Properties of Four-Quark states from functional methods

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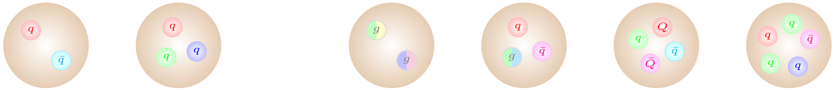
Motivation

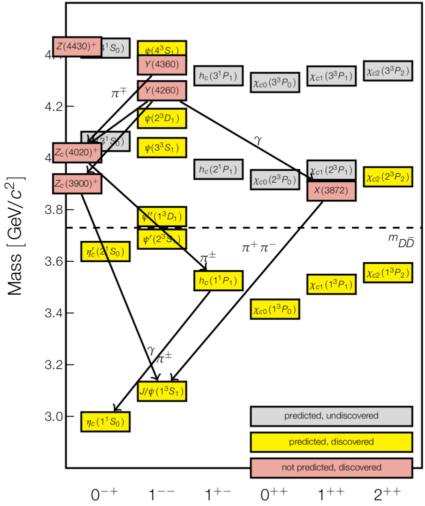
Conventional Hadrons: Exotic Hadrons:



Mesons Baryons Glueballs Hybrids Four-quark states Pentaquarks

- Powerful toolkit to classify conventional hadrons: Quark Model (QM).
- Lot of particles measured that do not fit into the QM picture, i.e., exotic hadrons.
- A few examples:
  - Light scalar mesons: σ, κ, a_0, f_0
  - Exotic XYZ-states: $X(3872)$, $X(3915)$, $Z_c(3900)$, $Z_c(4430)$

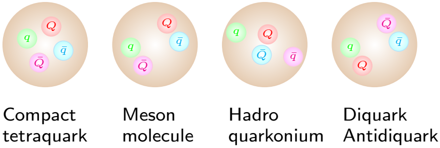
Tetraquark candidates in the charmonium region



Wolfgang Gradl, BESIII, St Goar 2015

Many unexpected states found by Belle, BARBAR, BES, LHCb, ...

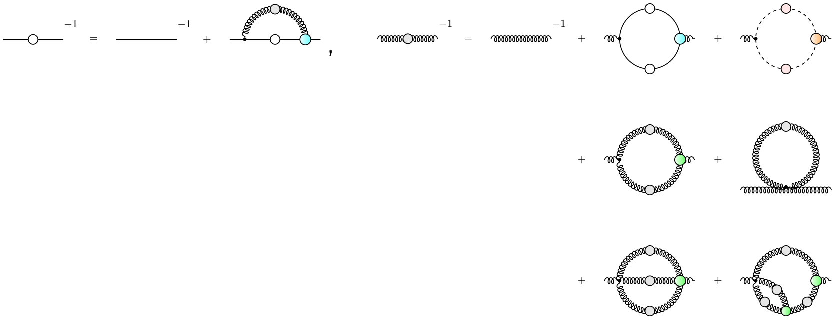
Internal structure??



Related to details of underlying QCD forces between quarks

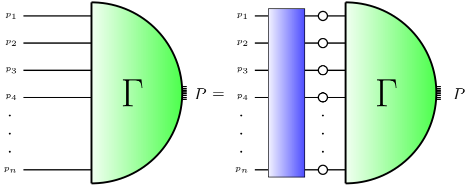
Functional Framework

- Non-perturbative, fully relativistic framework.
- To compute the properties of bound states, use combination of:
  - DSEs: The QCD quantum equations of motion,



Functional Framework

- Non-perturbative, fully relativistic framework.
- To compute the properties of bound states, use combination of:
  - DSEs: The QCD quantum equations of motion,
  - Hadronic bound state equations: BSEs, Faddeev eqs. .



Eigenvalue equation

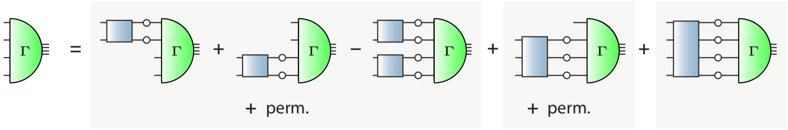
$$\lambda(P^2)\,\Gamma^{(n)} = K^{(n)} G^{(n)}\,\Gamma^{(n)}$$

with $\lambda(P^2 = -M^2) = 1$

Four-quark BSE

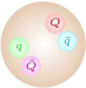
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, Fischer, PLB 718 (2012) 545-549
Eichmann, Fischer, Heupel, PLB 753 (2016) 282-287

Exact equation:

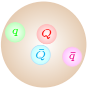


Two-body interactions

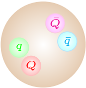
Three- and four-body interactions



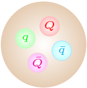
Meson molecule



Hadro quarkonium

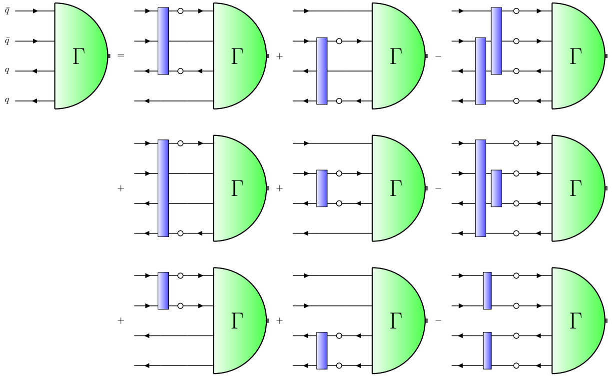


Diquark Antidiquark



Compact tetraquark

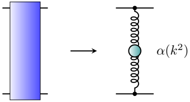
The Four-quark BSE:

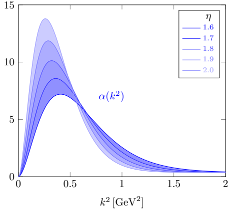


$$\Gamma(k,q,p,P) = \sum_i f_i \underbrace{(\ldots)}_{\text{Lorentz-invariants}} \tau_i(k,q,p,P) \otimes \Gamma_C \otimes \Gamma_F$$

Lorentz-invariants

Calculations are done in the *Rainbow-Ladder truncation*:



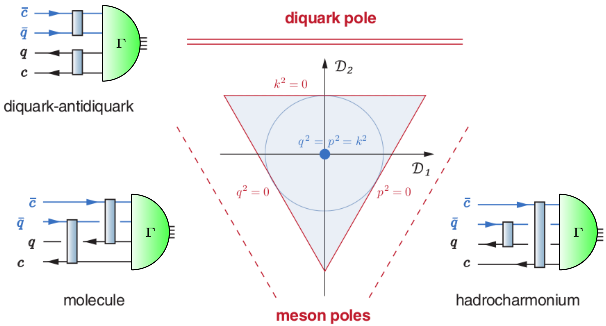


Maris, Tandy, PRC 60 (1999)
Qin et al., PRC 84 (2011)

- Can cast the Lorentz-invariants into multiplets of S_4:

  Eichmann, Fischer, Heupel, Phys.Lett.B 753 (2016) 282-287

  - One singlet: S_0
  - One doublet: $D = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}$
  - Two triples: T_0, T_1 $\rightarrow$ subleading
- Dressing functions: $f_i(S_0, D)$
- Poles dynamically generated in D
- "Physical basis": put poles in externally: $f_i(S_0, D) \rightarrow f_i(S_0) \cdot P_{ab} \cdot P_{cd}$



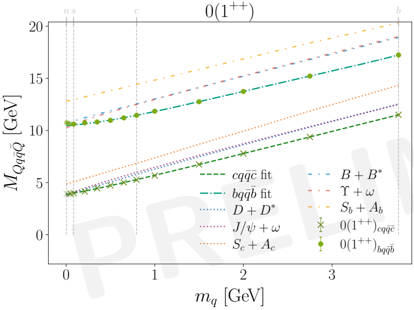
Eichmann, Fischer, Heupel, Santowsky, Wallbott,Few Body Syst. 61 (2020) 4, 38

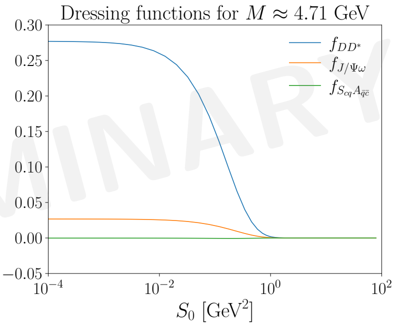
Physical amplitude

- Structure of the Amplitude Γ is determined according to the quantum numbers of the state in question, the quark content and the physical decay channels.
- Example for the $X(3872) \quad (I(J^{PC}) = 0(1^{++}))$:
  - Meson-molecule: $D\bar{D}^*$
  - Hadro-charmonium: $J/\psi\,\omega$
  - Diquark-Antidiquark: $S_c A_c$

  $\Rightarrow \Gamma = f_{DD^*} \cdot \tau_{DD^*} + f_{J/\Psi\omega} \cdot \tau_{J/\Psi\omega} + f_{S_{cq}A_{\bar{q}\bar{c}}} \cdot \tau_{S_{cq}A_{\bar{q}\bar{c}}}$
- No assumptions of a dominant substructure needed!
- The equation **dynamically** develops internal two-body pole structures.
- This introduces decay thresholds into the equation.

Example quark mass evolution for the $0(1^{++})$ state





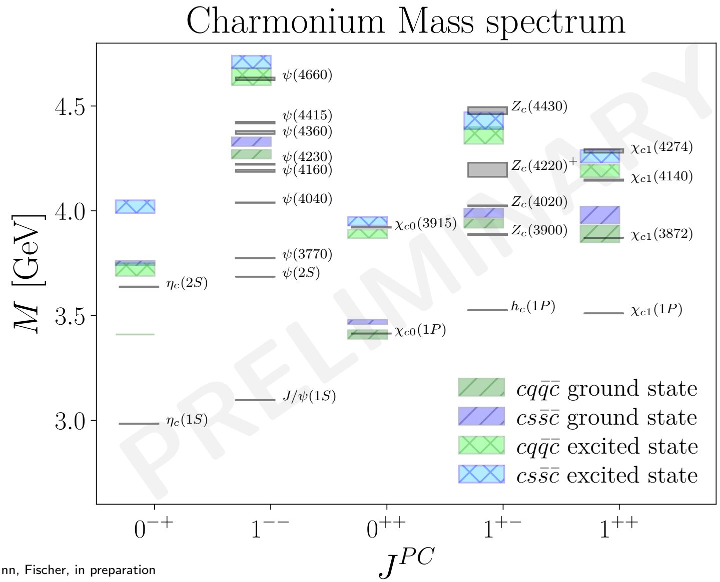
JH, Eichmann, Fischer, in preparation

Physical basis:

$$\Gamma = f_{DD^*} \cdot \tau_{DD^*} + f_{J/\Psi\omega} \cdot \tau_{J/\Psi\omega} + f_{S_{cq}A_{\bar{q}\bar{c}}} \cdot \tau_{S_{cq}A_{\bar{q}\bar{c}}}$$

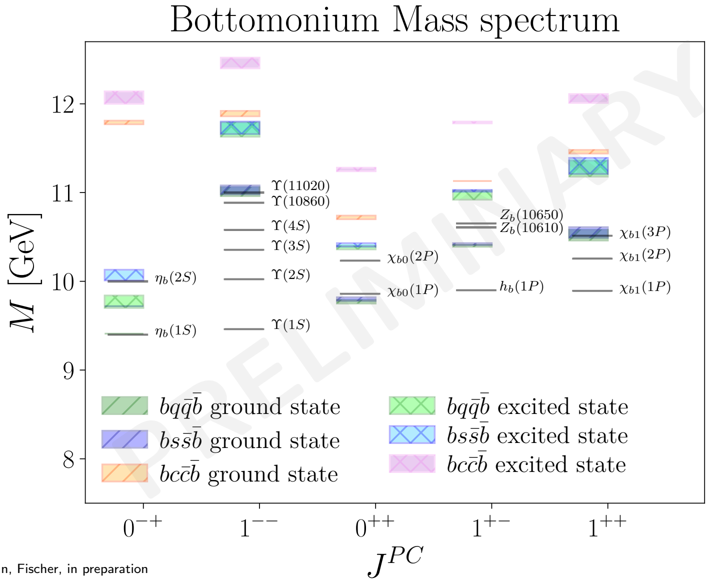
- $M_{1^{++}}^{cq\bar{q}\bar{c}} = 3.89 \pm 0.04$ GeV $\quad \rightarrow \quad X(3872)$
- $M_{1^{++}}^{bq\bar{q}\bar{b}} = 10.52 \pm 0.06$ GeV $\quad \rightarrow \quad ??$

Hidden-charm mass spectrum



JH, Eichmann, Fischer, in preparation

Hidden-bottom mass spectrum



JH, Eichmann, Fischer, in preparation

Hidden-flavour ground state masses

	$I(J^{PC})$	Physical components	GS Mass	Exp.
hidden charm	$0(0^{++})$	$D\bar{D}$, $J/\psi\omega$, S_cS_c	$3.41(2)$	–
$(cq\bar{q}\bar{c})$	$0(1^{++})$	$D\bar{D}^*$, $J/\psi\omega$, S_cA_c	$3.89(4)$	$\chi_{c1}(3872)$
	$1(1^{+-})$	$D\bar{D}^*$, $J/\psi\pi$, S_cA_c	$3.94(2)$	$Z_c(3900)$
	$0(1^{--})$	$D\bar{D}_1$, $\chi_{c0}\omega$, $J/\psi\sigma$	$4.27(2)$	$\psi(4230)$
	$0(0^{-+})$	$D\bar{D}_0$, $\eta_c\sigma$, $\chi_{c0}\eta$	$3.41(0)$	–
hidden bottom	$0(0^{++})$	$B\bar{B}$, $\Upsilon\omega$, S_bS_b	$9.77(2)$	–
$(bq\bar{q}\bar{b})$	$0(1^{++})$	$B\bar{B}^*$, $\Upsilon\omega$, S_bA_b	$10.52(6)$	–
	$1(1^{+-})$	$B\bar{B}^*$, $\Upsilon\pi$, S_bA_b	$10.40(1)$	$Z_b(10610)$
	$0(1^{--})$	$B\bar{B}_1$, $\chi_{b0}\omega$, $\Upsilon\sigma$	$11.01(5)$	–
	$0(0^{-+})$	$B\bar{B}_0$, $\eta_b\sigma$, $\chi_{b0}\eta$	$9.41(0)$	–

JH, Eichmann, Fischer, in preparation,
Wallbott, Eichmann, Fischer, Phys.Rev.D 100 (2019) 1, 014033, Wallbott Eichmann, Fischer, Phys.Rev.D 102 (2020) 5, 051501

Open-flavour ground state masses

	$I(J^{PC})$	Physical components	GS Mass	Exp.
open charm	$1(0^+)$	DD, D^*D^*, A_cA_c	$3.39(1)$	–
$(cc\bar{q}\bar{q})$	$0(1^+)$	DD^*, D^*D^*, S_cA_c	$3.79(1)$	T_{cc}^+
	$1(1^+)$	DD^*, A_cA_c	$4.25(2)$	–
open bottom	$1(0^+)$	BB, B^*B^*, A_bA_b	$9.60(1)$	-
$(bb\bar{q}\bar{q})$	$0(1^+)$	BB^*, B^*B^*, S_bA_b	$10.14(2)$	$(T_{bb}^+?)$
	$1(1^+)$	BB^*, A_bA_b	$11.0(2)$	–

JH, Eichmann, Fischer, in preparation
Wallbott, Eichmann, Fischer, Phys.Rev.D 100 (2019) 1, 014033,
Wallbott Eichmann, Fischer, Phys.Rev.D 102 (2020) 5, 051501

Lattice QCD, e.g.,
Leskovec, Meinel, Pflaumer, Wagner, Phys. Rev. D 100, 014503

Summary:

- DSE/BSE framework is a good tool to qualitatively analyse the charm and bottom four-quark state region.
- New results for the the 1^{--} and 0^{-+} four-quark states.
- Analysed the dressing function as a means to investigate the internal structure.

Outlook:

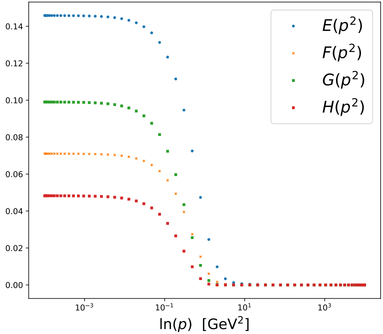
- Investigate the norm contributions to gain insight into the internal structure.
- Open-flavour states like
  - $0(1^+)$ and $0(0^+)$ with $bc\bar{q}\bar{q}$ and $cs\bar{q}\bar{q}$
  - $\frac{1}{2}(1^+)$ with $bb\bar{q}\bar{s}$
- Include the two-body quarkonium mixing.

Backup slides

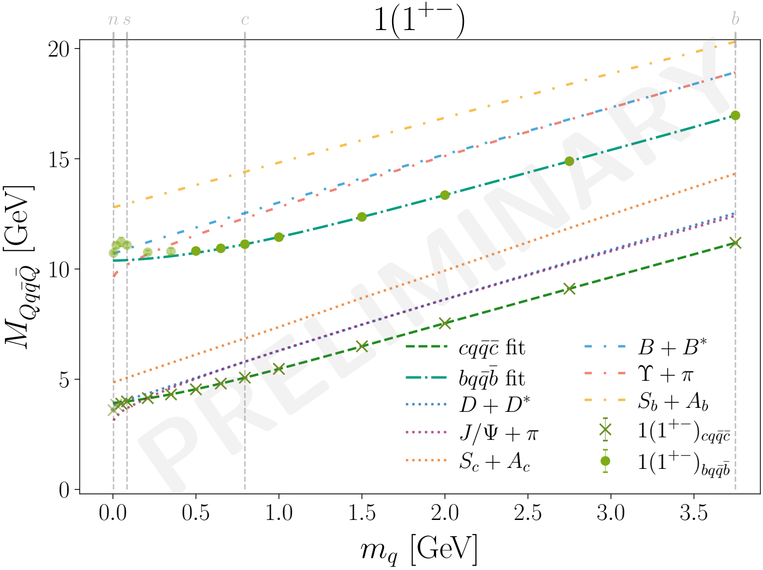
Pion BSE

Pion Bethe-Salpeter amplitude is given by:

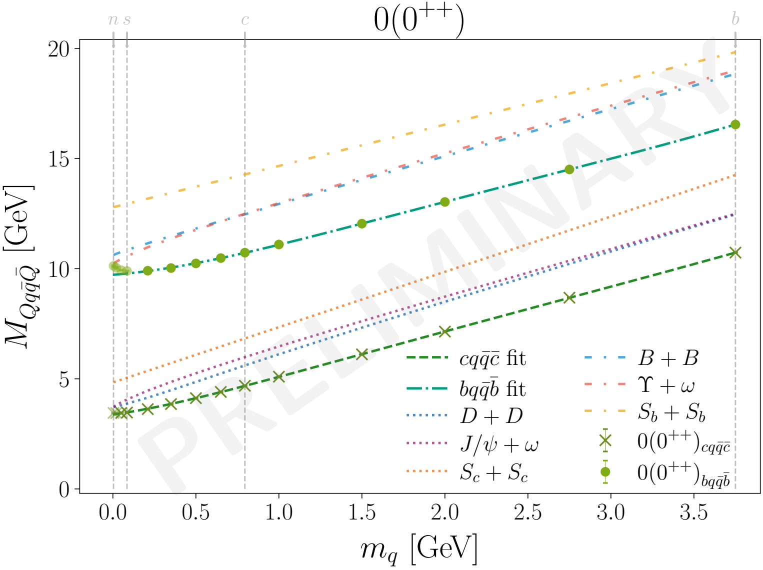
$$\Gamma_{\text{pion}}(p^2) = E(p^2)\cdot\tau_1(p,P) + F(p^2)\cdot\tau_2(p,P) + G(p^2)\cdot\tau_3(p,P) + H(p^2)\cdot\tau_4(p,P)$$



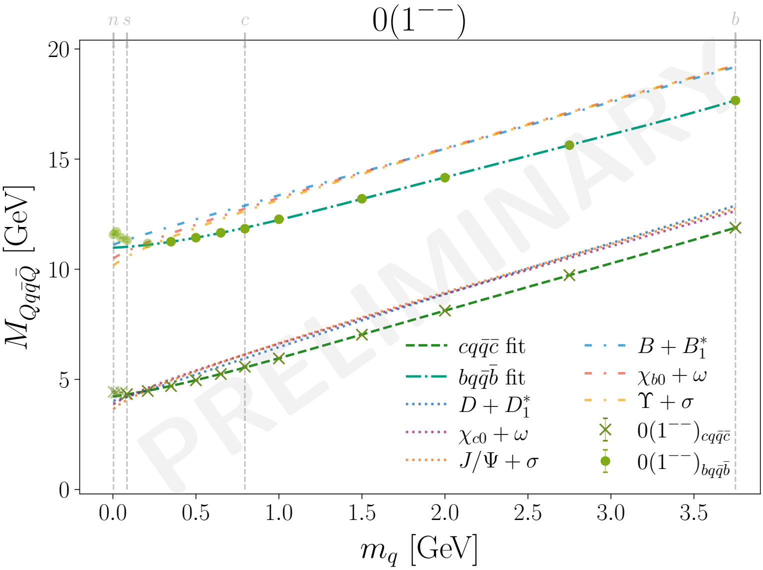
Quark mass evolution 1^{+-}



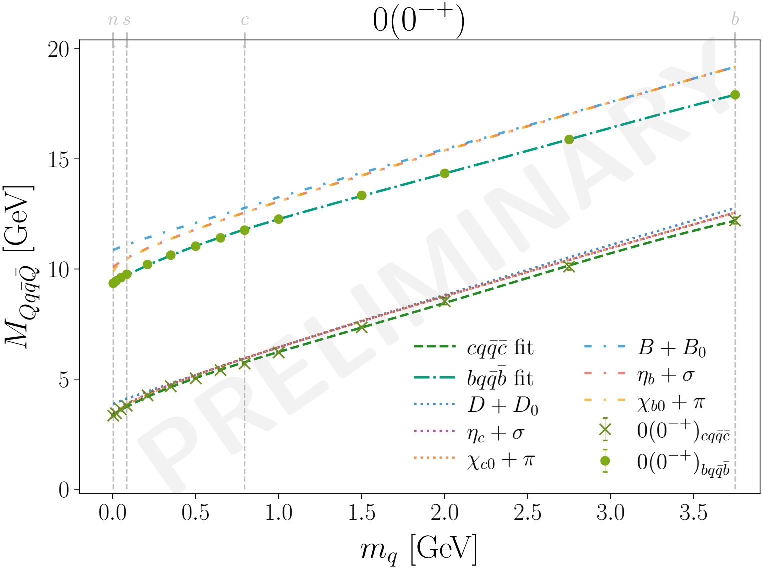
Quark mass evolution 0^{++}



Quark mass evolution 1^{--}

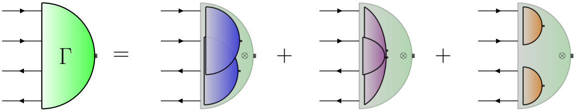


Quark mass evolution 0^{-+}



Norm contributions

Physical amplitude:



Norm contributions: