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QCD in the heavy + dense regime

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CRC-TR 211 Strong-interaction matter under extreme conditions

NIC

A disclaimer…

“Anyone who wants to analyze the properties of matter in a real problem might want to start by writing down the fundamental equations and then try to solve them mathematically. Although there are people who try to use such an approach, these people are the failures in the field….”

Richard Feynman

That is the program pursued here…. necessary for results based on QCD!

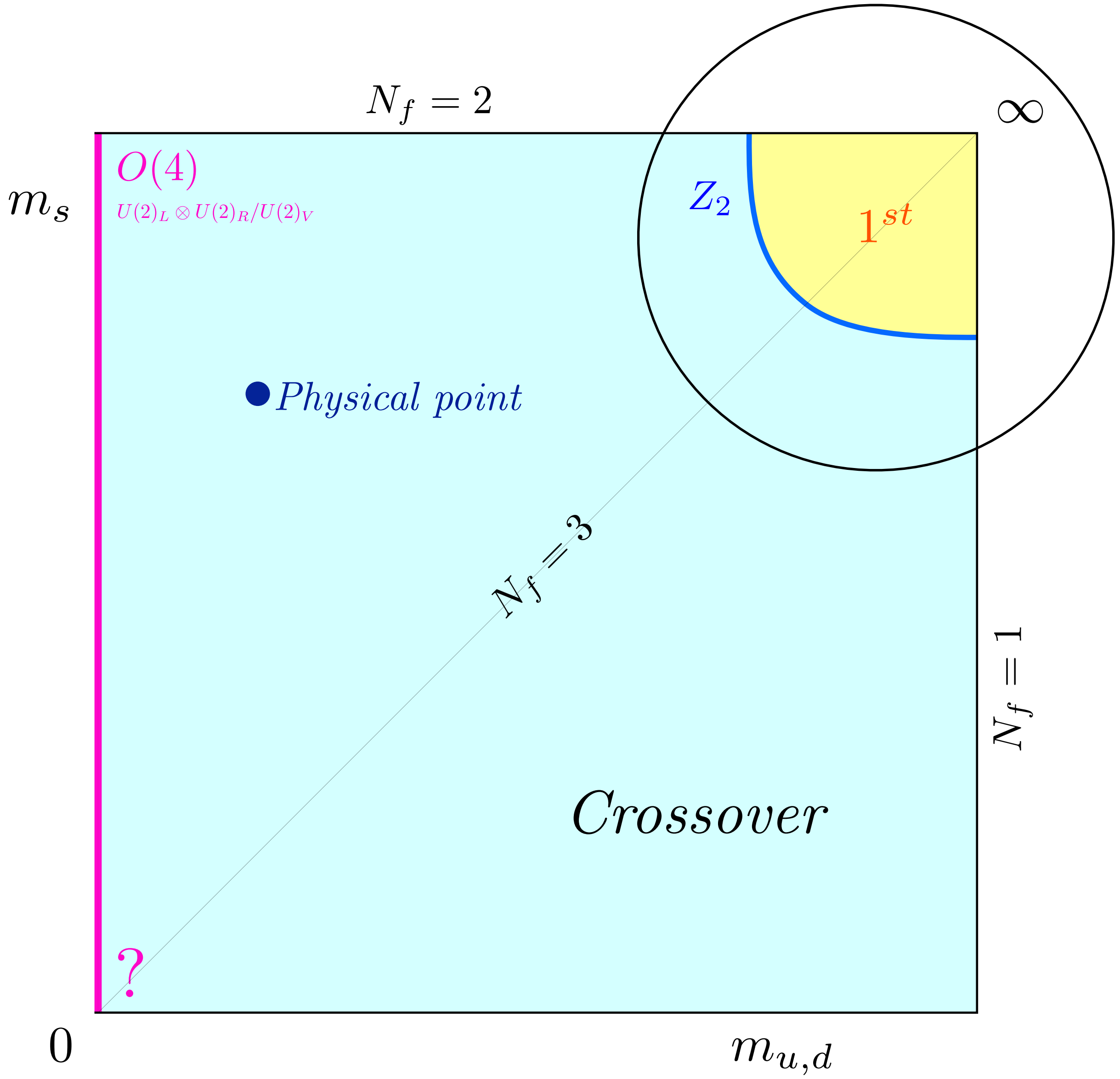
Goals here:

- Understand how bulk nuclear physics arises qualitatively from QCD
- Identify and constrain possible effective theories

Motivation: why heavy QCD?

- The sign problem of lattice QCD
- Static fermion determinant ($m = \infty$) known exactly: HDQCD
  better approximation than quenched, corrections computable;
  here: move also into finite mass plane
- Sign problem milder than in full QCD, MC possible
  [Blum, Hetrick, Toussaint, PRD 96]
- Testing ground for new algorithms
  (complex Langevin, density of states,…)
- Analytic approach viable (no sign problem)
- Interest in deconfinement transition:

  Disentangle Debye screening (medium) from screening by dynamical quarks (vacuum)



arxiv:2107.12739

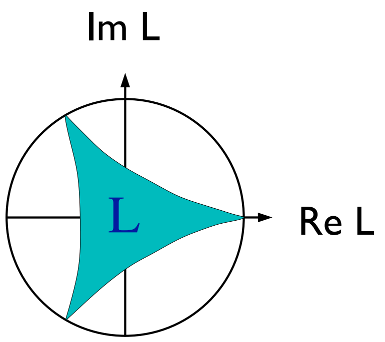
What is known at zero density?

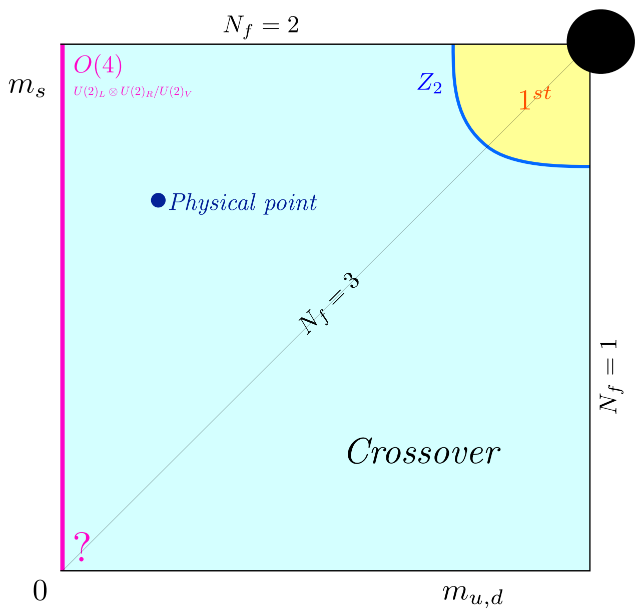
Pure gauge:

- Global Z(3) center symmetry
  Order parameter: Polyakov loop

$$\langle L \rangle = \frac{Z_Q}{Z} = \exp[-(F_Q - F_0)/T]$$

$$\langle L \rangle = 0 \Leftrightarrow F_Q = \infty$$





- Deconfinment transition: $\langle L \rangle \neq 0$

  spontaneous breaking of Z(3) center symmetry

- $T_c \approx 270 - \underline{300}\ \mathrm{MeV}$

  (uncertainty from scale setting, not calculation), equation of state [Boyd et al., NPB 96]

- Latent heat: [WHOT, PTEP 21] $\dfrac{\Delta\epsilon}{T^4} = 0.95(7)$

Including dynamical quarks

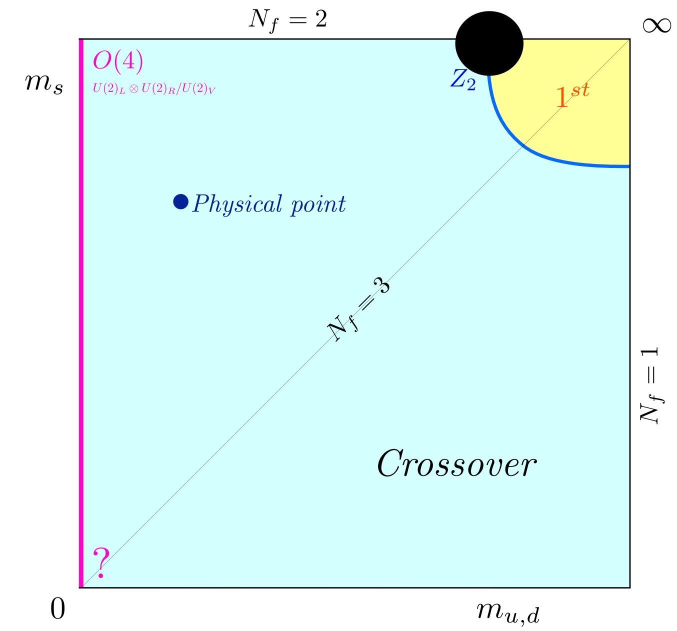
- Z(3) symmetry explicitly broken by $\frac{1}{m_q}$

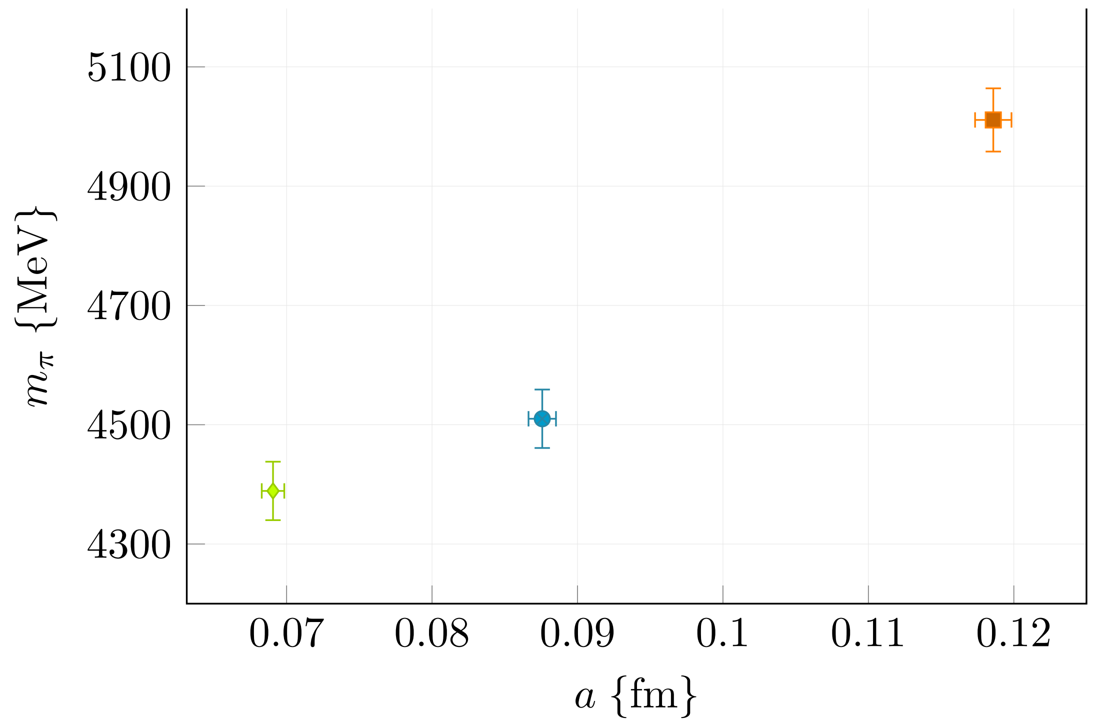
  $\langle L \rangle \neq 0$ always!

- Deconfinement transition weakens, disappears at $\frac{1}{m_q^c} \Leftrightarrow m_\pi^c$

- Lattice determination in progress: $m_\pi^c \approx 4$ GeV
  [WHOT, Frankfurt]

- Dyson-Schwinger study $m_q^c \approx 460$ MeV
  [Fischer, Luecker, Pawlowski]





Cuteri, O.P., Schön, Sciarra, PRD 21

Effective lattice theory for heavy + dense QCD

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: split temporal and spatial link integrations:

$$Z = \int DU_0 DU_i \; \det Q \; e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL \; e^{-S_{eff}[L]}$$

  Spatial integration after analytic strong coupling and hopping expansion $\sim \frac{1}{g^2}, \frac{1}{m_q}$

- Step II.: mild sign problem of effective theory

- Analytic solution by linked cluster expansion

Starting point: Wilson's lattice action

Pure gauge part: character expansion

$$u(\beta) = \frac{\beta}{18} + \frac{\beta^2}{216} + \ldots < 1$$

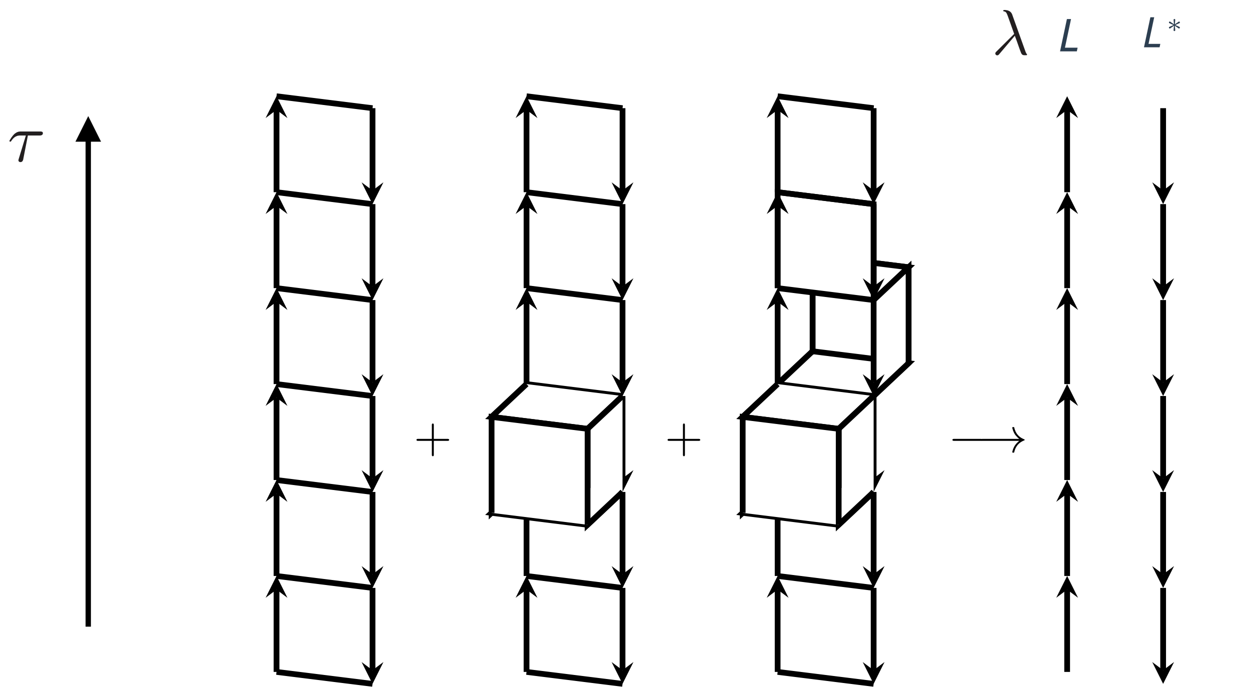
$$\beta = \frac{2N_c}{g^2} \qquad T = \frac{1}{aN_\tau}$$

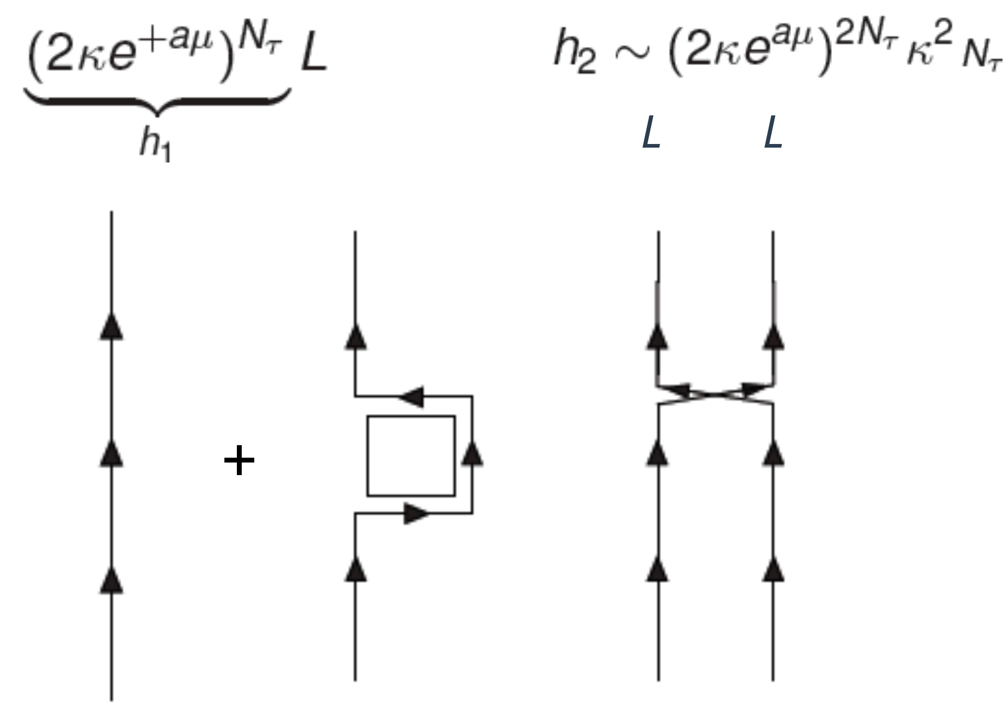
Fermion determinant: hopping expansion

$$\kappa = \frac{1}{2am+8}$$

both convergent series expansions!

Generates couplings over all distances, n-pt. couplings, higher reps…. :





$$\lambda(u, N_\tau \geq 5) \; = \; u^{N_\tau} \exp\left[N_\tau\left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1055797}{5120}u^{10}\right)\right]$$

Effective theory with leading interactions

$$Z = \int \mathcal{D}W \prod_{\langle \mathbf{x},\mathbf{y}\rangle} \left[1 + \lambda(L(\mathbf{x})L(\mathbf{y})^* + L(\mathbf{x})^* L(\mathbf{y}))\right]$$ pure gauge

$$\times \prod_{\mathbf{x}} \left[1 + h_1 L(\mathbf{x}) + h_1^2 L(\mathbf{x})^* + h_1^3\right]^{2N_f} \left[1 + \bar{h}_1 L(\mathbf{x}) + \bar{h}_1^2 L(\mathbf{x})^* + \bar{h}_1^3\right]^{2N_f}$$ stat. det.

$$\times \prod_{\langle \mathbf{x},\mathbf{y}\rangle} \left[1 - h_2 N_f \,\mathrm{tr}\left(\frac{h_1 W(\mathbf{x})}{1 + h_1 W(\mathbf{x})}\right) \mathrm{tr}\left(\frac{h_1 W(\mathbf{y})}{1 + h_1 W(\mathbf{y})}\right)\right]$$ kinetic det.

$$\times \left[1 - h_2 N_f \,\mathrm{tr}\left(\frac{\bar{h}_1 W(\mathbf{x})^\dagger}{1 + \bar{h}_1 W(\mathbf{x})^\dagger}\right) \mathrm{tr}\left(\frac{\bar{h}_1 W(\mathbf{y})^\dagger}{1 + \bar{h}_1 W(\mathbf{y})^\dagger}\right)\right] \quad \ldots.$$

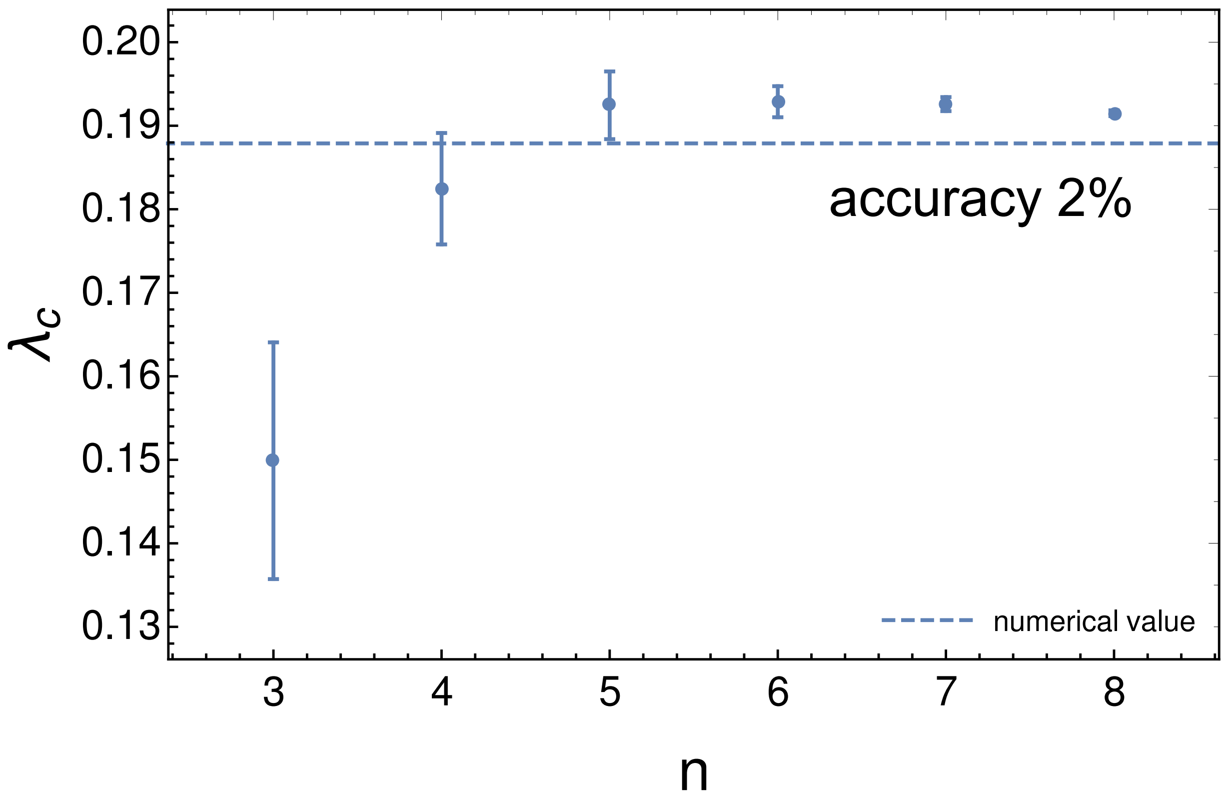
$$W(\mathbf{x}) = \prod_{\tau=0}^{N_\tau - 1} U_0(\tau, \mathbf{x}), \quad L(\mathbf{x}) = \mathrm{tr}(W(\mathbf{x})), \quad \mathcal{D}W = \prod_{\mathbf{x}\in\Lambda_s} \mathrm{d}W(x)\,.$$

This is a 3d continuous spin model!

- mild sign problem, MC simulable
- series expansion in eff. couplings

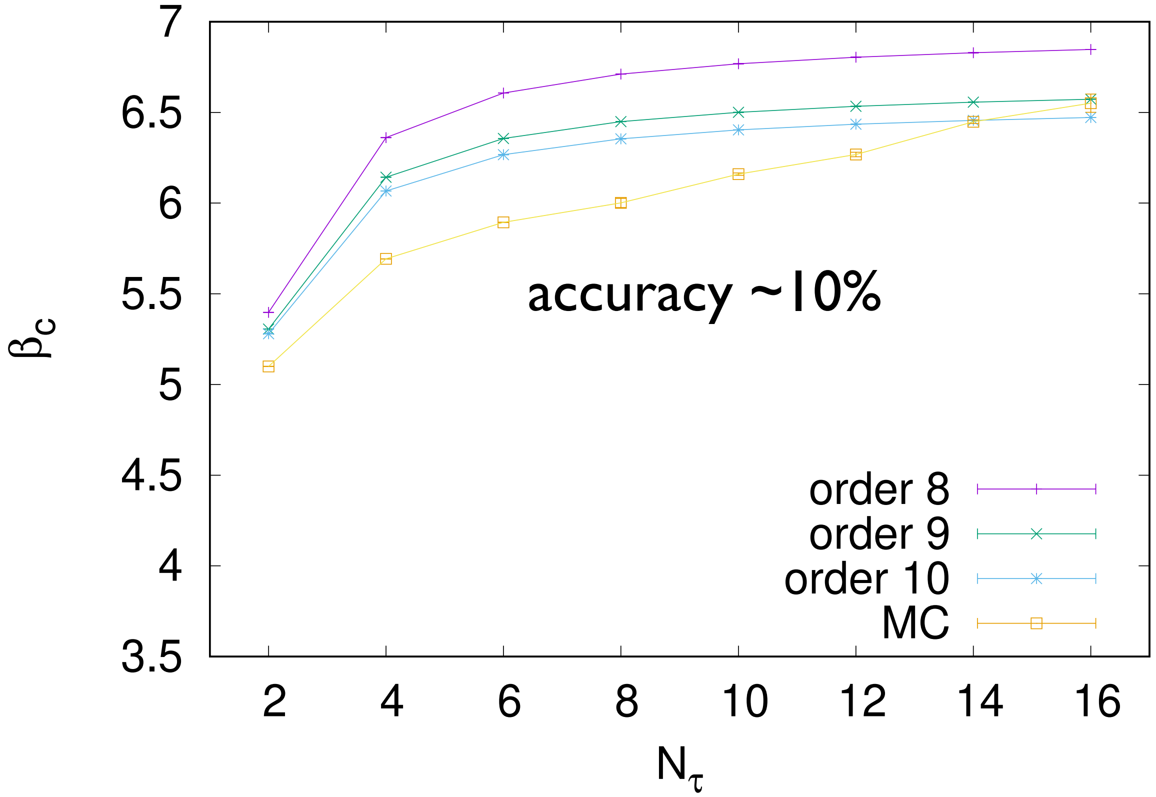
Yang-Mills deconfinement trans. from series expansion

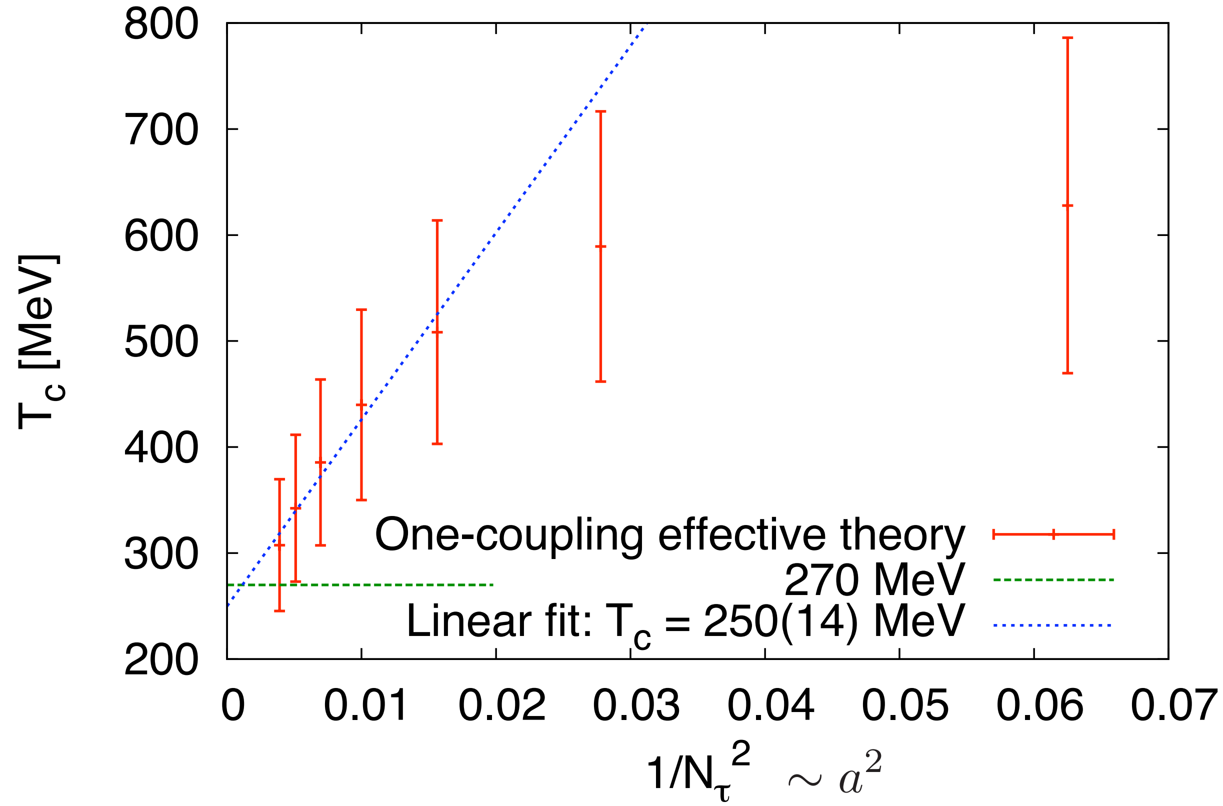
Solution of effective theory



order of expansion

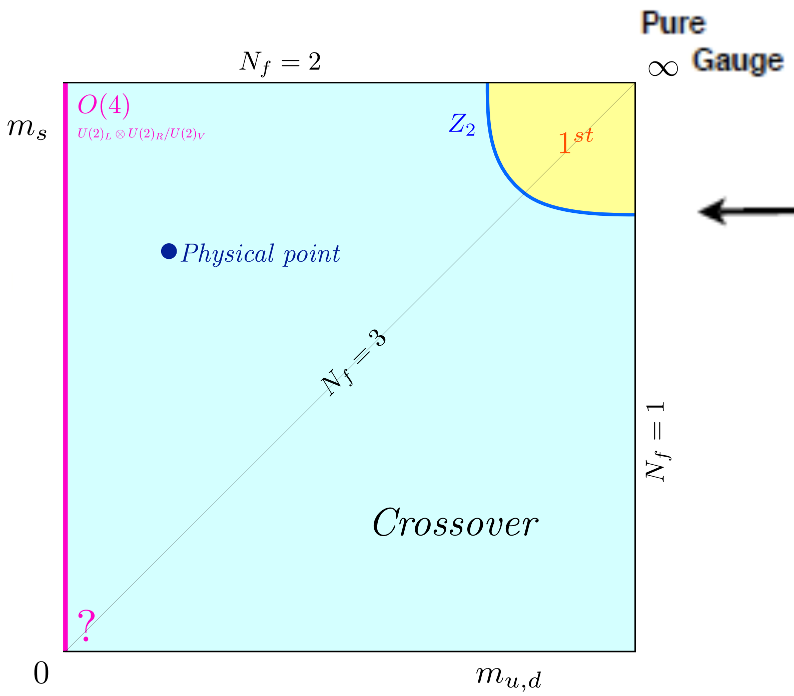
Conversion to 4d YM





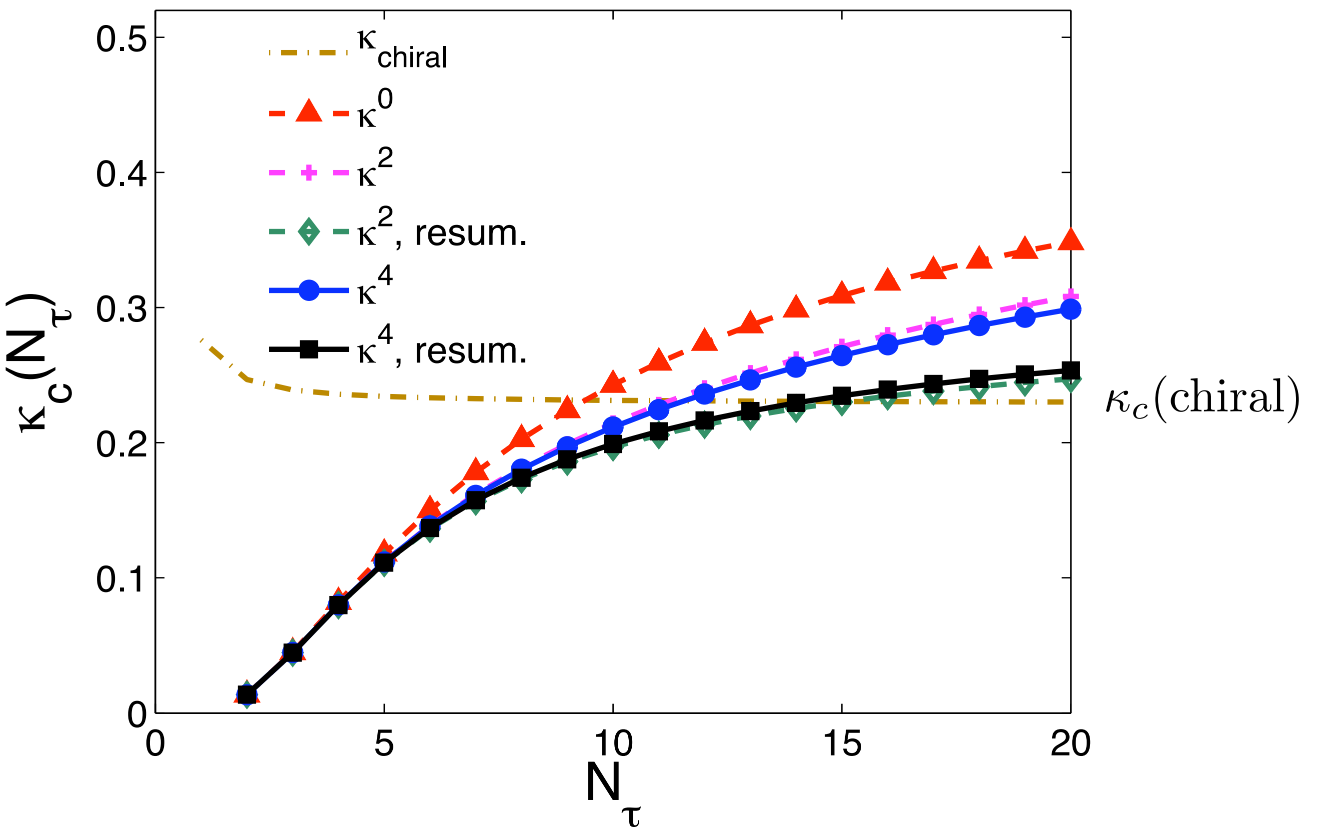
Two calculations:

1. by "hand" (Q. Pham, J. Scheunert, GU)
2. automatic graph generation (J. Kim, GU)



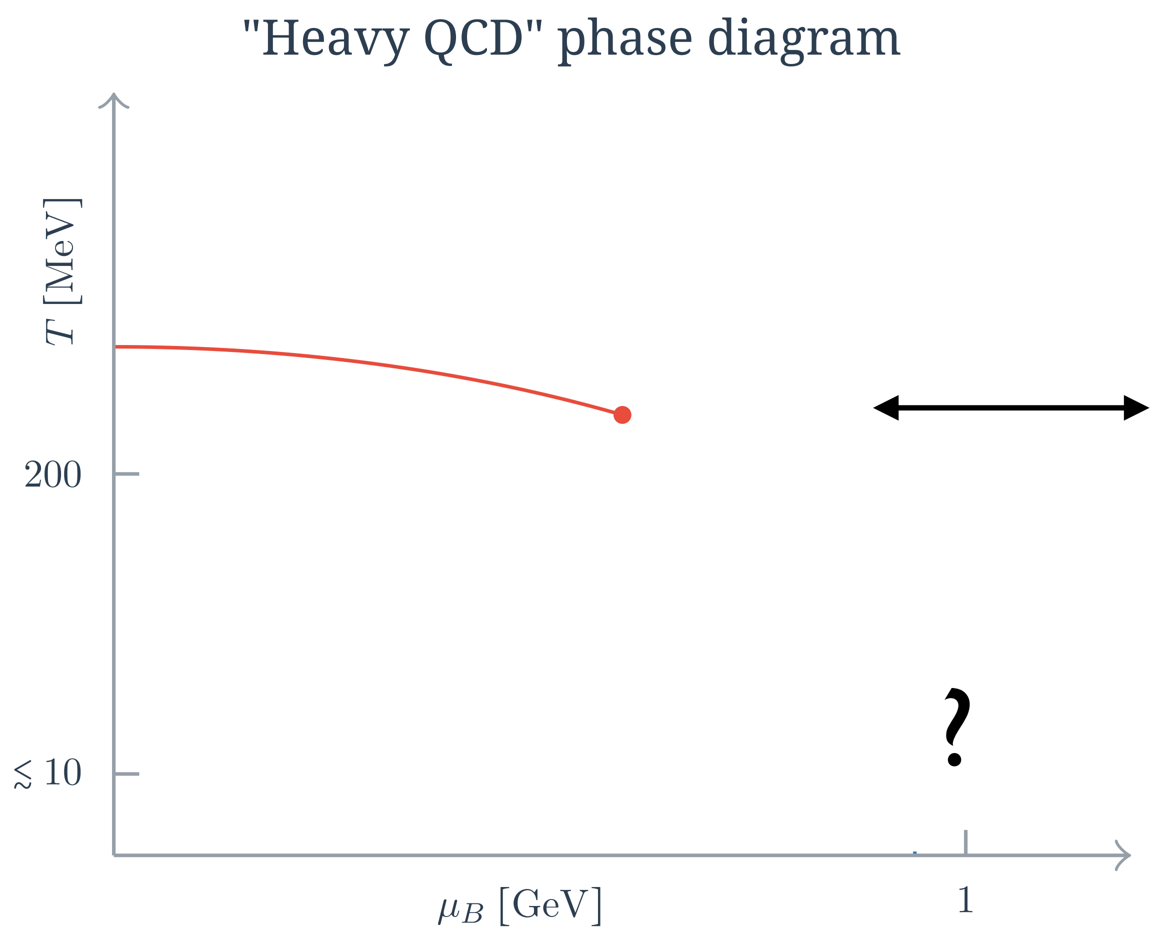
		eff. theory	4d MC, WHOT	4d MC, de Forcrand et al
N_f	M_c/T	$\kappa_c(N_\tau = 4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
1	7.22(5)	0.0822(11)	0.0783(4)	~ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

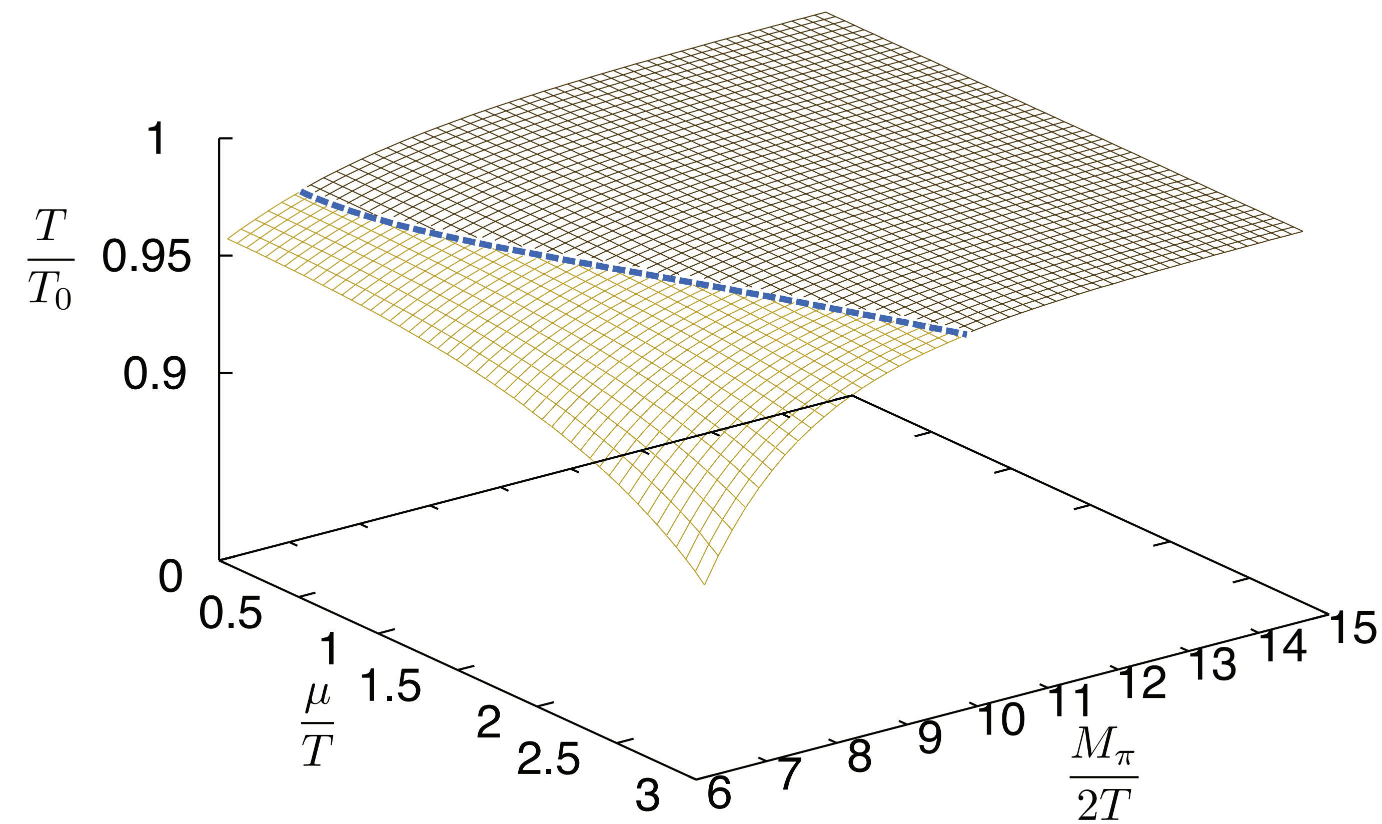
- Now deal with double series
- On coarse Nt=4 lattices good accuracy
- Finer lattices need higher orders
  + more couplings,
  automatisation required



Deconfinement transition at finite density

[Fromm, Langelage, Lottini, O.P. JHEP (2012)]





Same phase structure: continuum effective Polyakov loop theories, benchmarking possible!

[Fischer, Lücker, Pawlowski PRD (2015); Lo, Friman, Redlich PRD (2014)]

Cold and dense QCD: static strong coupling limit

[Fromm, Langelage, Lottini, Neuman, O.P. , PRL (2013)]

T=0: anti-fermions decouple:

$$\begin{aligned} h_1 &= (2\kappa e^{a\mu})^{N_\tau} = e^{\frac{\mu - m}{T}} \\ \bar{h}_1 &= (2\kappa e^{-a\mu})^{N_\tau} = e^{\frac{-\mu - m}{T}} \end{aligned}$$

$$Z(\beta=0) \xrightarrow{T\to 0} \left[\prod_f \int dW (1 + h_1 L + h_1^2 L^* + h_1^3)^2\right]^V = z_0^V$$

$$N_f = 1: \; z_0 = 1 + \underset{\uparrow}{4h_1^3} + \underset{\uparrow}{h_1^6}$$

spin 3/2, 0

free baryon gas (HRG) emerges!

cf. finite T [Langelage, O.P. JHEP (2010)]

Silver blaze phenomenon + Pauli principle:

$$\lim_{T\to 0} a^3 n = \begin{cases} 0, & \mu < m \\ 2N_c, & \mu > m \end{cases}$$

1st order phase transition from vacuum to saturated quark crystal

$N_f = 2:$ The baryon gas (or liquid)

Δ^- $\quad 2n + 4\Delta^0 \quad 2p + 4\Delta^+ \quad \Delta^{++}$

$$z_0 = (1 + 4h_d^3 + h_d^6) + (6h_d^2 + 4h_d^5)h_u + (6h_d + 10h_d^4)h_u^2 + (4 + 20h_d^3 + 4h_d^6)h_u^3$$
$$+(10h_d^2 + 6h_d^5)h_u^4 + (4h_d + 6h_d^4)h_u^5 + (1 + 4h_d^3 + h_d^6)h_u^6$$

"Di-baryons": 3 spin 1 triplets, 1 spin 0 singlet, $\Delta^{++}\Delta^0$, $\quad pp$

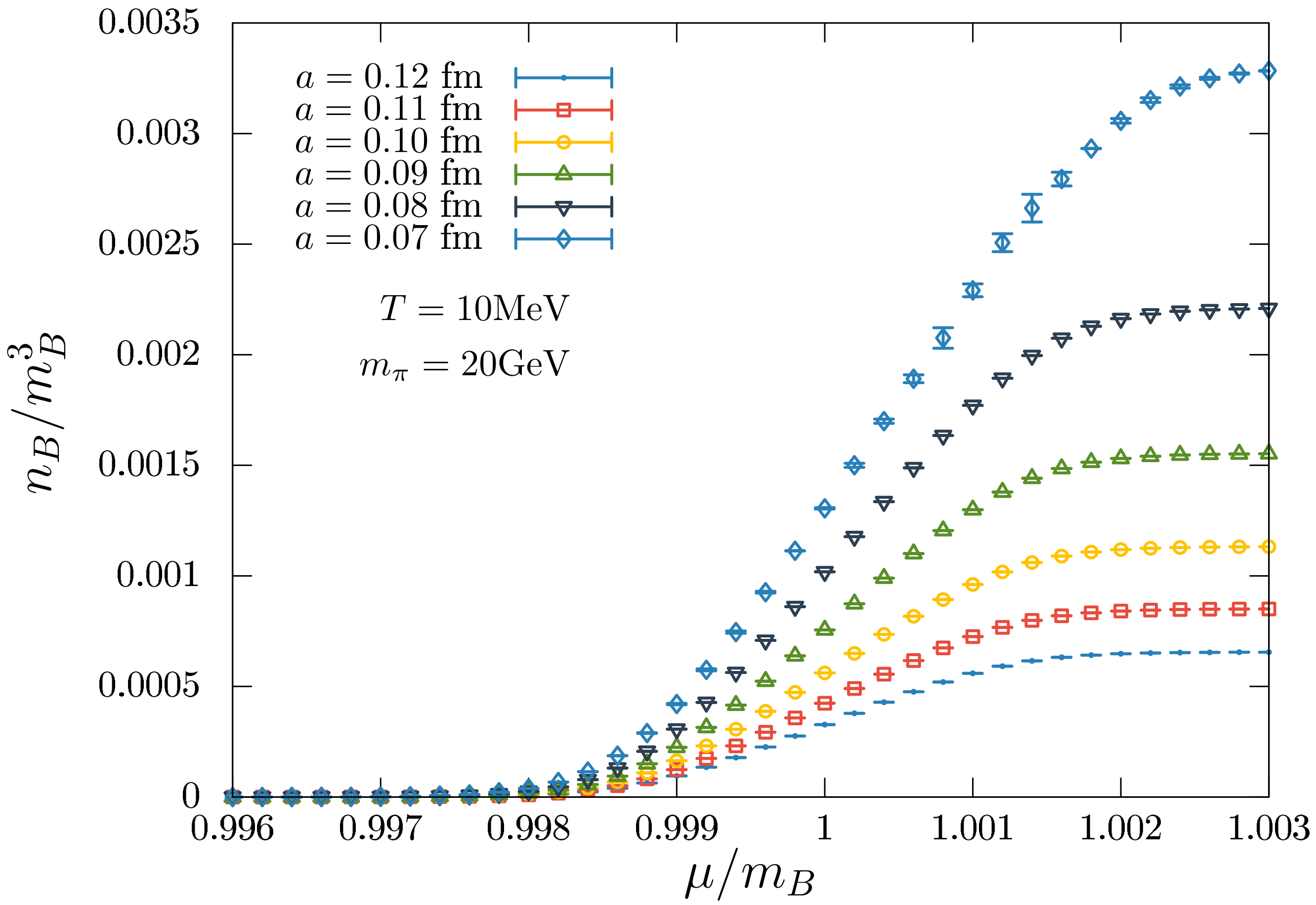
Complete spin-flavour structure of baryons (mesons for finite T or isospin chemical potential)

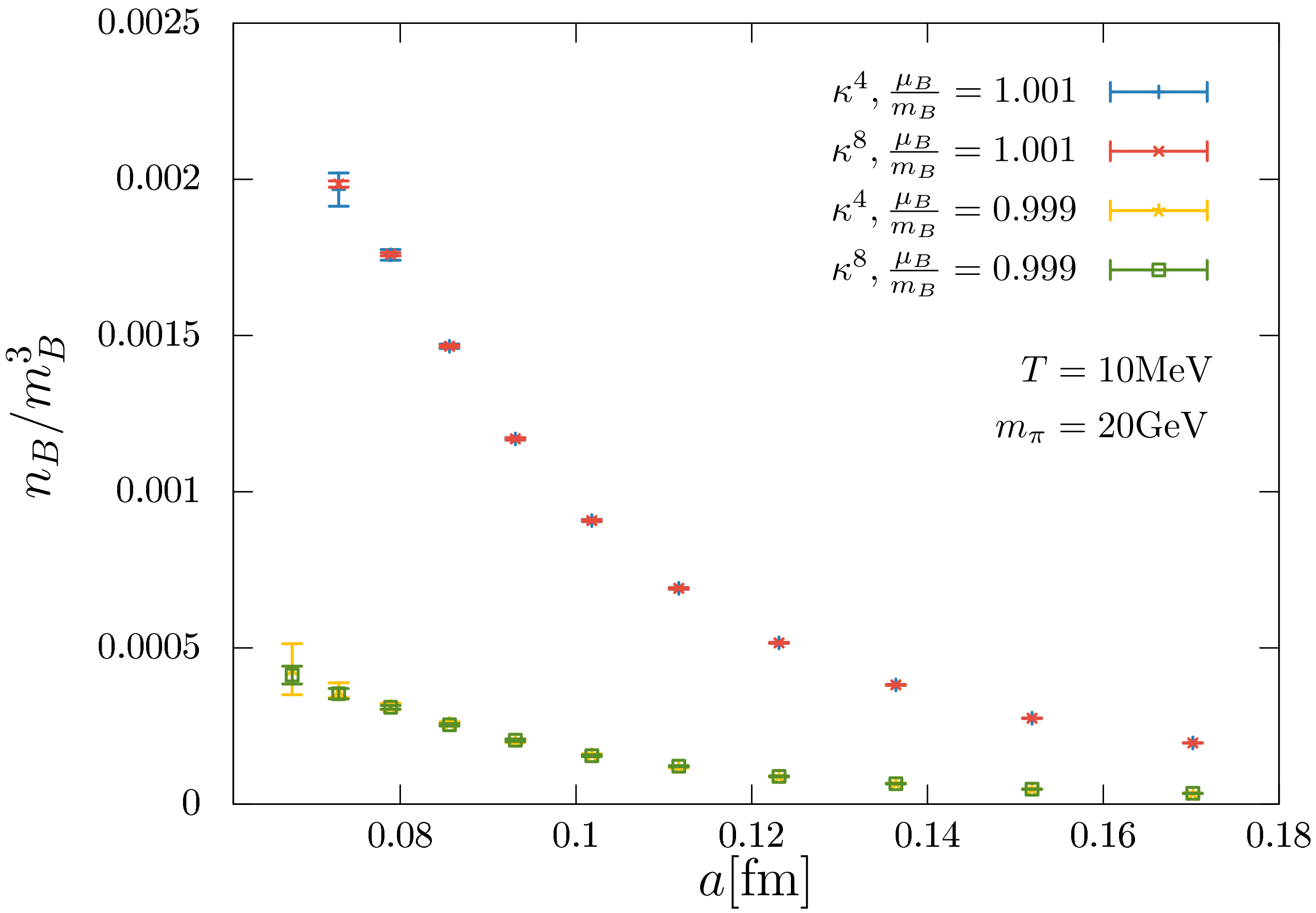
Gauge and Lorentz symmetries!

Cold and dense regime: onset of baryon matter

Accuracy: $\sim u^5 \kappa^8$

[Glesaaen, Neuman, O.P., JHEP 15]

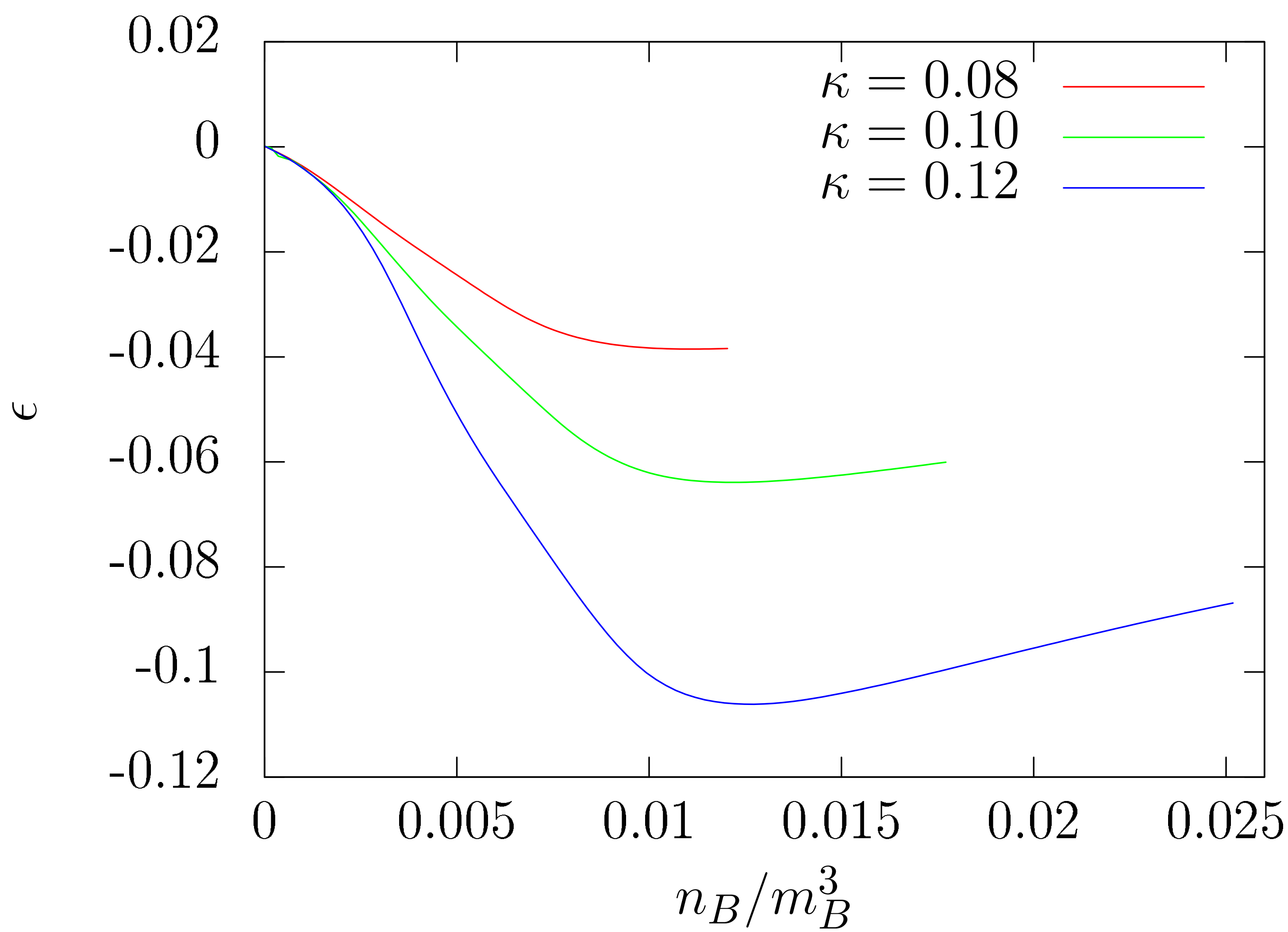




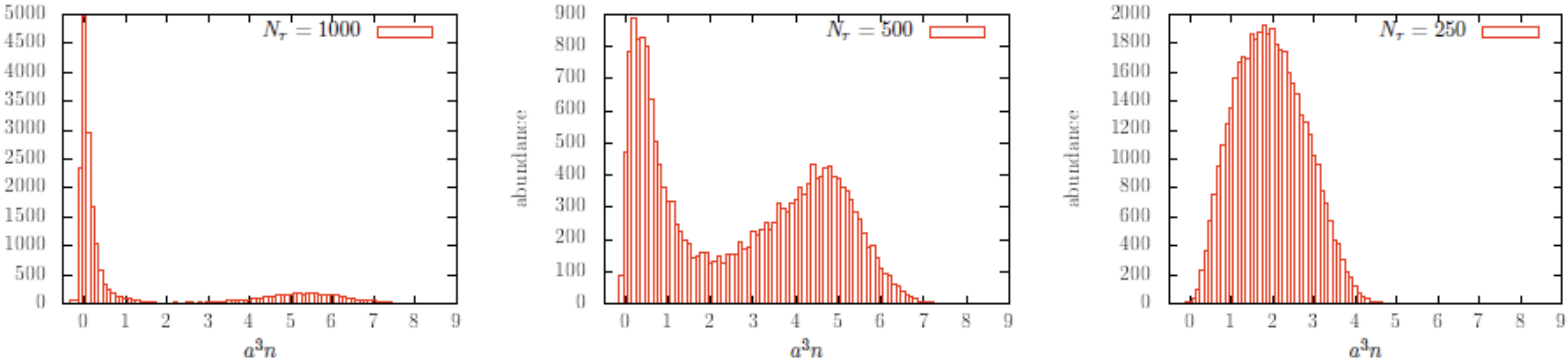
- Continuum approach ~a as expected for Wilson fermions
- Cut-off effects grow rapidly beyond onset transition: lattice saturation!
- Finer lattice necessary for larger density!

Binding energy per baryon, strong coupling limit

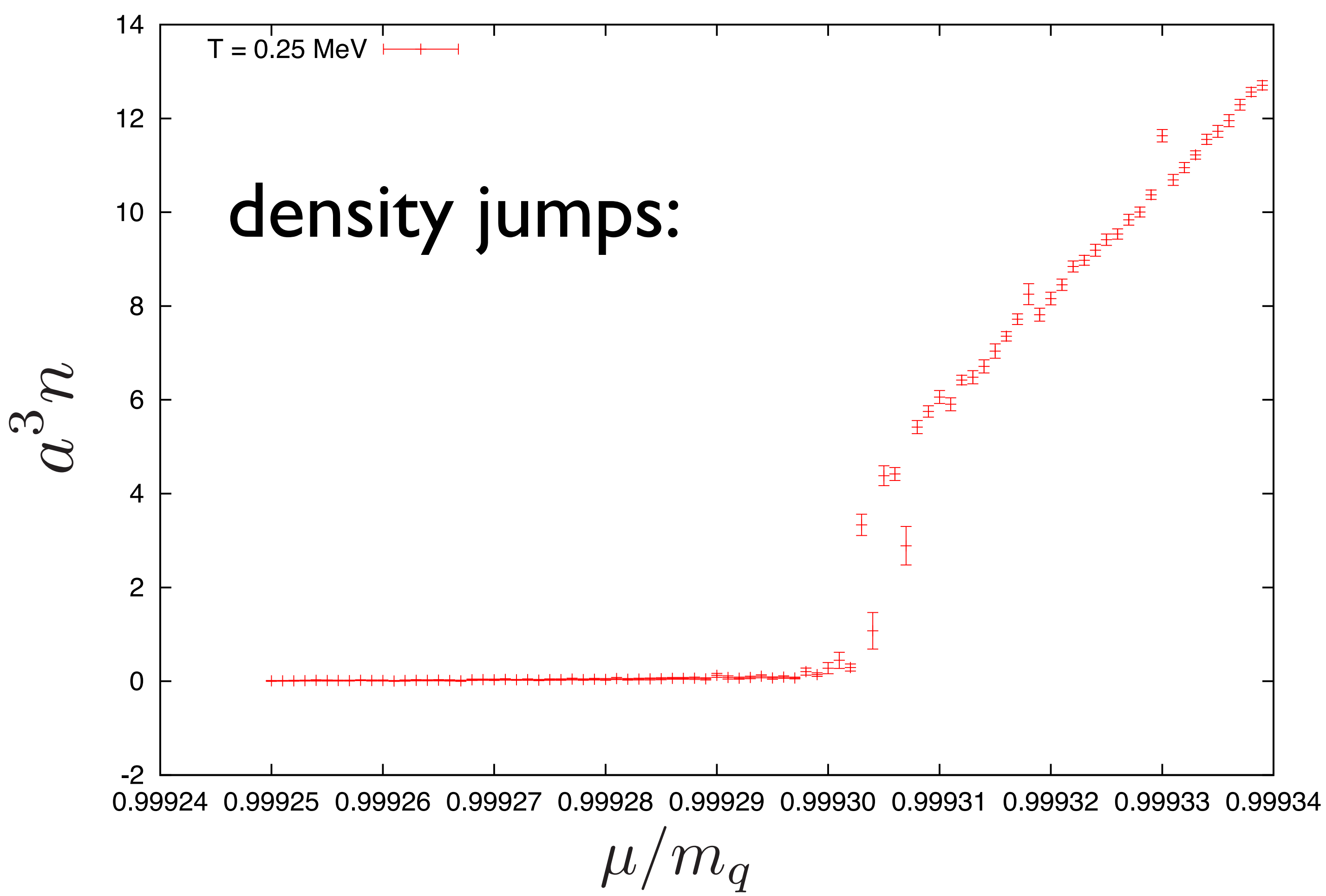
$$\epsilon \equiv \frac{e - n_B m_B}{n_B m_B} \stackrel{LO}{=} -\frac{4}{3}\frac{1}{a^3 n_B}\left(\frac{z_3}{z_0}\right)^2 \kappa^2$$



Light quarks: 1st order onset transition + endpoint

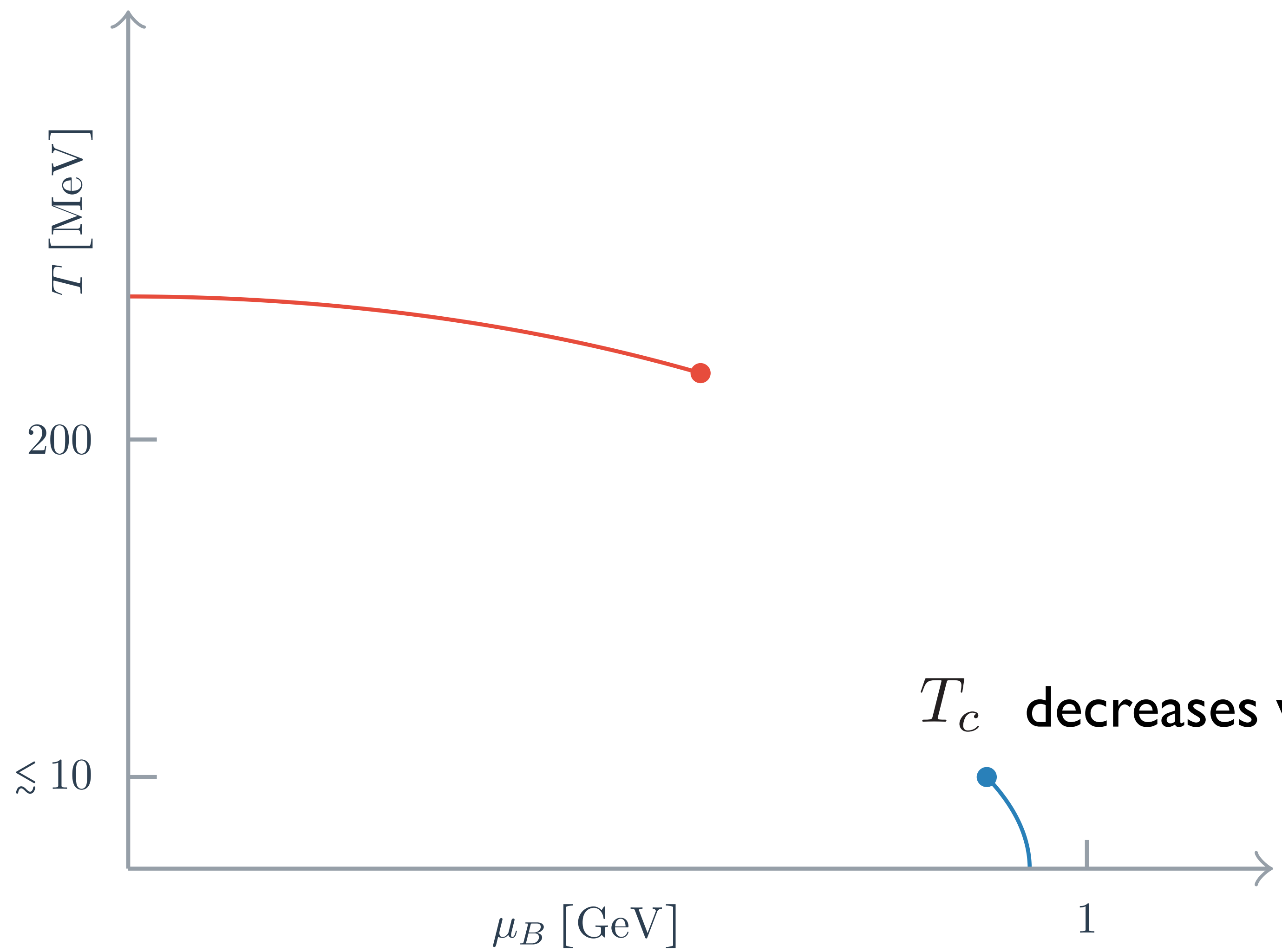


$\kappa = 0.12$



- phase coexistence: first order
- for higher $T = \frac{1}{aN_\tau}$ crossover
- nuclear liquid gas transition!

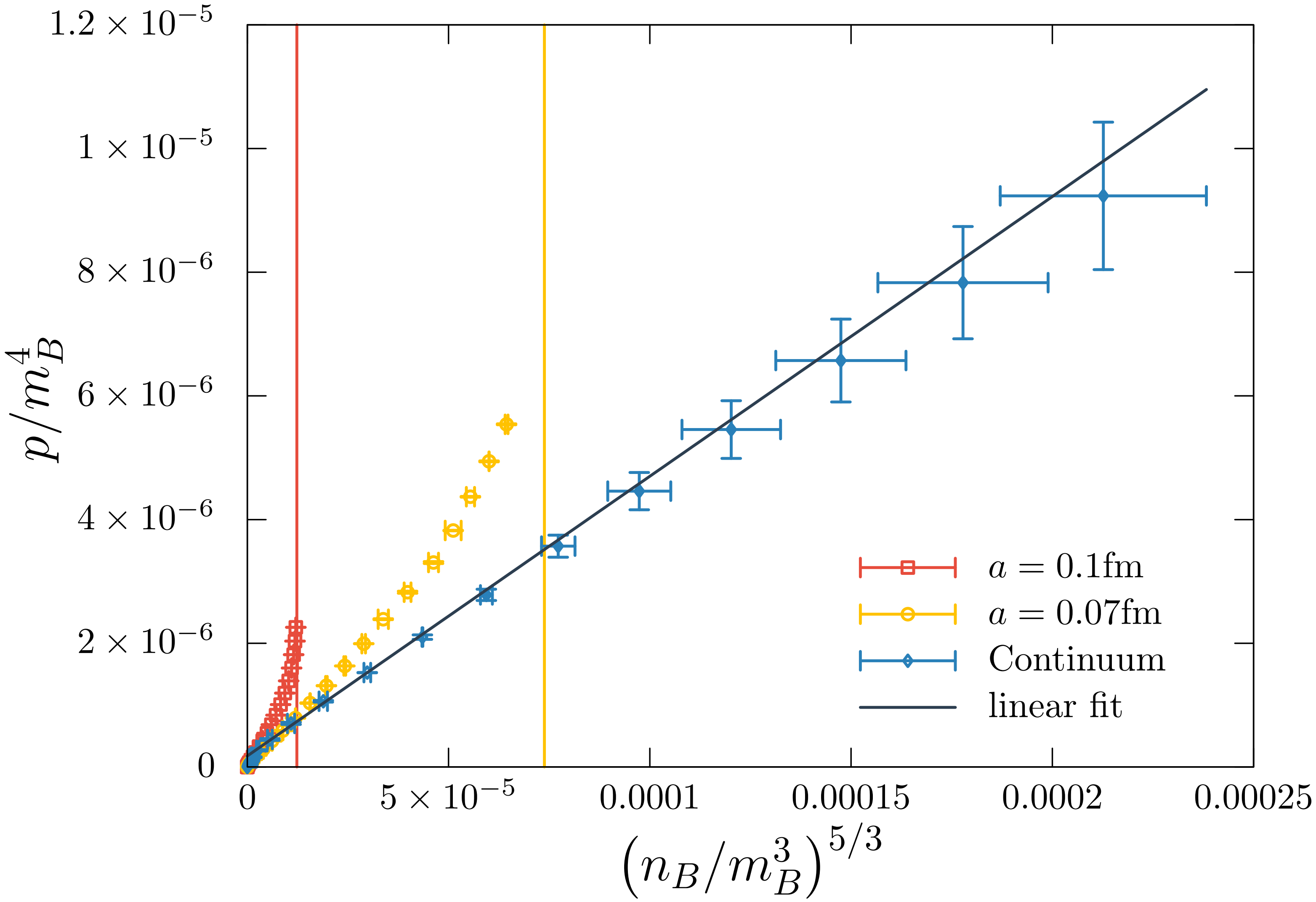
Phase diagram of heavy QCD



T_c decreases with growing quark mass

Equation of state for heavy baryon matter

[Glesaaen, Neuman, O.P., JHEP 15]



- EoS fitted by polytrope, non-relativistic fermions!
- Can we understand the pre-factor? Interactions, mass-dependence…

QCD for a large number of colours

Definition, 't Hooft 1974 : $N_c \longrightarrow \infty, \quad g^2 N_c = const.$

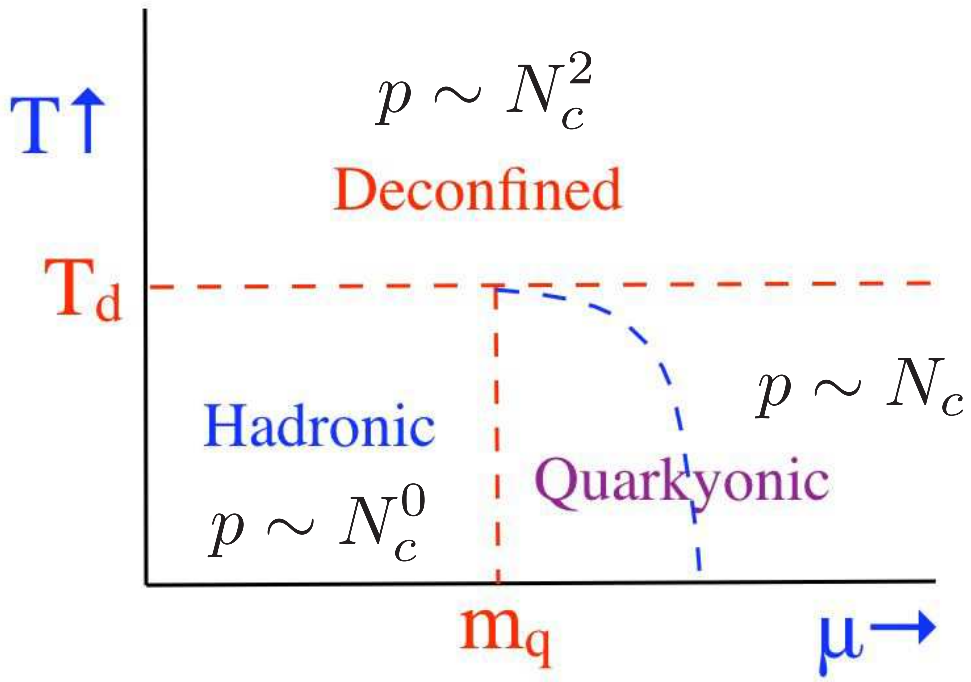
- suppresses quark loops in Feynman diagrams
- mesons are free;
  corrections: cubic interactions $\sim 1/\sqrt{N_c}$, quartic int. $\sim 1/N_c$
- meson masses $\sim \Lambda_{QCD}$
- baryons: N_c quarks, baryon masses $\sim N_c \Lambda_{QCD}$
- baryon interactions: $\sim N_c$

Witten 1979

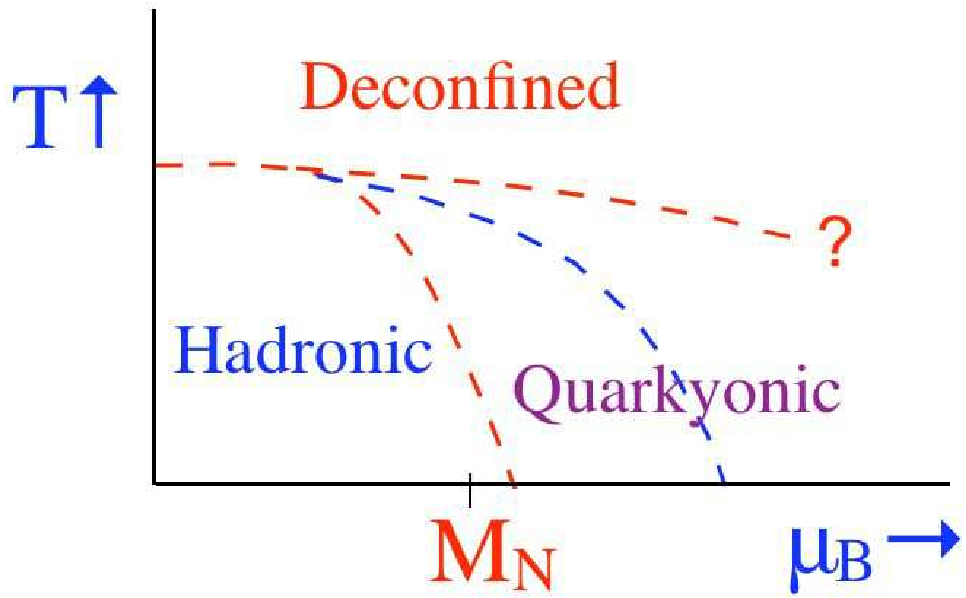
Conjectures for the QCD phase diagram

Pictures from [McLerran, Pisarski Nucl.Phys.A 796 (2007)]

large N_c



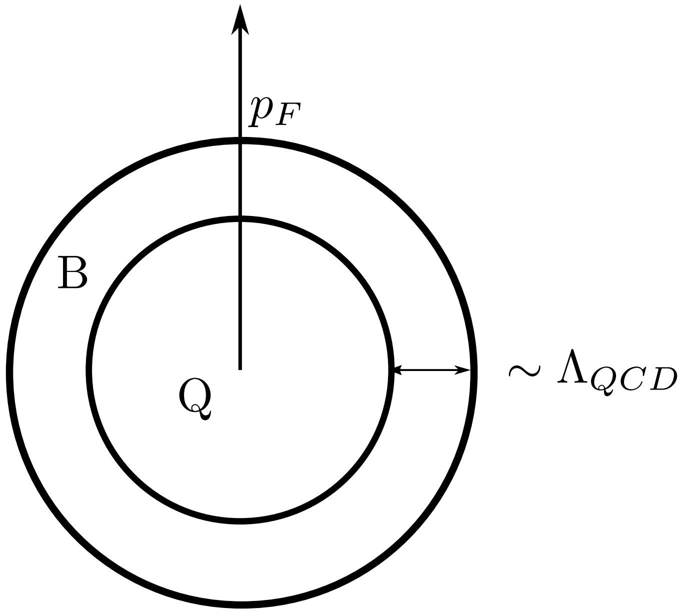
$N_c = 3$



Quarkyonic matter in momentum space:

Fermi sea of quarks, surrounded by Fermi shell of baryons;

$p_F \sim \mu$ interpolates from purely baryonic to quark matter



From conjecture to calculation

[O.P., Scheunert JHEP (2019)]

Investigate N_c - dependence in effective theory

"In principle" straight-forward: use character expansion for general N_c and recompute all integrals for general N_c, without approximation

For example, static strong coupling limit, baryon gas:

$$Z(\beta = 0) = \left(1 + (N_c + 1)h_1^{N_c} + h_1^{2N_c}\right)^V$$

Baryonic spin degeneracy depends on number of colours!

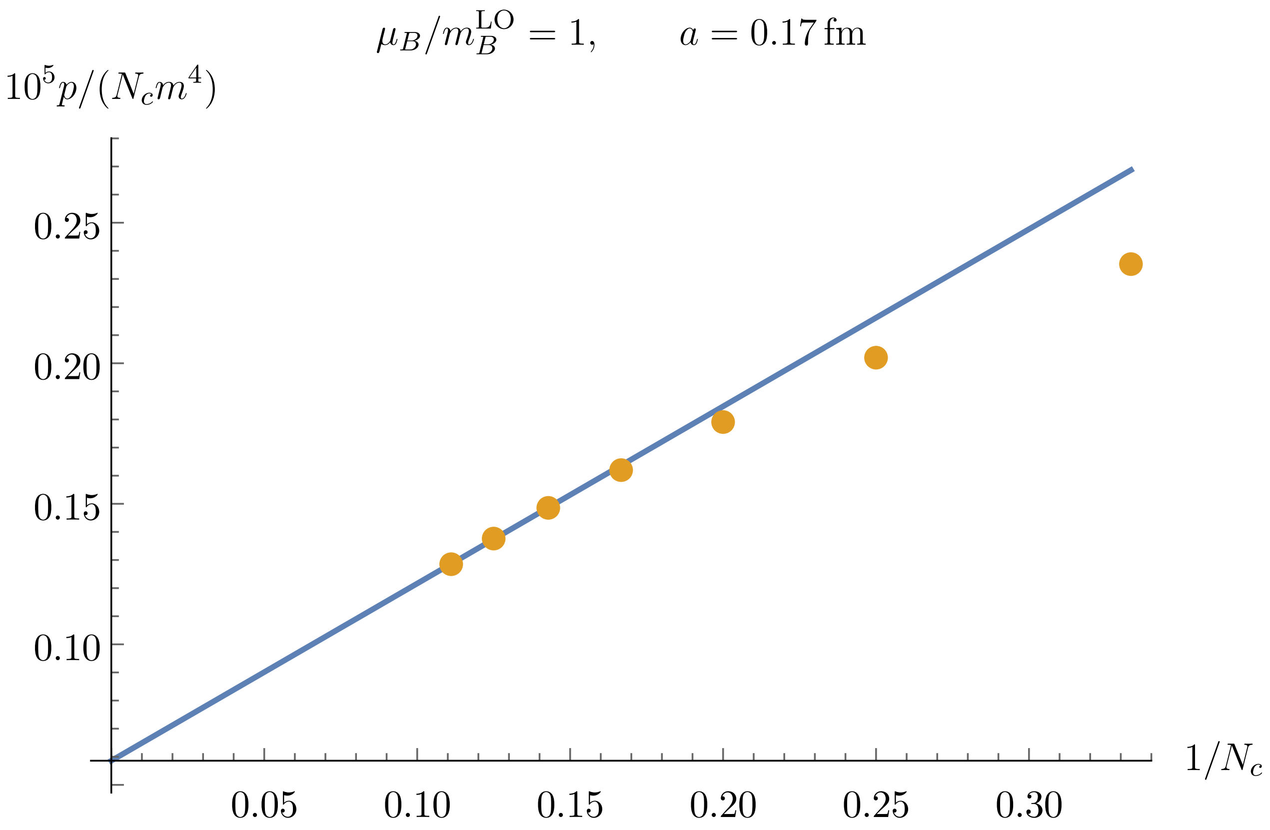
Thermodynamic functions for large Nc

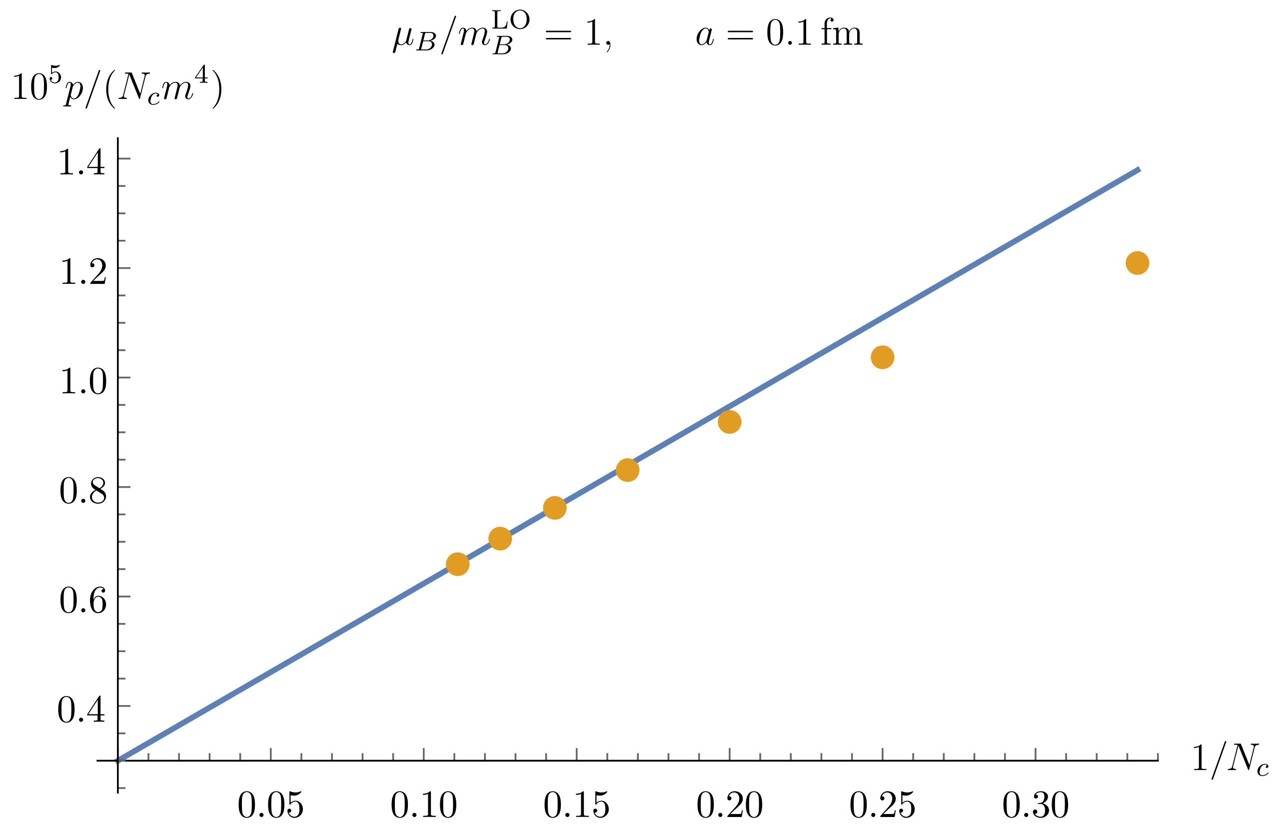
Strong coupling limit:

Order hopping expansion		κ^0	κ^2	κ^4
$h_1 < 1$	$a^4 p$	$\sim \frac{1}{6N_\tau} N_c^3 h_1^{N_c}$	$\sim -\frac{1}{48} N_c^7 h_1^{2N_c}$	$\sim \frac{3N_\tau \kappa^4}{800} N_c^8 h_1^{2N_c}$
	$a^3 n_B$	$\sim \frac{1}{6} N_c^3 h_1^{N_c}$	$\sim -\frac{N_\tau}{24} N_c^7 h_1^{2N_c}$	$\sim \frac{(9N_\tau+1)N_\tau}{1200} N_c^8 h_1^{2N_c}$
	$a^4 e$	$\sim -\frac{\ln(2\kappa)}{6} N_c^4 h_1^{N_c}$	$\sim \frac{N_\tau \ln(2\kappa)}{48} N_c^8 h_1^{2N_c}$	
	ϵ	0	$\sim -\frac{1}{4} N_c^3 h_1^{N_c}$	
$h_1 > 1$	$a^4 p$	$\sim \frac{4\ln(h_1)}{N_\tau} N_c$	$\sim -12 N_c$	$\sim 198 N_c$
	$a^3 n_B$	~ 4	$\sim -N_\tau \frac{N_c^4}{h_1^{N_c}}$	$\sim -\frac{(59N_\tau - 19)N_\tau}{20} \frac{N_c^5}{h_1^{N_c}}$
	$a^4 e$	$\sim -4\ln(2\kappa) N_c$	$\sim 24 \ln(2\kappa) N_c$	
	ϵ	0	~ -6	

Beyond the onset transition: $p \sim N_c$ definition of quarkyonic matter!

Pressure scaling, including gauge corrections



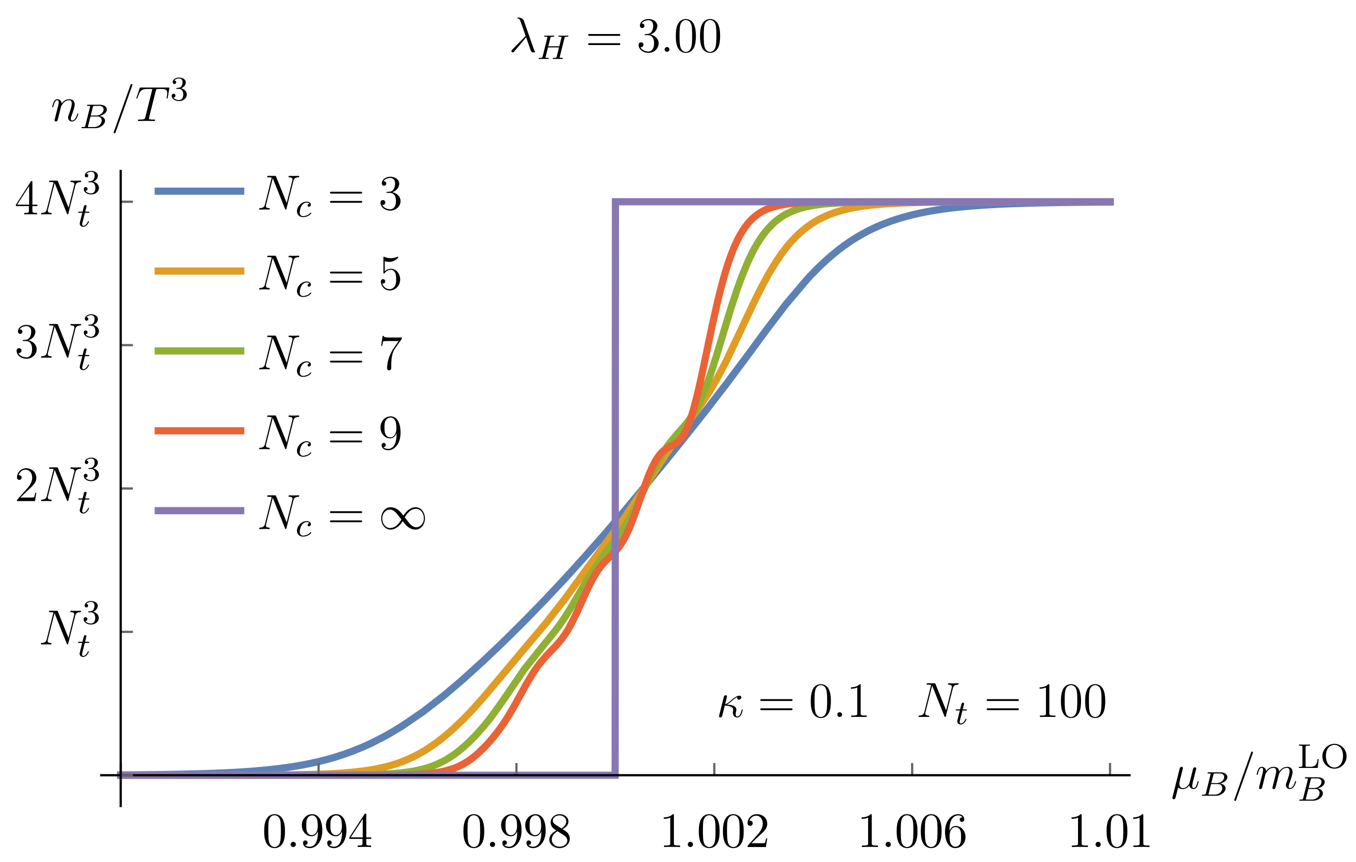


$$p \sim N_c(1 + \mathrm{const.} N_c^{-1})$$

Baryon onset with growing Nc

$$u(\beta) = \frac{1}{\lambda_H} = \frac{1}{g^2 N_c} < 1$$

Gross, Witten 80

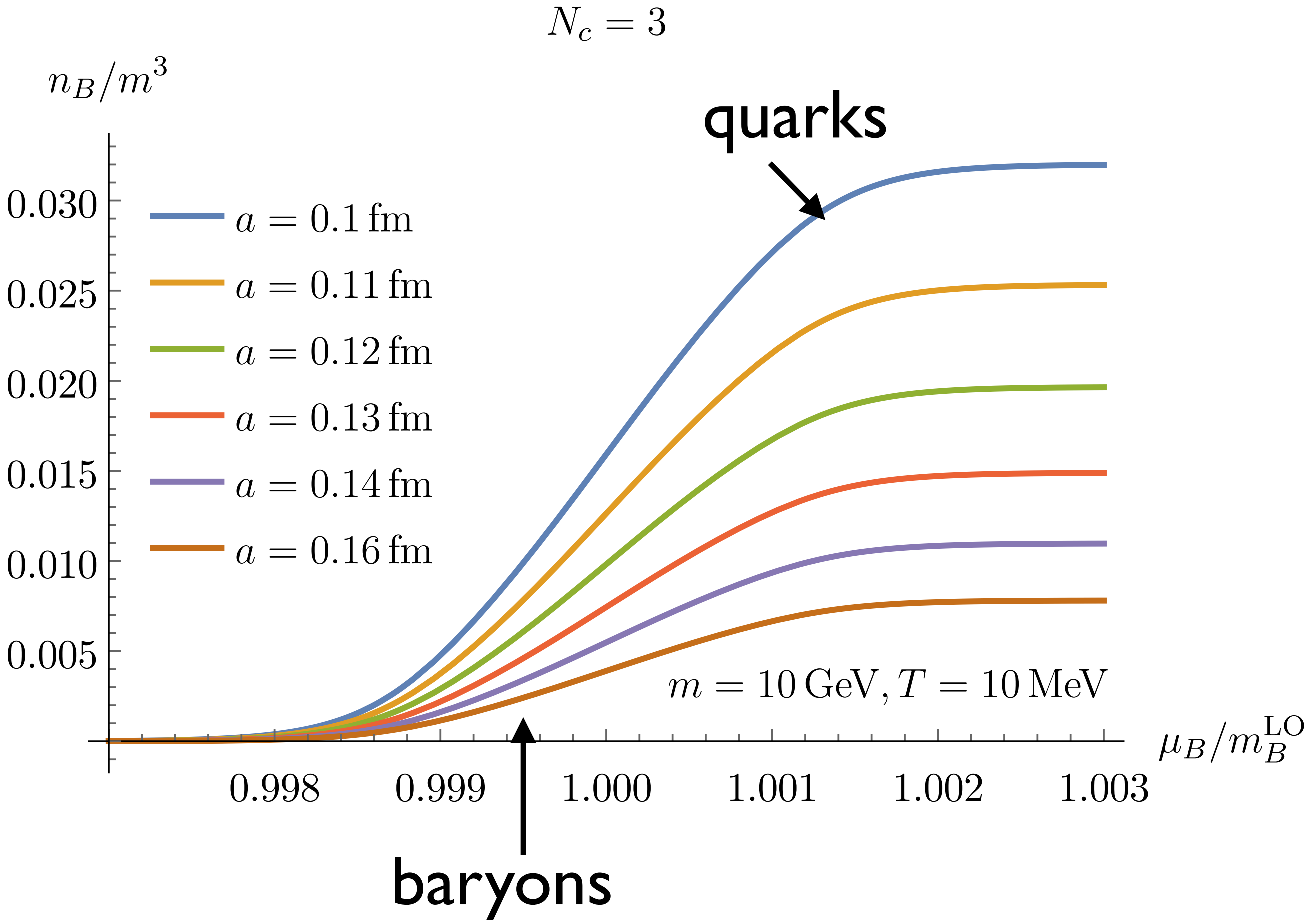


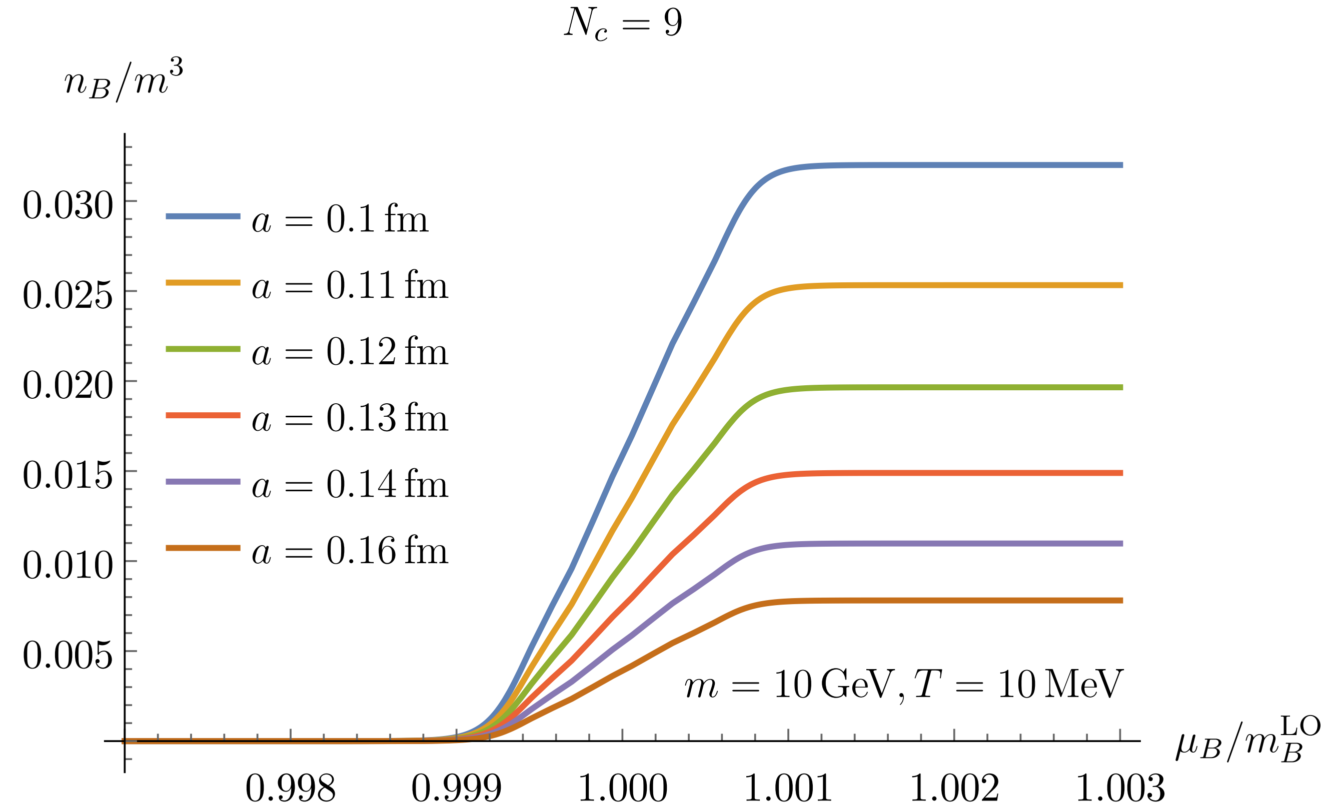
Transition steepens independent of Nt, asymptotically becomes first order

But: system immediately jumps to lattice saturation, unphysical

Continuum approach

Order of limits, take continuum limit first!

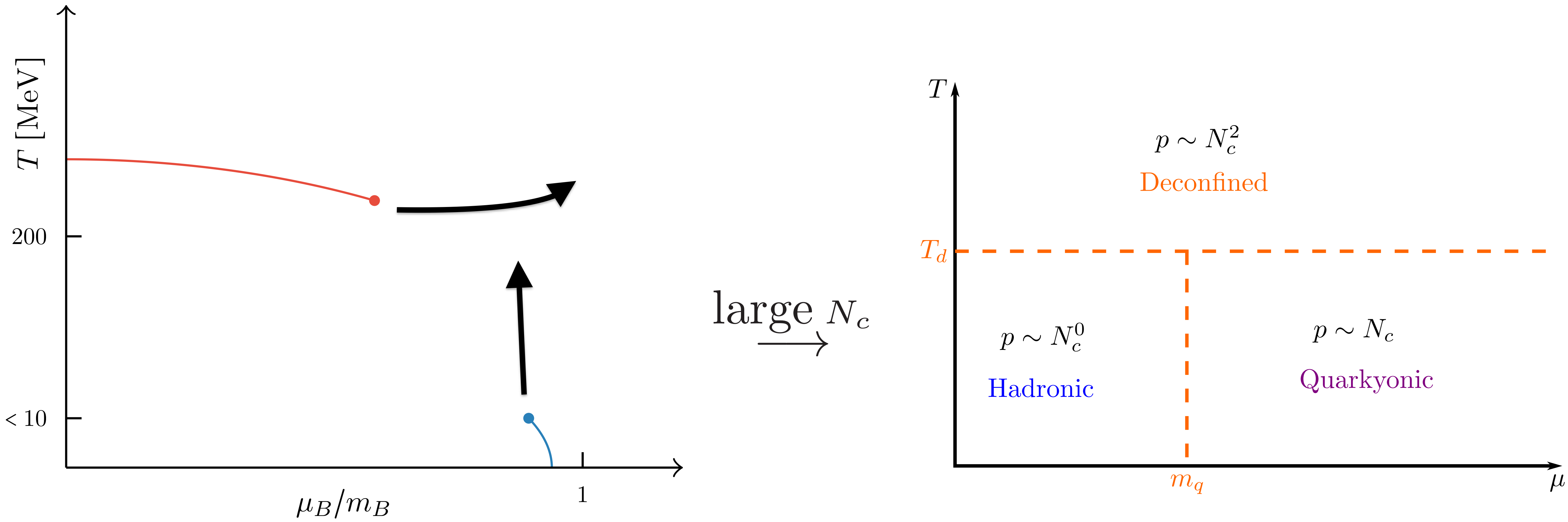




Not enough orders to take limits, but steepening of transition clearly observed!

Quarkyonic matter on the lattice!

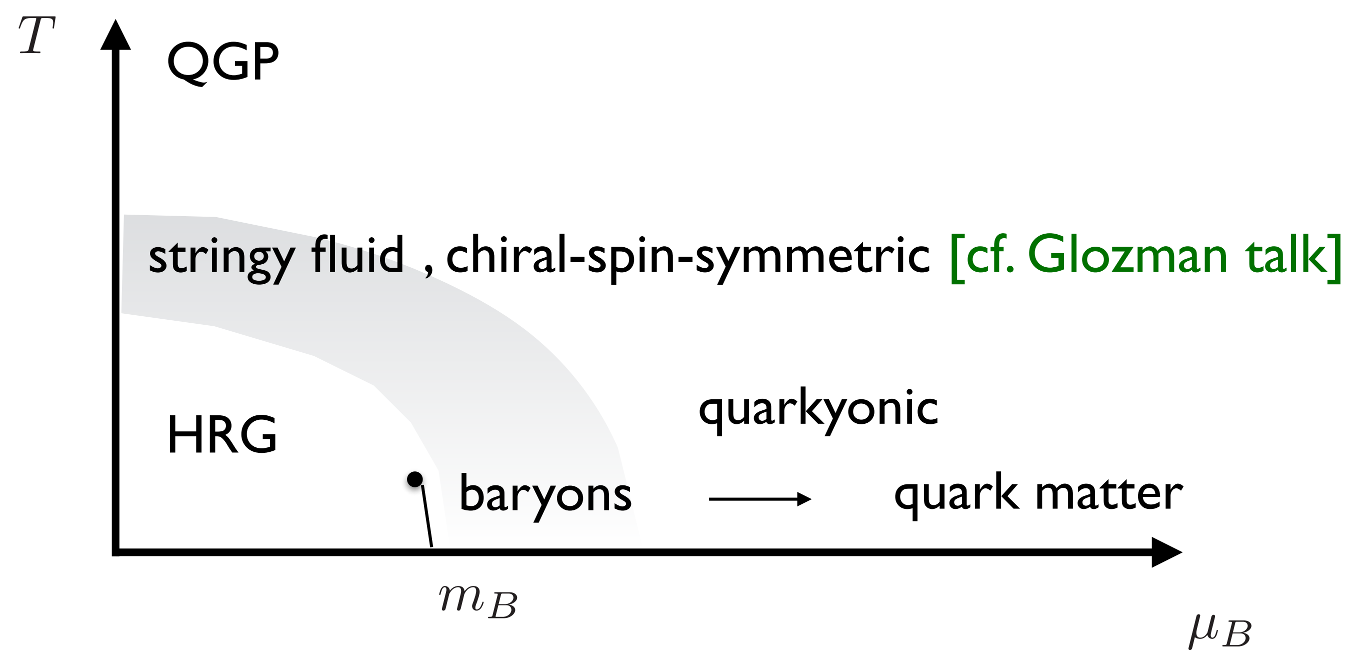
Phase diagram with increasing Nc



- Conjectured large N_c phase diagram emerges smoothly in heavy QCD
- Dense QCD is consistent with quarkyonic matter (=baryon matter for nucl. densities)
- No phase transition to quarkyonic matter besides nuclear liquid gas
- Should also hold for light quarks!

Implications for physical QCD ?

A possibility consistent with all lattice results so far:



Conclusions

(Semi-) analytic treatment of heavy dense Lattice QCD gives new insights:

- Deconfinement transition for any baryon density on coarse lattices
- Mathematical origin of silver blaze phenomenon
- Prediction of nuclear liquid gas transition
- Equation of state in baryon matter phase
- Prediction of quarkyonic matter, without additional phase transition

Mass dependence, possibility of extrapolations…?

Backup slides

Subleading couplings

Subleading contributions for next-to-nearest neighbours:

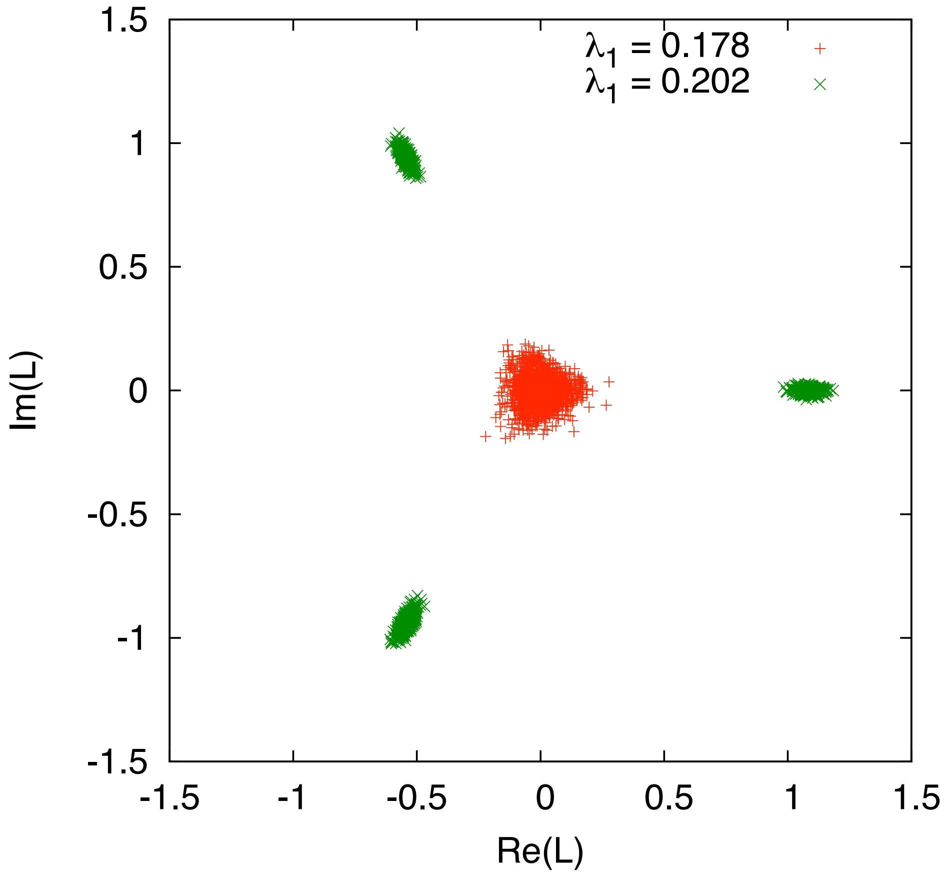
$$\lambda_2 S_2 \quad \propto \quad u^{2N_\tau+2} \sum_{[kl]}' 2\mathrm{Re}(L_k L_l^*) \quad \text{distance} \; = \sqrt{2}$$

$$\lambda_3 S_3 \quad \propto \quad u^{2N_\tau+6} \sum_{\{mn\}}'' 2\mathrm{Re}(L_m L_n^*) \quad \text{distance} \; = 2$$

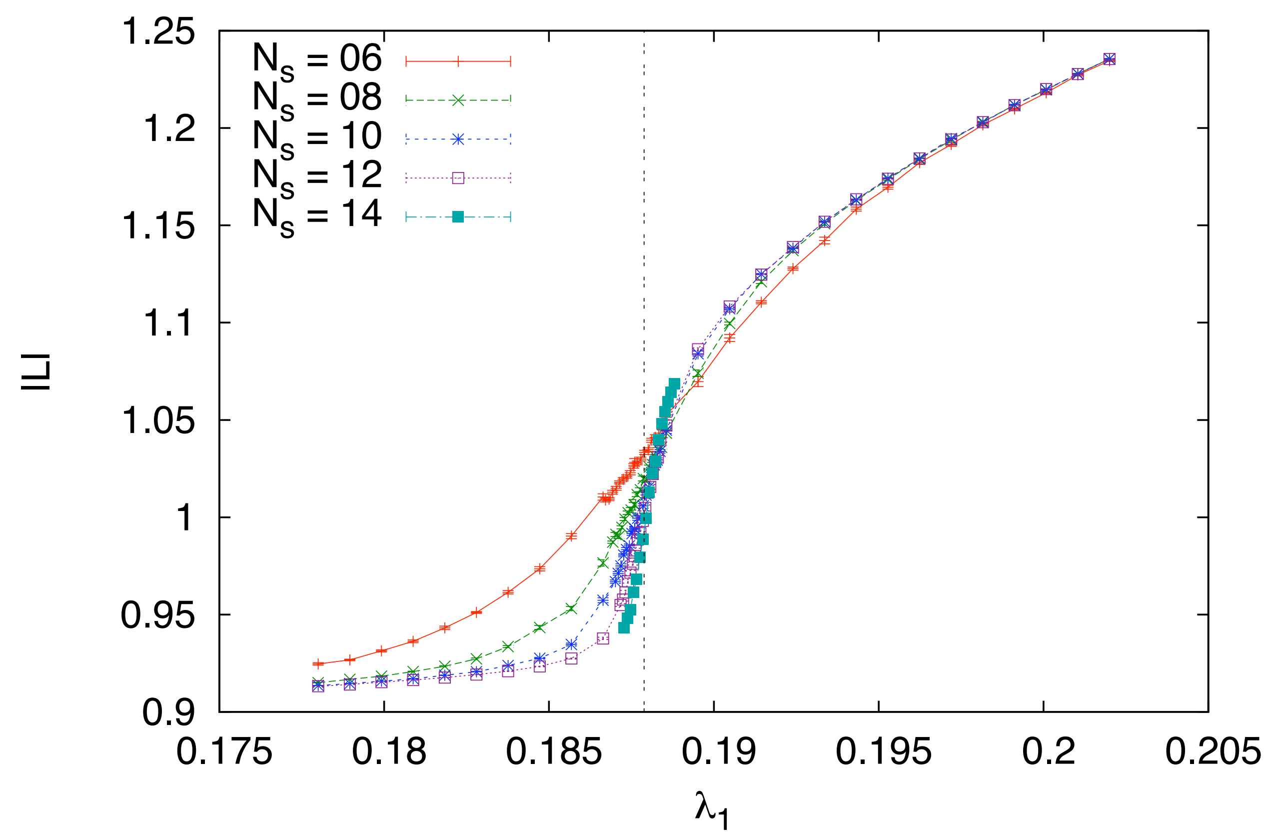
as well as terms from loops in the *adjoint* representation:

$$\lambda_a S_a \propto u^{2N_\tau} \sum_{<ij>} \mathrm{Tr}^{(a)} W_i \mathrm{Tr}^{(a)} W_j \quad ; \quad \mathrm{Tr}^{(a)} W = |L|^2 - 1$$

Numerical results for SU(3), one coupling



Order-disorder transition
=Z(3) breaking



Linked cluster expansion of effective theory

$$\mathcal{Z} = \int \mathcal{D}\phi \, e^{-S_0[\phi]+\frac{1}{2}\sum v_{ij}(x,y)\phi_i(x)\phi_j(y)+\frac{1}{3!}\sum u_{ijk}(x,y,z)\phi_i(x)\phi_j(y)\phi_k(z)+\ldots}$$

"perturbation theory" in effective couplings

Glesaaen, Neuman, O.P. 15

through $u^5\kappa^8$