

Short distance constraints from HLbL contribution to the muon anomalous magnetic moment

Daniel Gerardo Melo Porrás^a

In collaboration with:

Angelo Raffaele Fazio^a, Edilson Alfonso Reyes Rojas^b

^aUniversidad Nacional de Colombia, ^bUniversidad de Pamplona

International School of Nuclear Physics,
EMFCSC

Erice, Sicily
September 23, 2023

Outline

- 1 HLbL scattering: The basics
- 2 OPE for $\Pi^{\mu_1\mu_2\mu_3\mu_4}$
- 3 Quark loop computation
- 4 Conclusions

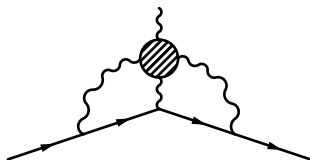
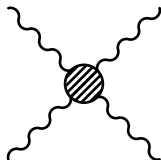
Outline for section 1

1 HLbL scattering: The basics

2 OPE for $\Pi^{\mu_1\mu_2\mu_3\mu_4}$

3 Quark loop computation

4 Conclusions

Basics of $\Pi^{\mu_1\mu_2\mu_3\mu_4}$ 

$$\Pi^{\mu_1\mu_2\mu_3\mu_4} = -i \int d^4x \int d^4y \int d^4z e^{-i(q_1x + q_2y + q_3z)} \\ \times \langle \Omega | J^{\mu_1}(x) J^{\mu_2}(y) J^{\mu_3}(z) J^{\mu_4}(0) | \Omega \rangle$$

- ➡ Loops on q_1 and q_2 sweep very different kinematic regions.
Alternate tools required near Λ_{QCD} and below it.

Dispersive computation of $\Pi^{\mu_1\mu_2\mu_3\mu_4}$

The “basis” of $\Pi^{\mu_1\mu_2\mu_3\mu_4}$ free of kinematic singularities and zeroes:

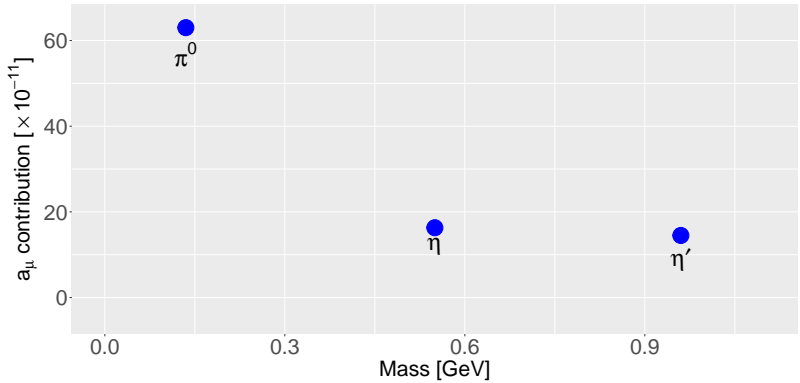
$$\Pi^{\mu_1\mu_2\mu_3\mu_4} = \sum_i^{54} \Pi_i T_i^{\mu_1\mu_2\mu_3\mu_4} \quad \text{Colangelo et al. (2017)}$$

- ➡ From 138 to 43 independent tensors vs. 1 in HVP.
- ➡ To avoid kinematic zeroes/singularities it is necessary to expand to 54 redundant Π_i . (Tarrach)
- ➡ Richer tensor structure made BTT basis much harder to find.

Low energy, data-driven a_μ^{HLbL}

Dispersive HLbL contributions to a_μ up to 1 GeV

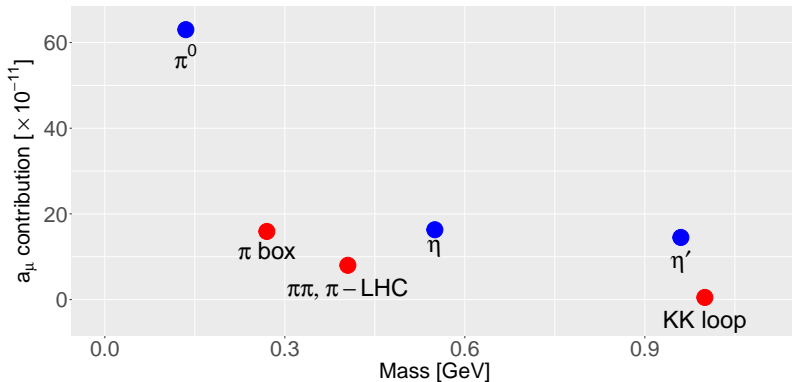
● Single particle



Low energy, data-driven a_μ^{HLbL}

Dispersive HLbL contributions to a_μ up to 1 GeV

● Single particle ● Multiple particles



Estimation of a_μ^{HLbL} in 1 GeV – 2 GeV

Hurdles for dispersive computation in the medium–energy region:

- ➔ Lack of data for: known form factors at high energy. and new unknown form factors.
- ➔ Increasing complexity of unitarity diagrams.
- ➔ Double counting issues.

The expected small size of contributions motivates narrow width approximation, which results in:

$$a_\mu^{\text{scalars+tensor}} = -1(3) \times 10^{-11}$$
$$a_\mu^{\text{axial vectors}} = 6(6) \times 10^{-11}$$

a_{μ}^{HLbL} at high energy and SDC

SDC = asymptotic behaviour of amplitudes.

Useful for:

- ➔ High energy contributions from light intermediate states. (Form factors)
- ➔ Full contributions from heavy intermediate states. (HLbL amplitude)

We focus in the latter case!

Two high virtualities regimes:

$$|q_1^2| \sim |q_2^2| \sim |q_3^2| \gg \Lambda_{\text{QCD}}^2 \qquad |q_1^2| \sim |q_2^2| \gg |q_3^2|, \Lambda_{\text{QCD}}^2$$

Outline for section 2

- 1 HLbL scattering: The basics
- 2 OPE for $\Pi^{\mu_1\mu_2\mu_3\mu_4}$
- 3 Quark loop computation
- 4 Conclusions

Symmetric asymptotic limit for $\Pi^{\mu_1\mu_2\mu_3\mu_4}$

- ➔ The OPE is an useful tool, meant for high energies, so one must be careful.
- ➔ Four-current OPE not suitable due to soft external photon:
- ➔ Mix of different scales is very problematic in QCD.

Bijnens et al.(2019, 2020, 2021) solved this with an OPE with a background soft photon for:

$$\begin{aligned}\Pi^{\mu_1\mu_2\mu_3} &= \frac{1}{e} \langle \Omega | J^{\mu_1}(q_1) J^{\mu_2}(q_2) J^{\mu_3}(0) | \gamma(q_4) \rangle \\ &= -\epsilon_{\mu_4} (q_4) \Pi^{\mu_1\mu_2\mu_3\mu_4}(q_1, q_2, q_3),\end{aligned}$$

OPE elements for $\Pi^{\mu_1\mu_2\mu_3}$

List of OPE elements:

$$S_{1,\mu\nu} \equiv ee_f F_{\mu\nu} ,$$

$$S_{2,\mu\nu} \equiv \bar{\Psi} \sigma_{\mu\nu} \Psi ,$$

$$S_{3,\mu\nu} \equiv ig_S \bar{\Psi} G_{\mu\nu} \Psi ,$$

$$S_{4,\mu\nu} \equiv ig_S \bar{\Psi} \bar{G}_{\mu\nu} \gamma_5 \Psi ,$$

$$S_{5,\mu\nu} \equiv \bar{\Psi} \Psi ee_f F_{\mu\nu} ,$$

$$S_{6,\mu\nu} \equiv \frac{\alpha_s}{\pi} G_a^{\alpha\beta} G_{\alpha\beta}^a ee_f F_{\mu\nu} ,$$

$$S_{7,\mu\nu} \equiv g_S \bar{\Psi} (G_{\mu\lambda} D_\nu + D_\nu G_{\mu\lambda}) \gamma^\lambda \Psi \\ + g_S \bar{\Psi} (G_{\nu\lambda} D_\mu + D_\mu G_{\nu\lambda}) \gamma^\lambda \Psi ,$$

$$S_{\{8\},\mu\nu} \equiv \alpha_s (\bar{\Psi} \Gamma \Psi \bar{\Psi} \Gamma \Psi)_{\mu\nu} ,$$

Same quantum numbers as $F_{\mu\nu}$:

➡ Second rank

➡ Antisymmetric

➡ Odd C-parity

Outline for section 3

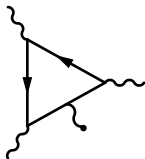
1 HLbL scattering: The basics

2 OPE for $\Pi^{\mu_1\mu_2\mu_3\mu_4}$

3 Quark loop computation

4 Conclusions

The quark loop



$$S_{1,\mu\nu} \rightarrow \Delta a_{\mu}^{\text{HLbL}} = 1.7 \times 10^{-10}$$

$$S_{2,\mu\nu} \rightarrow \Delta a_{\mu}^{\text{HLbL}} = -1.2 \times 10^{-12}$$

...

Wilson coefficient of $\langle F_{\mu\nu} \rangle$, proportional to:

$$i \frac{N_c}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{\partial}{\partial q_{4\nu_4}} \sum_{\sigma(1,2,4)} \left\{ \gamma^{\mu_3} S^0(p + q_1 + q_2 + q_4) \gamma^{\mu_4} \right. \\ \left. \times S^0(p + q_1 + q_2) \gamma^{\mu_1} S^0(p + q_2) \gamma^{\mu_2} S^0(p) \right\} \Big|_{q_4=0}$$

S^0 is the free fermion propagator. Due to mixing, its massless part is the real LO of the OPE.

Quark loop computation

Bijnens et al. (2020) uses the standard approach:

- ➔ Project $\bar{\Pi}_i$ out of the quark loop and compute the rest.
- ➔ Taylor expand massive propagators.
- ➔ Use Integration By Parts (IBP) to arrive at a small number of master integrals.

Quark loop computation

We tensor decompose the quark loop amplitude following Davydychev (1991), for instance

$$\begin{aligned} I_{\mu_1\mu_2}^{(2)}(d; \nu_1, \nu_2) = & -\frac{1}{2\pi} g_{\mu_1\mu_2} I^{(2)}(d+2; \{\nu_i\}) \\ & + \frac{1}{\pi^2} q_{1\mu_1} q_{1\mu_2} \nu_1(\nu_1+1) I^{(2)}(d+4; \nu_1+2, \nu_2) \\ & + \frac{1}{\pi^2} q_{2\mu_1} q_{2\mu_2} \nu_2(\nu_2+1) I^{(2)}(d+4; \nu_1, \nu_2+2) \\ & + \frac{1}{\pi^2} (q_{1\mu_1} q_{2\mu_2} + q_{1\mu_2} q_{2\mu_1}) \nu_1 \nu_2 I^{(2)}(d+4; \nu_1+2, \nu_2+1) \end{aligned}$$

It does not introduce **kinematic singularities**, but at the cost of **shifting dimensions**.

Superficial degree of divergence conserved!

Scalar integrals in shifted dimensions

We find 133 different scalar triangle and self-energy integrals in shifted dimensions.

Davydychev (1991) gives Mellin-Barnes (MB) representation for arbitrary scalar integrals:

$$I^{(2)}(d; \nu_1, \nu_2) = \int_{-i\infty}^{i\infty} ds \left(-\frac{q_1^2}{m^2} \right)^s \Gamma(-s) \frac{\Gamma(s + \nu_1) \Gamma(s + \nu_2)}{\Gamma(2s + \nu_1 + \nu_2)} \\ \times \Gamma\left(s + \nu_1 + \nu_2 - \frac{d}{2}\right)$$

Scalar integrals in shifted dimensions

Usual Taylor expansion of massive propagators:

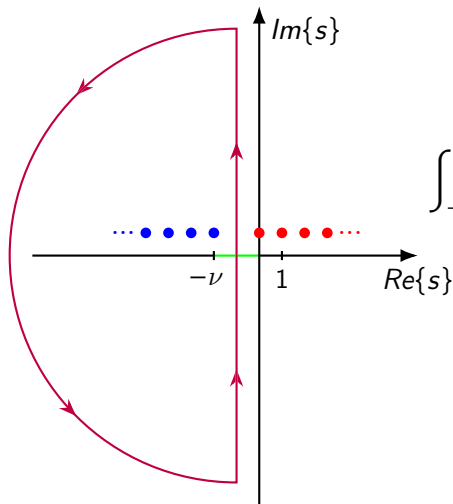
$$\begin{aligned}\frac{1}{(p^2 - m^2)^\nu} &= \frac{1}{p^{2\nu}} \frac{1}{\Gamma(\nu)} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{m^2}{p^2}\right)^n \Gamma(\nu + n) && \text{for } \left|\frac{m^2}{p^2}\right| < 1, \\ &= \frac{1}{(-m^2)^\nu} \frac{1}{\Gamma(\nu)} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{p^2}{m^2}\right)^n \Gamma(\nu + n) && \text{for } \left|\frac{p^2}{m^2}\right| < 1.\end{aligned}$$

MB representation is valid for any kinematic regime:

$$\frac{1}{(p^2 - m^2)^\nu} = \frac{1}{m^{2\nu}} \frac{1}{\Gamma(\nu)} \int_{-i\infty}^{i\infty} ds \left(-\frac{p^2}{m^2}\right)^s \Gamma(\nu + s) \Gamma(-s).$$

Full mass dependence conserved.

Scalar integrals in shifted dimensions

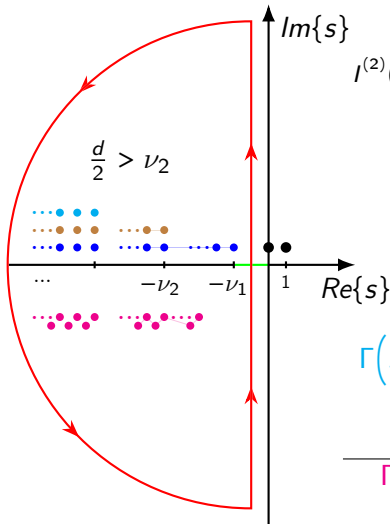


$$\int_{-i\infty}^{i\infty} ds \left(-\frac{p^2}{m^2}\right)^s \Gamma(\nu + s) \Gamma(-s)$$

$$\left|\frac{m^2}{p^2}\right| < 1$$

$$\left|\frac{p^2}{m^2}\right| < 1$$

Scalar integrals in shifted dimensions



$$I^{(2)}(d; \nu_1, \nu_2) = \int_s \left(-\frac{q_1^2}{m^2} \right)^s \Gamma(-s) \frac{\Gamma(s + \nu_1) \Gamma(s + \nu_2)}{\Gamma(2s + \nu_1 + \nu_2)} \times \Gamma\left(s + \nu_1 + \nu_2 - \frac{d}{2}\right)$$

$$\frac{\Gamma\left(s + \nu_1 + \nu_2 - \frac{d}{2}\right) \Gamma(s + \nu_2) \Gamma(s + \nu_1)}{\Gamma(2s + \nu_1 + \nu_2)} \Bigg| \Gamma(-s)$$

Scalar integrals in shifted dimensions

Nested MB integrals require multivariate residues (*MBConicHulls* (*B. Ananthanarayan et al, 2021*), *Multivariate residues* (*Larsen, 2017*))

- ➔ There are several different triple (almost) hypergeometric series representation for one integral.
- ➔ Convergence of hypergeometric series studied with Horn's theorem.
- ➔ Too many integrals with too many representations: We wrote a script to check the convergence region of series representations.

Scalar integrals in shifted dimensions

Two-point example for $|q^2| > 4m^2$:

$$\begin{aligned}
 I^{(2)}(6; \nu, \nu + 1) &= \frac{-i}{(4\pi)^{5/2}} \frac{(m^4)^{1-\nu}}{\Gamma(\nu)\Gamma(\nu+1)} \\
 &\times \left\{ \left(-\frac{4m^2}{q^2} \right)^\nu \sum_{k=0}^{\nu-3} \left(\frac{4m^2}{q^2} \right)^k \frac{\Gamma(\nu+k)}{\Gamma(-k+\frac{1}{2})} \frac{\Gamma(-k+\nu-2)}{\Gamma(k+1)} \right. \\
 &- \left(-\frac{16m^4}{q^4} \right)^{\nu-1} \sum_{l=0}^{\infty} \left(-\frac{4m^2}{q^2} \right)^l \frac{\Gamma(-2+2\nu+l)}{\Gamma(-\nu-l+\frac{5}{2})} \frac{1}{\Gamma(l+1)\Gamma(l+\nu-1)} \\
 &\times \left(\ln \left\{ -\frac{q^2}{4m^2} \right\} - \psi^{(0)}(-2+2\nu+l) - \psi^{(0)}(-\nu-l+\frac{5}{2}) - 3\psi^{(0)}(l+1) \right. \\
 &\quad \left. \left. + \psi^{(0)}(l+\nu-1) \right) \right\}
 \end{aligned}$$

Renormalization of the OPE and mixing

All Wilson coefficients are finite at LO, why is renormalization needed?

To deal with large logarithms: last step for separation of long and short distance effects.

$$\sum_n \alpha_S^n \ln^n \frac{|q_i^2|}{m_q^2} \longrightarrow \sum_n \alpha_S^n \ln^n \frac{|q_i^2|}{\mu^2}$$

μ : renormalization scale.

How does one renormalize a c-number background field operator?
OPE elements are also split in background + fluctuation.

Check of BTT tensor decomposition of $\Pi^{\mu_1\mu_2\mu_3\mu_4}$

$$\left. \frac{\partial \Pi^{\mu_1\mu_2\mu_3\mu_4}}{\partial q_{4\mu_5}} \right|_{q_4 \rightarrow 0} = \sum_i^{19} \Pi_i \partial^{\mu_5} T_i^{\mu_1\mu_2\mu_3\mu_4} \Big|_{q_4 \rightarrow 0}$$

- ➡ All form factors receive contributions from the quark loop? (Weak)

$$\Pi_i \neq 0$$

- ➡ Are there contributions to tensor structures not considered in BTT? (Strong)

$$\left. \frac{\partial \Pi^{\mu_1\mu_2\mu_3\mu_4}}{\partial q_{4\mu_5}} \right|_{q_4 \rightarrow 0} - \sum_i^{19} \Pi_i \partial^{\mu_5} T_i^{\mu_1\mu_2\mu_3\mu_4} \Big|_{q_4 \rightarrow 0} = 0$$

Outline for section 4

1 HLbL scattering: The basics

2 OPE for $\Pi^{\mu_1\mu_2\mu_3\mu_4}$

3 Quark loop computation

4 Conclusions

Conclusions

Main contributions:

- ➔ Independent check for the quark loop computation.
- ➔ Check of the complete span of the dispersive tensor basis for the quark loop amplitude.
- ➔ Complement for the *MBConicHulls* Mathematica package to automatize the study of convergence regions of multiple variables (almost) hypergeometric series.

Perspectives

Perspectives:

- ➔ Further perturbative corrections to $O(\alpha_S^2)$. Already done at $O(\alpha_S)$ by Bijmans et al. (2021)
- ➔ OPE for the HLbL tensor in the mixed–virtualities asymptotic regime. Axial anomaly gives lowest dimensional operator. Work in progress by Bijmans et al. (2023)

Thanks!