Studying the $^3P_0$ decay model from Landau gauge QCD

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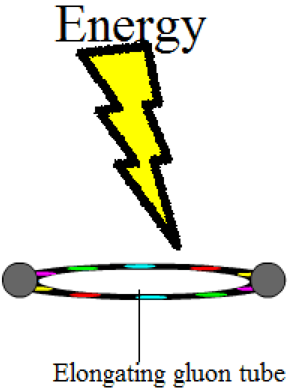
INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS
44th Course, Erice
23/09/2023

Usual picture for hadron decay:



Hadron decay and String Breaking

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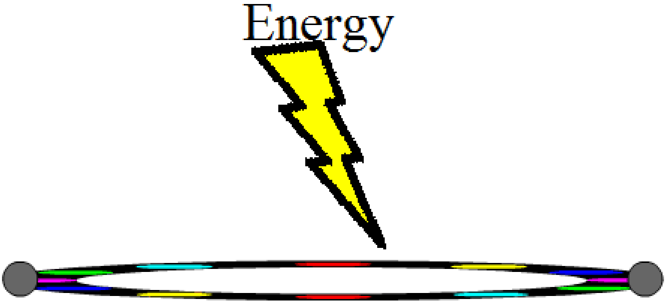

flux tube energy grows with inter-quark separation and creates a $q\bar{q}$ breaking the tube.

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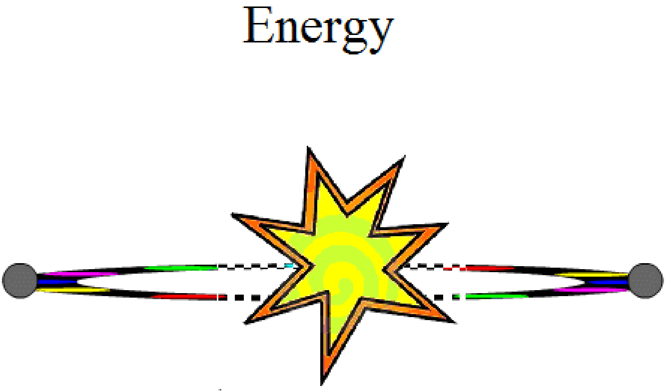

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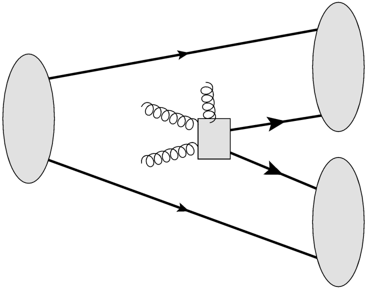
flux tube energy grows with inter-quark separation and creates a $q\bar{q}$ breaking the tube.

$q\bar{q}$ OZI-allowed meson decays



L. Micu, NPB 10 (1969) 521-526

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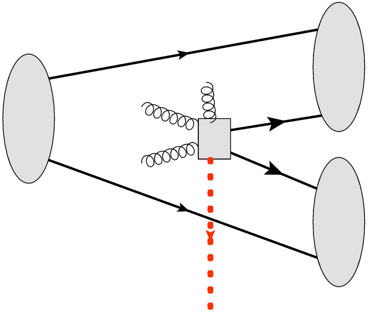

$^1S_0, ^1P_1, ^3S_1, ^3P_0, ^3P_1, ^3P_2 \ldots ^{2S+1}L_J$

Possible Q# of produced pair

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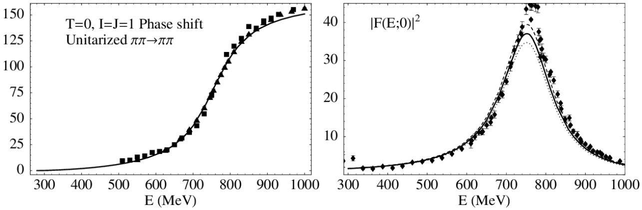


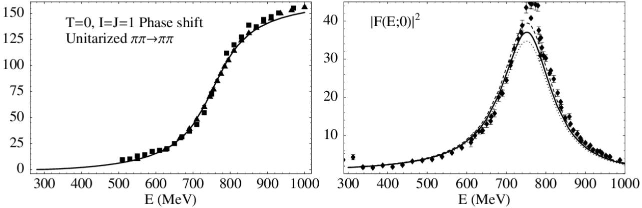
$\cancel{^1S_0}, \cancel{^1P_1}, ^3S_1, ^3P_0, ^3P_1, ^3P_2 \dots$

Think of $\rho(\uparrow\uparrow) \to \pi(\uparrow\downarrow)\pi(\uparrow\downarrow)$

Transition amplitude must have spin 1.

A. Gómez-Nicola *et al.* PLB **606** 351-360 (2005)

$q\bar{q}$ OZI-allowed meson decays



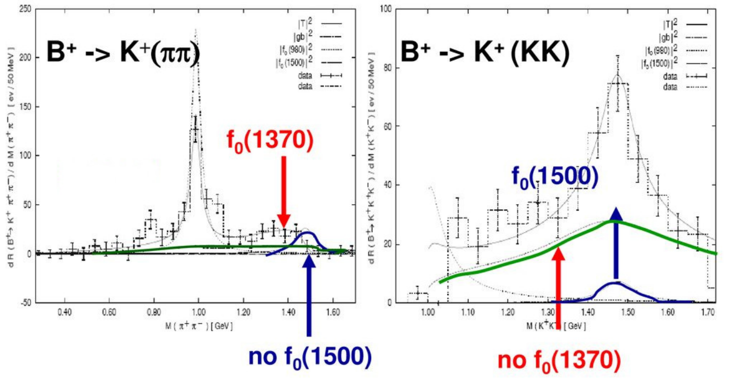
$$^1S_0, {}^1P_1, {}^3\not{S}_1, {}^3P_0, {}^3P_1, {}^3P_2 \ldots$$

$$\text{Think of } \rho(\text{s-wave}) \to \underbrace{\pi(\text{s-wave})\pi(\text{s-wave})}_{L=1}$$

Transition amplitude must carry a P-wave.

A. Gómez-Nicola *et al.* PLB **606** 351-360 (2005)

$q\bar{q}$ OZI-allowed meson decays

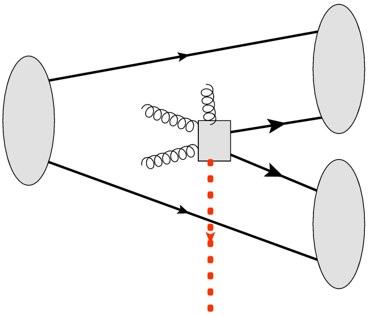


$$^1S_0, {}^1P_1, {}^3S_1, {}^3P_0, {}^3\cancel{P_1}, {}^3\cancel{P_2} \ldots$$

Move on to $\underbrace{f_0}_{\text{mainly } ^3P_0} \rightarrow \underbrace{\pi\pi}_{J=0}$

E. Klempt `https://slideplayer.com/slide/14648261/`

Lore: important $^3P_0$ pair production mechanism



$^1S_0, ^1P_1, ^3S_1, ^3P_0, ^3P_1, ^3P_2 \ldots ^{2S+1}L_J$

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Not visible in QCD (or QED)

- $\int d^3x \; \bar{\psi} \; \boldsymbol{\gamma} \cdot \mathbf{A} \; \psi$ seems ${}^3S_1$

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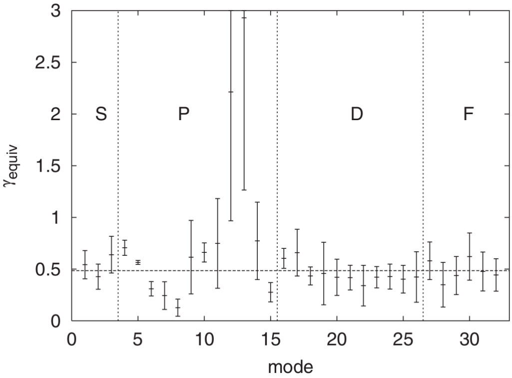
- $\int d^3x \; \bar{\psi} \; \gamma \cdot \mathbf{A} \; \psi$ seems ${}^3S_1$
- Chiral-symmetry respecting at all orders in perturbation theory
- But ${}^3P_0$ breaks chiral symmetry

Modelling the D/D_s spectrum with $^3P_0$: 32 modes studied by Close and Swanson



F. Close and E.S.Swanson PRD**72** 094004 (2005)

Effective Hamiltonian for $^3P_0$

$$H_{^3P_0} = \sqrt{3} g_s \int d^3\boldsymbol{x} \bar{\psi}(\boldsymbol{x}) \psi(\boldsymbol{x})$$

$\gamma = \frac{g_s}{2m}$

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Chiral-symmetry breaking:

$$[Q_5, H_{^3P_0}] = \left[\int d^3\boldsymbol{x} \psi^\dagger(\boldsymbol{x})\gamma_5\psi(\boldsymbol{x}), H_{^3P_0}\right] \neq 0$$

Effective Hamiltonian for ${}^3P_0$

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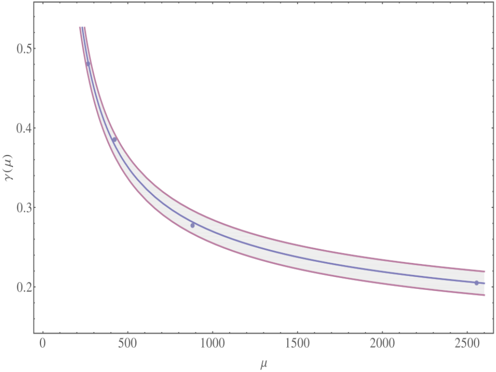
$$[Q_5, H_{{}^3P_0}] = \left[\int d^3\boldsymbol{x}\psi^\dagger(\boldsymbol{x})\gamma_5\psi(\boldsymbol{x}), H_{{}^3P_0}\right] \neq 0$$

$$i(2\pi)^4\delta^{(4)}(p+q)\mathcal{M}^{ss'}_{{}^3P_0}(p,q) = \langle \boldsymbol{p}s, \boldsymbol{q}s'|iT_{{}^3P_0}|0\rangle =$$
$$= (i\,(2\pi)^4\delta^{(4)}(p+q))(-\sqrt{3}g_s)\bar{u}^s(p)v^{s'}(q)$$

$$\boxed{\Rightarrow \bar{u}^s(\boldsymbol{p})v^{s'}(-\boldsymbol{p}) = 2\boldsymbol{p}\cdot\boldsymbol{\sigma}^{ss'}}$$

Dependence on the quark mass by Salamanca group



J. Segovia, D. R. Entem, F. Fernández Phys.Lett.B **715** (2012) 322-327

Ongoing work: connect Quark-model pheno w. Landau gauge QCD

Can we obtain the $^3P_0$ effective Hamiltonian from *ab initio* QCD calculations?

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Can we obtain the $^3P_0$ effective Hamiltonian from *ab initio* QCD calculations?

- N-gluon to $\bar{q}q$ kernel not known from first principles
- What to do with the information at hand?

Strategy: couple dynamical quarks to the flux tube background

Flux tube $\longleftrightarrow$ Dynamical quarks

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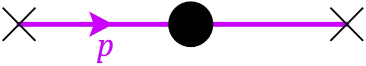
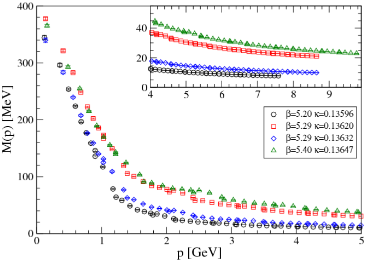
Flux tube $\longleftrightarrow$ Dynamical quarks

Through Landau-gauge DSE primitive QCD Green's functions

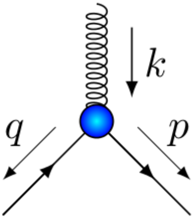
Extensive lattice+DSE work on Landau gauge primitive Green's functions

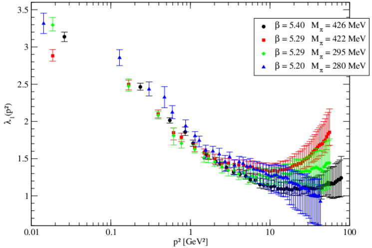




Lattice data from O. Oliveira *et al.* Acta Phys.Polon.Supp. 9 (2016) 363-368

Extensive lattice+DSE work on Landau gauge primitive Green's functions

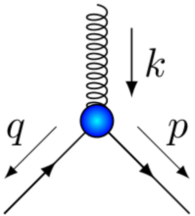




And also the pure Yang-Mills primitive Green's functions...

Lattice data from O. Oliveira *et al.* Acta Phys.Polon.Supp. 9 (2016) 363-368

Quark-Gluon vertex spin structure



$$\Gamma^{\mu}_{T}(q_E, p_E; k_E) = \sum_{i=1}^{8} g_i(\bar{p}_E^2)\rho_i^{\mu}(q_E, p_E)$$

$$k^{\mu} = p^{\mu} + q^{\mu}$$

- It includes chiral-symmetry respecting and breaking pieces

Parametrization of transverse part of the vertex

- The tree-level vertex $\rho^{\mu}_{1,E} = (\delta^{\mu\nu} - \hat{k}^{\mu}_{E}\hat{k}^{\nu}_{E})\gamma^{\mu}_{E} \equiv \gamma^{\mu}_{T,E}$

  with $\quad g_1(x) = 1 + \frac{1.67+0.204x}{1+0.683x+0.000851x^2}$

Parametrization of transverse part of the vertex

- Chiral-symmetry breaking structures ($s_E^\mu = (\delta^{\mu\nu} - \hat{k}_E^\mu \hat{k}_E^\nu)\bar{p}_E^\nu$)

$$\rho_{2,E}^\mu = i\hat{s}_E^\mu \quad \text{and} \quad \rho_{3,E}^\mu = i\hat{\not{k}}_E \gamma_{T,E}^\mu$$

$$\text{with} \quad g_3(x) = -1.45 g_2(x) = \frac{0.365x}{0.0187 + 0.353x + x^2} \ ;$$

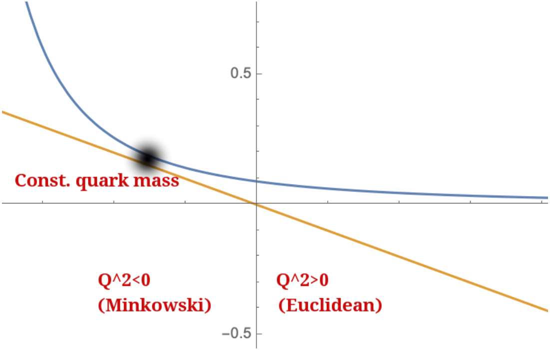
Parametrization of Landau-transverse part of the vertex

- The chirally symmetric structures

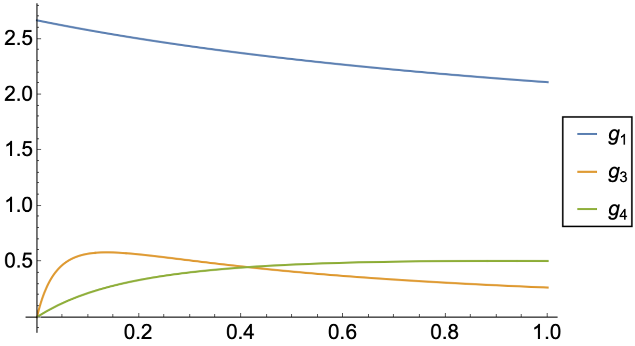
$$\rho^{\mu}_{4,E} = \hat{\not{k}}_E s^{\mu}_E \quad \text{and} \quad \rho^{\mu}_{7,E} = \hat{\not{s}}_E \hat{\not{k}}_E \gamma^{\mu}_{T,E}$$

$$\text{with} \quad g_4(x) = g_7(x) = \frac{2.59x}{0.859+3.27x+x^2} \ .$$

First, the propagator mass function:

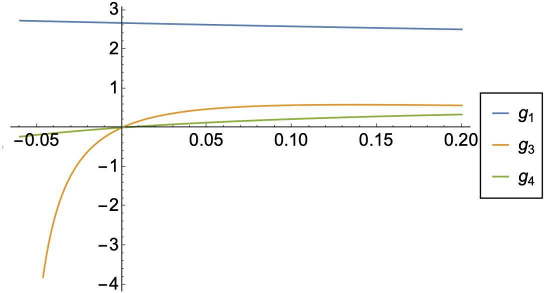


Euclidean q_E^2 functions (input from lattice, DSEs)



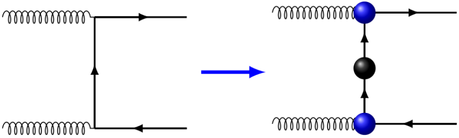
Extension to physical Minkowski space

Next, the vertex dressing form factors:



Note the $Q^2 < 0$ enhancement of the chiral symmetry breaking piece!

Breit-Wheeler process for $q\bar{q}$ creation



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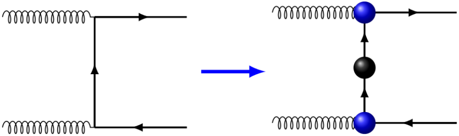
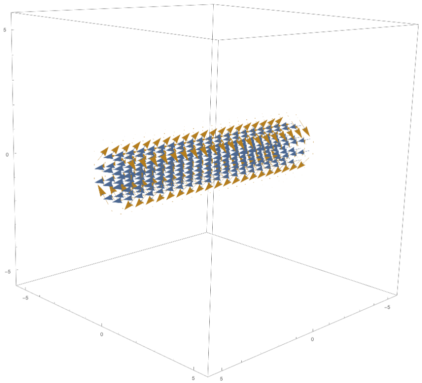


We couple flux-tube “gluons” to the quarks with the DSE functions

In a constant chromoelectric flux tube:



- Simplify to a constant chromo-E (parallel-plate capacitor)
  Background Landau-gauge field $(A_\rho, A_\theta, A_z, A_0) = (0, 0, 0, -Ez)$
- Think of the Schwinger pair-creation mechanism in QED

$$\langle \boldsymbol{p}s, \boldsymbol{q}s' | iT_{\text{singlet}} | 0 \rangle =$$

$$\langle \boldsymbol{p}s, \boldsymbol{q}s' | -\frac{g^2}{2} \int d^4x \bar{\psi}_i(x) T^a_{ij} A^a_\mu(x) \Gamma^\mu \psi_j(x) \int d^4y \bar{\psi}_i(y) T^a_{ij} A^a_\nu(y) \Gamma^\nu \psi_j(y) | 0 \rangle =$$

$$= -g^2 \int d^4x d^4y \int \frac{d^4t}{(2\pi)^4} \tilde{A}^a_0(p-t) \tilde{A}^a_0(q+t) \mathcal{K}^{ss'}_{ab}(p,q,t)$$

$$\langle \boldsymbol{p}s, \boldsymbol{q}s'|iT_{\text{singlet}}|0\rangle =$$

$$\langle \boldsymbol{p}s, \boldsymbol{q}s'| - \frac{g^2}{2}\int d^4x\bar{\psi}_i(x)T^a_{ij}A^a_\mu(x)\Gamma^\mu\psi_j(x)\int d^4y\bar{\psi}_i(y)T^a_{ij}A^a_\nu(y)\Gamma^\nu\psi_j(y)|0\rangle =$$

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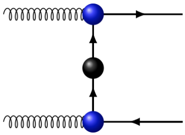
where, in the "skeleton" expansion,

$$\mathcal{K}^{ss'}_{ab}(p,q,t) \equiv \left[\bar{u}^s_i(p)T^a_{ij}\Gamma^0(p,-t)S(t)T^b_{jk}\Gamma^0(q,t)v^{s'}_k(q)\right]$$

and $S(t)$ is the dressed fermion propagator.

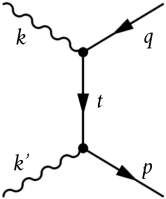


A relation between the gluon to quark kernel $\mathcal{K}$ and the pair production amplitude

$$\langle \boldsymbol{p}s, \boldsymbol{q}s' | iT_{\text{singlet}} | 0 \rangle = -(2\pi)^4 \delta^{(4)}(p+q)(gE)^2 \left[\frac{\partial}{\partial p^3} \frac{\partial}{\partial q^3} \mathcal{K}_{ab}^{ss'}(p,q,t) \right] \Big|_{t=-q}$$

- With the primitive Green's functions construct this skeleton kernel ✓
- Project it over ${}^{2S+1}L_J$ and numerically compare
  (But you can see that the chiral symmetry breaking part will be important, perhaps even dominant)

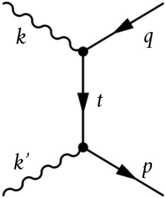
QED computation



For a $(A_\rho, A_\theta, A_z, A_0) = (0, 0, 0, -Ez)$ background field

$$\mathcal{M}^{ss'}(\boldsymbol{p}) \propto \left[\frac{\partial}{\partial p^3}\frac{\partial}{\partial q^3}\left(\bar{u}^s(\boldsymbol{p})(\gamma^0_T(p-t))(\not{t}+m)(\gamma^0_T(q+t))v^{s'}(\boldsymbol{q})\right)\right]\Big|_{\boldsymbol{t}=-\boldsymbol{q}=\boldsymbol{p}}$$

QED computation



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At symmetric point: $k^2 = (p-t)^2 = E_{\boldsymbol{p}}^2$ and $k'^2 = (q+t)^2 = E_{\boldsymbol{q}}^2$...

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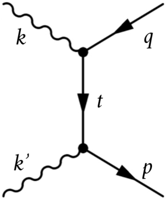

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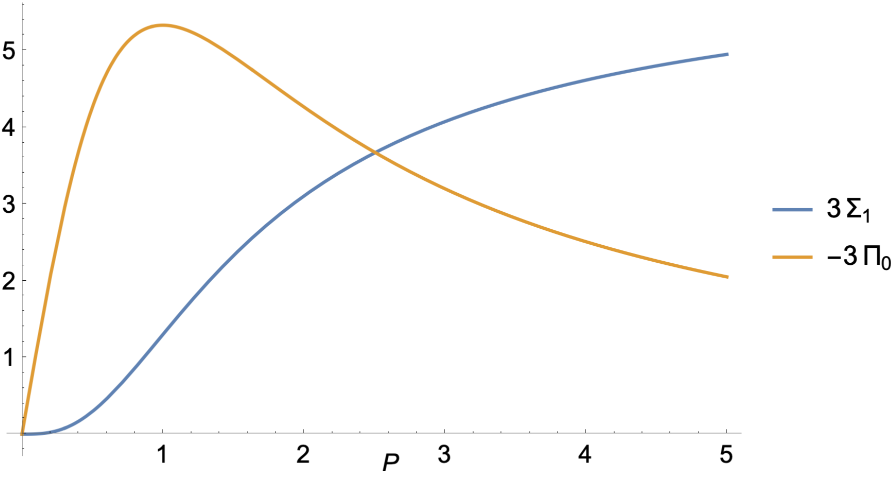
At symmetric point: $k^2 = (p-t)^2 = E_{\boldsymbol{p}}^2$ and $k'^2 = (q+t)^2 = E_{\boldsymbol{q}}^2$...
This amplitude has a ${}^3P_0$ contribution!:

$$\mathcal{M}_{\text{QED}}^{{}^3\Pi_0}(|\boldsymbol{p}|) \propto -\frac{16m|\boldsymbol{p}|}{3E_{\boldsymbol{p}}^2} \quad \mathcal{M}_{\text{QED}}^{{}^3\Sigma_1}(|\boldsymbol{p}|) \propto \pi|\boldsymbol{p}|\left(\frac{E_{\boldsymbol{p}}-m}{E_{\boldsymbol{p}}^2}\right)$$

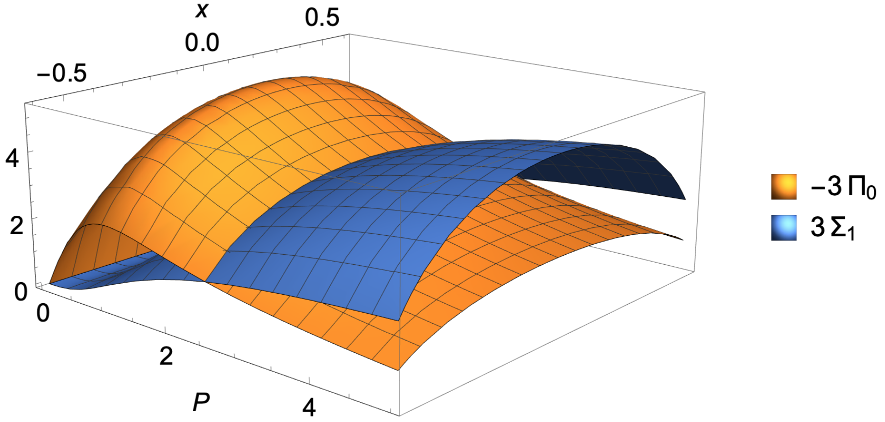
Diatomic-molecule notation.

Comparison of spins structures in QED



Comparison of spins structures in QED, around symmetric point

$k^2 = (p-t)^2 = (1-x)E_{\boldsymbol{p}}^2$ and $k'^2 = (q+t)^2 = (1+x)E_{\boldsymbol{q}}^2$



Currently constructing QCD amplitudes.

Currently constructing QCD amplitudes. Stay tuned!!

Recap

- Historical $^3P_0$ mechanism of strong decays needs QCD grounding.

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- Historical $^3P_0$ mechanism of strong decays needs QCD grounding.
- Working on it from Landau gauge Green's functions and coupling $q\bar{q}$ to a flux tube (ideas welcome)
- It appears that the chiral symmetry breaking piece is indeed important.
- QED already has $^3P_0$ contribution.

Acknowledgments

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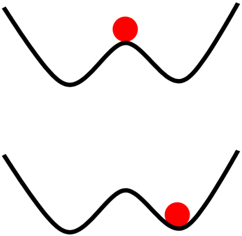
Chiral symmetry breaking



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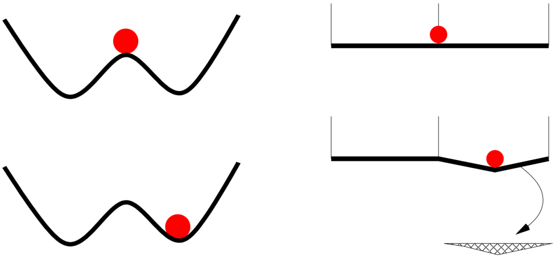

R. Alkofer *et al.* Annals Phys. **324** (2009) 106-172