Hyperon-anti-hyperon polarization asymmetry in relativistic heavy-ion collisions as an interplay between chiral and helical vortical effects

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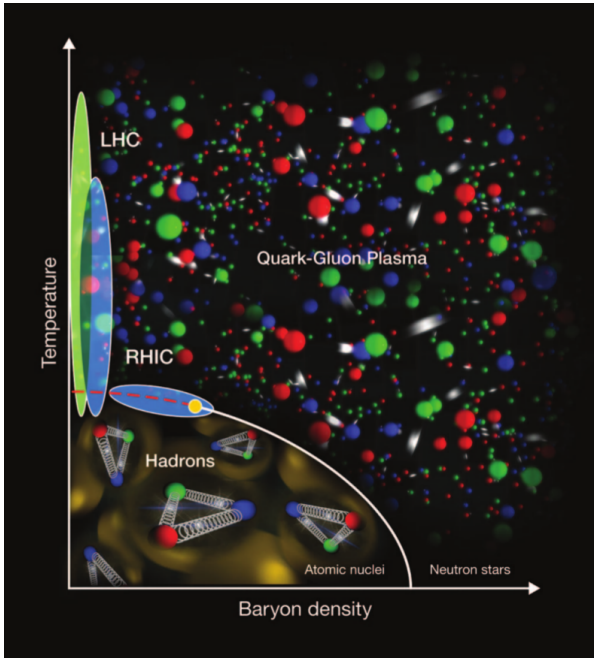
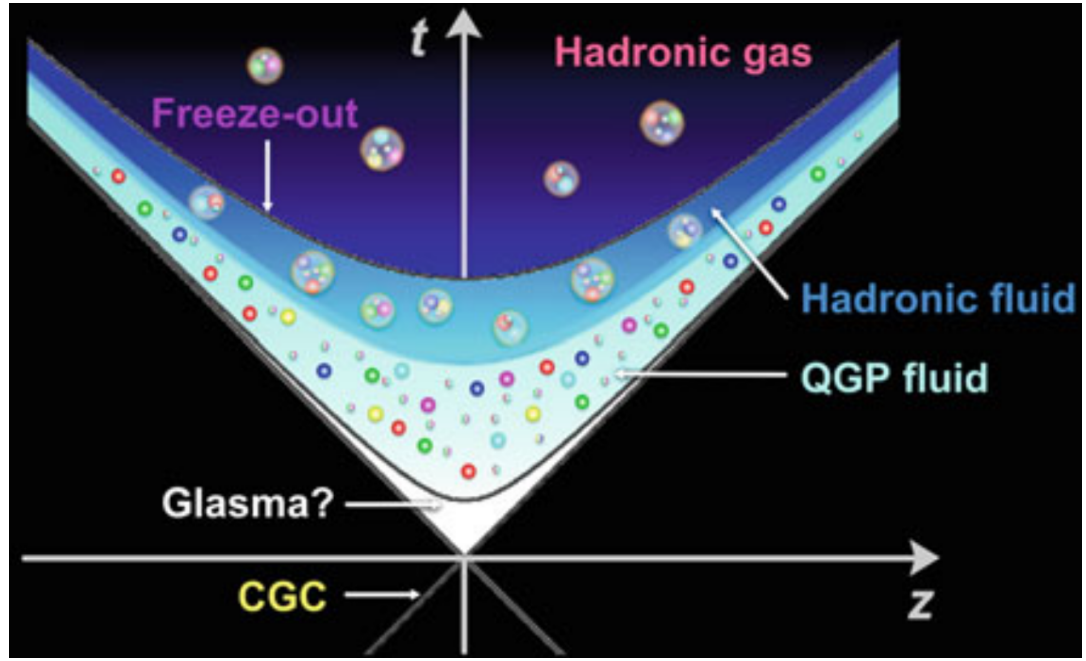
Outline

Quark-gluon plasma: hydrodynamic phase





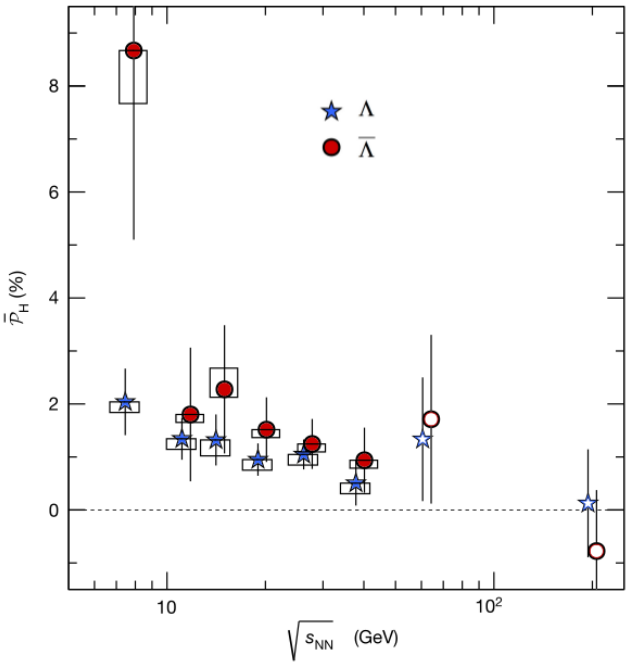
[B. V. Jacak, B. Muller, Science **337** (2012) 310.

A. Monnai, PhD Thesis (Tokyo, 2014).

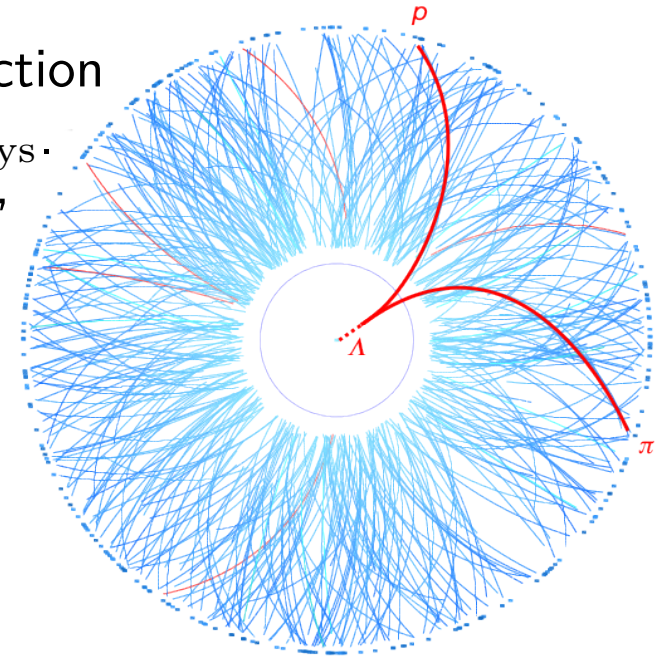
- The QGP produced at RHIC is...
  - the hottest ($k_B T \gtrsim 0.2$ GeV $\Leftrightarrow T \gtrsim 2.3 \times 10^{12}$ K),
  - densest ($p \gtrsim 10$ GeV/fm$^3 \simeq 1.6 \times 10^{36}$ Pa)
  - and most vortical ($\omega \simeq 10^{22}$ s^{-1})...

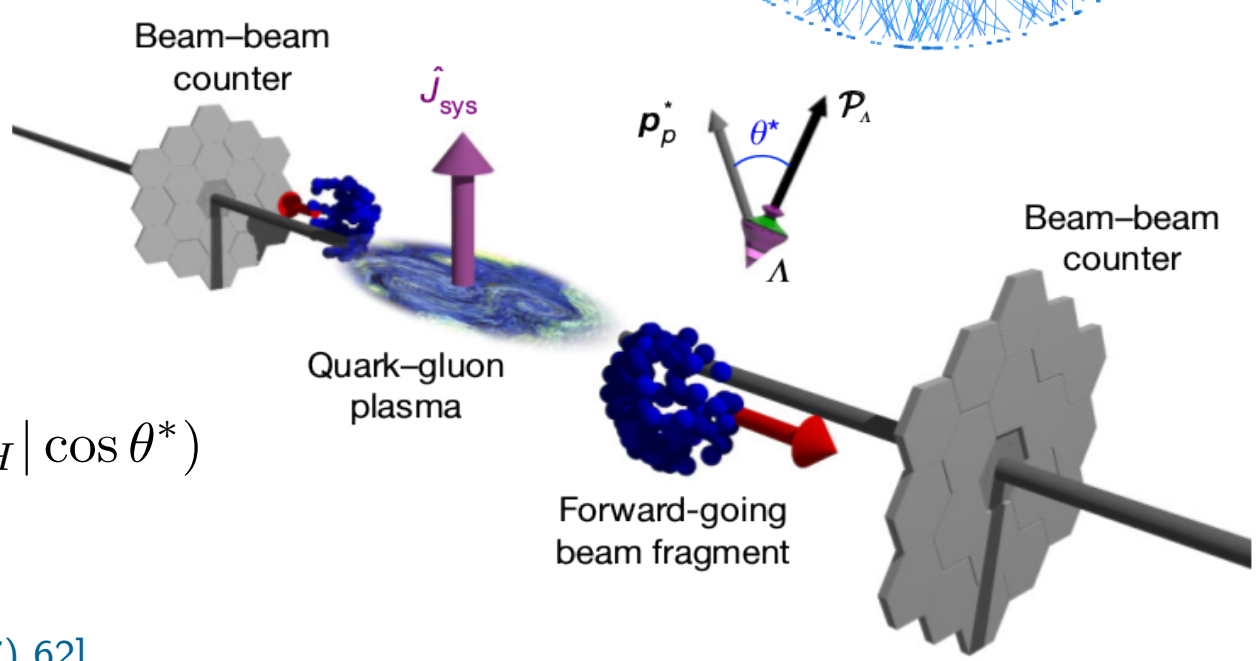
...fluid produced in the laboratory. [STAR Collaboration, Nature **548** (2017) 62]

QGP: Polarisation of Λ-hyperons



- $\overline{\mathcal{P}}_H \equiv$ average projection of polarization on $\hat{J}_{\rm sys}$.
- $\Lambda \equiv$ "self-analysing:" proton emitted preferentially along spin.

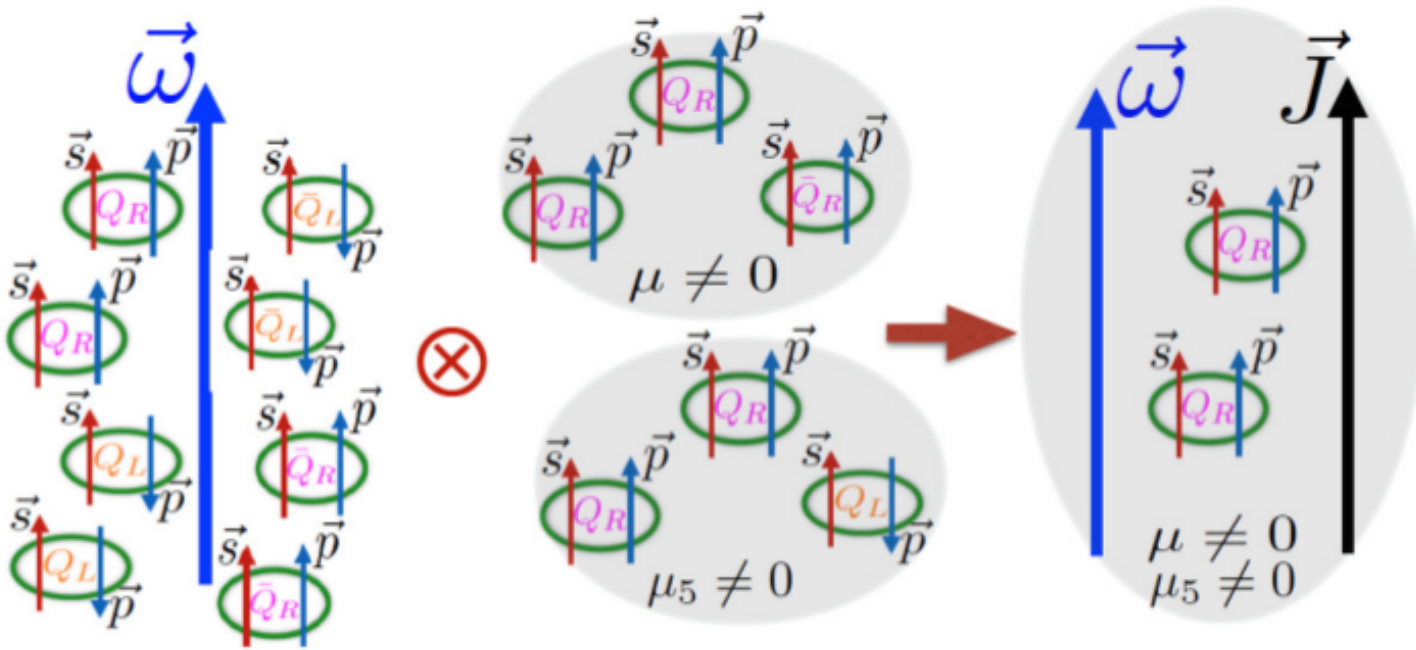




- $|\omega| \approx \dfrac{k_B T}{\hbar}(\overline{\mathcal{P}}_{\Lambda'} + \overline{\mathcal{P}}_{\bar{\Lambda}'})$.
- $\dfrac{dN_H}{d\cos\theta^*} = \dfrac{1}{2}(1 + \alpha_H |\mathcal{P}_H| \cos\theta^*)$

[STAR Collaboration, Nature **548** (2017) 62]

Known mechanism: Chiral vortical effect (CVE)



$$\boldsymbol{J}_V = \sigma_V \boldsymbol{\omega}, \qquad \boldsymbol{J}_A = \sigma_A \boldsymbol{\omega},$$

$$\sigma_V = \frac{\mu_V \mu_A}{\pi^2}, \qquad \sigma_A = \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2}.$$

$\boldsymbol{J}_A \neq 0$ even when $\mu_A = 0$!

[D. E. Kharzeev *et al.*, Nucl. Phys. **88** (2016) 1]

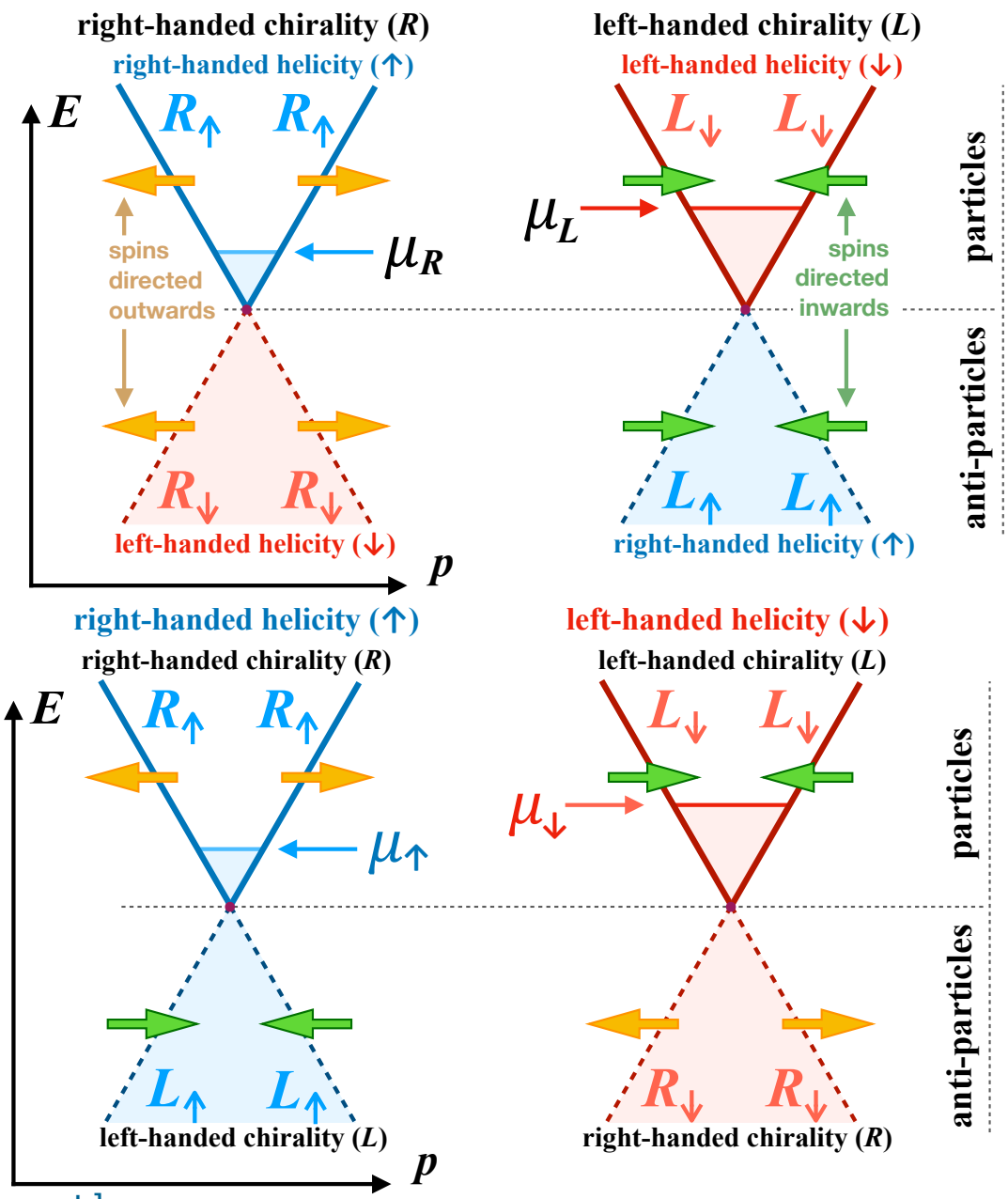
New mechanism: Helical vortical effect (HVE)

- Split particles into four groups:
  - $\mu_\uparrow^R$: particle: $R \Rightarrow \uparrow$
  - $\mu_\downarrow^L$: particle: $L \Rightarrow \downarrow$
  - $\bar{\mu}_\downarrow^R$: anti-particle: $R \Rightarrow \downarrow$
  - $\bar{\mu}_\uparrow^L$: anti-particle: $L \Rightarrow \uparrow$
- Charge densities:

$$Q_V \equiv (n_\uparrow^R + n_\downarrow^L) - (\bar{n}_\downarrow^R + \bar{n}_\uparrow^L),$$
$$Q_A \equiv (n_\uparrow^R + \bar{n}_\downarrow^R) - (n_\uparrow^L + \bar{n}_\downarrow^L),$$
$$Q_H \equiv (n_\uparrow^R + \bar{n}_\uparrow^L) - (n_\downarrow^L + \bar{n}_\downarrow^R).$$

- Vortical conductivities:

$$\sigma_V \simeq \frac{2\mu_H T}{\pi^2} \ln 2 + \frac{\mu_V \mu_A}{\pi^2},$$
$$\sigma_A \simeq \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2 + \mu_H^2}{2\pi^2},$$
$$\sigma_H \simeq \frac{2\mu_V T}{\pi^2} \ln 2 + \frac{\mu_H \mu_A}{\pi^2}.$$



VEA, M. N. Chernodub, arXiv:1912.11034 [hep-th].

Chirality (γ^5)

- For particles ($U_{R/L}$) and anti-particles ($V_{R/L} = i\gamma^2 U^*_{R/L}$):

$$\gamma^5 U_R = +\,U_R, \qquad \gamma^5 U_L = -U_L,$$
$$\gamma^5 V_R = -\,V_R, \qquad \gamma^5 V_L = +V_L. \tag{1}$$

- The axial current $J^\mu_A = \bar{\psi}\gamma^\mu\gamma^5\psi$ satisfies (classically)

$$\partial_\mu J^\mu_A = 2im\bar{\psi}\gamma^5\psi,$$

  and is hence conserved when $m = 0$.
- $Q_A = \int d^3x\, J^0_A$ can be promoted to a quantum operator:

$$:\widehat{Q}_A := \sum_j \chi_j(\hat{b}^\dagger_j\hat{b}_j + \hat{d}^\dagger_j\hat{d}_j), \qquad \chi_R = +1, \qquad \chi_L = -1, \tag{2}$$

  which satisfies $[\widehat{Q}_A, \widehat{H}] = 0 \Rightarrow$ when $m = 0$.

Helicity (h)

- The polarisation of ψ can be characterised using $h = \dfrac{\boldsymbol{S}\cdot\boldsymbol{P}}{p}$, with

$$hU_\lambda = \lambda U_\lambda, \qquad hV_\lambda = \lambda V_\lambda, \qquad \lambda = \pm\frac{1}{2}. \tag{3}$$

- The helicity current $J_H^\mu = \bar{\psi}\gamma^\mu h\psi + \overline{h\psi}\gamma^\mu\psi$ is conserved $\forall m$:

$$\partial_\mu J_H^\mu = 0.$$

- A comparison between Eqs. (1) and Eq. (3) shows that for a given mode U_j,

$$2\lambda_j = \chi_j. \tag{4}$$

- $Q_H = \int d^3x\, J_H^0$ can also be represented as a quantum operator:

$$:\widehat{Q}_H := \sum_j 2\lambda_j(\hat{b}_j^\dagger\hat{b}_j - \hat{d}_j^\dagger\hat{d}_j), \tag{5}$$

satisfying $[\widehat{Q}_H, \widehat{H}] = 0$.

CPT symmetries

	Q_V	Q_A	Q_H	$\boldsymbol{J}_V$	$\boldsymbol{J}_A$	$\boldsymbol{J}_H$	$\boldsymbol{\omega}$
C	$-$	$+$	$-$	$-$	$+$	$-$	$+$
P	$+$	$-$	$-$	$-$	$+$	$+$	$+$
T	$+$	$+$	$+$	$-$	$-$	$-$	$-$

$$:\widehat{Q}_V := \sum_j (\hat{b}_j^\dagger \hat{b}_j - \hat{d}_j^\dagger \hat{d}_j),$$

$$:\widehat{Q}_A := \sum_j 2\lambda_j (\hat{b}_j^\dagger \hat{b}_j + \hat{d}_j^\dagger \hat{d}_j),$$

$$:\widehat{Q}_H := \sum_j 2\lambda_j (\hat{b}_j^\dagger \hat{b}_j - \hat{d}_j^\dagger \hat{d}_j),$$

- J_V^μ, J_A^μ and J_H^μ form a triad: same T, different C and P.
- New vortical effects $\boldsymbol{J}_\ell = \sigma_\ell \boldsymbol{\omega}$ allowed by CPT symmetries:

$$\sigma_V \simeq \frac{2\mu_H T}{\pi^2} \ln 2 + \frac{\mu_V \mu_A}{\pi^2}, \quad \sigma_A \simeq \frac{T^2}{6} + \frac{\boldsymbol{\mu}^2}{2\pi^2}, \quad \sigma_H \simeq \frac{2\mu_V T}{\pi^2} \ln 2 + \frac{\mu_H \mu_A}{\pi^2}. \quad (6)$$

Classical results: RKT

- Fermions in equilibrium can be described using

$$f = f^{(\mathrm{eq})}(\beta \cdot k - \alpha), \qquad \nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0, \qquad \nabla_\mu \alpha = 0, \tag{7}$$

where $\beta^\mu = T^{-1} u^\mu$ and $\alpha = \mu/T$.

- According to the relativistic Boltzmann equation $k^\mu \partial_\mu f = C[f]$, global equilibrium is achieved when

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0, \qquad \nabla_\mu \alpha = 0. \tag{8}$$

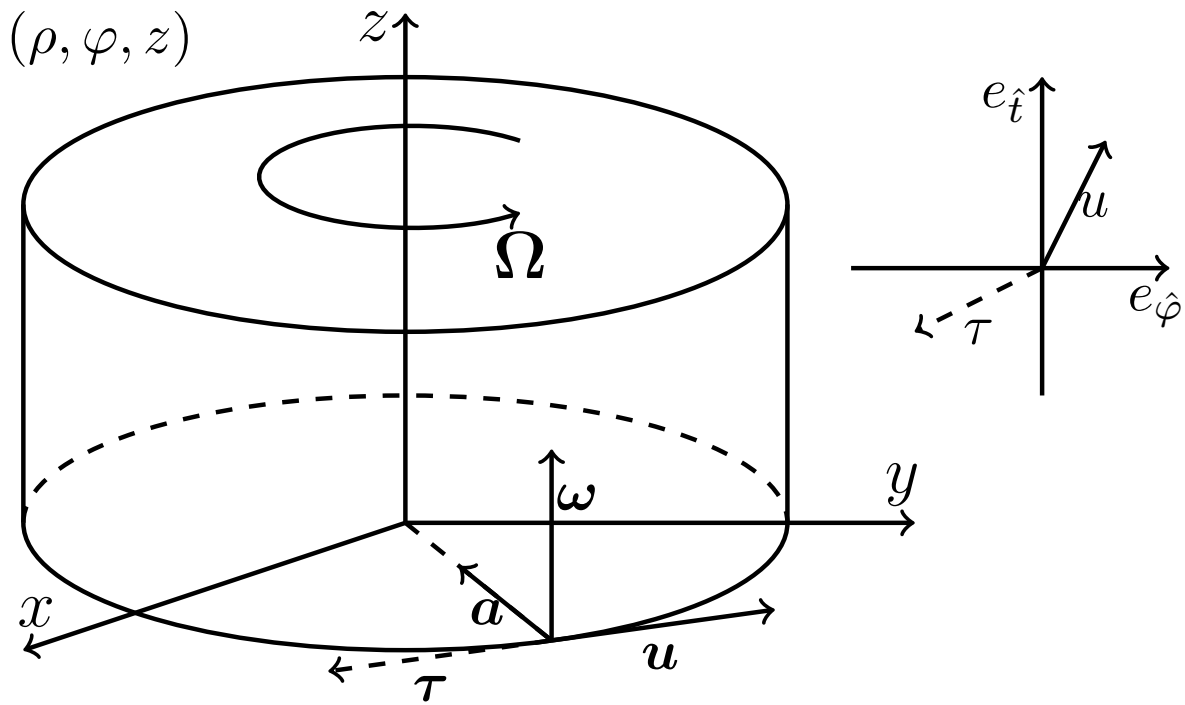
- One possible solution of the *Killing eq.* corresponds to rigid rotation:

$$\beta = \beta_0 (\partial_t + \Omega \partial_\varphi), \tag{9}$$

giving rise to

$$u = \Gamma(\partial_t + \Omega \partial_\varphi), \qquad \begin{pmatrix} T \\ \mu \end{pmatrix} = \Gamma \begin{pmatrix} T_0 \\ \mu_0 \end{pmatrix}, \qquad \Gamma = (1 - \rho^2 \Omega^2)^{-1/2}. \tag{10}$$

Kinematic frame for rigid rotation



The *kinematic tetrad* is given by: [Becattini, Grossi, PRD 2015]

Velocity : $u = \Gamma(e_{\hat{t}} + \rho\Omega e_{\hat{\varphi}}), \qquad \Gamma = (1-\rho^2\Omega^2)^{-1/2},$

Acceleration : $a = \nabla_u u = -\rho\Omega^2\Gamma^2 e_{\hat{\rho}},$

Vorticity : $\omega = \frac{1}{2}\varepsilon^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\sigma}} e_{\hat{\alpha}} u_{\hat{\beta}} (\nabla_{\hat{\gamma}} u_{\hat{\sigma}}) = \Omega\Gamma^2 e_{\hat{z}},$

Fourth vector : $\tau = \varepsilon^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\sigma}} e_{\hat{\alpha}} u_{\hat{\beta}} a_{\hat{\gamma}} \omega_{\hat{\sigma}} = -\rho\Omega^3\Gamma^5(\rho\Omega e_{\hat{t}} + e_{\hat{\varphi}}).$

Quantum rigidly-rotating thermal states

- Thermal states can be constructed using

$$\langle \hat{A} \rangle = Z^{-1}\mathrm{Tr}(\hat{\varrho}\hat{A}), \qquad \hat{\varrho} = \exp\left[-\frac{1}{T_0}\left(\widehat{H} - \Omega\widehat{M}^z - \sum_\ell \mu_{\ell;0}\widehat{Q}_\ell\right)\right],$$

where $Z = \mathrm{Tr}(\hat{\varrho})$ is the partition function.

- Expanding $\widehat{\Psi}$ w.r.t. U_j and $V_j = i\gamma^2 U_j^*$,

$$\widehat{\Psi}(x) = \sum_j [U_j(x)\hat{b}_j + V_j(x)\hat{d}_j^\dagger], \tag{11}$$

we are interested in the following t.e.v.s:

$$J_V^\mu = \langle : \widehat{\overline{\Psi}}\gamma^\mu\widehat{\Psi} : \rangle, \qquad J_A^\mu = \langle : \widehat{\overline{\Psi}}\gamma^\mu\gamma^5\widehat{\Psi} : \rangle, \qquad J_H^\mu = \langle : \widehat{\overline{\Psi}}\gamma^\mu 2h\widehat{\Psi} : \rangle. \tag{12}$$

- The charge currents deviate from the *perfect fluid form*,

$$J_\ell^\mu = Q_\ell u^\mu + \sigma_\ell^\omega \omega^\mu + \sigma_\ell^\tau \tau^\mu. \tag{13}$$

- The circular terms are suppressed since $\tau = -\Omega^3\Gamma^5(\rho^2\Omega\partial_t + \partial_\varphi)$.

Vortical conductivities: mode sums

- Noting that

$$\langle \hat{b}_j^\dagger \hat{b}_j \rangle = n_{\beta_0}(\tilde{p}, 1, 2\lambda, 2\lambda), \qquad \langle \hat{d}_j^\dagger \hat{d}_j \rangle = n_{\beta_0}(\tilde{p}, -1, 2\lambda, -2\lambda), \tag{14}$$

where $\tilde{p} = p - \Omega m$ and

$$n_{\beta_0}(\tilde{p}, q_V, q_A, q_H) = [\exp(\beta_0(\tilde{p} - q_\ell \mu_\ell)) + 1]^{-1}, \tag{15}$$

...the vortical conductivities can be expressed as mode sums:

$$\sigma_V^\omega = \sum_{\sigma=\pm 1} \sum_{\lambda=\pm\frac{1}{2}} \frac{2\lambda\sigma}{4\pi^2\Omega} \sum_{m=-\infty}^{\infty} \int_0^\infty dp\, n_{\beta_0}(\tilde{p}, \sigma, 2\lambda, 2\lambda\sigma) \int_0^p dk\, p J_m^-(q\rho),$$

$$\sigma_A^\omega = \sum_{\sigma=\pm 1} \sum_{\lambda=\pm\frac{1}{2}} \frac{1}{4\pi^2\Omega} \sum_{m=-\infty}^{\infty} \int_0^\infty dp\, n_{\beta_0}(\tilde{p}, \sigma, 2\lambda, 2\lambda\sigma) \int_0^p dk\, p J_m^-(q\rho),$$

$$\sigma_H^\omega = \sum_{\sigma=\pm 1} \sum_{\lambda=\pm\frac{1}{2}} \frac{\sigma}{4\pi^2\Omega} \sum_{m=-\infty}^{\infty} \int_0^\infty dp\, n_{\beta_0}(\tilde{p}, \sigma, 2\lambda, 2\lambda\sigma) \int_0^p dk\, p J_m^-(q\rho),$$

where ρ is the distance to the rotation axis, $q = \sqrt{p^2 - k^2}$ and

$$J_m^-(q\rho) = J^2_{m-\frac{1}{2}}(q\rho) - J^2_{m+\frac{1}{2}}(q\rho). \tag{16}$$

Leading order in Ω

- At small Ω,

$$n_{\beta_0}(\tilde{p}, q_V, q_A, q_H) = n_{\beta_0}(p, q_V, q_A, q_H) - m\Omega\partial_p n_{\beta_0}(p, q_V, q_A, q_H) \tag{17}$$

...and the sum over m can be performed using

$$\sum_{m=-\infty}^{\infty} J_m^-(q\rho) = 0, \qquad \sum_{m=-\infty}^{\infty} m J_m^-(q\rho) = 1, \tag{18}$$

such that the k integral is trivially $\int_0^p dk\, p \sum_m m J_m^- = p^2$.

- Integrating by parts w.r.t. p gives

$$\begin{aligned}\sigma_\ell^\omega &= \sum_{\sigma=\pm 1} \sum_{\lambda=\pm\frac{1}{2}} \frac{2\lambda q_\ell}{2\pi^2} \int_0^\infty dp\, p\, n_{\beta_0}(p, \sigma, 2\lambda, 2\lambda\sigma), \\ &= -\sum_{\sigma=\pm 1} \sum_{\lambda=\pm\frac{1}{2}} \frac{q_A q_\ell T^2}{2\pi^2} \mathrm{Li}_2\left[-\exp\left(\frac{\boldsymbol{q}\cdot\boldsymbol{\mu}}{T}\right)\right],\end{aligned}$$

where $(q_V, q_A, q_H) = (\sigma, 2\lambda, 2\lambda\sigma)$.

Axial/helical vortical effects: Consitutive relations

- For high temperatures, the vortical conductivities are

$$
\begin{aligned}
\sigma_V^\omega &= \frac{2\mu_H T}{\pi^2}\ln 2 + \frac{\mu_A\mu_V}{\pi^2} + O(T^{-1}),\\
\sigma_A^\omega &= \frac{T^2}{6} + \frac{\mu_V^2+\mu_A^2+\mu_H^2}{2\pi^2} + O(T^{-1}),\\
\sigma_H^\omega &= \frac{2\mu_V T}{\pi^2}\ln 2 + \frac{\mu_A\mu_H}{\pi^2} + O(T^{-1}).
\end{aligned}
$$

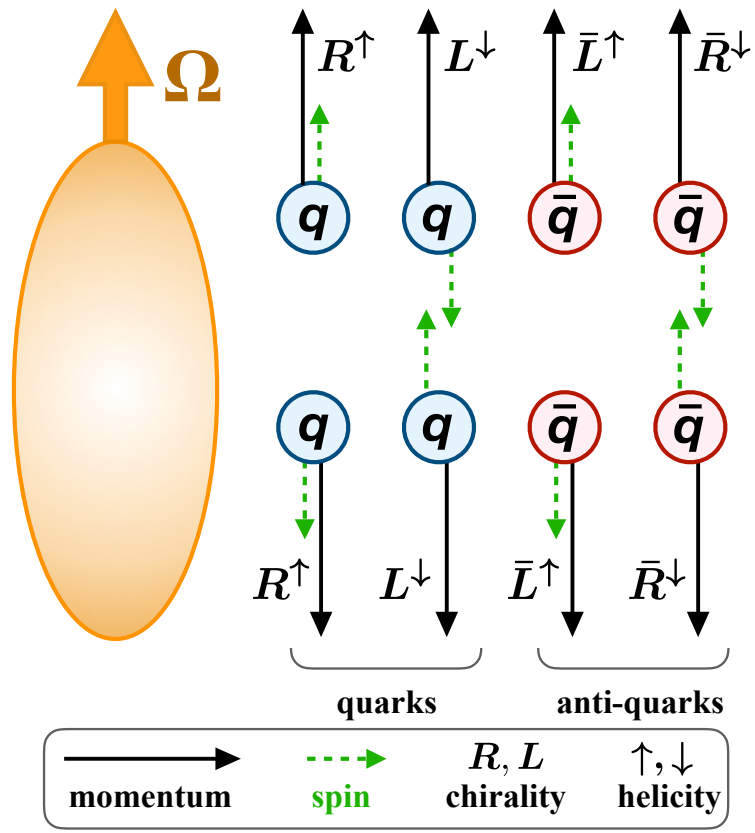
- At finite T and μ_V, $\boldsymbol{\omega}$ generates both $\boldsymbol{J}_A$ **<u>and</u>** $\boldsymbol{J}_H$!

Particle/anti-particle polarisation from $\boldsymbol{J}_A \pm \boldsymbol{J}_H$

- Considering now a system with $\boldsymbol{\Omega}_{\rm sys} = \boldsymbol{n}_{\rm sys}|\boldsymbol{\Omega}_{\rm sys}|$ and $J_\ell \equiv \boldsymbol{J}_\ell \cdot \boldsymbol{n}$, we can identify:

$$\begin{aligned} J_V &= J_\uparrow + J_\downarrow - \bar{J}_\uparrow - \bar{J}_\downarrow, \\ J_A &= J_\uparrow + \bar{J}_\uparrow - J_\downarrow - \bar{J}_\downarrow, \\ J_H &= J_\uparrow + \bar{J}_\downarrow - J_\downarrow - \bar{J}_\uparrow, \end{aligned}$$

where $\_(\uparrow,\downarrow) \equiv$ (right-, left-)handed helicity, while $\bar{\_} \equiv$ anti-particles.



- The net helicity current of particles and anti-particles can be obtained as

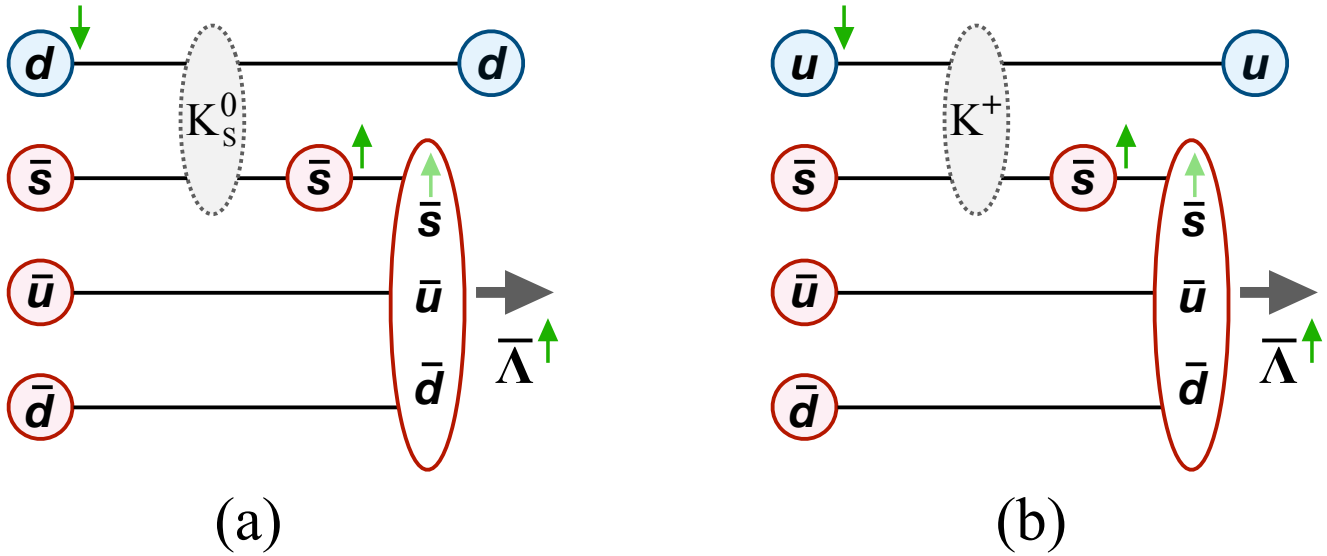
$$J_\uparrow - J_\downarrow = \frac{J_A + J_H}{2}, \qquad \bar{J}_\uparrow - \bar{J}_\downarrow = \frac{J_A - J_H}{2}. \tag{19}$$

- The polarisation of (light flavour) quarks / anti-quarks can be related to the above via

$$\mathcal{P}_q = \kappa_{qj}(J_\uparrow - J_\downarrow), \qquad \mathcal{P}_{\bar{q}} = \kappa_{\bar{q}\bar{j}}(\bar{J}_\uparrow - \bar{J}_\downarrow), \tag{20}$$

where $\kappa_{qj} = \kappa_{\bar{q}\bar{j}}$ are (C-even) kinematical factors.

(Anti-)hyperon polarisation from $q/\bar{q}$



- The discussion above applies to $q = (u, d)$. [strange-neutrality requires $\mu_s = 0$]
- $\mathcal{P}_\Lambda$ comes predominantly from $\mathcal{P}_s$. [QCDSF Collaboration, PLB **545** (2002) 112.]
- $\mathcal{P}_q$ can be transferred to $\mathcal{P}_{\bar{s}}$ via intermediate K_S^0, K^+ states:

$$\mathcal{P}_s = \kappa_{s\bar{q}}\mathcal{P}_{\bar{q}}, \qquad \mathcal{P}_{\bar{s}} = \kappa_{\bar{s}q}\mathcal{P}_q, \qquad \kappa_{s\bar{q}} = \kappa_{\bar{s}q}. \tag{21}$$

- The intermediate Kaons donate their $\bar{s}$ quarks to the antihyperon:

$$\mathcal{P}_\Lambda = \kappa_{\Lambda s}\mathcal{P}_s, \qquad \mathcal{P}_{\bar{\Lambda}} = \kappa_{\bar{\Lambda}\bar{s}}\mathcal{P}_{\bar{s}}, \qquad \kappa_{\Lambda s} = \kappa_{\bar{\Lambda}\bar{s}}. \tag{22}$$

Freezeout calculation

a, GeV	b, GeV	c, GeV	d, GeV	f, GeV^{-1}
$0.166(2)$	$0.139(16)$	$0.053(21)$	$1.308(28)$	$0.273(8)$

- Applying the vortical effects for $\mathcal{P}_{q/\bar{q}}$, we get

$$\mathcal{P}_{\Lambda} = \frac{1}{2}\kappa_{\Lambda s}\kappa_{s\bar{q}}\kappa\bar{q}\bar{j}(\sigma_A^{\omega} - \sigma_H^{\omega})\omega, \qquad \mathcal{P}_{\bar{\Lambda}} = \frac{1}{2}\kappa_{\bar{\Lambda}\bar{s}}\kappa_{\bar{s}q}\kappa q j(\sigma_A^{\omega} + \sigma_H^{\omega})\omega. \quad (23)$$

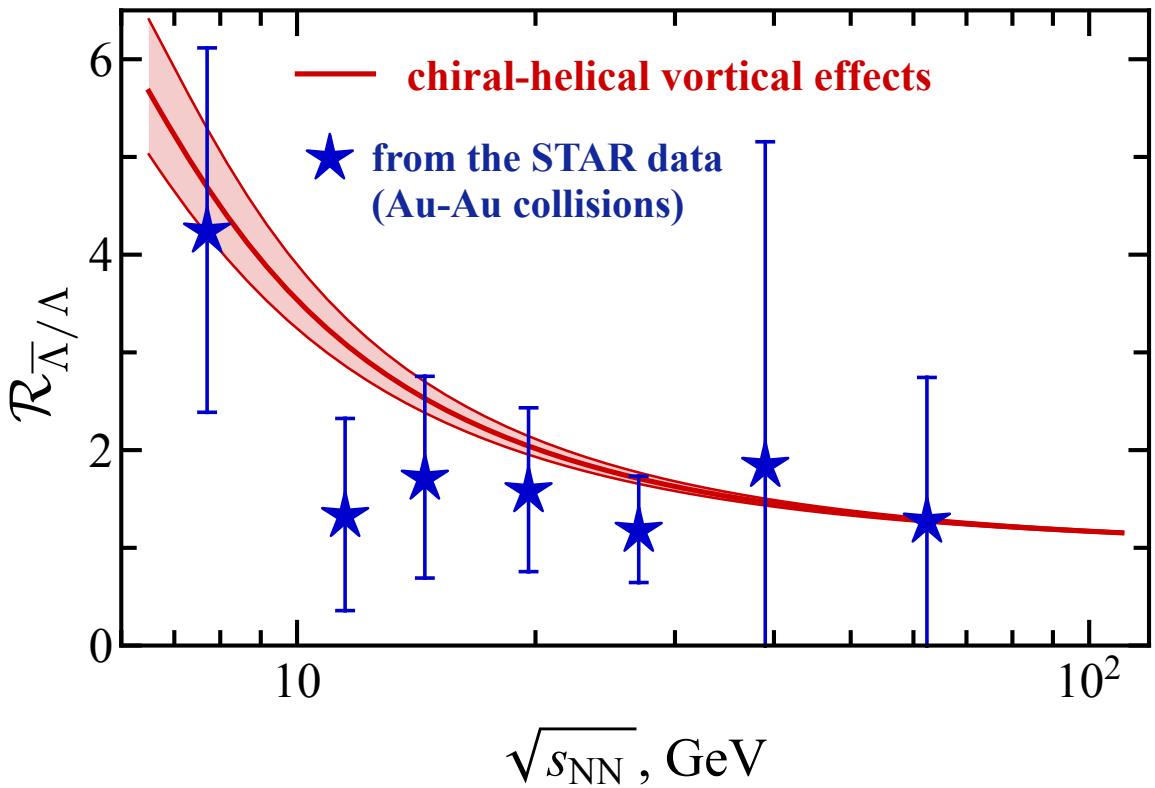
- At freezeout, [Cleymans, Oeschler, Redlich, Wheaton, PRC **73** (2006) 034905]

$$T \equiv T(\mu_B) = a - b\mu_B^2 - c\mu_B^4, \qquad \mu_B(\sqrt{s}) = \frac{d}{1 + f\sqrt{s}}. \quad (24)$$

- The total polarisation can be obtained by integrating $\mathcal{P}$ over the FO hypersurface:

$$\mathcal{P}_{q/\bar{q}} = \frac{1}{2}\kappa_{qj}(\sigma_A^{\omega} \pm \sigma_H^{\omega}) \int d\Sigma_{\mu}\omega^{\mu}. \quad (25)$$

Result



- The anti-hyperon / hyperon polarisation ratio becomes simply

$$\mathcal{R}_{\bar{\Lambda}/\Lambda} = \frac{\mathcal{P}_{\bar{\Lambda}}}{\mathcal{P}_{\Lambda}} = \frac{\mathcal{P}_q}{\mathcal{P}_{\bar{q}}} = \frac{\sigma_A^\omega + \sigma_H^\omega}{\sigma_A^\omega - \sigma_H^\omega} = 1 + \frac{8\ln 2}{\pi^2}\frac{\mu_B}{T} + O(\mu_B^2/T^2). \qquad (26)$$

Conclusion

- The (V, A, H) triad uncovers the *helical vortical effects* (HVE).
- $\boldsymbol{J}_A$ generated at finite T and/or finite μ_V, even when $\mu_A = \mu_H = 0$.
- $\boldsymbol{J}_H$ generated at finite T and μ_V, even when $\mu_A = \mu_H = 0$.
- Polarisation of light quarks /antiquarks can be expressed via $J_A \pm J_H$.
- Assuming $\mathcal{P}_{q/\bar{q}} \to \mathcal{P}_{\bar{s}/s} \to \mathcal{P}_{\bar{\Lambda}/\Lambda}$, it is easy to derive $\mathcal{R}_{\bar{\Lambda}/\Lambda} \simeq 1 + \frac{8 \ln 2}{\pi^2} \frac{\mu_B}{T}$.

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THANK YOU!