Erice International School on Nuclear Physics - 44th course

Modified NJL models and their applications

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Outline

* Introduction / Motivation
* NJL model
* Thermo-magnetic NJL coupling
* Thermodynamics with the new coupling
* Meson properties at zero temperature
* Magnetization
* Final remarks

Collaborators

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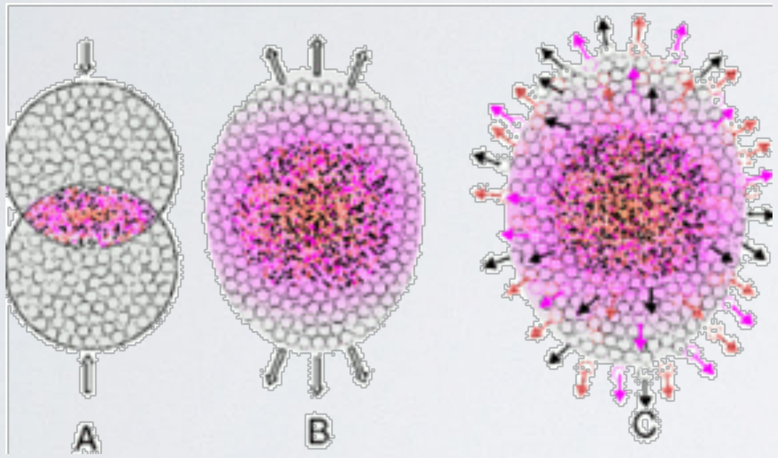
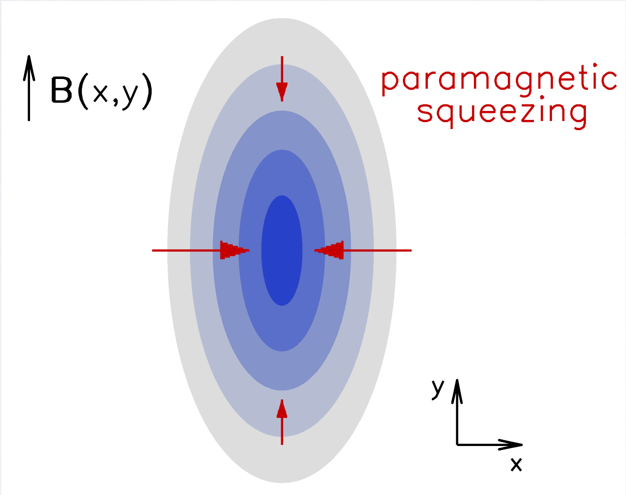
Financial Support

Introduction / Motivation

LHC / RHIC





Phys. Rev. Lett. 112 (2014) 042301

G. S. Bali, F. Bruckmann, G. Endrődi, and A. Schäfer

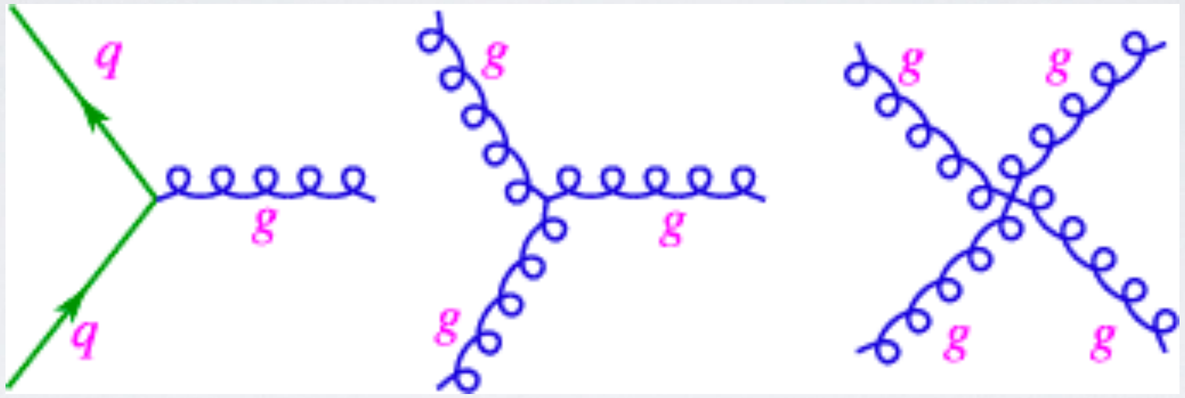
Magnetars

neutron stars

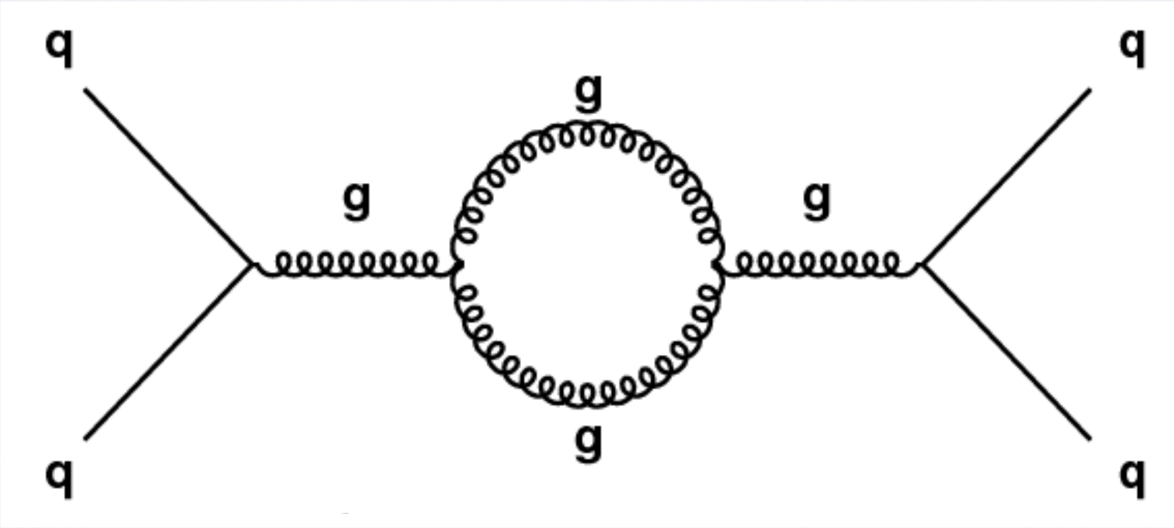
hadronic matter

QCD complications

Force carriers interact directly (increases the number of possible processes)



Couplings are strong (high order processes are not necessarily less important)



Effective Models

Models are less powerful than theories

They are used when the fundamental theory is too complicated

Many examples: meson exchange, van der Waals, quark-meson coupling, …

Built to explain part of the features of a complex theory

QCD	**NJL**
Confinement	No confinement
Asymptotic freedom	No asymptotic freedom
Symmetry breaking	Symmetry breaking
Mass generation	Mass generation

Nambu–Jona-Lasinio Models

Buballa, Bernard, Klevansky, Ratti, Weise,...

SU(2)

$$\mathcal{L}_{\mathrm{NJL}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} \left(\not{D} - m \right) \psi + G \left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \tau \psi)^2 \right]$$

SU(3) + PL

$$\mathcal{L} = \bar{q} \left[i\gamma_\mu D^\mu - \hat{m}_c \right] q \; + \; \mathcal{L}_{sym} \; + \; \mathcal{L}_{det}$$
$$+ \mathcal{U}\left(\Phi, \bar{\Phi}; T\right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\mathcal{L}_{sym} = \frac{G_s}{2} \sum_{a=0}^{8} \left[(\bar{q}\lambda_a q)^2 + (\bar{q} i \gamma_5 \lambda_a q)^2 \right]$$

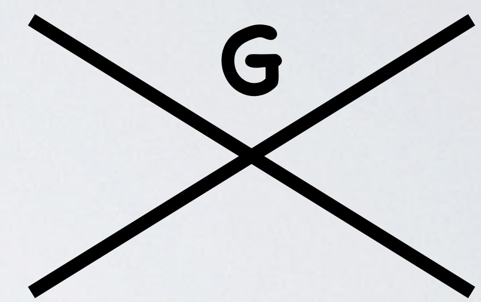
$$\mathcal{L}_{det} = -K \left\{ \det \left[\bar{q}(1+\gamma_5) q \right] + \det \left[\bar{q}(1-\gamma_5) q \right] \right\}$$

NJL gap equation: simple view

see review by Weise

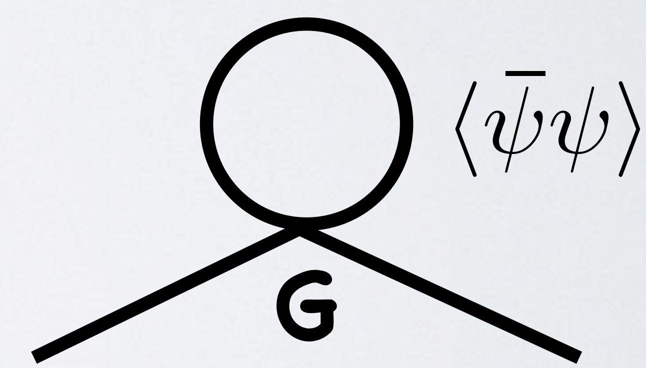
$$\mathcal{L}_D = \bar{\psi}\, i\gamma^\mu \partial_\mu\, \psi \;-\; m\, (\bar{\psi}\psi)$$

$$(\, i\gamma^\mu \partial_\mu - m \,)\, \psi \;=\; 0$$



$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}\, i\gamma^\mu \partial_\mu\, \psi \;+\; G\, (\bar{\psi}\psi)^2$$

$$(\, i\gamma^\mu \partial_\mu + 2G\langle\bar{\psi}\psi\rangle \,)\, \psi \;=\; 0$$



$$m \;=\; -\,2G\, \langle\bar{\psi}\psi\rangle$$

$$\langle\bar{\psi}\psi\rangle \sim \int d^4p \frac{1}{p^2 + m^2} \sim \int^{\Lambda} dp\; p^2 \frac{m}{\sqrt{p^2 + m^2}}$$

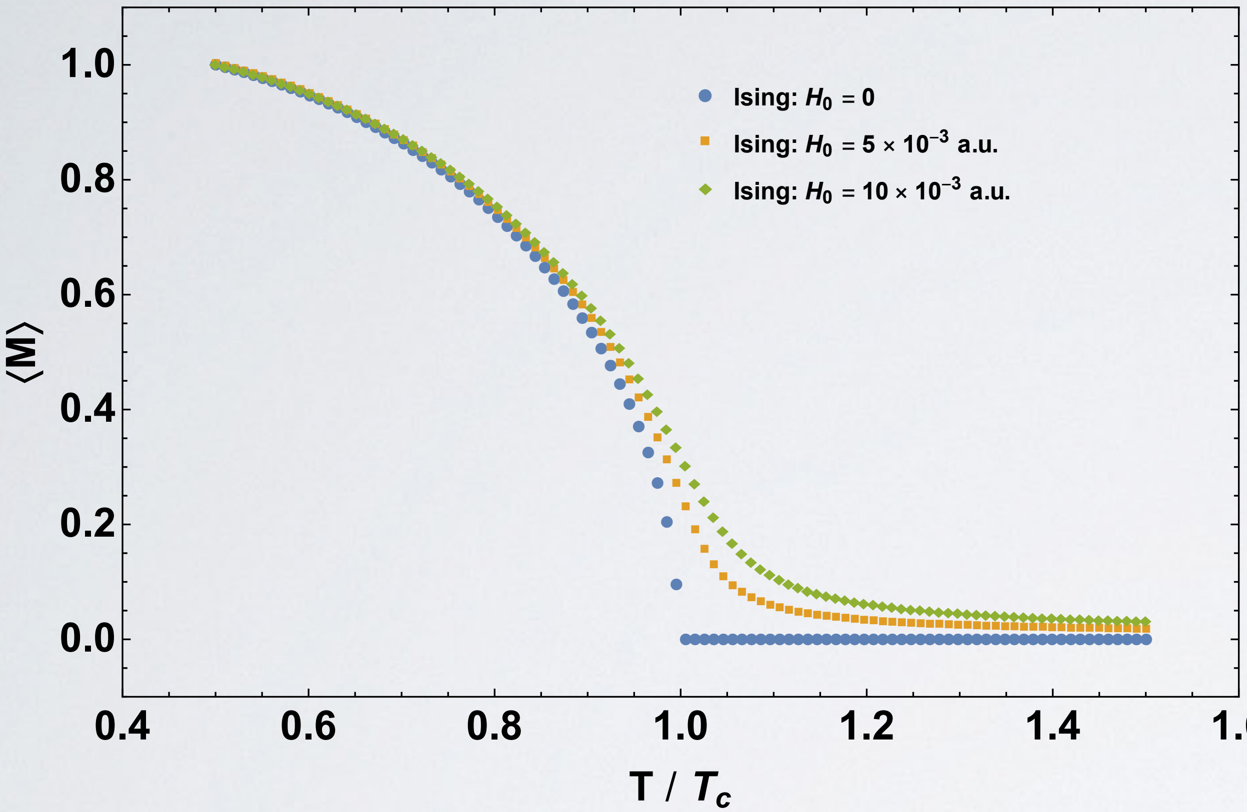
$\langle M \rangle$ and $\langle \bar{q}q \rangle$

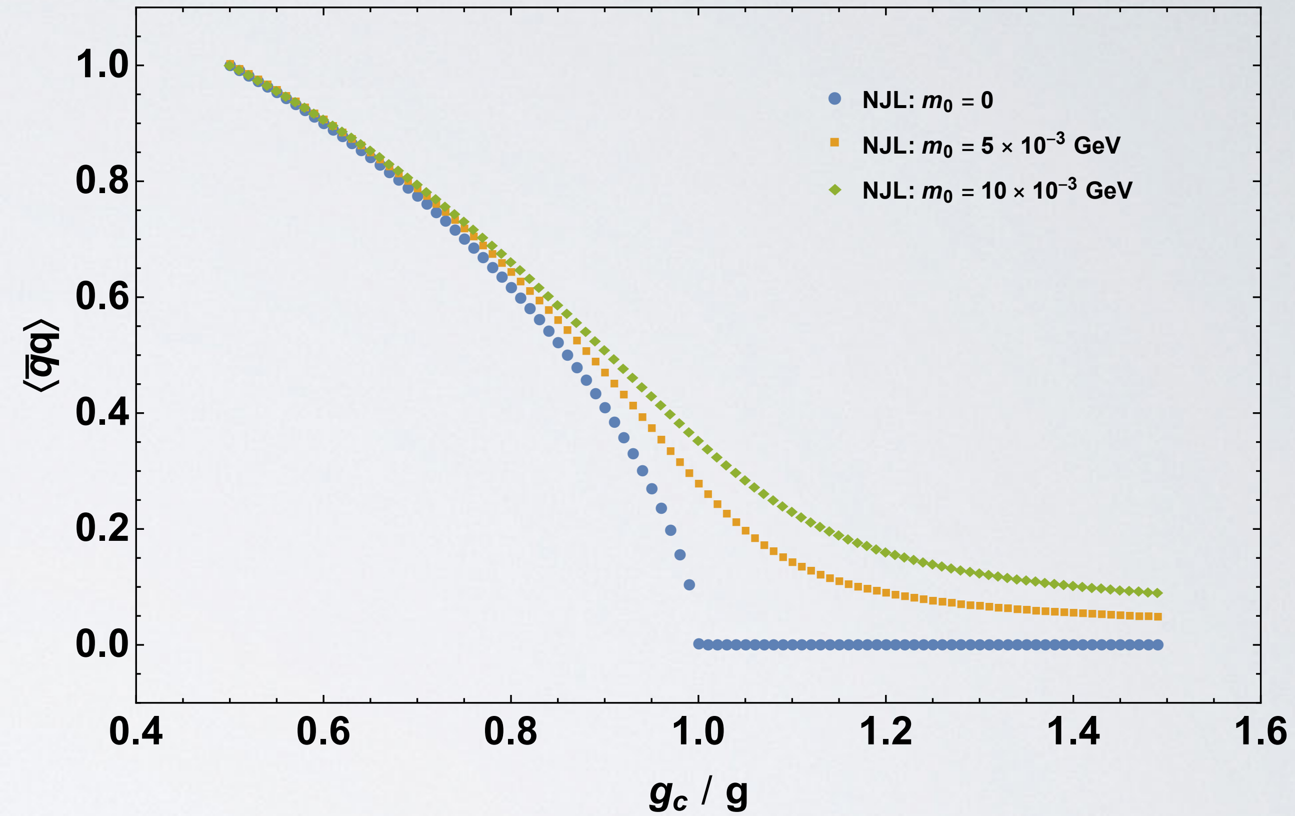
2D Ising model

Magnetization

2F NJL model

Chiral condensate





Rotational symmetry is broken

Chiral symmetry is broken

Collective Goldstone mode: spin waves

Collective Goldstone mode: pions

External mag. field h0 (explicit breaking)

Current quark mass m0 (explicit breaking)

Gap equations at finite temperature and magnetic field

$$M_u = m_u - 2G\langle\bar{u}u\rangle - 2K\langle\bar{d}d\rangle\langle\bar{s}s\rangle$$

$$M_d = m_d - 2G\langle\bar{d}d\rangle - 2K\langle\bar{s}s\rangle\langle\bar{u}u\rangle$$

$$M_s = m_s - 2G\langle\bar{s}s\rangle - 2K\langle\bar{u}u\rangle\langle\bar{d}d\rangle$$

$$\boxed{\langle\bar{q}q\rangle \rightarrow \langle\bar{q}q\rangle_{\rm vac} + \langle\bar{q}q\rangle_{\rm mag} + \langle\bar{q}q\rangle_{\rm Tmag}}$$

Condensates

$$\langle\bar{\psi}_f\psi_f\rangle^{vac} = -\frac{MN_c}{2\pi^2}\left[\Lambda\,\epsilon_\Lambda - M^2\ln\left(\frac{\Lambda+\epsilon_\Lambda}{M}\right)\right]$$

$$\langle\bar{\psi}_f\psi_f\rangle^{mag} = -\frac{M|q_f|BN_c}{2\pi^2}\left[\ln\Gamma(x_f) - \frac{1}{2}\ln(2\pi) + x_f - \frac{1}{2}\left(2x_f - 1\right)\ln(x_f)\right]$$

$$\langle\bar{\psi}_f\psi_f\rangle^{Tmag} = \sum_{k=0}^{\infty}\alpha_k\frac{M|q_f|BN_c}{2\pi^2}\int_{-\infty}^{+\infty}\mathrm{d}p\,\frac{n(E_f)}{E_f}$$

$$\epsilon_\Lambda = \left(\Lambda^2 + M^2\right)^{1/2}$$

$$E_f = \left(p^2 + M^2 + 2|q_f|Bk\right)^{1/2}$$

$$x_f = \frac{M^2}{2|q_f|B}$$

$$n(E_f) = \frac{1}{1+\exp(E_f/T)}$$

$\mu = 0$

Grand canonical potential in MFA

$$\Omega = -\,T\,ln\,\mathcal{Z} \qquad \mathcal{Z} = Tr\,e^{-\beta(H-\mu N)}$$

$$\Omega(T,\mu) = G_s \sum_{f=u,d,s} \langle \bar{q}_f q_f \rangle^2 + 4K \langle \bar{q}_u q_u \rangle \langle \bar{q}_d q_d \rangle \langle \bar{q}_s q_s \rangle + \mathcal{U}(\Phi,\bar{\Phi},T) + \sum_{f=u,d,s} \left(\Omega^f_{\rm vac} + \Omega^f_{\rm med} + \Omega^f_{\rm mag} \right)$$

$$\Omega^f_{\rm vac} = -6 \int_\Lambda \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + M_f^2}$$

$$\zeta'(-1,x_f) = \left. \frac{{\rm d}\zeta(z,x_f)}{{\rm d}z} \right|_{z=-1}$$

$$\Omega^f_{\rm mag} = -\frac{3(|q_f|B)^2}{2\pi^2} \left[\zeta'(-1,x_f) - \frac{1}{2}(x_f^2 - x_f)\ln x_f + \frac{x_f^2}{4} \right]$$

$$\Omega^f_{\rm T,B} = -T \frac{|q_f B|}{2\pi} \sum_{k=0} \alpha_k \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} \ln\{1 + \exp[-(E_f/T)]\}$$

$$\mu = 0$$

Thermodynamics

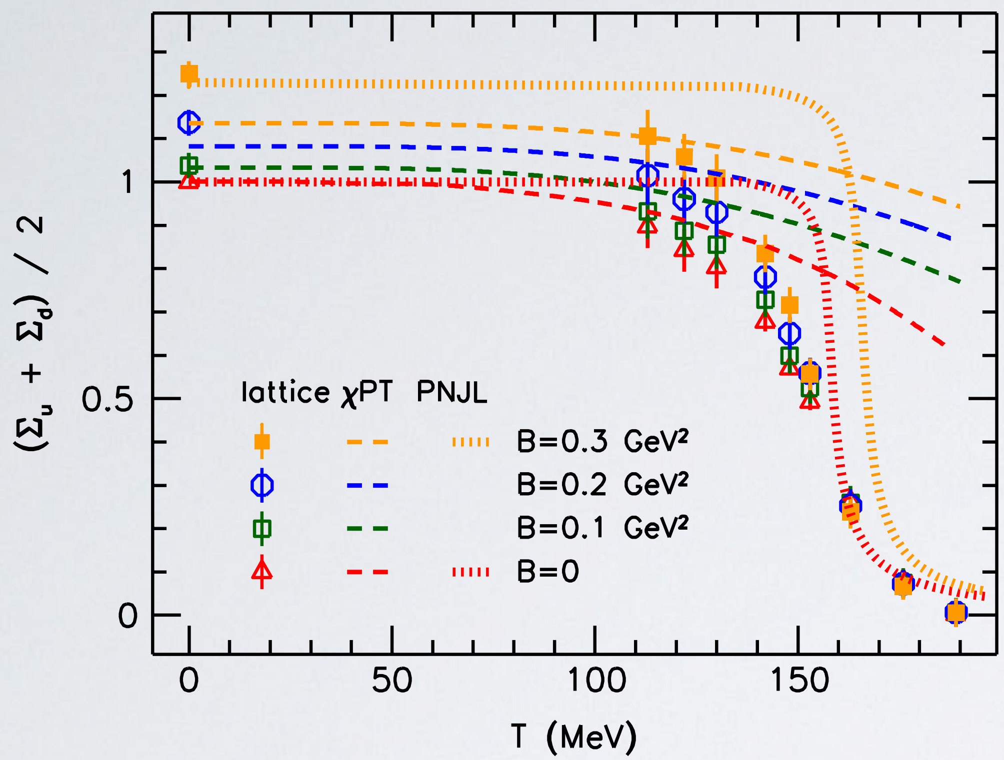
$$\Omega = - T \, ln \, \mathcal{Z} \qquad \mathcal{Z} = Tr \, e^{-\beta(H-\mu N)}$$

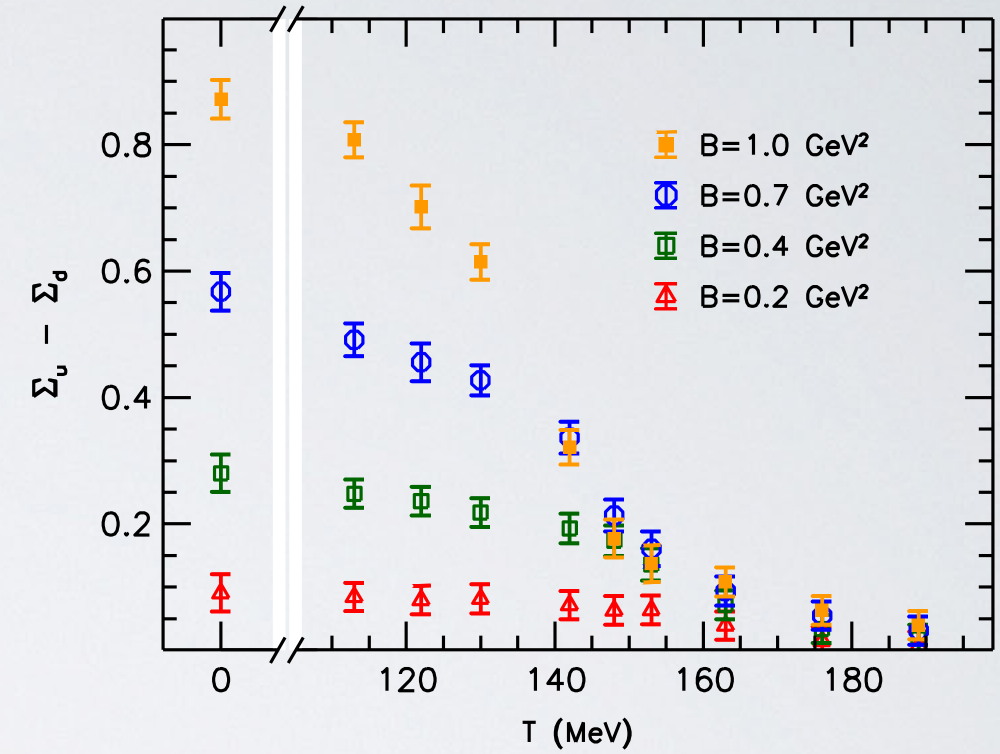
$$\epsilon = \Omega + T \, s + \mu \, \rho$$

$$p = -\Omega \qquad s = -\frac{\partial \Omega}{\partial T} \qquad c_v = T\frac{dS}{dT}$$

$$\Delta = \epsilon - 3 \, p \qquad v_s^2 = \frac{dp}{d\epsilon} \qquad \mathcal{M} = -\frac{\partial \Omega}{\partial B}$$

Some lattice QCD results



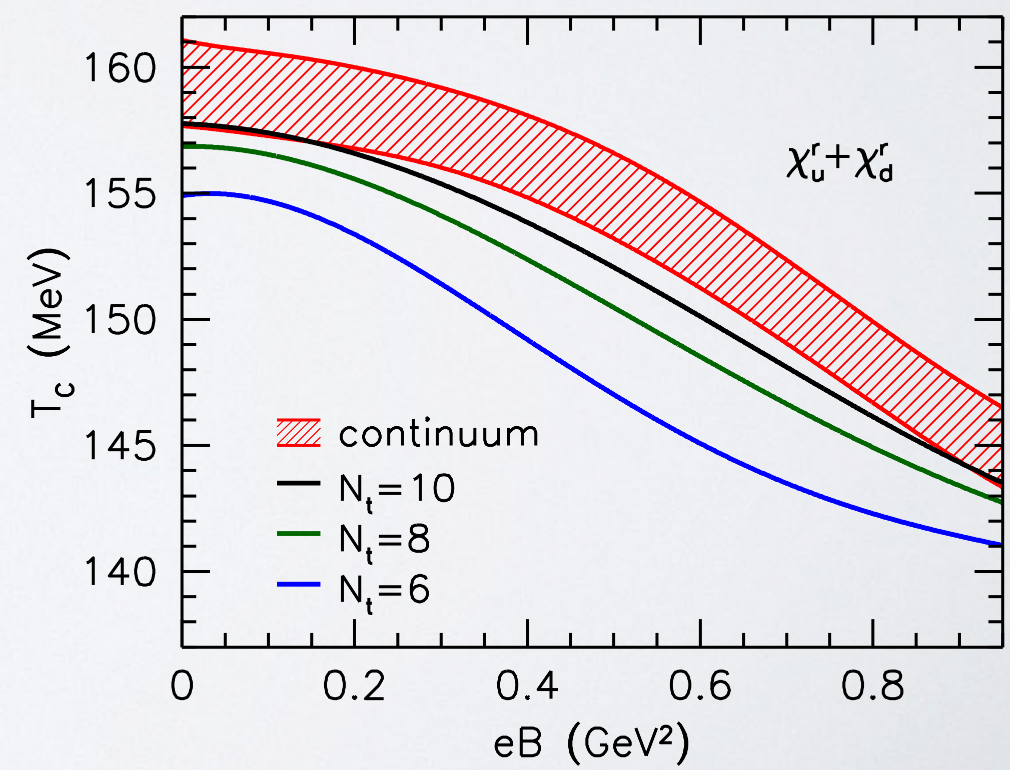


G. Bali et al.

JHEP 02 (2012) 044

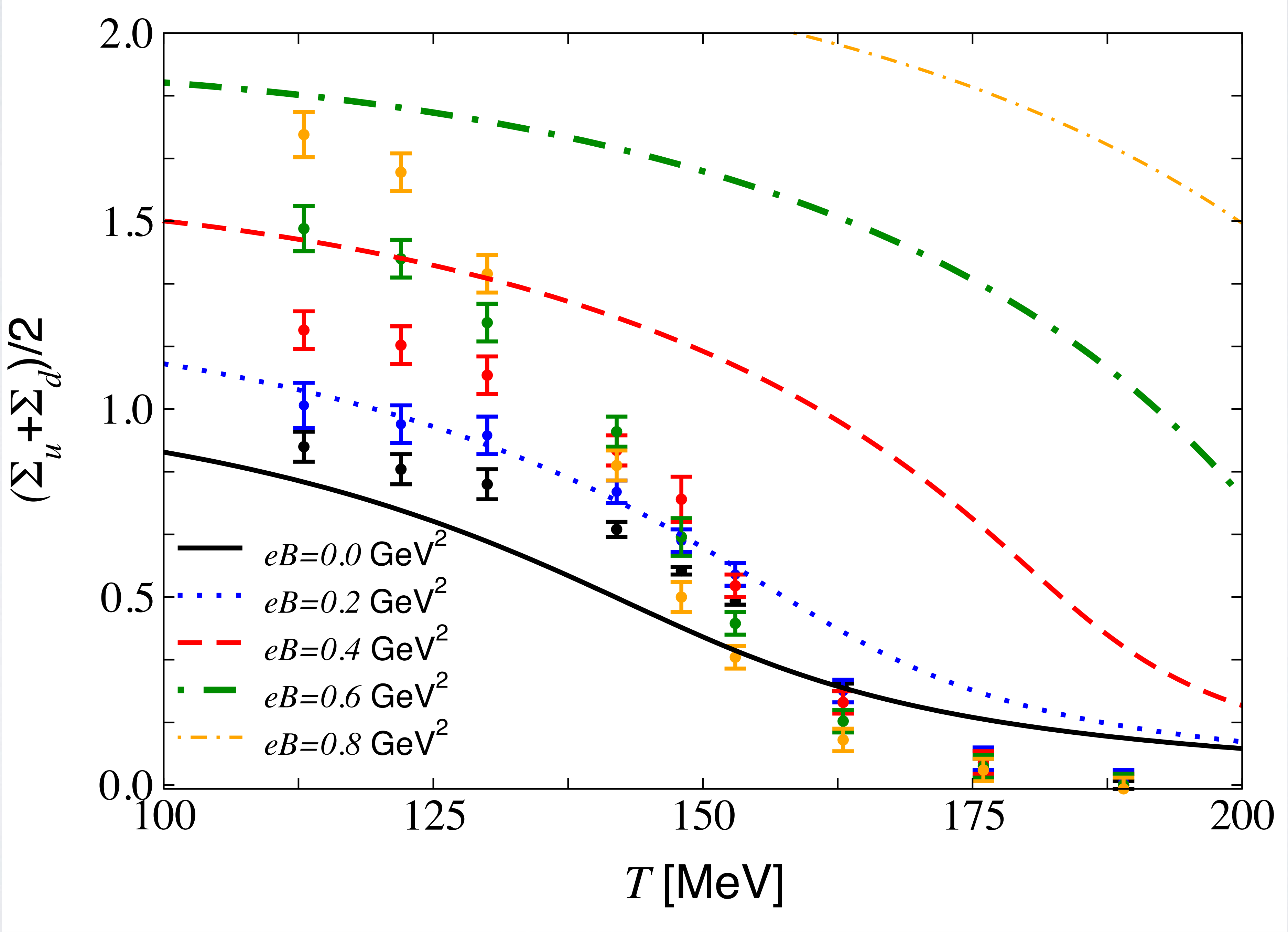
Phys. Rev. D 86 (2012) 071502

JHEP 08 (2014) 177



Constant coupling: SU(2) NJL model

G



Thermo-magnetic coupling: prototype

$$eB >> \Lambda^2_{\text{QCD}}$$

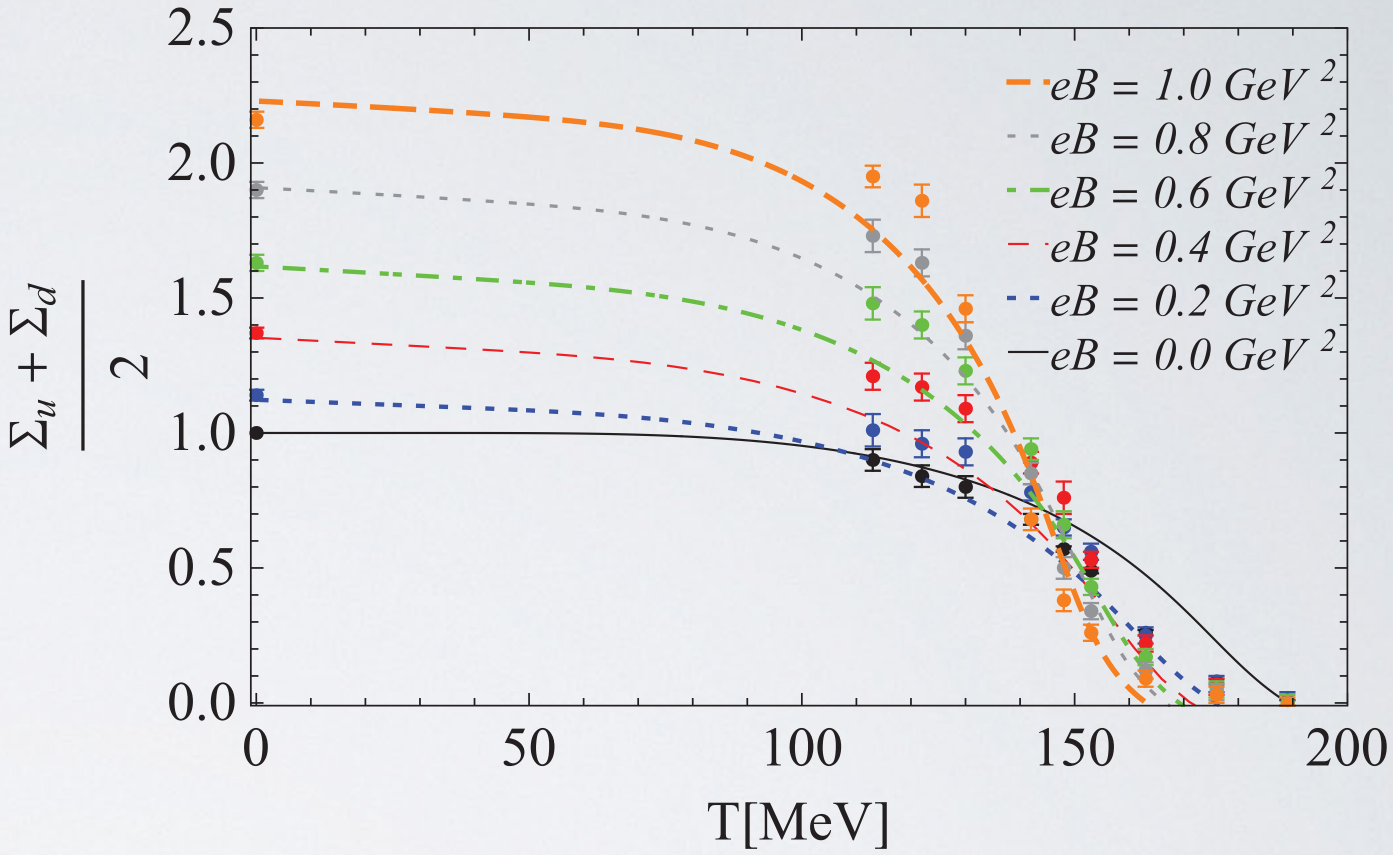
$$\alpha_s \sim \frac{1}{b \ln(eB/\Lambda^2_{\text{QCD}})}$$

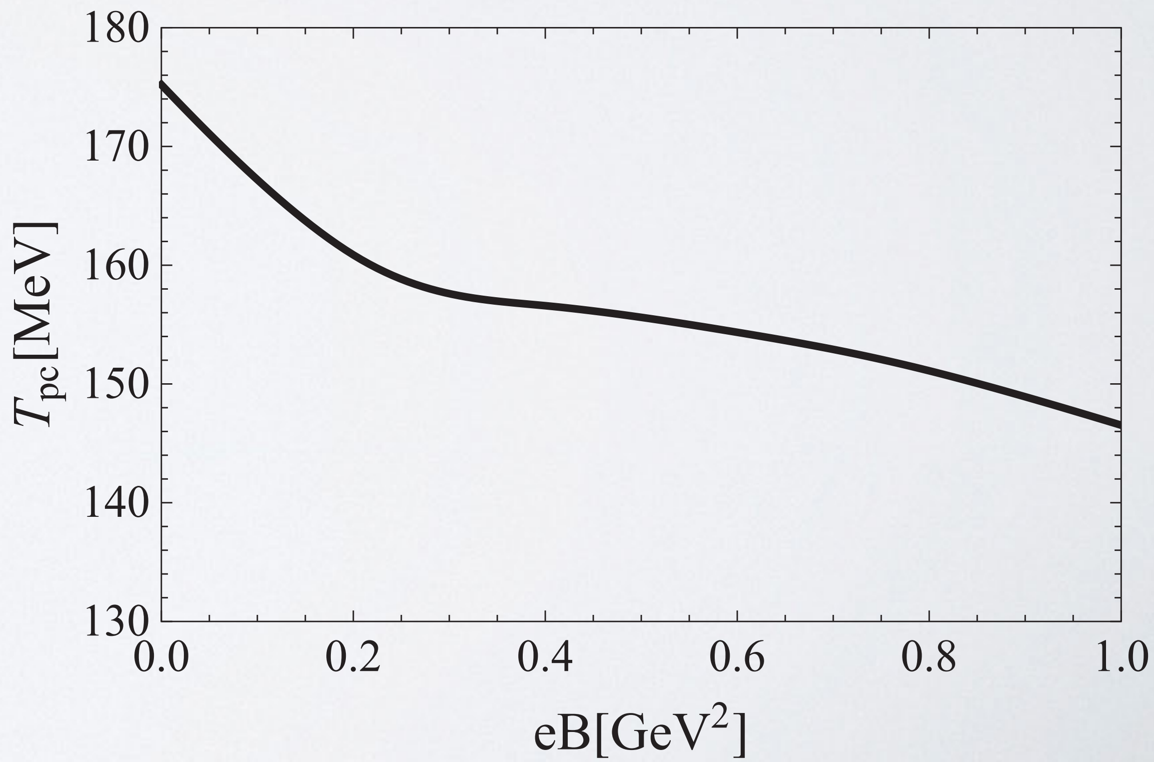
$T = 0$

$$G(B) = \frac{G_0}{1 + \alpha \ln\left(1 + \beta \frac{eB}{\Lambda^2_{\text{QCD}}}\right)}$$

$T > 0$

$$G(B,T) = G(B)\left(1 - \gamma \frac{|eB|}{\Lambda^2_{\text{QCD}}} \frac{T}{\Lambda_{\text{QCD}}}\right)$$





Farias, Gomes, Krein, Pinto
Phys. Rev. C **90**, 025203 (2014)

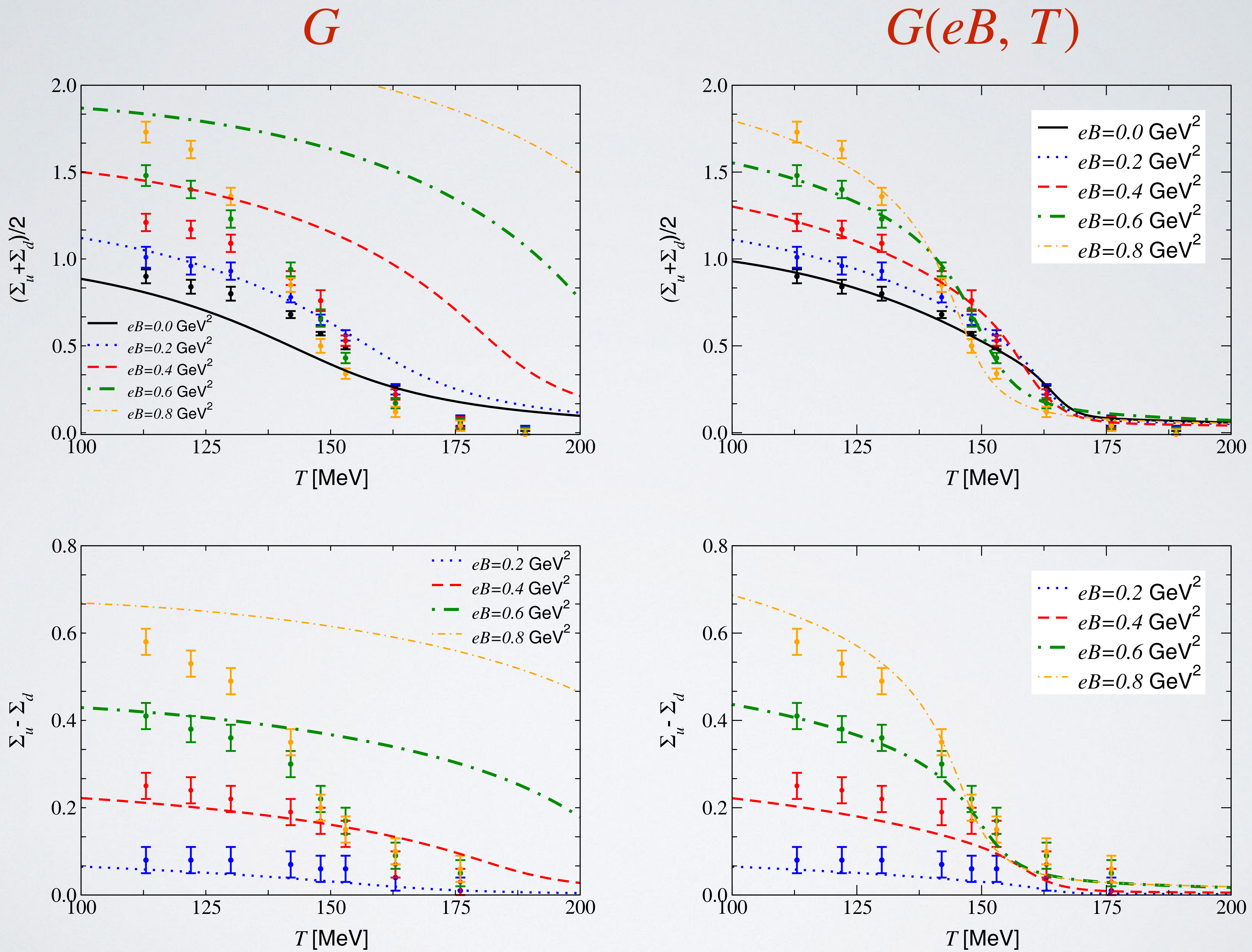
Matching the NJL model to lattice QCD

Build a thermo-magnetic coupling for the NJL model from lattice QCD results

For given values of T and eB:

- start with an initial attempt for $G(T, eB)$
- for this G, make an initial guess for M
- solve the gap equation
- with M, compute the condensate averages
- compare to lattice QCD result for that T and eB
- repeat until the best $G(T, eB)$ is found

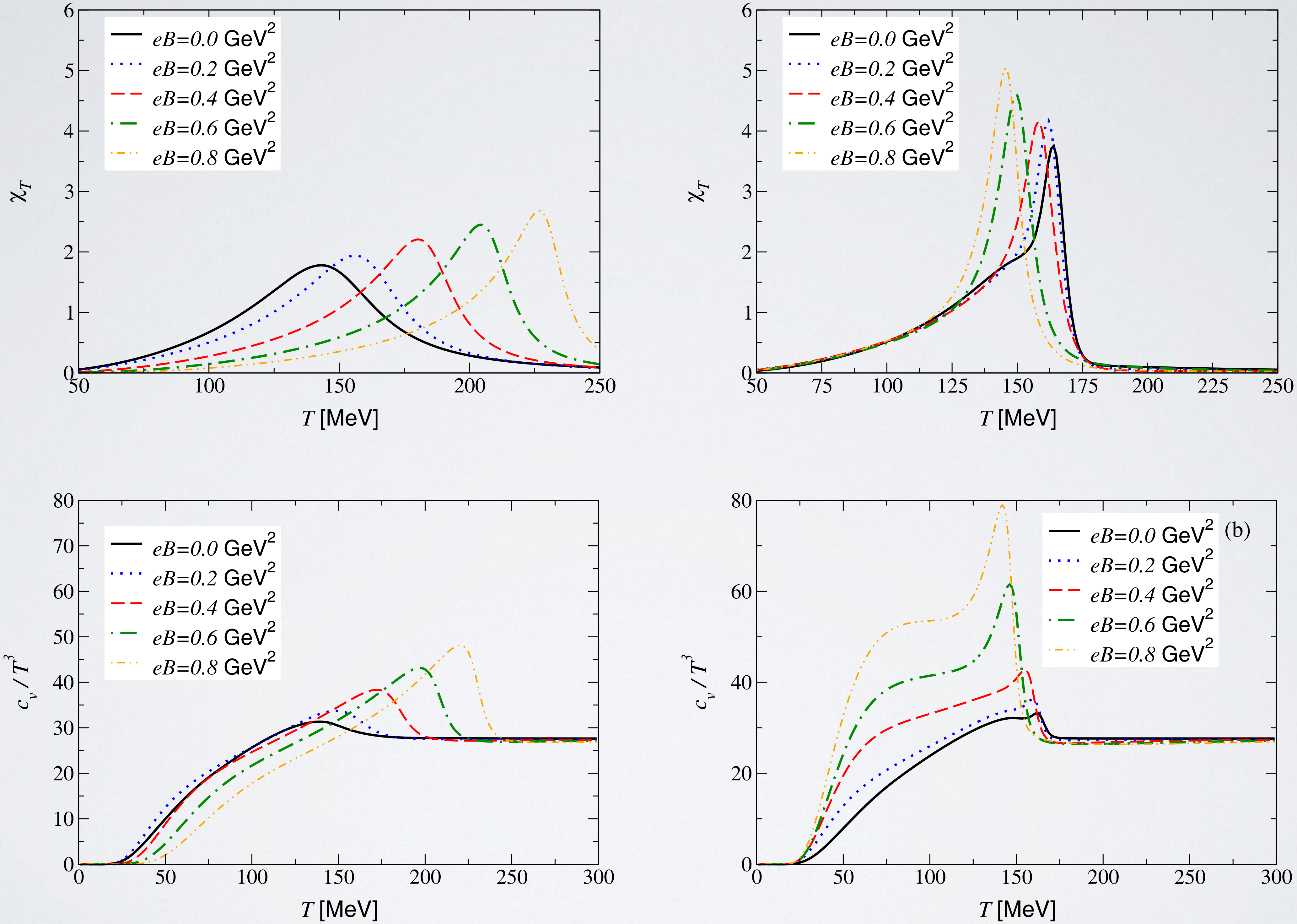
Thermo-magnetic dependent coupling



Eur. Phys. J. A 53 (2017) 101

Thermal Susceptibilities and Specific Heat

G | $G(eB, T)$



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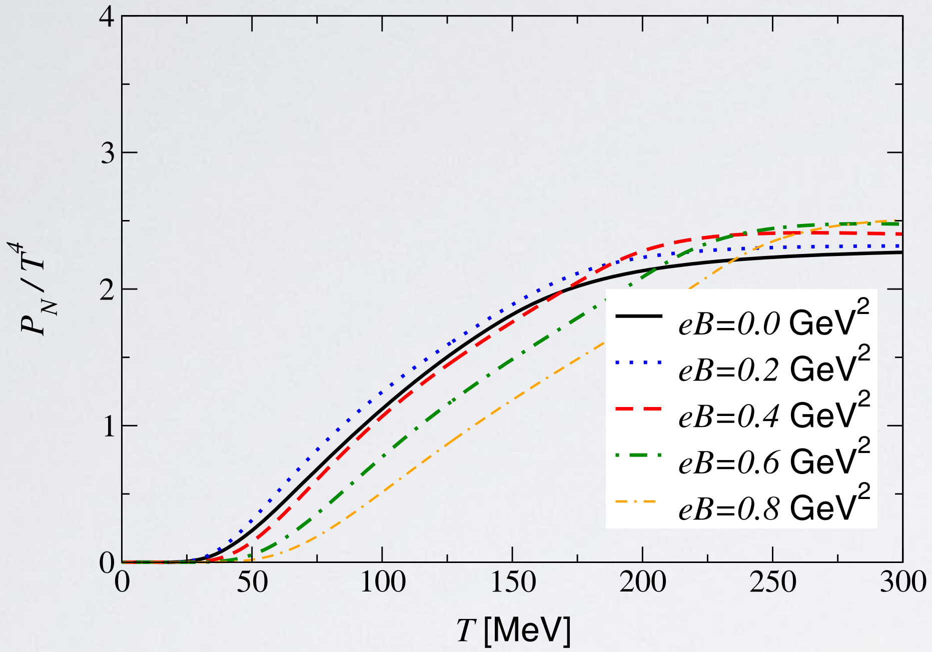
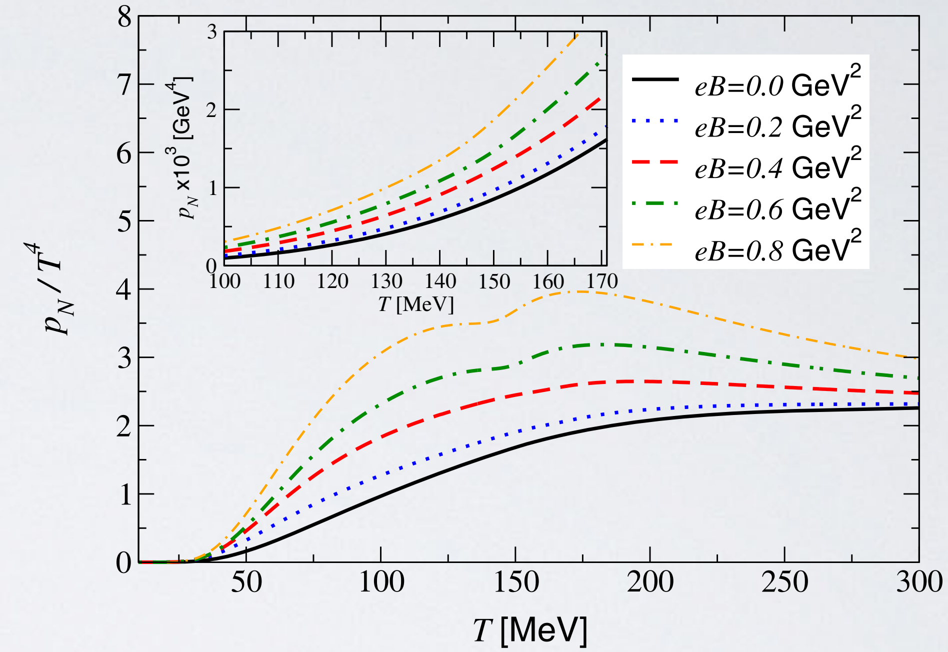
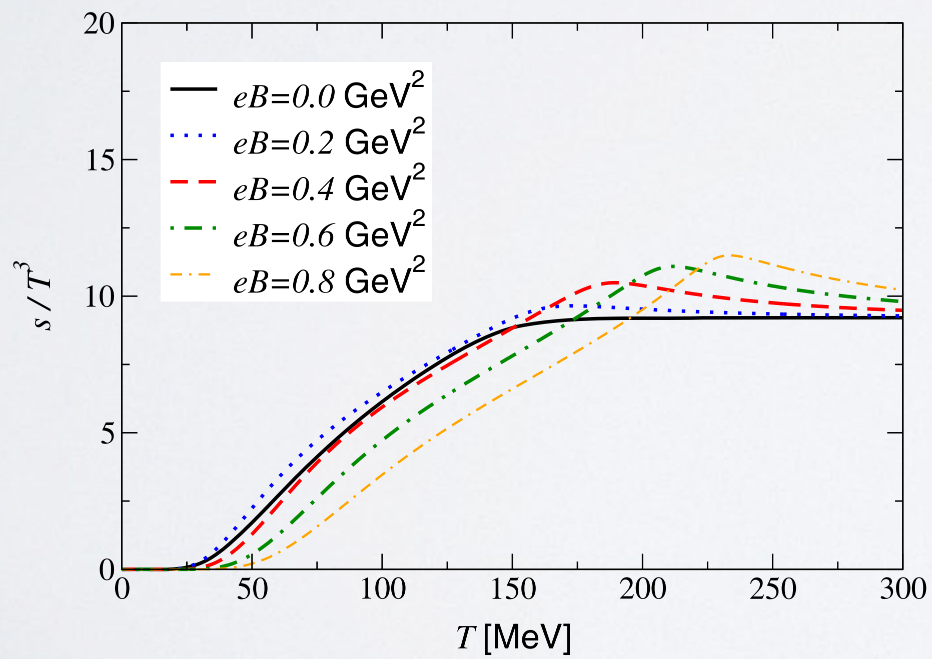
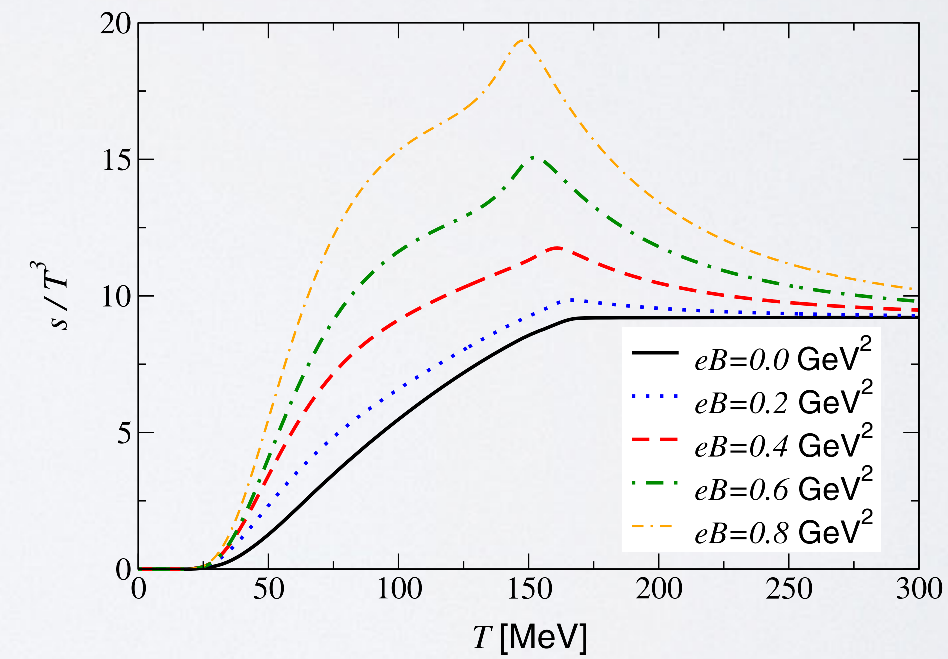
Pressure and Entropy

G $G(eB, T)$









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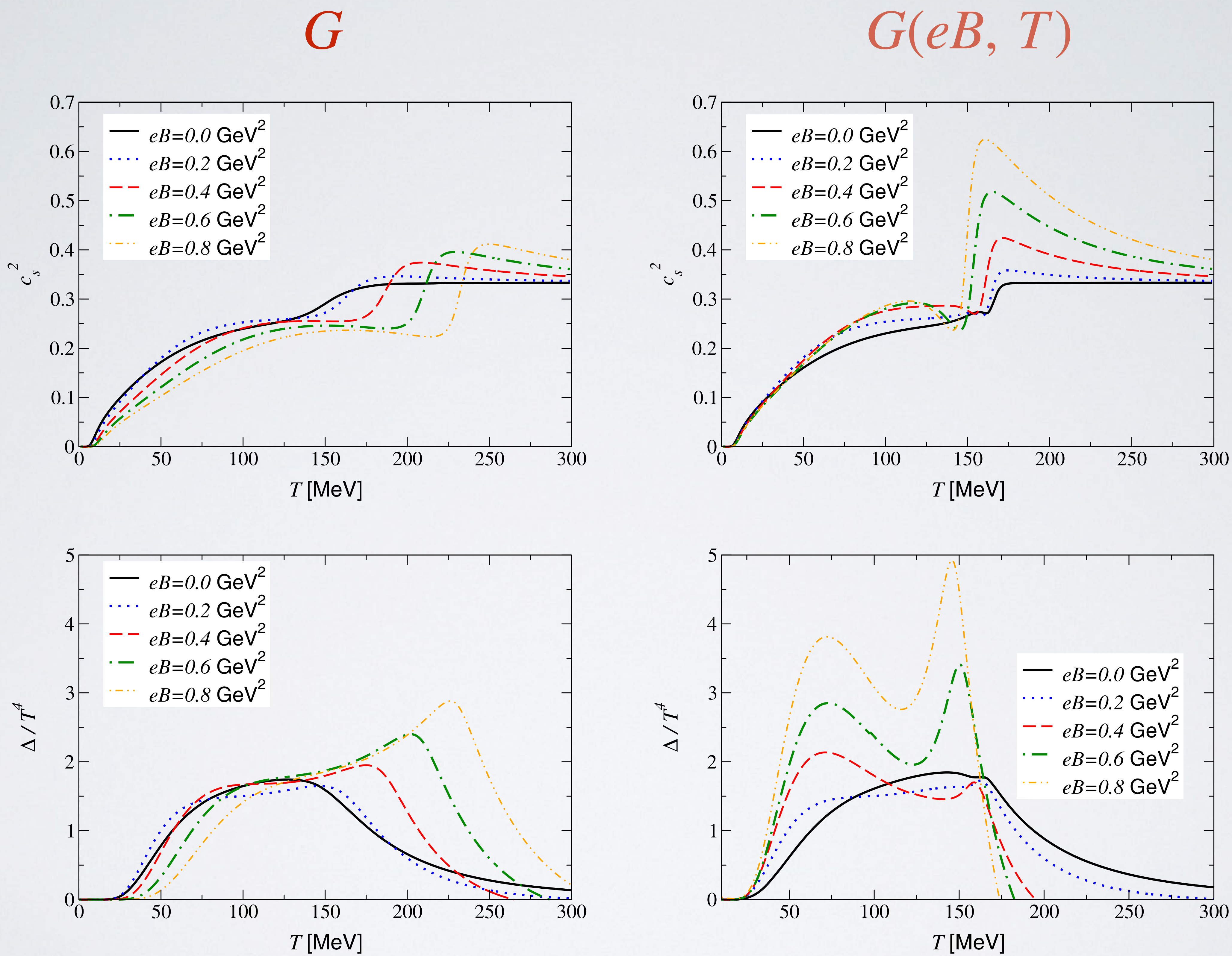
Sound Velocity and Interaction Measure



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Pseudo-critical temperature

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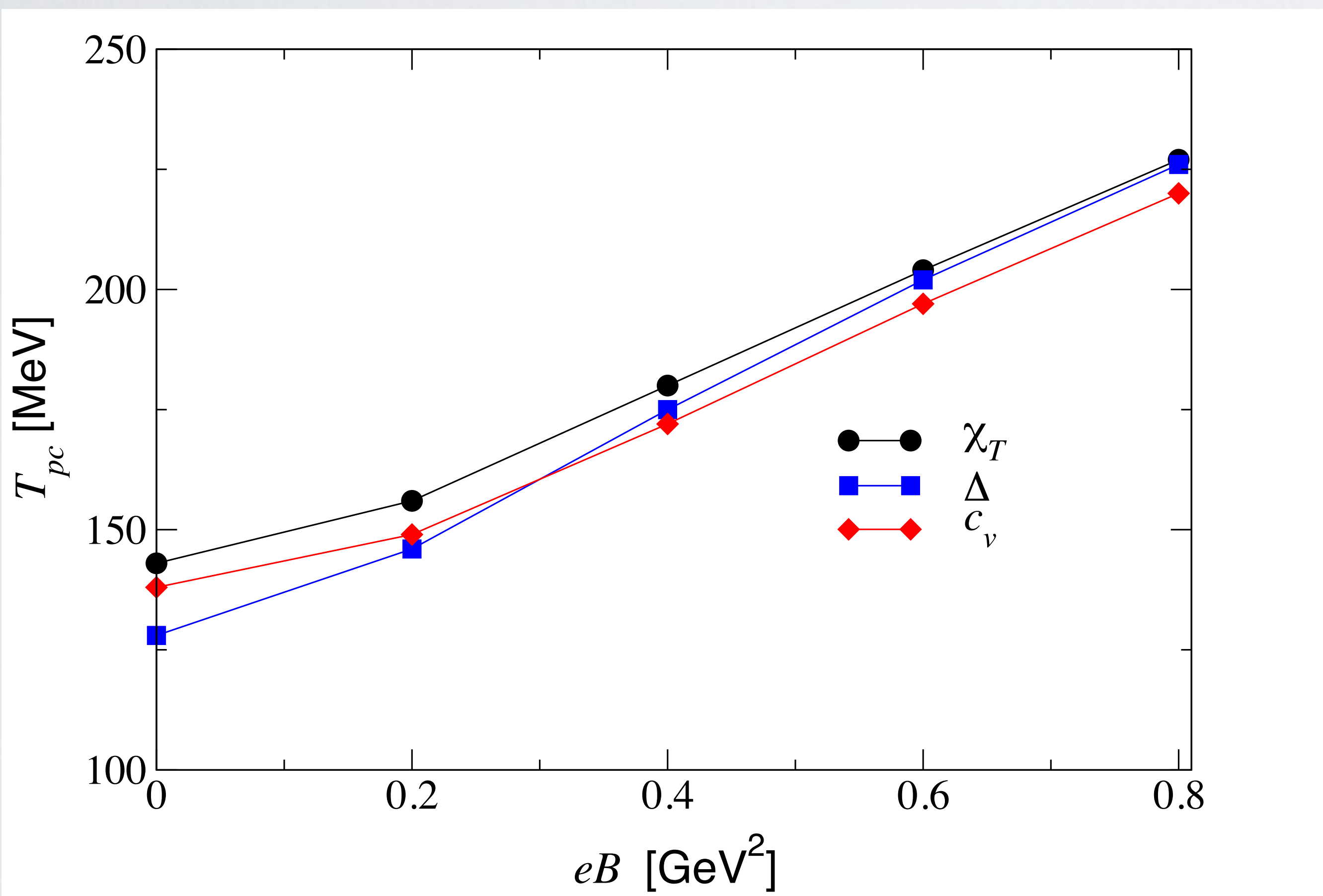
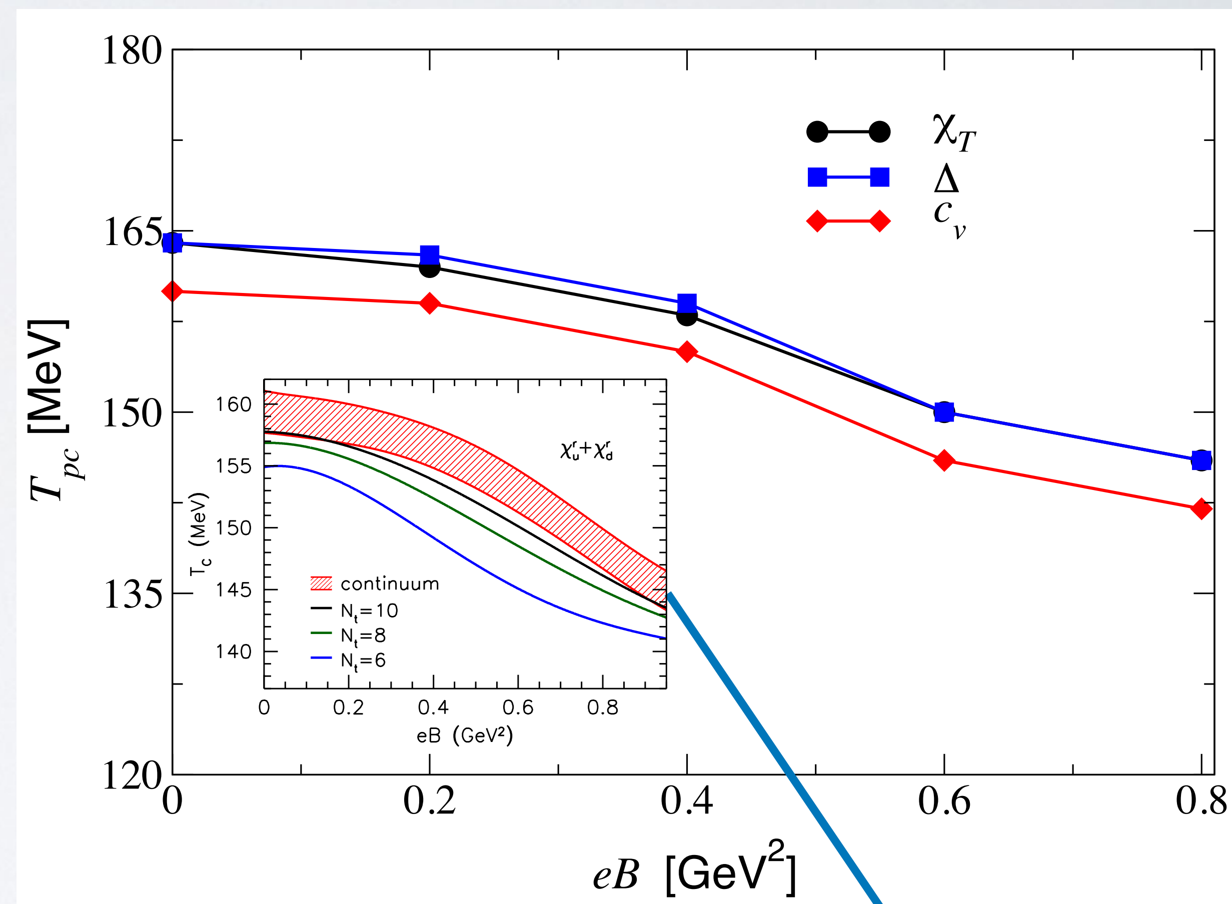
G



$G(eB, T)$



Bali et al.
JHEP 02 (2012) 044

Magnetization

$$\mathcal{M} = -\left.\frac{\partial \Omega}{\partial B}\right|_{\{\phi_f\},\rho} - \frac{\partial \Omega}{\partial \phi_f}\frac{\partial \phi_f}{\partial B} - \frac{\partial \Omega}{\partial \rho}\frac{\partial \rho}{\partial B}$$

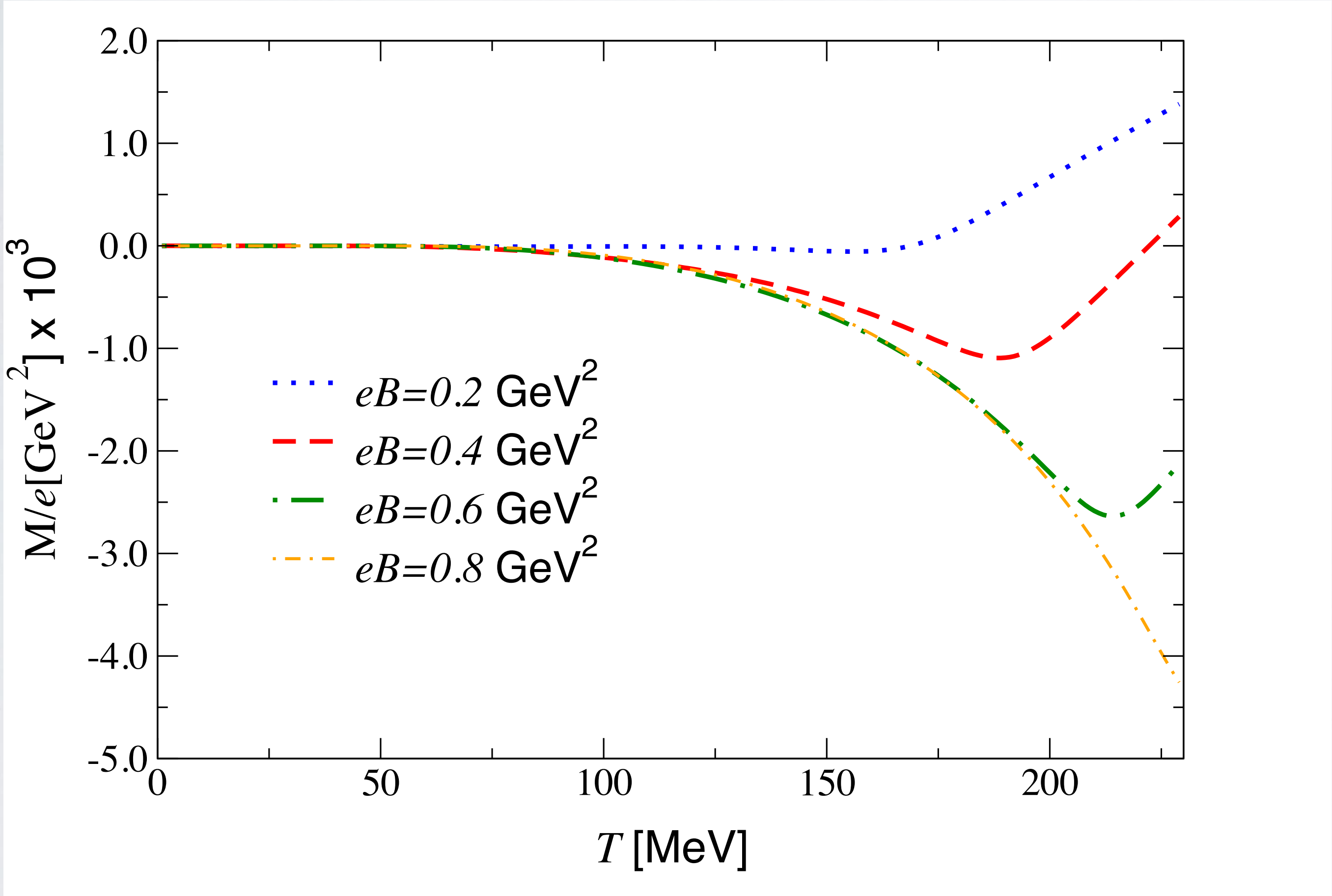
$$\frac{\partial \Omega}{\partial \phi_f} = 0 \qquad \frac{\partial \Omega}{\partial \rho} = 0$$

$$\mathcal{M} = \sum_f \left(\frac{\partial P_f^{\mathrm{mag}}}{\partial B} + \frac{\partial P_f^{\mathrm{Tmag}}}{\partial B} \right)$$

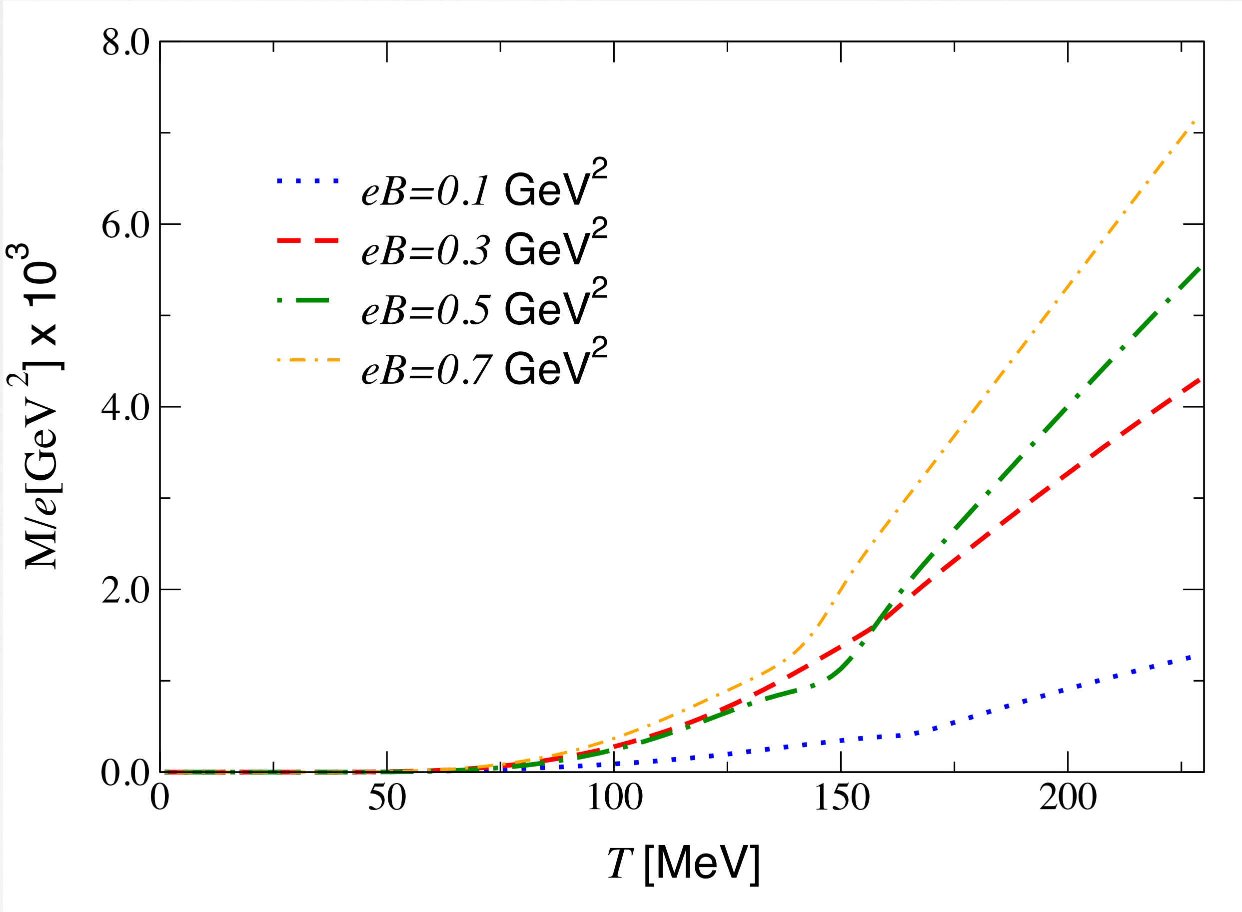
$$P = -\Omega$$

Magnetization

G



$G(eB, T)$



Meson properties under strong magnetic fields

$T = 0$

$$(ig_{\pi_0 qq})^2\, iD_{\pi_0}(k^2) = \frac{2iG}{1 - 2G\Pi_{PS}(k^2)}$$

$$D_{\pi_0}(k^2) = \frac{1}{k^2 - m_{\pi_0}^2}$$

$$\mathcal{L}_{\pi qq} = ig_{\pi qq}\bar{\psi}\gamma_5\vec{\tau}\cdot\vec{\pi}\,\psi$$

$$S_q(x,x') = e^{i\Phi_q(x,x')}\sum_{n=0}^{\infty} S_{q,n}(x-x')\ ,\ q = u,d$$

$$\beta_q = |q_q|B$$
$$k_\parallel = k_0 - k_3$$
$$g_n = 2 - \delta_{n0}$$

$$\frac{1}{i}\Pi_{PS}(k^2) = -\sum_{q=u,d}\int\frac{d^4p}{(2\pi)^4}Tr\left[i\gamma_5 iS_q\left(p+\frac{k}{2}\right)i\gamma_5\, iS_q\left(p-\frac{k}{2}\right)\right]$$

$$\frac{1}{i}\Pi_{PS}(k_\parallel^2) = -i\left(\frac{M-m}{2MG}\right) - \sum_{q=u,d}\beta_q N_c\frac{k_\parallel^2}{(2\pi)^3}\sum_{n=0}^{\infty} g_n I_{q,n}(k_\parallel^2)$$

$$I_{q,n}(k_\parallel^2) = \int d^2p_\parallel\frac{1}{[p_\parallel^2 - M^2 - 2\beta_q n][(p+k)_\parallel^2 - M^2 - 2\beta_q n]}$$

$$1 - 2G\,\Pi_{PS}(k^2)|_{k^2 = m_{\pi_0}^2} = 0$$

$$I({k_\parallel}^2, B) = I_{vac}(k_\parallel^2) + I(k_\parallel^2, B)$$

$$m_{\pi_0}^2(B) = -\frac{m}{M(B)}\frac{1}{4iGN_cN_f I(m_{\pi_0}^2, B)}$$

$$I(m_{\pi_0}^2, B) = \frac{1}{4(2\pi)^3}\sum_{q=u,d}\beta_q\sum_{n=0}^{\infty} g_n I_{q,n}(k_\parallel^2 = m_{\pi_0}^2)$$

Phys. Rev. D 93 (2016) 014010

Physics Letters B 767 (2017) 247-252

Simple $G(eB)$ at $T = 0$

- fit to lattice QCD condensates (few values of eB)
- interpolation to generate a larger set
- fit of the larger set to a shifted gaussian

$$G(eB) = \alpha + \beta\, e^{-\gamma\, (eB)^2}$$

$$\alpha = 1.44373 \text{ GeV}^{-2},\ \beta = 3.06 \text{ GeV}^{-2} \text{ and } \gamma = 1.31 \text{ GeV}^{-4}$$

$$G(0) = \alpha + \beta$$

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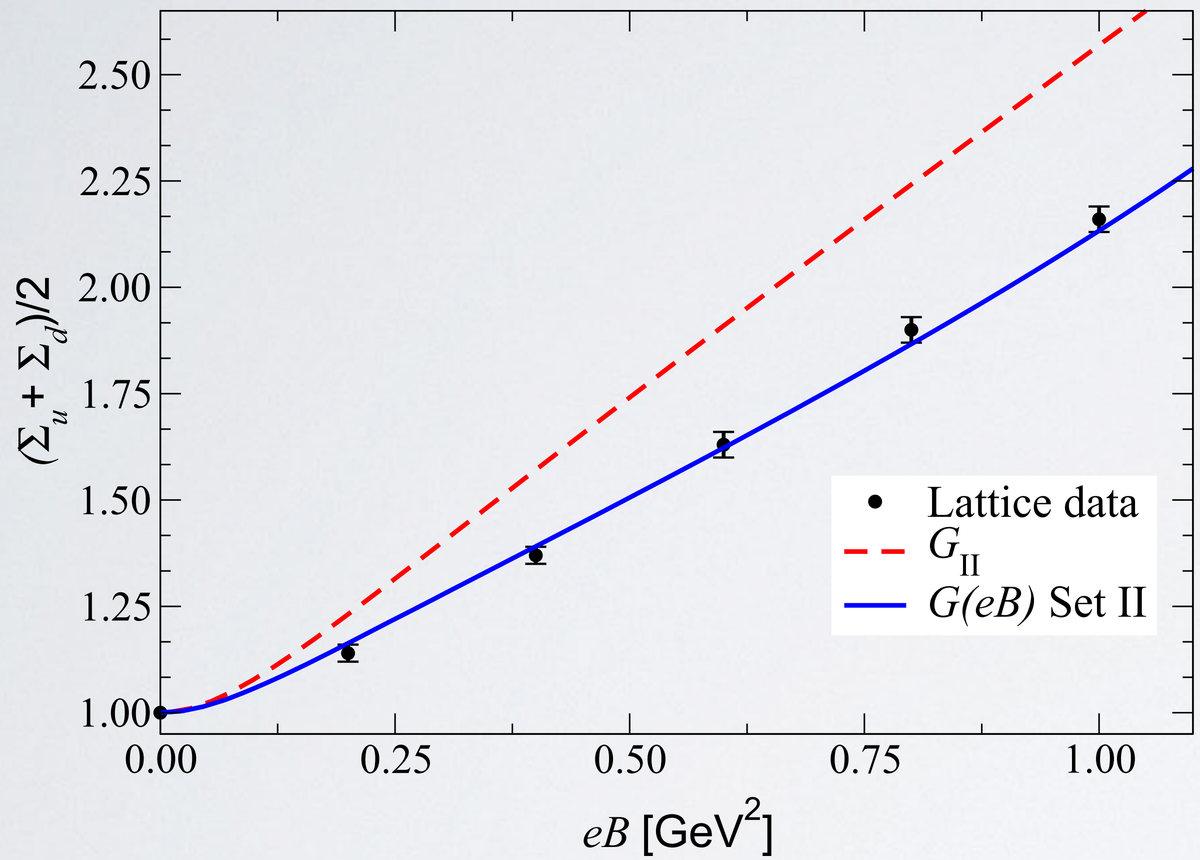
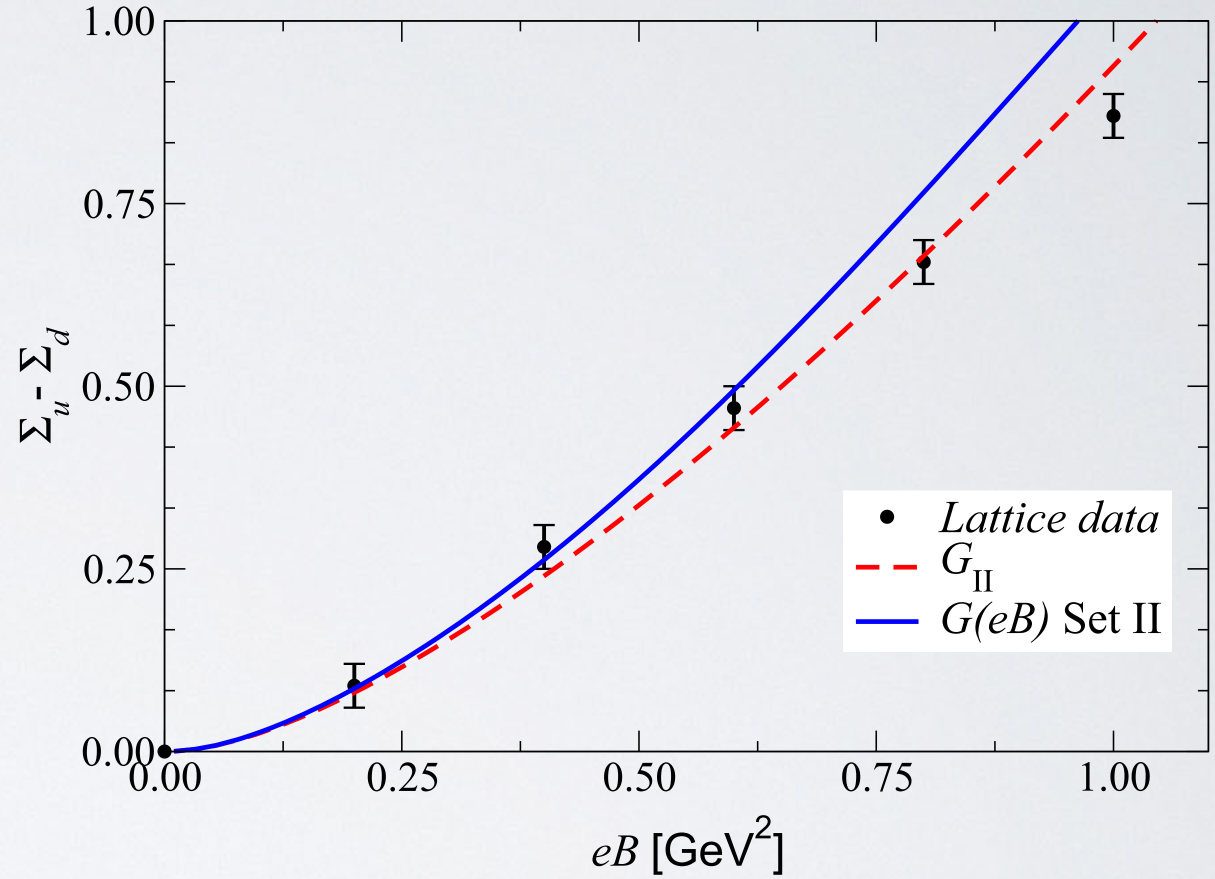
Condensates at $T = 0$





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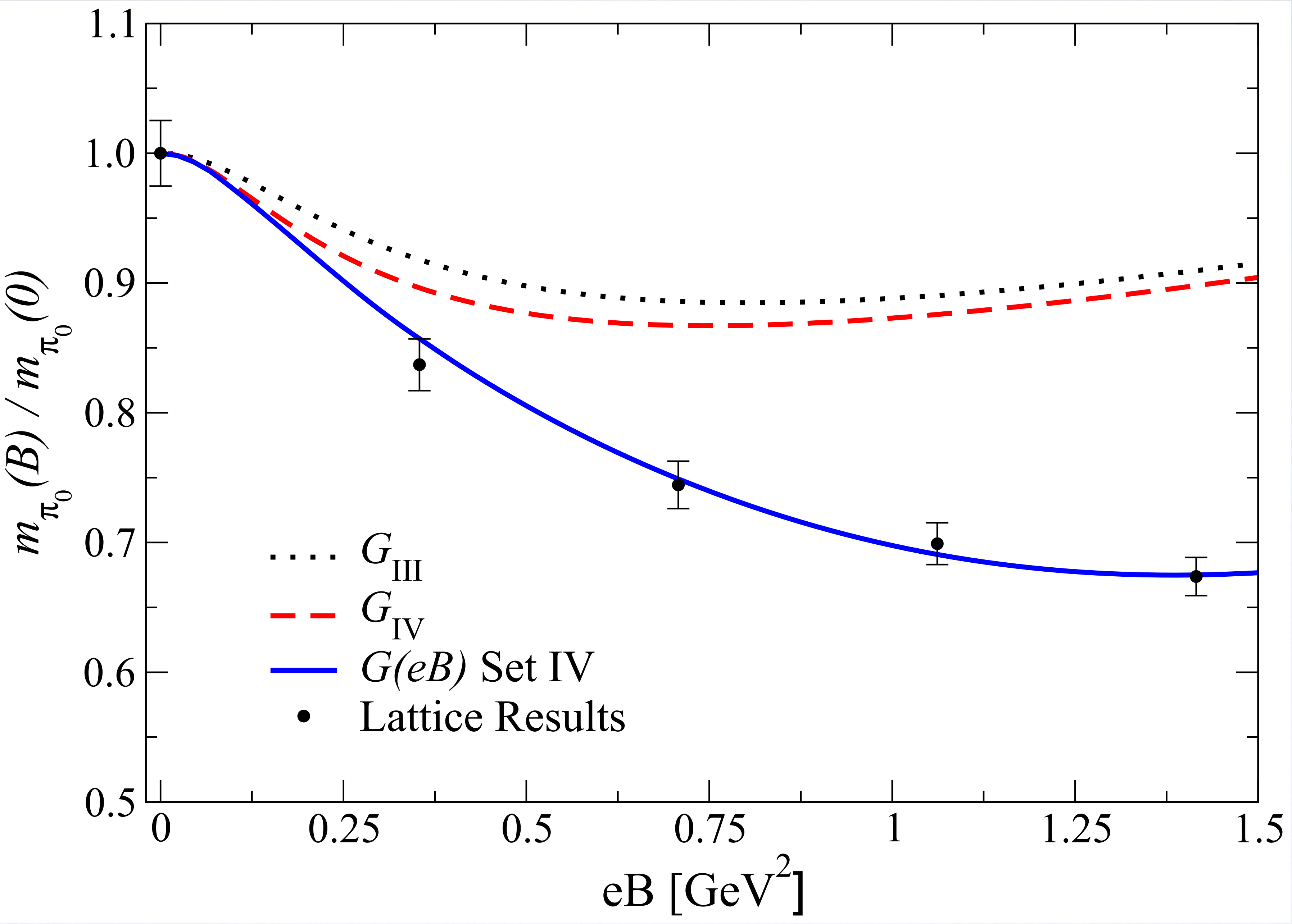
π_0 mass at $T = 0$



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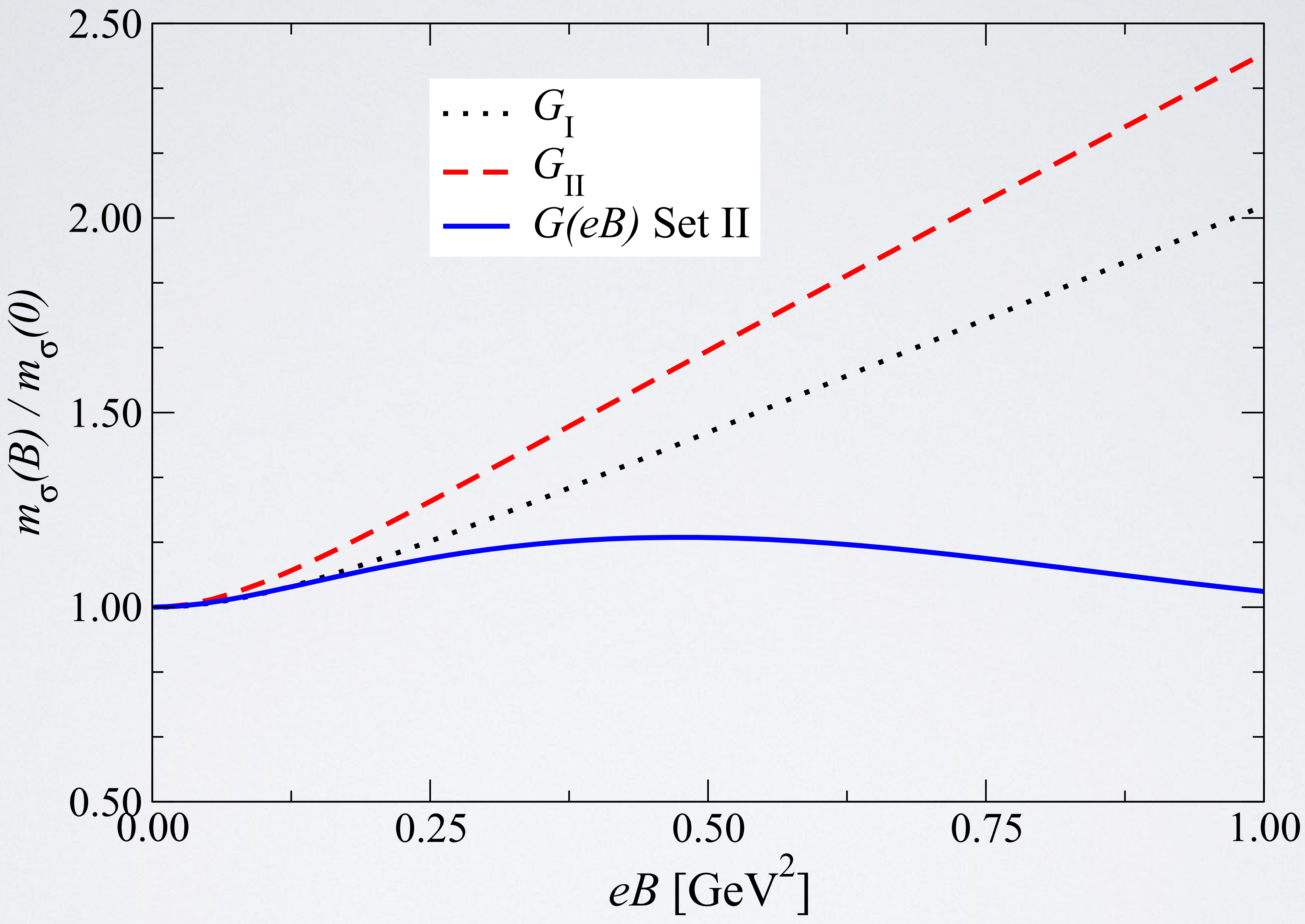
σ mass at $T = 0$



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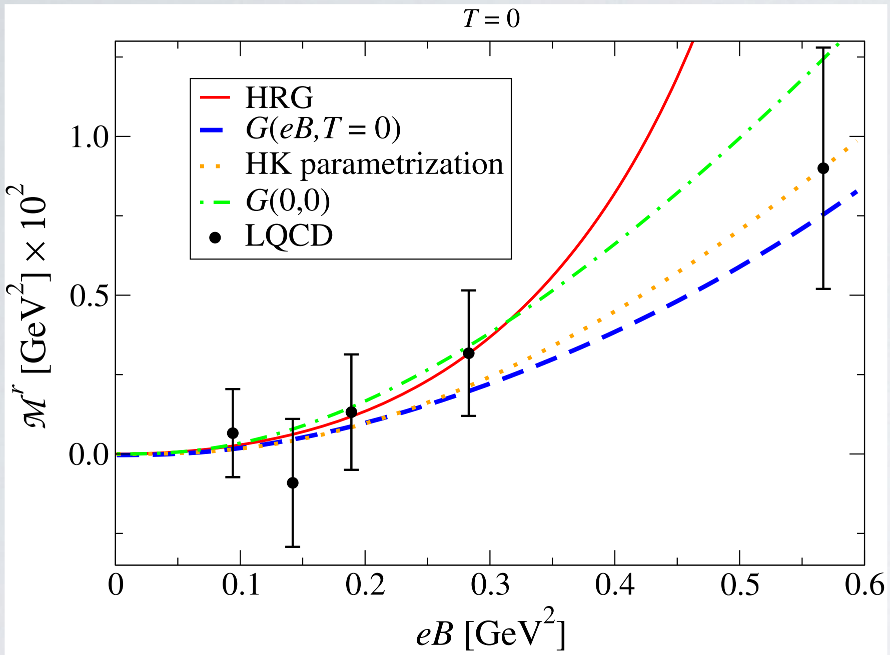
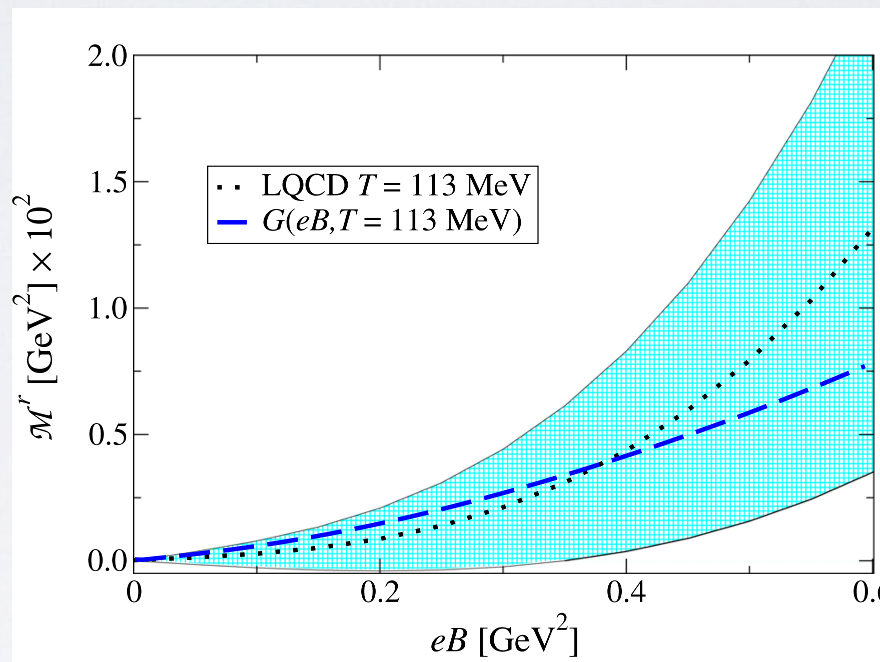
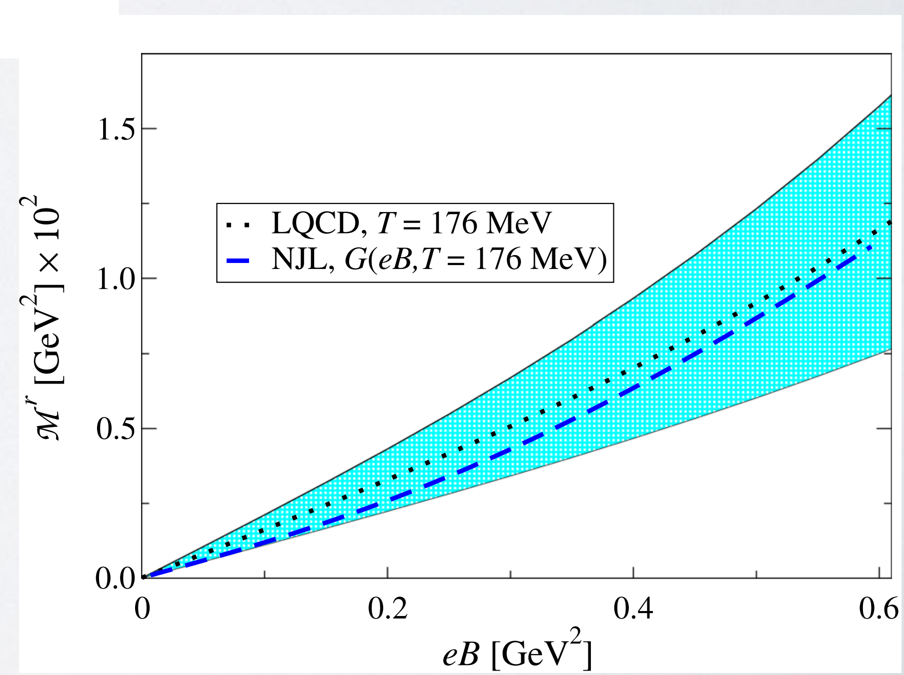
Renormalized Magnetization



$$\mathcal{M}^r \cdot eB = \mathcal{M} \cdot eB - (eB)^2 \lim_{eB \to 0} \frac{\mathcal{M} \cdot eB}{(eB)^2}\bigg|_{T=0}$$



$SU(3)$



Eur. Phys. J. A (2021) 57:278

Final remarks

* NJL models with fixed coupling fails to describe lattice QCD calculations

* Thermo-magnetic coupling seems to be adequate to improve NJL results

* Thermodynamic quantities are all affected by the variation of the coupling

* Sign of magnetization changes when $G \to G(eB, T)$

* Pion mass at $T = 0$ matches lattice QCD calculations with $G(eB)$