The critical endpoint of QCD in a finite volume

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Based on:
JB, Fischer, Isserstedt, Schaefer (arXiv:2107.05504)

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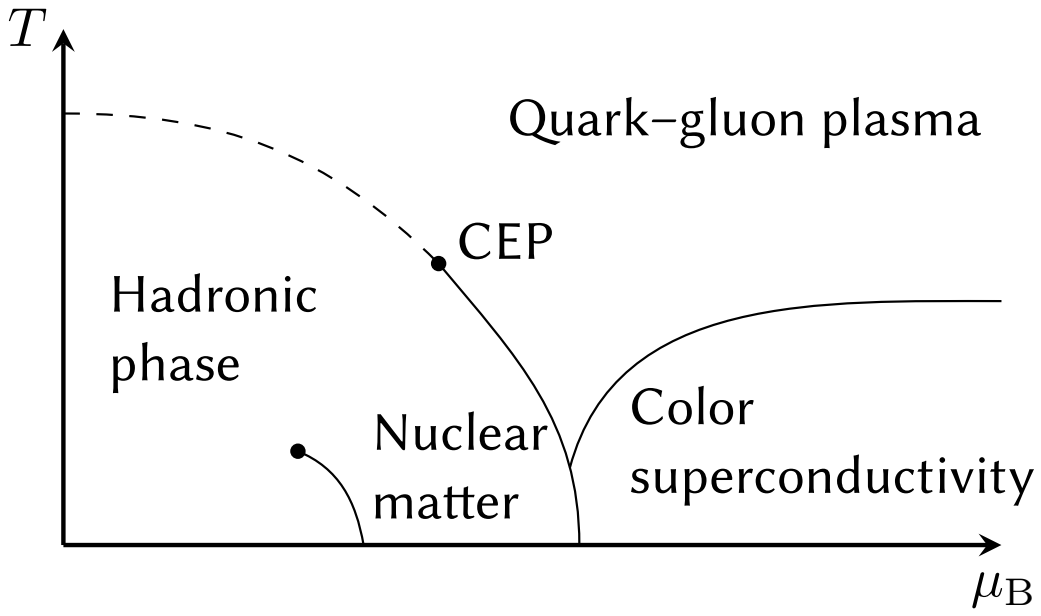
Outline

Why Finite Volume?

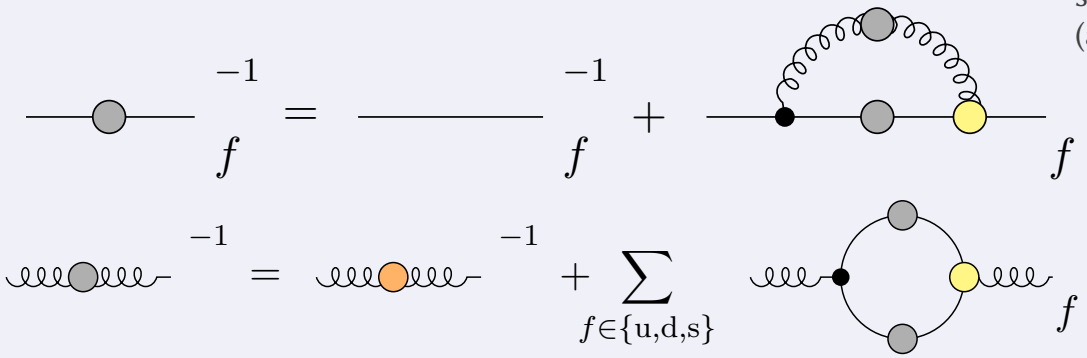


- Goal of many experiments is to locate CEP in QCD phase diagram
- "Fireball" of heavy-ion collisions has finite spatial extent
- Impact of volume effects on CEP is important for comparison between experiment and theory
- Lattice QCD is by construction formulated in a finite volume

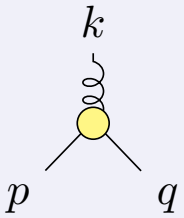
Truncated Set of DSEs

Truncated DSEs for quarks and gluons

see Fischer, PPNP 105 (2019) 1
(and references therein)



Quark–gluon vertex ansatz



$$\Gamma^f_\mu(k,p,q) = \gamma_\mu \Gamma(k,p,q) \Gamma^{f,\mathrm{BC}}_\mu(p,q)$$ (Information about quarks)

Quenched gluon propagator

$$D^{\mathrm{que}}_{\mu\nu}(k) = D^{\mathrm{que}}_{\mu\nu}(k;T)$$ (Temperature-dependent fit to lattice data)

reference for lattice data: Fischer, Maas, Müller, EPJC 68 (2010) 165-181
Maas, Pawlowski, von Smekal, Spielmann, PRD 85 (2012) 034037

Finite Volume Framework I

- Feasible shape as ansatz: cube with edge length L

$$\int_{\mathbb{R}^3} \mathrm{d}^3x\, \mathcal{L} \to \int_{[0,L]^3} \mathrm{d}^3x\, \mathcal{L}$$

- For quarks, free to choose between

$$\psi(\boldsymbol{x} + L\boldsymbol{e}_i) = +\psi(\boldsymbol{x}) \quad \text{periodic boundary conditions (PBC)}$$
$$\psi(\boldsymbol{x} + L\boldsymbol{e}_i) = -\psi(\boldsymbol{x}) \quad \text{antiperiodic boundary conditions (ABC)}$$

- For gluons, need PBC for kinematic reasons

$\to$ Only discrete values possible in momentum space

Finite Volume Framework II

Spatial Matsubara modes

$$\omega_n^L = \begin{cases} 2n\pi/L & \text{for PBC}\,, \\ (2n+1)\pi/L & \text{for ABC}\,, \end{cases} \qquad n \in \mathbb{Z}$$

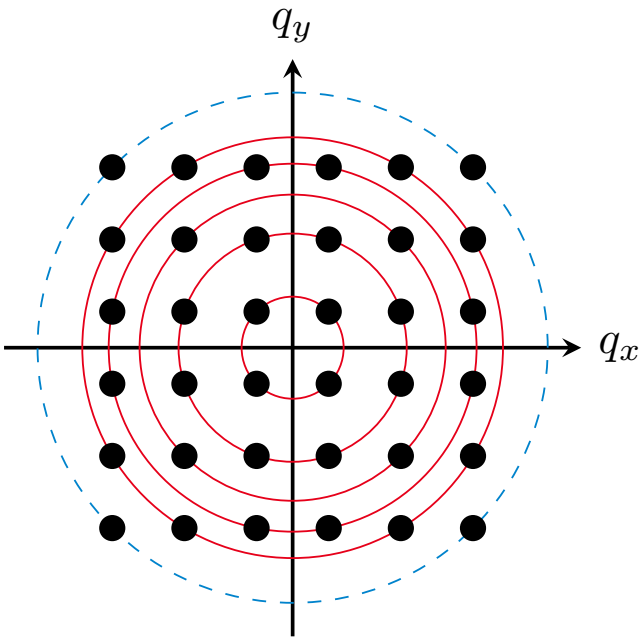
- Momentum integrals become sums

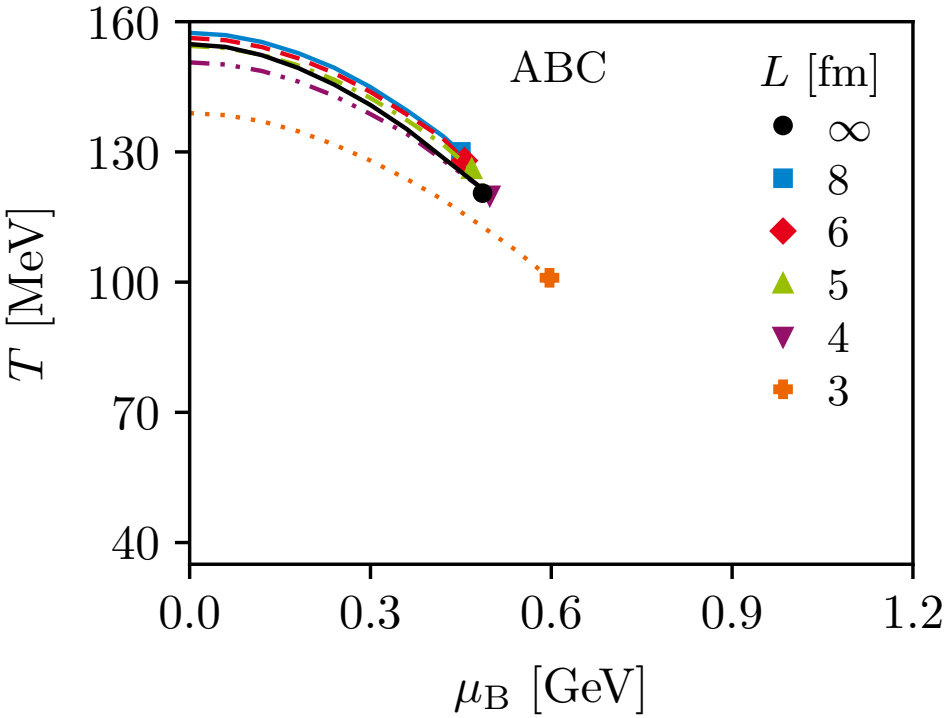
$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3} K(\boldsymbol{q}) \to \frac{1}{L^3} \sum_{\boldsymbol{n}\in\mathbb{Z}^3} K(\boldsymbol{q_n})\,,$$

  where $\boldsymbol{q_n} := \sum_{i=1}^3 \omega_{n_i}^L \boldsymbol{e}_i$ are allowed momentum vectors

- Also consider PBC without zero mode, $\boldsymbol{q} = \boldsymbol{0}$: referred to as PBC*

Intricacies of Momentum Summation





- Need to save all possible momenta

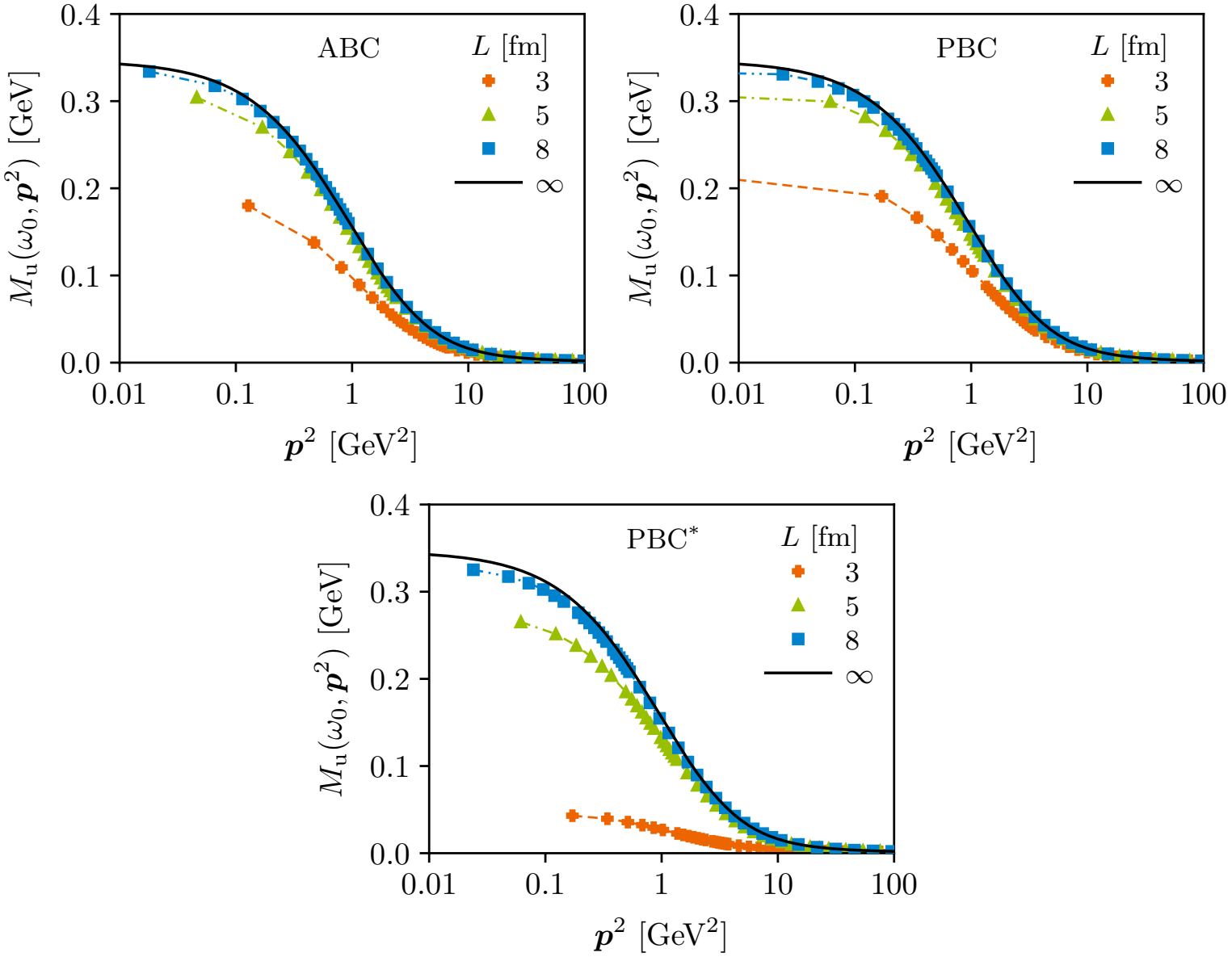
→ Large cutoffs unfeasible

- No consistent infinite-volume limit!

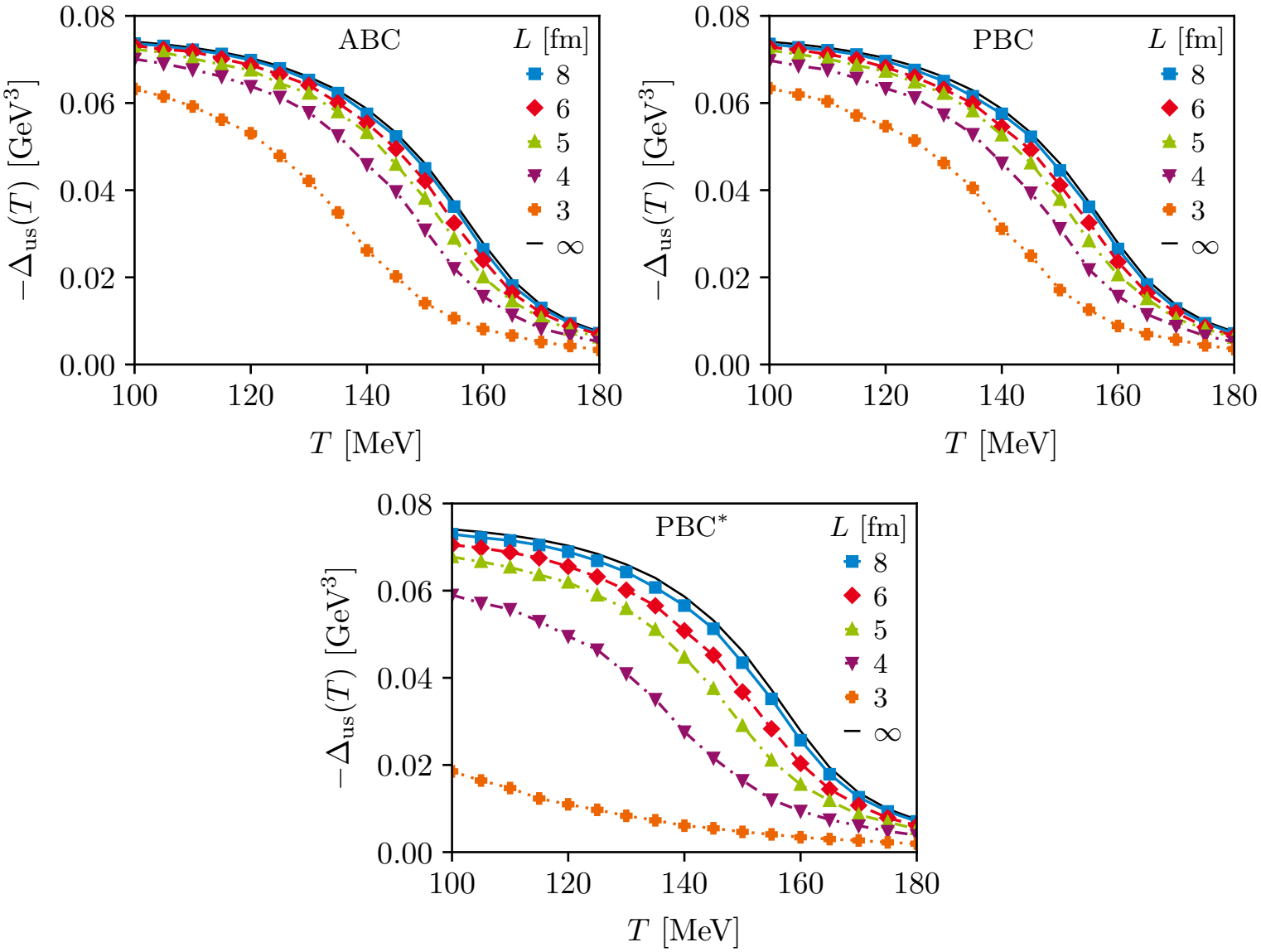
→ Can be improved by adding momentum integration in UV

JB, Fischer, Isserstedt, Schaefer (arXiv:2107.05504)

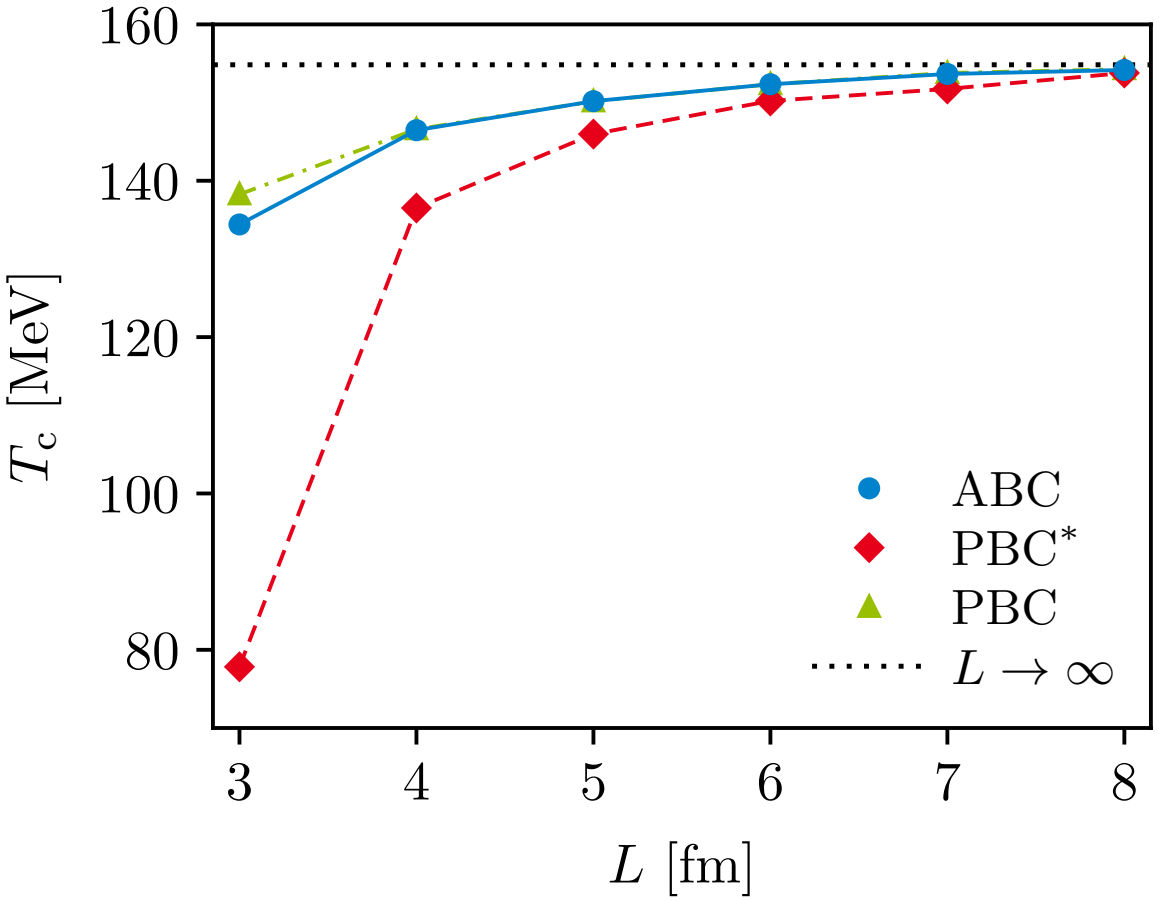
Results: Mass Function at $T = 130\,\mathrm{MeV}$ and $\mu_\mathrm{B} = 0$



Results: Subtracted Quark Condensate at $\mu_B = 0$

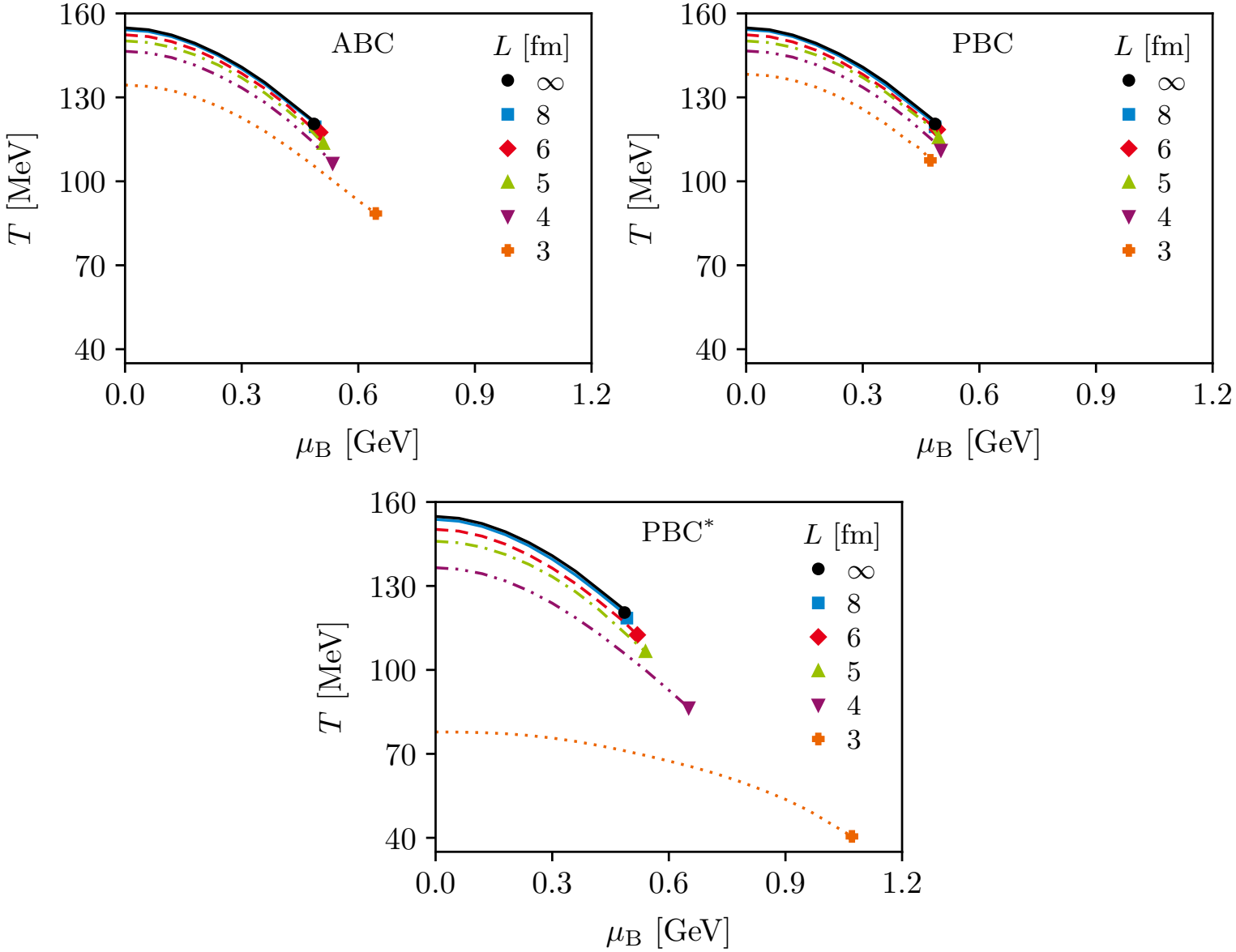


Results: Pseudocritical Temperature at $\mu_B = 0$



JB, Fischer, Isserstedt, Schaefer (arXiv:2107.05504)

Results: QCD Phase Diagram



JB, Fischer, Isserstedt, Schaefer (arXiv:2107.05504)

Reminder: Fluctuations

Fluctuations from QCD's grand-canonical potential

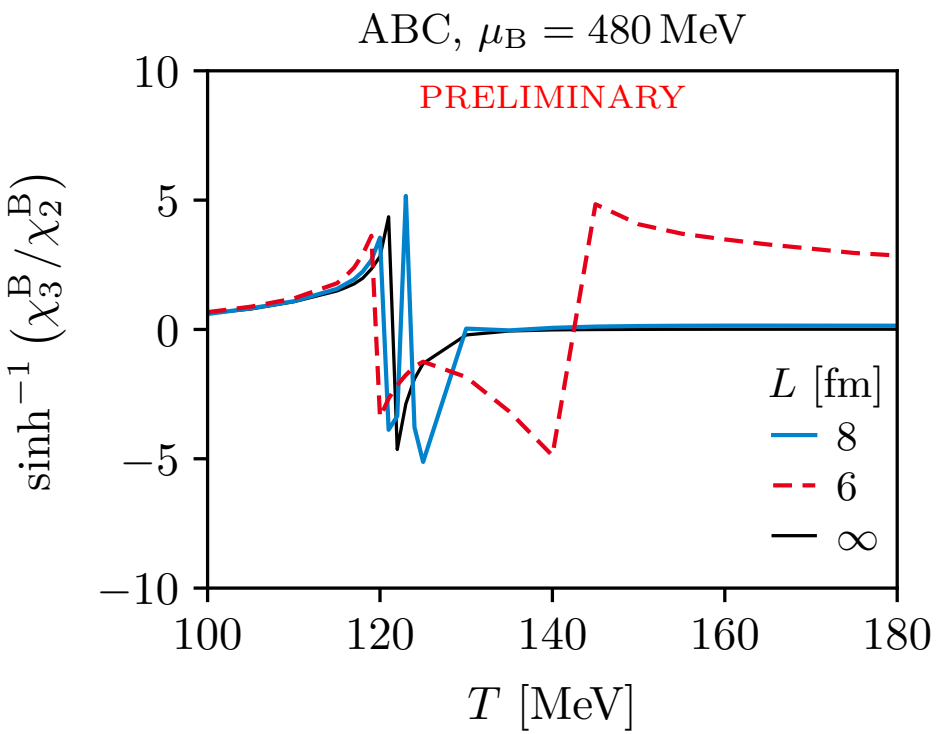
$$\chi_{ijk}^{\mathrm{uds}} = -\frac{1}{T^{4-(i+j+k)}} \frac{\partial^{i+j+k}\,\Omega}{\partial\mu_{\mathrm{u}}^{i}\,\partial\mu_{\mathrm{d}}^{j}\,\partial\mu_{\mathrm{s}}^{k}}$$

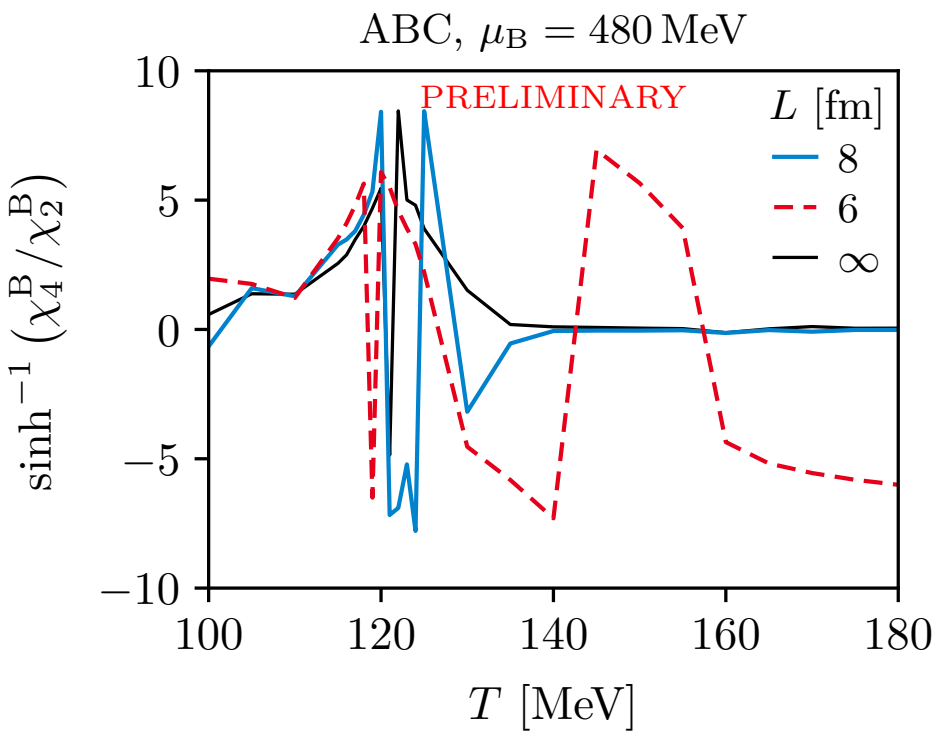
- Relation to cumulants of probability distribution: $C_n^X = V T^3 \chi_n^X$
- Statistical quantities: $M_X = C_1^X$, $S_X = C_3^X (C_2^X)^{-3/2}, \ldots$
- Prominent quantities are ratios:

$$\chi_3^{\mathrm{B}}/\chi_2^{\mathrm{B}} = \sigma_{\mathrm{B}}^2/M_{\mathrm{B}}\,, \qquad \chi_4^{\mathrm{B}}/\chi_2^{\mathrm{B}} = \kappa_{\mathrm{B}}\,\sigma_{\mathrm{B}}^2$$

Reviews: Luo, Xu, Nucl. Sci. Tech. 28 (2017) 112
Bzdak, Esumi, Koch, Liao, Stephanov, Xu, Phys. Rep. 853 (2020) 1

Preview: Baryon Number Fluctuation Ratios





Conclusion and Outlook

Conclusion:

- Studied finite-volume effects on QCD phase diagram using DSEs beyond rainbow–ladder truncation for ABC, PBC and PBC*
- Consistent infinite-volume limit
- Strong volume effects for $L \leq 4\,\mathrm{fm}$ (especially for PBC*)
- Qualitative agreement with FRG findings

Braun, Klein, Schaefer, PLB 713 (2012) 216-223
Tripolt, Braun, Klein, Schaefer, PRD 90 (2014) 054012

Outlook:

- Baryon number fluctuations in finite volume
- Spherical volume (MIT bag model)
- Spectral functions