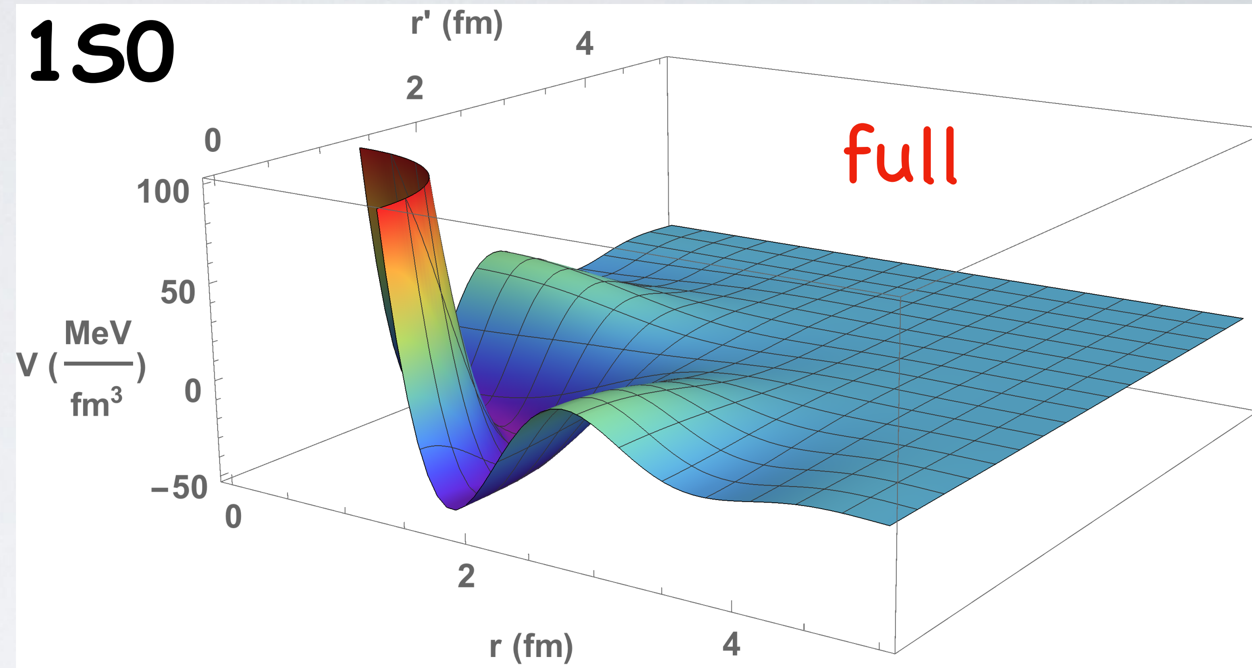
E. F. Batista, S. Szpigel and VST, in preparation

for details on the subtractive renormalization, see:

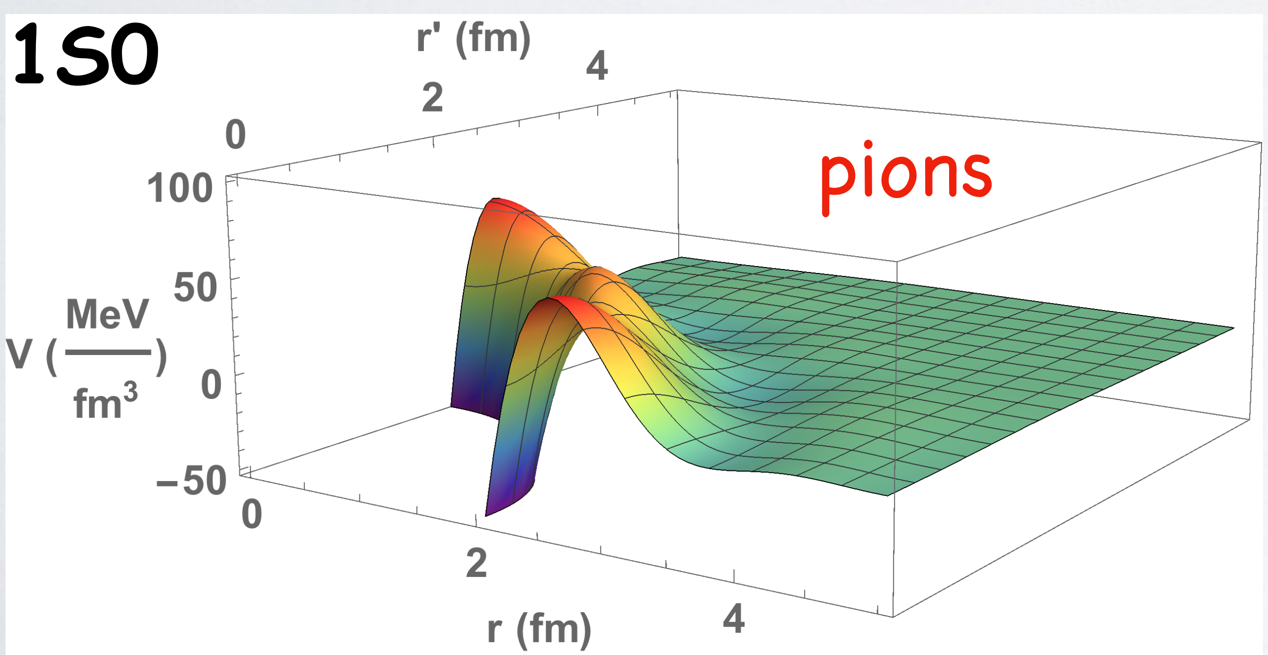
Frederico, VST, Delfino, Nucl. Phys. A 653 (1999) 209
VST, Frederico, Delfino, Tomio, Phys. Lett. B 621 (2005) 109
VST, Frederico, Delfino, Tomio, Phys.. Rev. C 83 (2011) 064005

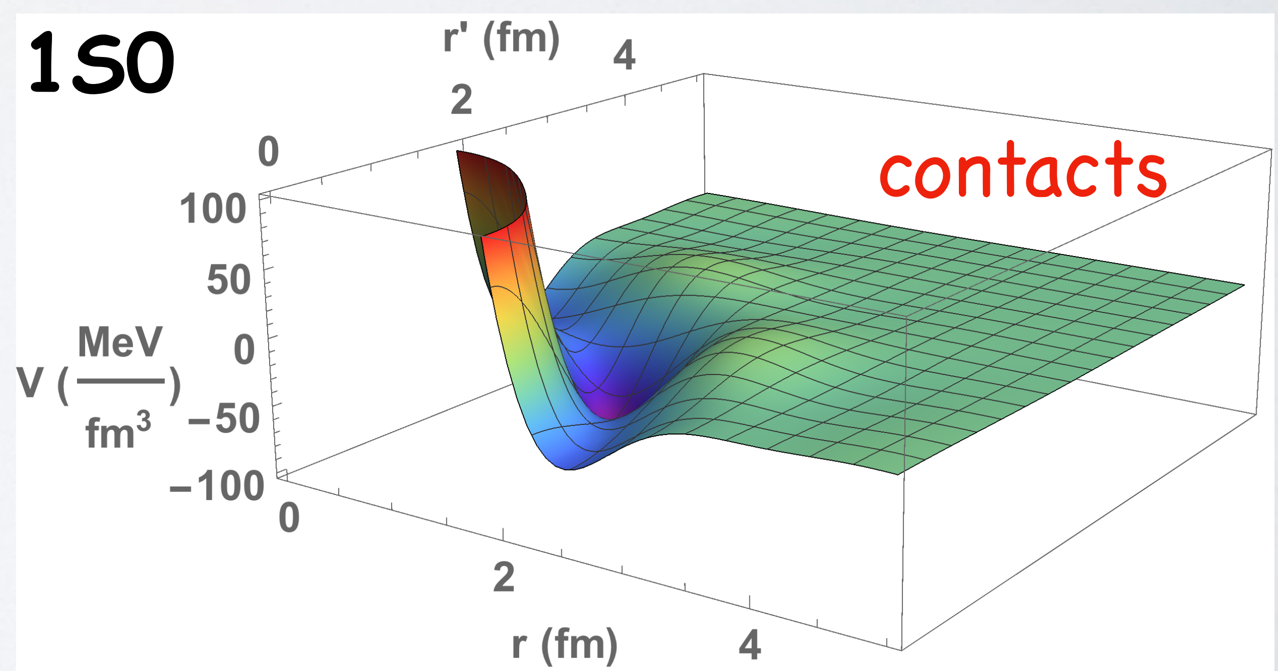
Central channel: pion vs contacts



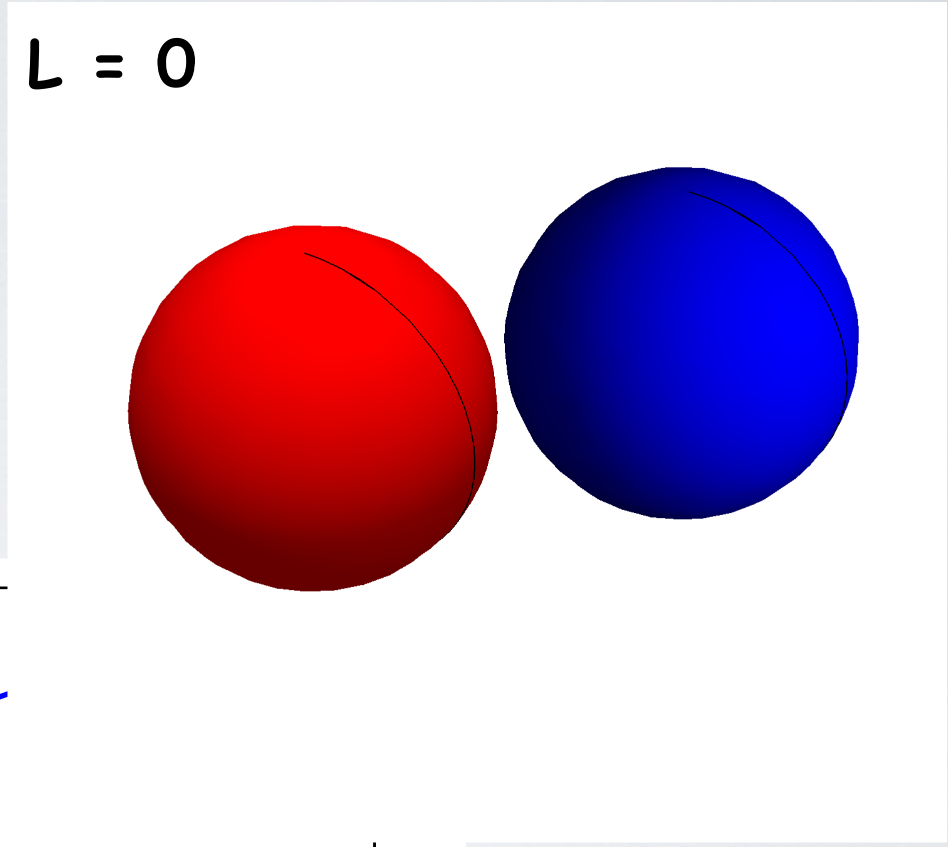


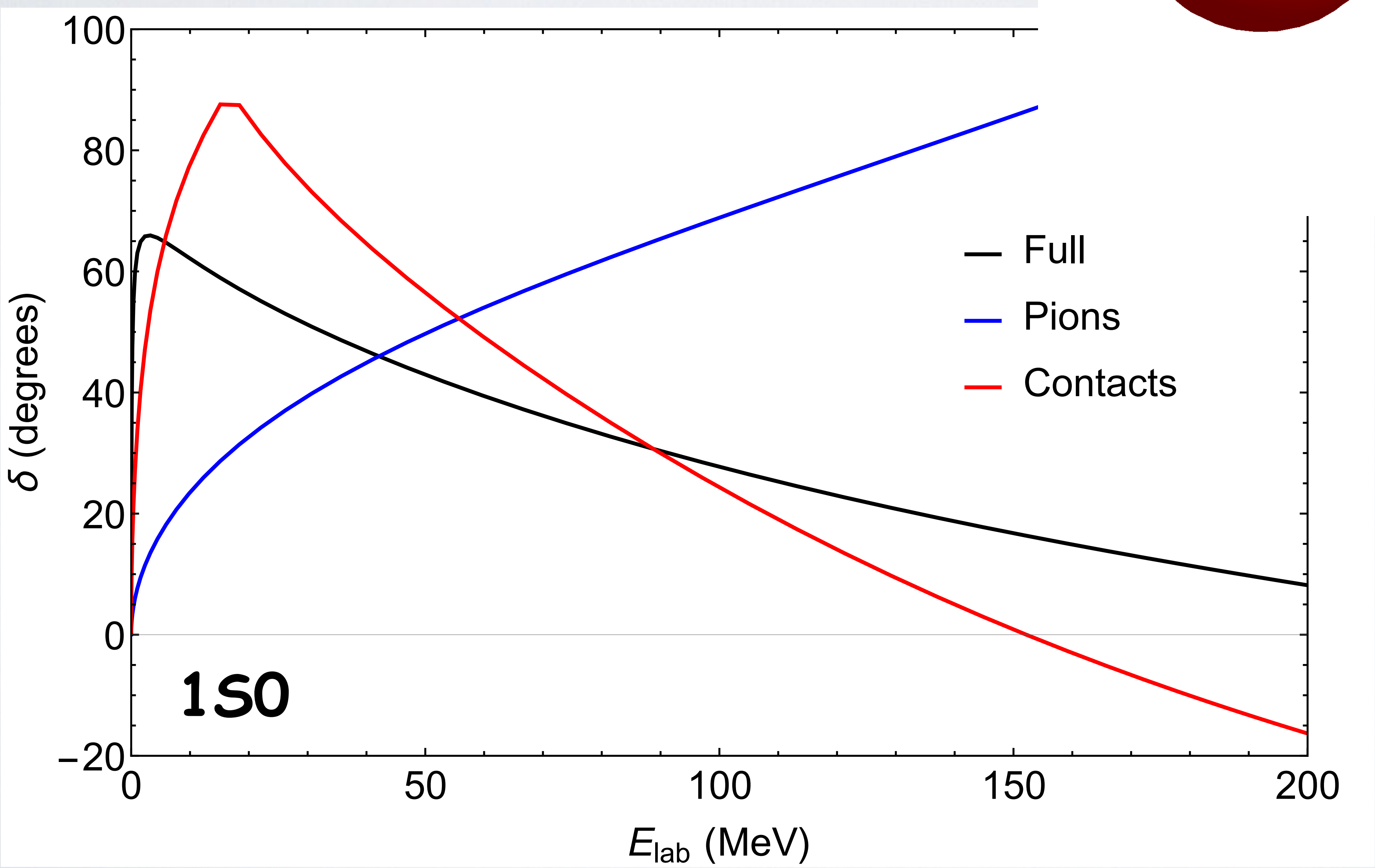
Interaction in this channel is dominated by the contacts !!!





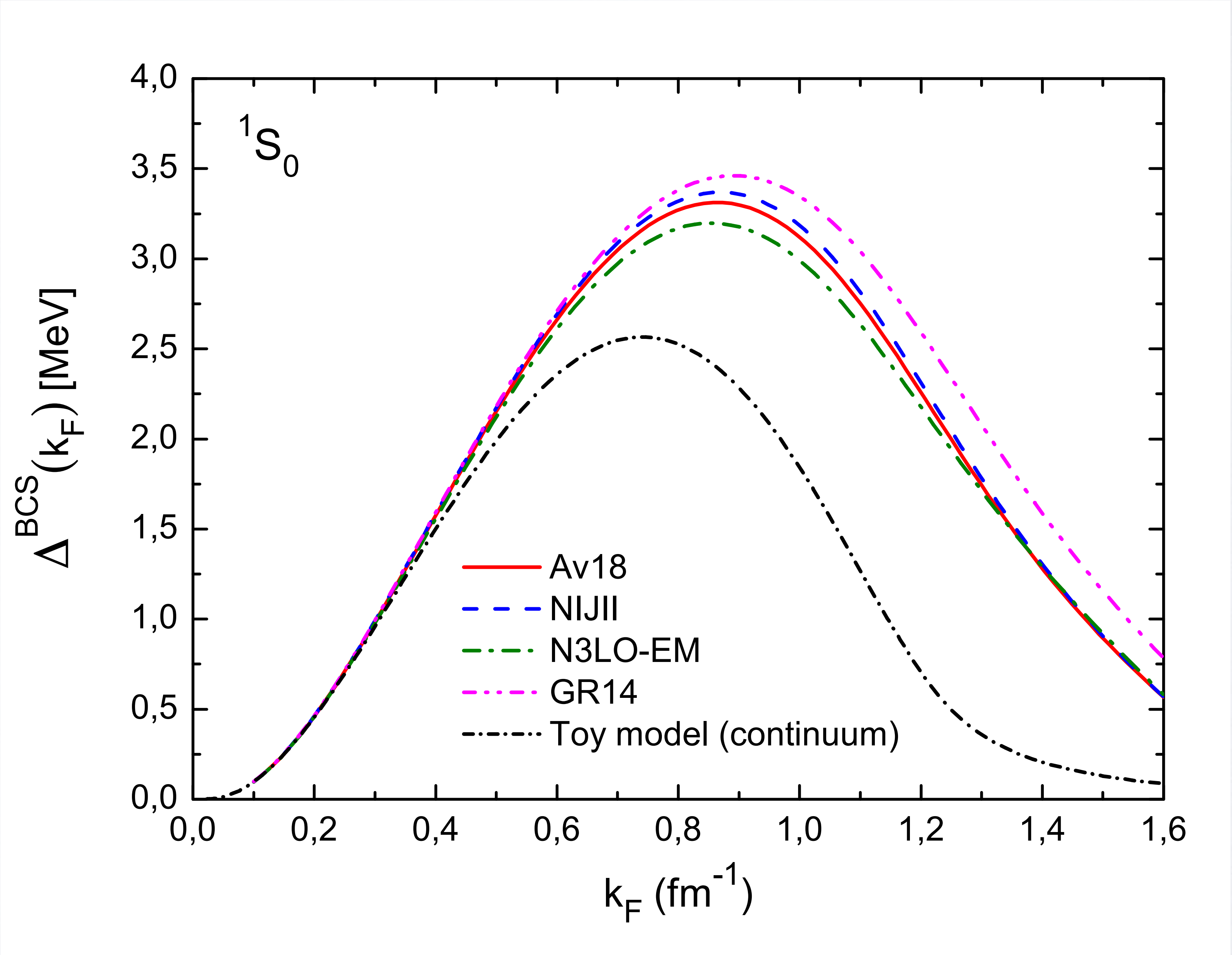
Central channel: pion vs contacts





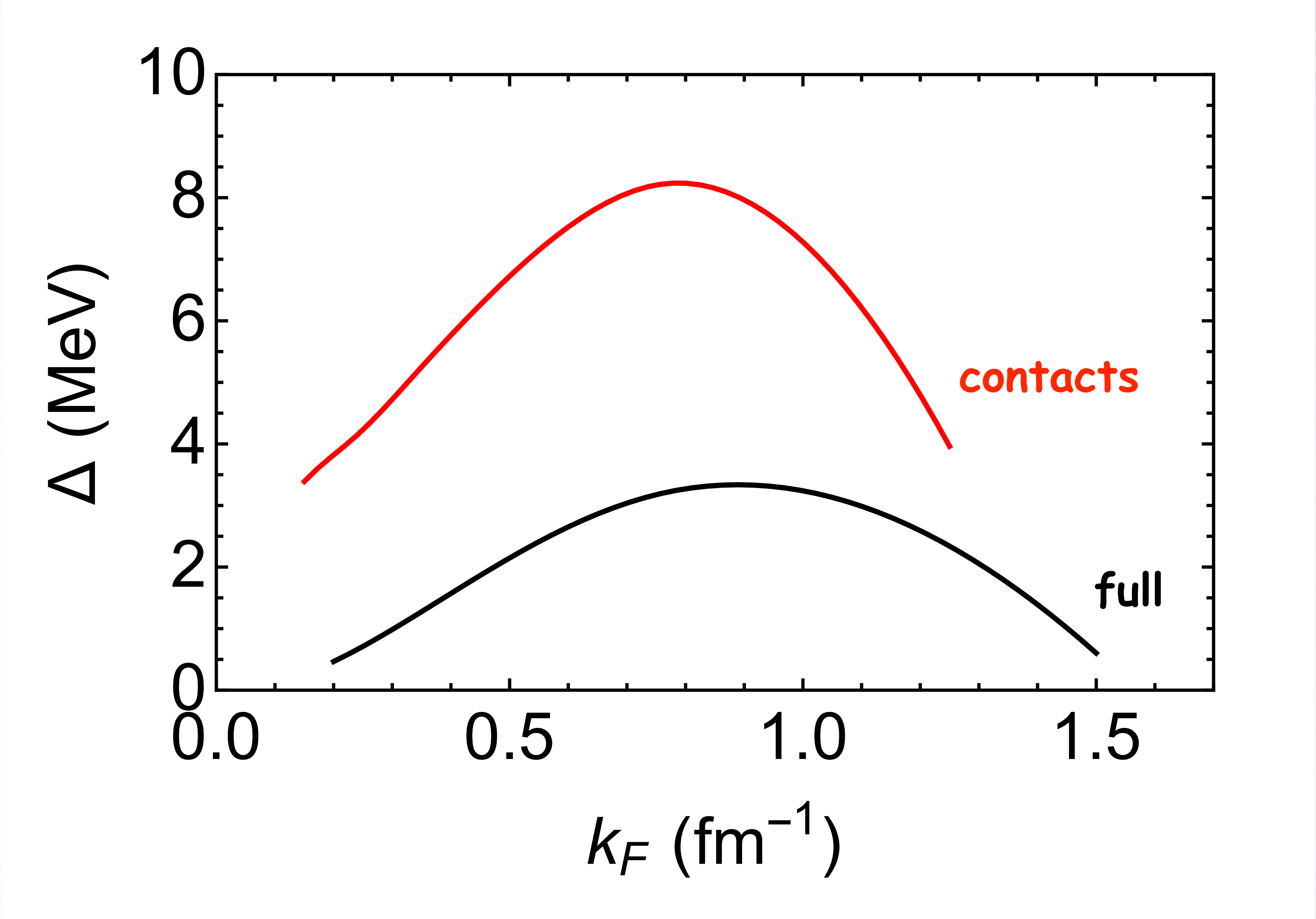
BCS pairing gap with different interactions

$$\Delta(k) = -\frac{1}{m\pi}\int_0^\infty dp\, p^2\ V(k,p)\ \underbrace{\frac{\Delta(p)}{\sqrt{[(p^2-p_F^2)/(2m)]^2+\Delta(p)}}}_{E(p)}$$



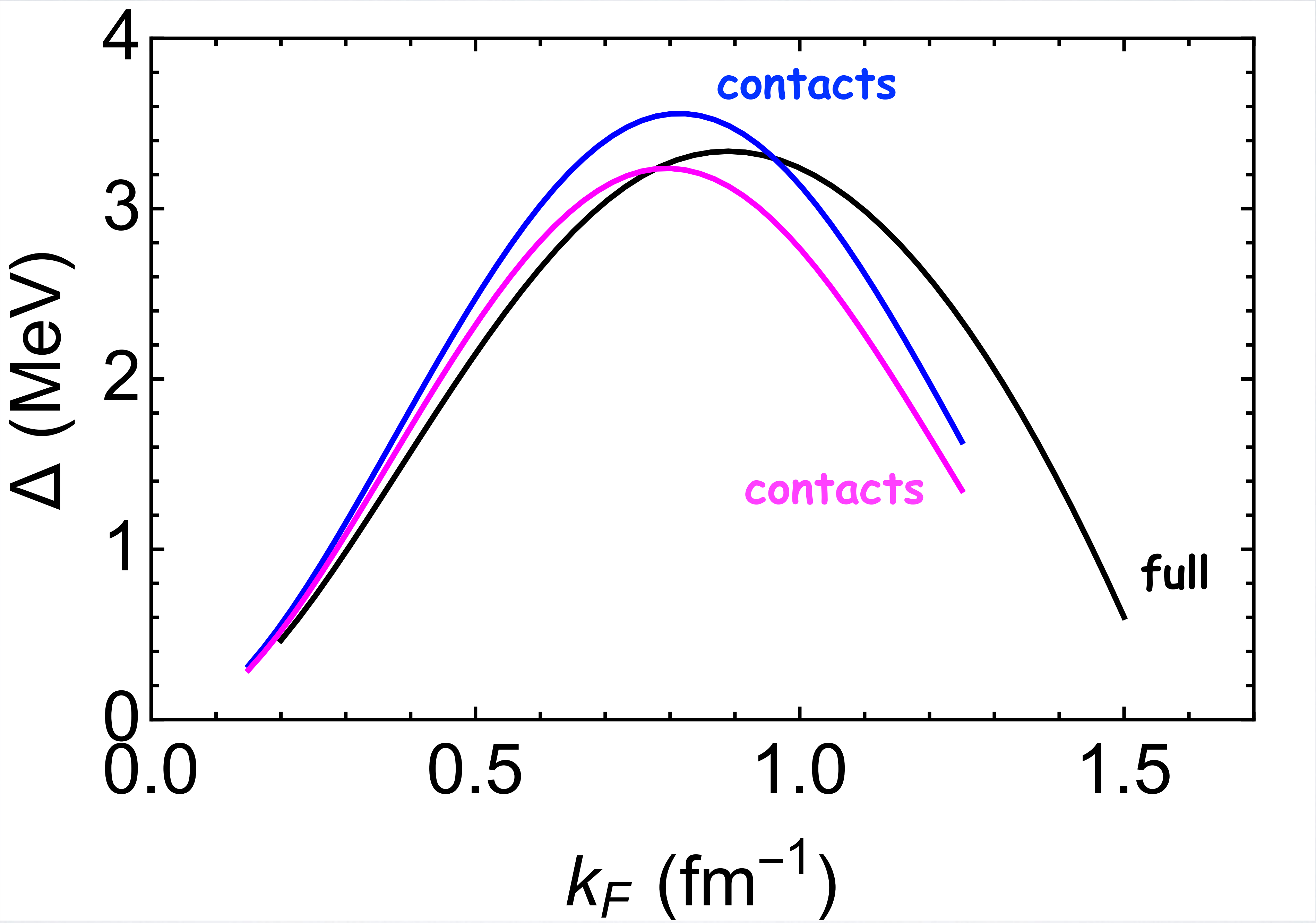
Pairing gap without pions

pions + contacts = full



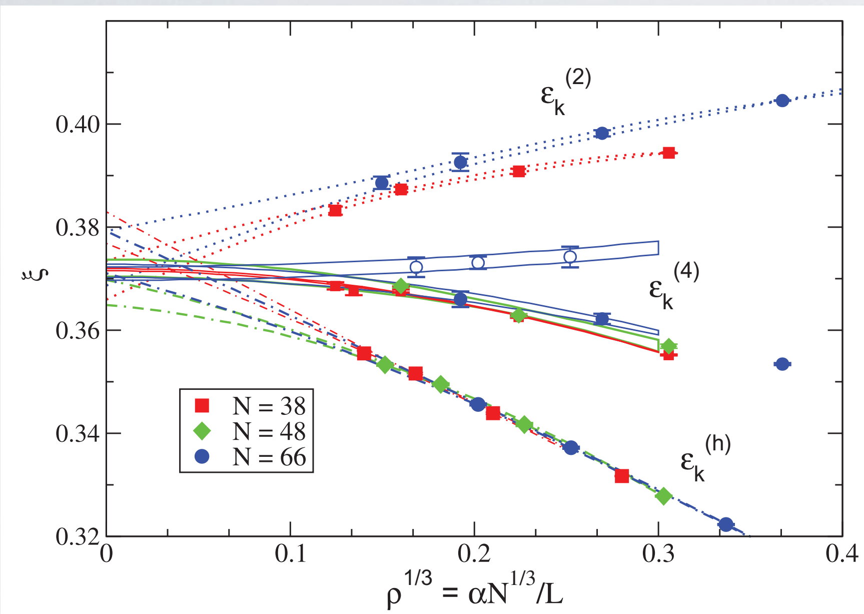
Incorporating pions into contacts

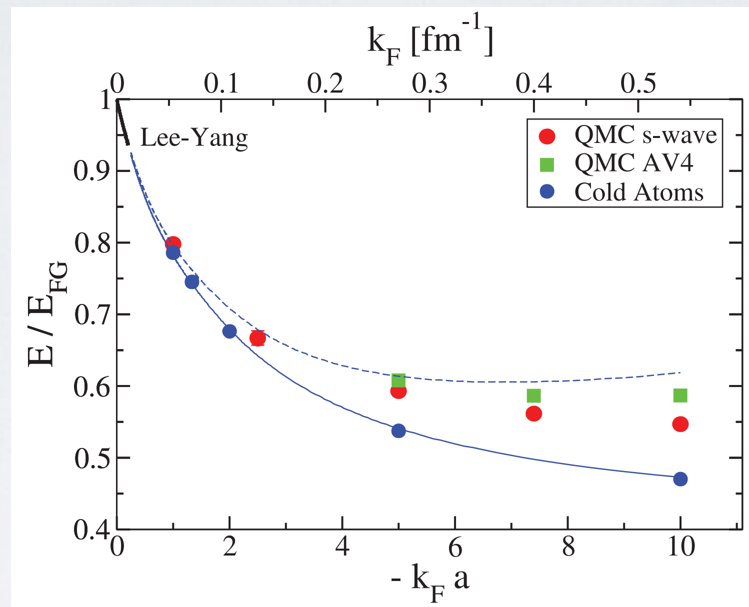
contacts ~ contacts ~ full

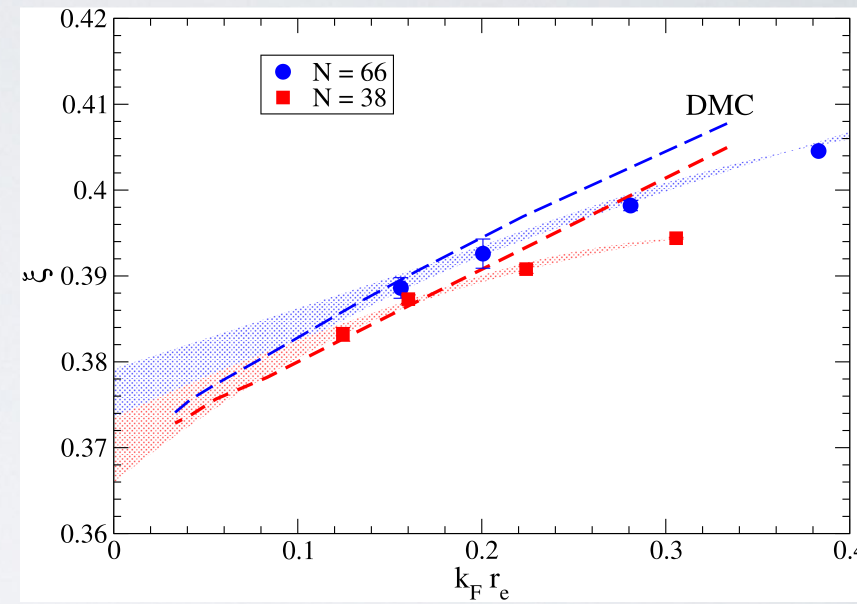


Neutron matter & Cold Atoms

Monte Carlo simulations







J. Carlson, S. Gandolfi and A. Gezerlis, Prog. Theor. Exp. Phys., 01A209 (2012).

"We report quantum Monte Carlo calculations of superfluid Fermi gases with short-range two-body attractive interactions with infinite scattering length. The energy of such gases is estimated to be (0.44 ± 0.01) times that of the noninteracting gas, and their pairing gap is approximately twice the energy per particle."

J. Carlson, S. Y. Chang, V. R. Pandharipande and K. E. Schmidt, Phys. Rev. Lett. 91, 050401 (2003)

Neutron matter with only contact interactions

(in the Unitarity Limit)

$$\xi(K_F) = \frac{T(k_F) + U(k_F)}{T(k_F)} = 1 + \frac{U(k_F)}{T(k_F)}$$

$$T(k_F) = \frac{3k_F^2}{10m_n}$$

$$U(k_F) = \frac{4}{m_n}\frac{2}{\pi}\int_0^{k_F} dk\ k^2 \left(1 - \frac{3k}{2k_F} + \frac{k^3}{2k_F^3}\right)\ \underline{V_{NN}(k,k)}$$

$$V(p,p') = C_0 + C_2\ (p^2 + p'^2) + C_4\ (p^4 + p'^4) + C_4'\ p^2 p'^2 + \cdots$$

LO NLO NNLO

Constraining LECs to unitarity condition

$$V(p,p') = C_0 + C_2\,(p^2 + p'^2) + C_4\,(p^4 + p'^4) + C_4'\,p^2 p'^2 + \cdots$$

LO NLO NNLO

Compute two-body T-matrix with V(p,p')

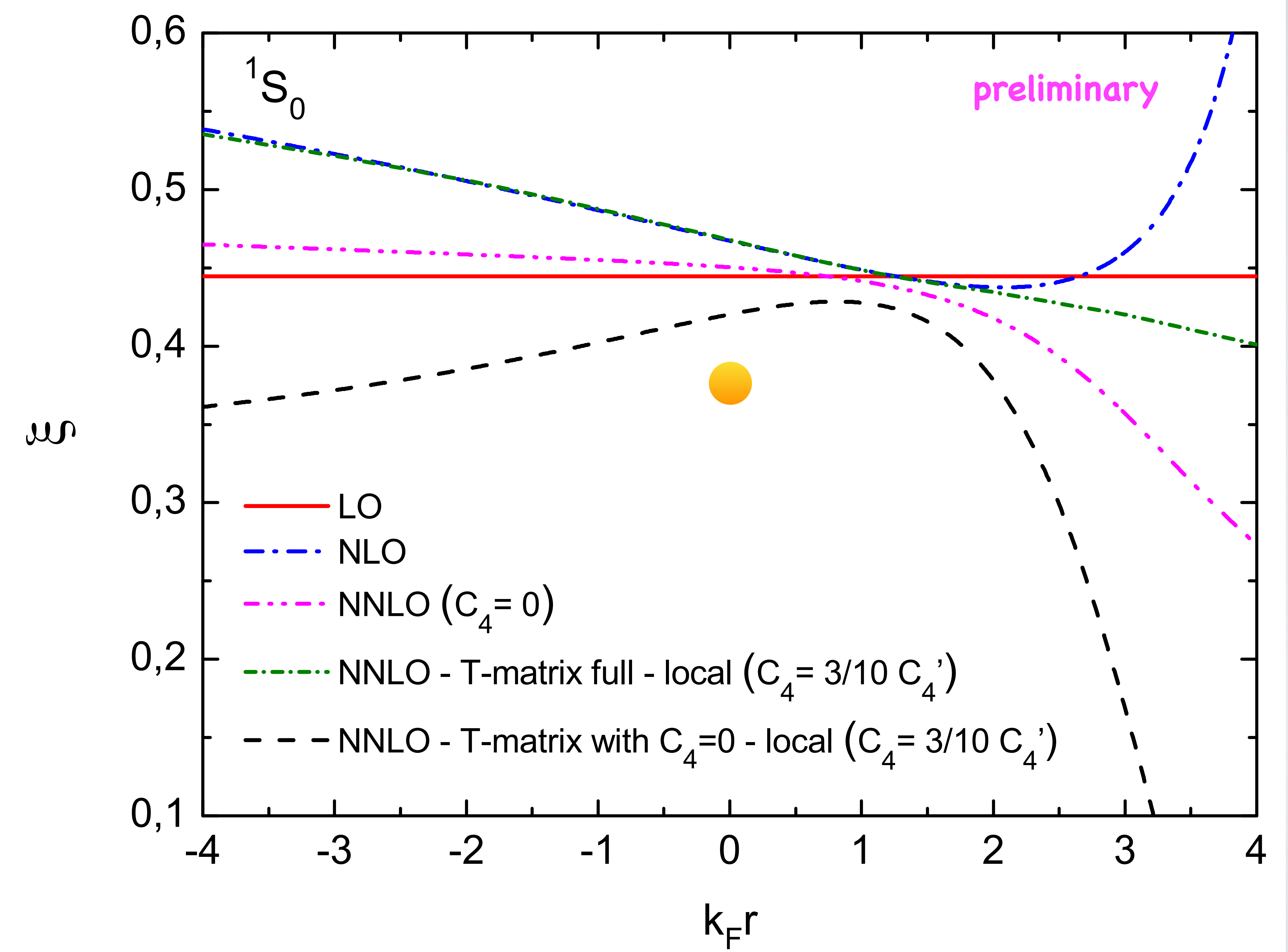
Match to Effective Range Expansion

Impose unitarity condition -1/a = 0 and r = 0

$$\xi_{\mathrm{LO}}(x) = \frac{4}{9} = 0.444\ldots \qquad \xi_{\mathrm{NLO}}(x) = \frac{\left(3\pi\,x - 6\sqrt{48 - 3\pi\,x} - 64\right)}{3\pi\,x - 48}$$

$$x = k_F r$$

Neutron matter (unitary limit)



Final Remarks

- Peripheral waves displays pure pionic effect
- S-wave completely dominated by the “unknown" part of the nuclear force, which is fitted to 2N observables
- Neutron matter in the unitary limit can be reasonably described by lower order contact interactions
- N2LO (N3LO) results are close but indicate that N3LO (N5LO) high order terms seems to be required