Transverse-momentum-dependent distributions

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AdT

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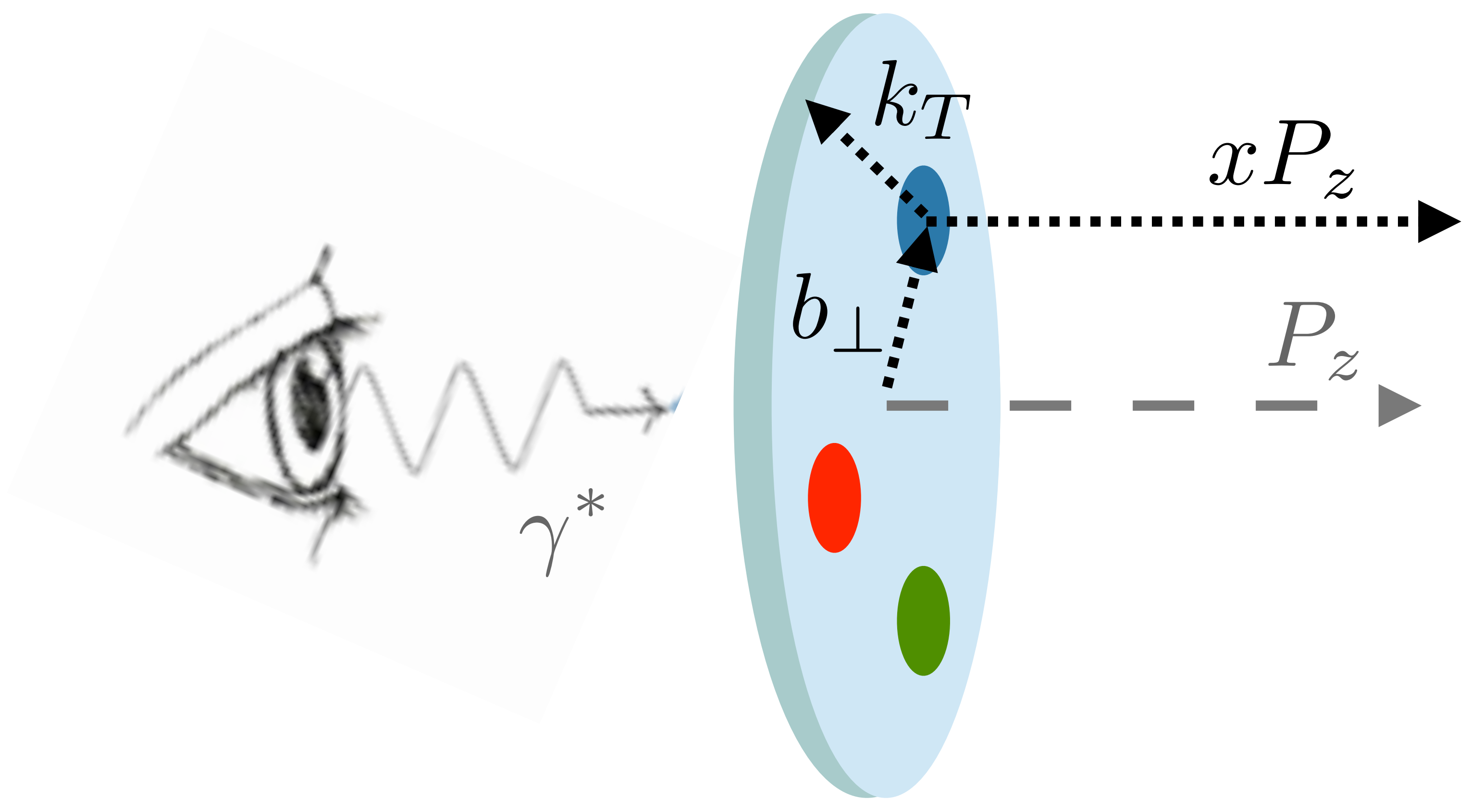
International School of Nuclear Physics
From quarks and gluons to hadrons and nuclei
Erice, Sicily
September 18-24, 2023

The various dimensions of the nucleon structure

Wigner distributions $W(x, \vec{k}_T, \vec{b}_\perp)$

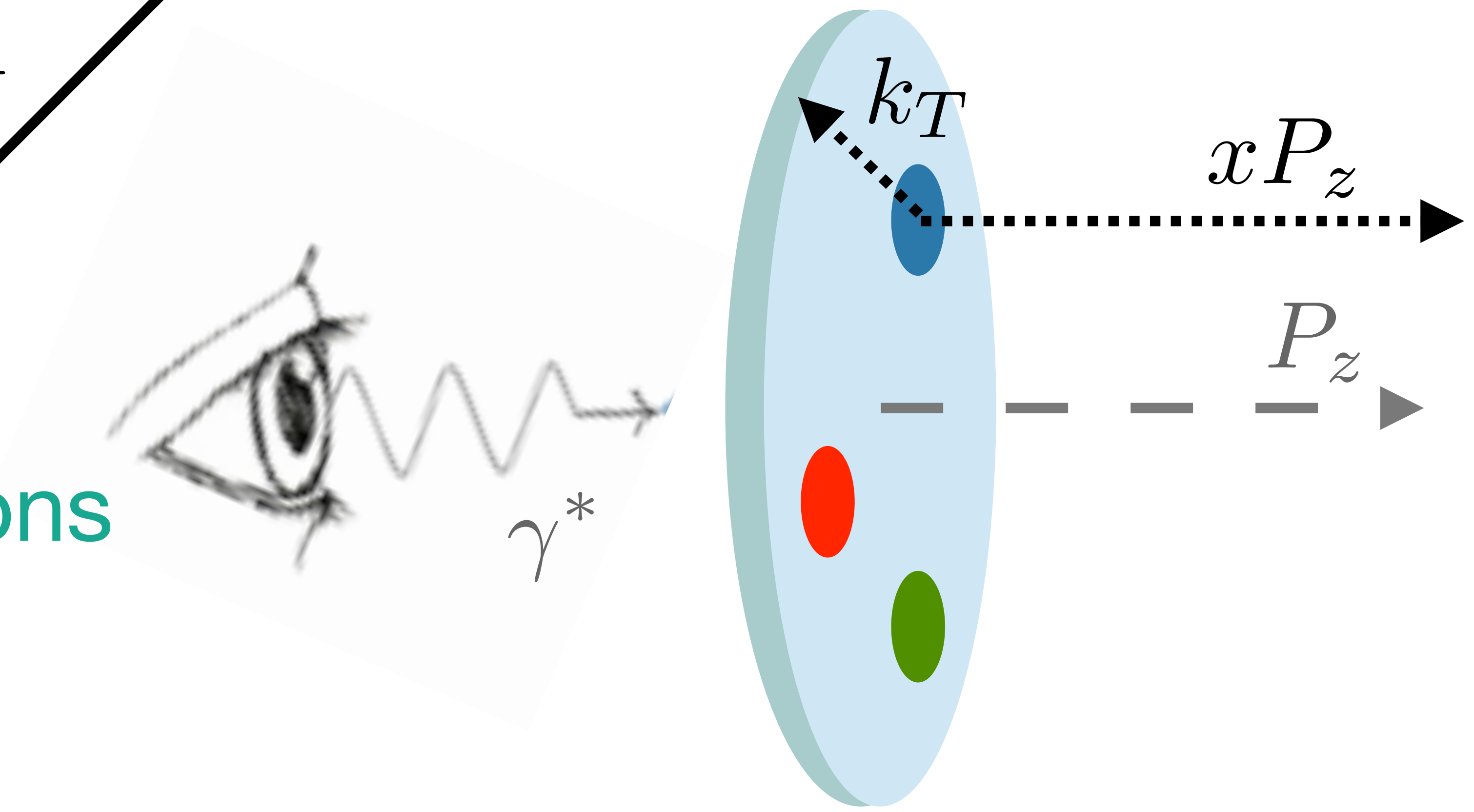


The various dimensions of the nucleon structure

Wigner distributions $W(x, \vec{k}_T, \vec{b}_\perp)$

$\int d^2\vec{b}_\perp$

transverse-momentum
dependent (TMD)
parton distribution functions
(PDFs)



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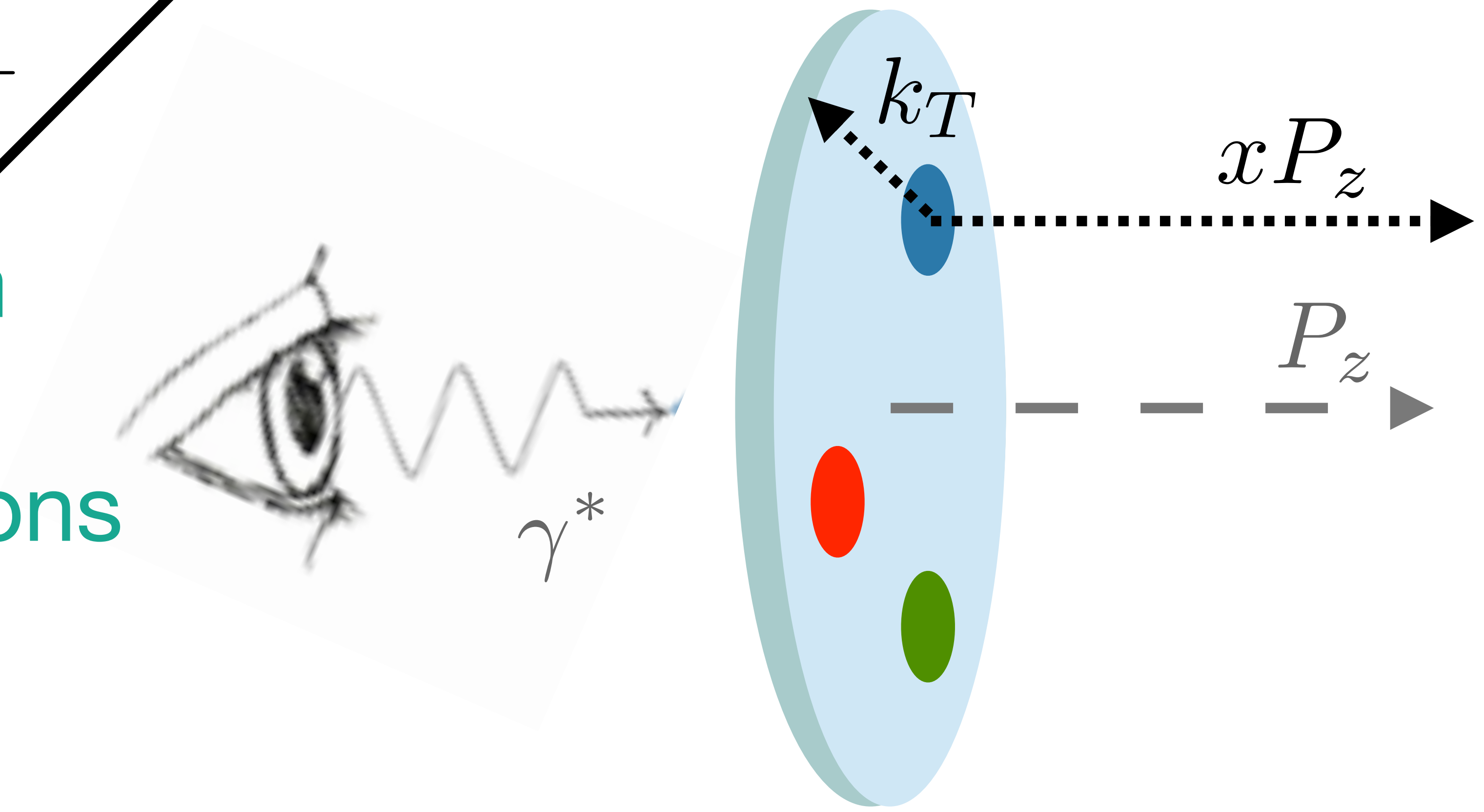
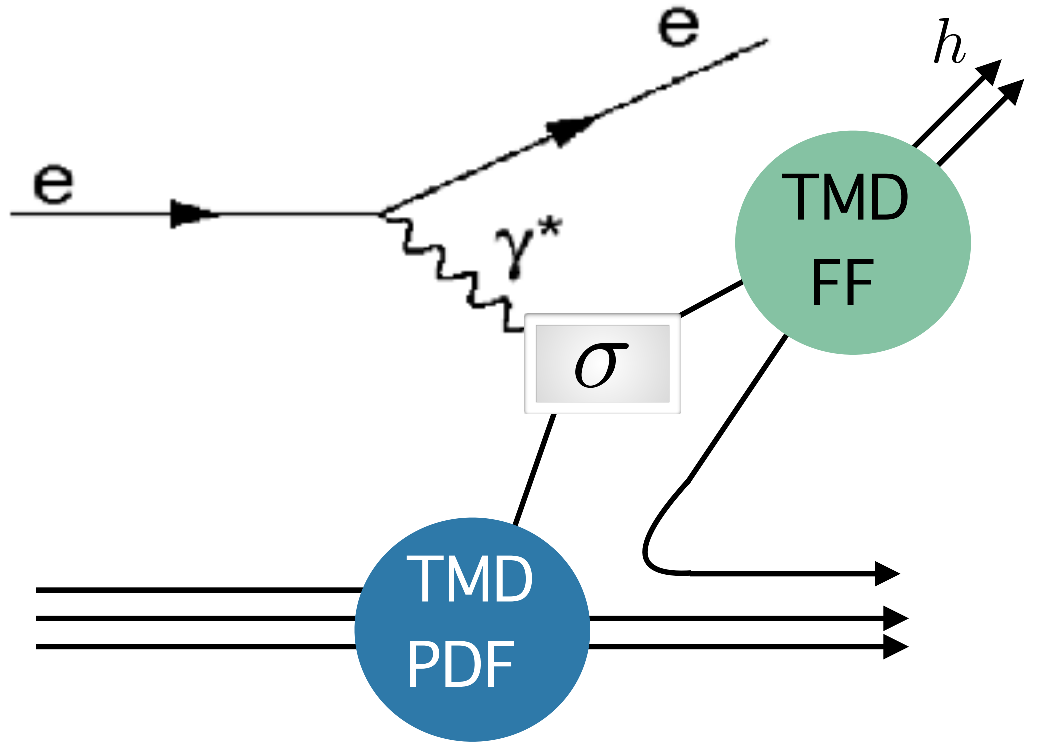


Semi-inclusive production



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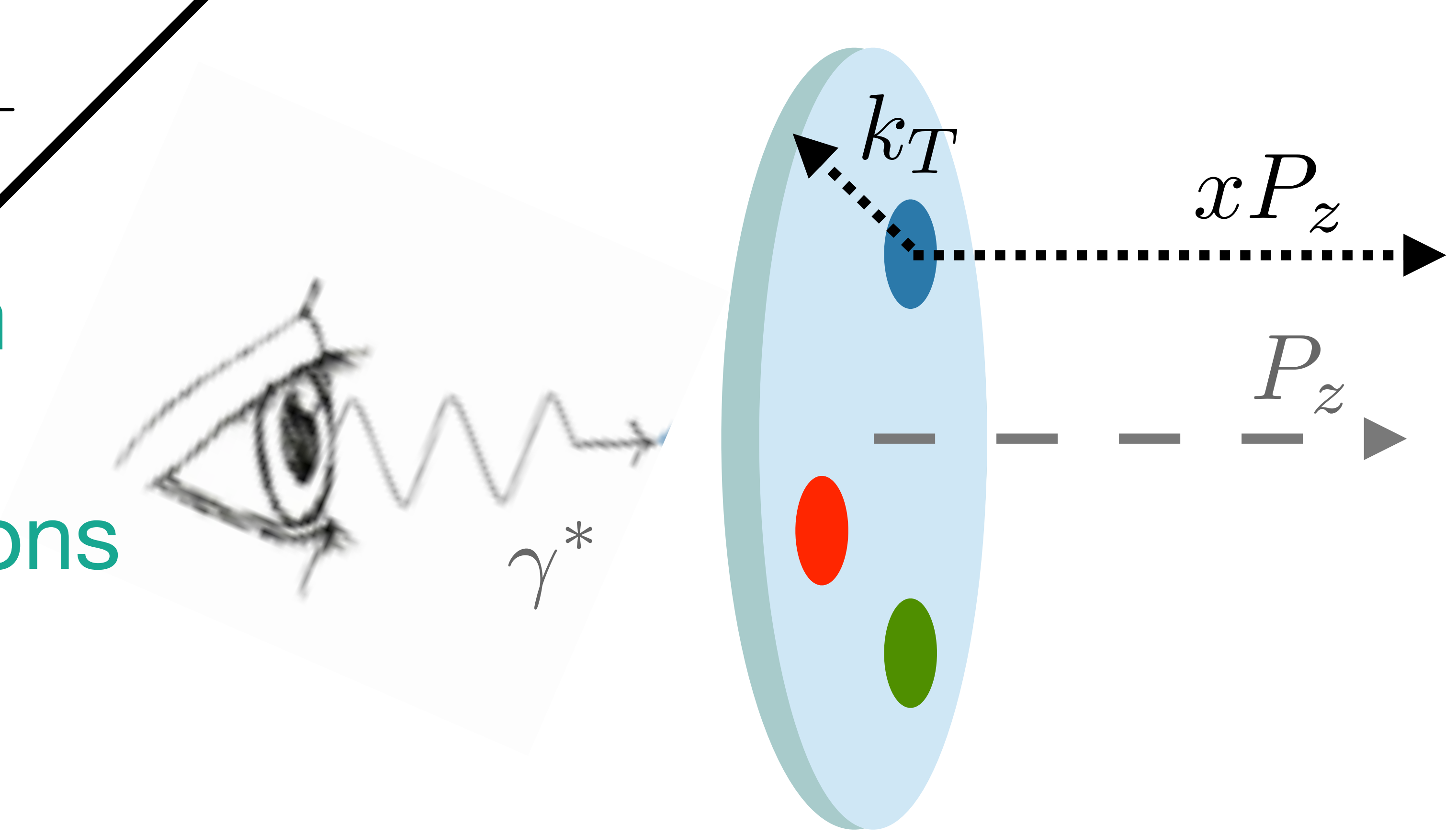

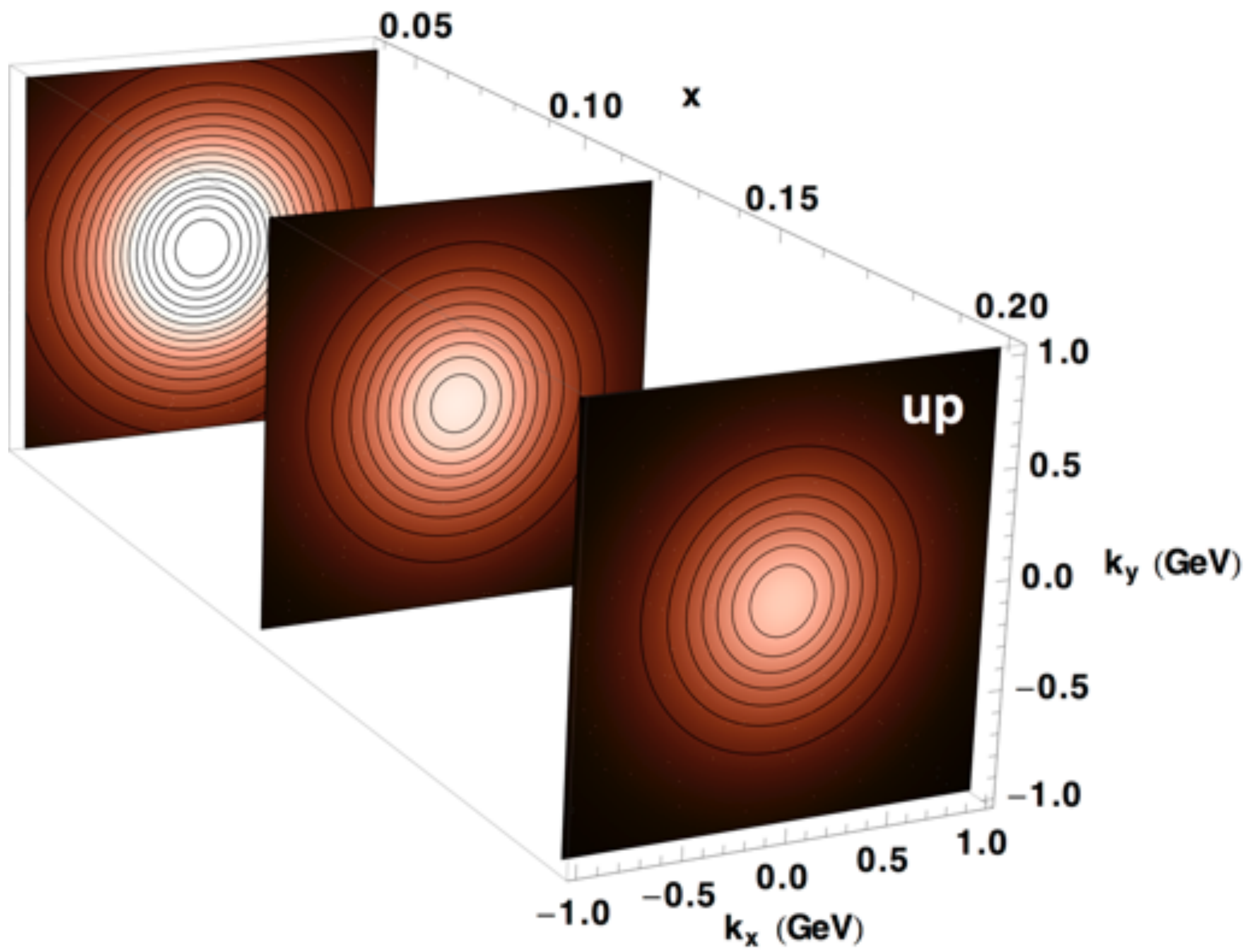

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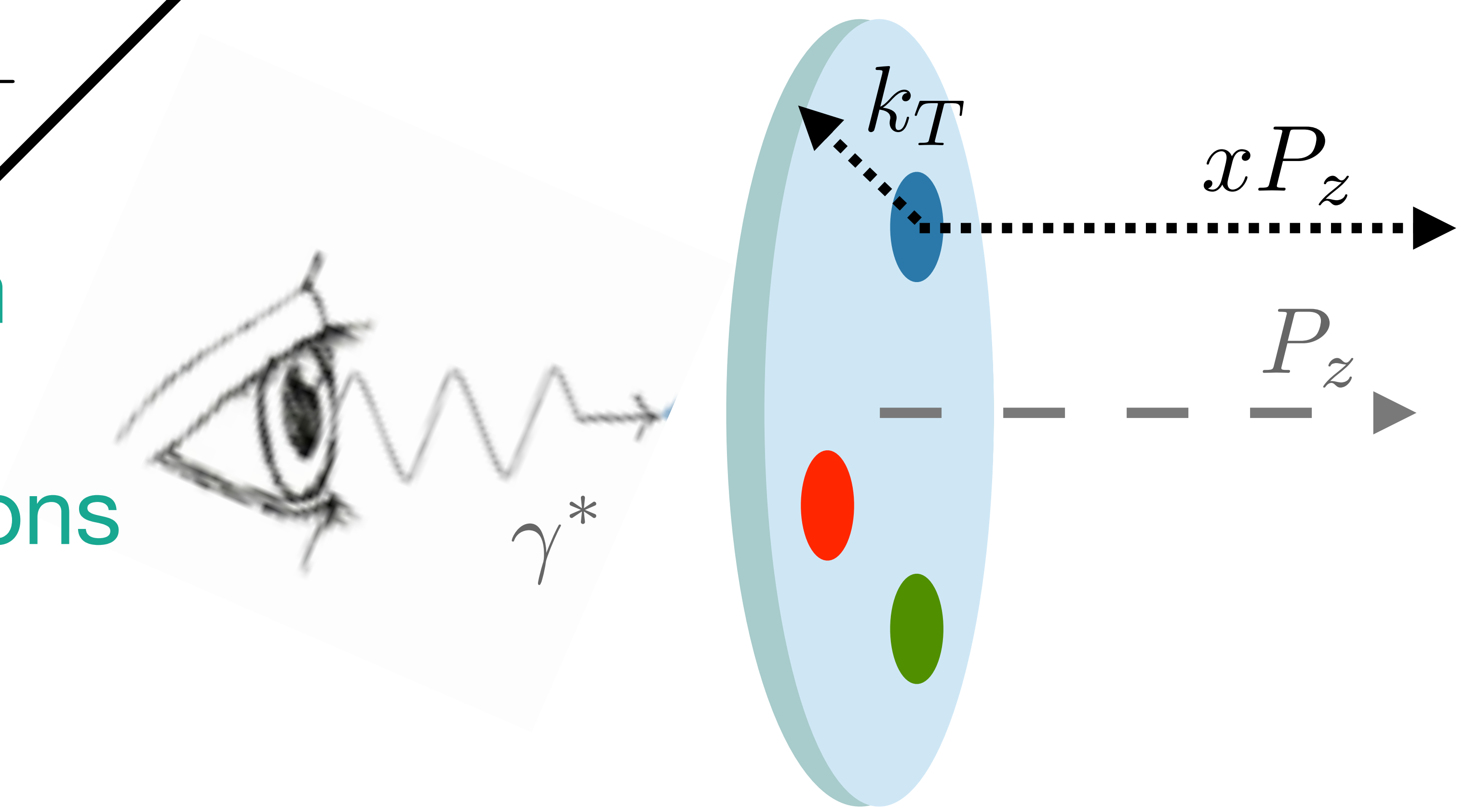

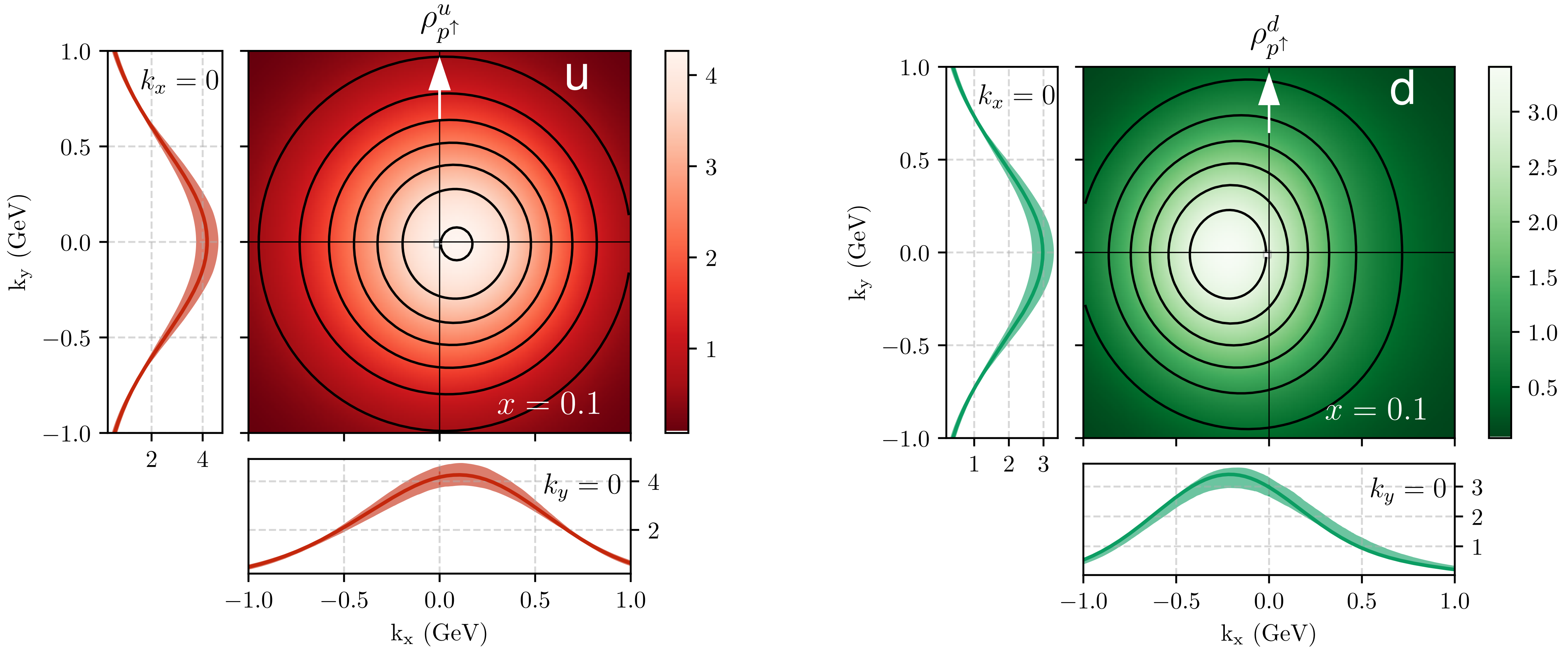

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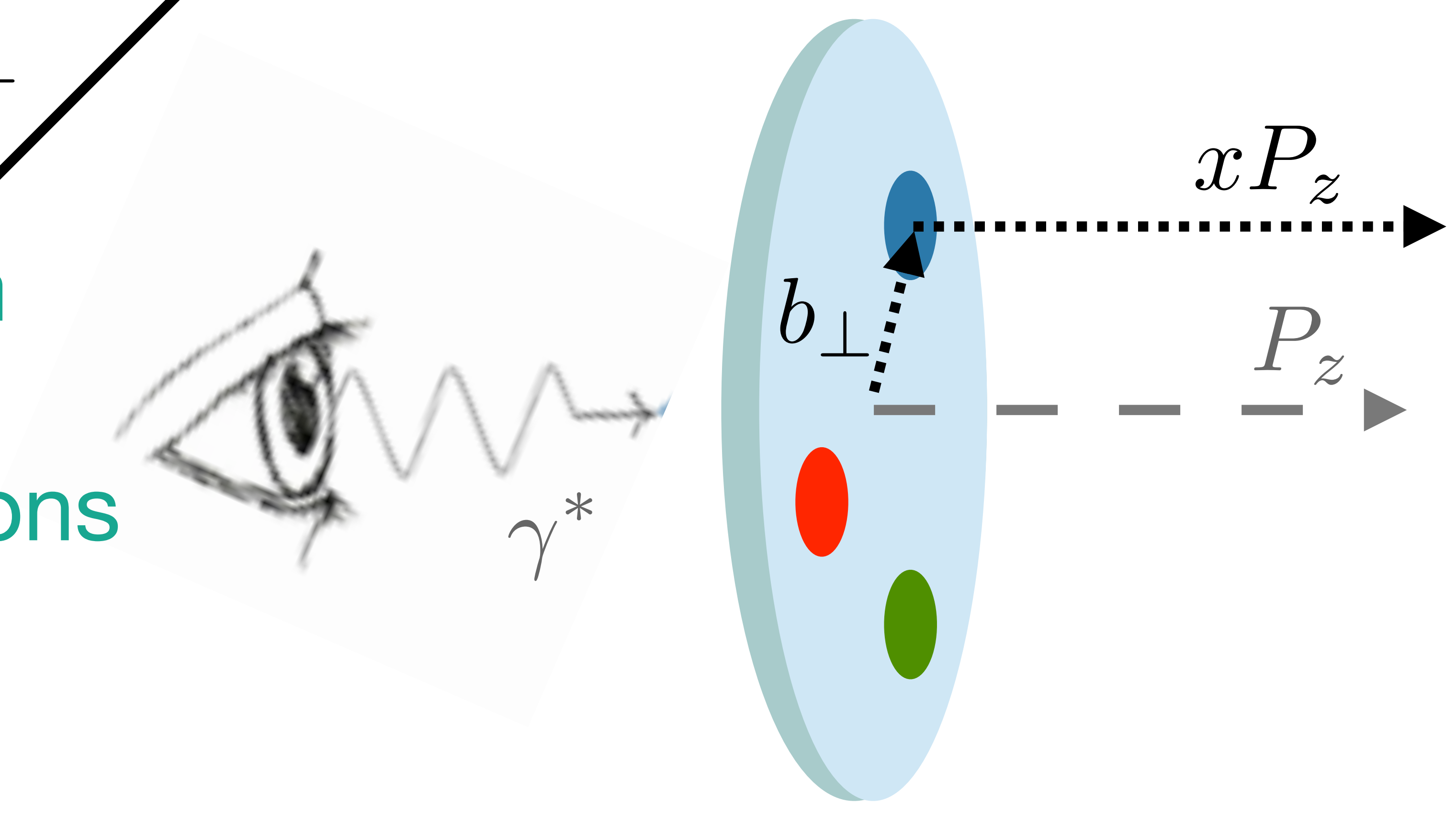
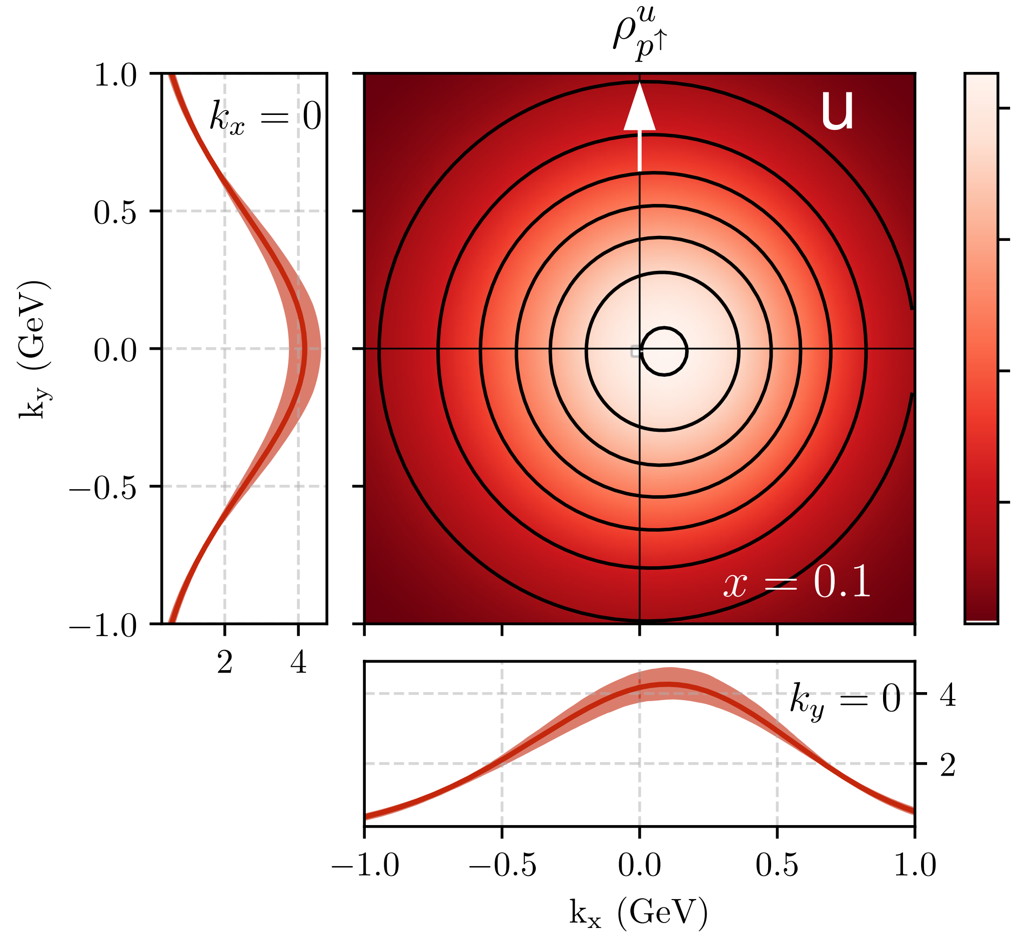
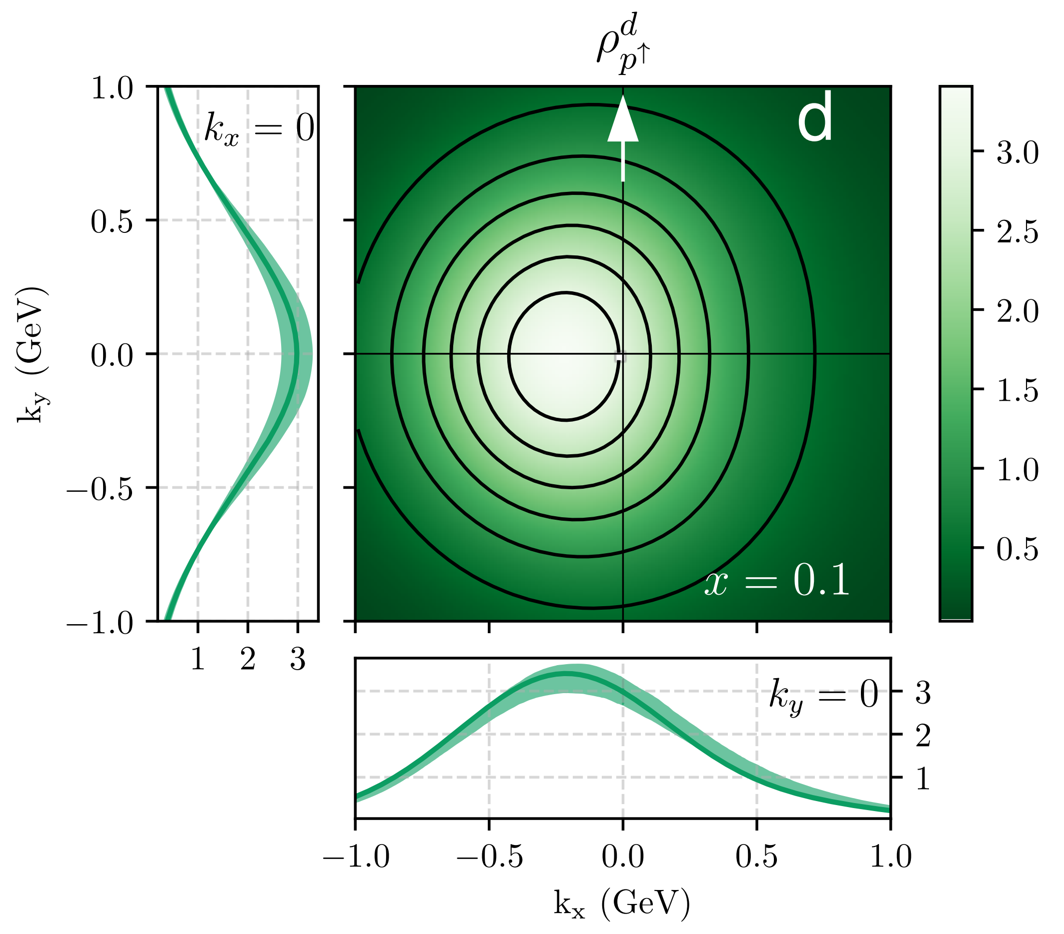


impact-parameter dependent PDFs





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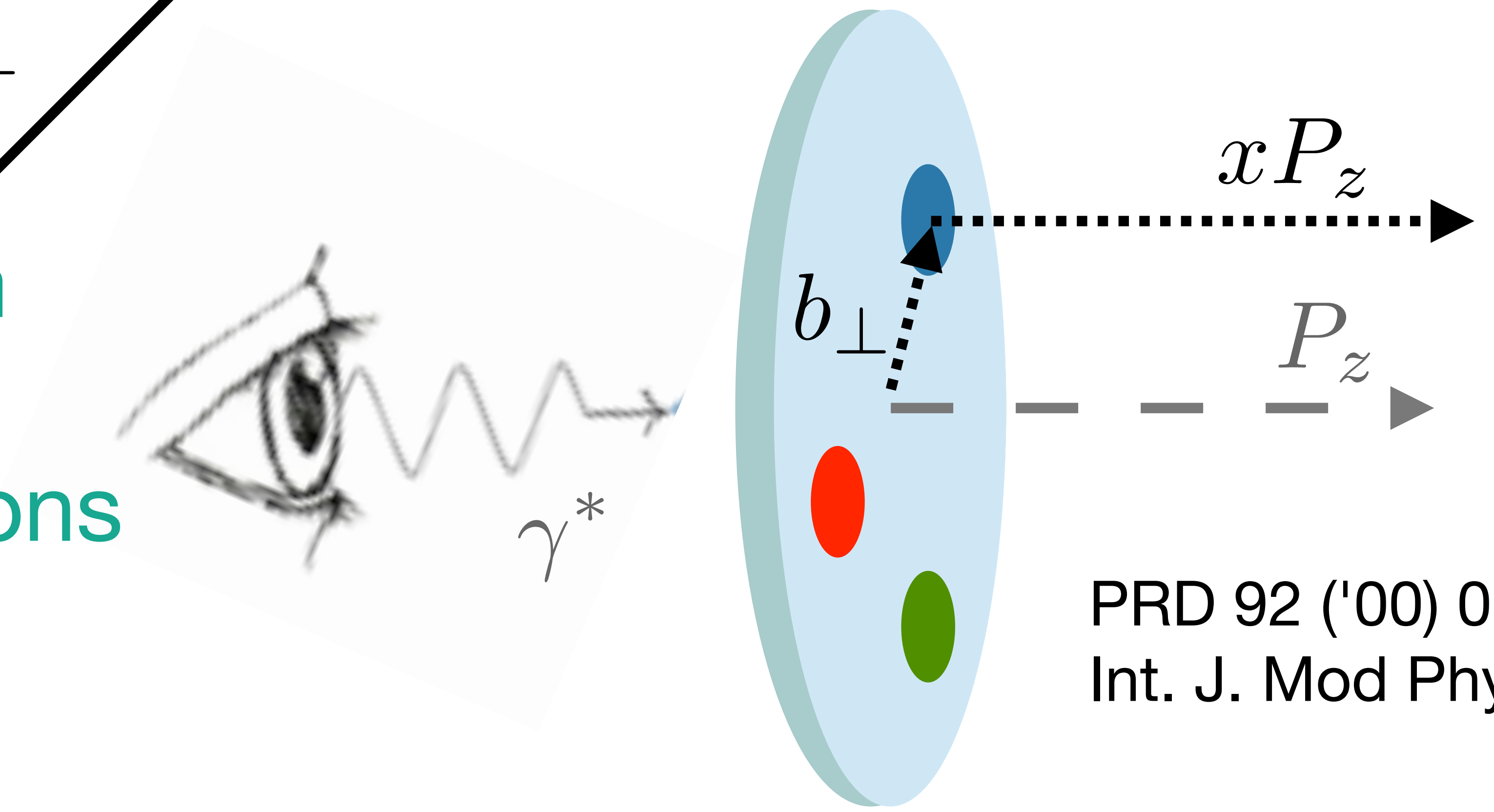

$\int d^2\vec{k}_T$

impact-parameter dependent PDFs

Fourier transform

generalised parton distributions (GPDs)

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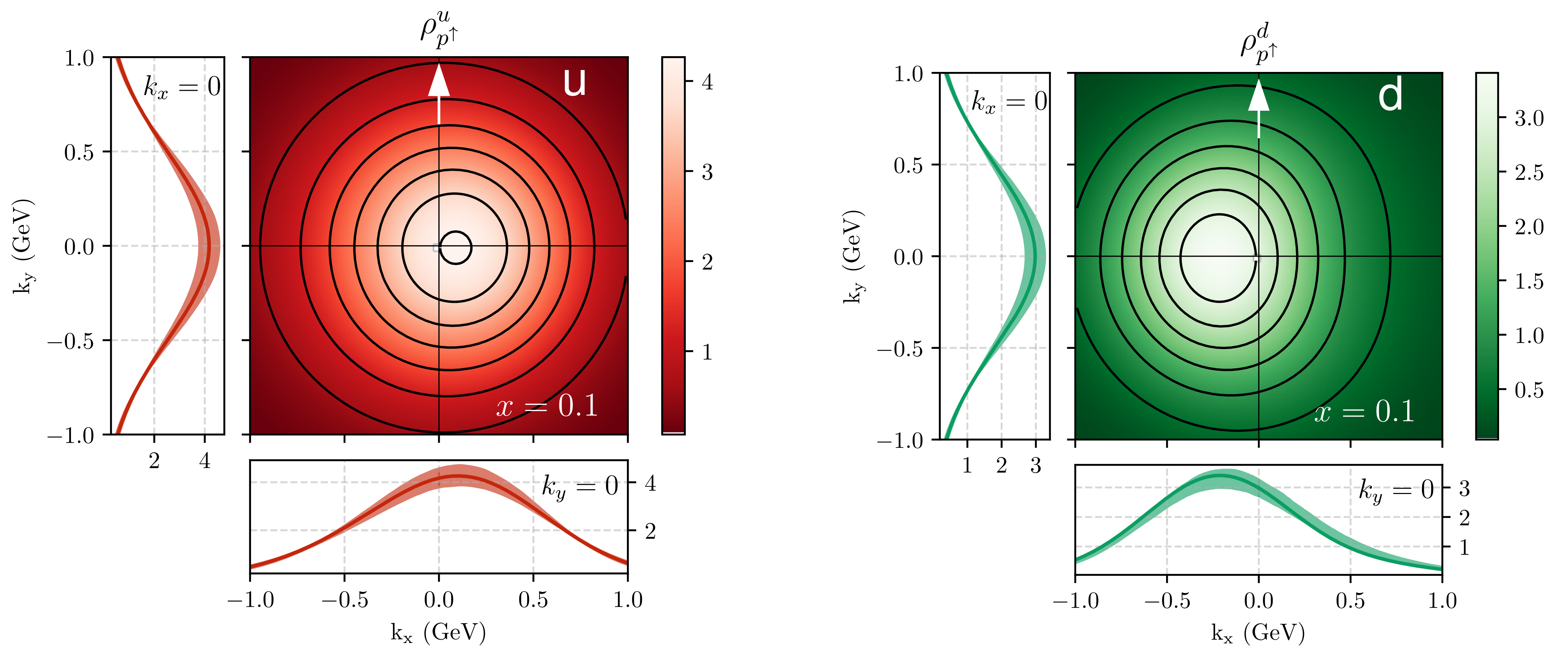
Int. J. Mod Phys. A 18 ('03) 173



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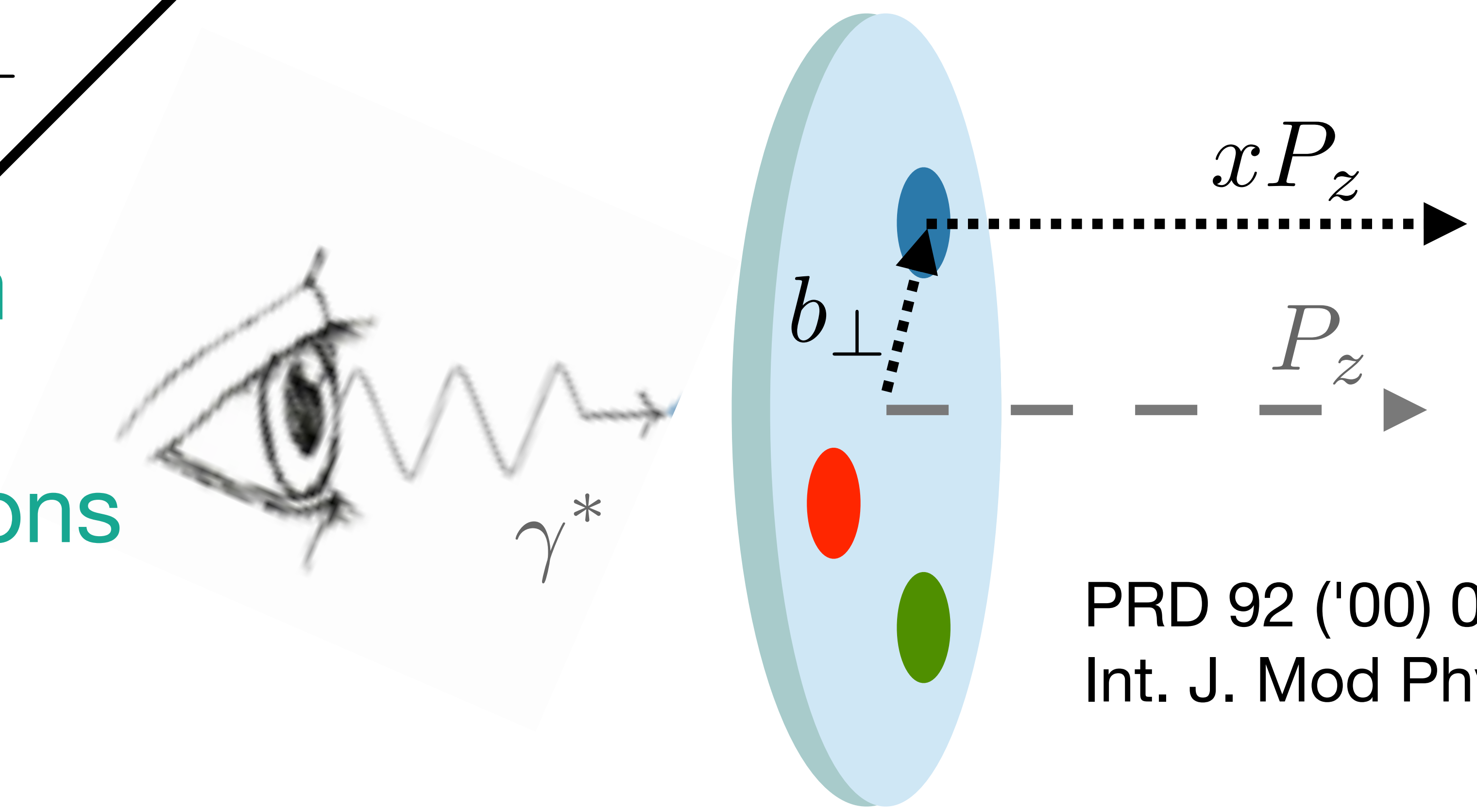

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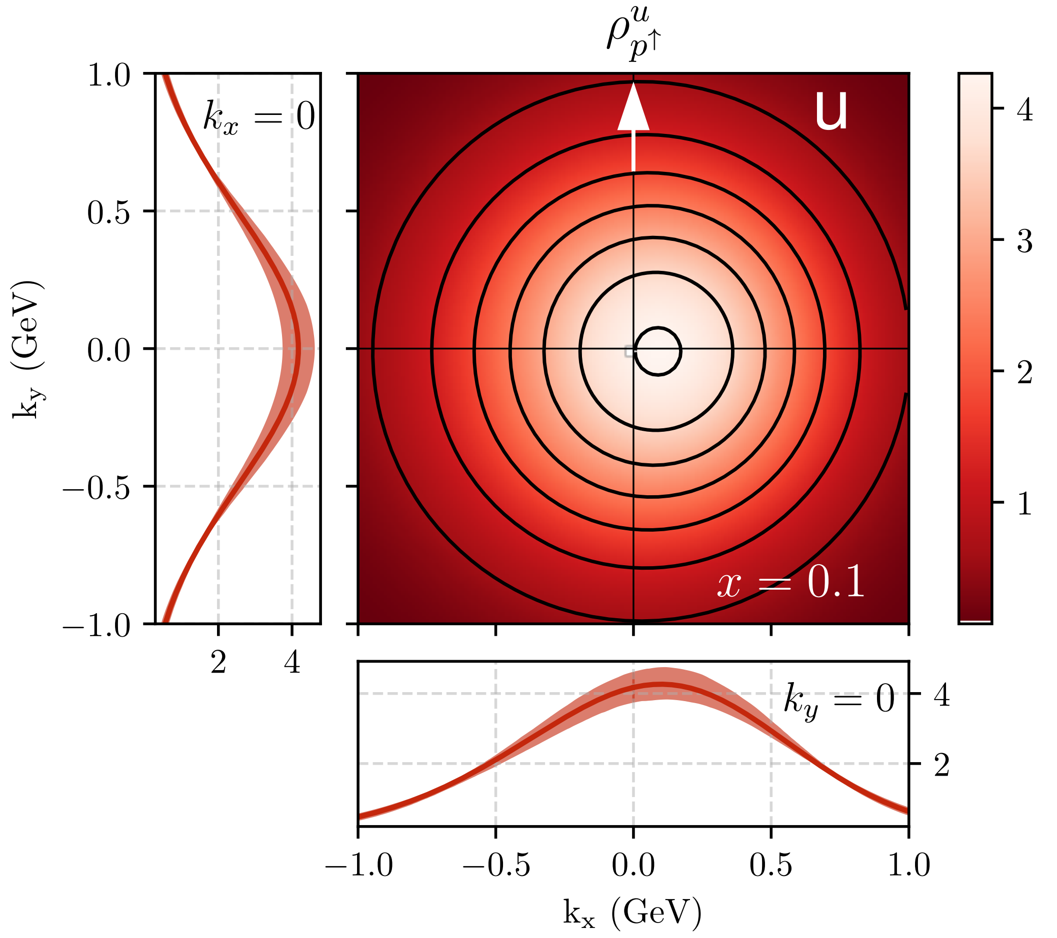
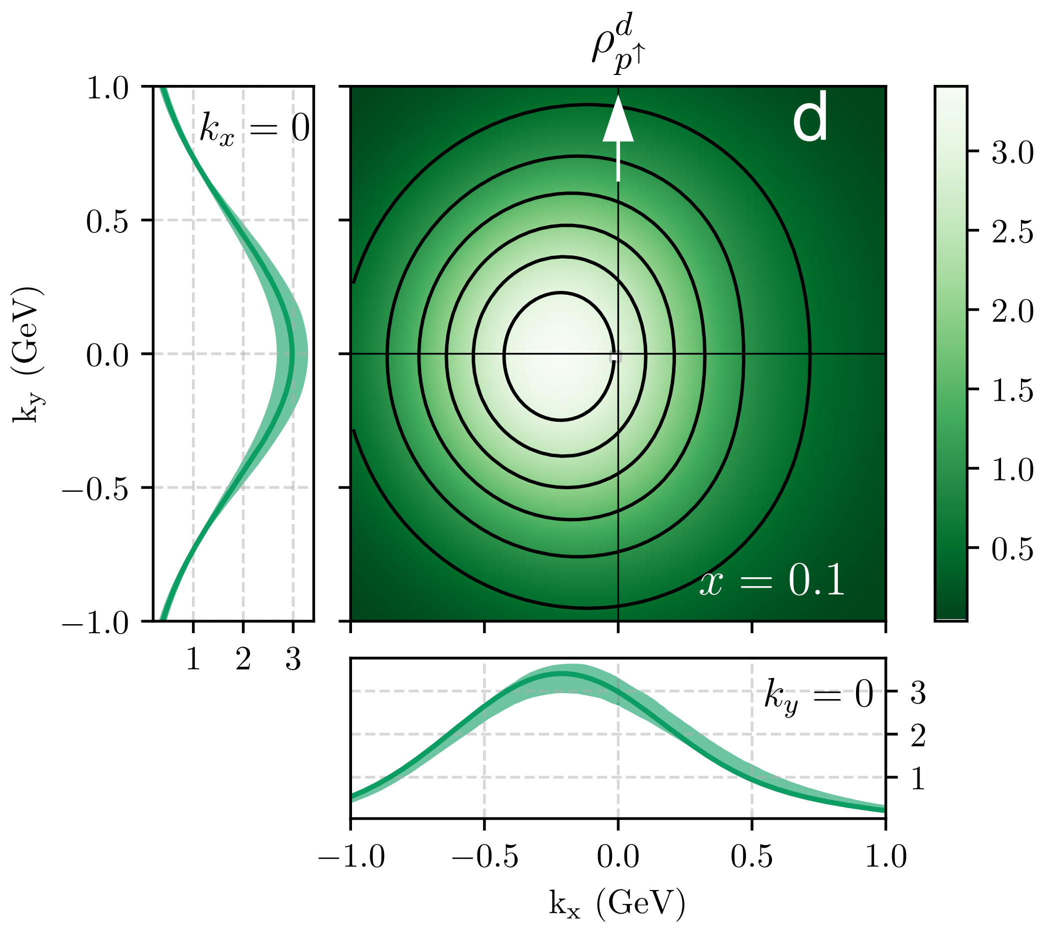
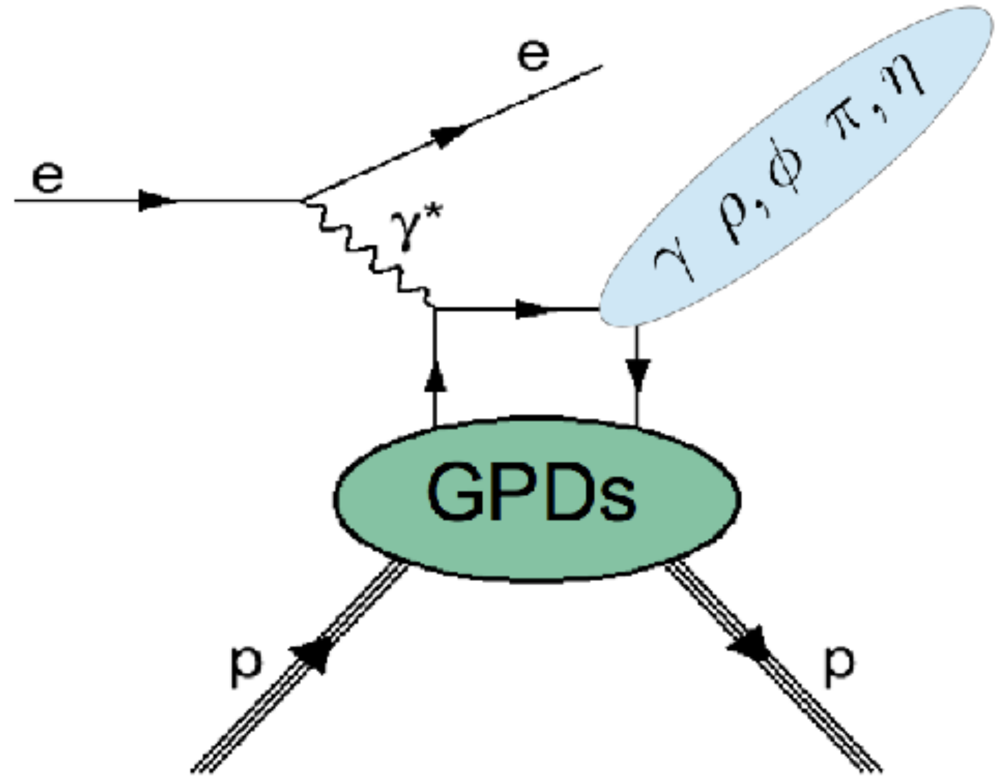




Exclusive production



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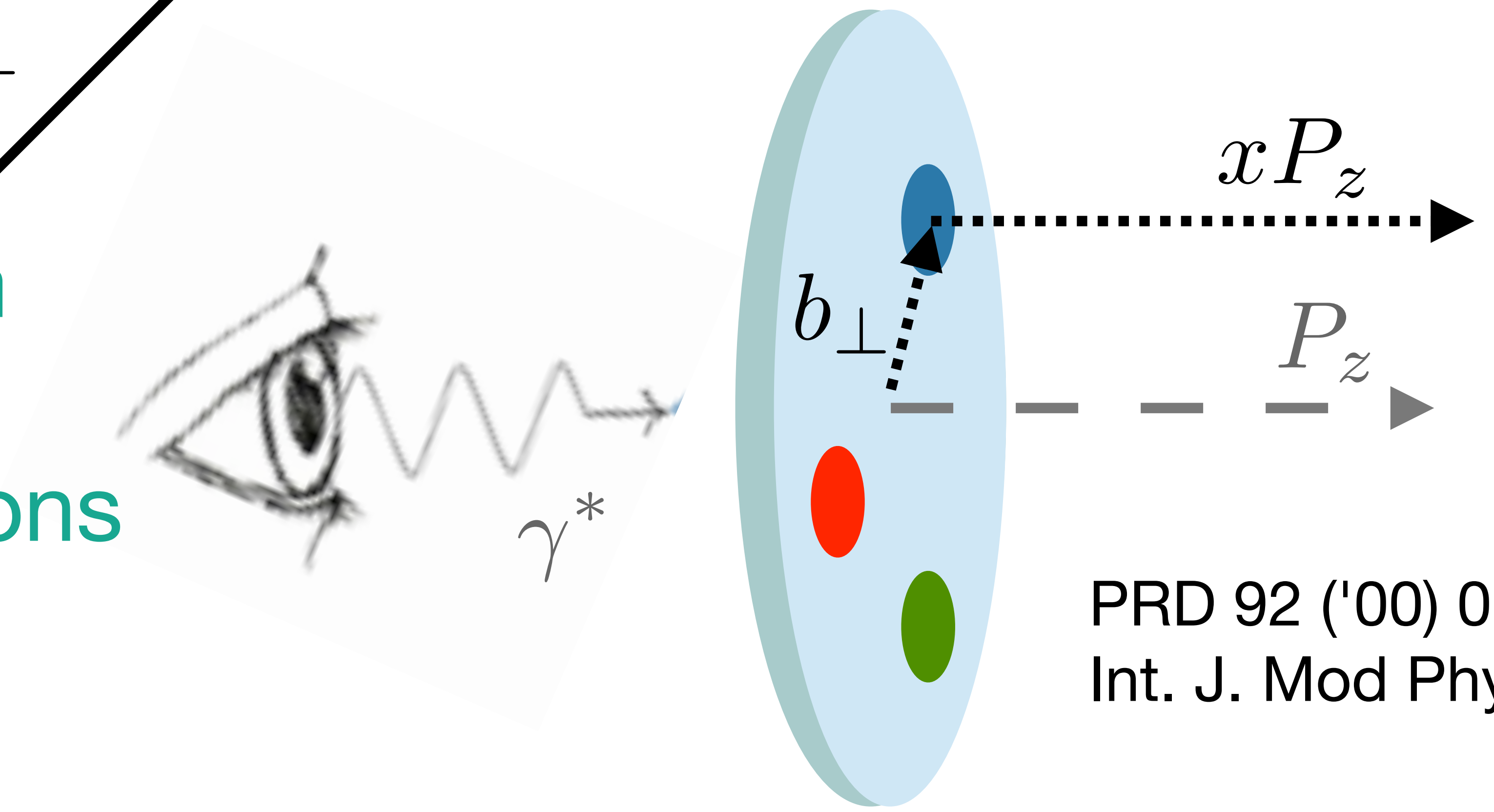

$\int d^2\vec{k}_T$ → impact-parameter dependent PDFs

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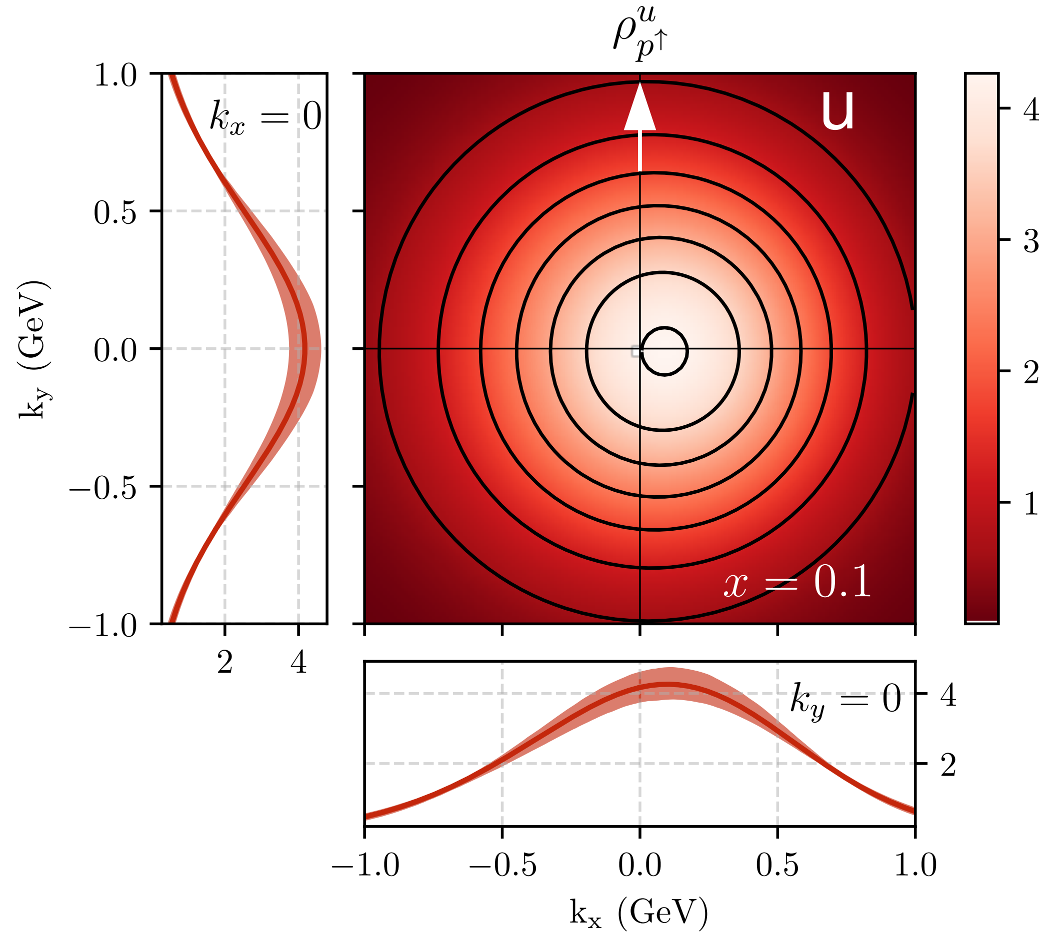
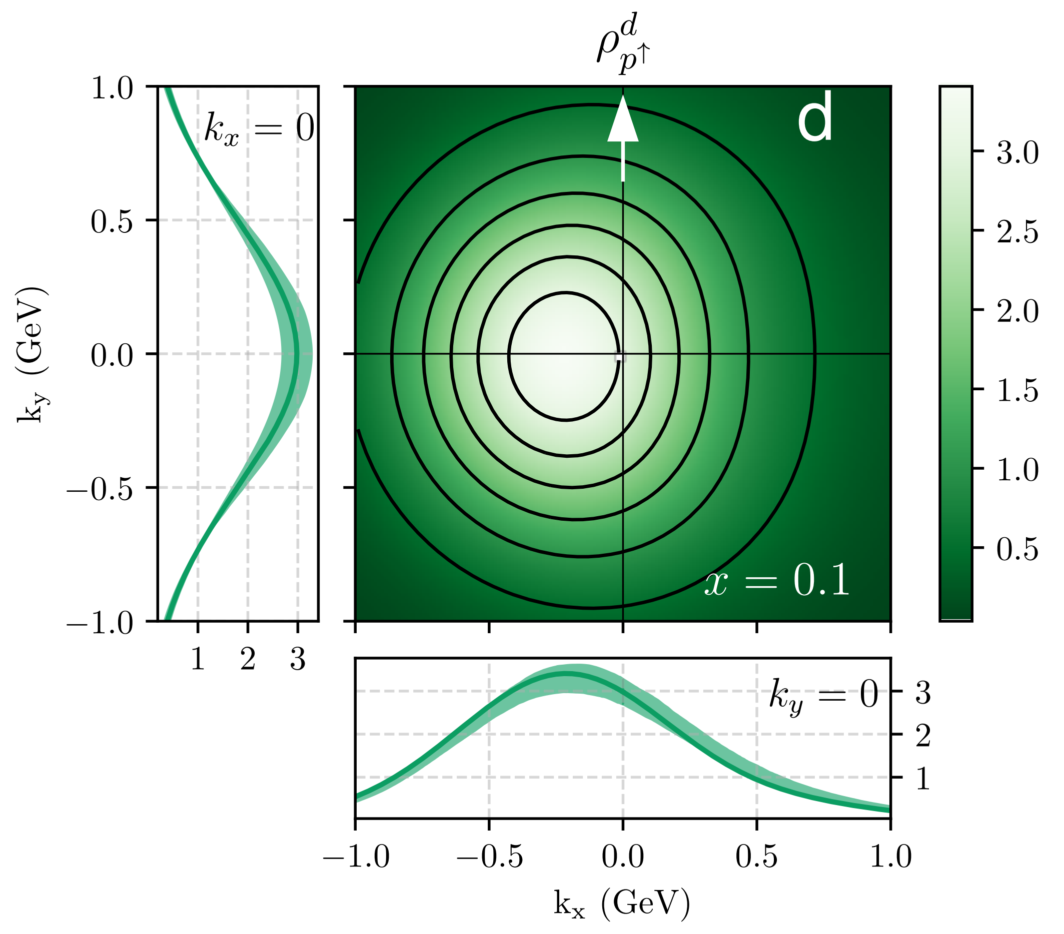
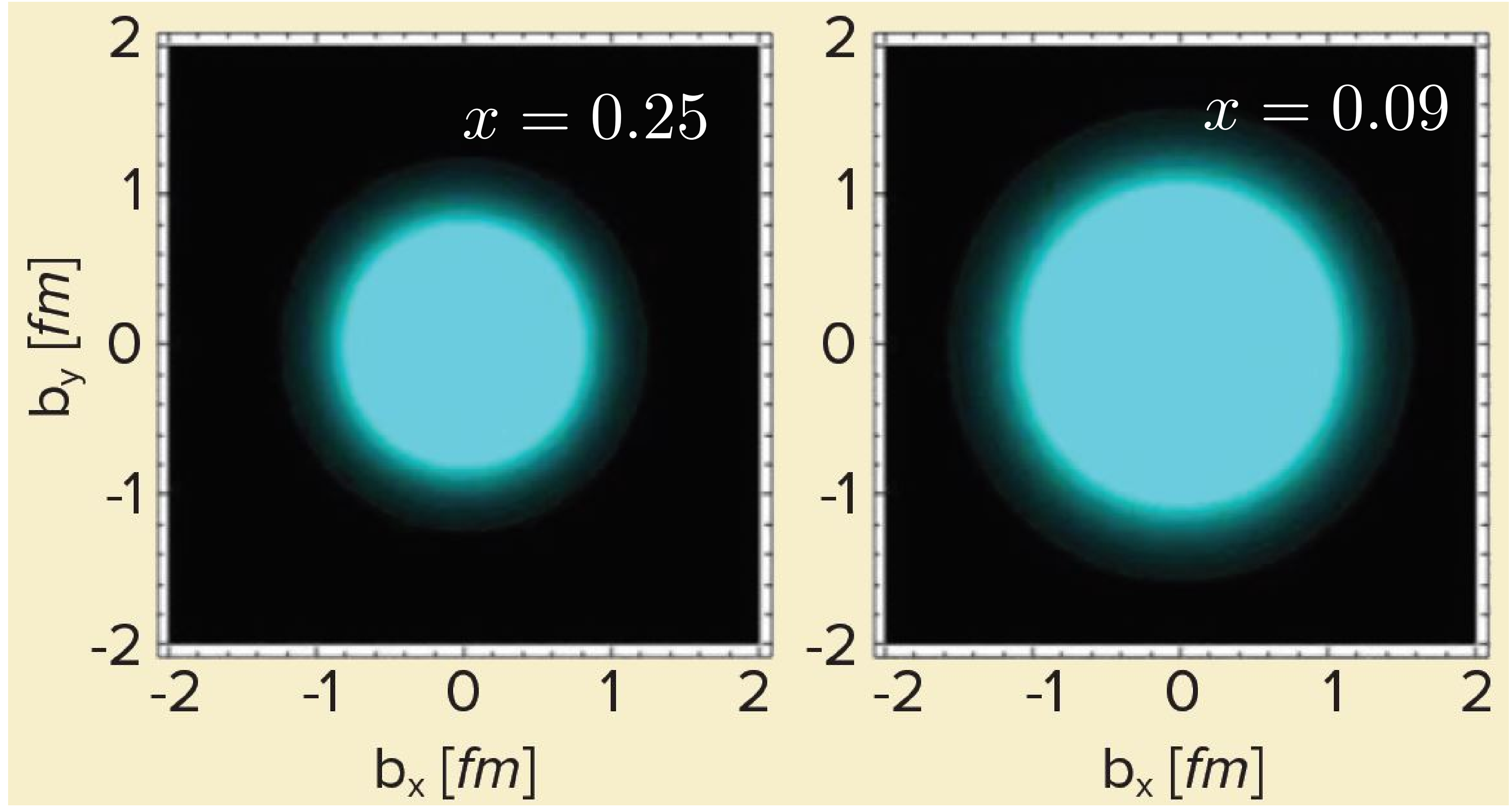
Fourier transform → generalised parton distributions (GPDs)







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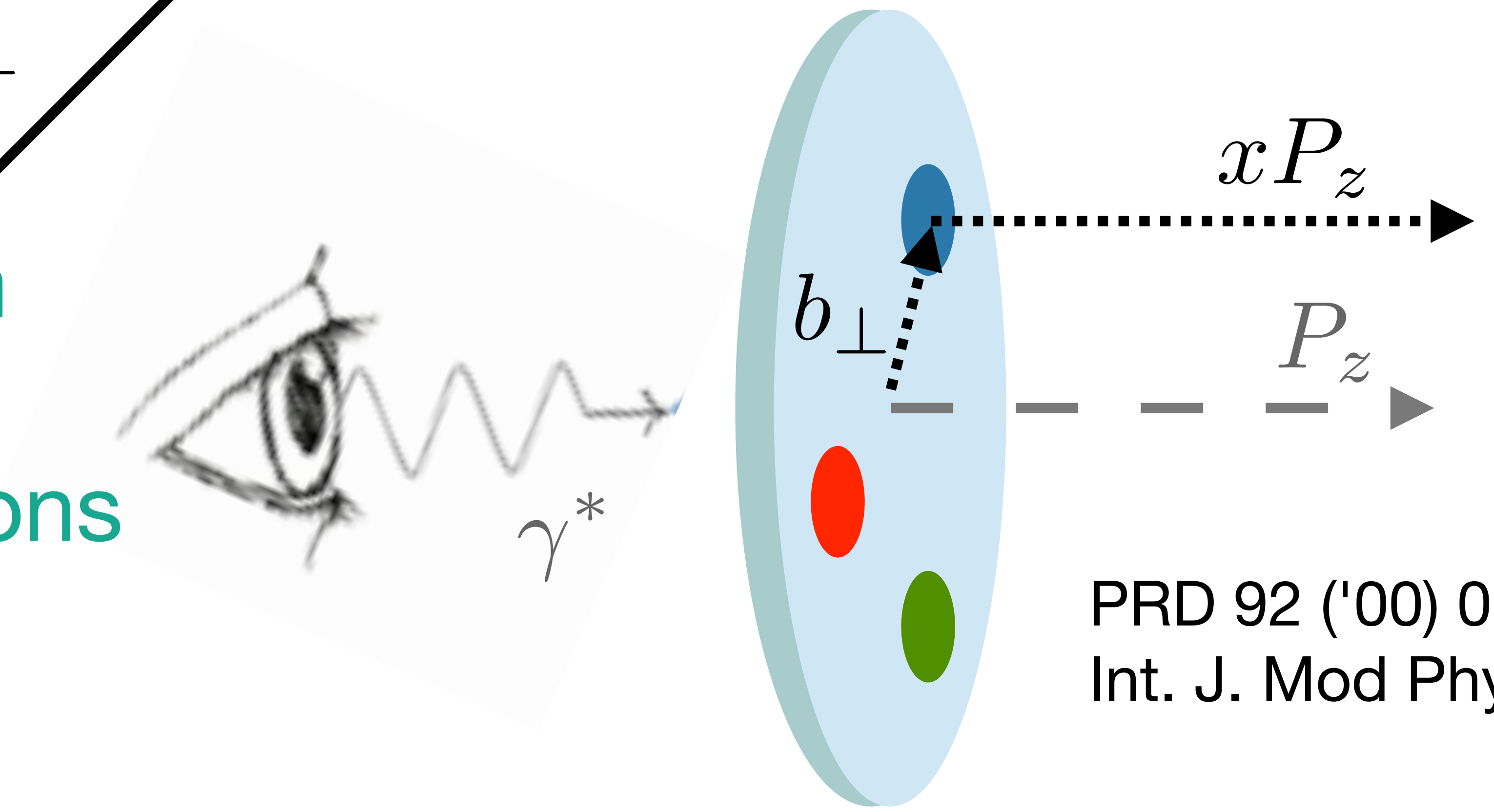

$\int d^2\vec{k}_T$ → impact-parameter dependent PDFs

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Fourier transform ↓

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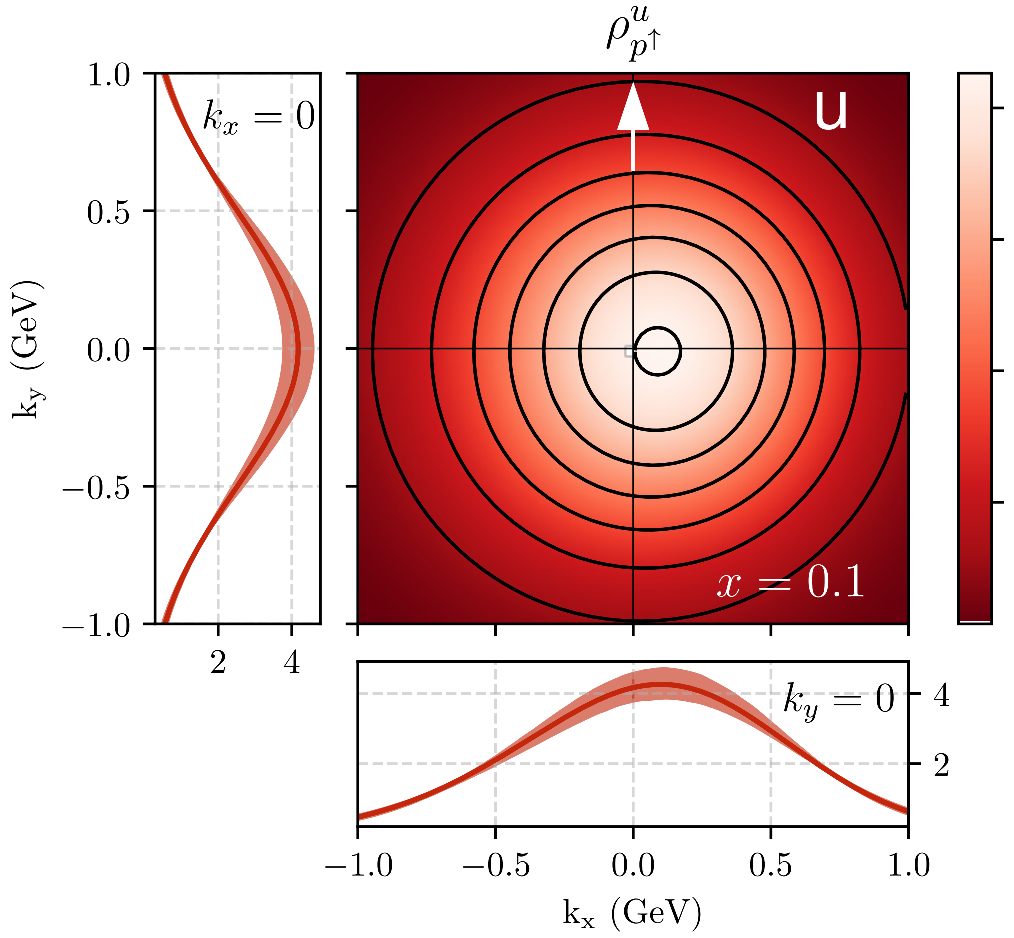

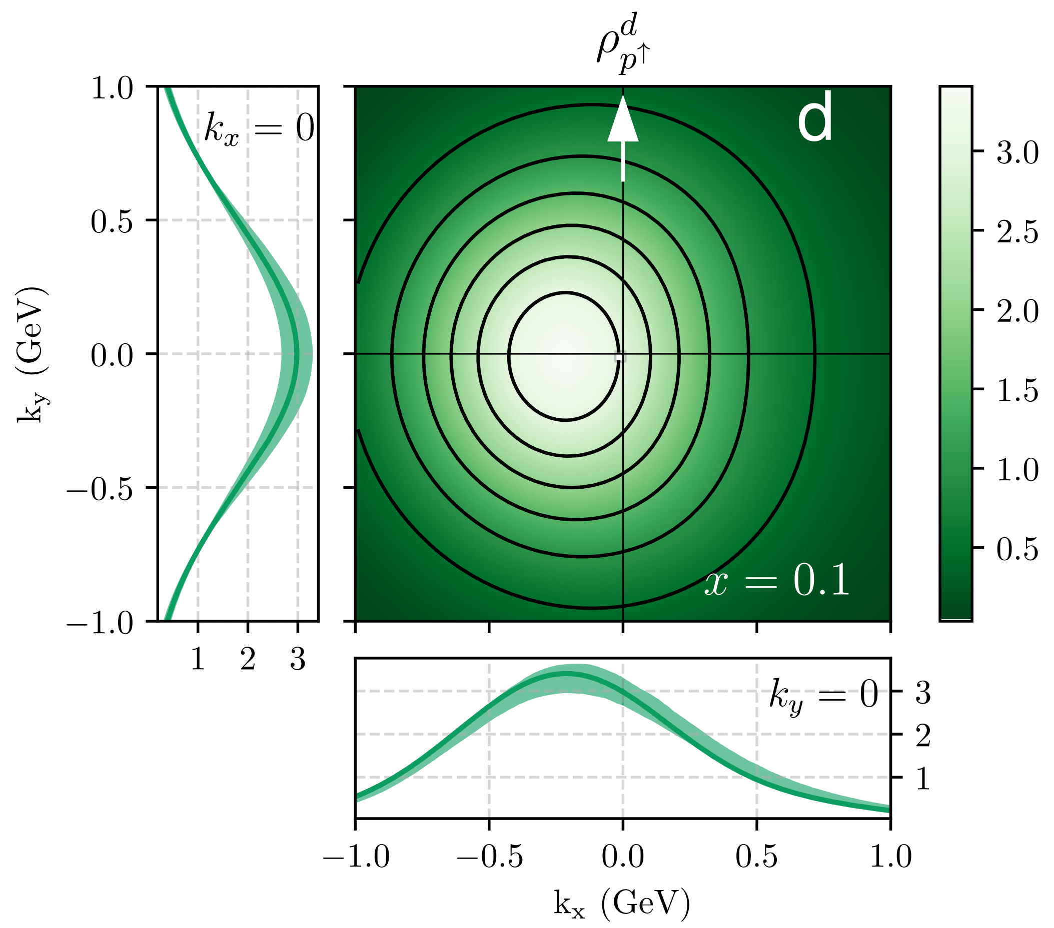

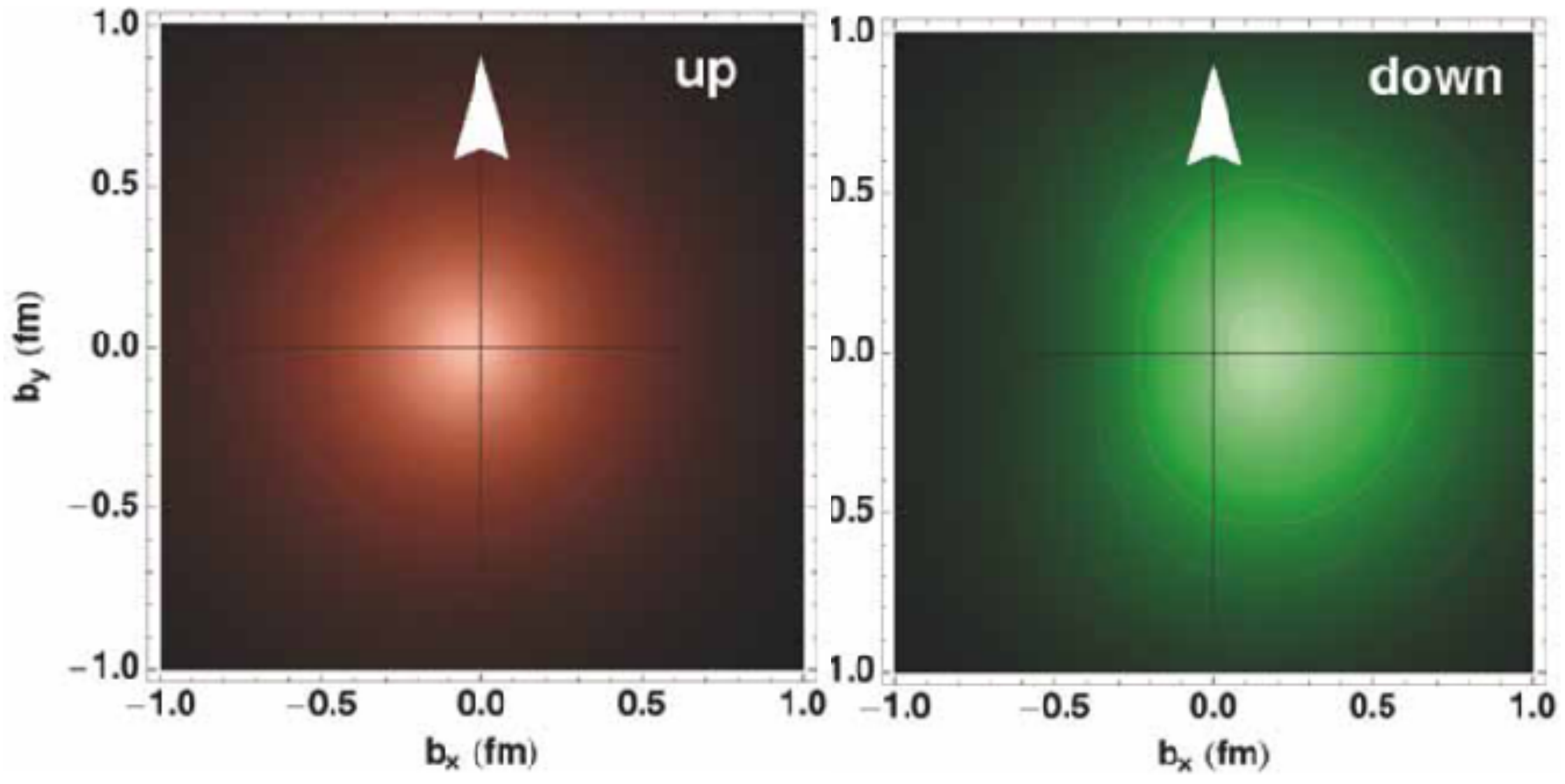

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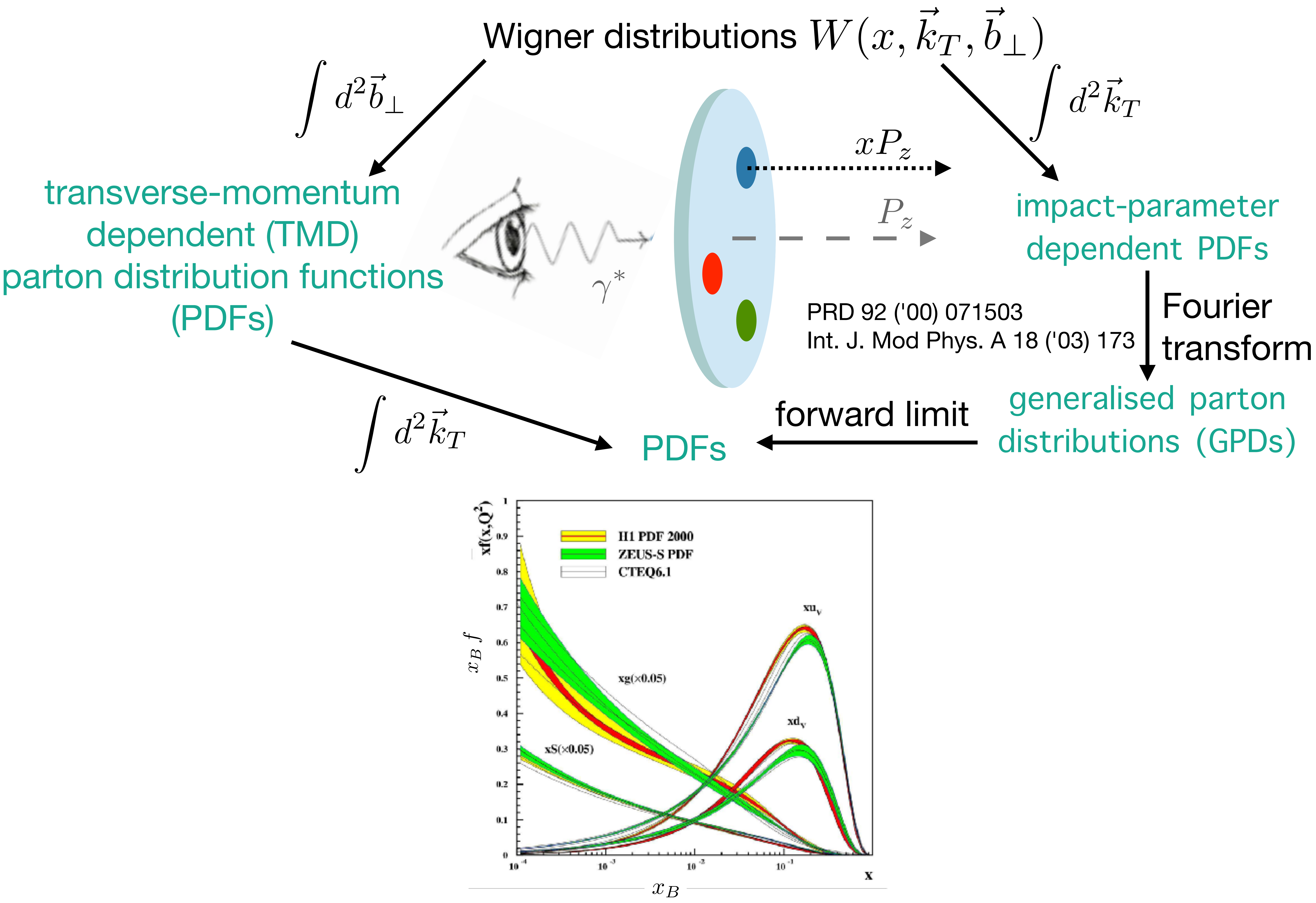
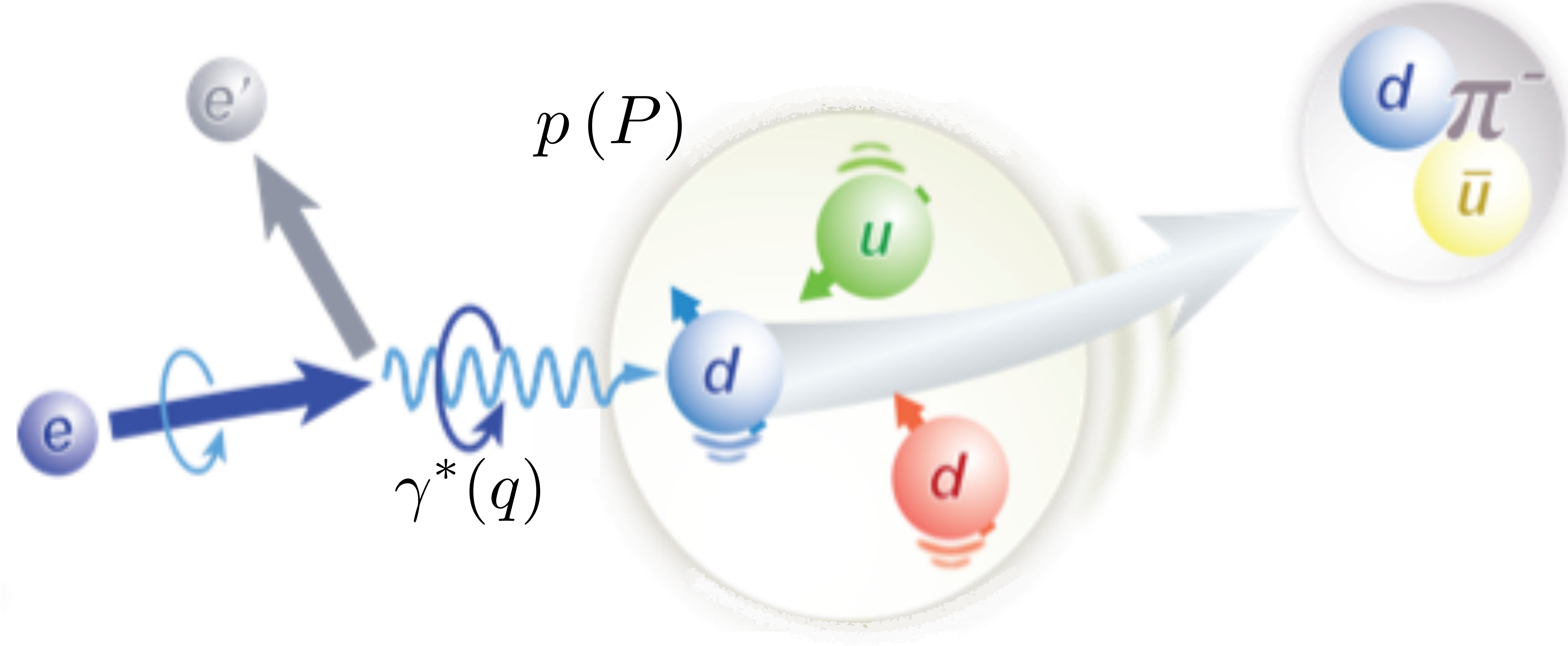


Single-hadron production in semi-inclusive DIS

$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2P \cdot q}$$



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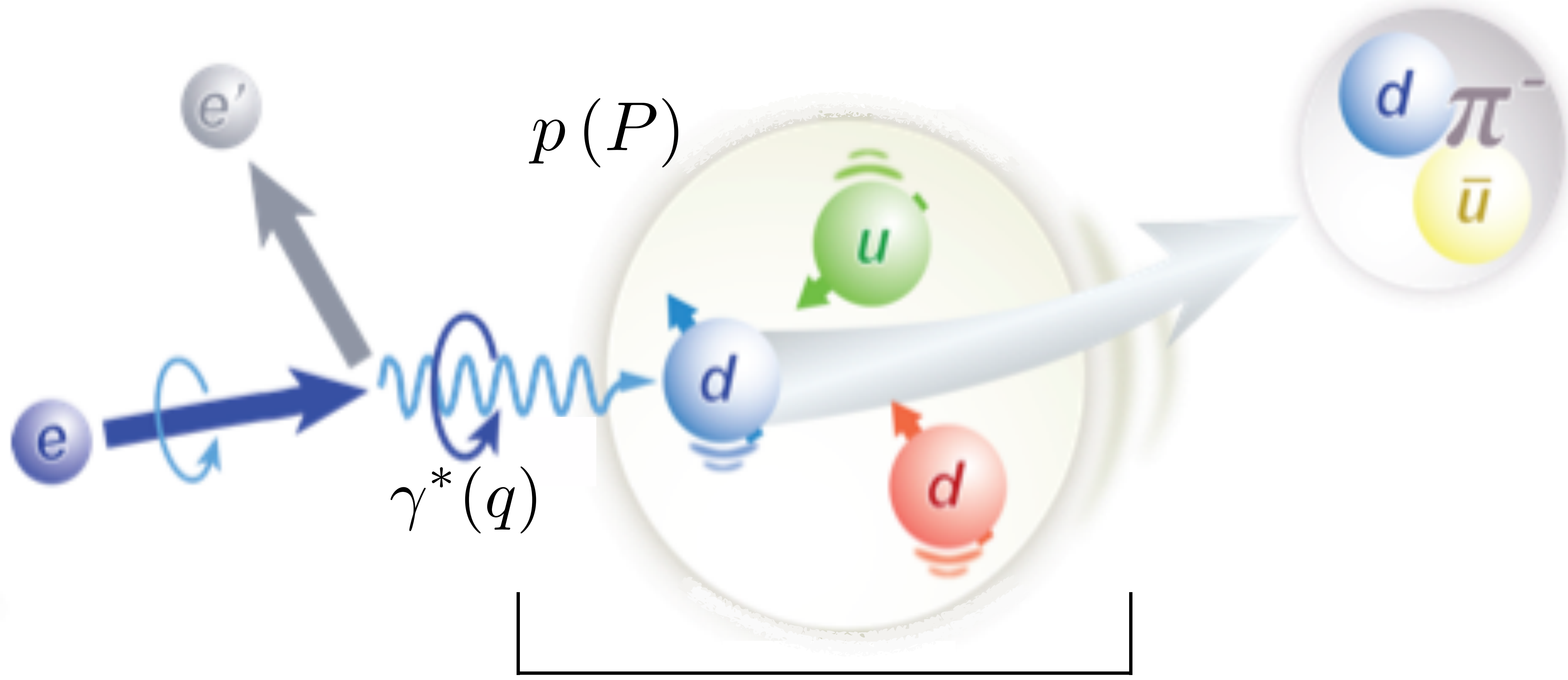

parton distribution function $PDF(x_B)$

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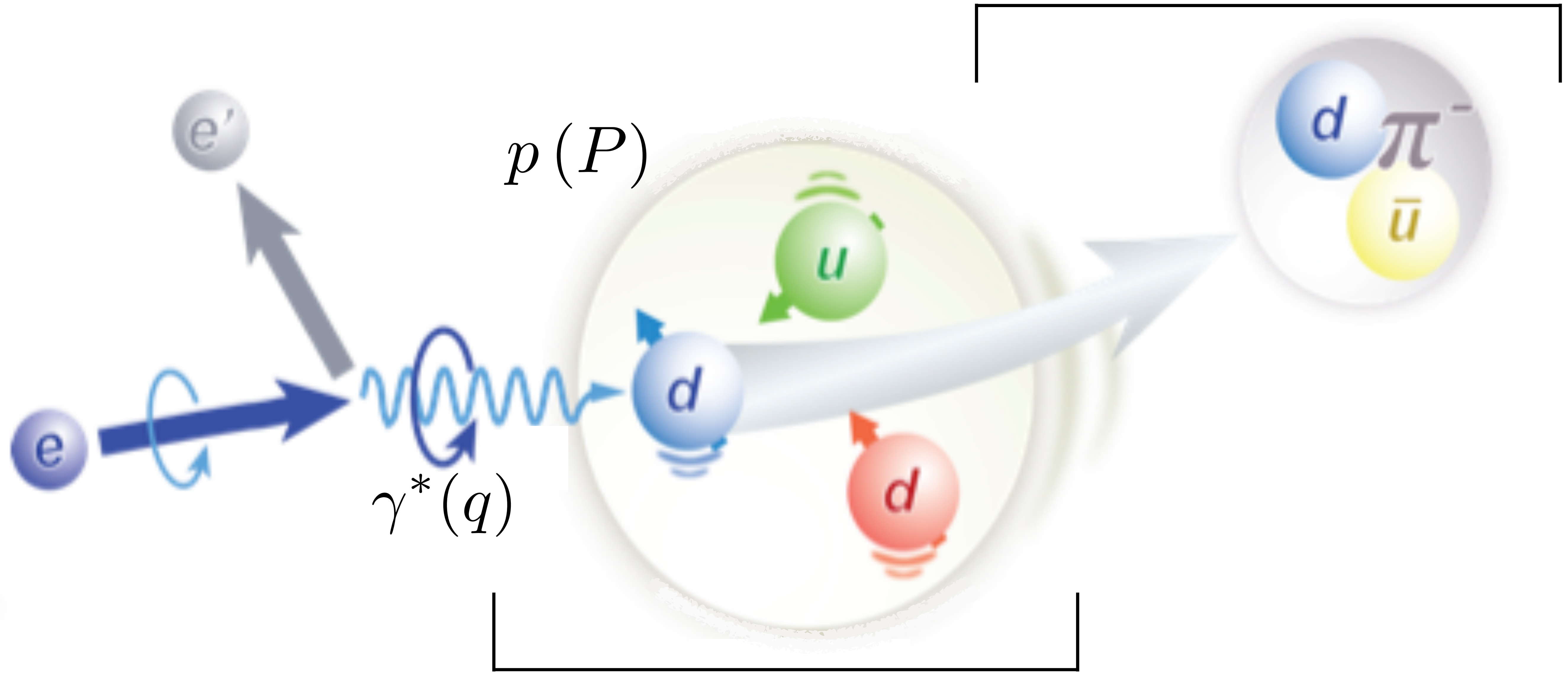
$$z \overset{\text{lab}}{=} \frac{E_h}{E_{\gamma*}}$$

fragmentation function $FF(z)$



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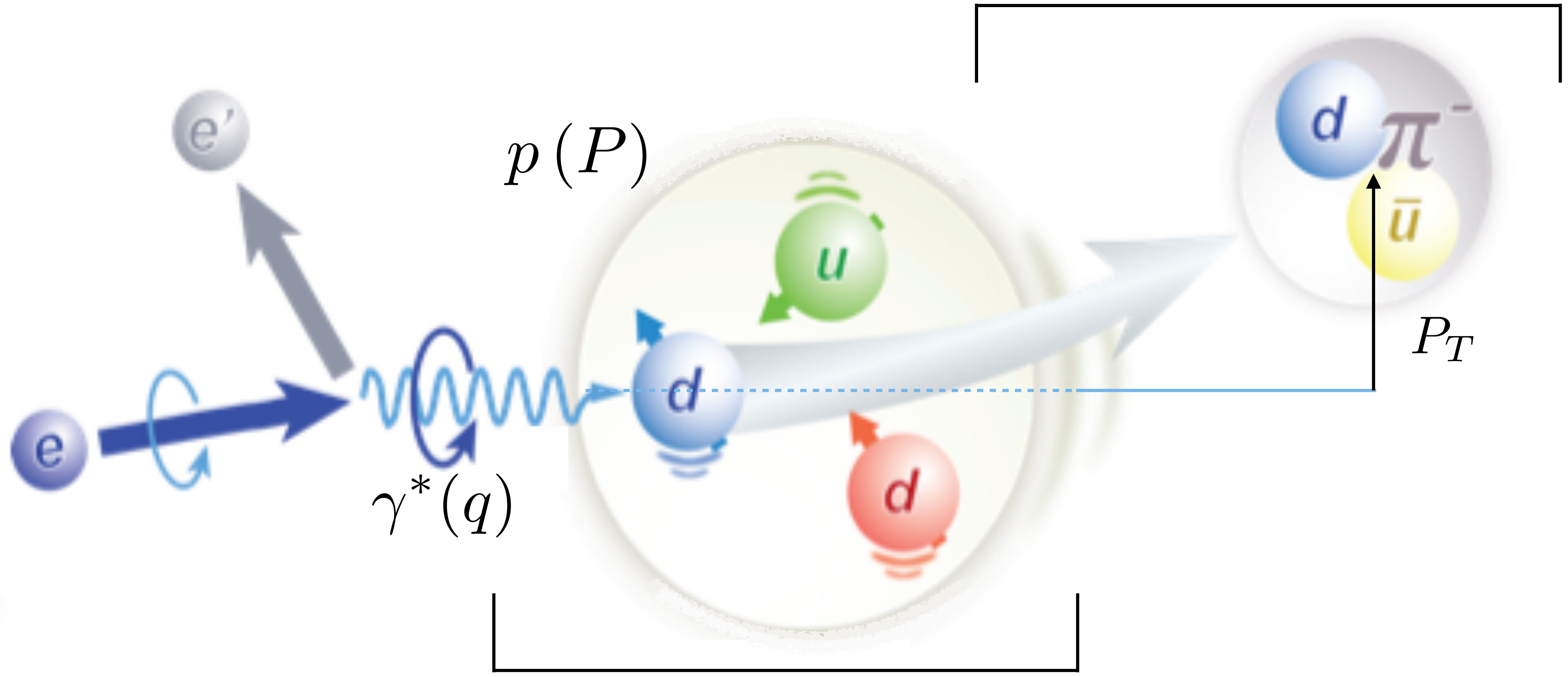
Transverse-momentum-dependent (TMD) fragmentation function $FF(z, p_\perp)$



Transverse-momentum-dependent (TMD) parton distribution function $PDF(x_B, k_\perp)$

Single-hadron production in semi-inclusive DIS

Transverse-momentum-dependent (TMD)

fragmentation function $FF(z, p_\perp, Q^2)$

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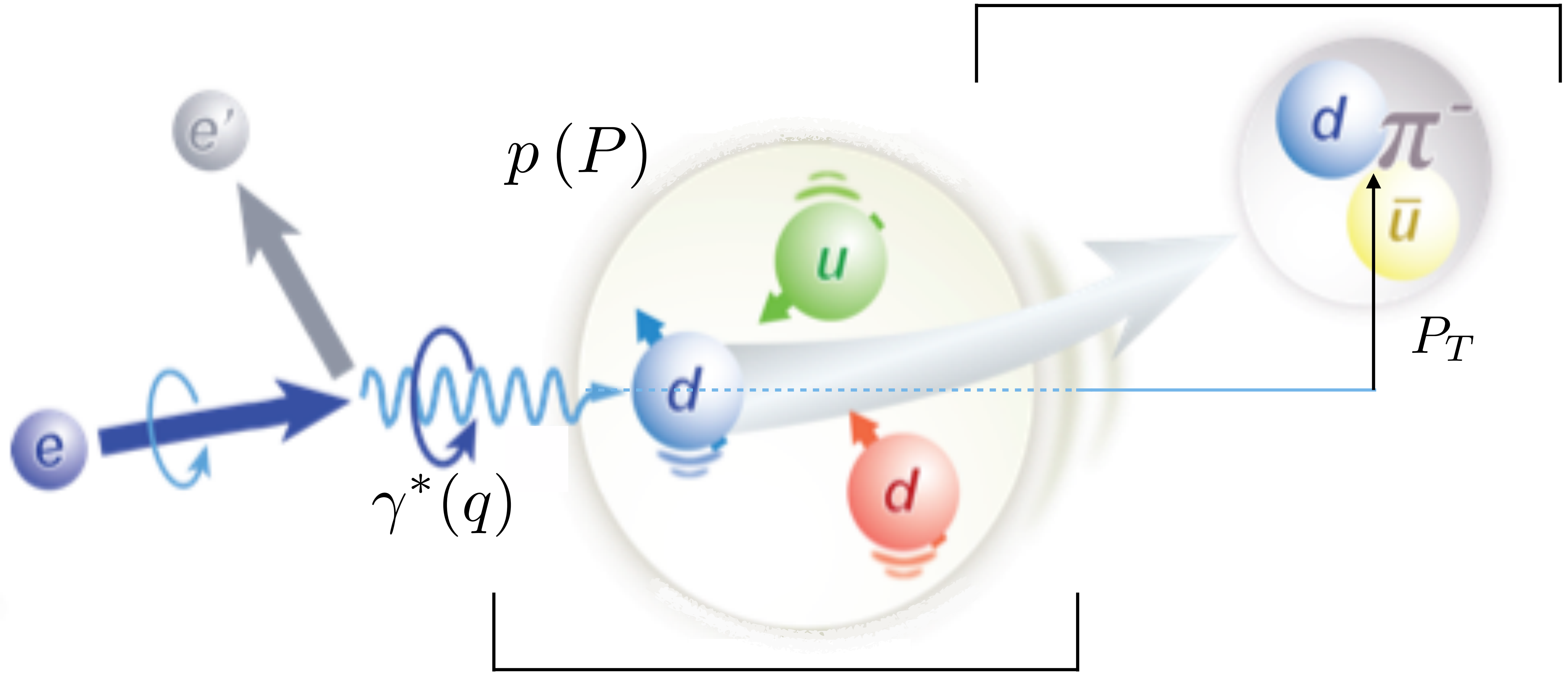

TMD evolution

Transverse-momentum-dependent (TMD)

parton distribution function $PDF(x_B, k_\perp, Q^2)$

Single-hadron production in semi-inclusive DIS

$Q^2 = -q^2$

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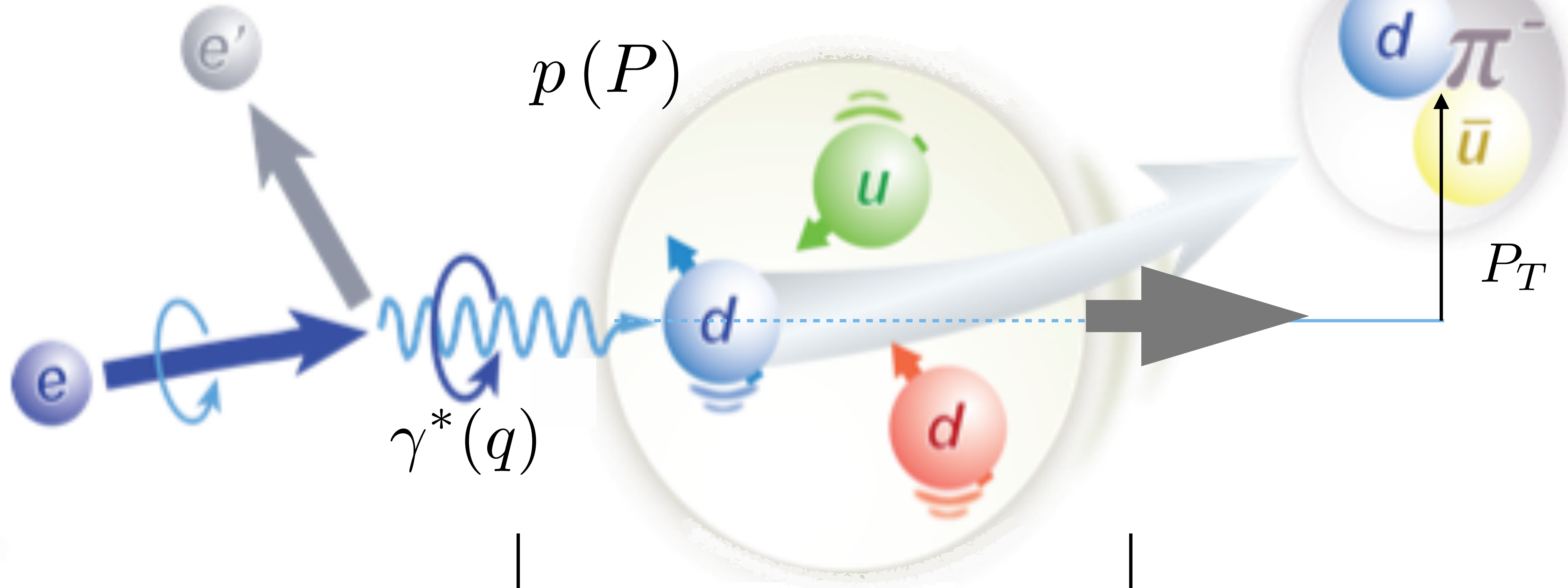
Transverse-momentum-dependent (TMD) fragmentation function $FF(z, p_\perp, Q^2)$



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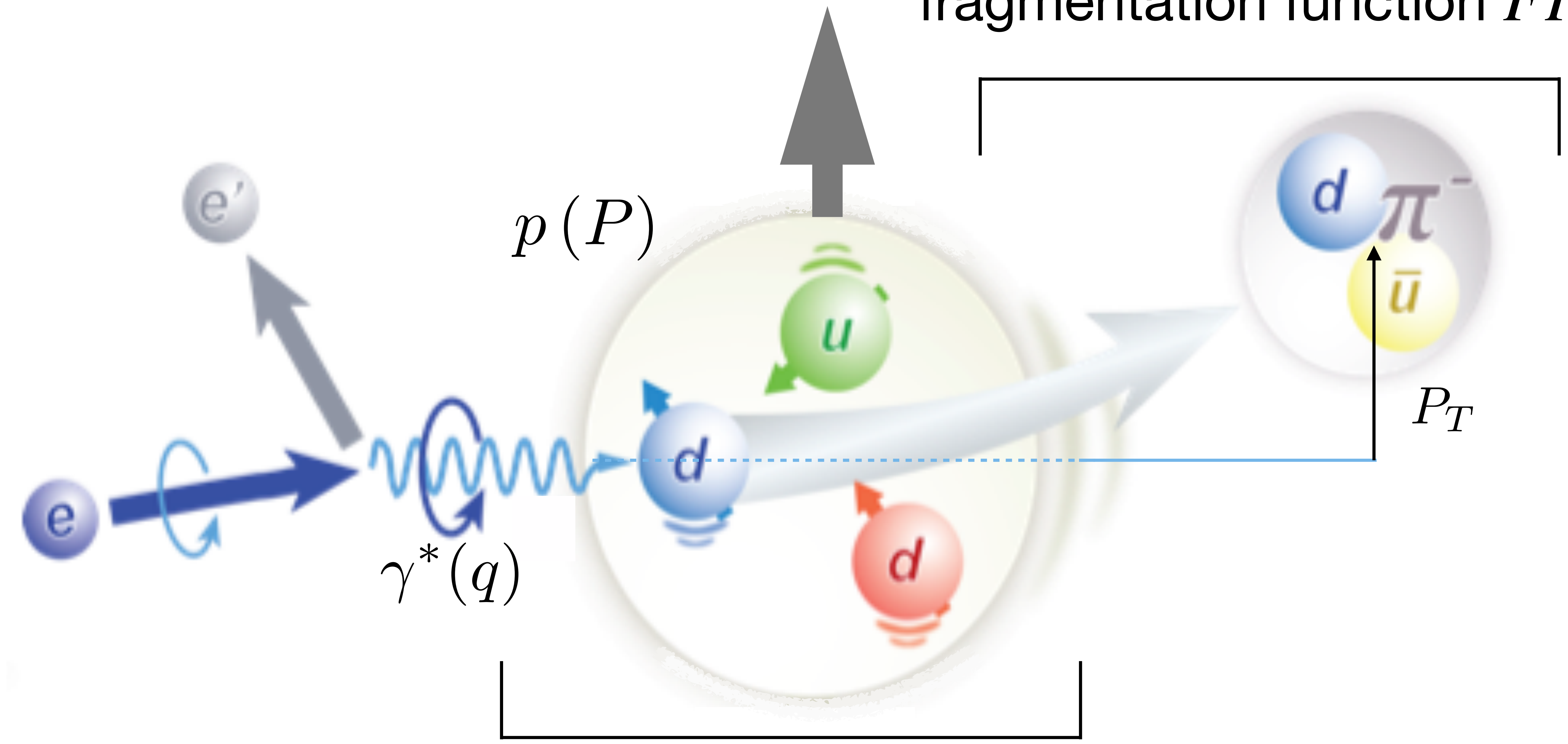
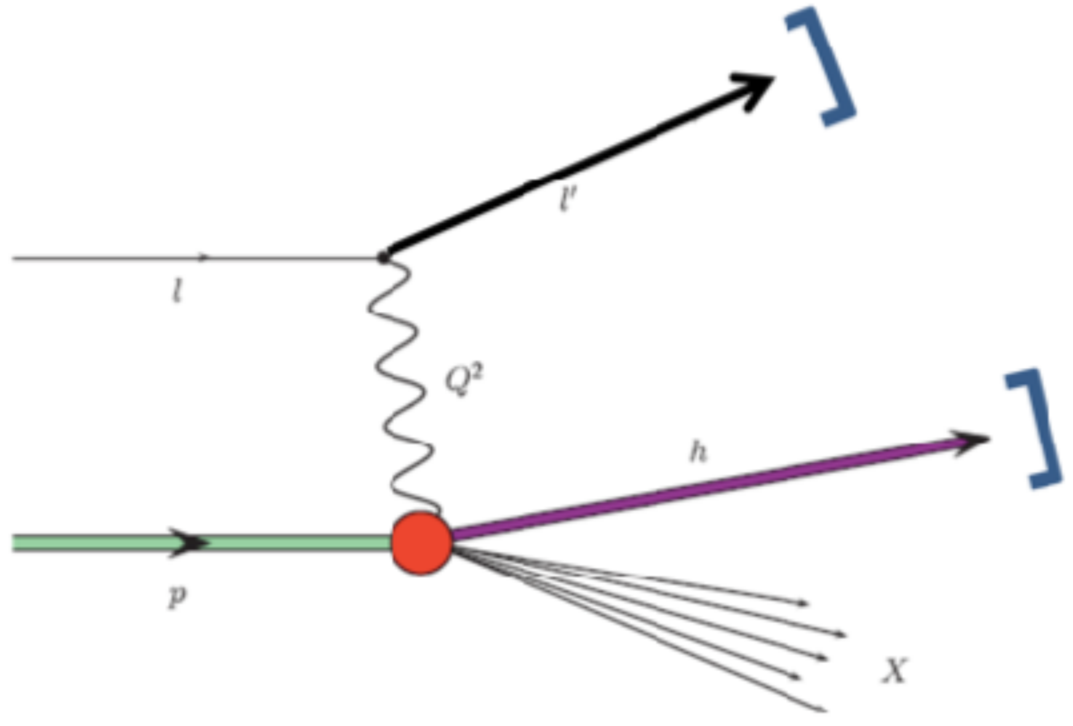


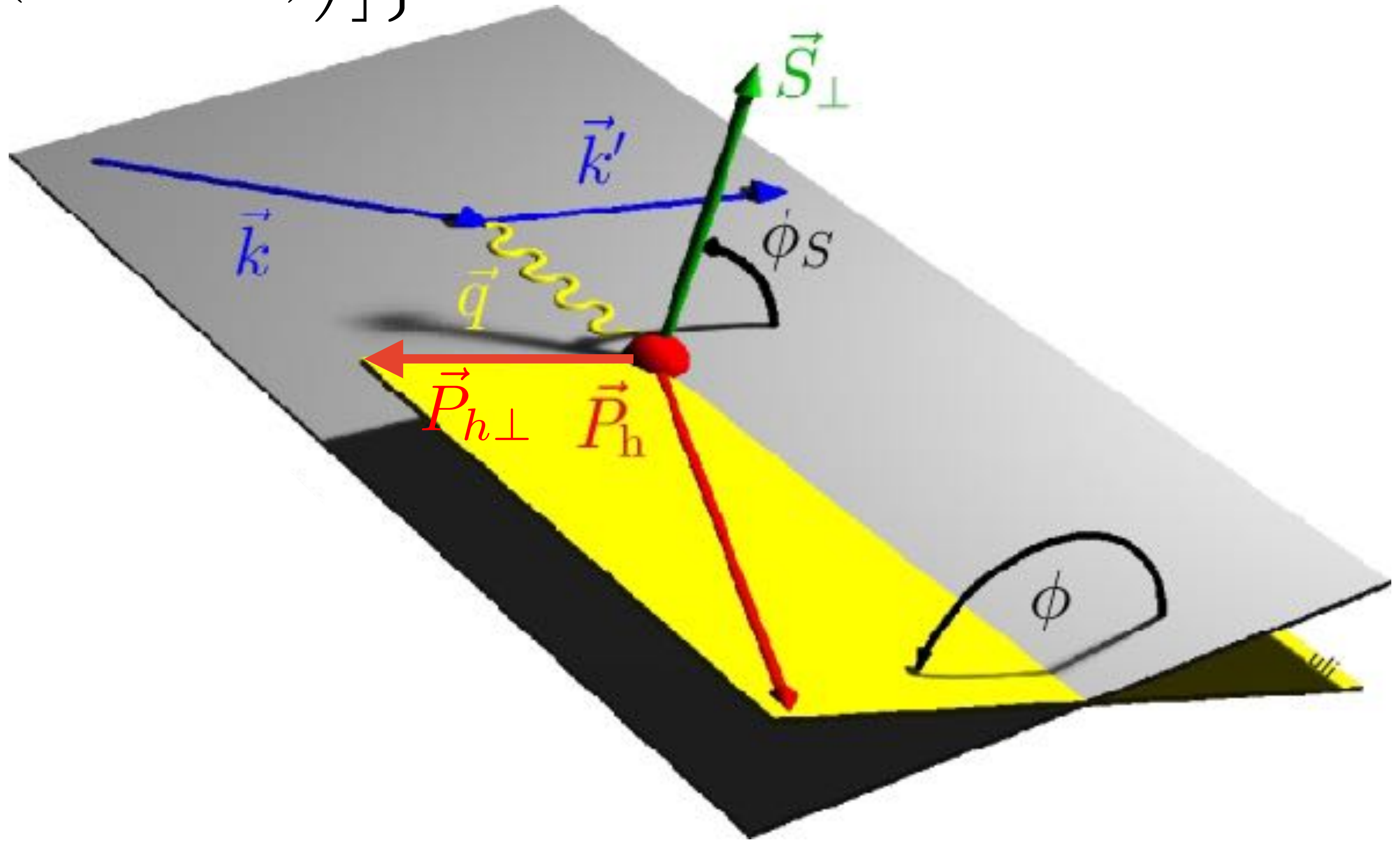
Transverse-momentum-dependent (TMD)
parton distribution function $PDF(x_B, k_\perp, Q^2)$

TMD evolution

Semi-inclusive DIS cross section



$$
\begin{aligned}
\sigma^h(\phi,\phi_S) \;=\;& \sigma^h_{UU}\Big\{1+2\langle\cos(\phi)\rangle^h_{UU}\,\cos(\phi)+2\langle\cos(2\phi)\rangle^h_{UU}\,\cos(2\phi)\\
+\;& \lambda_l\,2\langle\sin(\phi)\rangle^h_{LU}\,\sin(\phi)\\
+\;& S_L\Big[2\langle\sin(\phi)\rangle^h_{UL}\,\sin(\phi)+2\langle\sin(2\phi)\rangle^h_{UL}\,\sin(2\phi)\\
+\;& \lambda_l\Big(2\langle\cos(0\phi)\rangle^h_{LL}\,\cos(0\phi)+2\langle\cos(\phi)\rangle^h_{LL}\,\cos(\phi)\Big)\Big]\\
+\;& S_T\Big[2\langle\sin(\phi-\phi_S)\rangle^h_{UT}\,\sin(\phi-\phi_S)+2\langle\,\sin(\phi+\phi_S)\rangle^h_{UT}\,\sin(\phi+\phi_S)\\
+\;& 2\langle\sin(3\phi-\phi_S)\rangle^h_{UT}\,\sin(3\phi-\phi_S)+2\langle\sin(\phi_S)\rangle^h_{UT}\,\sin(\phi_S)\\
+\;& 2\langle\sin(2\phi-\phi_S)\rangle^h_{UT}\,\sin(2\phi-\phi_S)\\
+\;& \lambda_l\Big(2\langle\cos(\phi-\phi_S)\rangle^h_{LT}\,\cos(\phi-\phi_S)\\
+\;& 2\langle\cos(\phi_S)\rangle^h_{LT}\,\cos(\phi_S)+2\langle\cos(2\phi-\phi_S)\rangle^h_{LT}\,\cos(2\phi-\phi_S)\Big)\Big]\Big\}
\end{aligned}
$$



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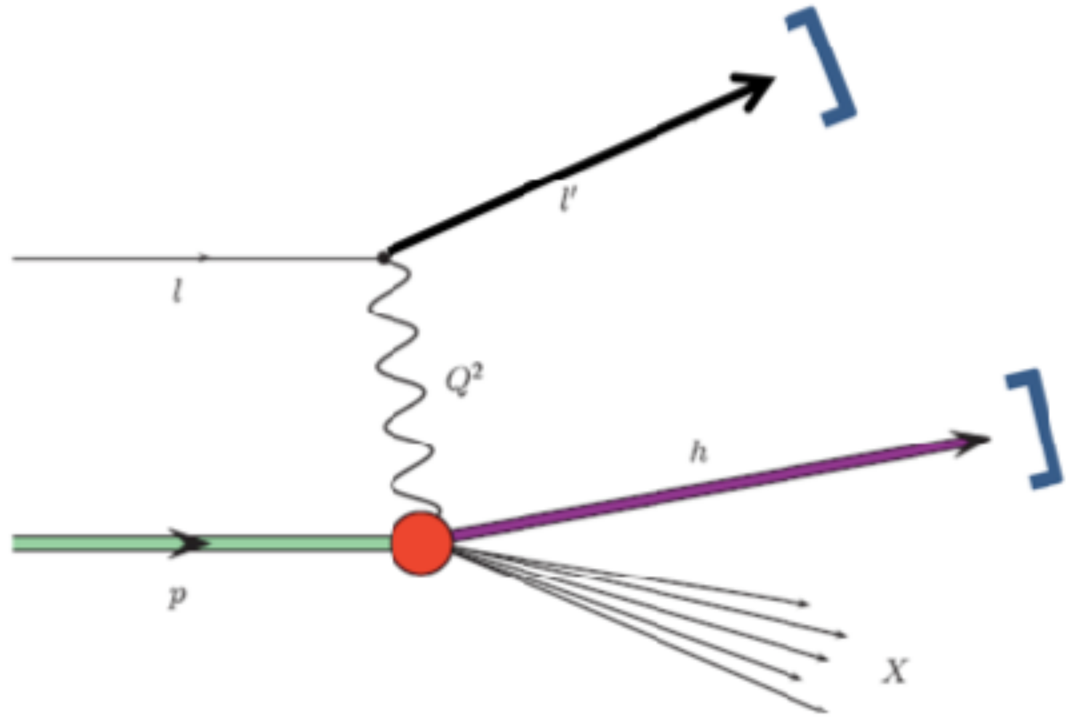

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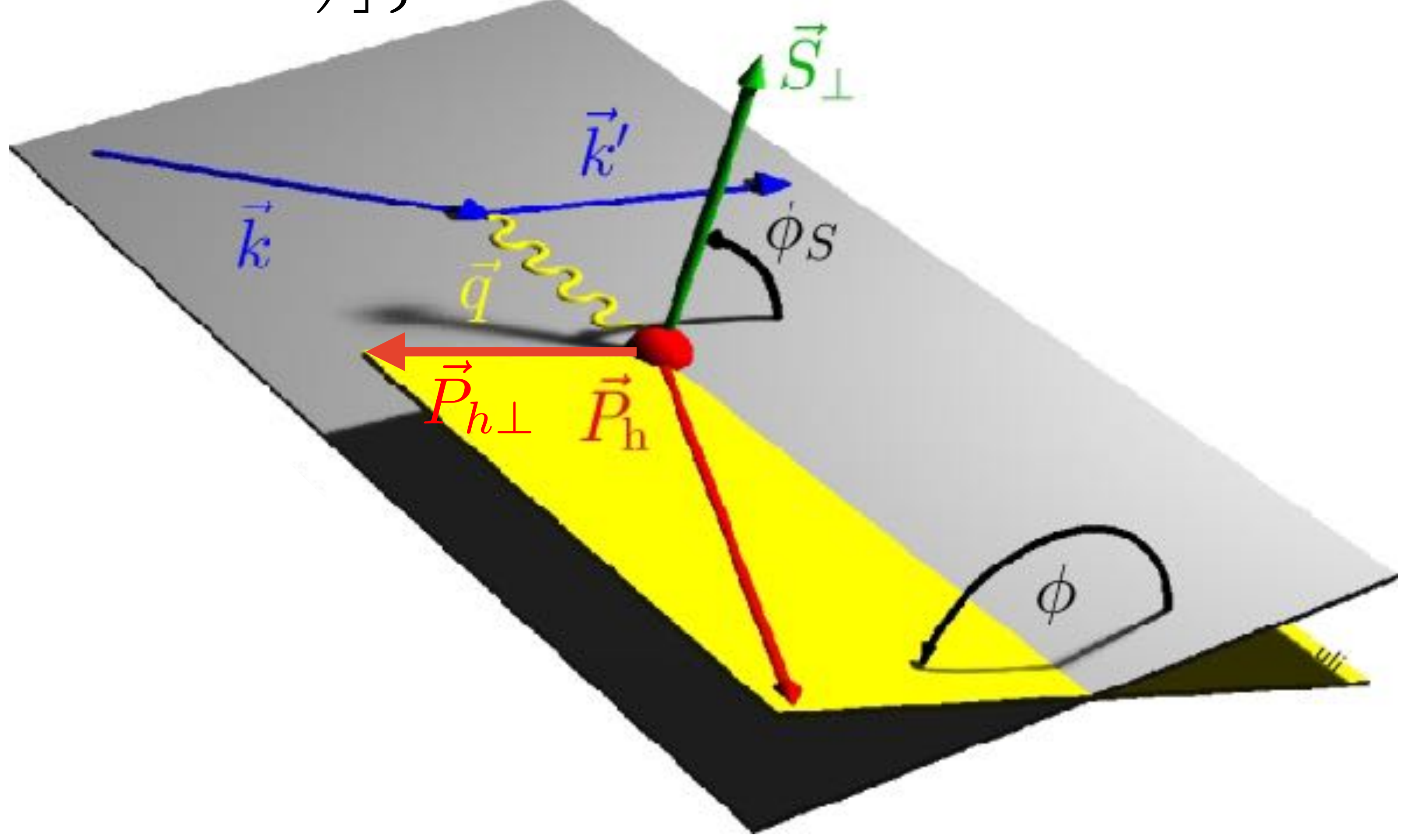
&+ S_L\Big[2\langle\sin(\phi)\rangle^h_{UL}\,\sin(\phi) + 2\langle\sin(2\phi)\rangle^h_{UL}\,\sin(2\phi)\\
&+ \lambda_l\Big(2\langle\cos(0\phi)\rangle^h_{LL}\,\cos(0\phi) + 2\langle\cos(\phi)\rangle^h_{LL}\,\cos(\phi)\Big)\Big]\\
&+ S_T\Big[2\langle\sin(\phi-\phi_S)\rangle^h_{UT}\,\sin(\phi-\phi_S) + 2\langle\sin(\phi+\phi_S)\rangle^h_{UT}\,\sin(\phi+\phi_S)\\
&+ 2\langle\sin(3\phi-\phi_S)\rangle^h_{UT}\,\sin(3\phi-\phi_S) + 2\langle\sin(\phi_S)\rangle^h_{UT}\,\sin(\phi_S)\\
&+ 2\langle\sin(2\phi-\phi_S)\rangle^h_{UT}\,\sin(2\phi-\phi_S)\\
&+ \lambda_l\Big(2\langle\cos(\phi-\phi_S)\rangle^h_{LT}\,\cos(\phi-\phi_S)\\
&+ 2\langle\cos(\phi_S)\rangle^h_{LT}\,\cos(\phi_S) + 2\langle\cos(2\phi-\phi_S)\rangle^h_{LT}\,\cos(2\phi-\phi_S)\Big)\Big]\Big\}
\end{aligned}
$$

longitudinal target polarisation (S_L)

transverse target polarisation (S_T)

beam polarisation (λ_l)

LT: beam polarisation, target polarisation



TMD PDFs and fragmentation functions (FFs)

Azimuthal amplitudes related to structure functions F_{XY} :

$$2\langle \sin(\phi + \phi_S) \rangle^h_{UT} = \epsilon \, F^{\sin(\phi+\phi_S)}_{UT}$$

TMD PDFs and fragmentation functions (FFs)

Azimuthal amplitudes related to structure functions F_{XY} :

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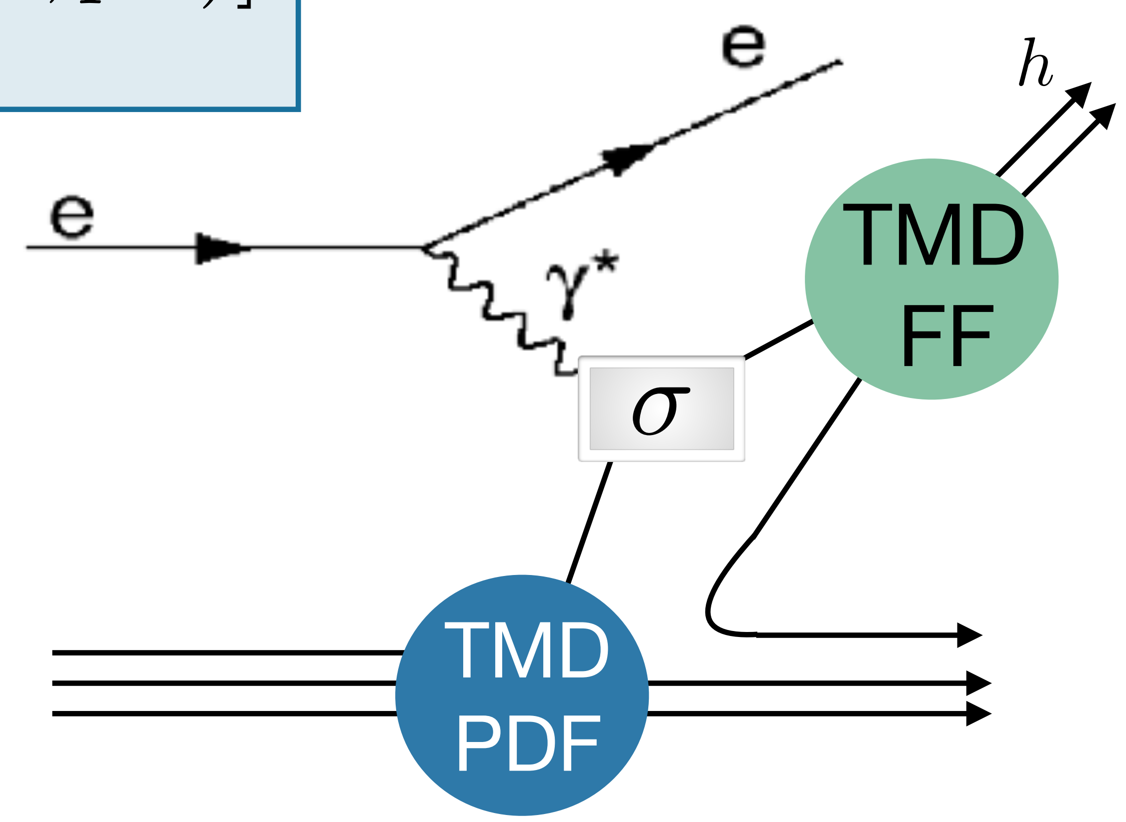
$$F_{XY} \propto \mathcal{C}\,[\mathrm{TMD\ PDF}(x, k_\perp) \times\ \mathrm{TMD\ FF}\ (z, p_\perp)]$$

$$z \overset{\mathrm{lab}}{=} \frac{E_h}{E_{\gamma*}}$$



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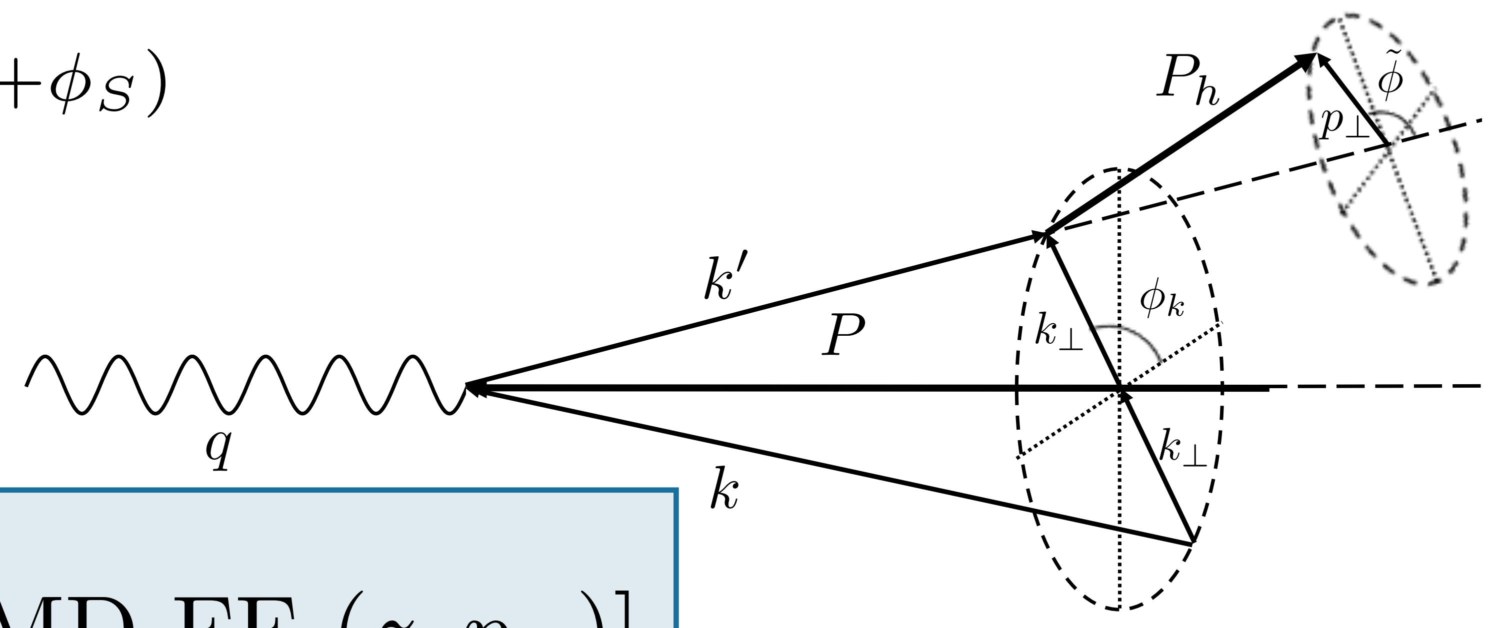

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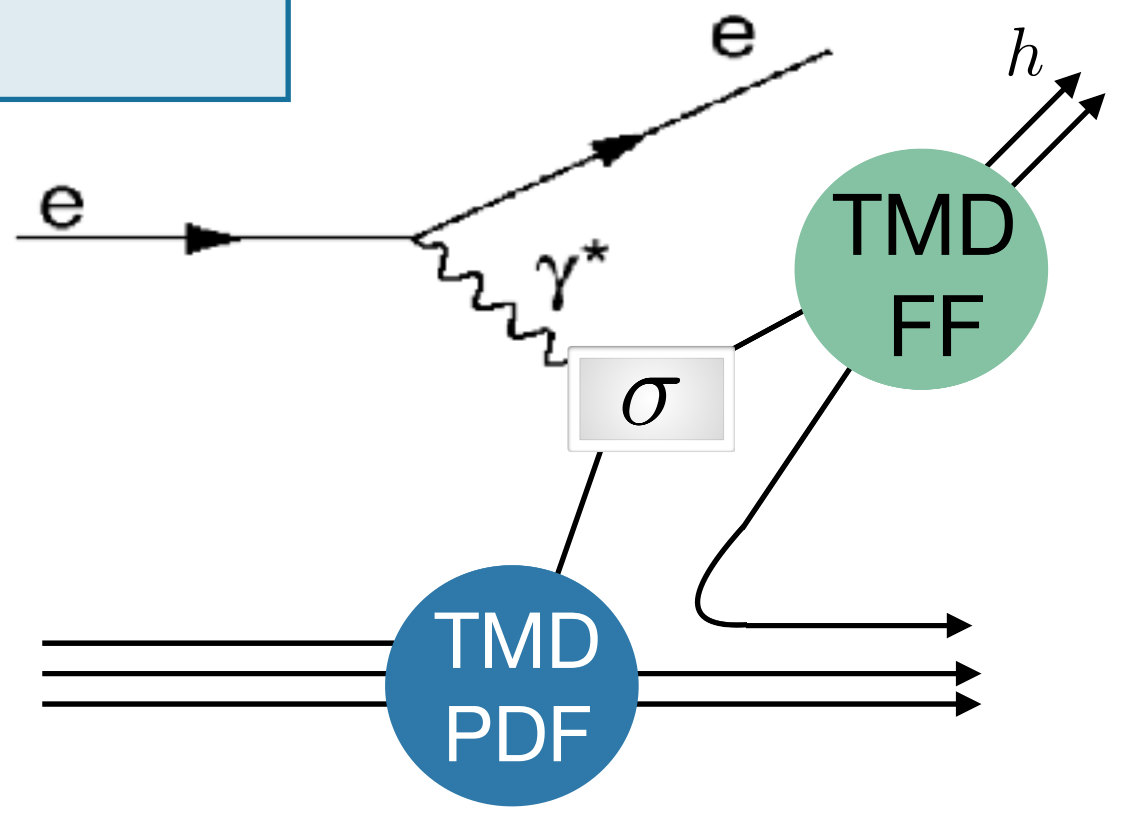

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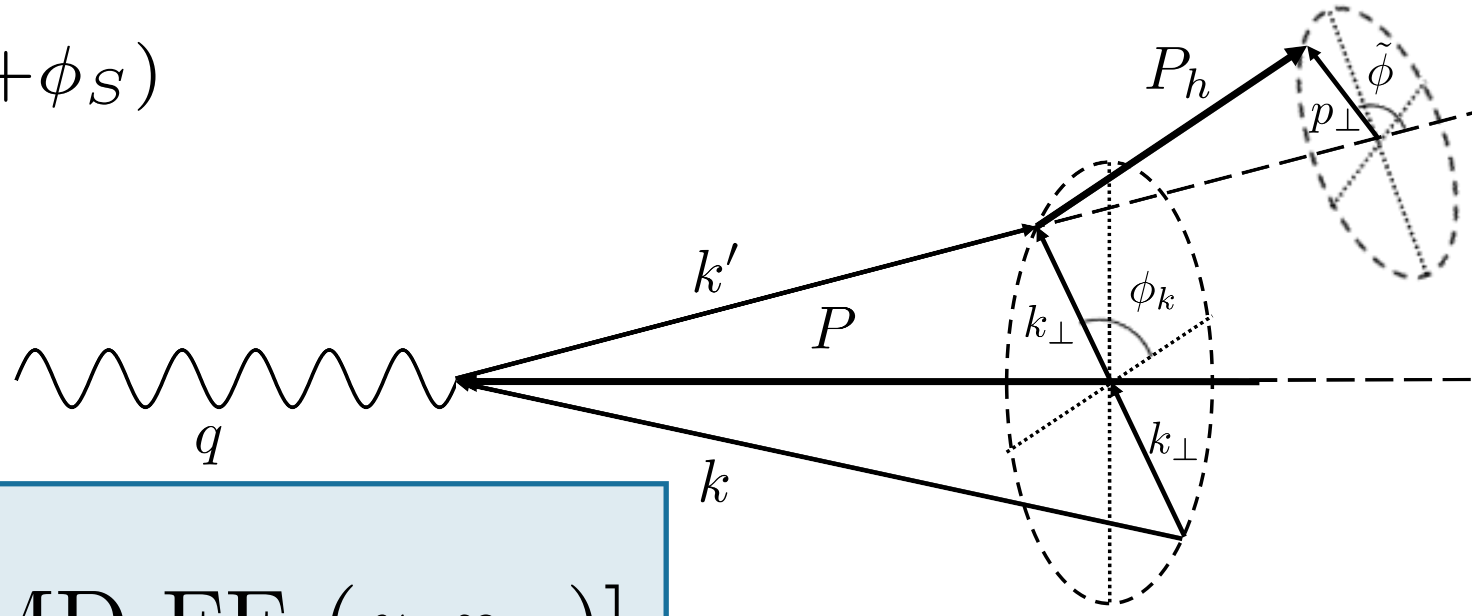

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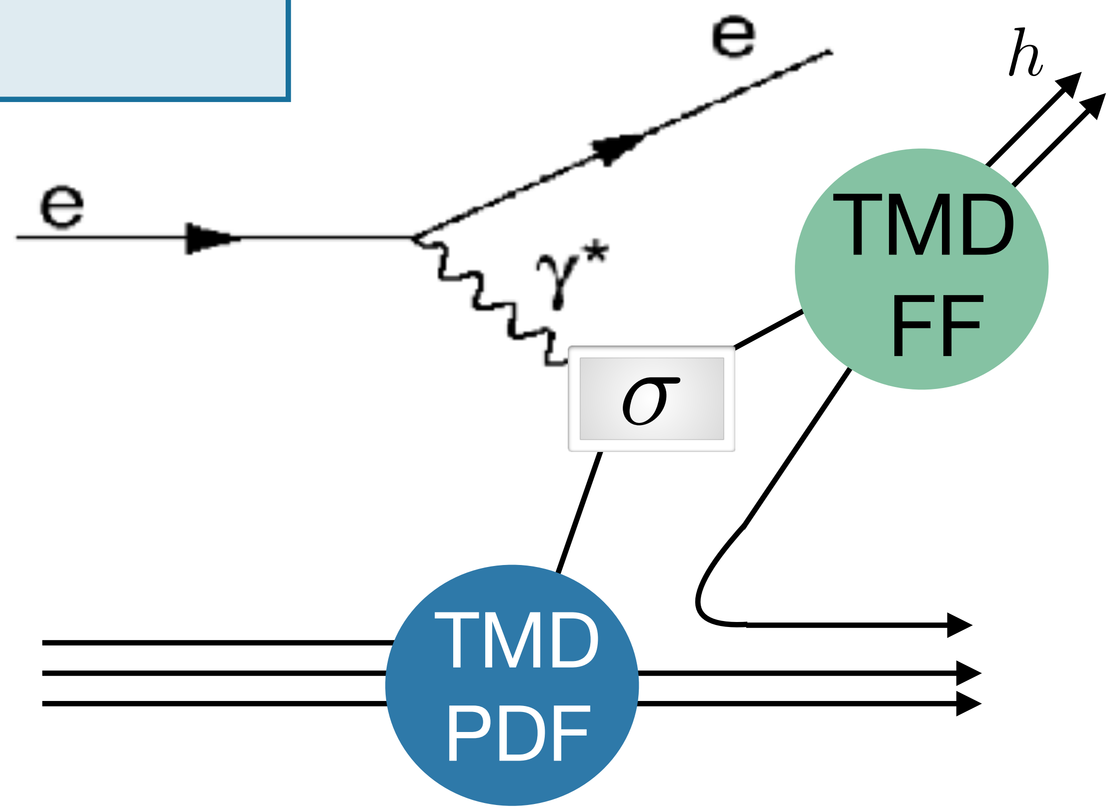

quark polarisation

nucleon polarisation

	U	L	T
U	f_1		
L		g_{1L}	
T			h_{1T}

survive integration of parton transverse momentum

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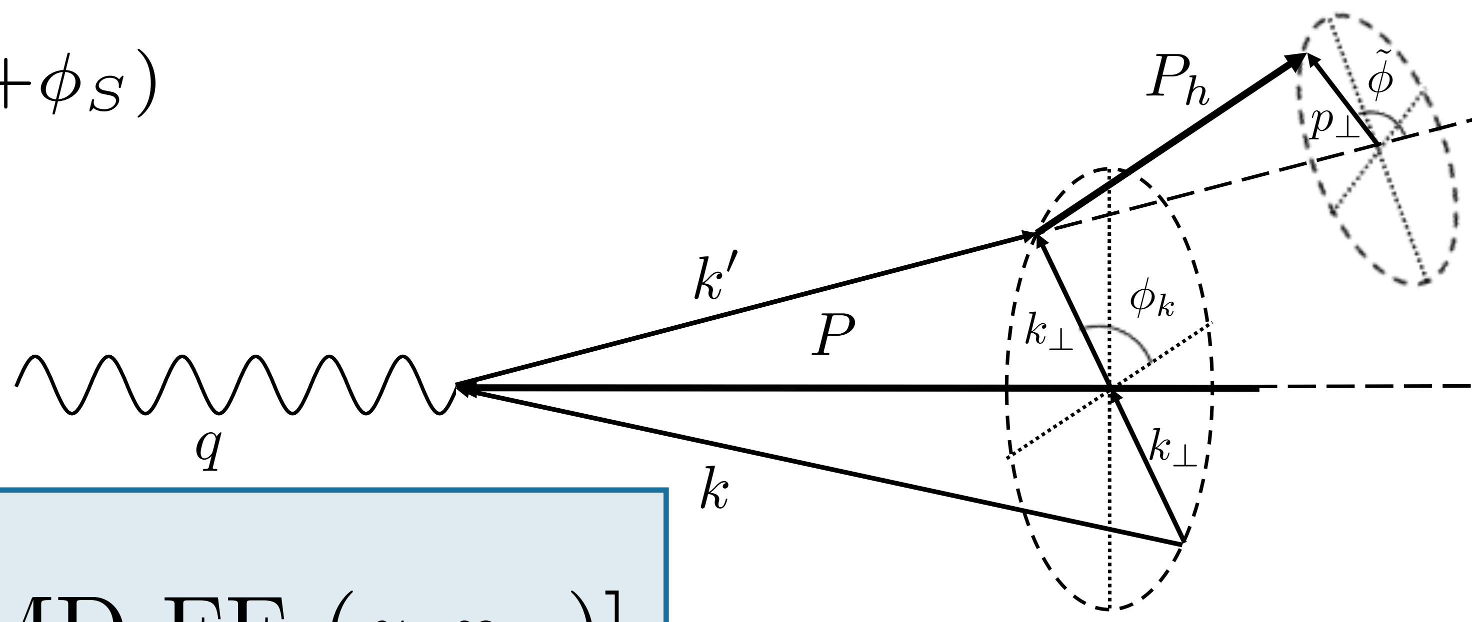

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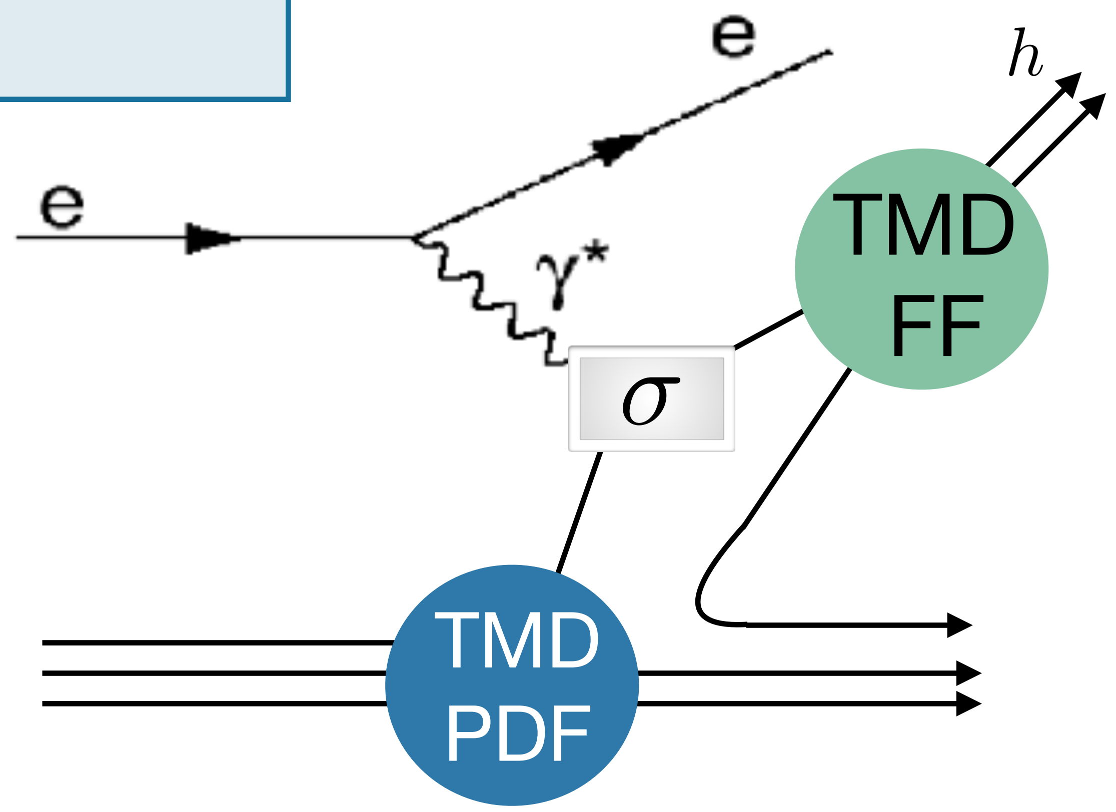

quark polarisation

nucleon polarisation

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_{1T}\, h_{1T}^\perp$

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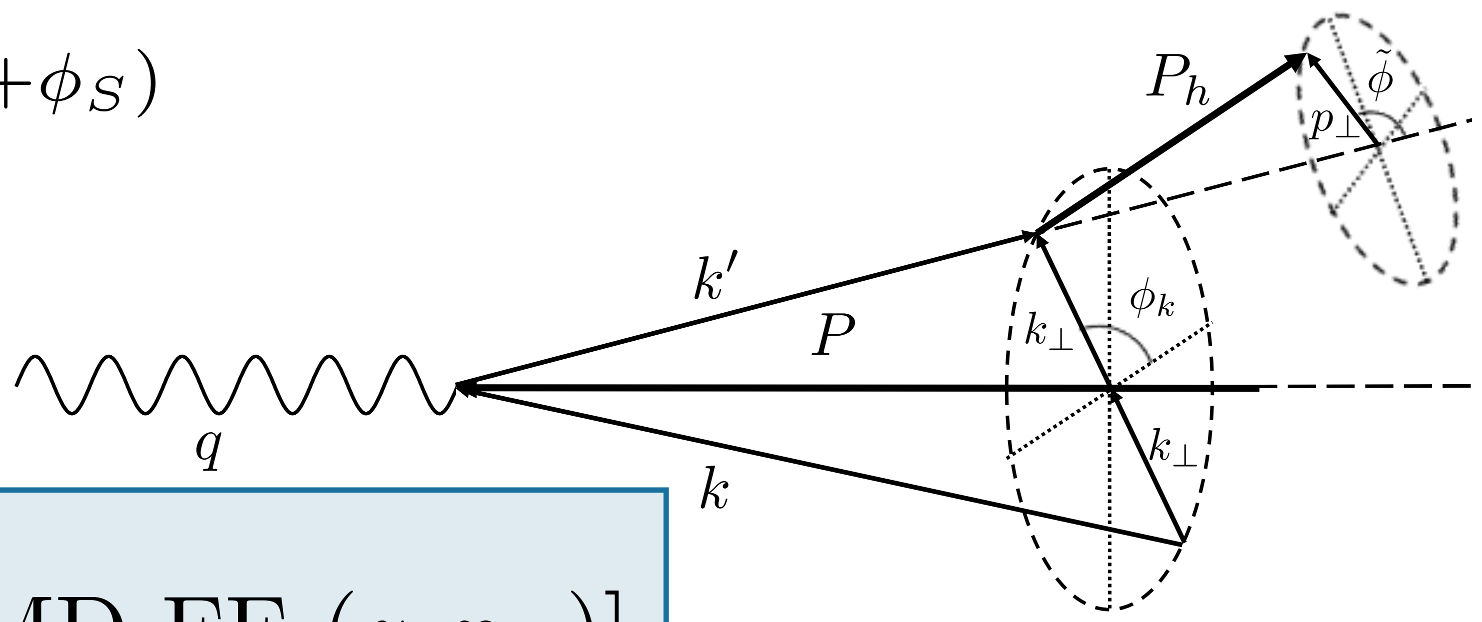
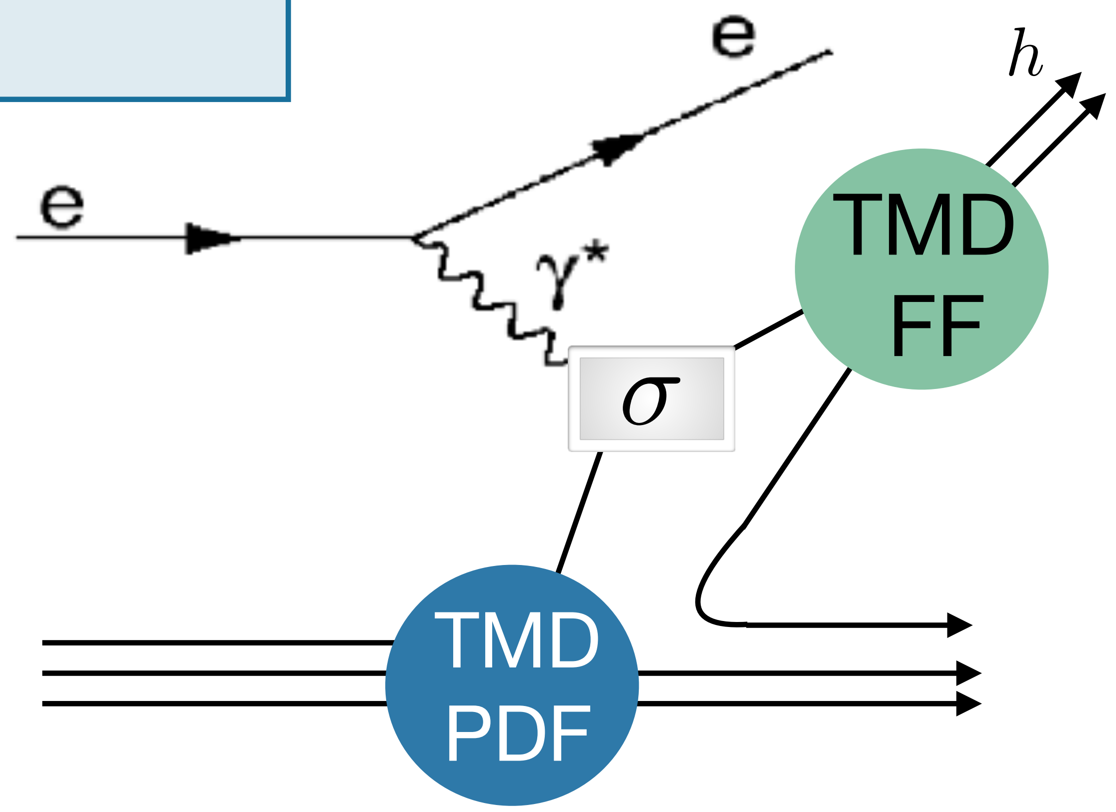
Azimuthal amplitudes related to structure functions F_{XY} :

$$2\langle\,\sin(\phi+\phi_S)\rangle^h_{UT} = \epsilon\, F_{UT}^{\sin(\phi+\phi_S)}$$



$$F_{XY} \propto \mathcal{C}\,[\mathrm{TMD\ PDF}(x,k_\perp)\times\ \mathrm{TMD\ FF}\ (z,p_\perp)]$$

$$z \overset{\mathrm{lab}}{=} \frac{E_h}{E_{\gamma *}}$$



quark polarisation (columns) / nucleon polarisation (rows)

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_{1T}\,h_{1T}^\perp$

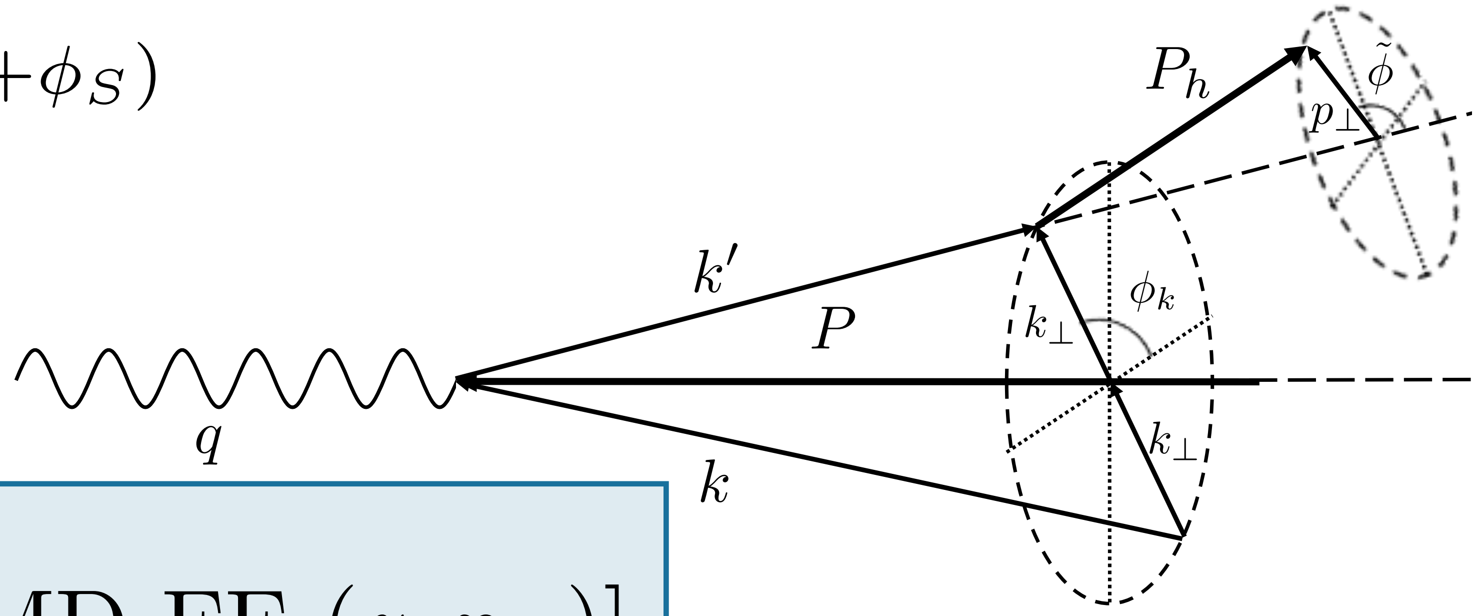
quark polarisation (columns) / hadron polarisation (rows)

	U	L	T
U	D_1		$H_1^\perp$

TMD PDFs and fragmentation functions (FFs)

Azimuthal amplitudes related to structure functions F_{XY} :

$$2\langle \sin(\phi+\phi_S)\rangle^h_{UT} = \epsilon\, F_{UT}^{\sin(\phi+\phi_S)}$$



$$F_{XY} \propto \mathcal{C}\,[\text{TMD PDF}(x, k_\perp) \times \ \text{TMD FF}\ (z, p_\perp)]$$

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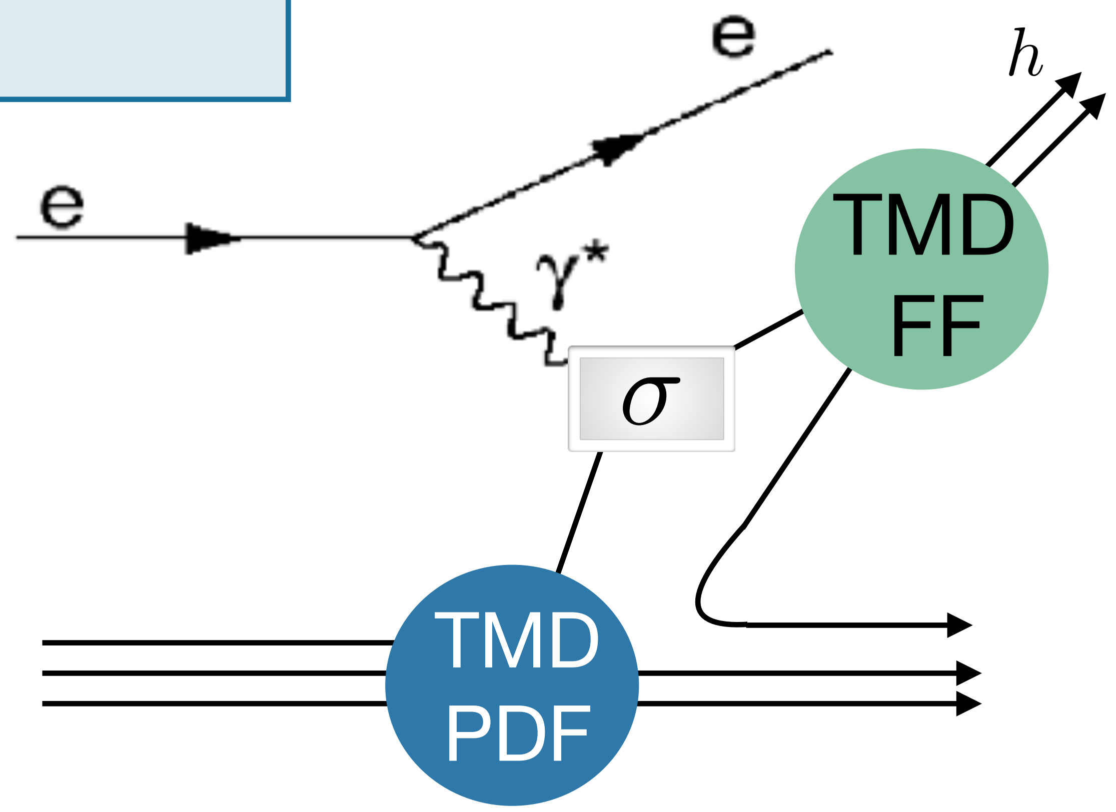

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quark polarisation (columns) / hadron polarisation (rows)

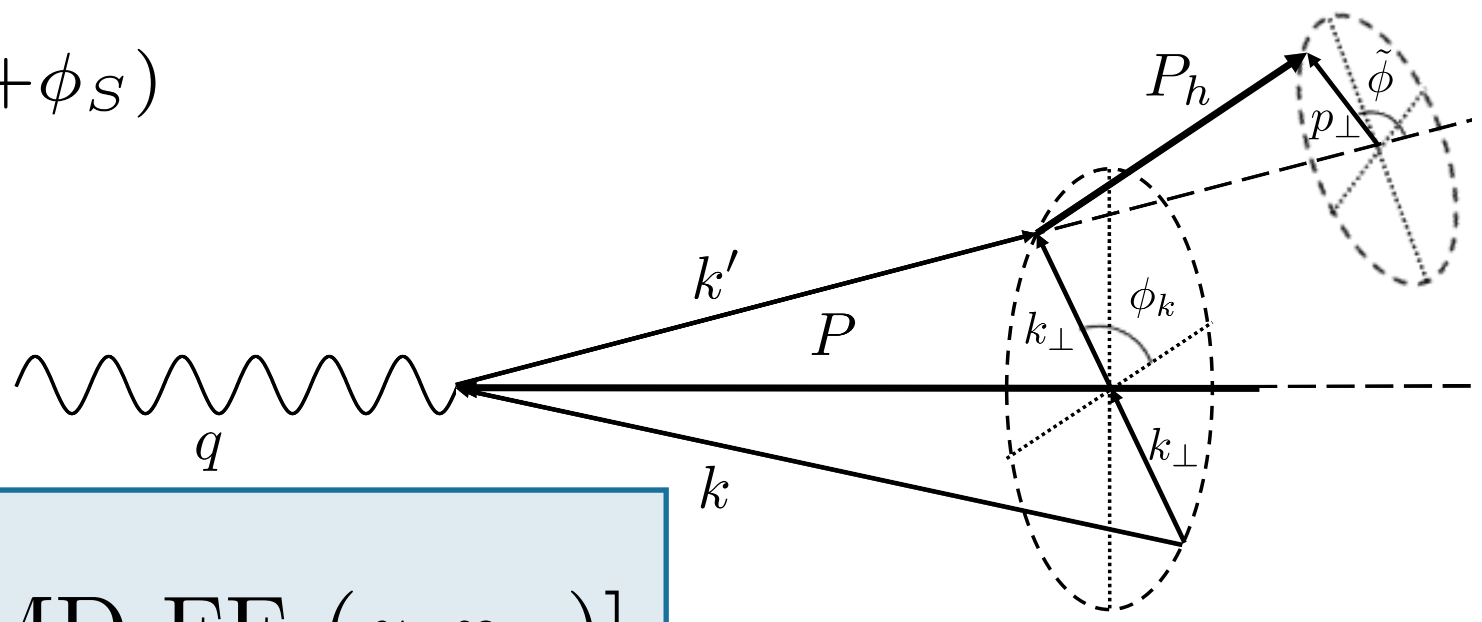
	U	L	T
U	D_1		$H_1^\perp$

Chiral odd

TMD PDFs and fragmentation functions (FFs)

Azimuthal amplitudes related to structure functions F_{XY} :

$$2\langle \sin(\phi + \phi_S)\rangle^h_{UT} = \epsilon\, F^{\sin(\phi+\phi_S)}_{UT}$$



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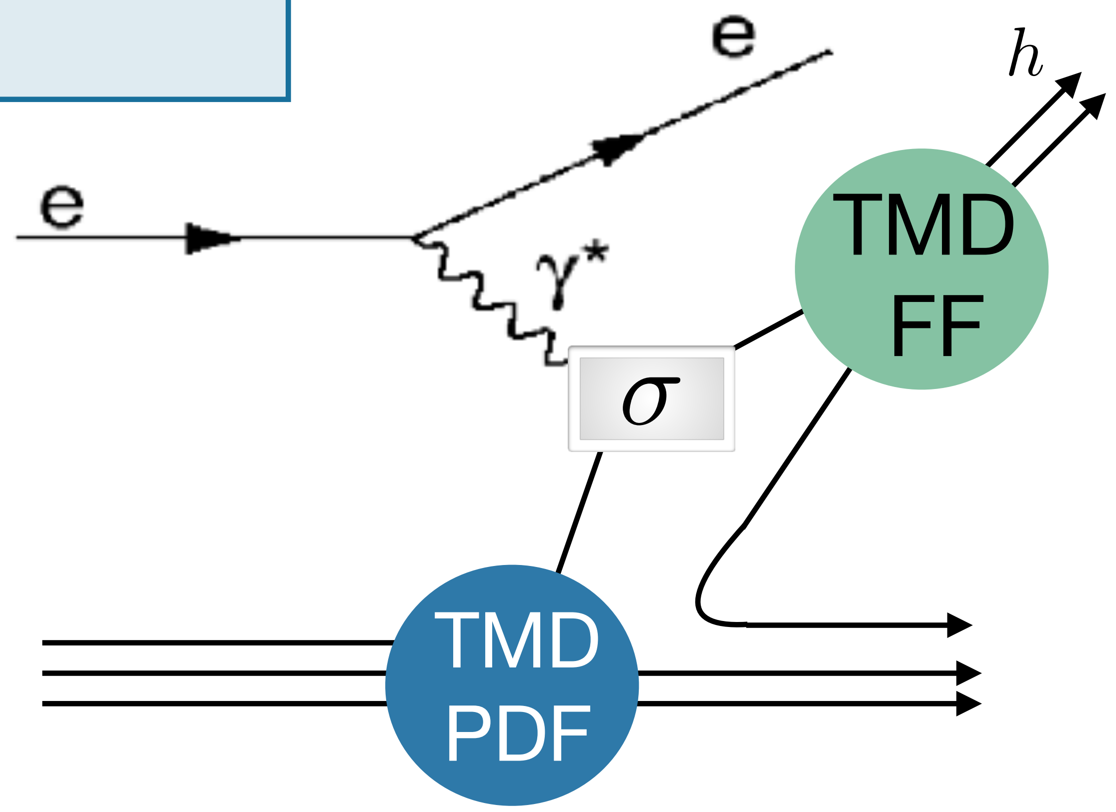

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quark polarisation (columns); hadron polarisation (rows)

	U	L	T
U	D_1		$H_1^\perp$

Chiral odd

Naive T-odd

TMD PDFs and fragmentation functions (FFs)

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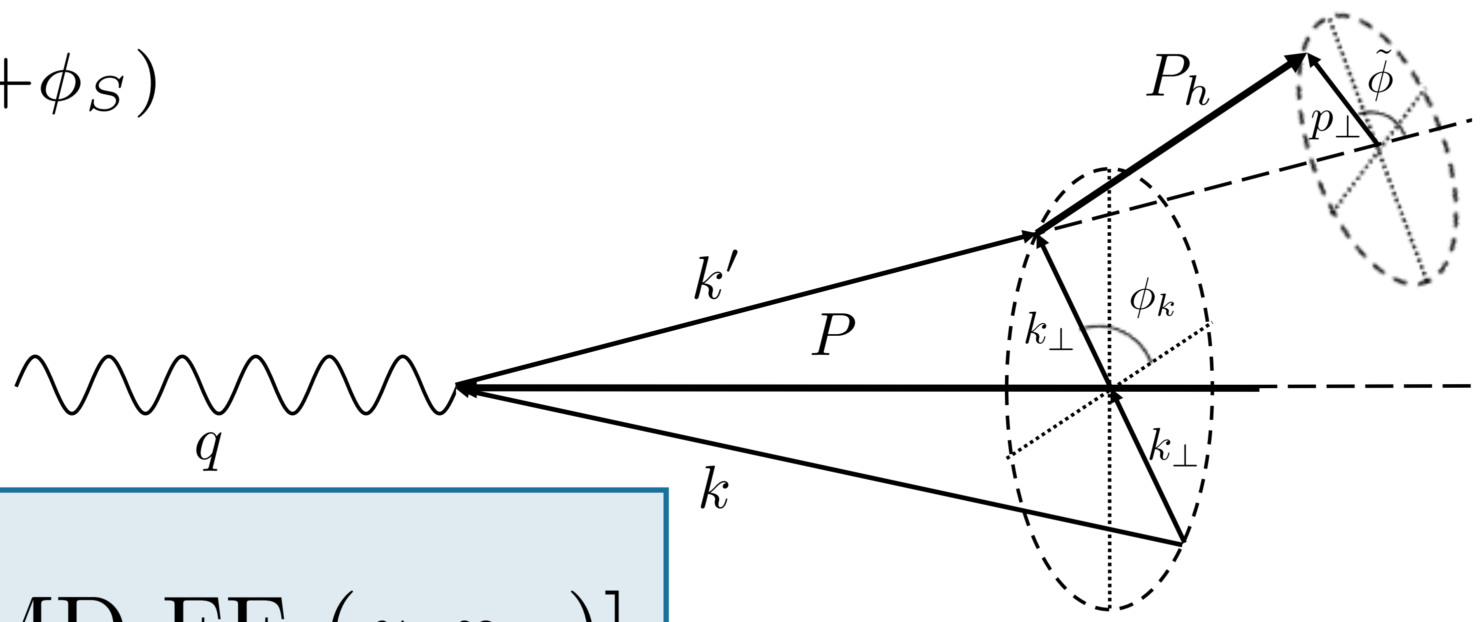

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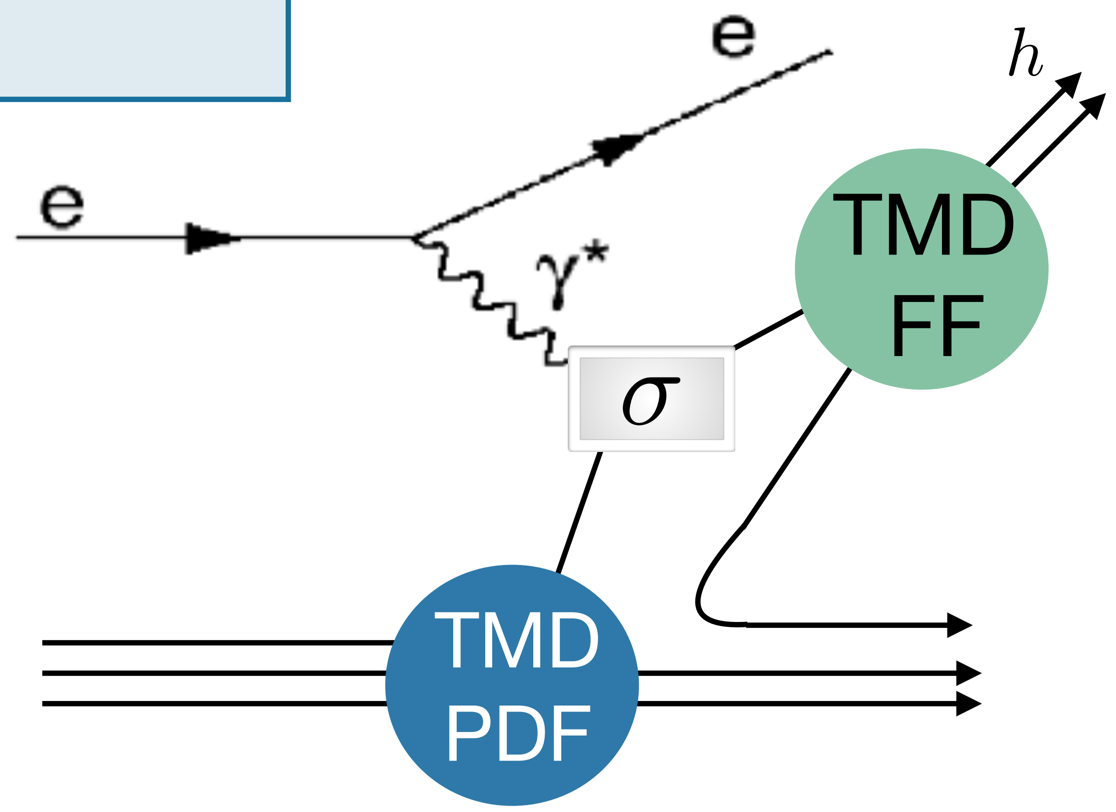

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quark polarisation (columns) / hadron polarisation (rows)

	U	L	T
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Chiral odd

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TMD PDFs and fragmentation functions (FFs)

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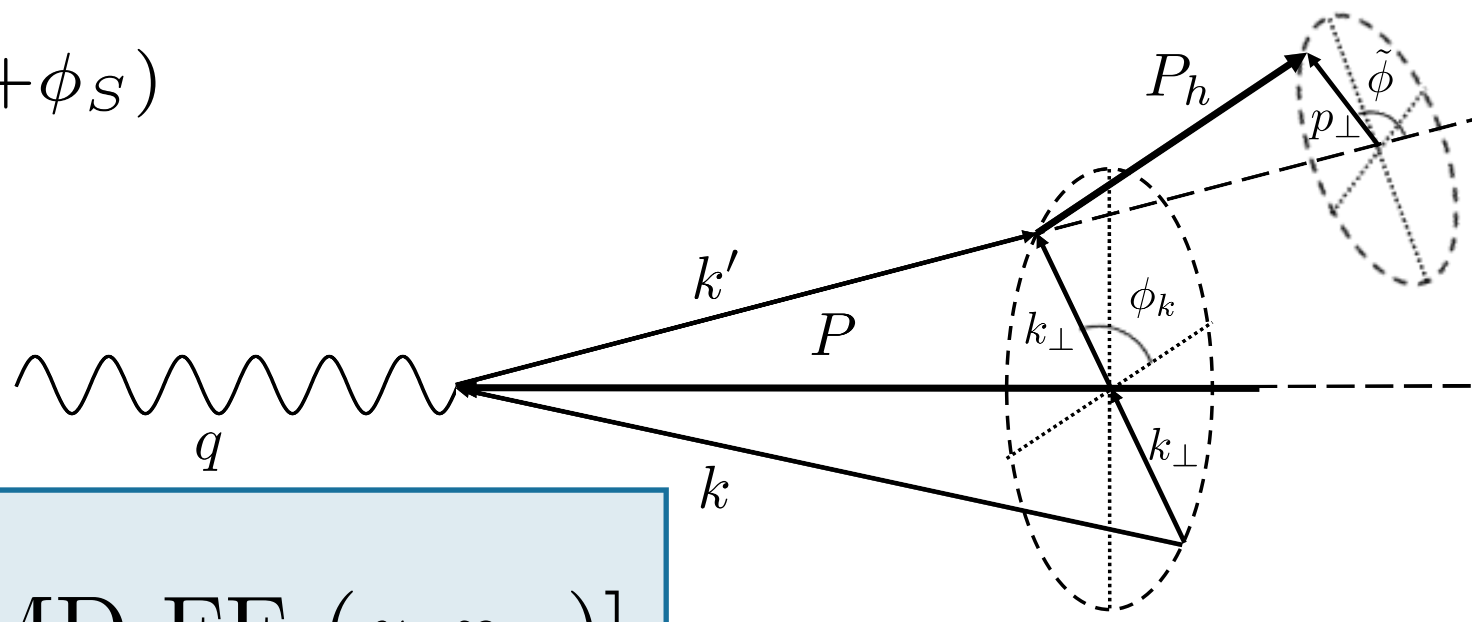

$$F_{XY} \propto \mathcal{C}\,[\text{TMD PDF}(x, k_\perp) \times\ \text{TMD FF}\ (z, p_\perp)]$$

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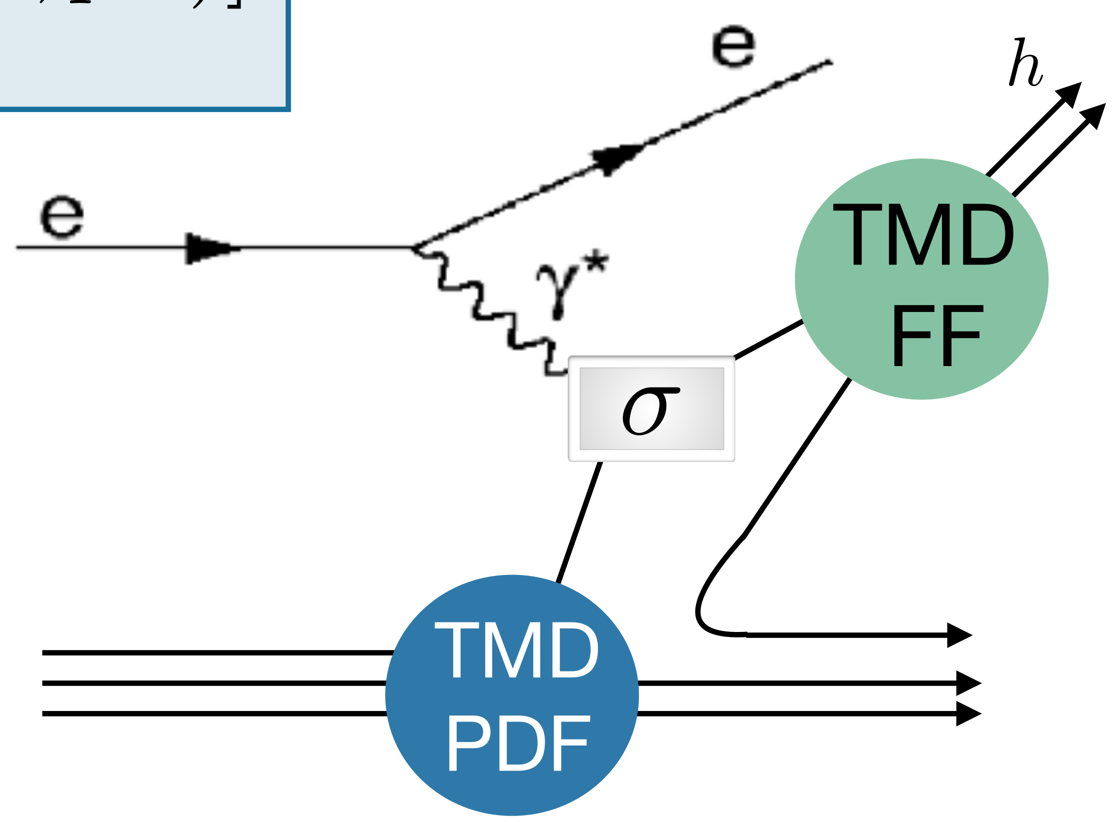

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quark polarisation (columns) / hadron polarisation (rows)

	U	L	T
U	D_1		$H_1^\perp$

Chiral odd

Naive T-odd

Semi-inclusive DIS cross section

$$
\begin{aligned}
\sigma^h(\phi,\phi_S) \;=\;& \sigma^h_{UU}\Big\{1 + 2\langle\cos(\phi)\rangle^h_{UU}\,\cos(\phi) + 2\langle\cos(2\phi)\rangle^h_{UU}\,\cos(2\phi)\\
&+\lambda_l\,2\langle\sin(\phi)\rangle^h_{LU}\,\sin(\phi)\\
&+S_L\Big[2\langle\sin(\phi)\rangle^h_{UL}\,\sin(\phi) + 2\langle\sin(2\phi)\rangle^h_{UL}\,\sin(2\phi)\\
&+\lambda_l\Big(2\langle\cos(0\phi)\rangle^h_{LL}\,\cos(0\phi) + 2\langle\cos(\phi)\rangle^h_{LL}\,\cos(\phi)\Big)\Big]\\
&+S_T\Big[2\langle\sin(\phi-\phi_S)\rangle^h_{UT}\,\sin(\phi-\phi_S) + 2\langle\sin(\phi+\phi_S)\rangle^h_{UT}\,\sin(\phi+\phi_S)\\
&+2\langle\sin(3\phi-\phi_S)\rangle^h_{UT}\,\sin(3\phi-\phi_S) + 2\langle\sin(\phi_S)\rangle^h_{UT}\,\sin(\phi_S)\\
&+2\langle\sin(2\phi-\phi_S)\rangle^h_{UT}\,\sin(2\phi-\phi_S)\\
&+\lambda_l\Big(2\langle\cos(\phi-\phi_S)\rangle^h_{LT}\,\cos(\phi-\phi_S)\\
&+2\langle\cos(\phi_S)\rangle^h_{LT}\,\cos(\phi_S) + 2\langle\cos(2\phi-\phi_S)\rangle^h_{LT}\,\cos(2\phi-\phi_S)\Big)\Big]\Big\}
\end{aligned}
$$

longitudinal target polarisation

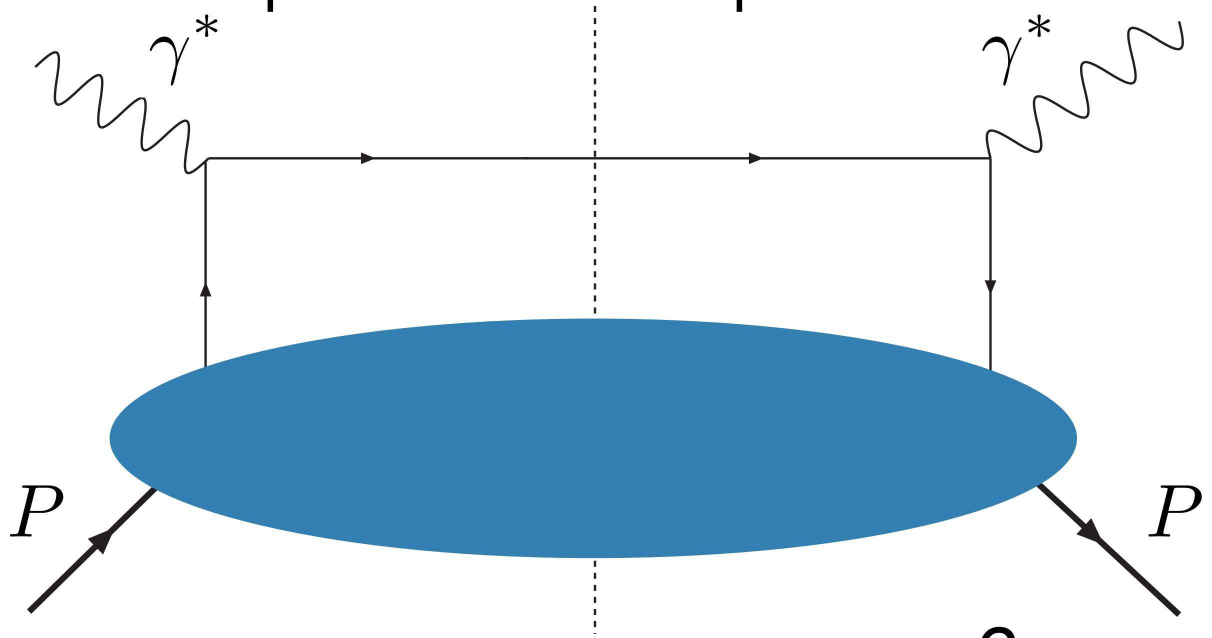
transverse target polarisation

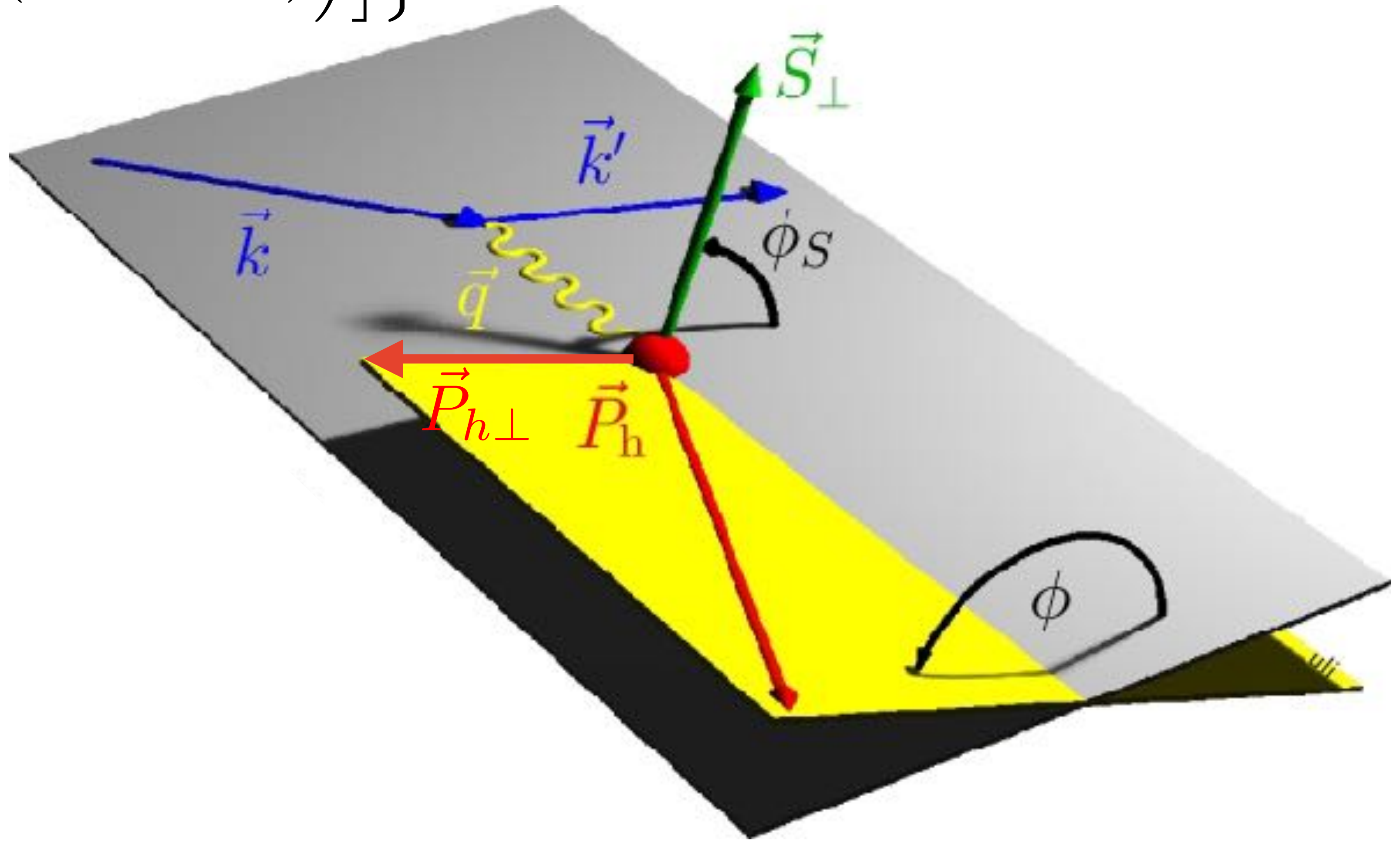
beam polarisation

beam polarisation

target polarisation

leading twist





Semi-inclusive DIS cross section

$$
\begin{aligned}
\sigma^h(\phi,\phi_S) = \; & \sigma^h_{UU}\Big\{1 + 2\langle\cos(\phi)\rangle^h_{UU}\cos(\phi) + 2\langle\cos(2\phi)\rangle^h_{UU}\cos(2\phi) \\
& + \lambda_l\, 2\langle\sin(\phi)\rangle^h_{LU}\sin(\phi) \\
& + S_L\Big[2\langle\sin(\phi)\rangle^h_{UL}\sin(\phi) + 2\langle\sin(2\phi)\rangle^h_{UL}\sin(2\phi) \\
& + \lambda_l\Big(2\langle\cos(0\phi)\rangle^h_{LL}\cos(0\phi) + 2\langle\cos(\phi)\rangle^h_{LL}\cos(\phi)\Big)\Big] \\
& + S_T\Big[2\langle\sin(\phi-\phi_S)\rangle^h_{UT}\sin(\phi-\phi_S) + 2\langle\sin(\phi+\phi_S)\rangle^h_{UT}\sin(\phi+\phi_S) \\
& + 2\langle\sin(3\phi-\phi_S)\rangle^h_{UT}\sin(3\phi-\phi_S) + 2\langle\sin(\phi_S)\rangle^h_{UT}\sin(\phi_S) \\
& + 2\langle\sin(2\phi-\phi_S)\rangle^h_{UT}\sin(2\phi-\phi_S) \\
& + \lambda_l\Big(2\langle\cos(\phi-\phi_S)\rangle^h_{LT}\cos(\phi-\phi_S) \\
& + 2\langle\cos(\phi_S)\rangle^h_{LT}\cos(\phi_S) + 2\langle\cos(2\phi-\phi_S)\rangle^h_{LT}\cos(2\phi-\phi_S)\Big)\Big]\Big\}
\end{aligned}
$$

longitudinal target polarisation

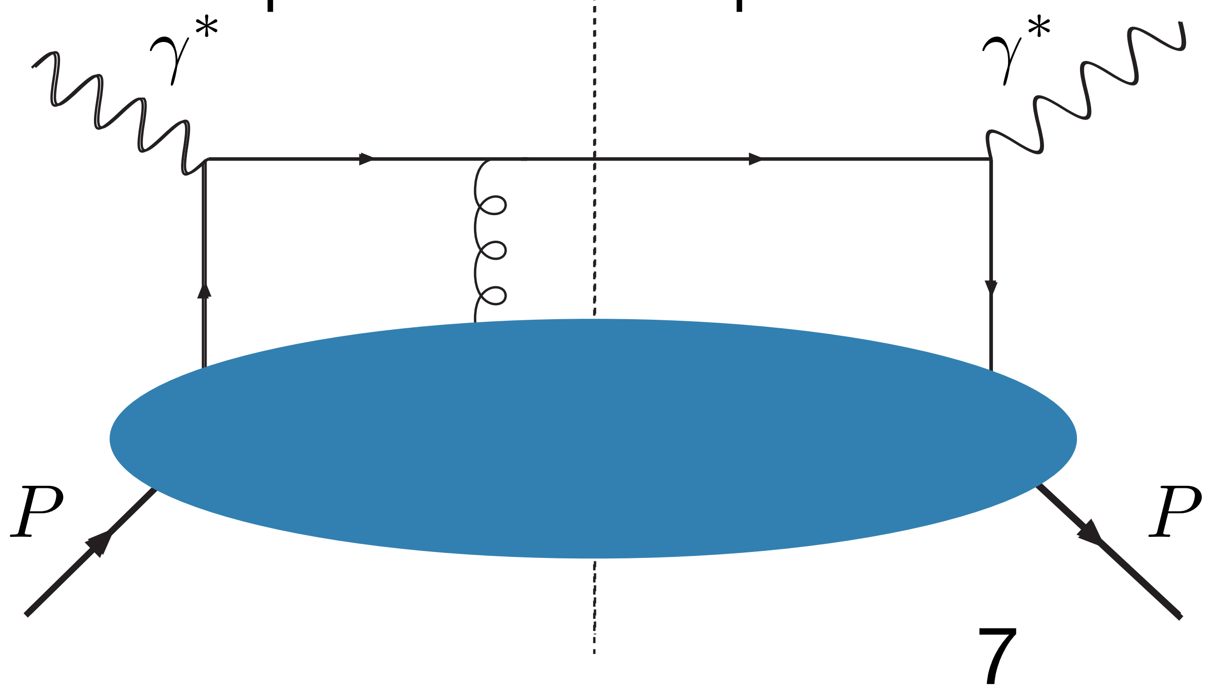
transverse target polarisation

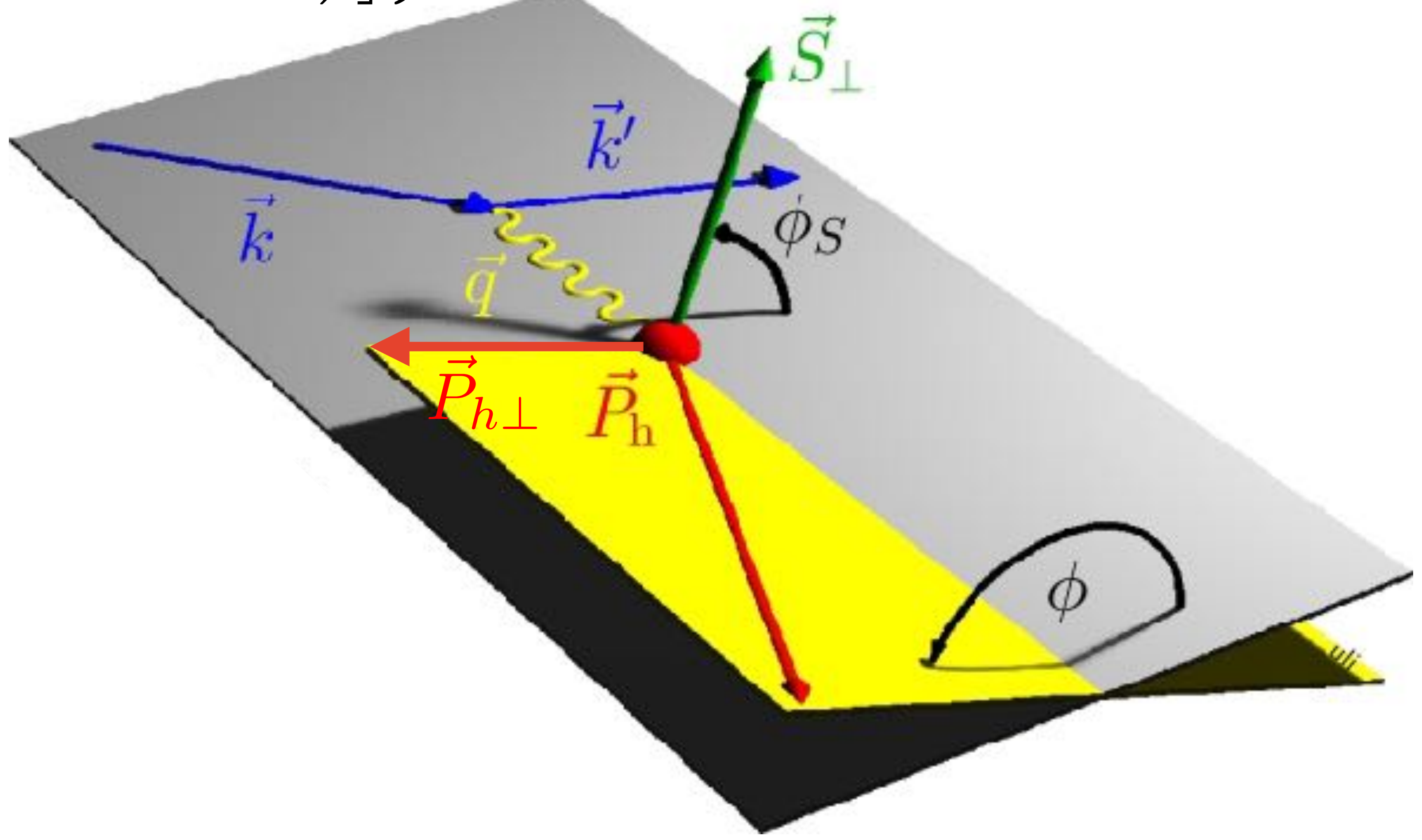
beam polarisation

sub-leading twist

beam polarisation

target polarisation

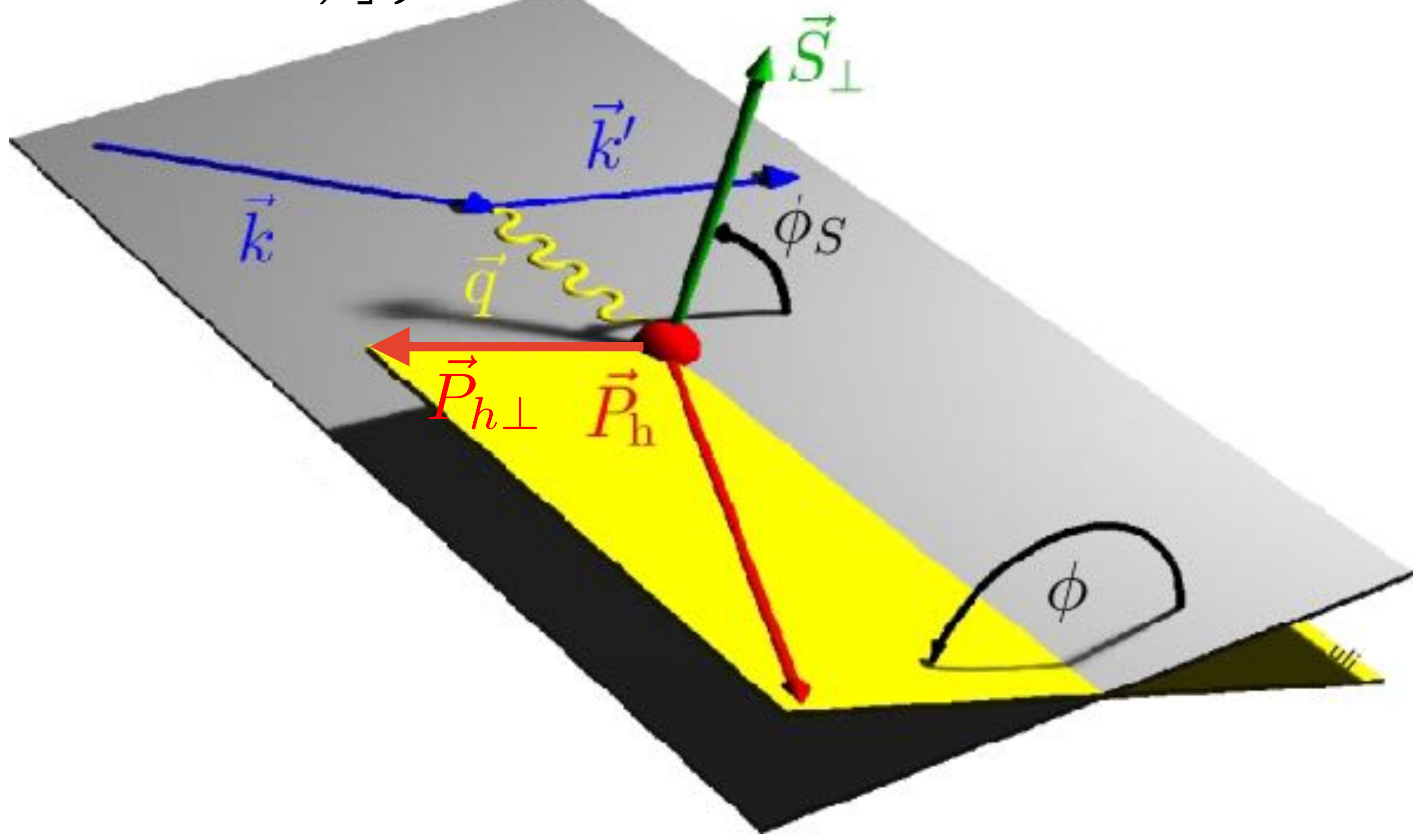




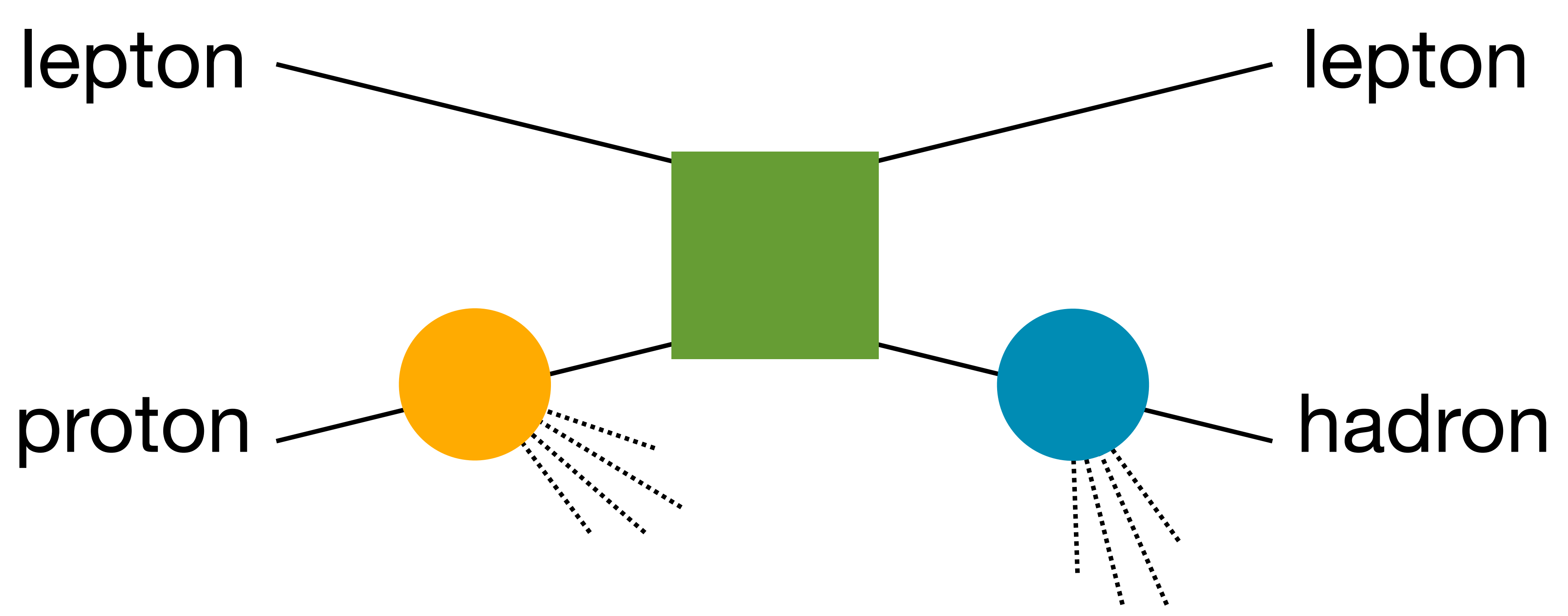
Presented amplitudes

$$
\begin{aligned}
\sigma^h(\phi, \phi_S) \;=\; & \sigma^h_{UU} \Big\{ 1 + 2\langle\cos(\phi)\rangle^h_{UU} \cos(\phi) + 2\langle\cos(2\phi)\rangle^h_{UU} \cos(2\phi) \\
+ \; & \lambda_l \, 2\langle\sin(\phi)\rangle^h_{LU} \sin(\phi) \\
+ \; & S_L \Big[2\langle\sin(\phi)\rangle^h_{UL} \sin(\phi) + 2\langle\sin(2\phi)\rangle^h_{UL} \sin(2\phi) \\
+ \; & \lambda_l \left(2\langle\cos(0\phi)\rangle^h_{LL} \cos(0\phi) + 2\langle\cos(\phi)\rangle^h_{LL} \cos(\phi) \right) \Big] \\
+ \; & S_T \Big[2\langle\sin(\phi - \phi_S)\rangle^h_{UT} \sin(\phi - \phi_S) + 2\langle\sin(\phi + \phi_S)\rangle^h_{UT} \sin(\phi + \phi_S) \\
+ \; & 2\langle\sin(3\phi - \phi_S)\rangle^h_{UT} \sin(3\phi - \phi_S) + 2\langle\sin(\phi_S)\rangle^h_{UT} \sin(\phi_S) \\
+ \; & 2\langle\sin(2\phi - \phi_S)\rangle^h_{UT} \sin(2\phi - \phi_S) \\
+ \; & \lambda_l \Big(2\langle\cos(\phi - \phi_S)\rangle^h_{LT} \cos(\phi - \phi_S) \\
+ \; & 2\langle\cos(\phi_S)\rangle^h_{LT} \cos(\phi_S) + 2\langle\cos(2\phi - \phi_S)\rangle^h_{LT} \cos(2\phi - \phi_S) \Big) \Big] \Big\}
\end{aligned}
$$

Presented here



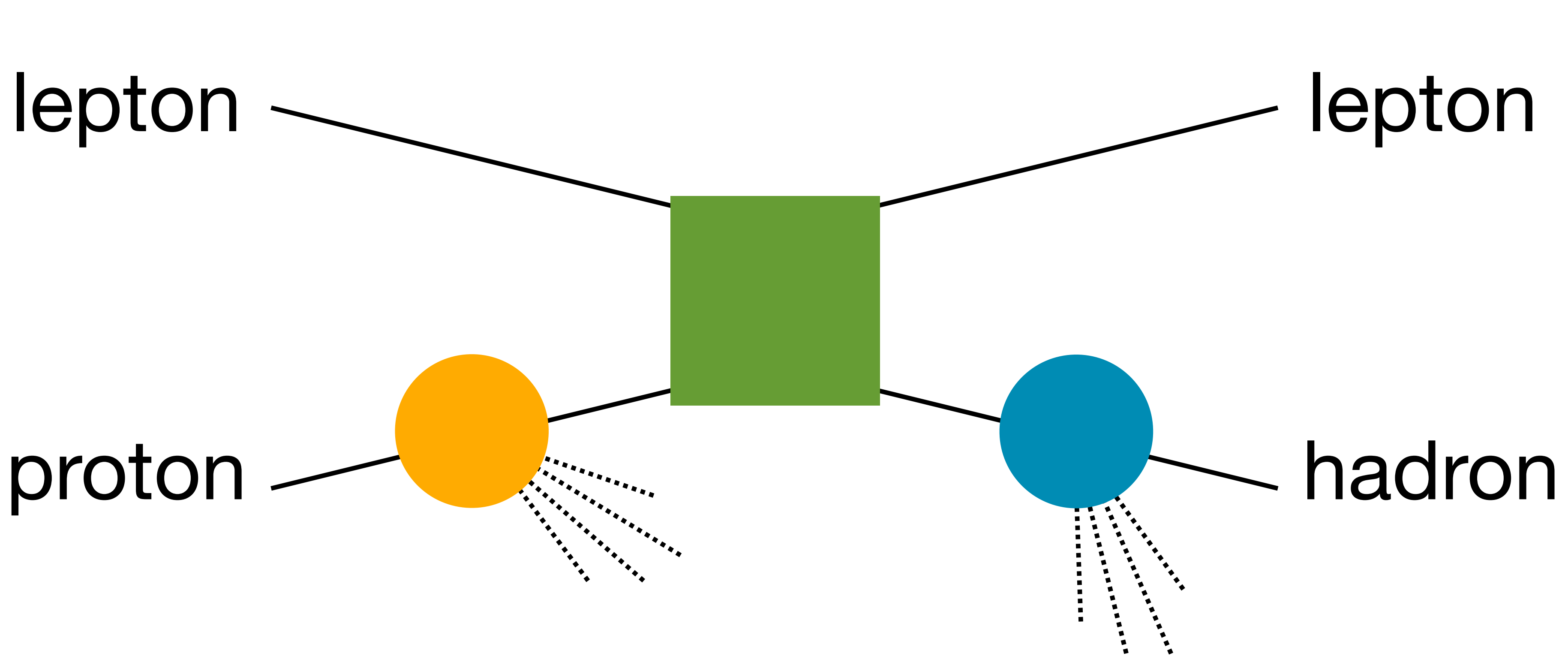
Factorisation and universality



semi-inclusive DIS

Adapted from A. Bacchetta

Factorisation and universality



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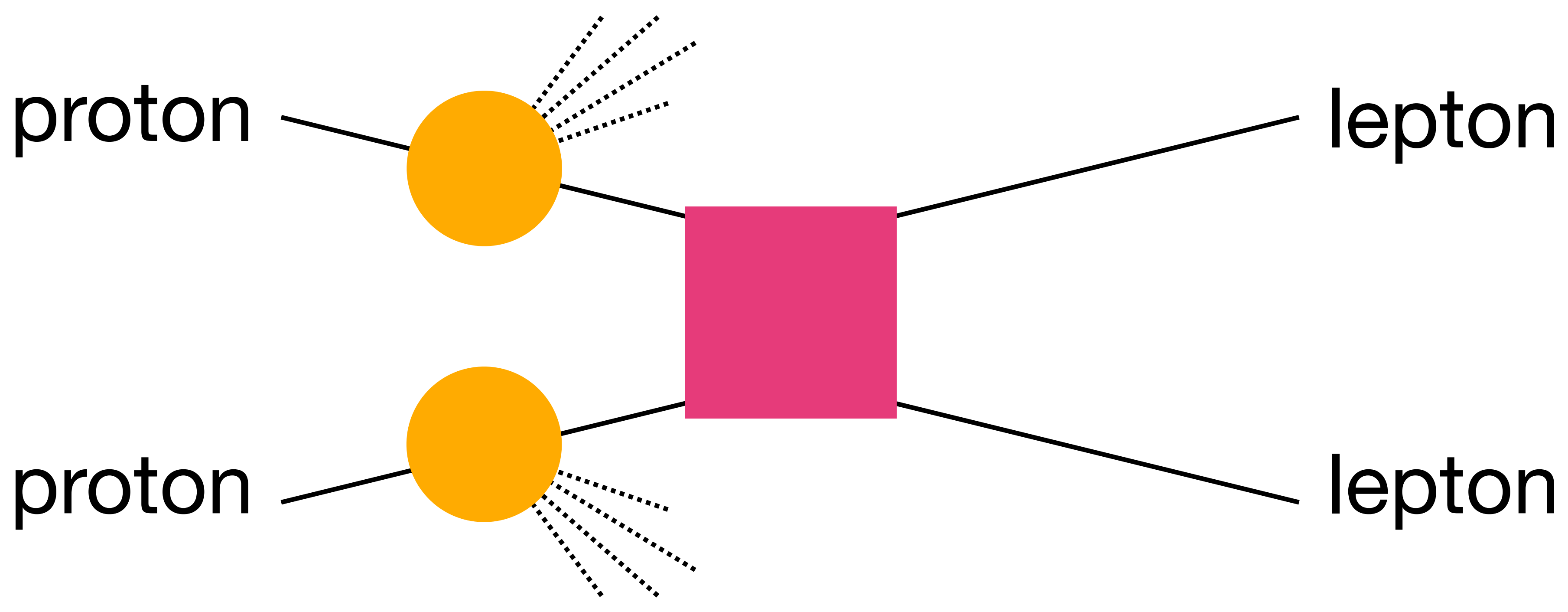
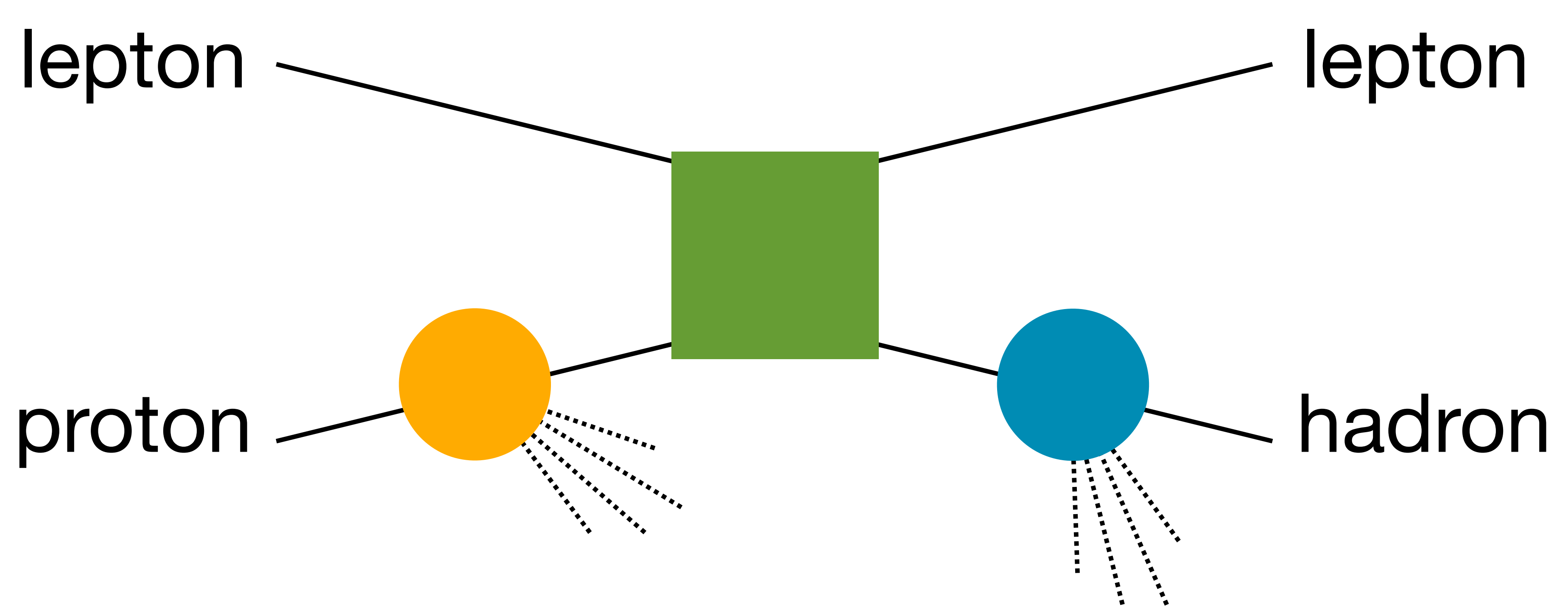


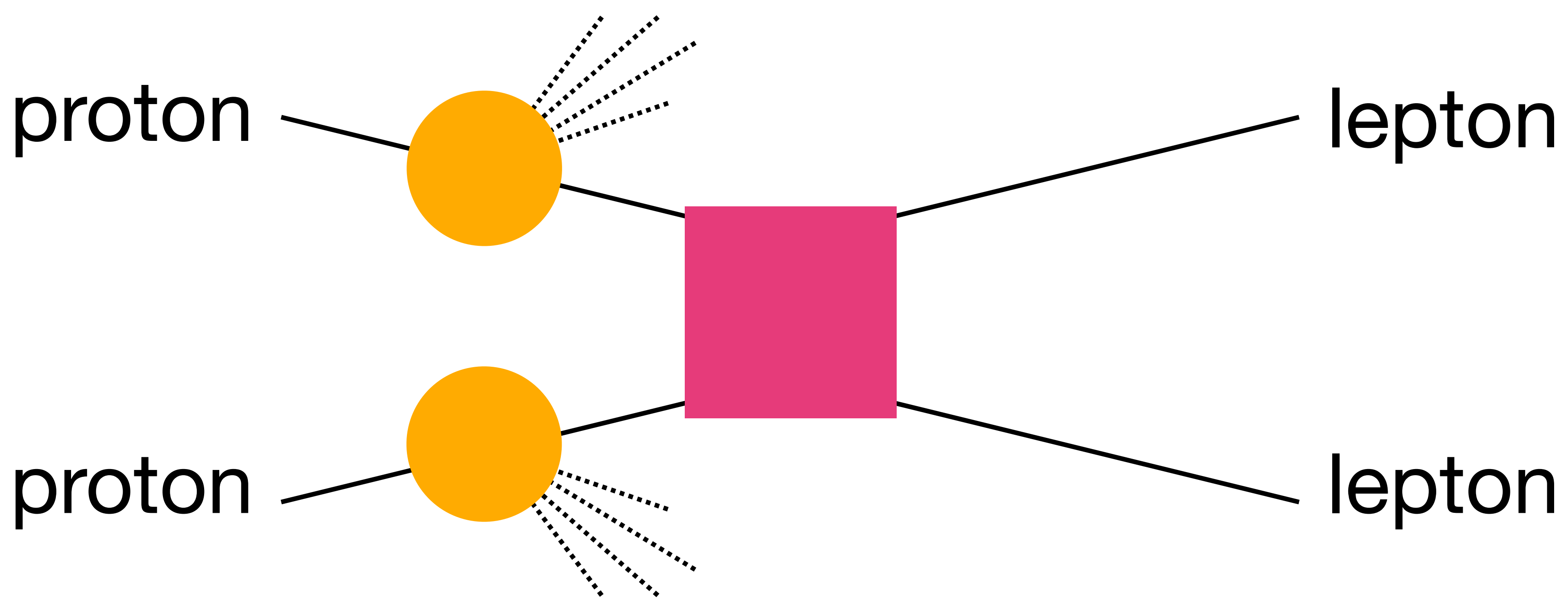
Drell-Yan

Adapted from A. Bacchetta

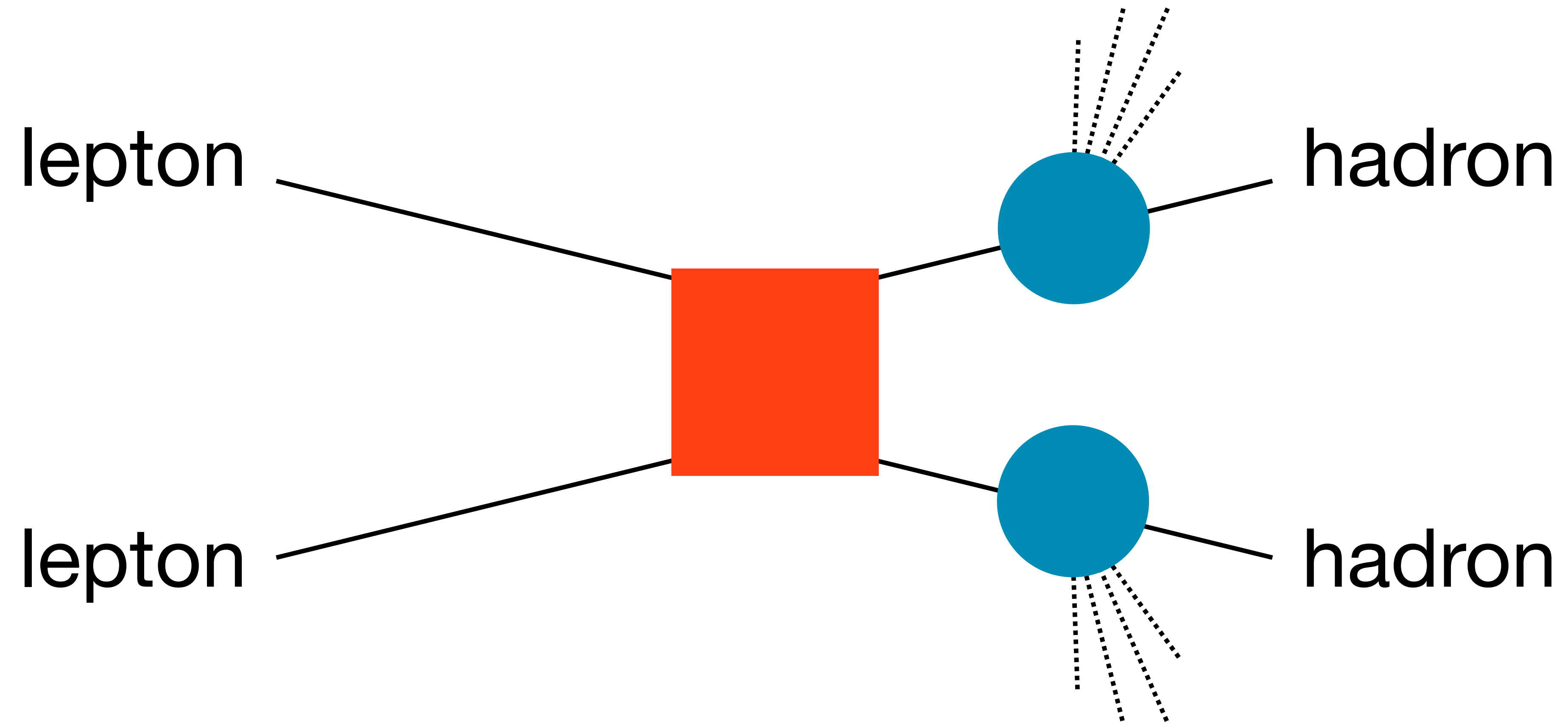
Factorisation and universality



semi-inclusive DIS



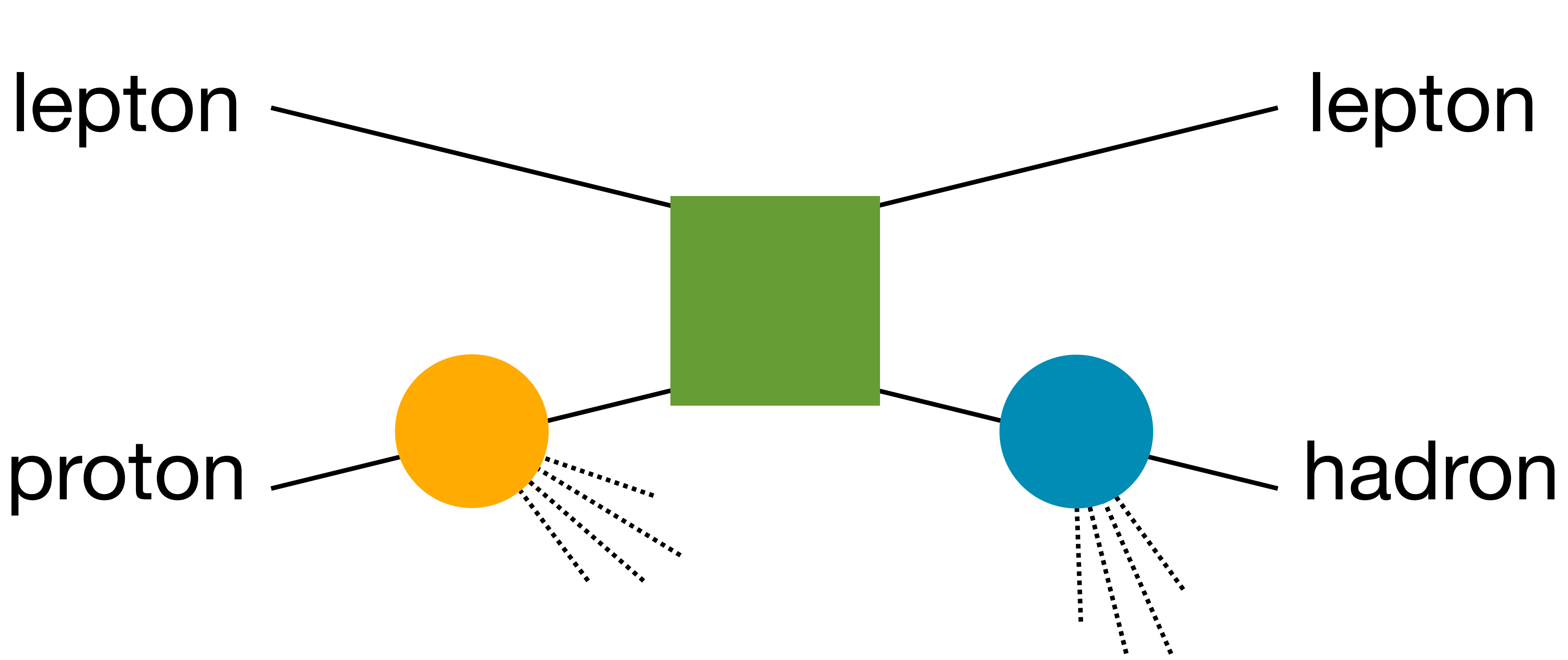
Drell-Yan



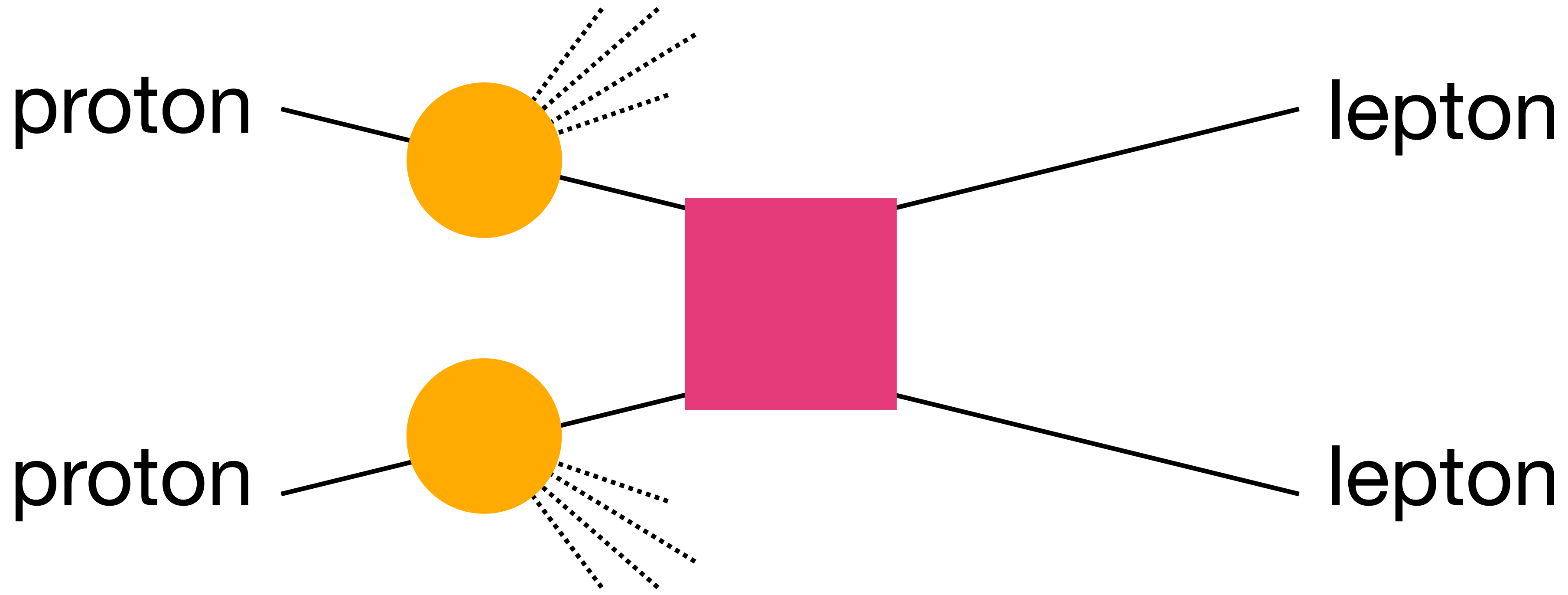
e^+e^- annihilation

Adapted from A. Bacchetta

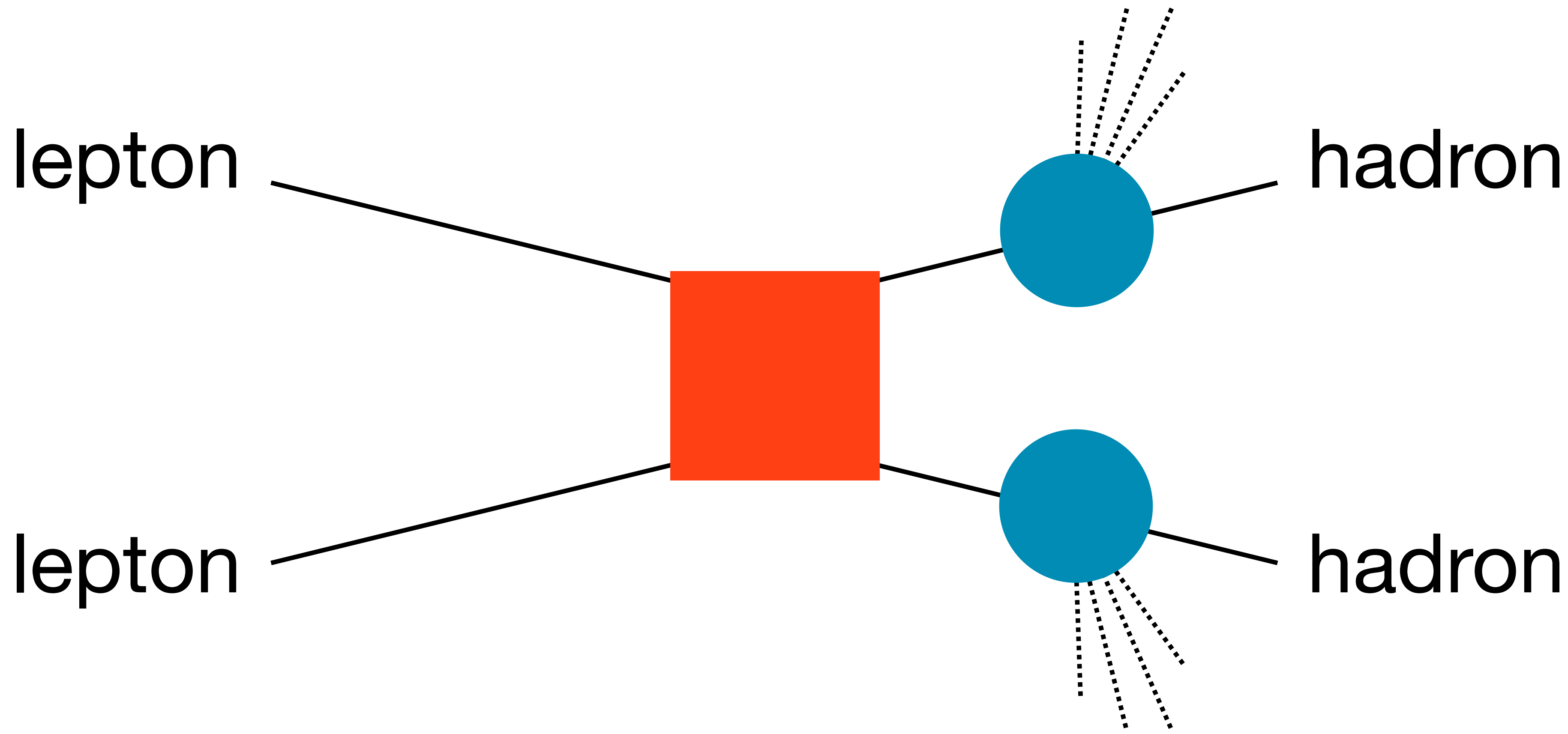
Factorisation and universality



semi-inclusive DIS



Drell-Yan



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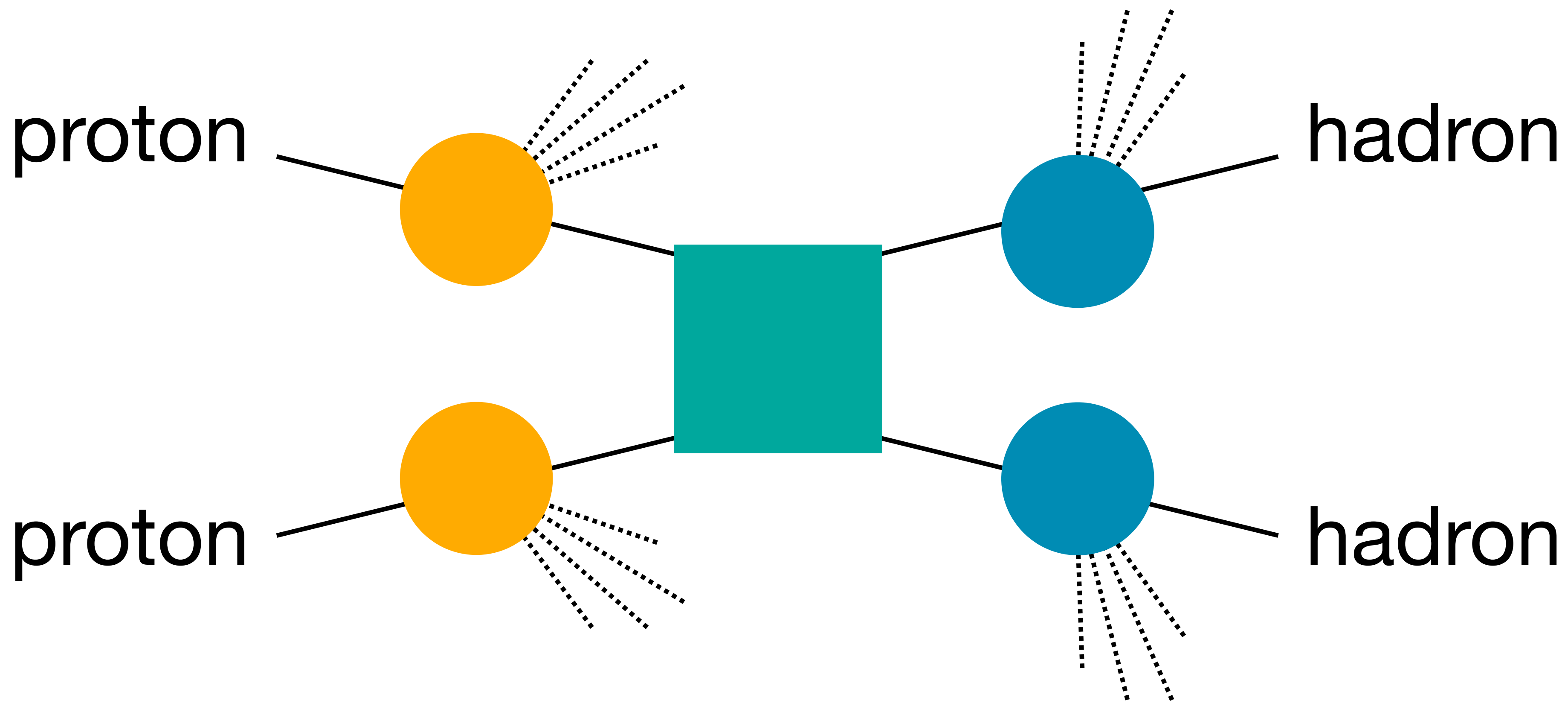
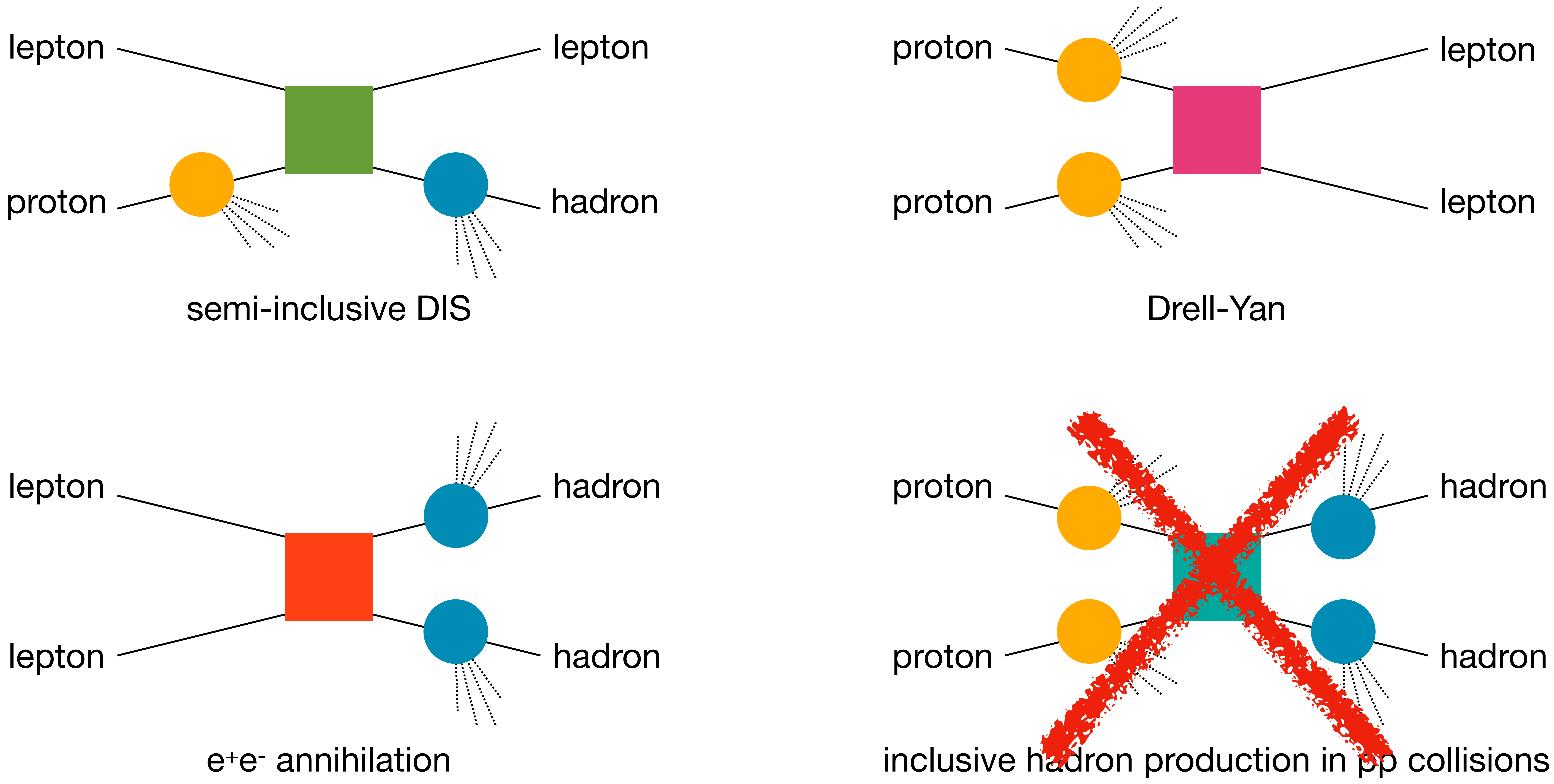


inclusive hadron production in pp collisions

Adapted from A. Bacchetta

Factorisation and universality

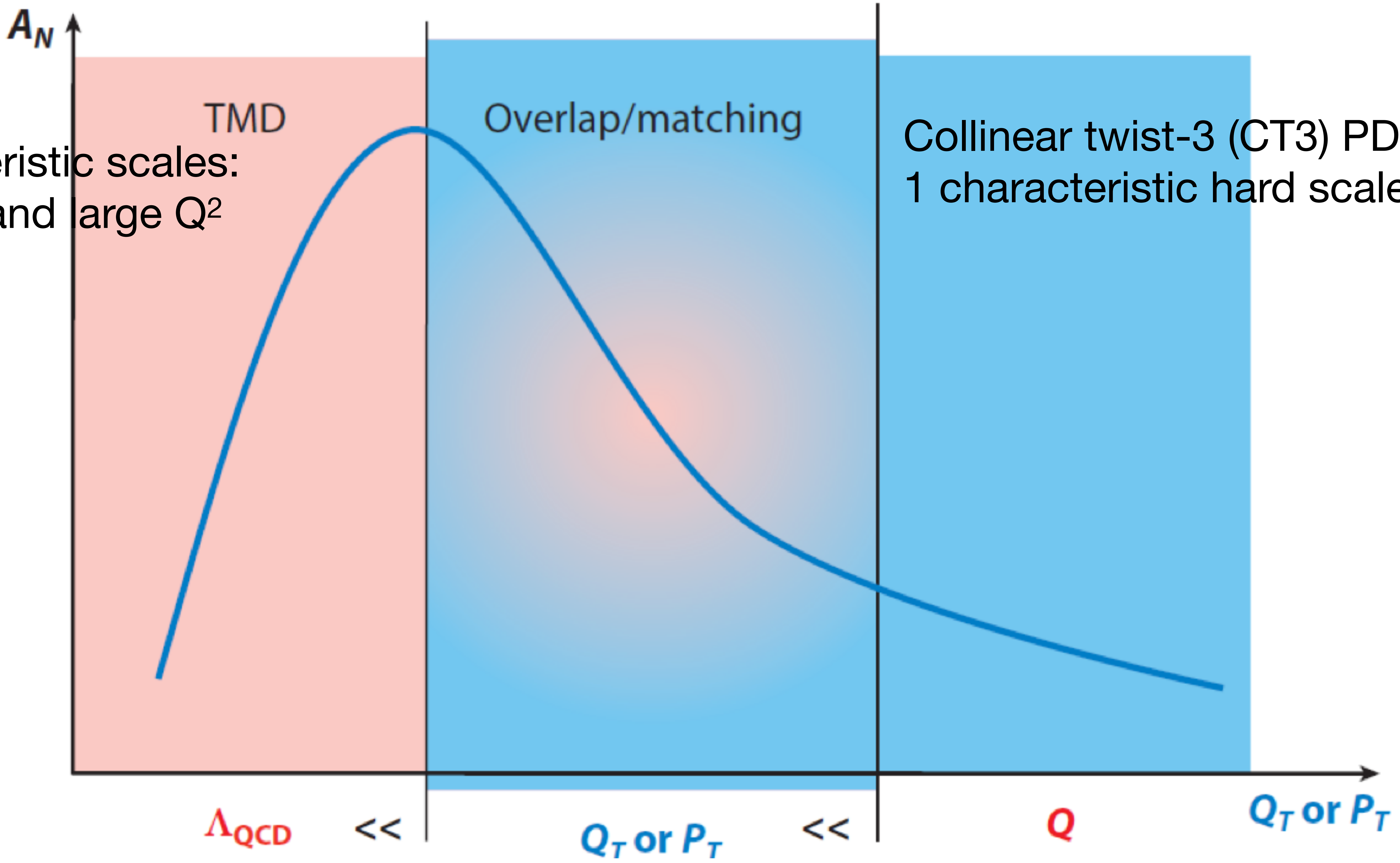


Adapted from A. Bacchetta

Validity of TMD description

2 characteristic scales:
small P_{hT} and large Q^2

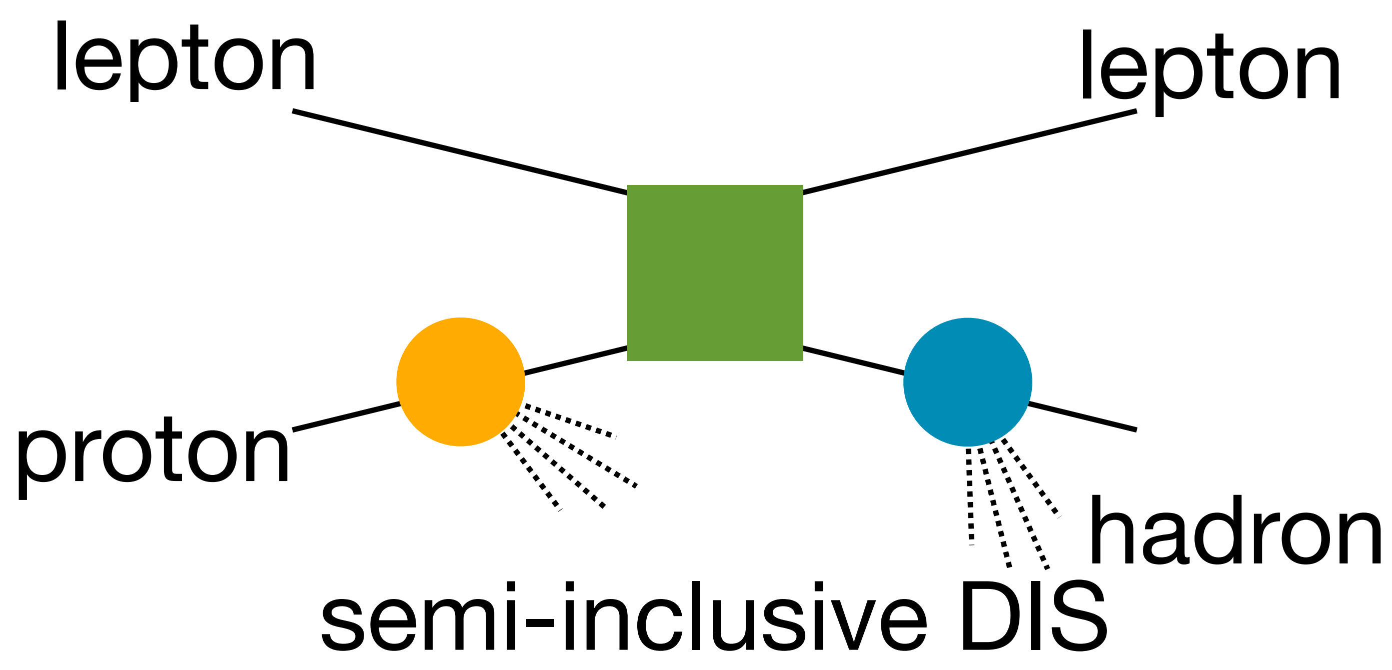
Collinear twist-3 (CT3) PDFs.
1 characteristic hard scale, e.g., P_{hT}



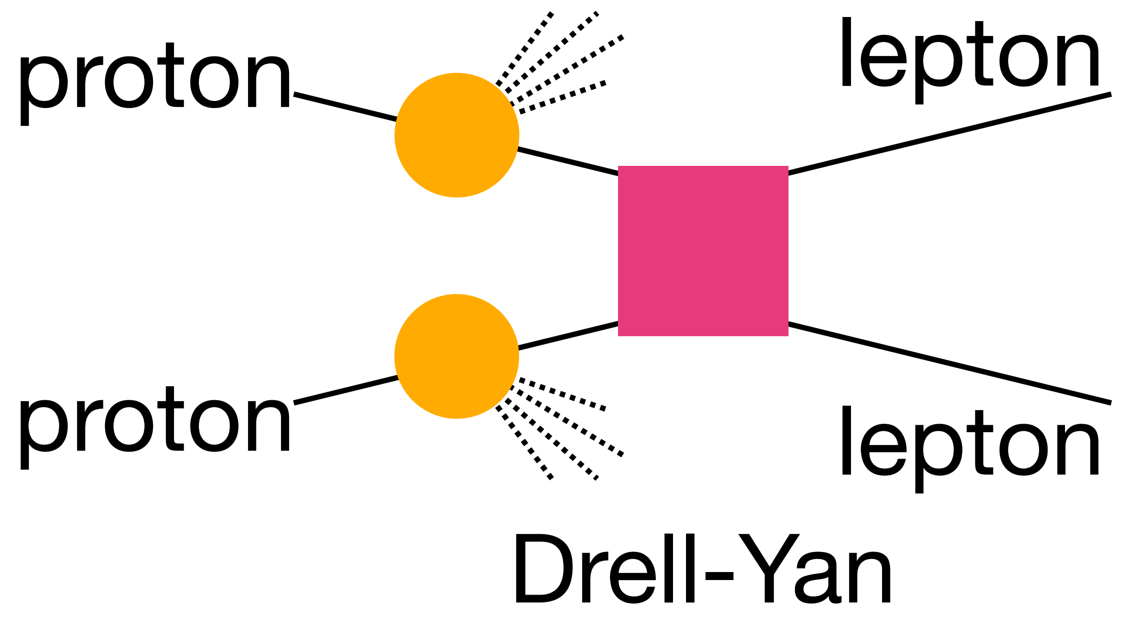
Consistent results for TMD
and CT3 in overlap region

Experiments investigating TMD PDFs and TMD FFs

Spin-independent TMD PDFs: global analysis



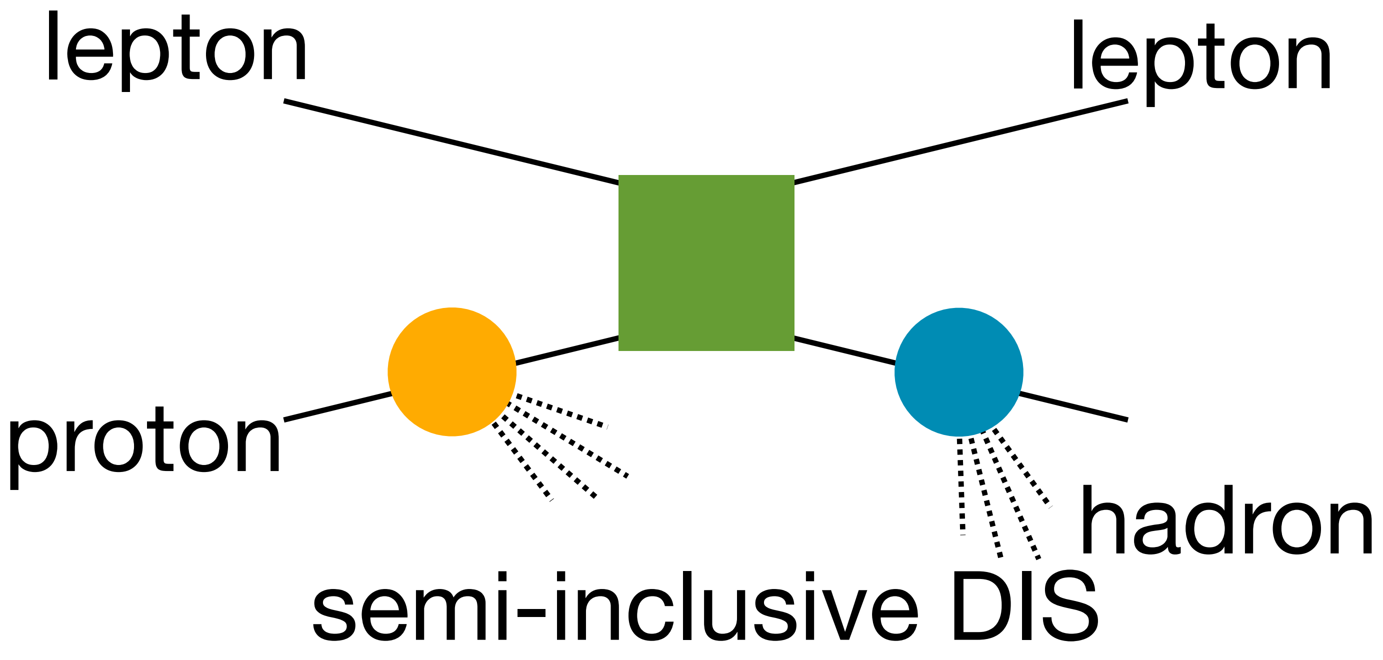
Experiment	Reaction	ref.	Kinematics	N_{pt} after cuts
HERMES	$p \to \pi^+$	[67]	$0.023 < x < 0.6$ (6 bins)	24
	$p \to \pi^-$		$0.2 < z < 0.8$ (6 bins)	24
	$p \to K^+$		$1.0 < Q < \sqrt{20}\,\text{GeV}$	24
	$p \to K^-$			24
	$D \to \pi^+$		$W^2 > 10\text{GeV}^2$	24
	$D \to \pi^-$		$0.1 < y < 0.85$	24
	$D \to K^+$			24
	$D \to K^-$			24
COMPASS	$d \to h^+$	[68]	$0.003 < x < 0.4$ (8 bins)	195
	$d \to h^-$		$0.2 < z < 0.8$ (4 bins)	195
			$1.0 < Q \simeq 9\,\text{GeV}$ (5 bins)	
Total				582

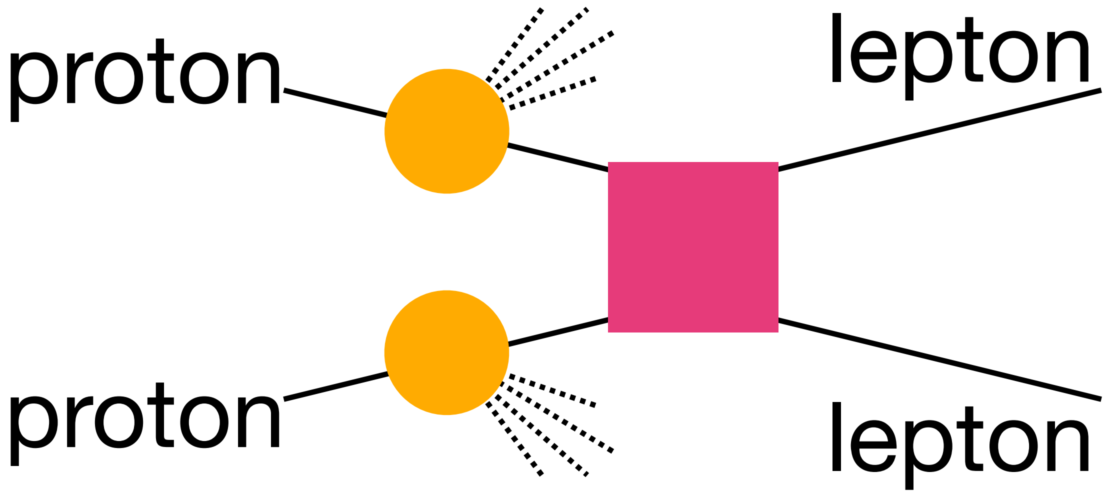


I. Scimemi, A. Vladimirov JHEP **06** (2020)137

Experiment	ref.	$\sqrt{s}$ [GeV]	Q [GeV]	y/x_F	fiducial region	N_{pt} after cuts
E288 (200)	[73]	19.4	4–9 in 1 GeV bins*	$0.1 < x_F < 0.7$		43
E288 (300)	[73]	23.8	4–12 in 1 GeV bins*	$-0.09 < x_F < 0.51$	—	53
E288 (400)	[73]	27.4	5–14 in 1 GeV bins*	$-0.27 < x_F < 0.33$	—	76
E605	[74]	38.8	7–18 in 5 bins*	$-0.1 < x_F < 0.2$	—	53
E772	[75]	38.8	5–15 in 8 bins*	$0.1 < x_F < 0.3$	—	35
PHENIX	[76]	200	4.8–8.2	$1.2 < y < 2.2$	—	3
CDF (run1)	[77]	1800	66–116	—	—	33
CDF (run2)	[78]	1960	66–116	—	–	39
D0 (run1)	[79]	1800	75–105			16
D0 (run2)	[80]	1960	70–110	—	—	8
D0 (run2)$_\mu$	[81]	1960	65–115	$\|y\| < 1.7$	$p_T > 15$ GeV $\|\eta\| < 1.7$	3
ATLAS (7 TeV)	[47]	7000	66–116	$\|y\| < 1$ $1 < \|y\| < 2$ $2 < \|y\| < 2.4$	$p_T > 20$ GeV $\|\eta\| < 2.4$	15
ATLAS (8 TeV)	[48]	8000	66–116	$\|y\| < 2.4$ in 6 bins	$p_T > 20$ GeV $\|\eta\| < 2.4$	30
ATLAS (8 TeV)	[48]	8000	46–66	$\|y\| < 2.4$	$p_T > 20$ GeV $\|\eta\| < 2.4$	3
ATLAS (8 TeV)	[48]	8000	116–150	$\|y\| < 2.4$	$p_T > 20$ GeV $\|\eta\| < 2.4$	7
CMS (7 TeV)	[49]	7000	60–120	$\|y\| < 2.1$	$p_T > 20$ GeV $\|\eta\| < 2.1$	8
CMS (8 TeV)	[50]	8000	60–120	$\|y\| < 2.1$	$p_T > 20$ GeV $\|\eta\| < 2.1$	8
LHCb (7 TeV)	[82]	7000	60–120	$2 < y < 4.5$	$p_T > 20$ GeV $2 < \eta < 4.5$	8
LHCb (8 TeV)	[83]	8000	60–120	$2 < y < 4.5$	$p_T > 20$ GeV $2 < \eta < 4.5$	7
LHCb (13 TeV)	[84]	13000	60–120	$2 < y < 4.5$	$p_T > 20$ GeV $2 < \eta < 4.5$	9
Total						457

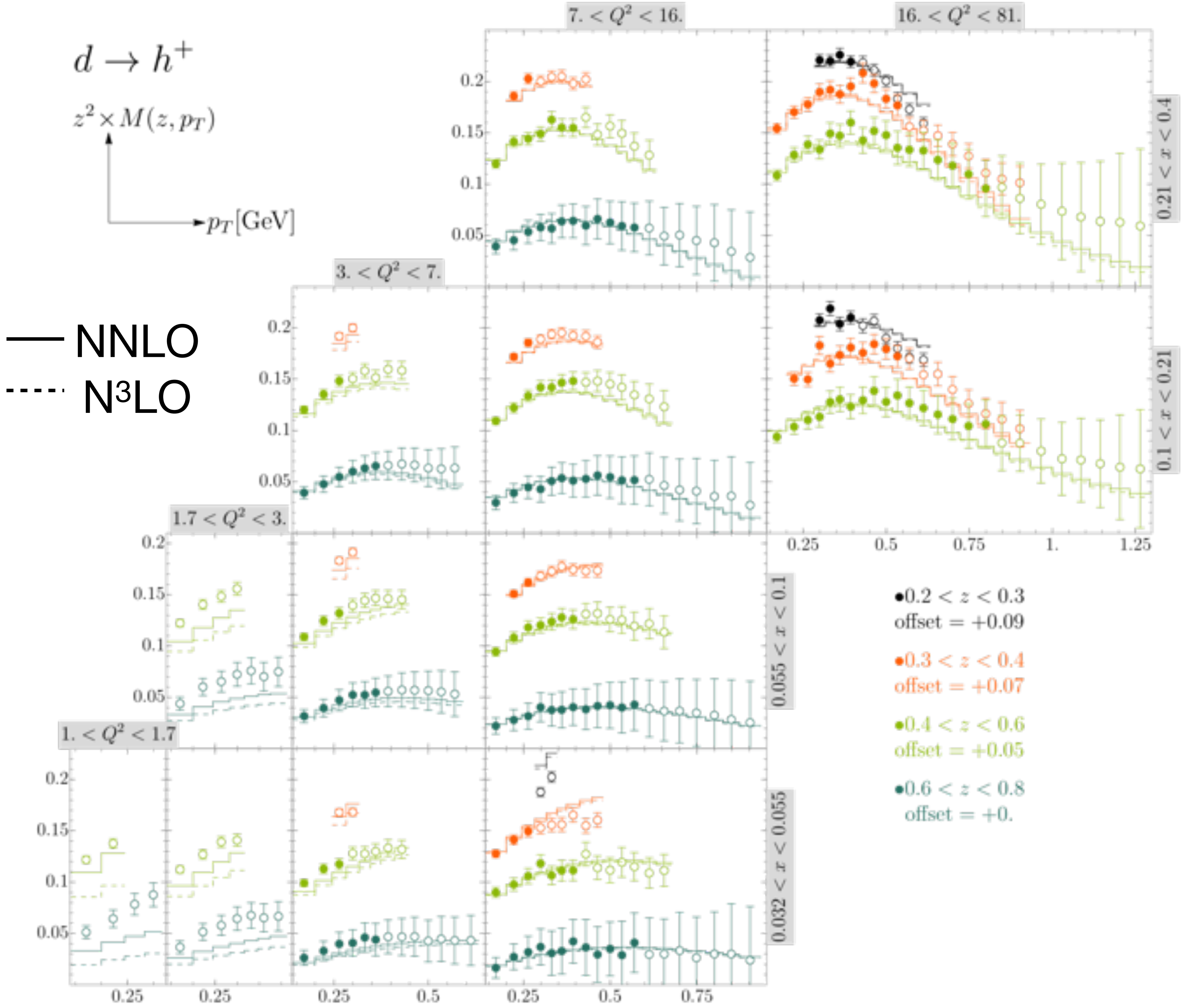
Spin-independent TMD PDFs: global analysis



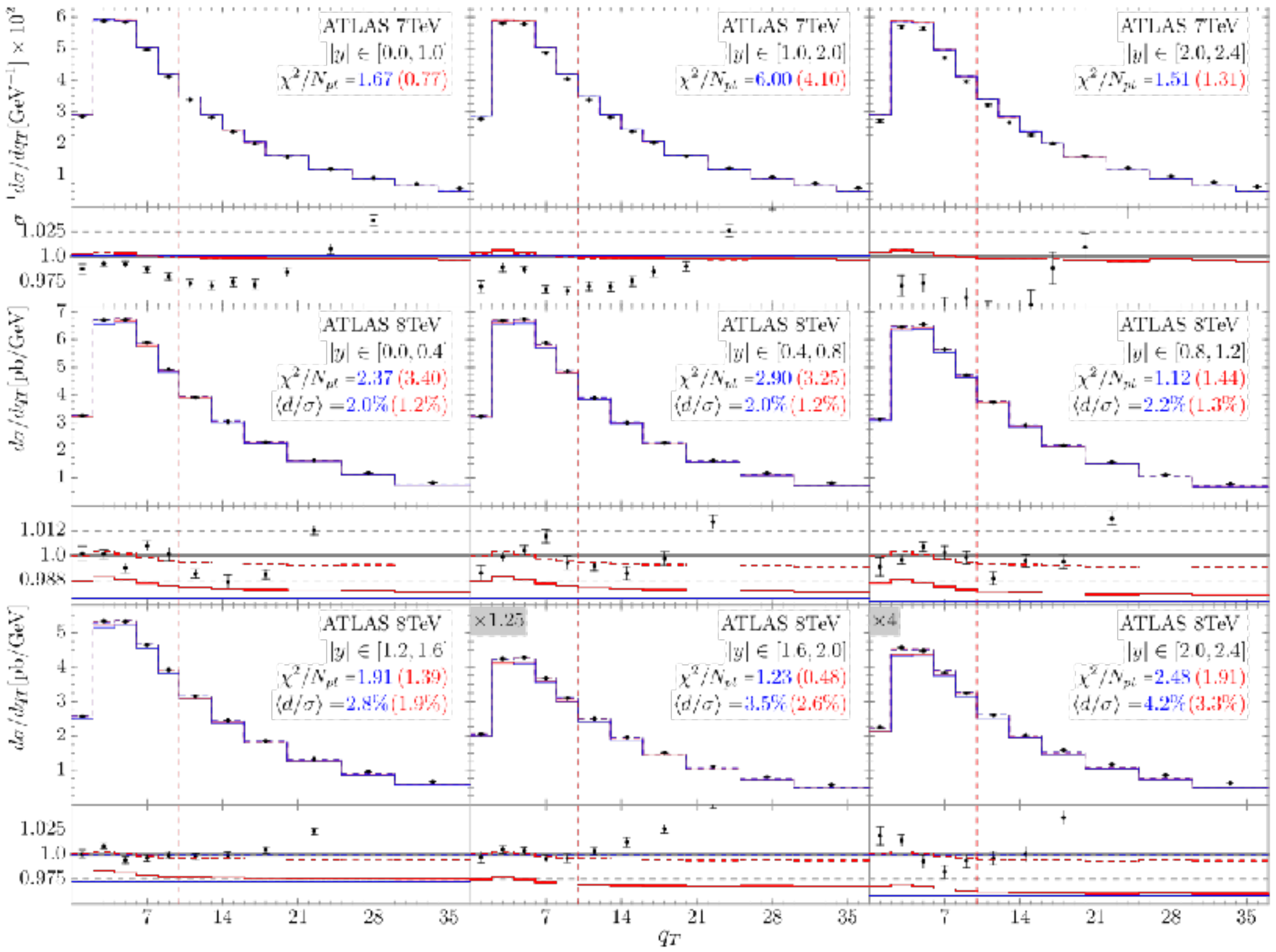


I. Scimemi, A. Vladimirov JHEP **06** (2020)137

Experiment	ref.	$\sqrt{s}$ [GeV]	Q [GeV]	y/x_F	fiducial region	N_{pt} after cuts
E288 (200)	[73]	19.4	4–9 in 1 GeV bins*	$0.1 < x_F < 0.7$		43
E288 (300)	[73]	23.8	4–12 in 1 GeV bins*	$-0.09 < x_F < 0.51$	—	53
[illegible]						
Total						457

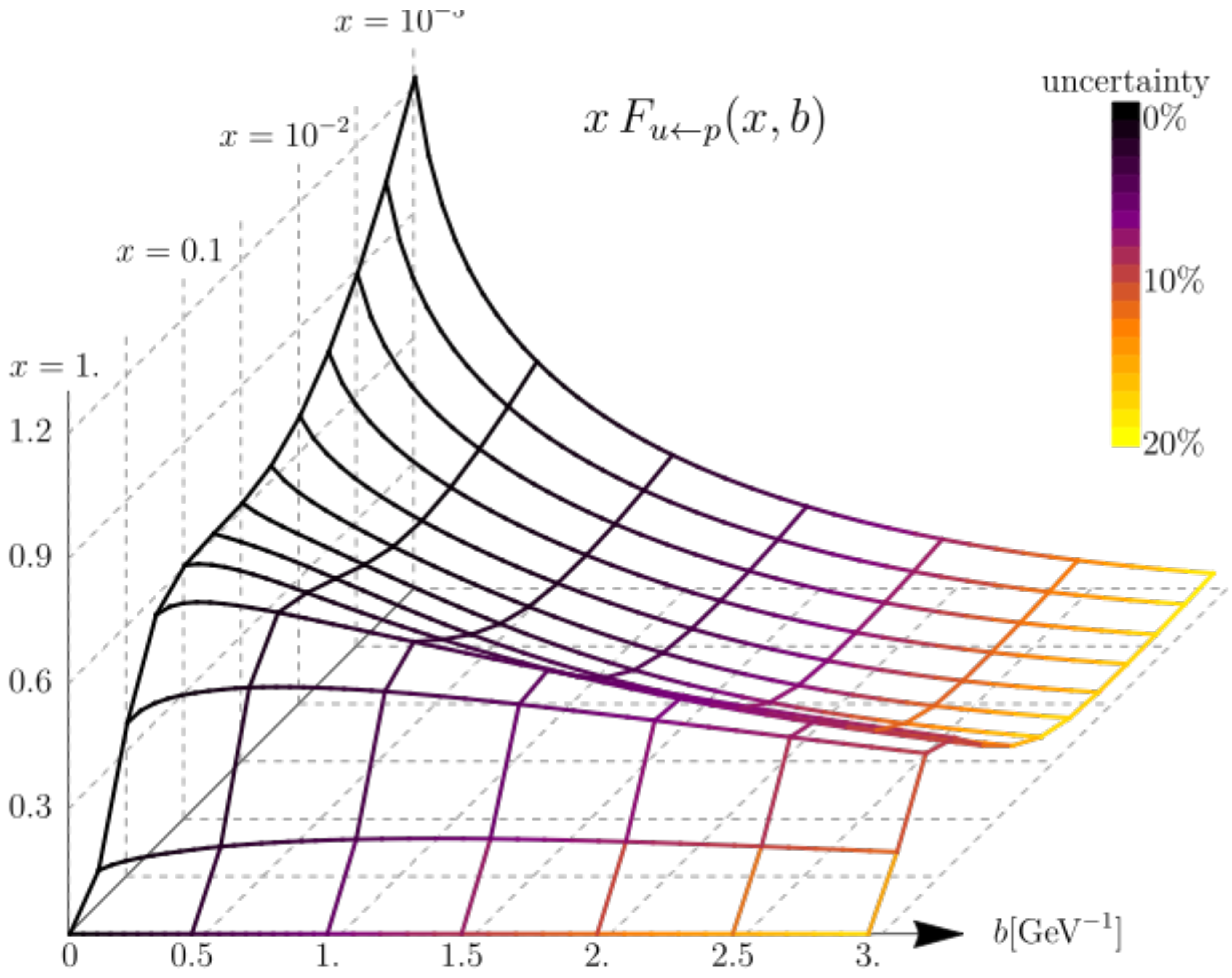


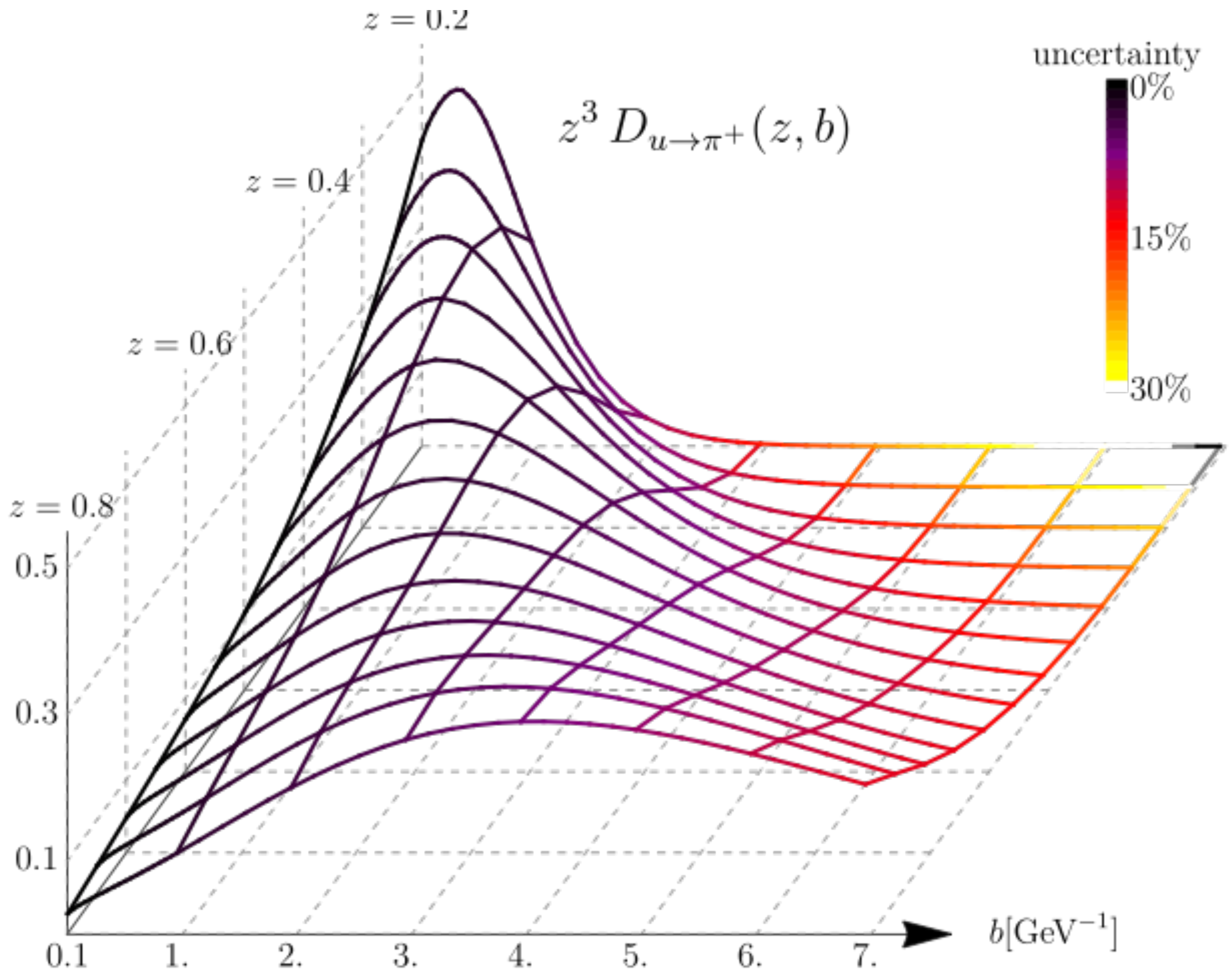
Description of the data



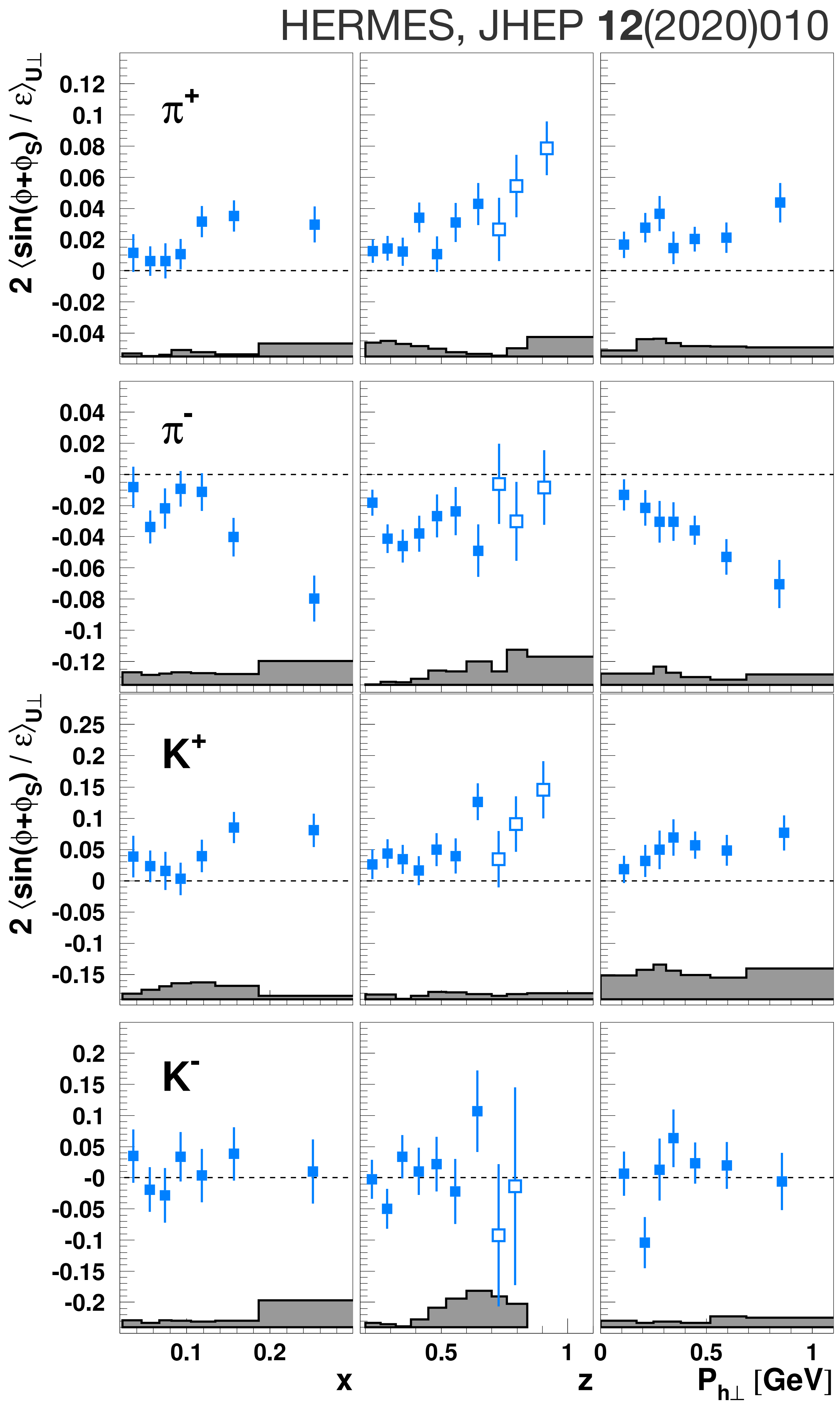
NNLO/N3LO

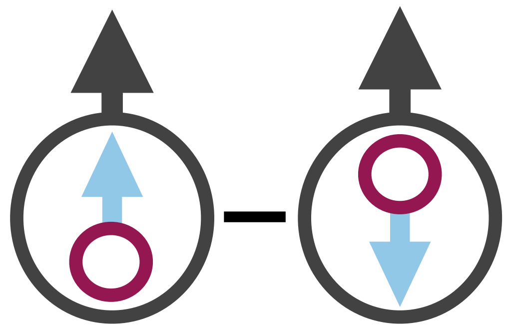
Spin-independent TMD PDFs: global analysis





Collins amplitudes





$\mathcal{C}\left[h_{1T}^{q} \times H_1^{\perp,q}\right]$

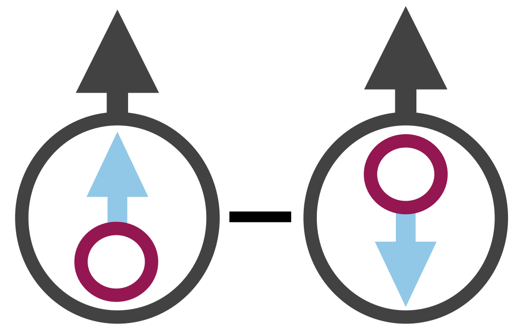
- Oppositely signed amplitudes for π+ and π-:

$$H_1^{\perp,u\to\pi^+} \approx -H_1^{\perp,u\to\pi^-}$$

- Amplitudes for K+ larger than for π+:

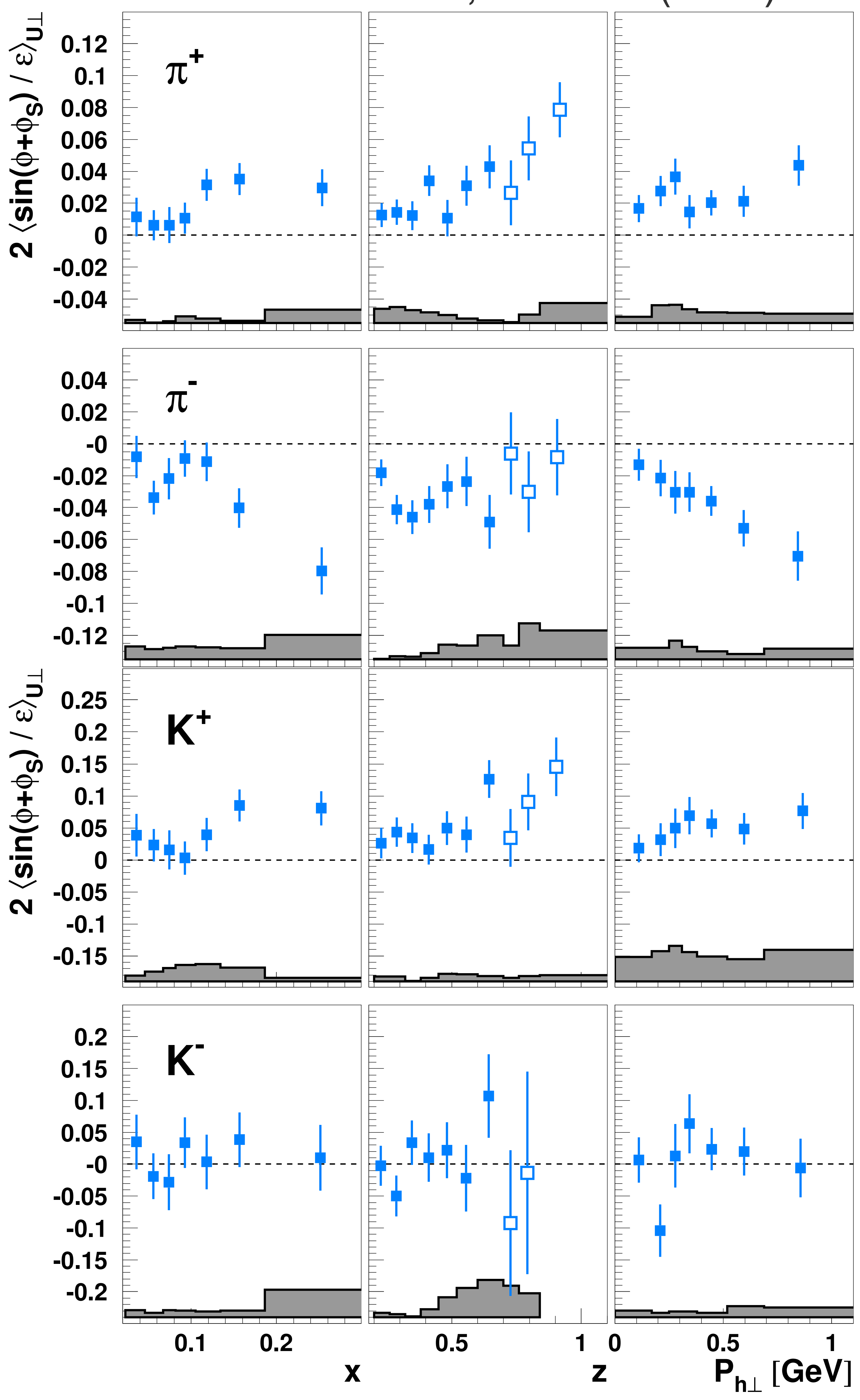
$$H_1^{\perp,u\to K^+} > H_1^{\perp,u\to\pi^+}$$

Collins amplitudes



$\mathscr{C}\left[h_{1T}^{q} \times H_{1}^{\perp,q}\right]$

HERMES, JHEP **12**(2020)010



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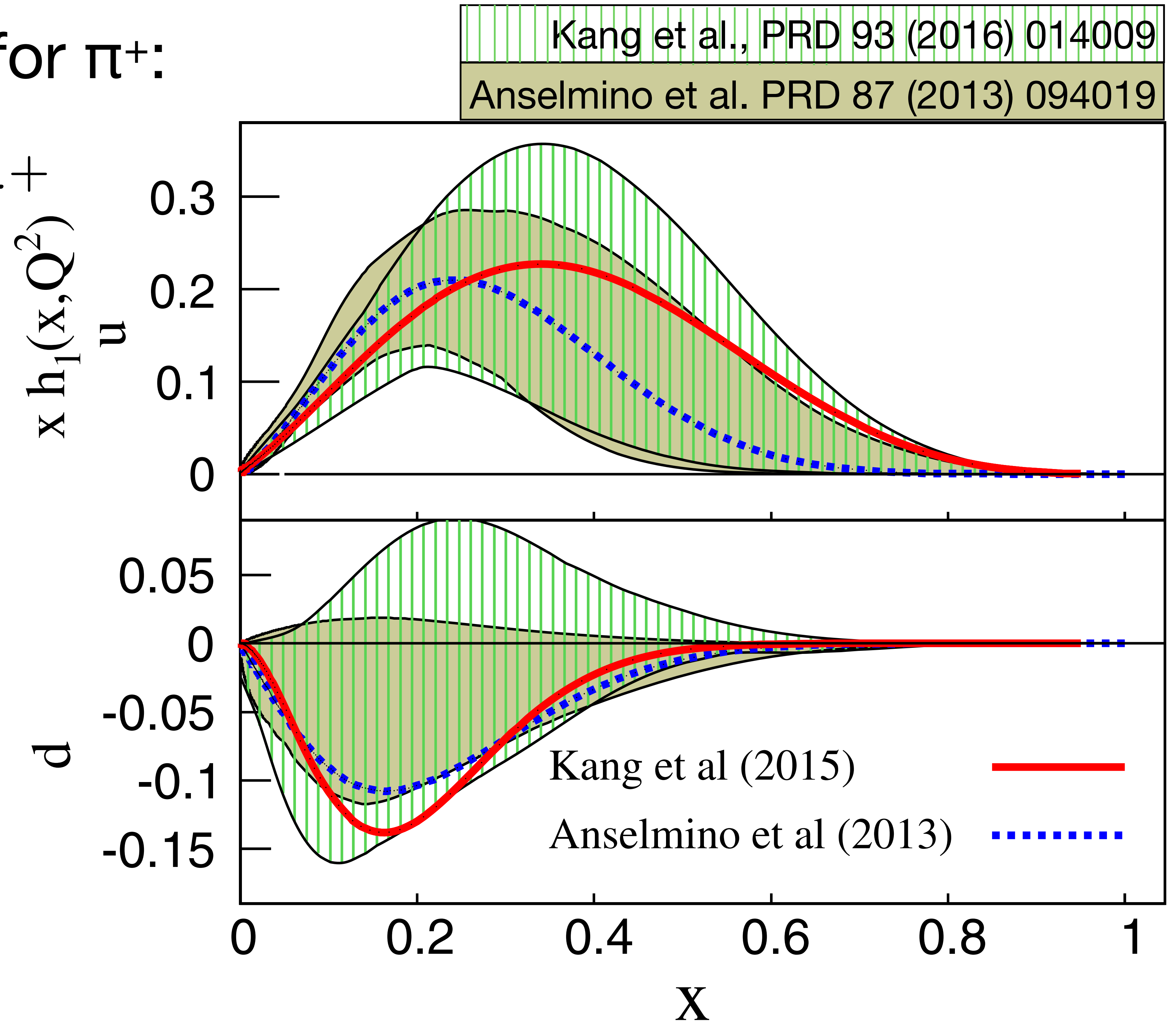
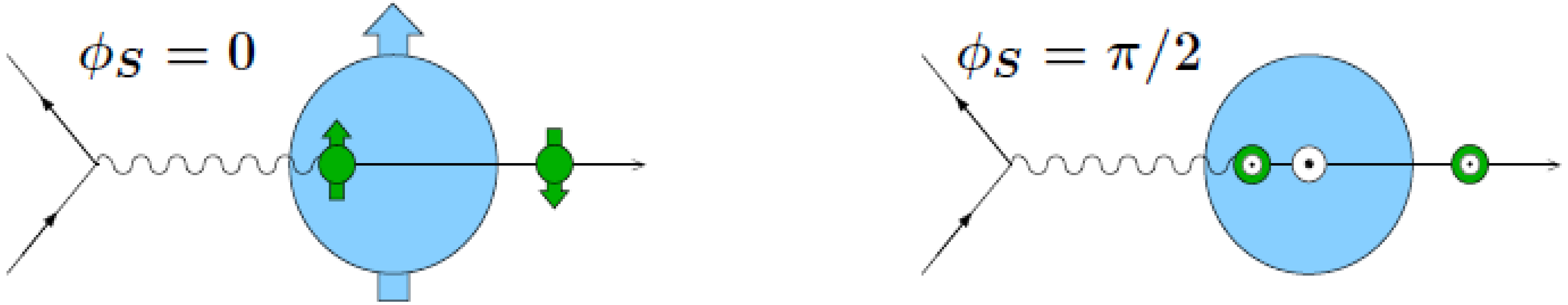


data from Belle, Babar, COMPASS, HERMES, Jefferson Lab Hall A

Artru model

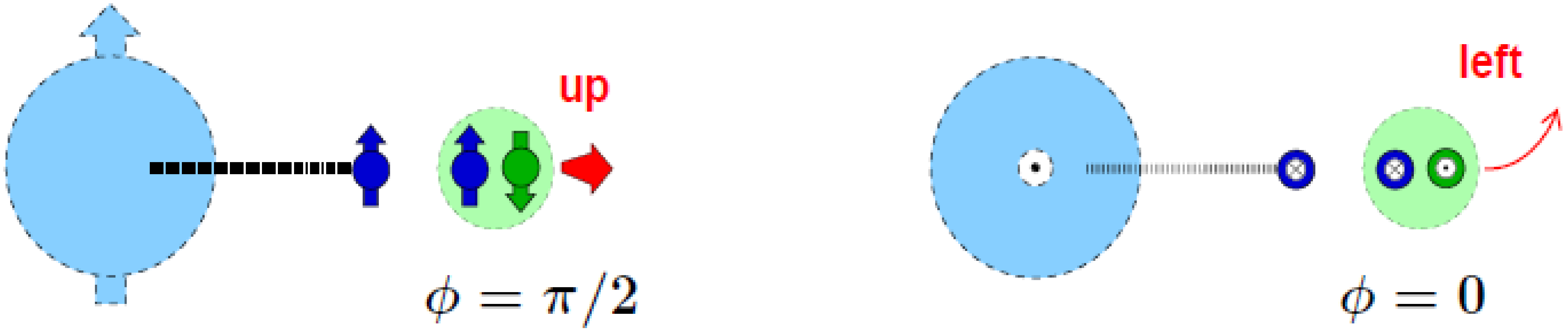
X. Artru et al., Z. Phys. C**73** (1997) 527

polarisation component in lepton scattering plane reversed by photoabsorption:



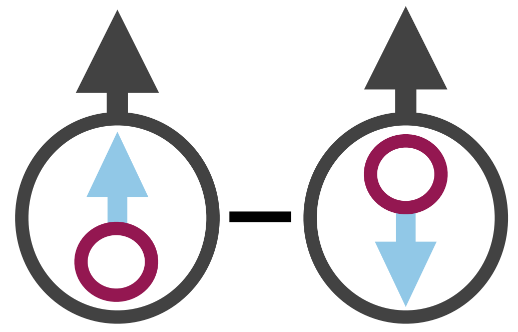
string break, quark-antiquark pair with vacuum numbers:

orbital angular momentum creates transverse momentum:

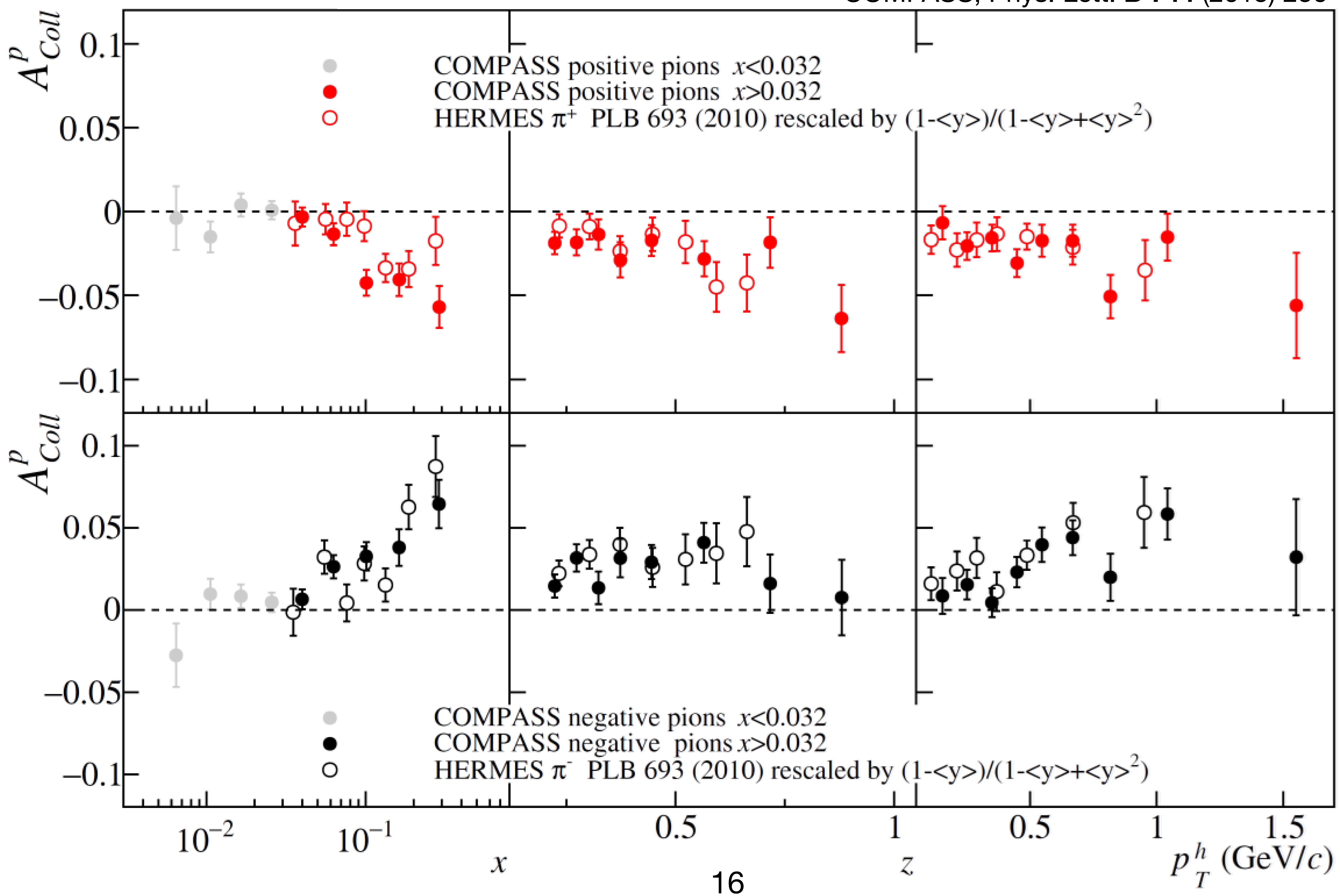


Courtesy U. Elschenbroich

Collins amplitudes: QCD evolution



COMPASS, Phys. Lett. **B 744** (2015) 250



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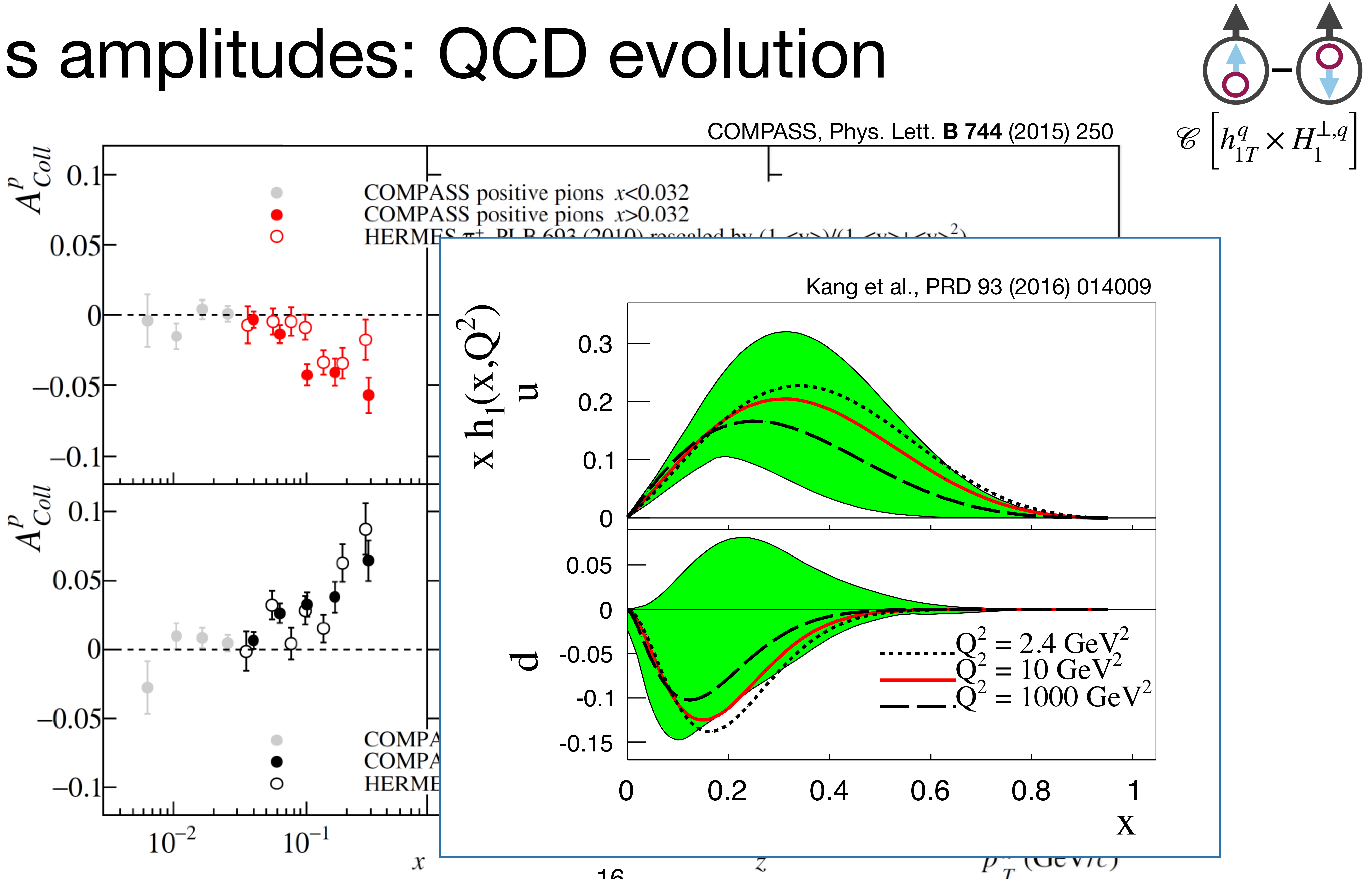
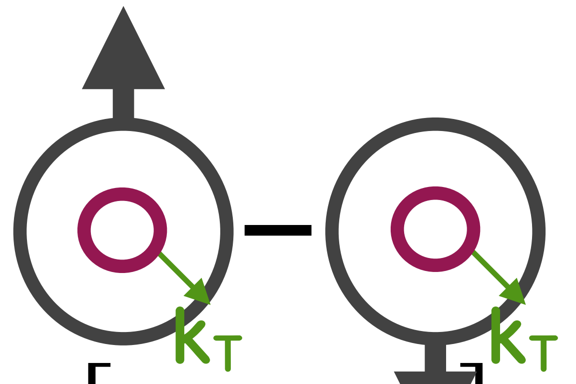
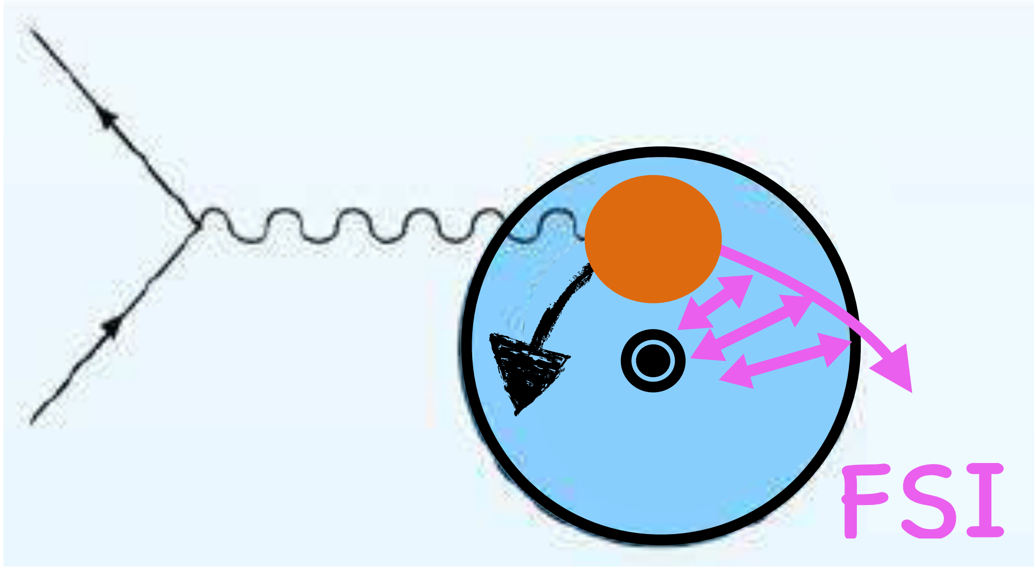


Sivers amplitudes

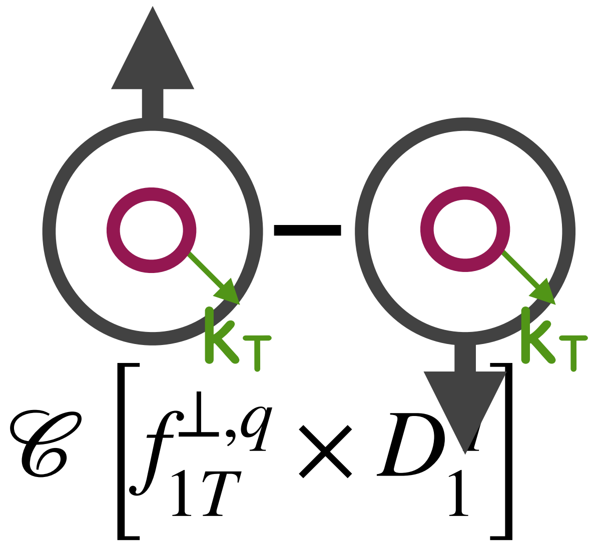


$\mathscr{C}\left[f_{1T}^{\perp,q} \times D_1^q\right]$

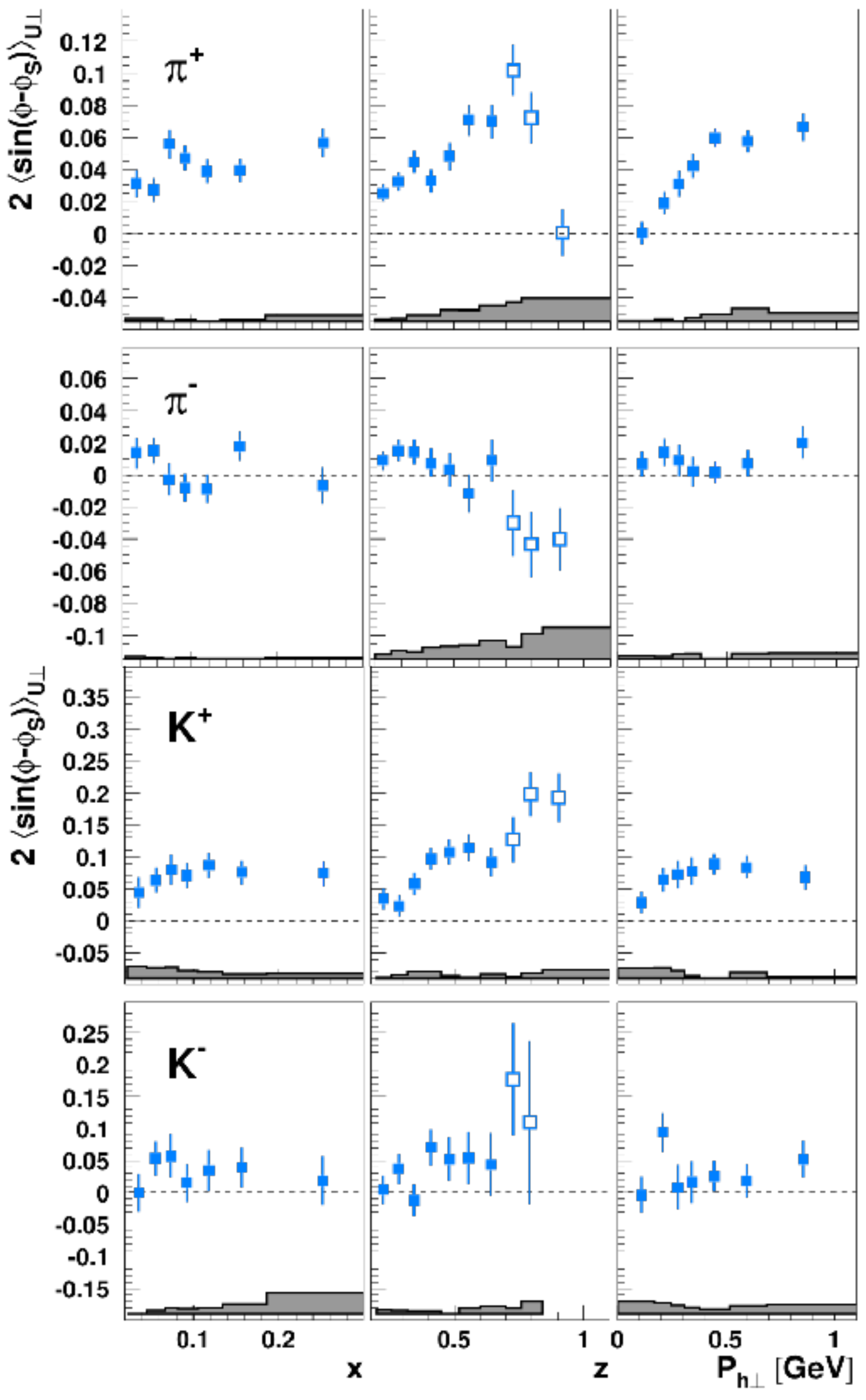
- Sivers function:
  - requires non-zero orbital angular momentum
  - final-state interactions ⟶ azimuthal asymmetries



Sivers amplitudes



HERMES, JHEP **12**(2020)010



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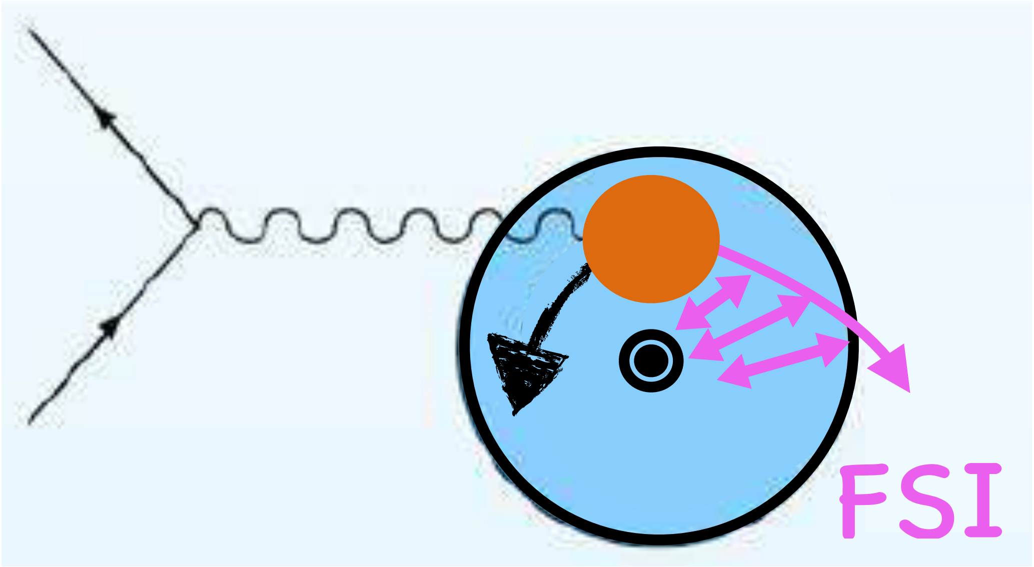
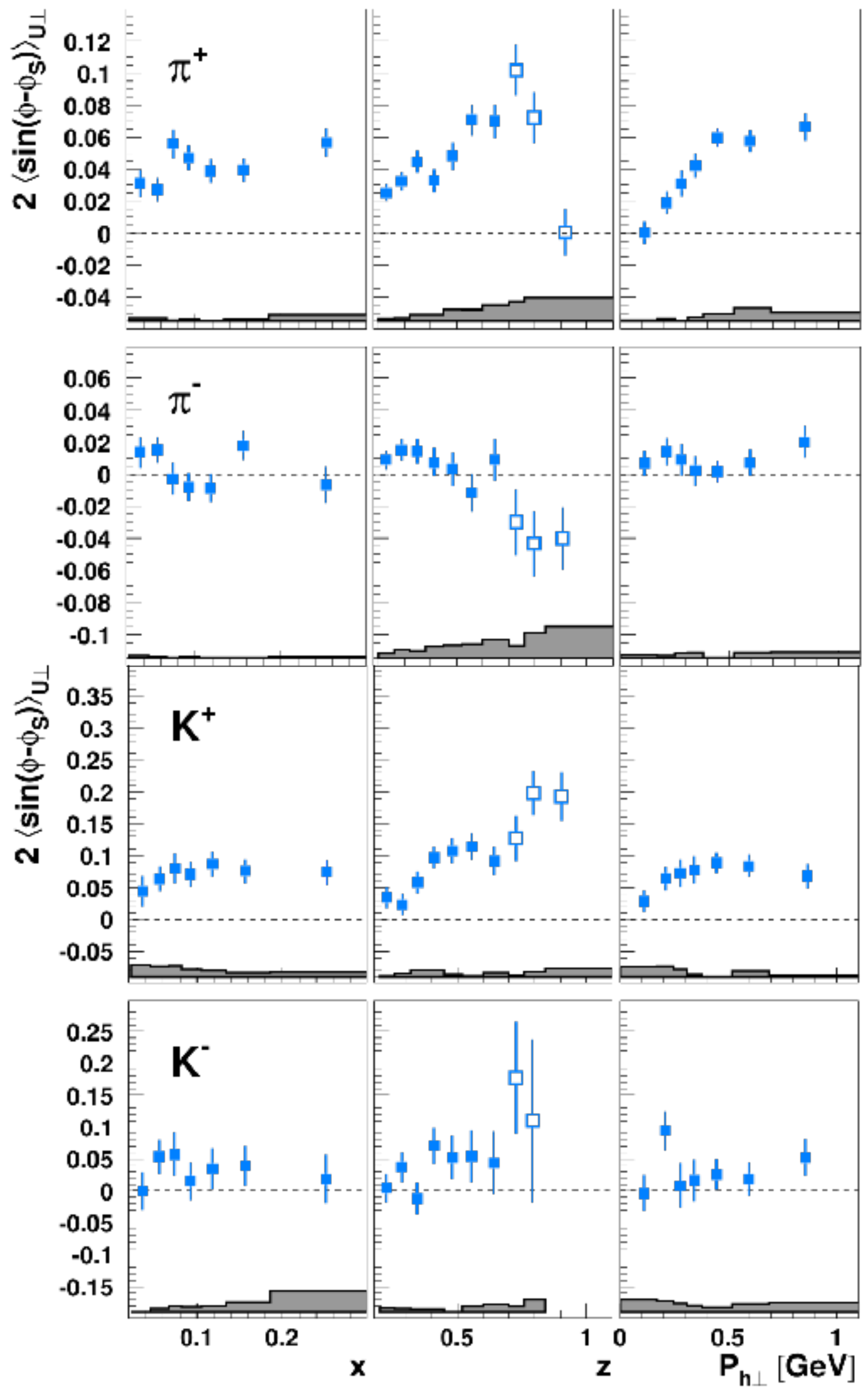
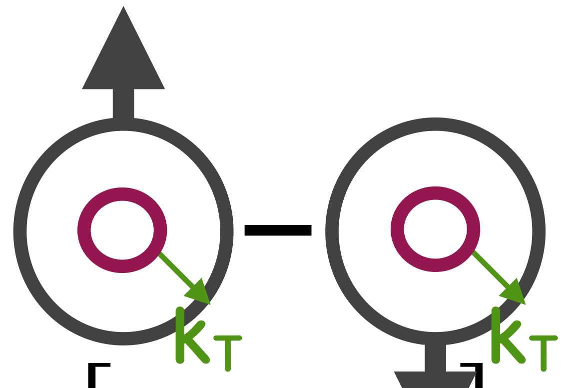


Sivers amplitudes

HERMES, JHEP **12**(2020)010





$\mathscr{C}\left[f_{1T}^{\perp,q} \times D_1\right]$

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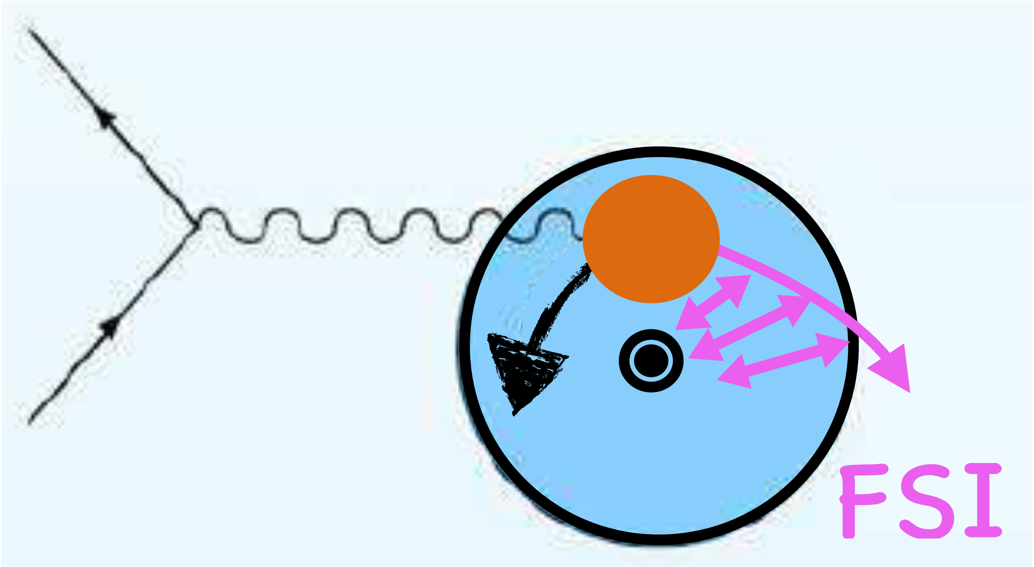
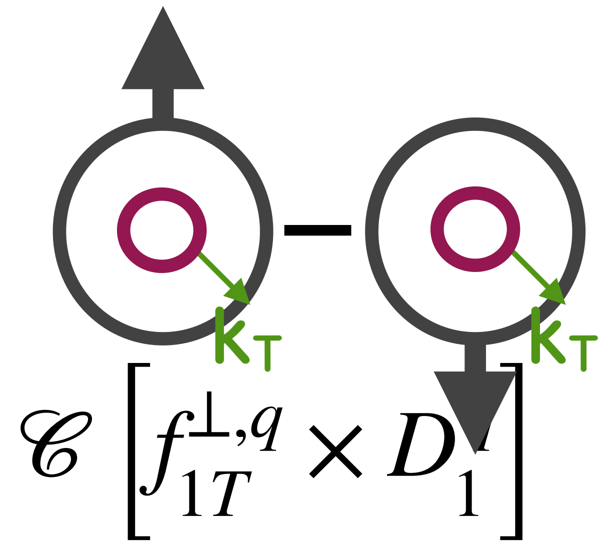


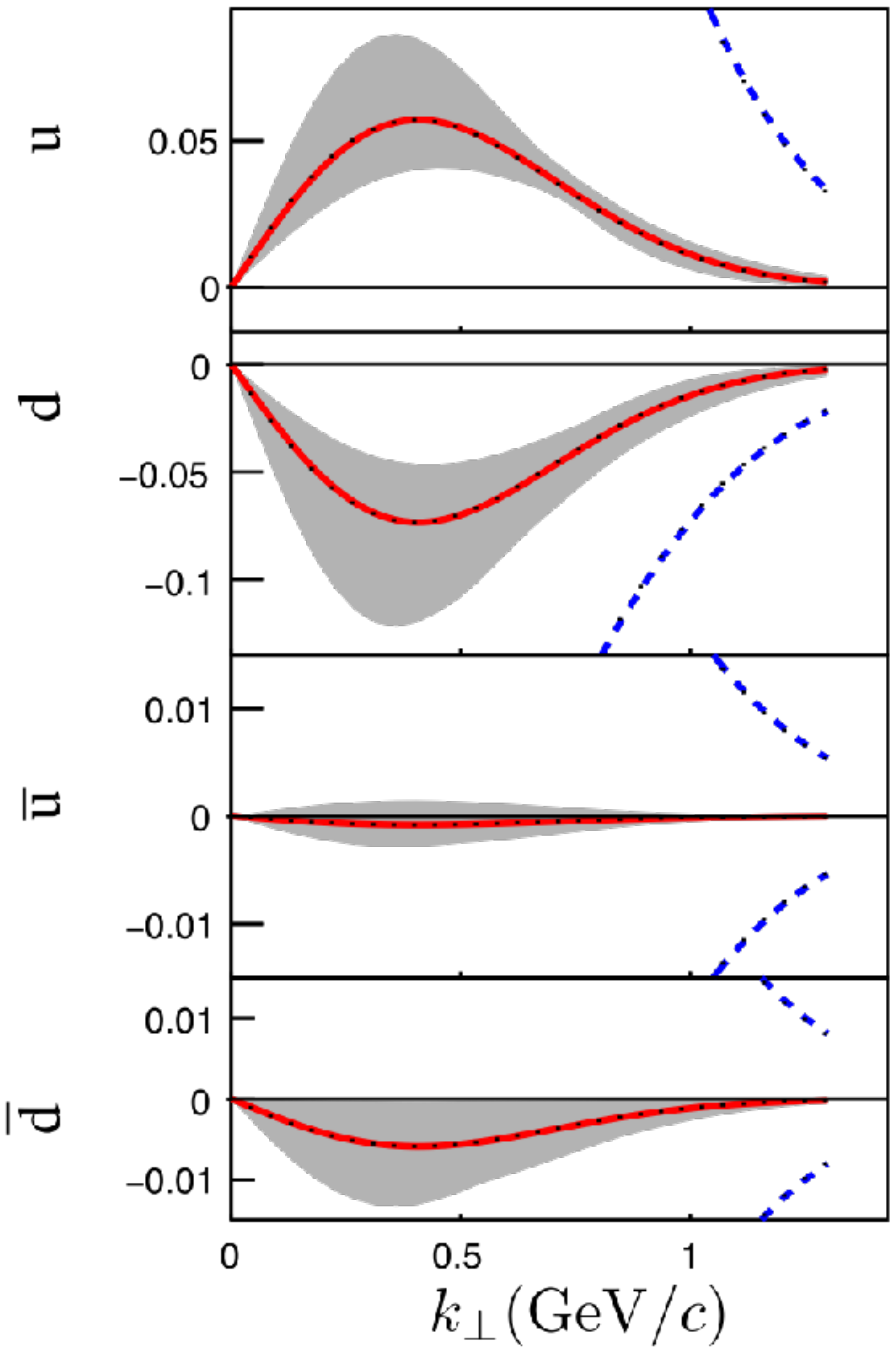
- π^+:
  - positive -> non-zero orbital angular momentum

- π^-:
  - consistent with zero → u and d quark cancelation

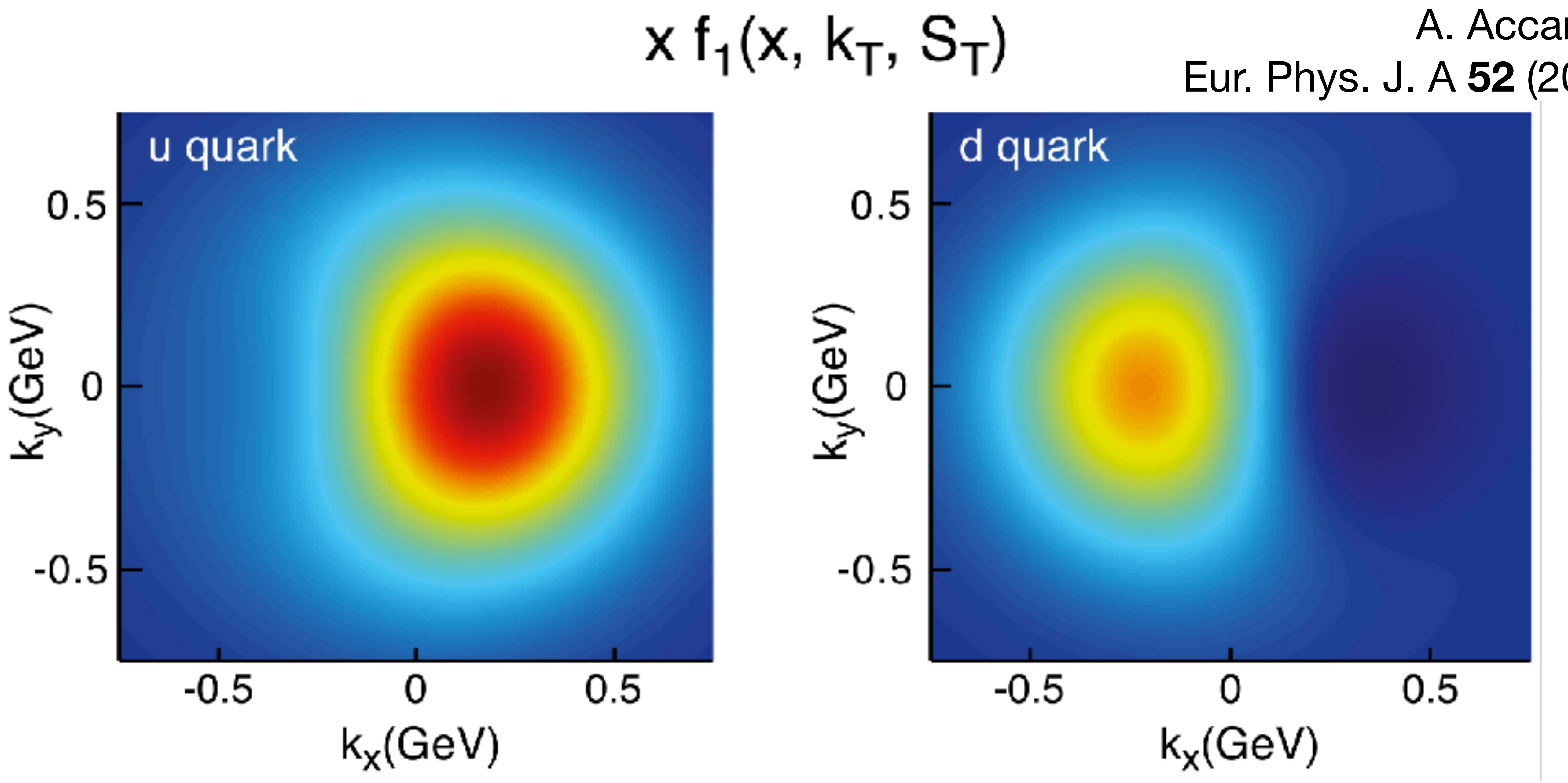
Sivers function



M. Anselmino et al., JHEP **04** (2017) 046



A. Accardi et al., Eur. Phys. J. A **52** (2016) 268

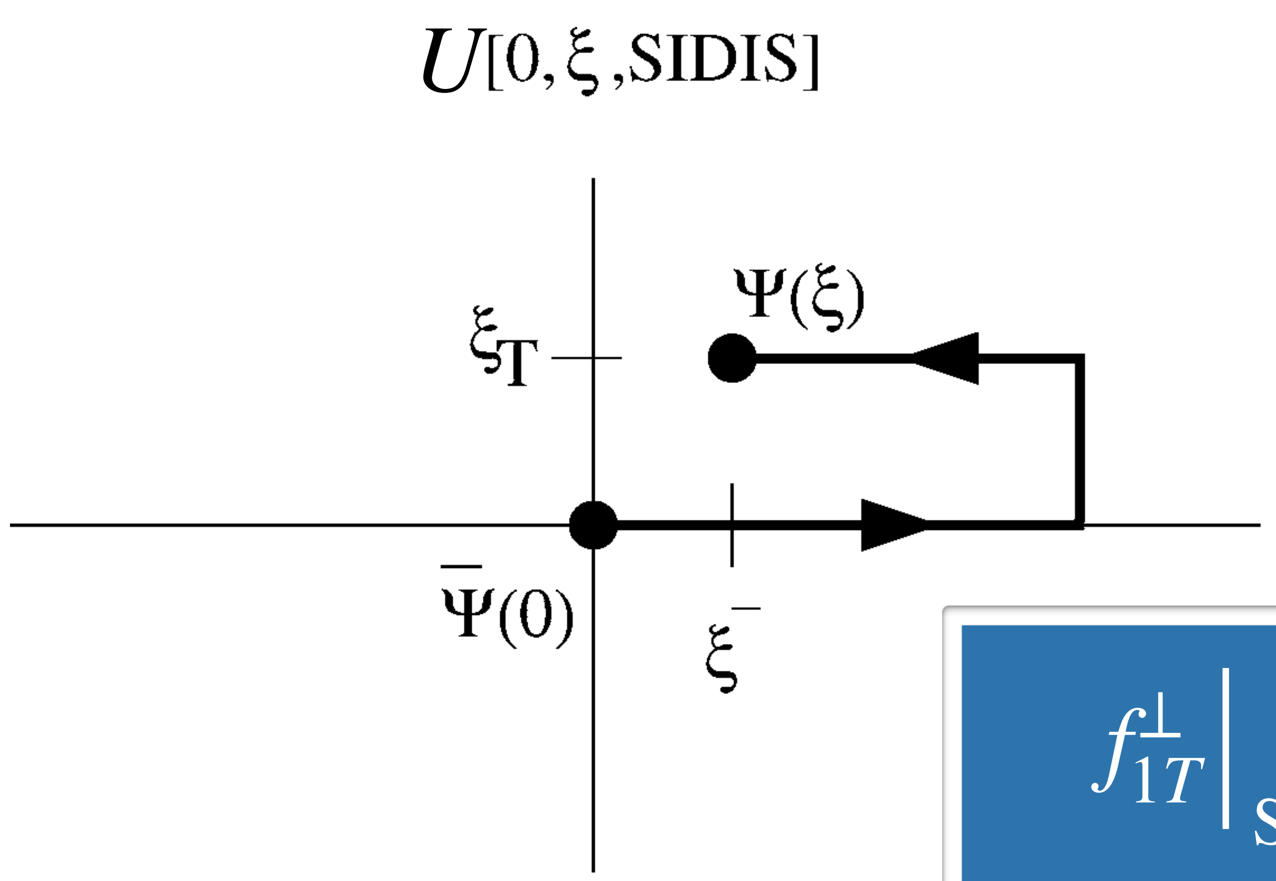


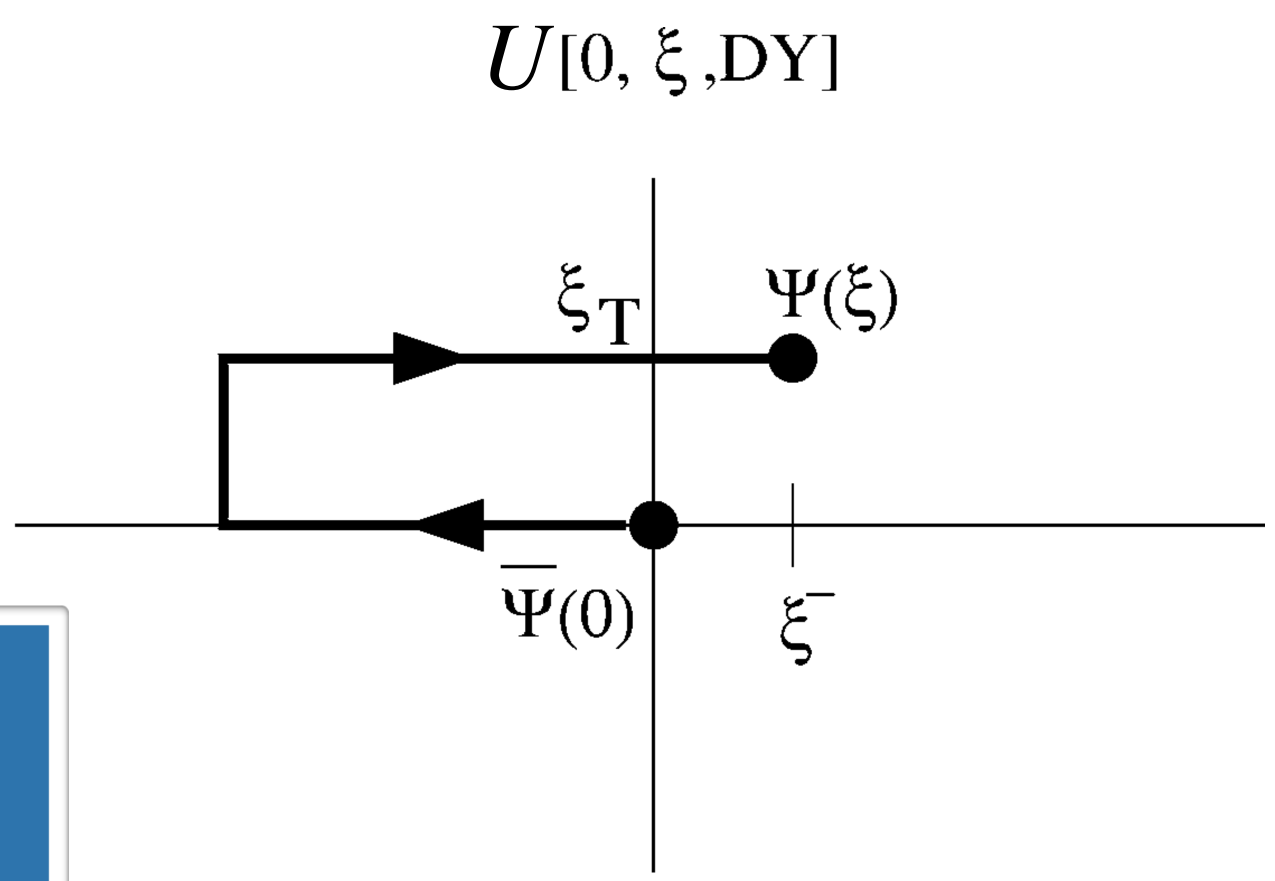
nucleon polarised along $\hat{y}$

Predicted Sivers sign change for SIDIS and Drell-Yan

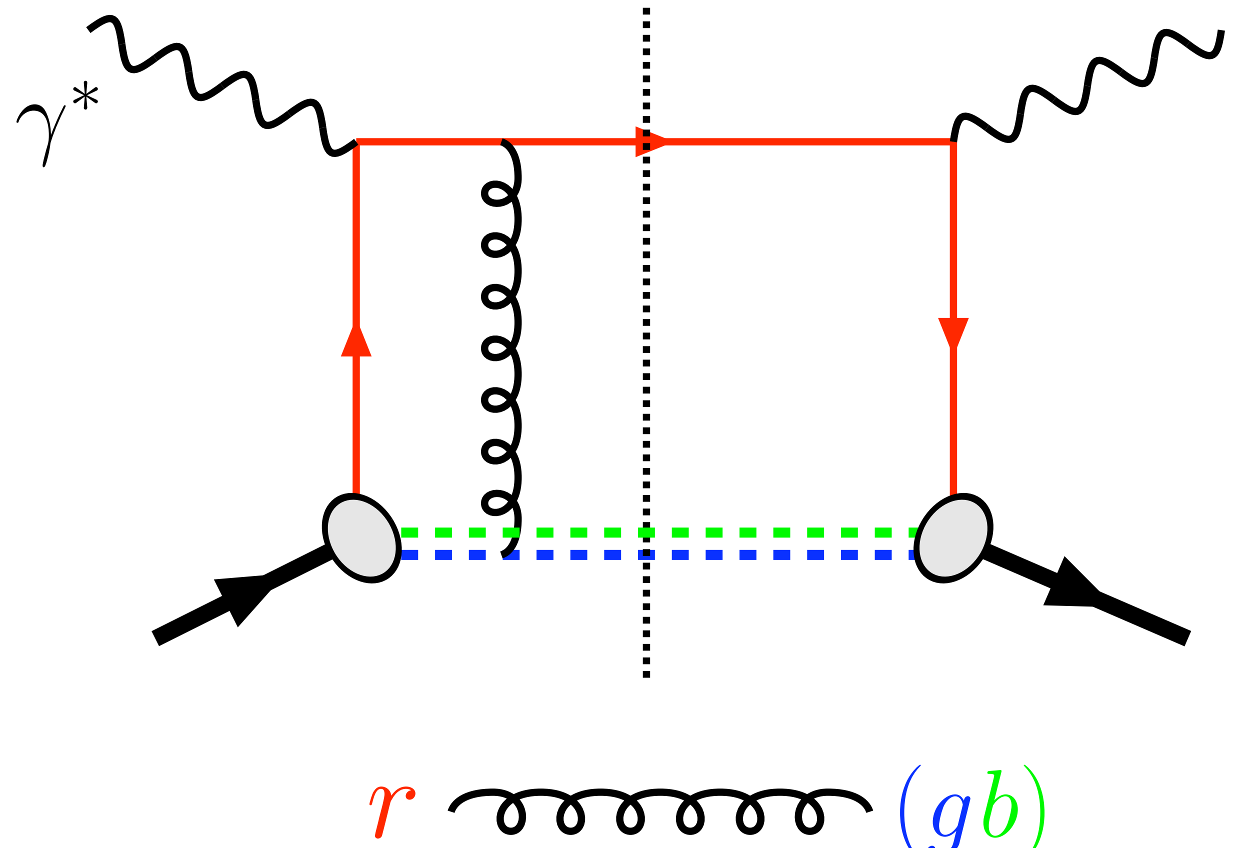
J. C. Collins, Phys. Lett. B 536 (2002) 43

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\langle P,S|\,\bar{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,|P,S\rangle$$

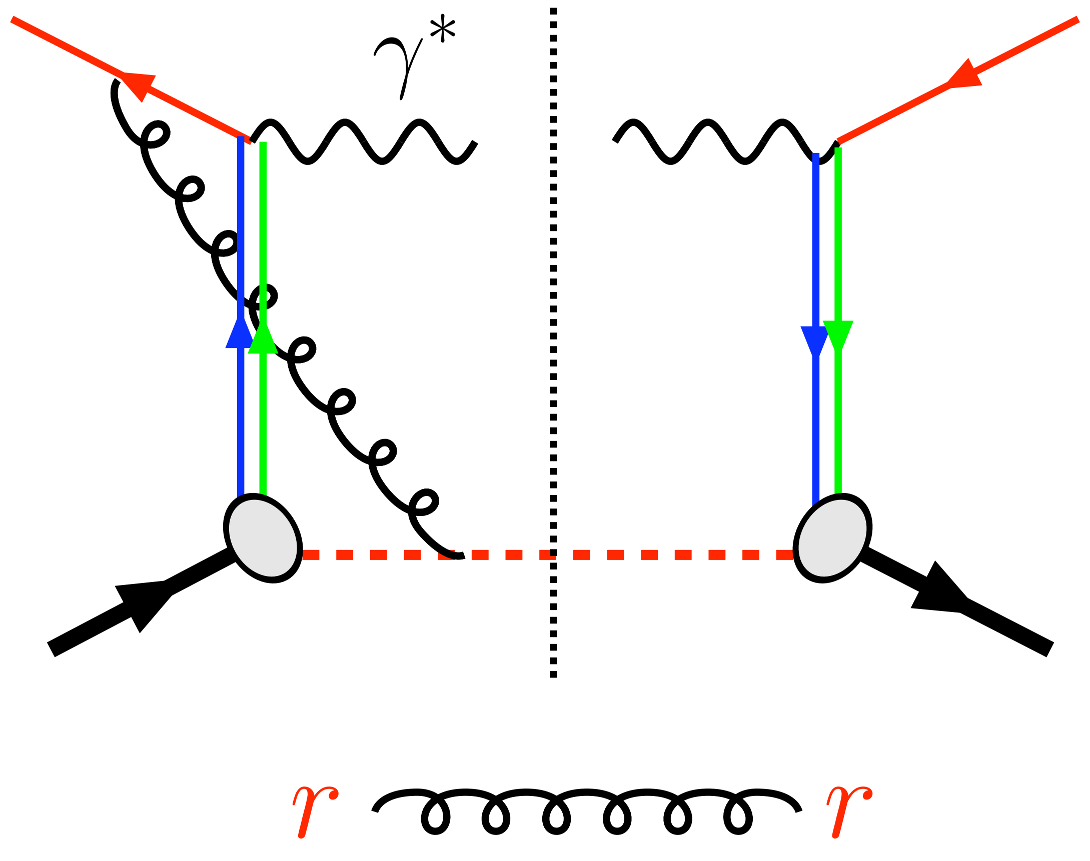




$$f_{1T}^{\perp}\Big|_{\text{SIDIS}} = -f_{1T}^{\perp}\Big|_{\text{DY}}$$

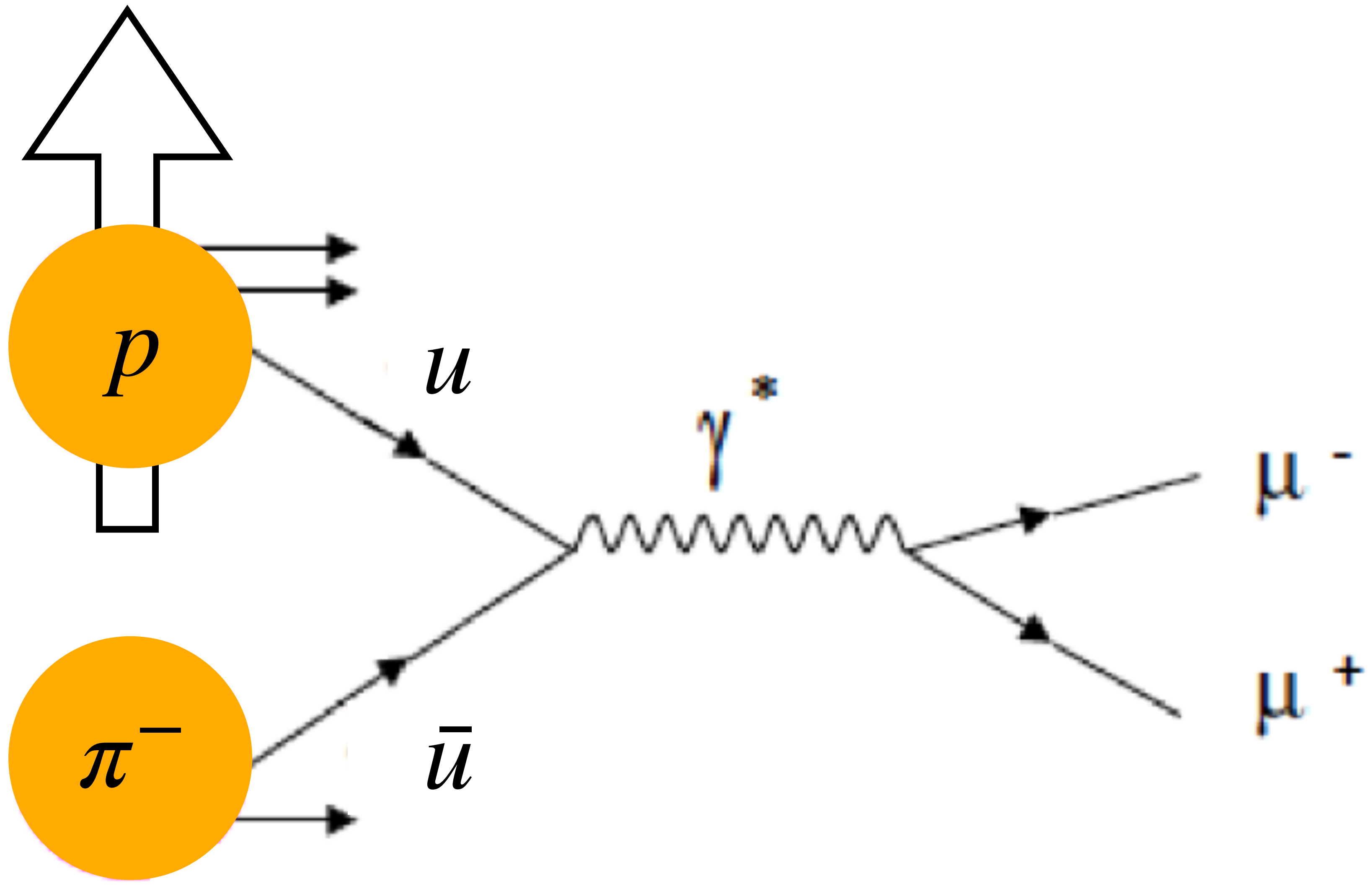


SIDIS

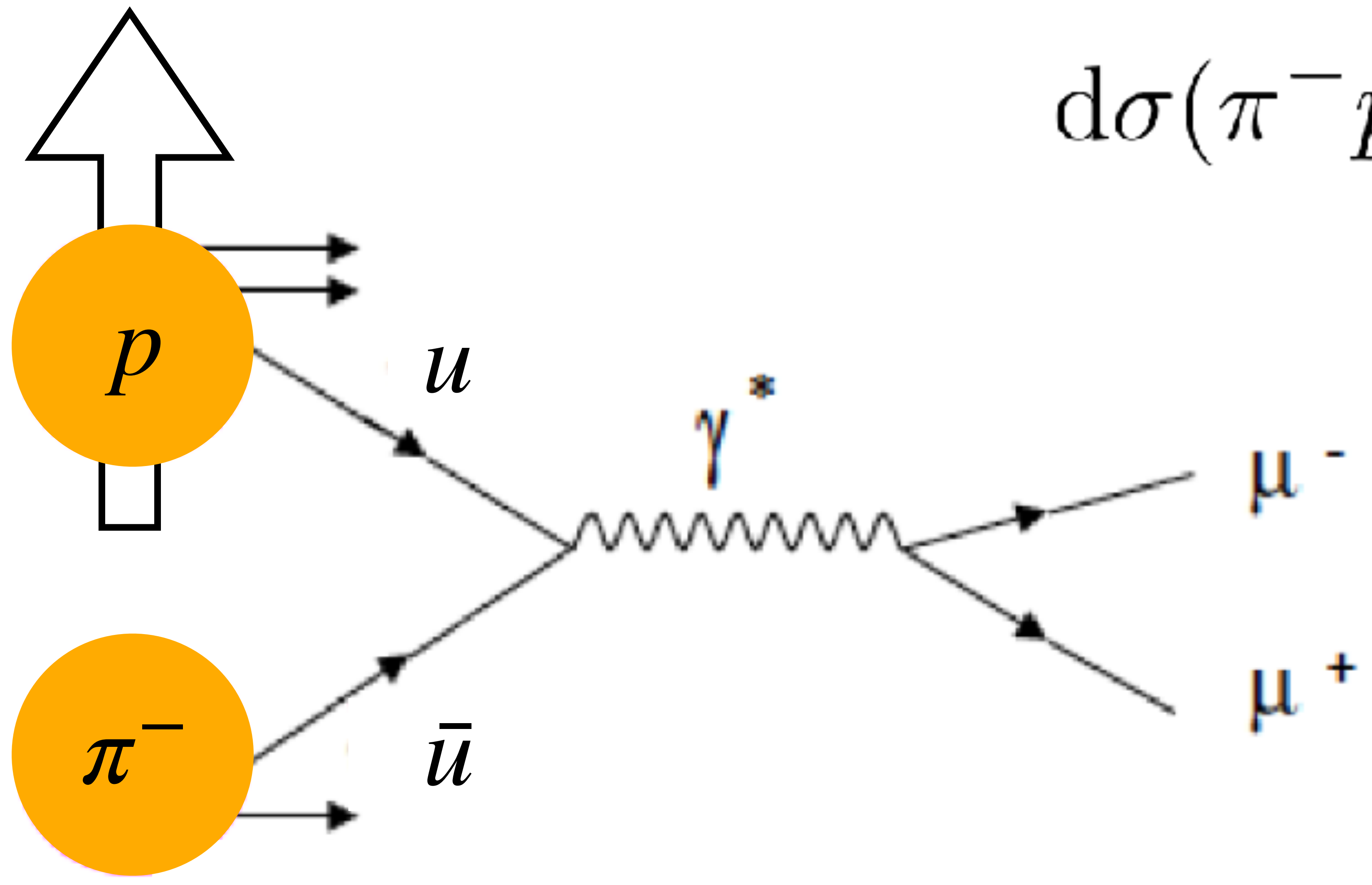


Drell-Yan

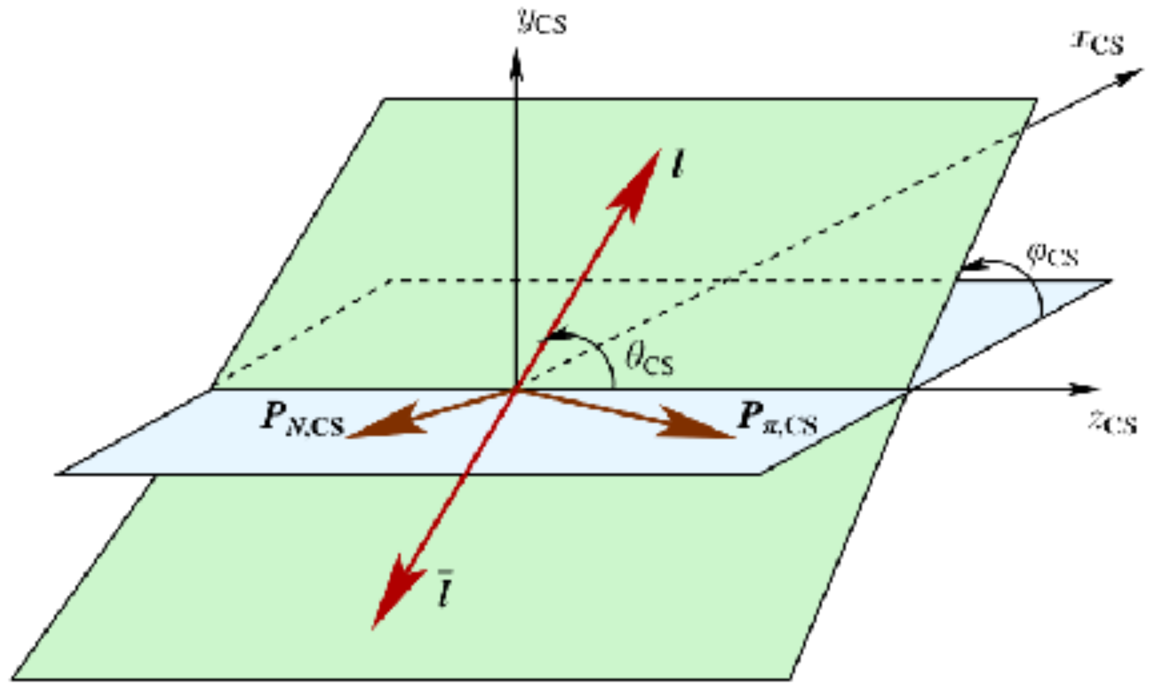
Experimental access to Sivers in Drell-Yan

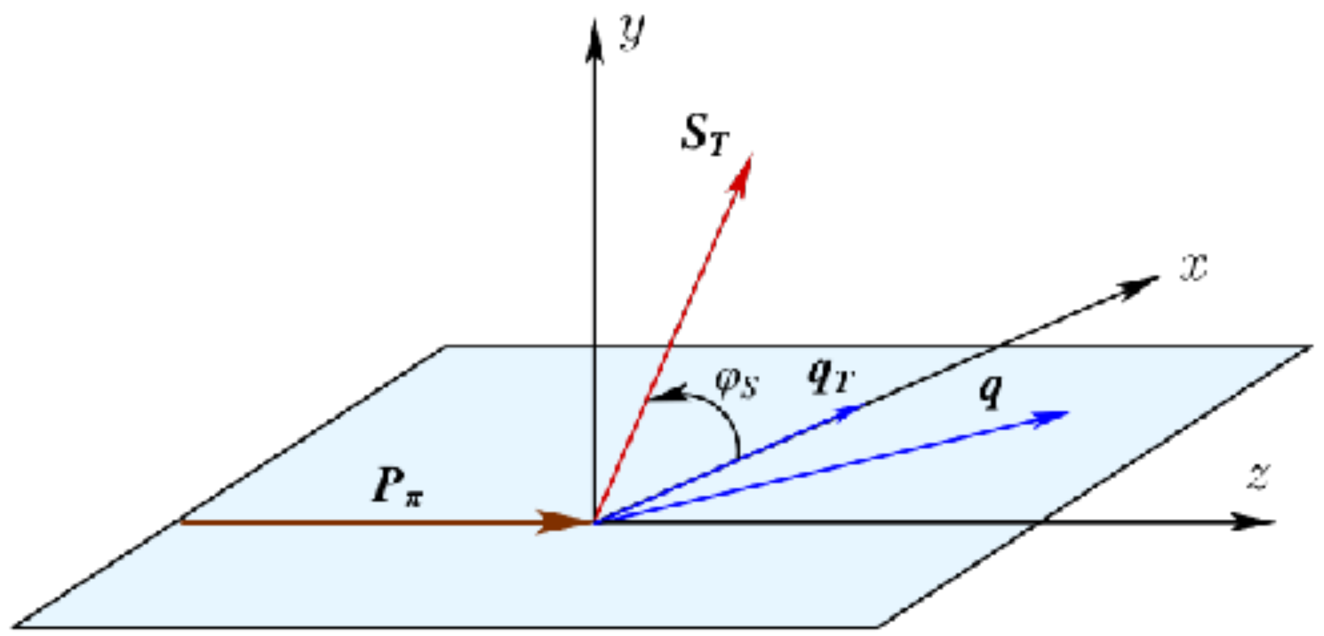


Experimental access to Sivers in Drell-Yan



$$d\sigma(\pi^- p^\uparrow \to \mu^+\mu^- X) \sim 1 + \overline{h}_1^\perp \otimes h_1^\perp \cos(2\phi)$$
$$+ |S_T| \;\; \overline{f}_1 \otimes f_{1T}^\perp \sin\phi_S$$
$$+ |S_T| \;\; \overline{h}_1^\perp \otimes h_{1T}^\perp \sin(2\phi + \phi_S)$$
$$+ |S_T| \;\; \overline{h}_1^\perp \otimes h_{1T} \sin(2\phi - \phi_S)$$





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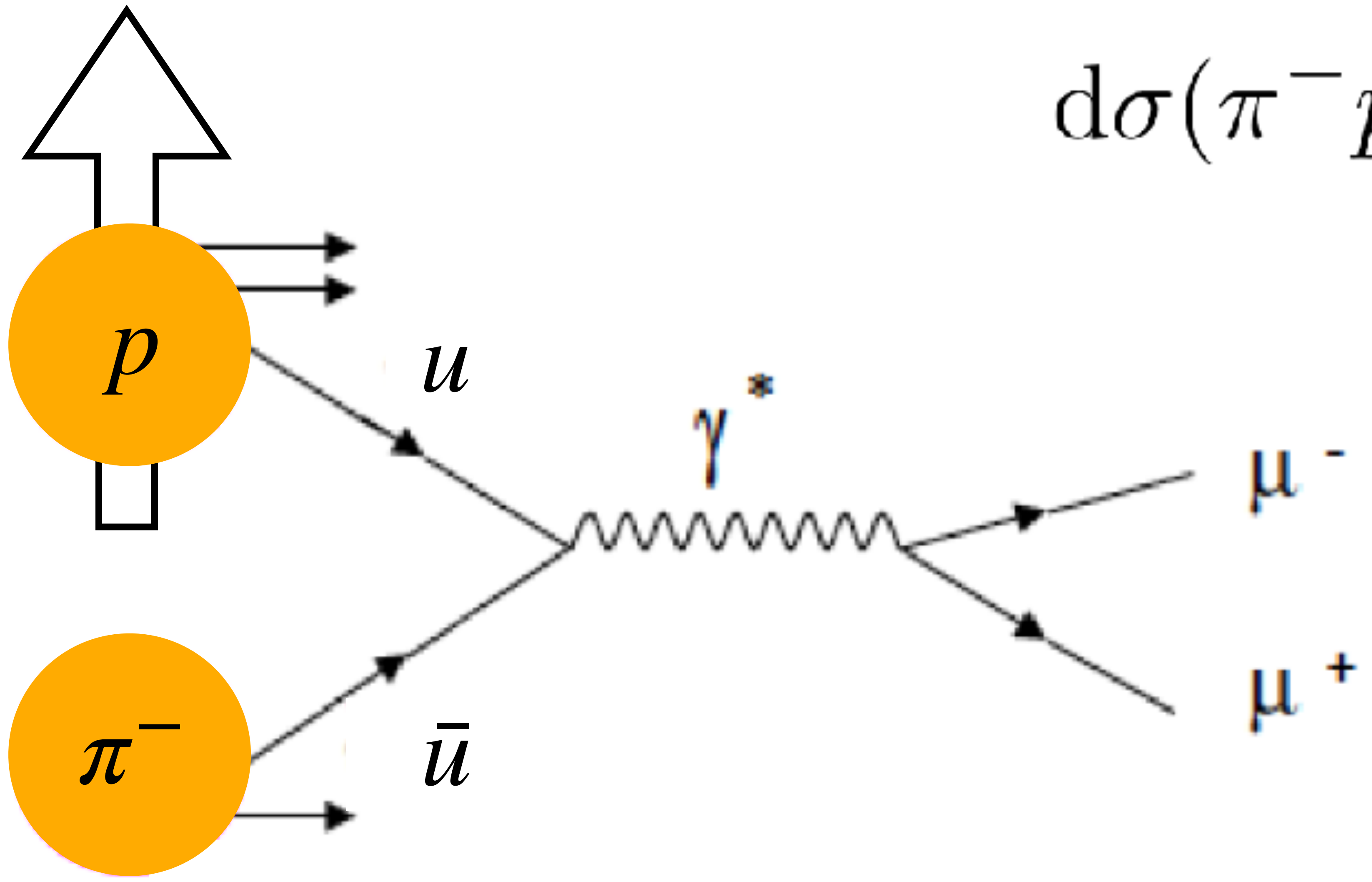

$$
\begin{aligned}
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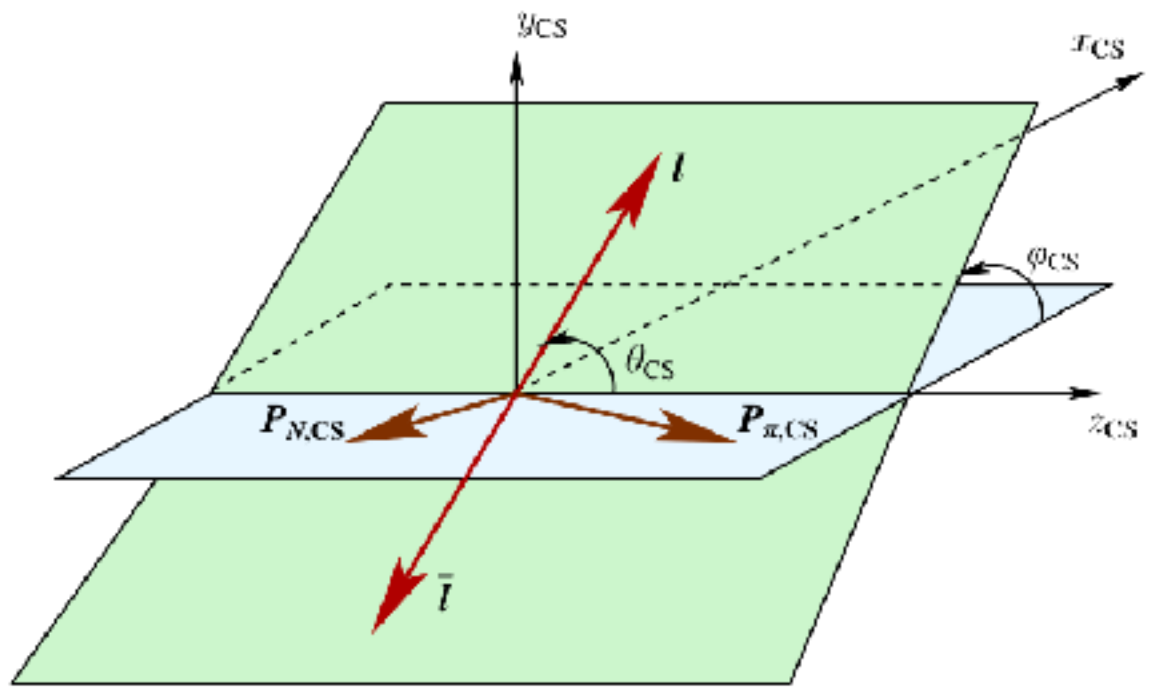
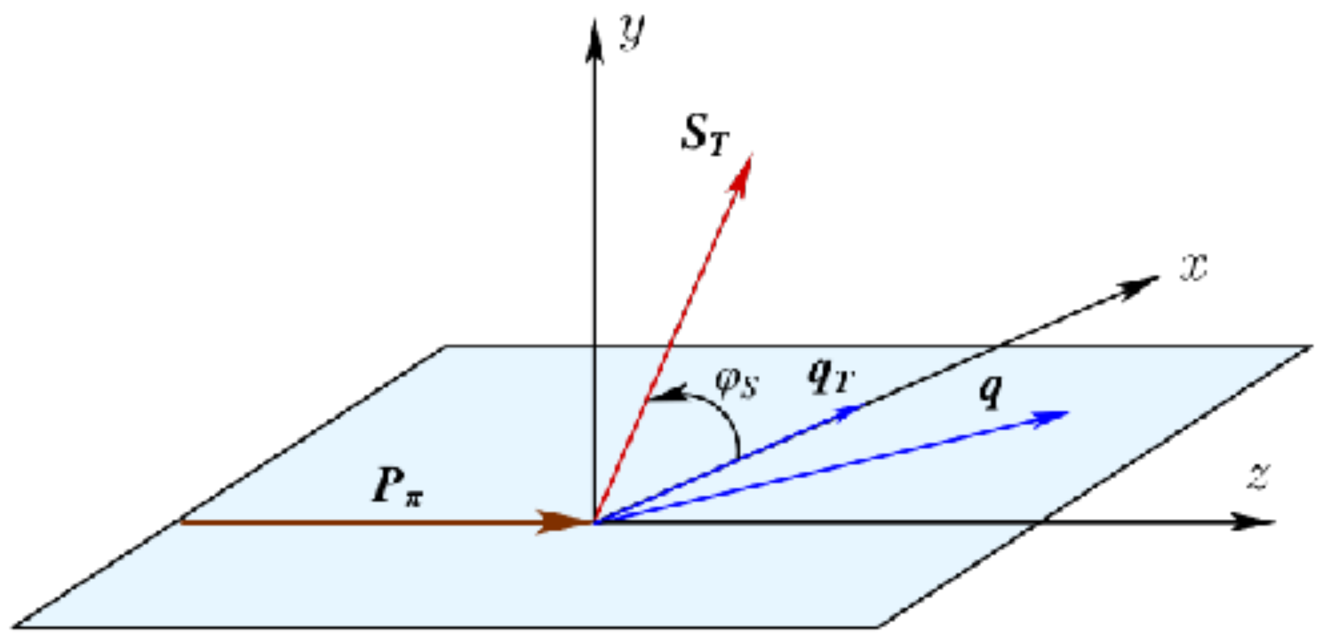
\end{aligned}
$$





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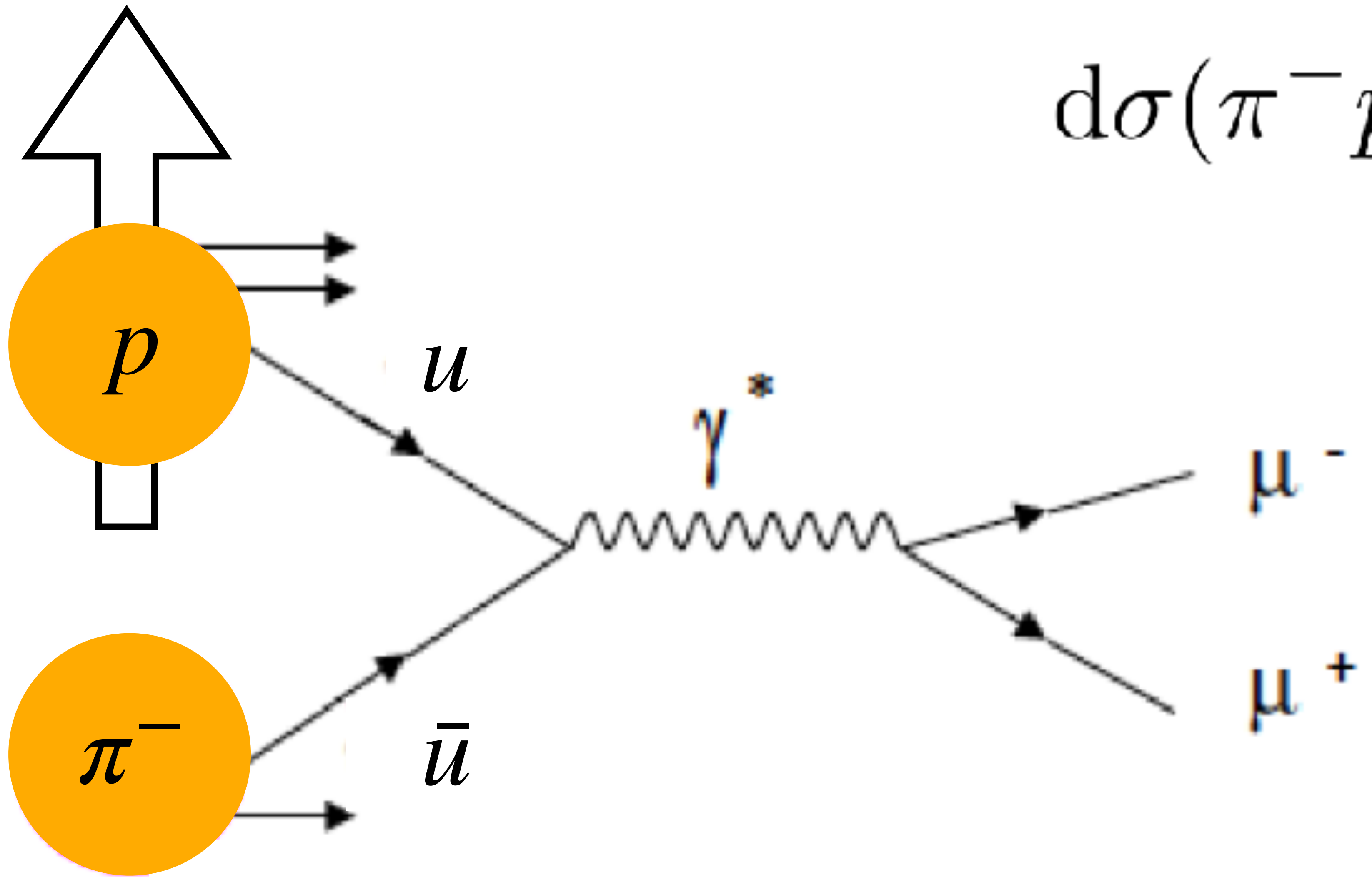

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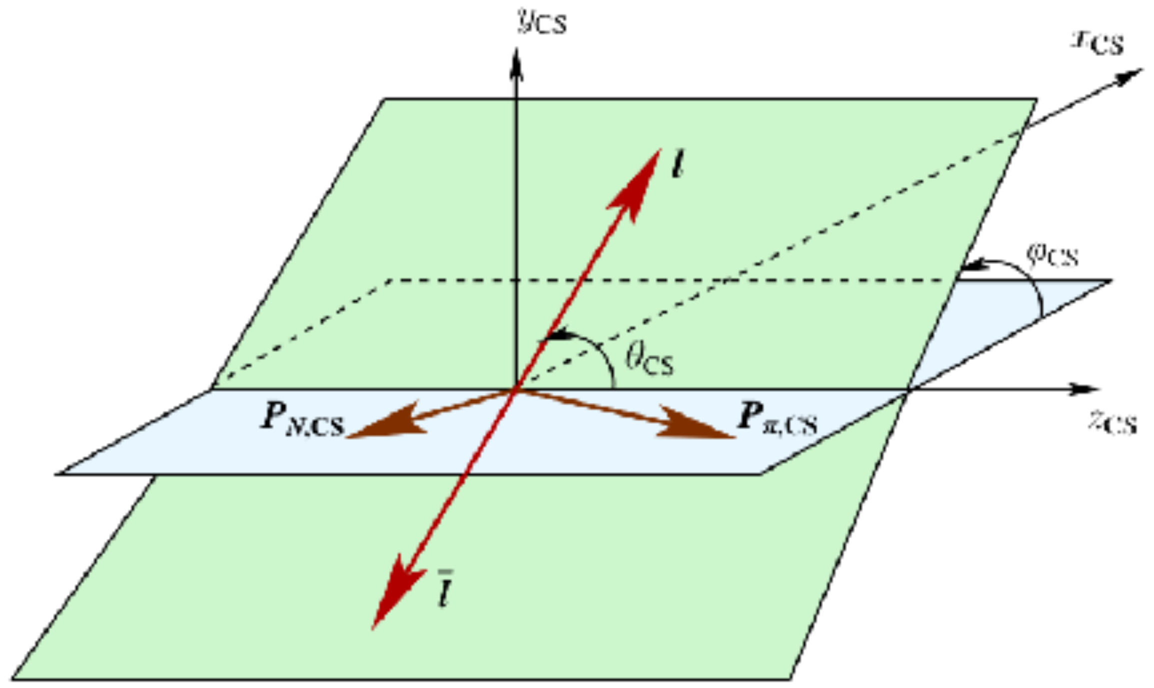
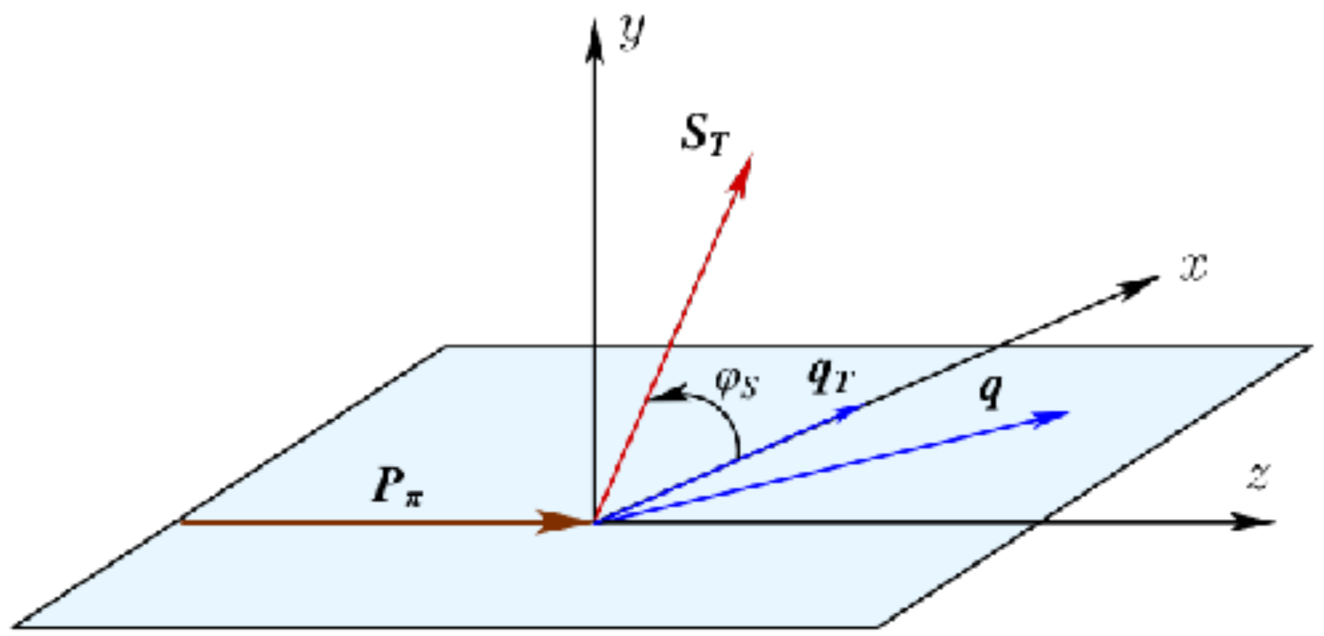




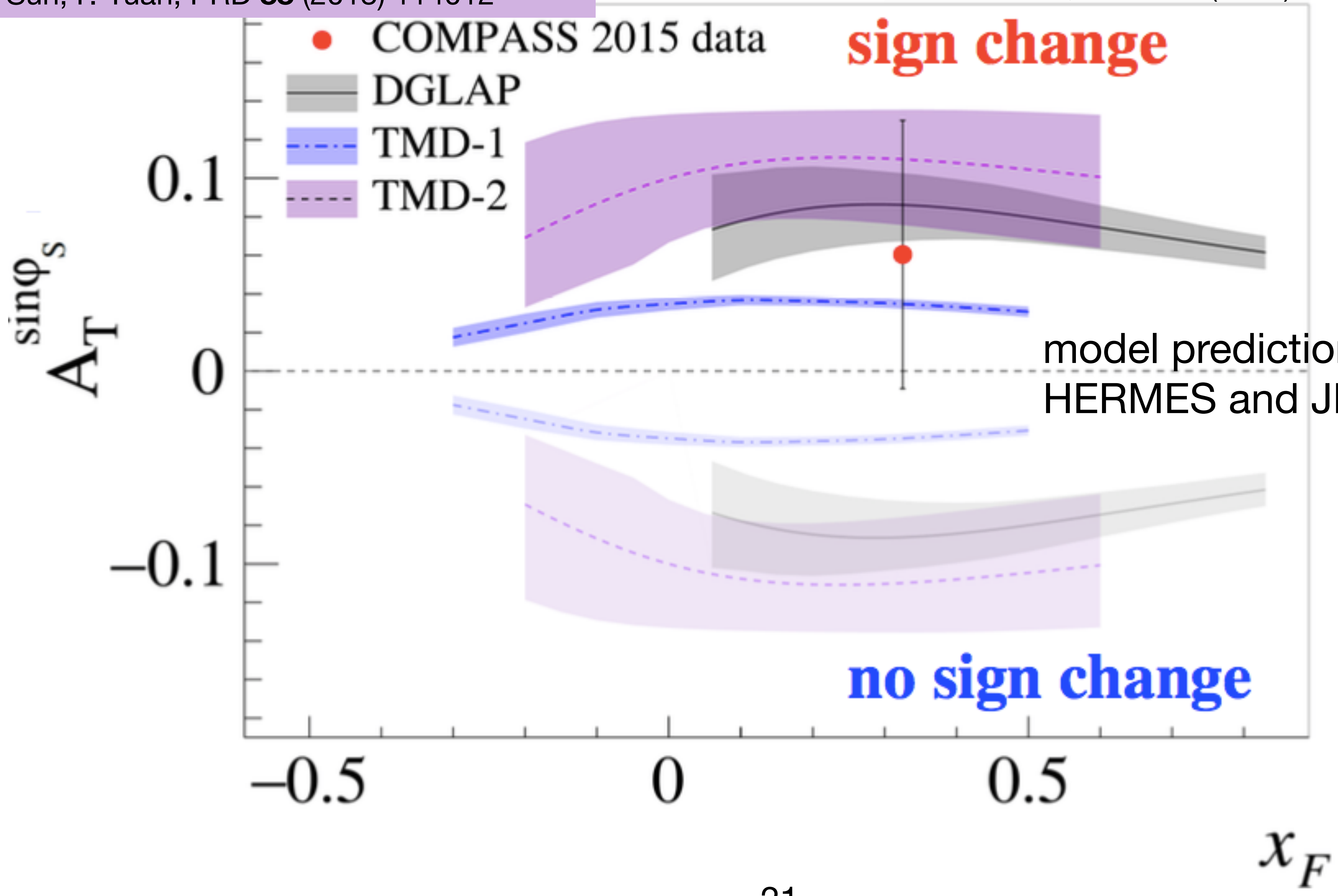
Investigation of the Sivers sign change in $p^{\uparrow}\pi^{-}$ collisions

M. Anselmino et al., JHEP **04** (2017) 046

M. G. Echevarria et al. PRD **89** (2014)074013

P. Sun, F. Yuan, PRD **88** (2013) 114012

COMPASS, PRL **119** (2017) 112002



model predictions based on COMPASS, HERMES and JLab semi-inclusive DIS data

Investigation of the Sivers sign change in $p^{\uparrow}\pi^{-}$ collisions

M. Anselmino et al., JHEP **04** (2017) 046

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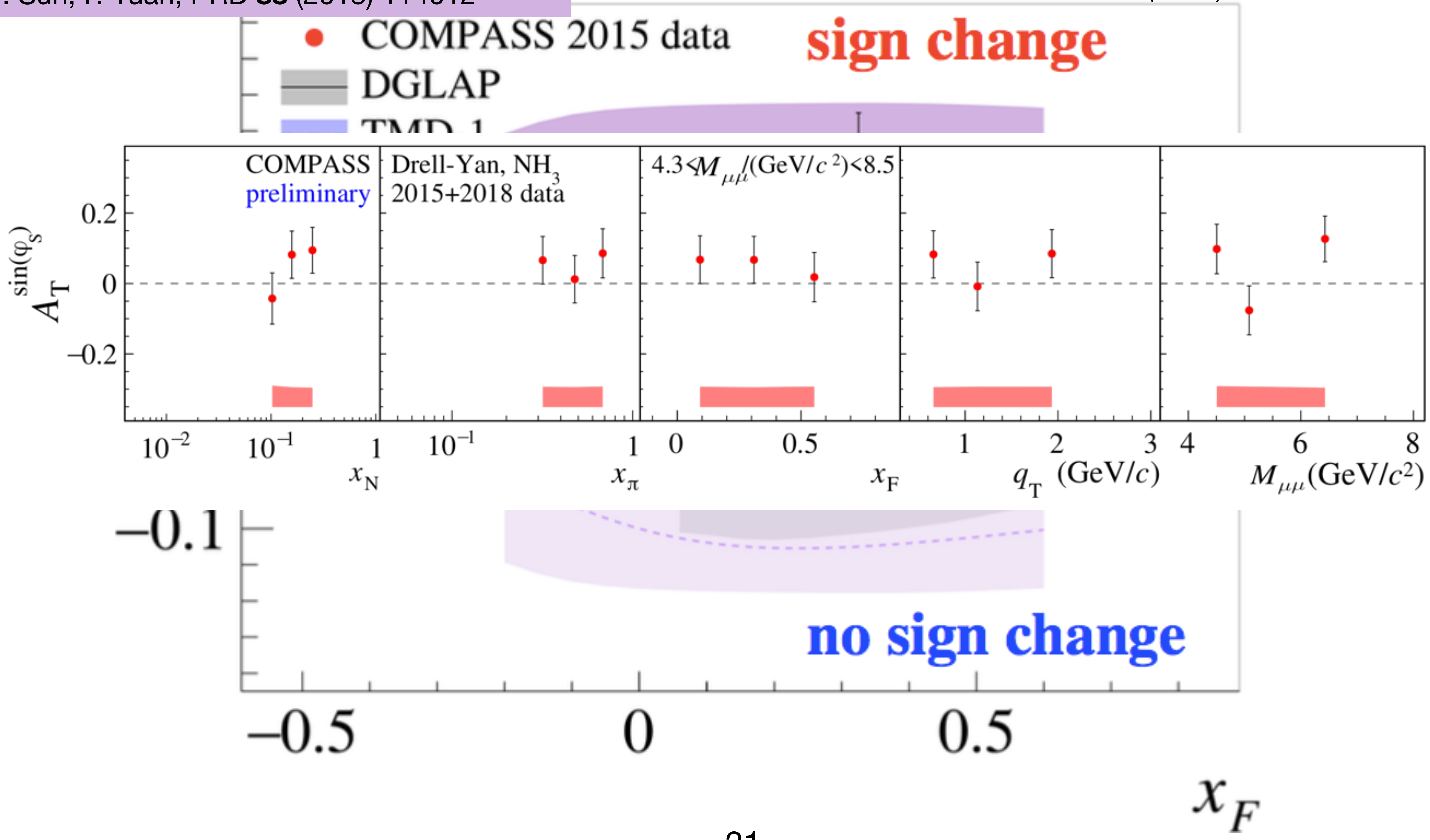
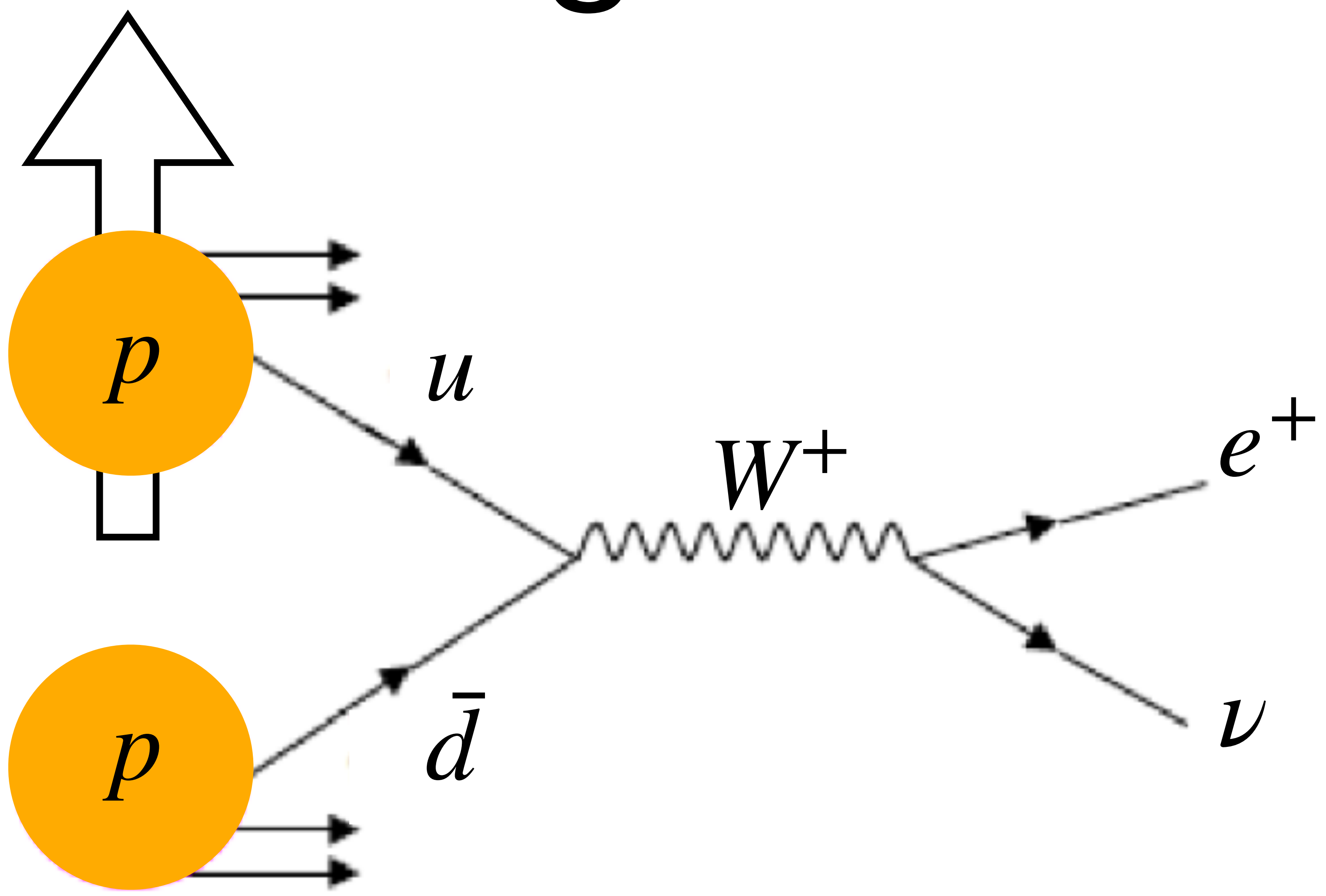
COMPASS, PRL **119** (2017) 112002

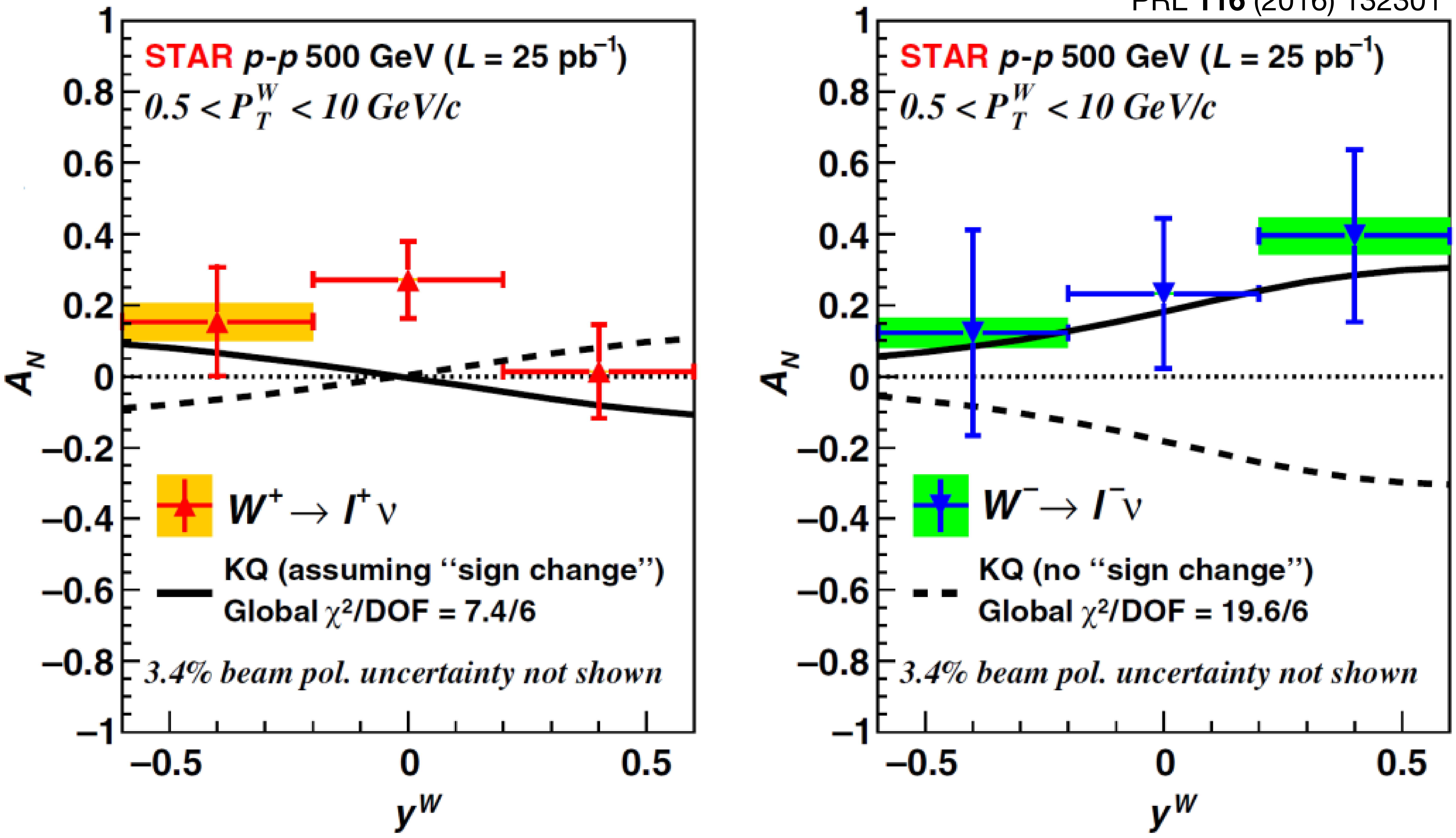


on COMPASS, inclusive DIS data

Investigation of the Sivers sign change in $p^{\uparrow}p$ collisions

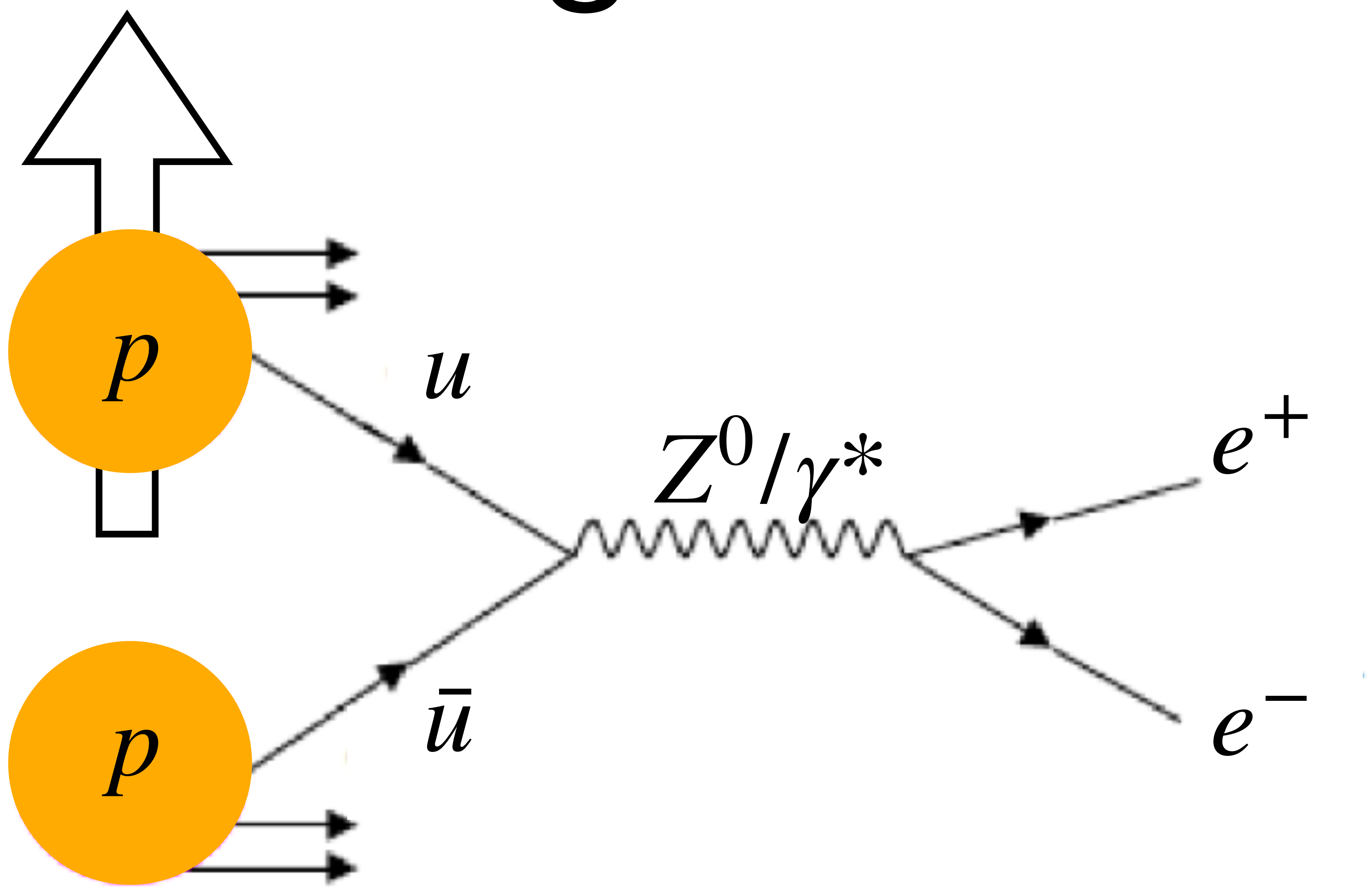


PRL **116** (2016) 132301

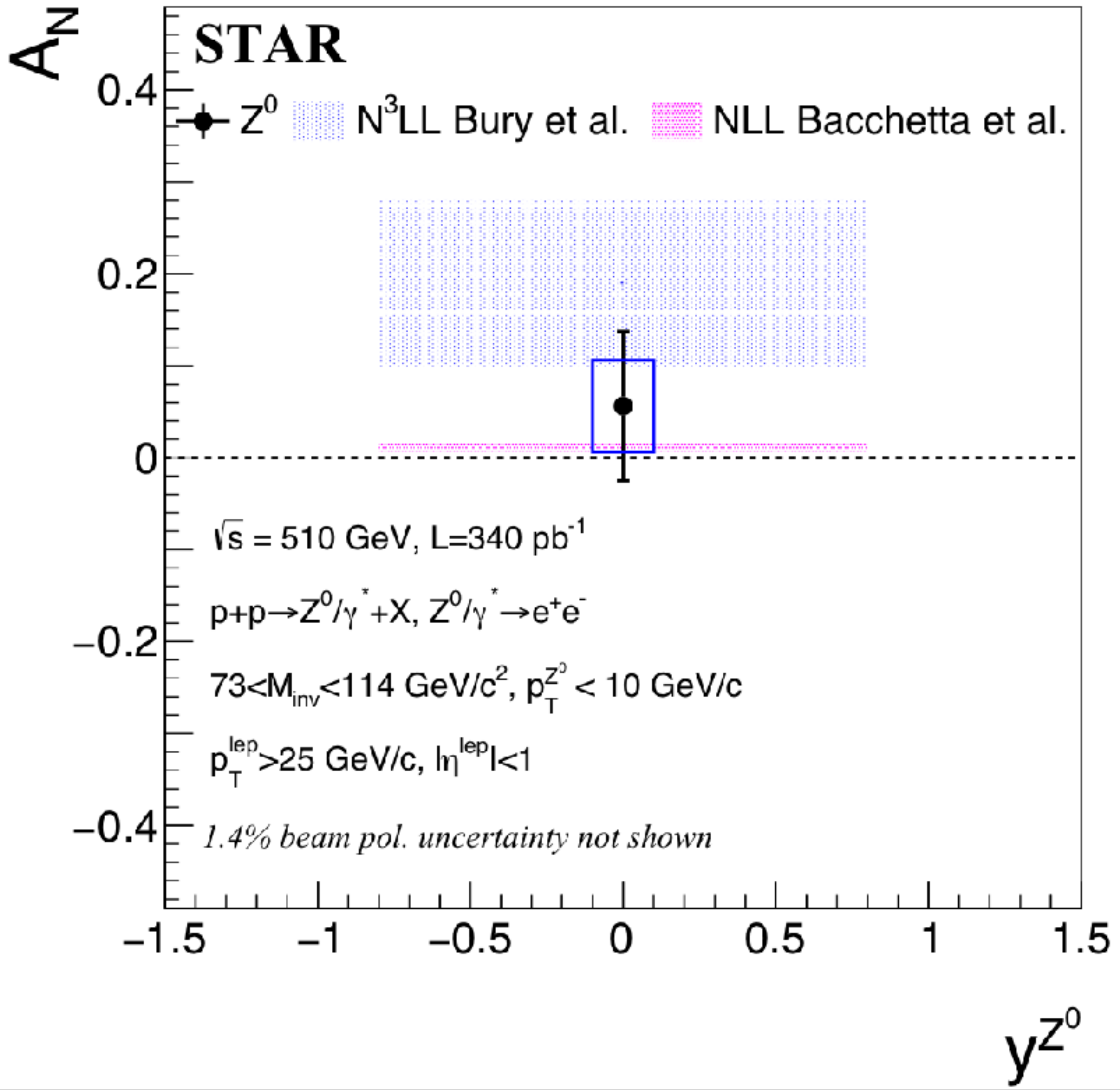


$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y}$$

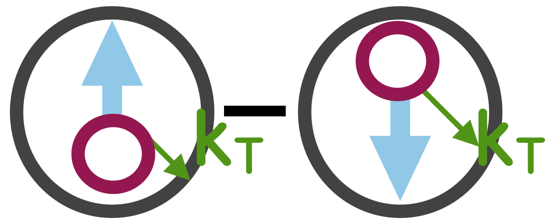
Investigation of the Sivers sign change in $p^{\uparrow}p$ collisions



arXiv:2308.15496v1



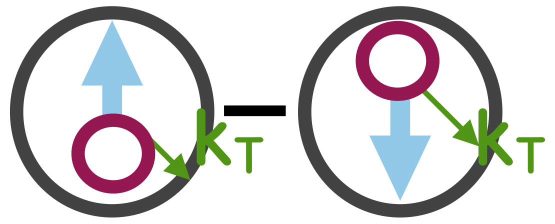
Boer-Mulders asymmetries



$$\mathscr{C}\left[h_1^{\perp,q} \times H_1^{\perp,q}\right]$$

Spin-dependence with unpolarised hadrons!

Boer-Mulders asymmetries



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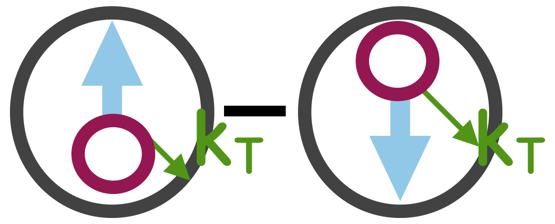
Spin-dependence with unpolarised hadrons!

Measurement in ep: $\langle\cos(2\phi_h)\rangle_{Born}(j)$ $\langle\cos(2\phi_h)\rangle_{meas}(i)$

Boer-Mulders asymmetries



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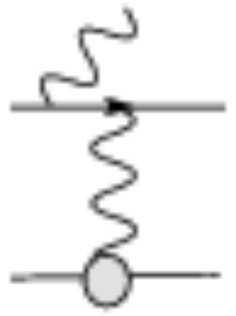
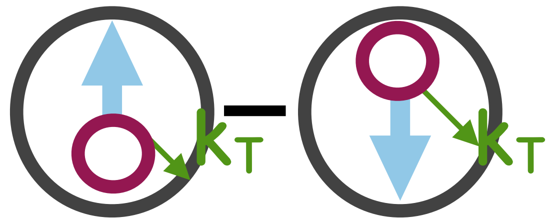
- QED radiate effects



Boer-Mulders asymmetries



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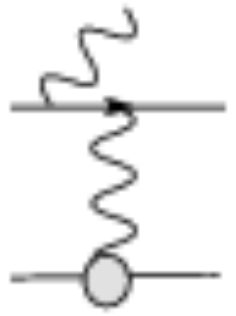
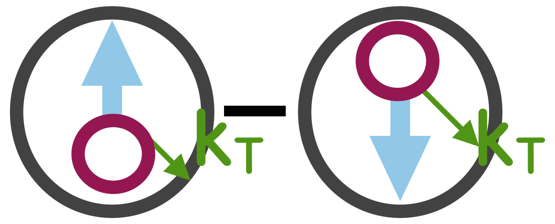
- QED radiate effects 
- limited geometric and kinematic acceptance of detector

Boer-Mulders asymmetries



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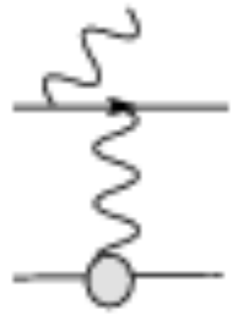

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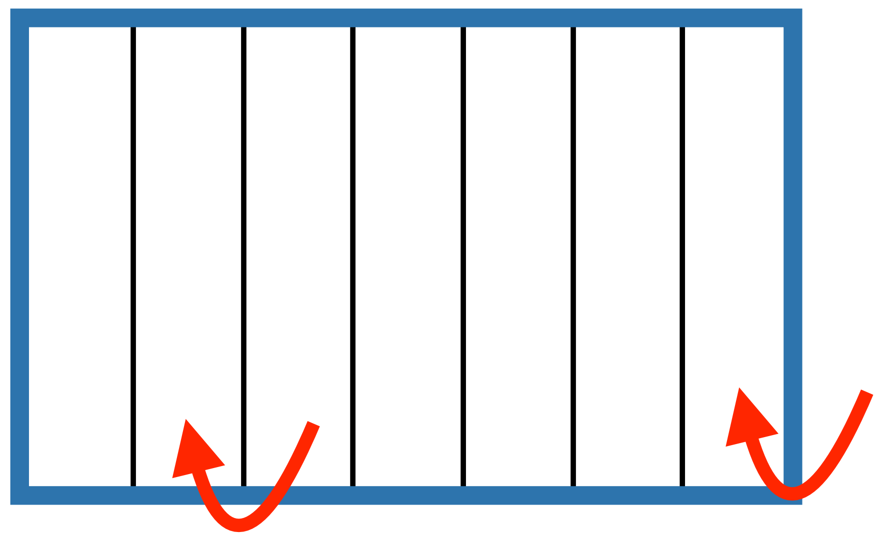
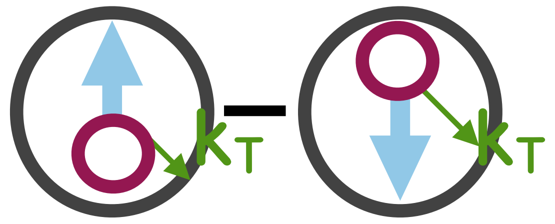
- limited detector resolution



Boer-Mulders asymmetries



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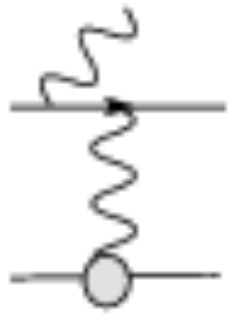

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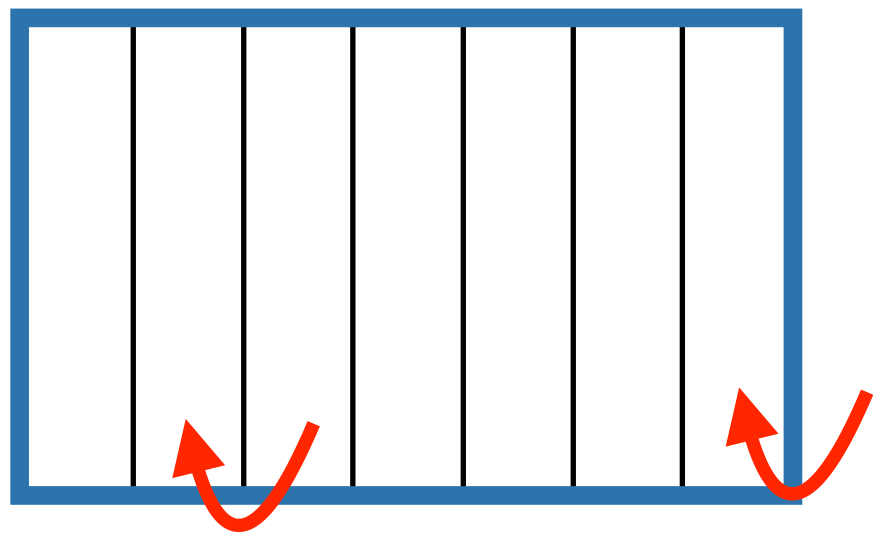
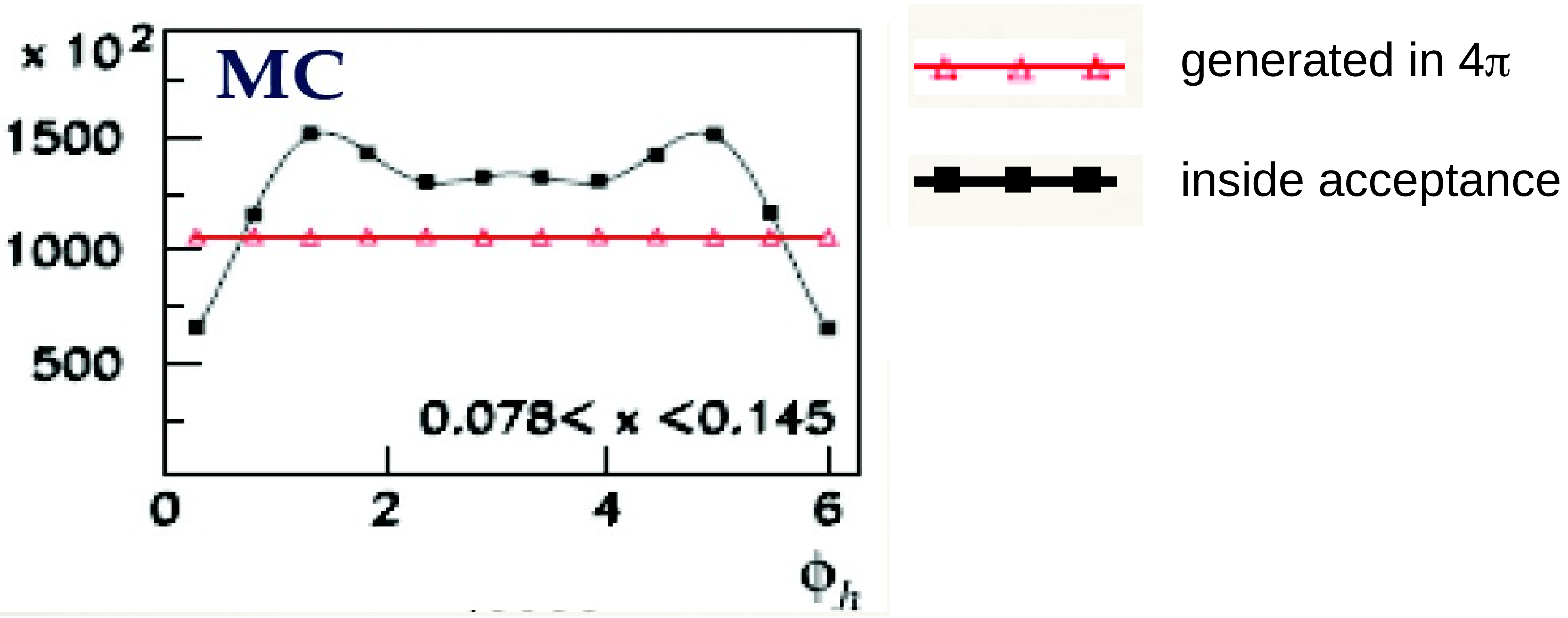
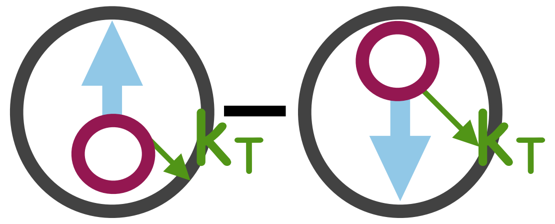




Boer-Mulders asymmetries



$$\mathscr{C}\left[h_1^{\perp,q}\times H_1^{\perp,q}\right]$$

Spin-dependence with unpolarised hadrons!

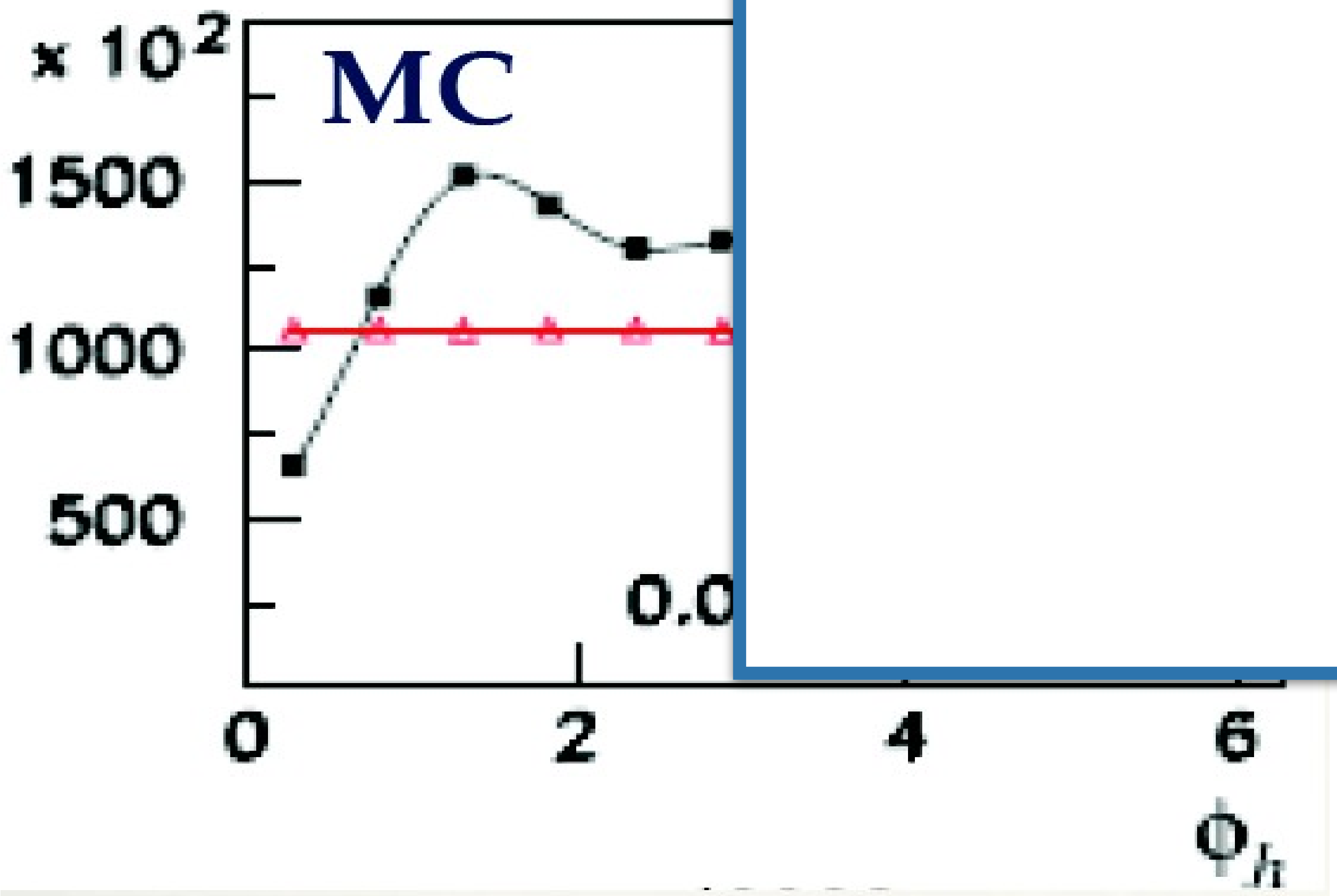
Measurement in ep:

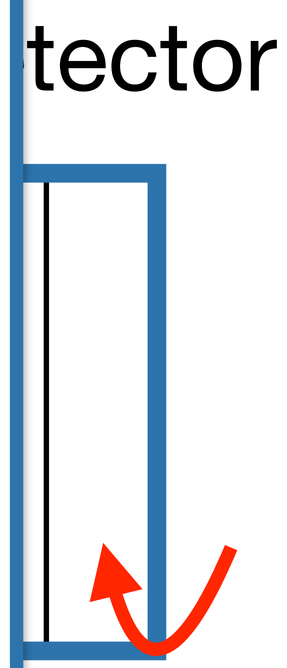
Fully differential analysis

Unfolding in 400 x 12 bins

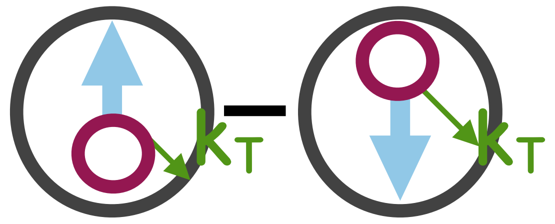
4D

BINNING 400 kinematic bins x 12 ϕ-bins							
Variable	Bin limits						#
x	0.023	0.042	0.078	0.145	0.27	1	5
y	0.3	0.45	0.6	0.7	0.85		4
z	0.2	0.3	0.45	0.6	0.75	1	5
P_{hT}	0.05	0.2	0.35	0.5	0.75		4



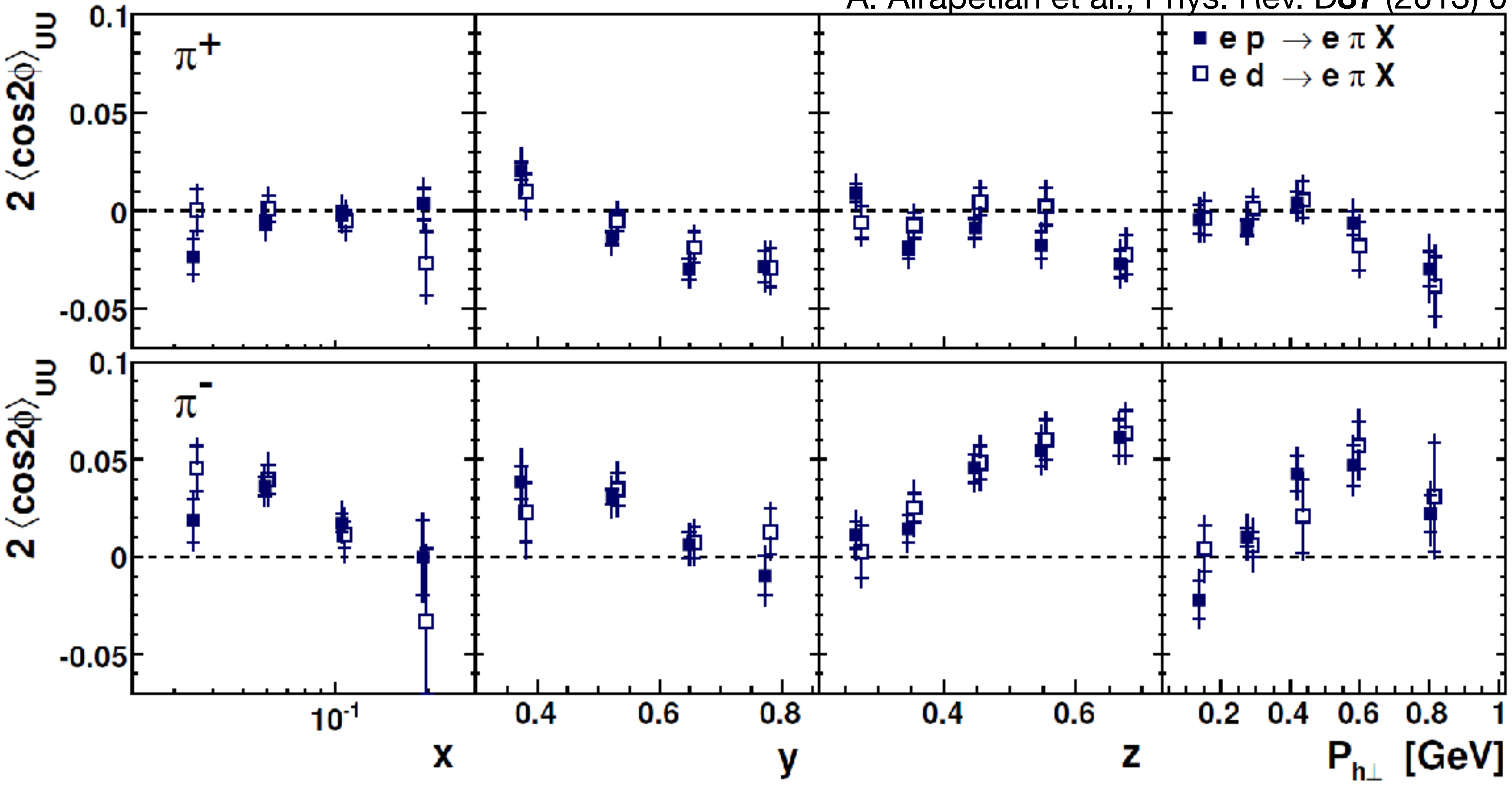


Boer-Mulders asymmetries



$\mathcal{C}\left[h_1^{\perp,q} \times H_1^{\perp,q}\right]$

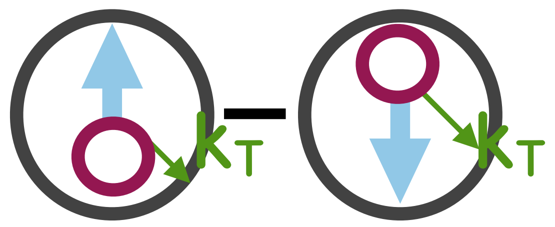
A. Airapetian et al., Phys. Rev. D**87** (2013) 012010



H–D comparison: $h_1^{\perp,u} \approx h_1^{\perp,d}$

Negative for π^+; positive for π^- $\rightarrow H_1^{\perp,fav} \approx -H_1^{\perp,disfav}$

Boer-Mulders asymmetries



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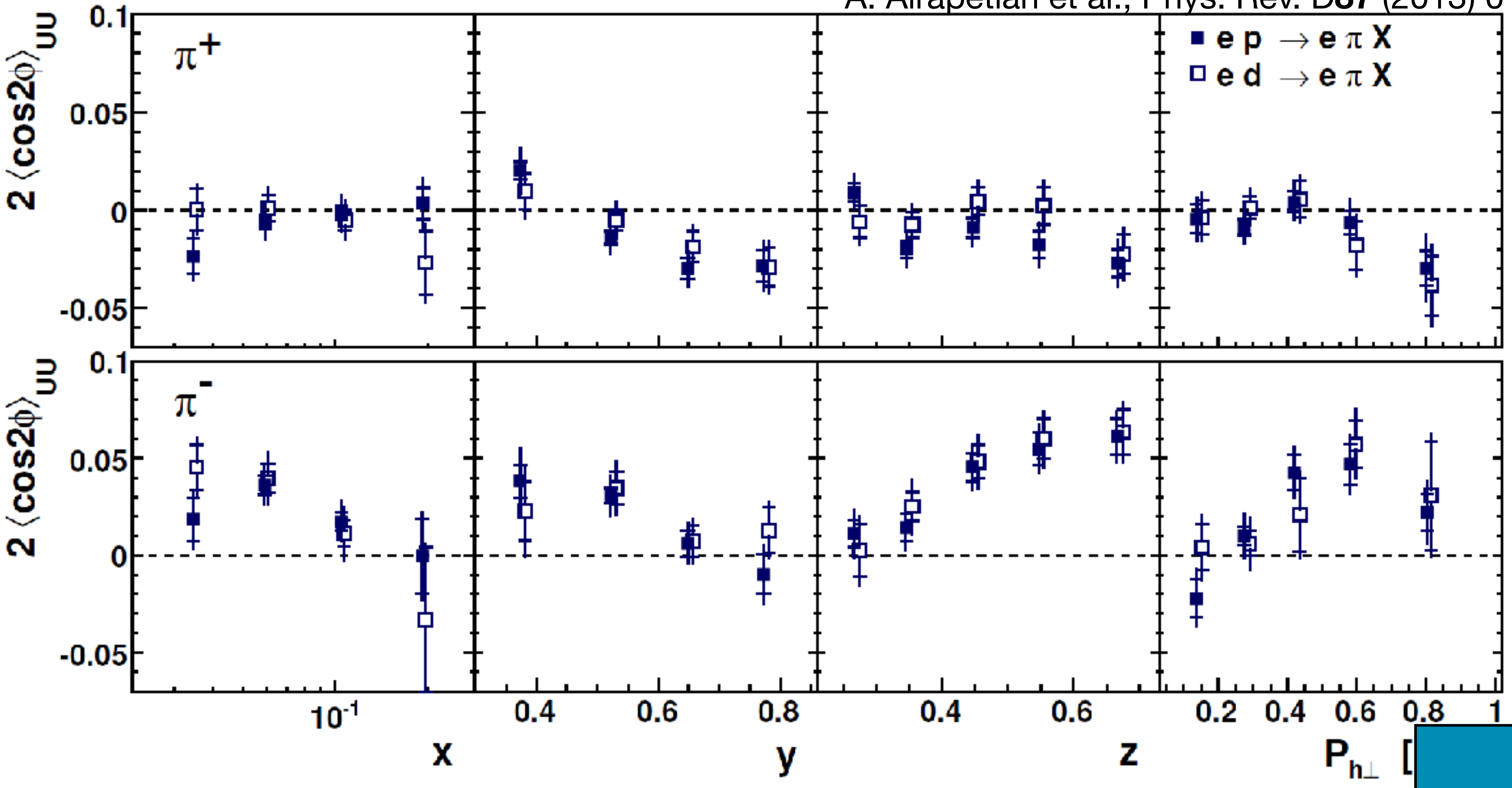
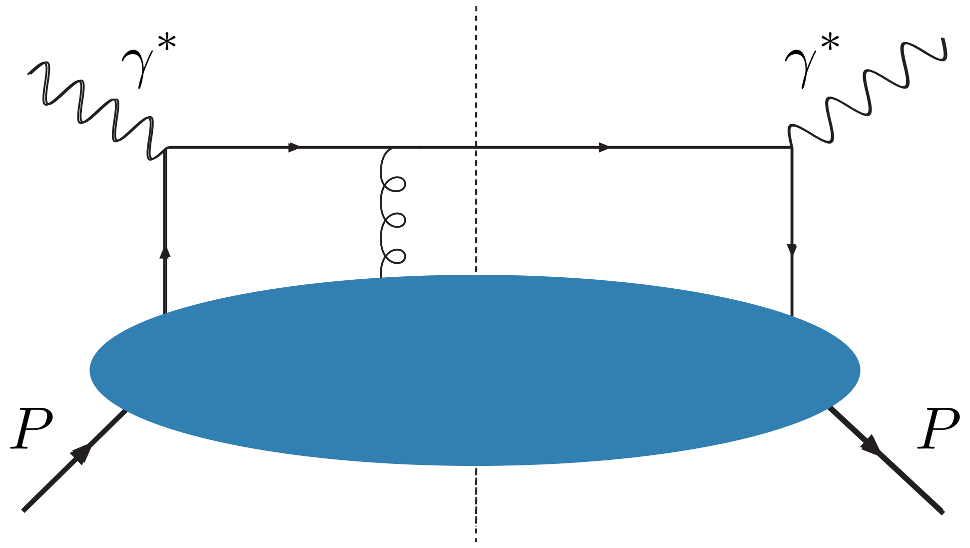


H–D comparison: $h_1^{\perp,u} \approx h_1^{\perp,d}$

Negative for π^+; positive for π^- $\rightarrow H_1^{\perp,fav} \approx -H_1^{\perp,disfav}$

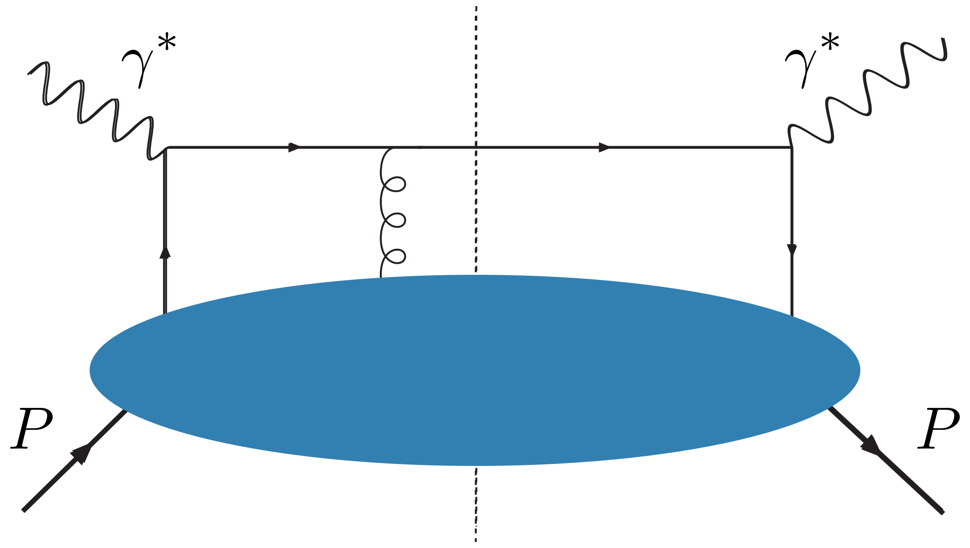
Measurement also possible in Drell Yan.

Twist-3: $\langle \sin(\phi) \rangle_{LU}^{h}$



$$\langle \sin(\phi) \rangle_{LU}^{h} \propto \mathcal{C}\left[h_1^{\perp} \times \tilde{E}, e \times H_1^{\perp}, g^{\perp} \times D_1, f_1 \times \tilde{G}^{\perp}\right]$$

Twist-3: $\langle \sin(\phi) \rangle_{LU}^{h}$



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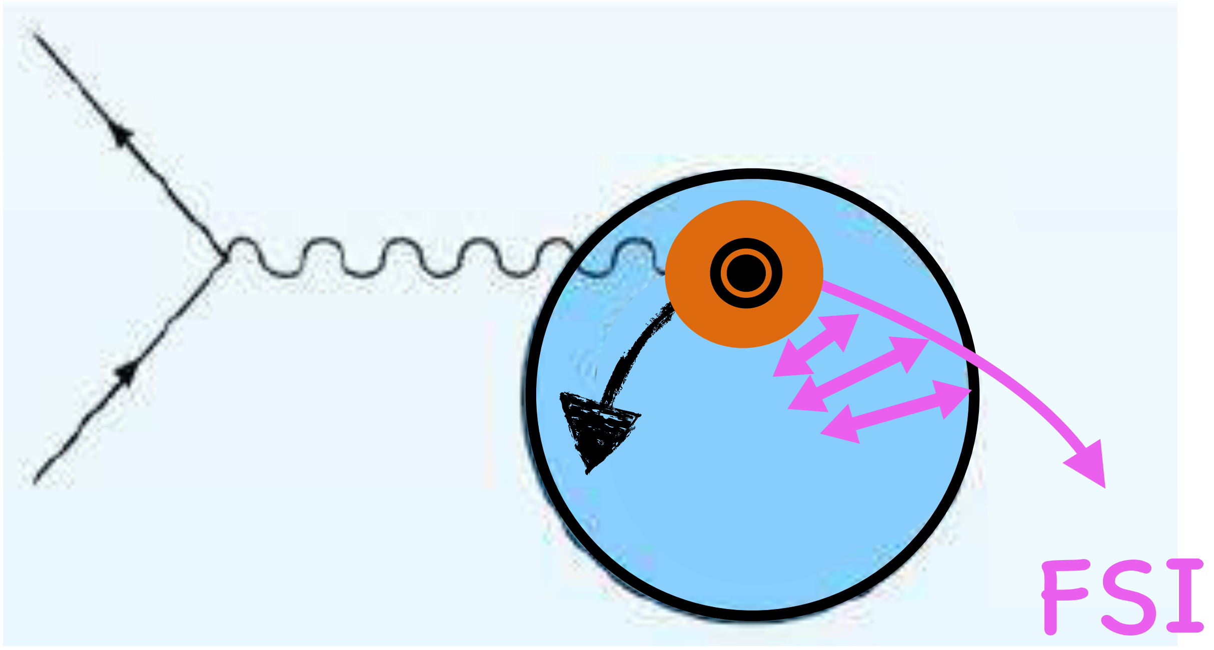
Boer-Mulders PDF

Chiral-odd T-even twist-3 FF

Boer-Mulders PDF



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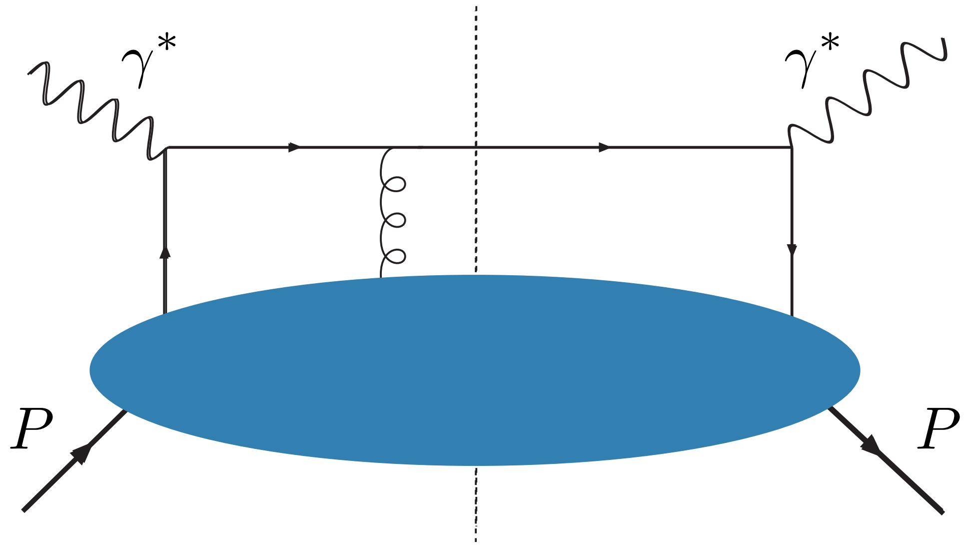

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Chiral-odd T-even twist-3 PDF

Collins FF

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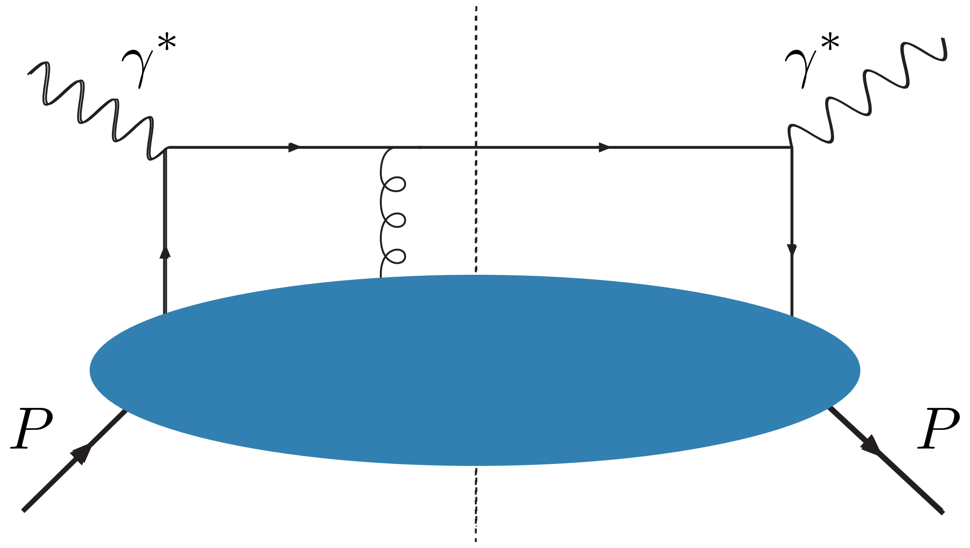

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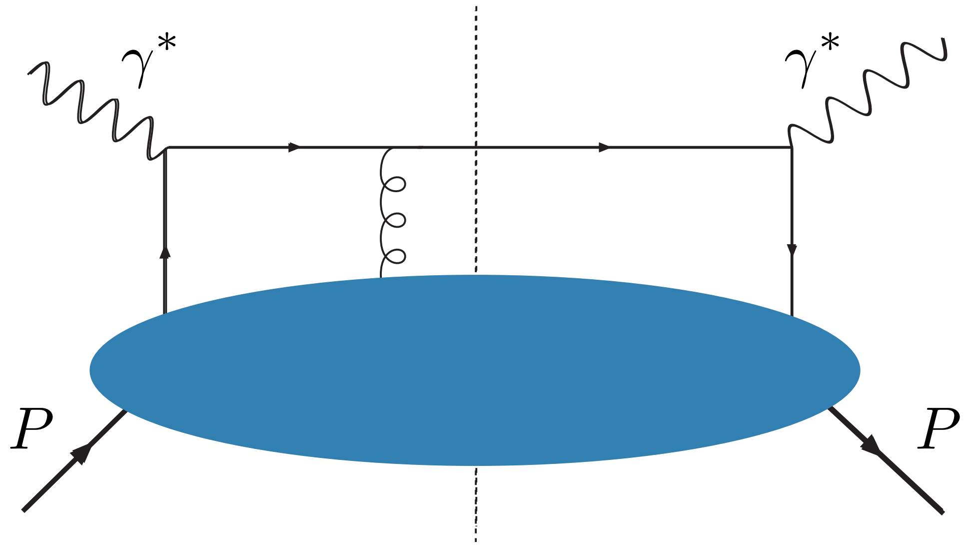
Chiral-odd T-even twist-3 PDF

Collins FF

$$e(x) = e^{\mathrm{WW}}(x) + \bar{e}(x)$$

Twist-3: $\langle \sin(\phi) \rangle_{LU}^{h}$



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Chiral-odd T-even twist-3 PDF

Collins FF

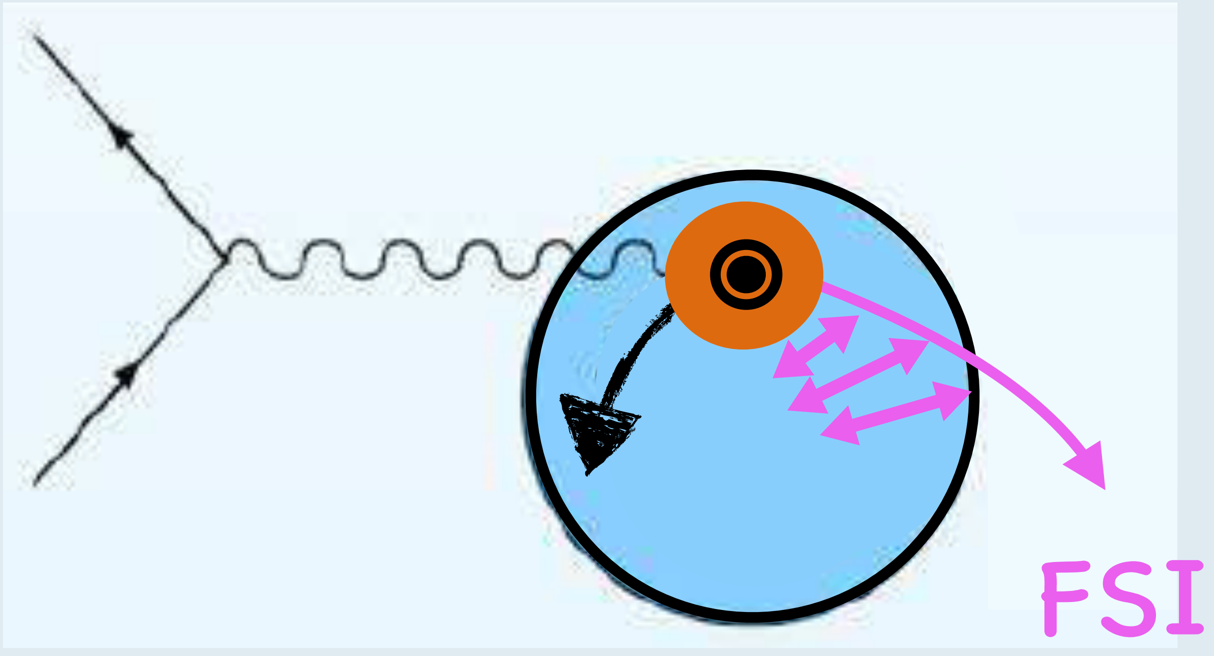
$$e(x) = e^{\mathrm{WW}}(x) + \bar{e}(x)$$

$$e_2 \equiv \int_0^1 dx\, x^2 \bar{e}(x)$$

→ force on struck quark at t=0

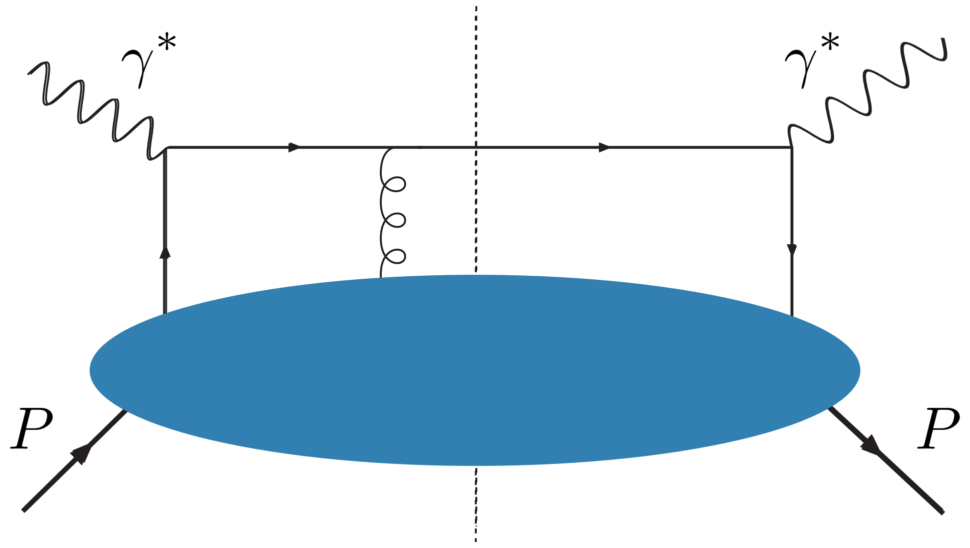
M. Burkardt, arXiv:0810.3589

Boer-Mulders PDF



FSI: t=0→∞

Twist-3: $\langle \sin(\phi) \rangle_{LU}^{h}$



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Chiral-even T-odd twist-3 PDF

spin-independent FF

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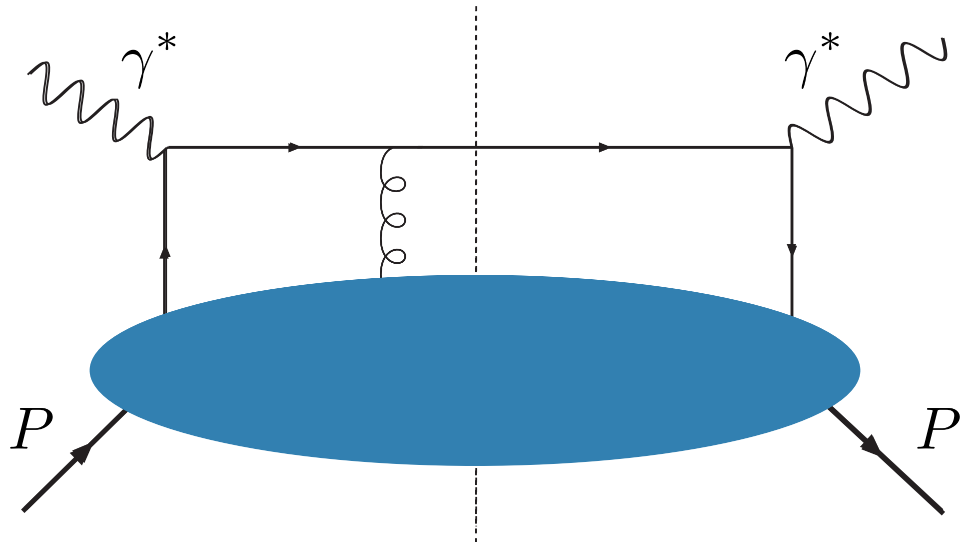

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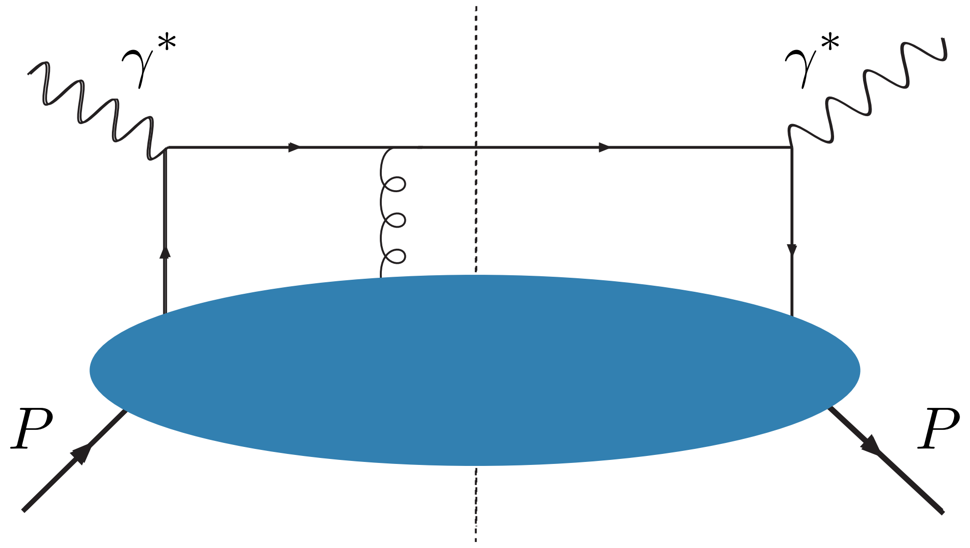
Chiral-even T-odd twist-3 PDF

spin-independent FF

Only term to survive in TMD single-jet inclusive DIS

$$e + p \to e' + \text{jet} + X$$

Twist-3: $\langle \sin(\phi) \rangle_{LU}^{h}$



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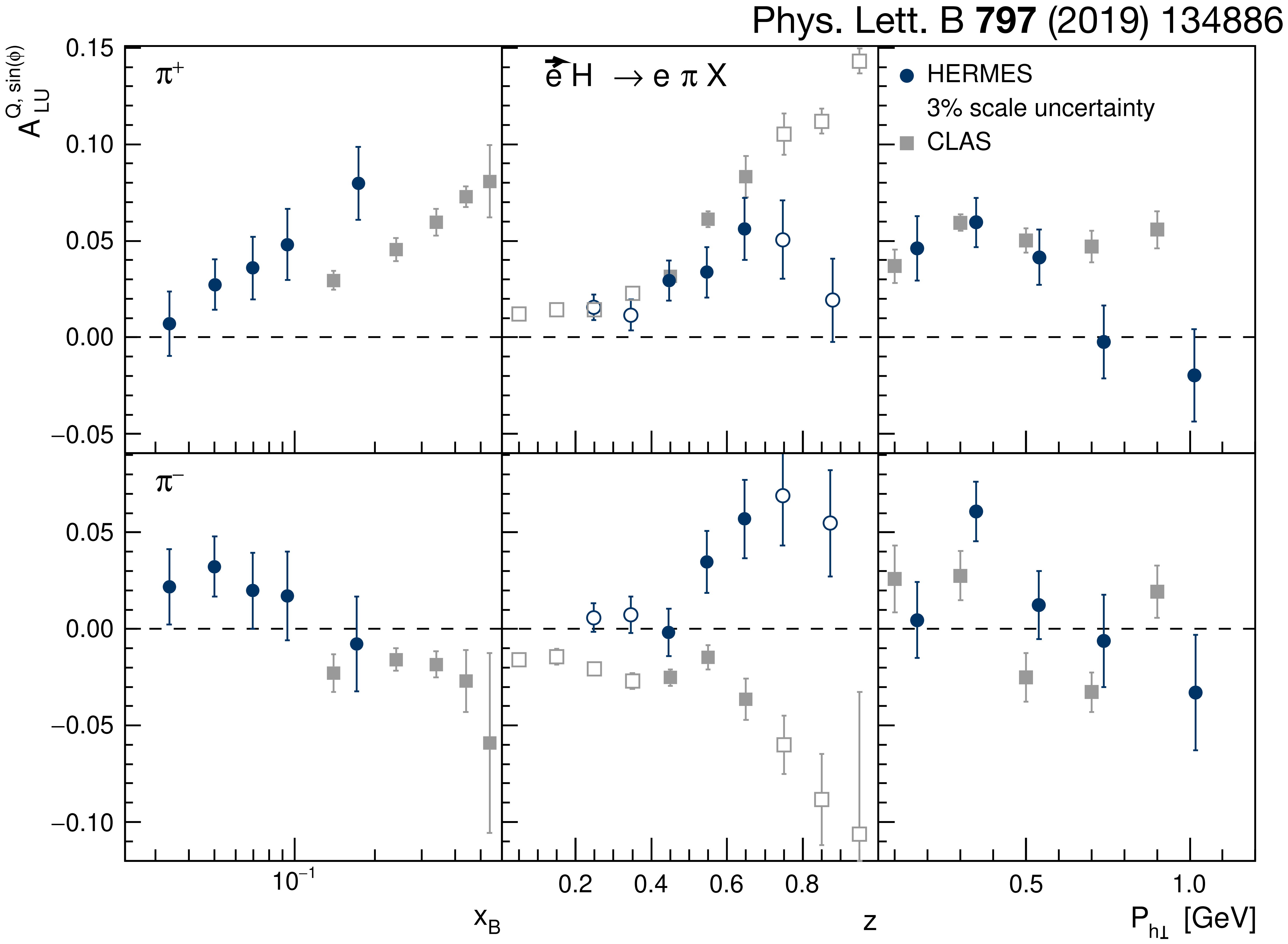
spin-independent PDF

chiral-even, T-odd twist-3 FF

Twist-3: $\langle \sin(\phi) \rangle^h_{LU}$

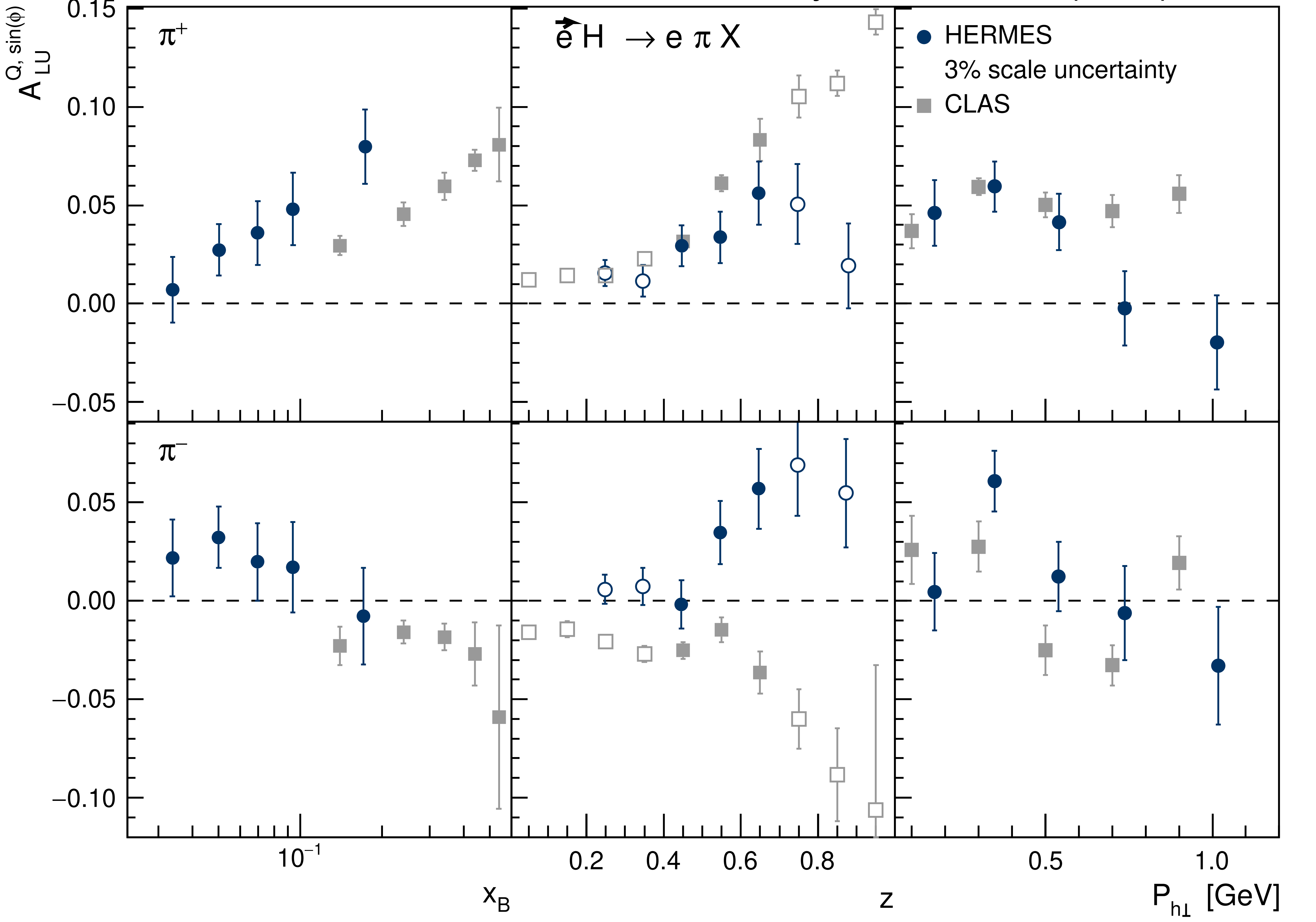


- Opposite behaviour for π- z projection due to different x range probed
- CLAS probes higher x region: more sensitive to $e \times H_1^{\perp}$?

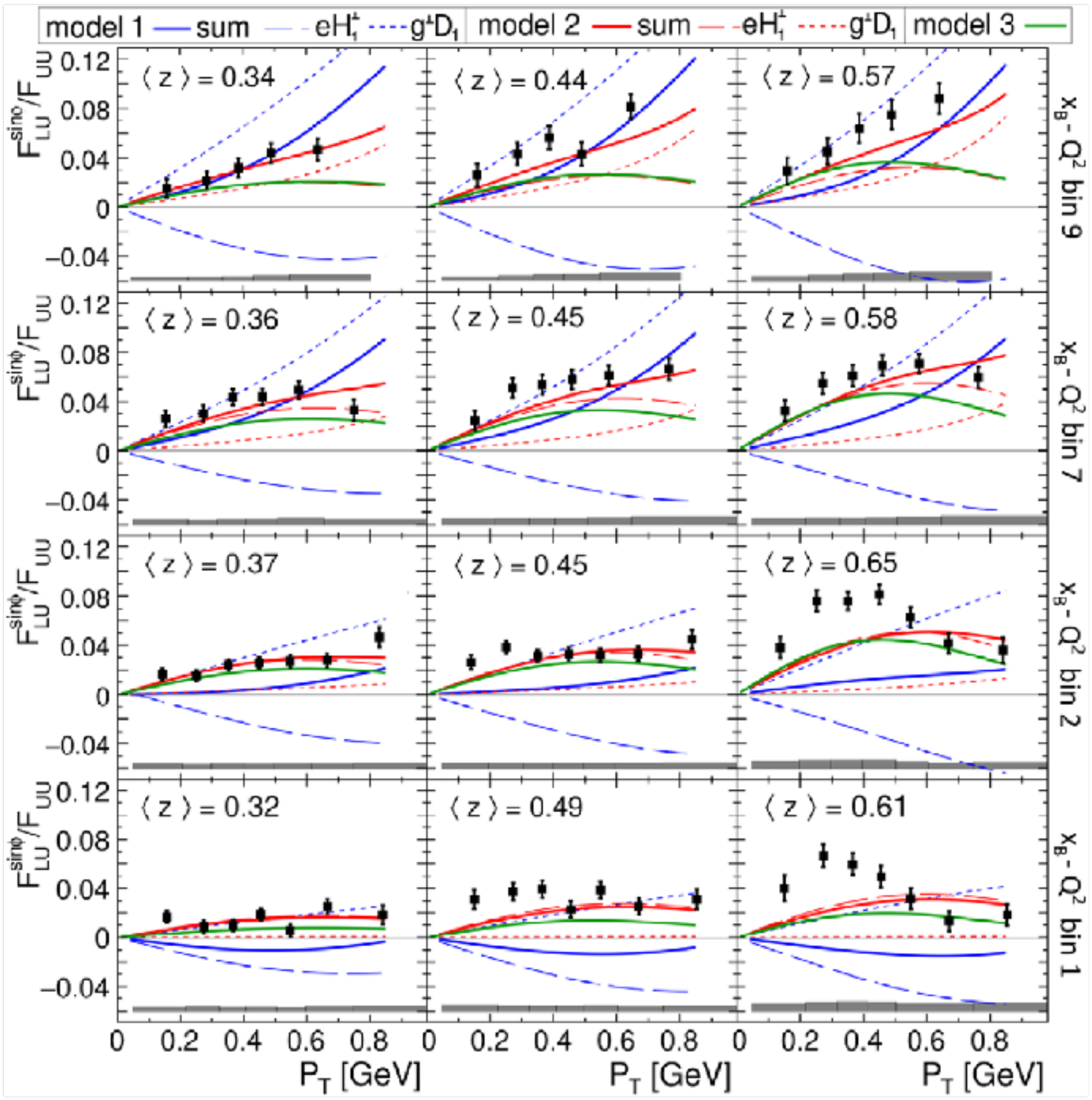
$$\langle \sin(\phi) \rangle^h_{LU} \propto \mathcal{C}\left[h_1^{\perp} \times \tilde{E}, x\, e \times H_1^{\perp}, x\, g^{\perp} \times D_1, f_1 \times \tilde{G}^{\perp}\right]$$

Twist-3: $\langle \sin(\phi) \rangle_{LU}^{h}$

Phys. Lett. B **797** (2019) 134886



CLAS12, Phys. Rev. Lett. **128** (2022) 062005



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Gluons

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

- In contrast to quark TMDs,
  gluon TMDs are almost unknown

- Accessible through production of dijets,
  high-P_T hadron pairs, quarkonia

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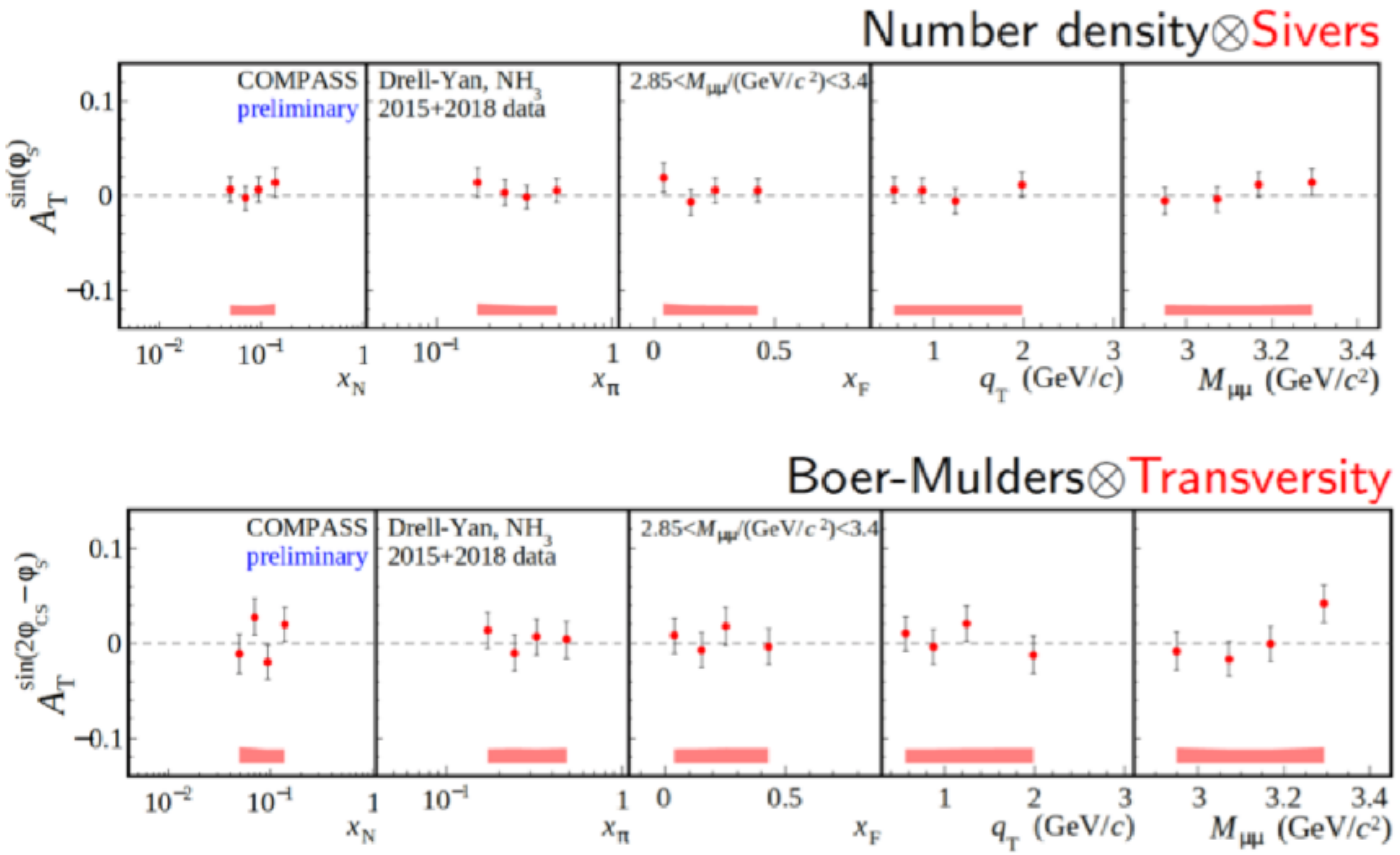
Drell-Yan with lepton pair in J/ψ mass region:
$q\bar{q}$ annihilation or gluon-gluon fusion



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T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g,\ h_{1T}^{\perp g}$

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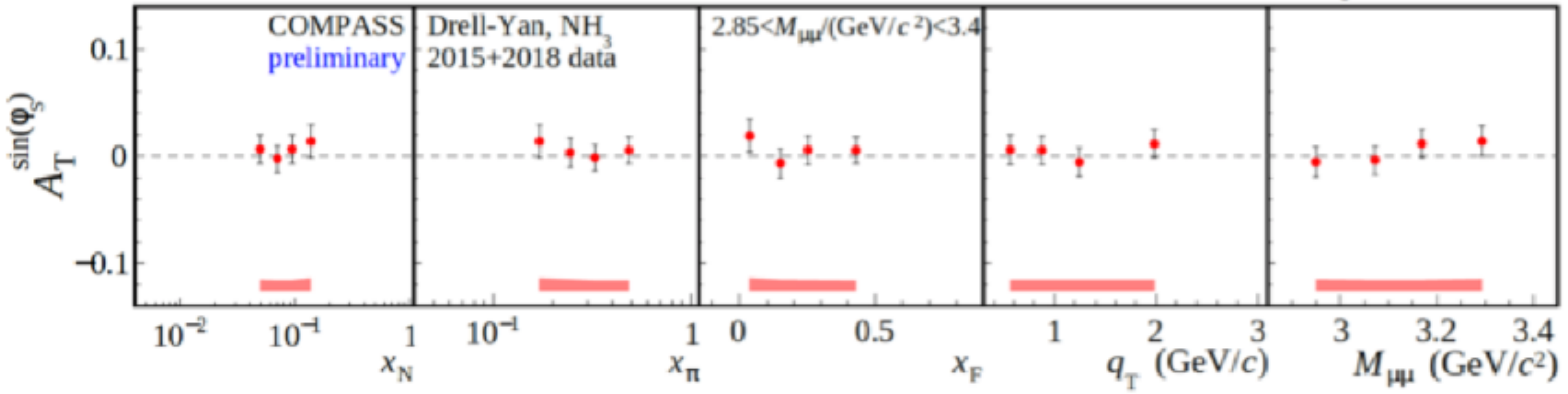
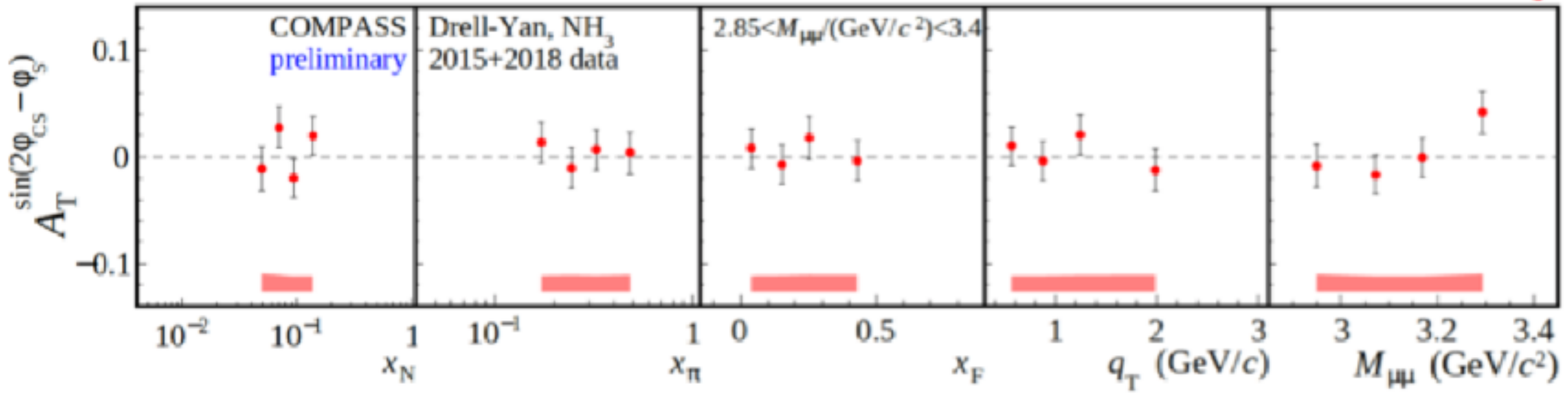
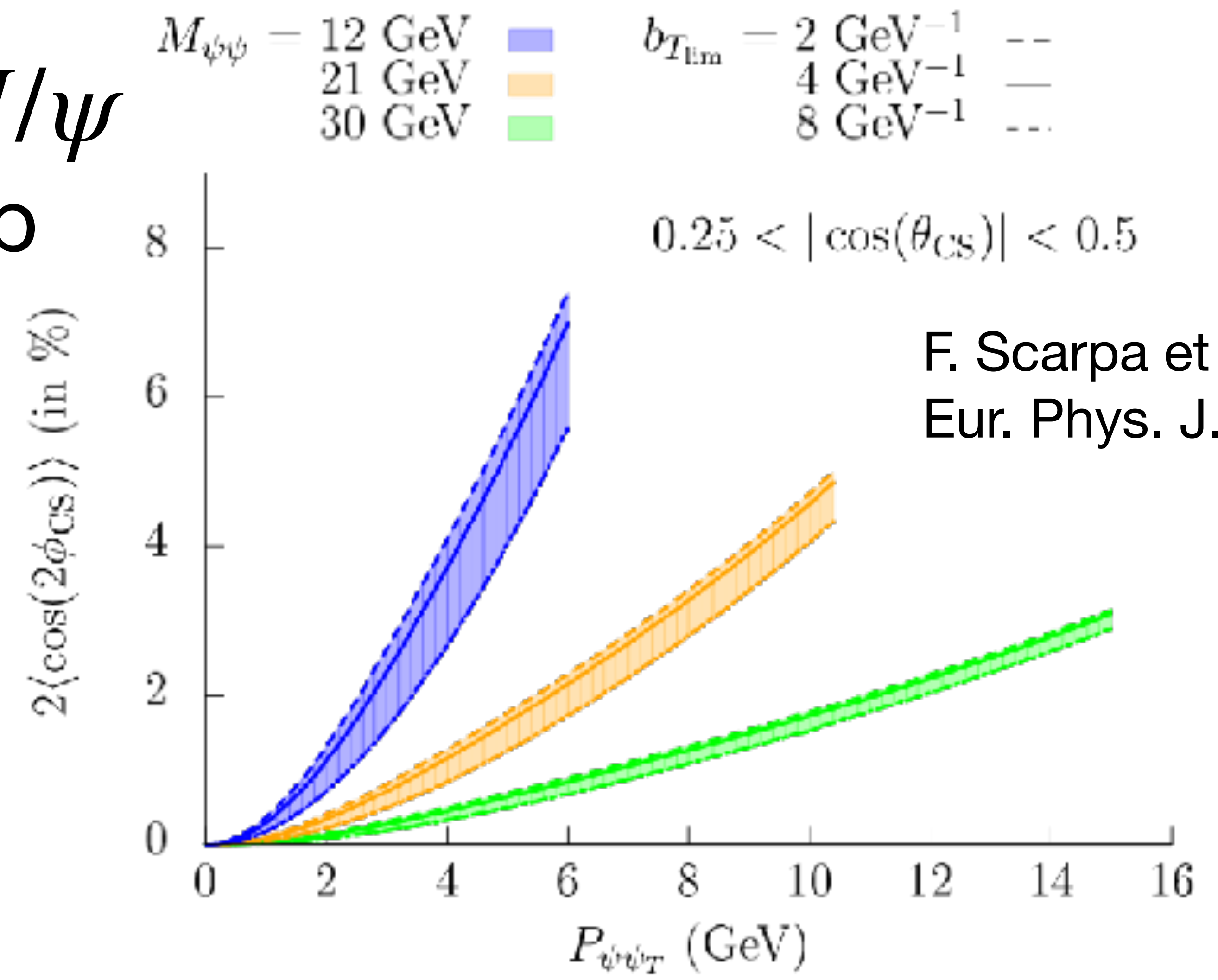
Number density⊗Sivers



Boer-Mulders⊗Transversity



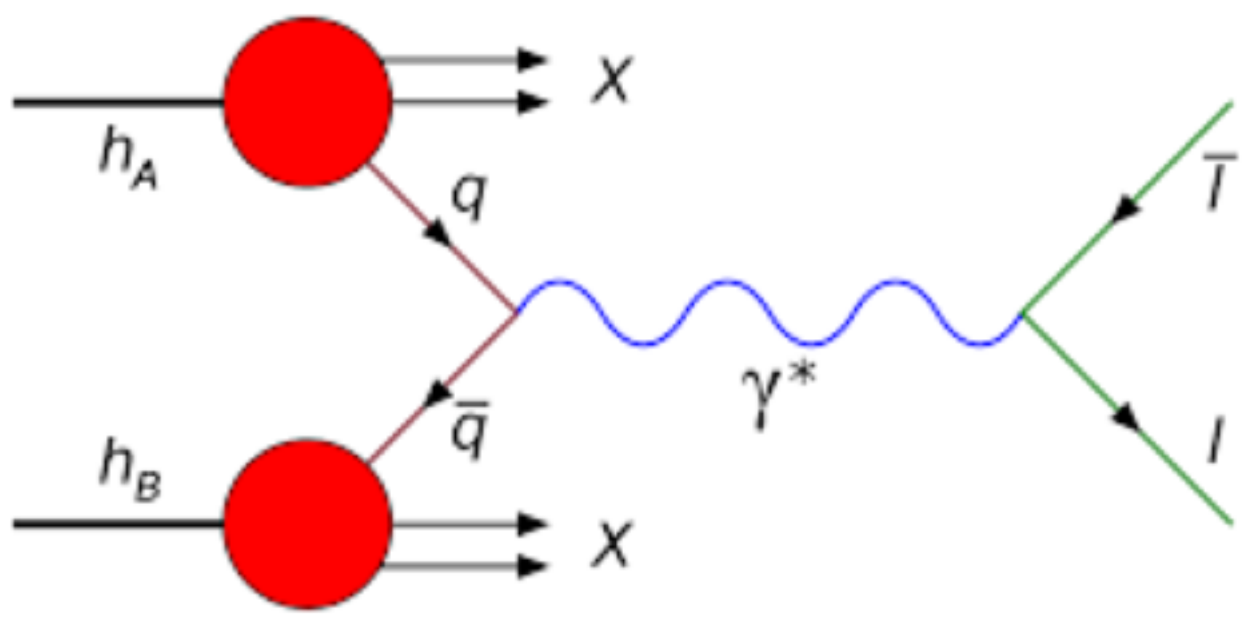
Predictions for di-J/ψ production at LHCb



F. Scarpa et al.,
Eur. Phys. J. C **80** (2020) 87

Upcoming

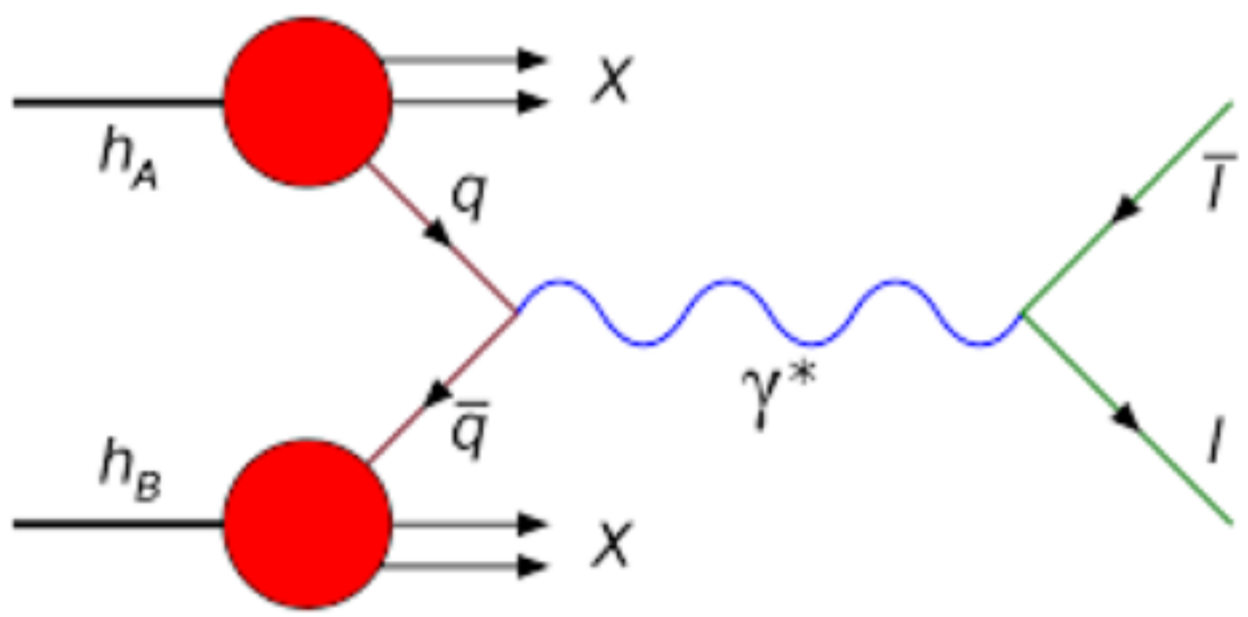
AMBER
Apparatus for Meson and Baryon Experimental Research



Meson structure

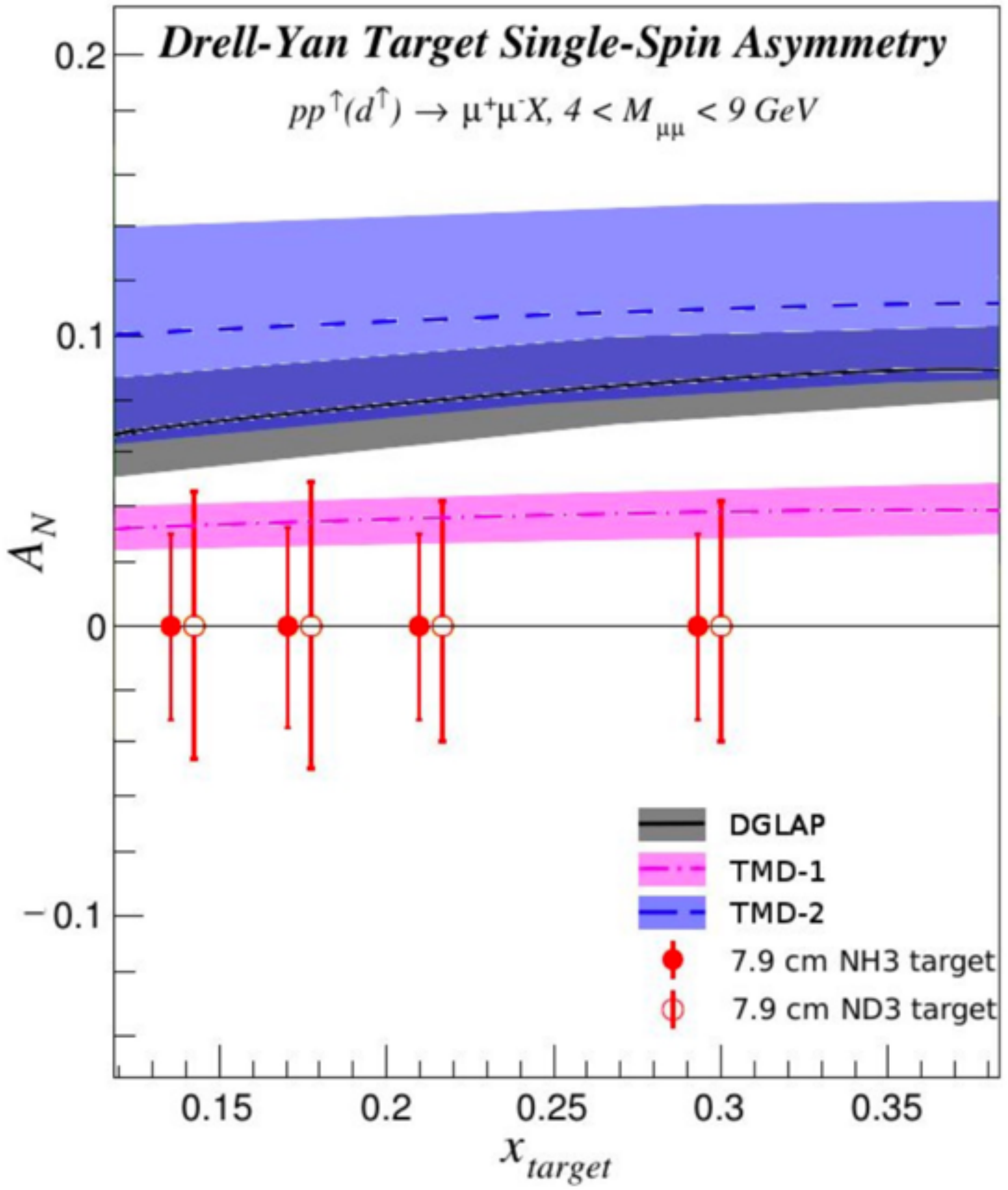
Upcoming

AMBER
Apparatus for Meson and Baryon Experimental Research



Meson structure

SpinQuest ⟶ Sivers function



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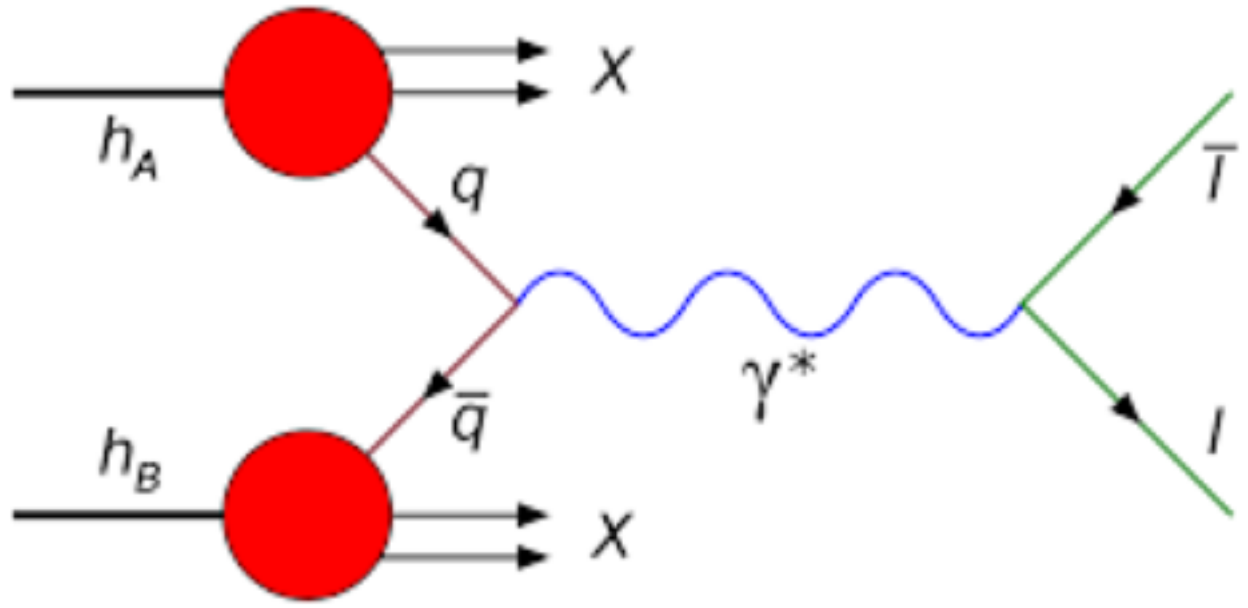

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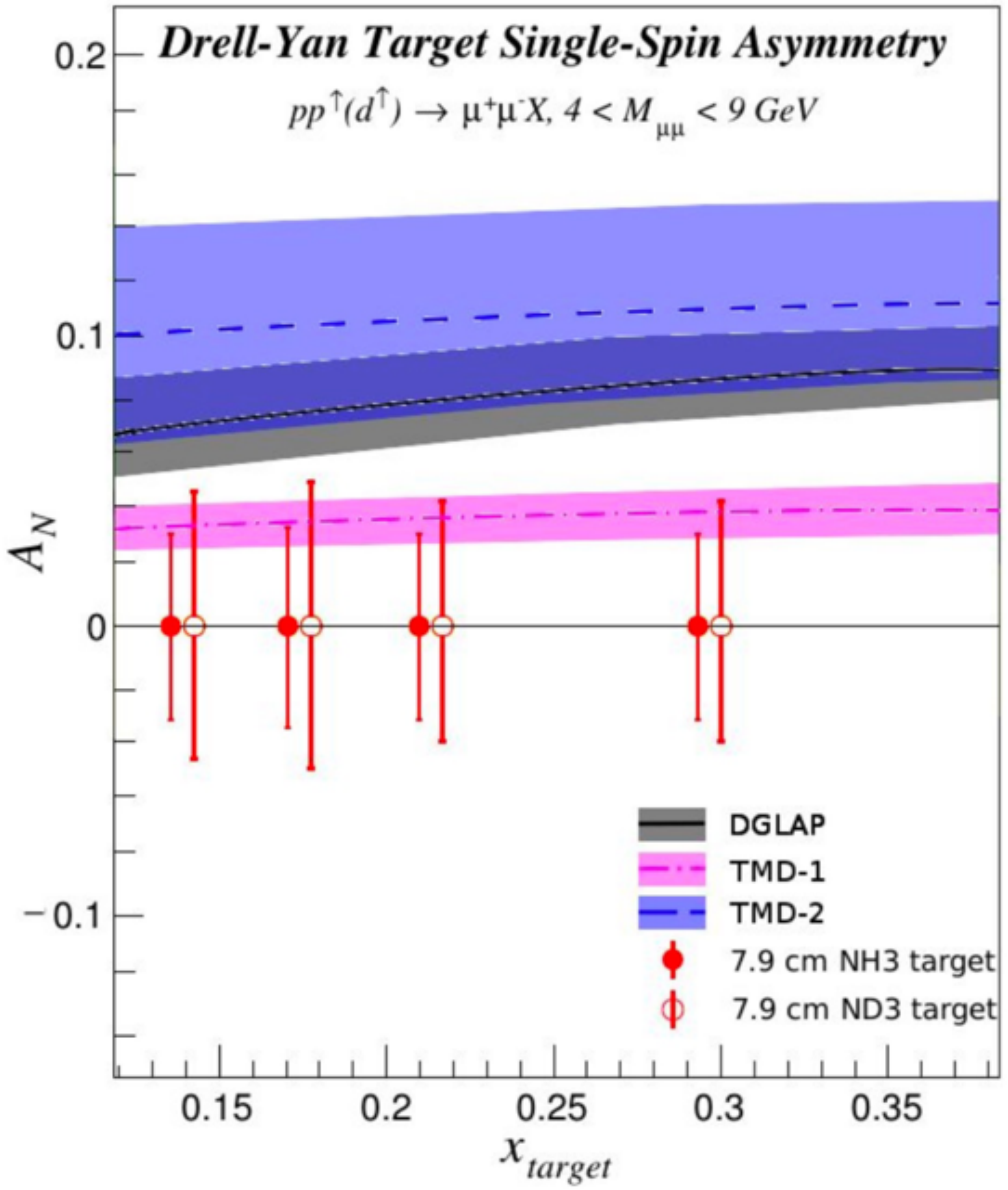
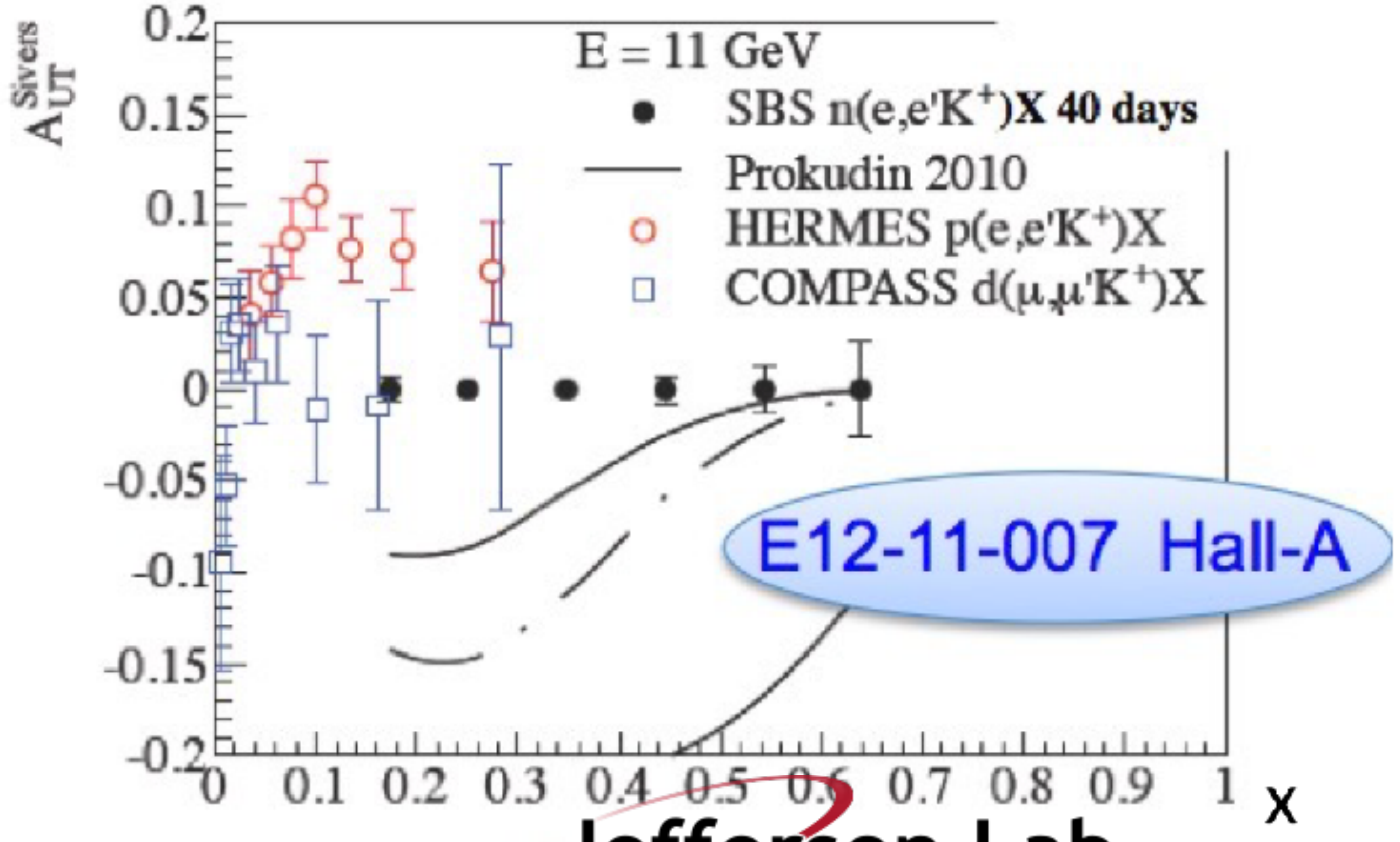
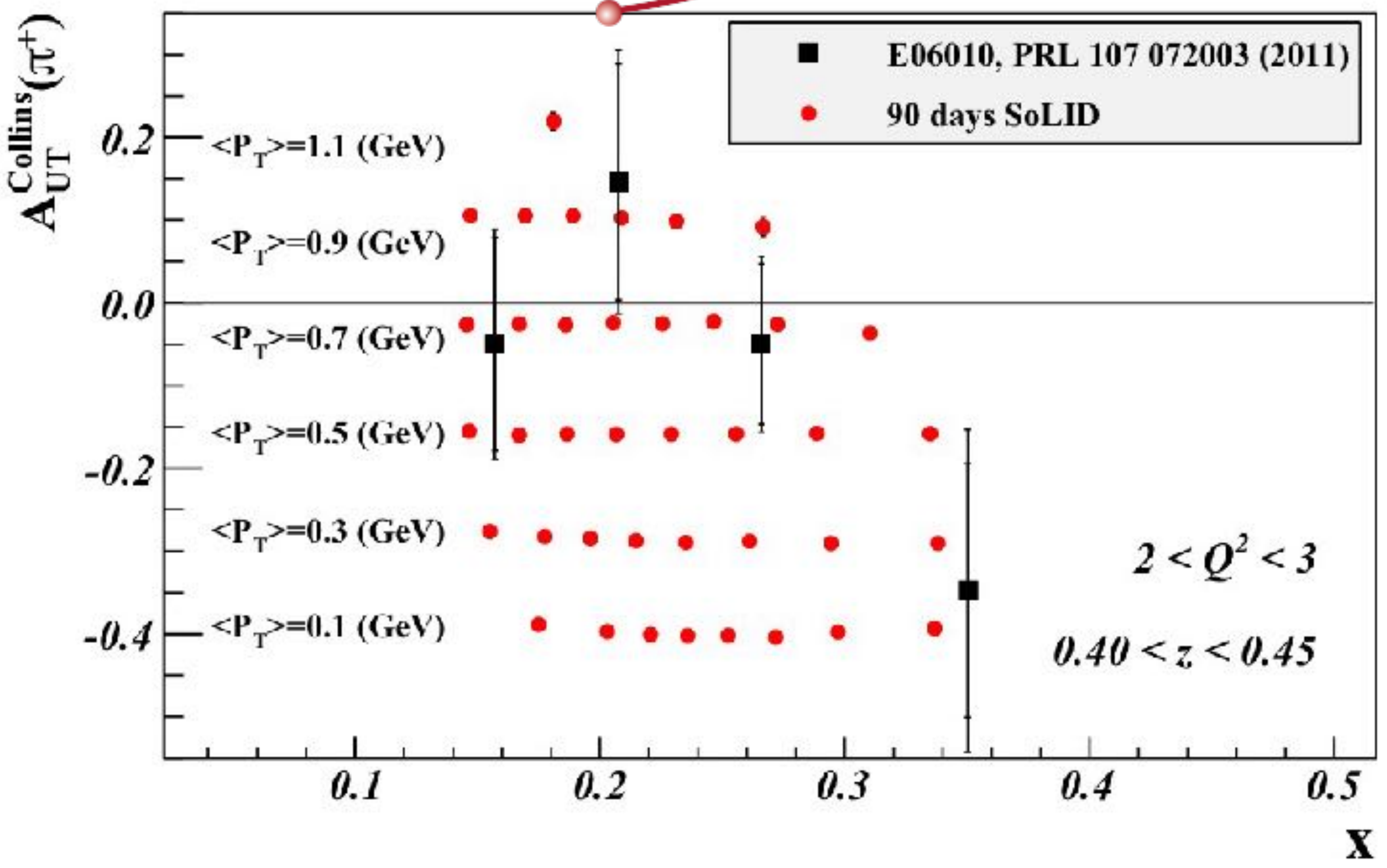
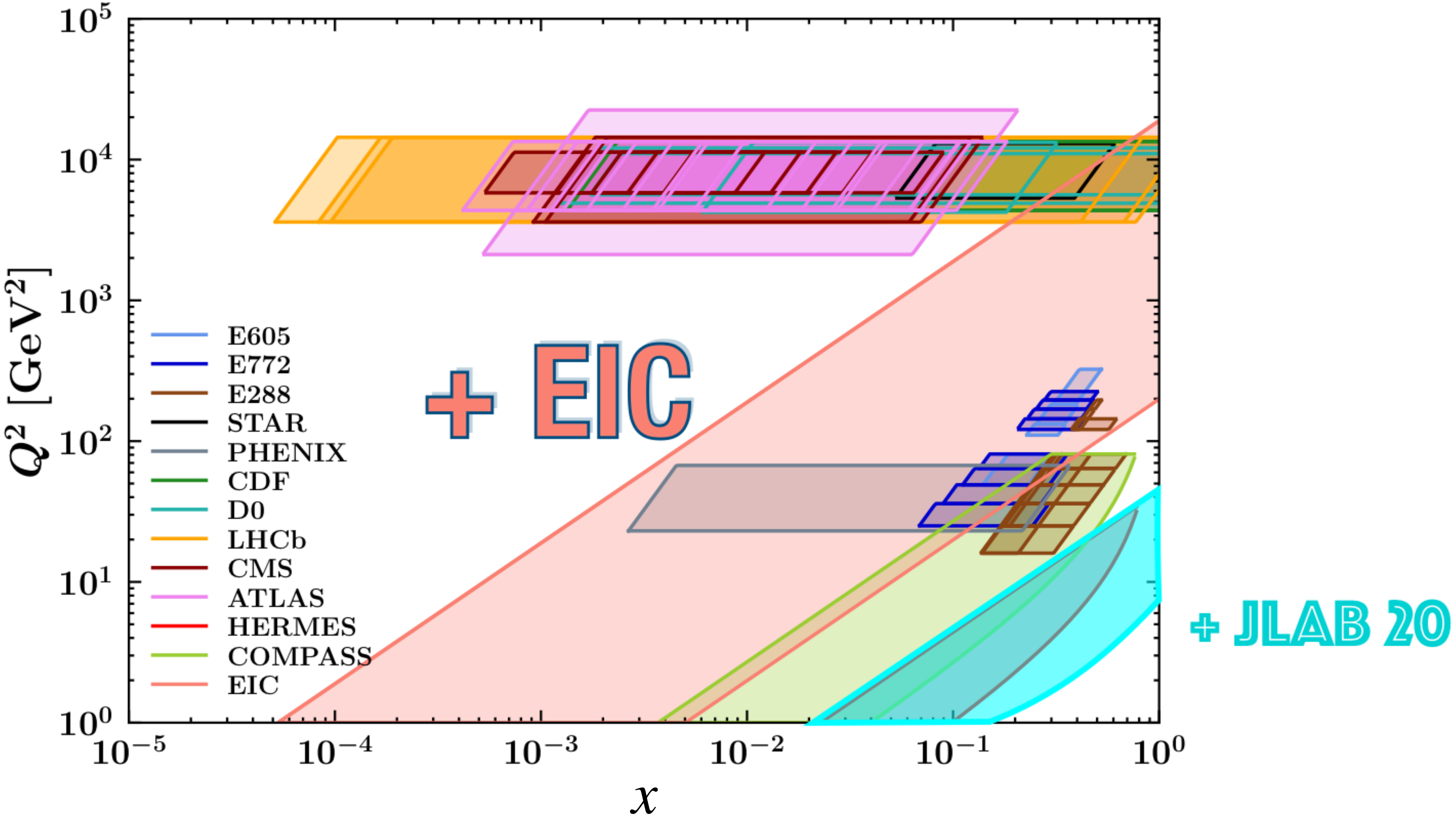




Jefferson Lab

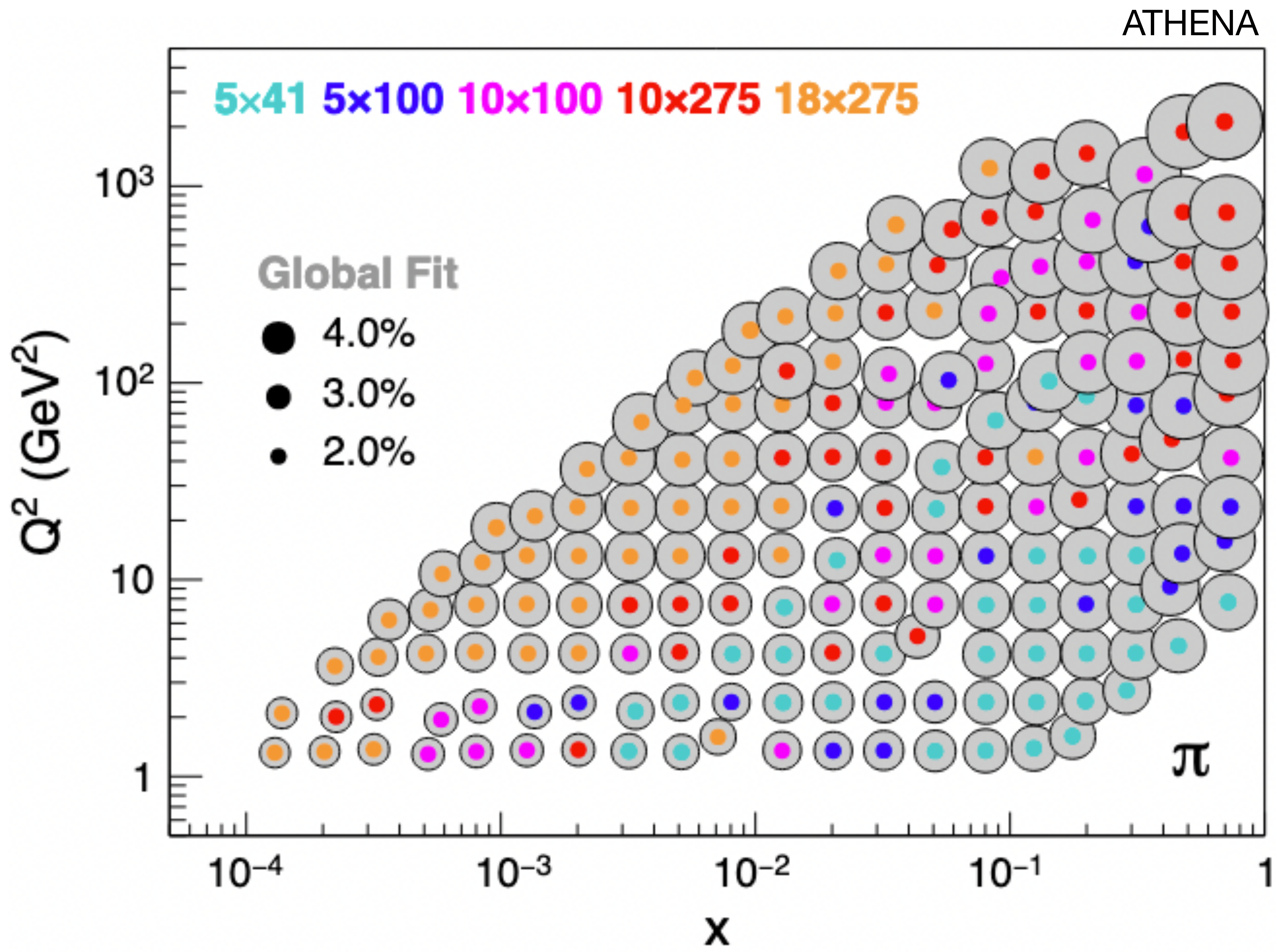


Future



Q^2 [GeV^2]
10^0
10^1
10^2
10^3
10^4
10^5
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
x
E605
E772
E288
STAR
PHENIX
CDF
D0
LHCb
CMS
ATLAS
HERMES
COMPASS
EIC
+ EIC
+ JLAB 20

Spin-independent TMD PDFs at EIC



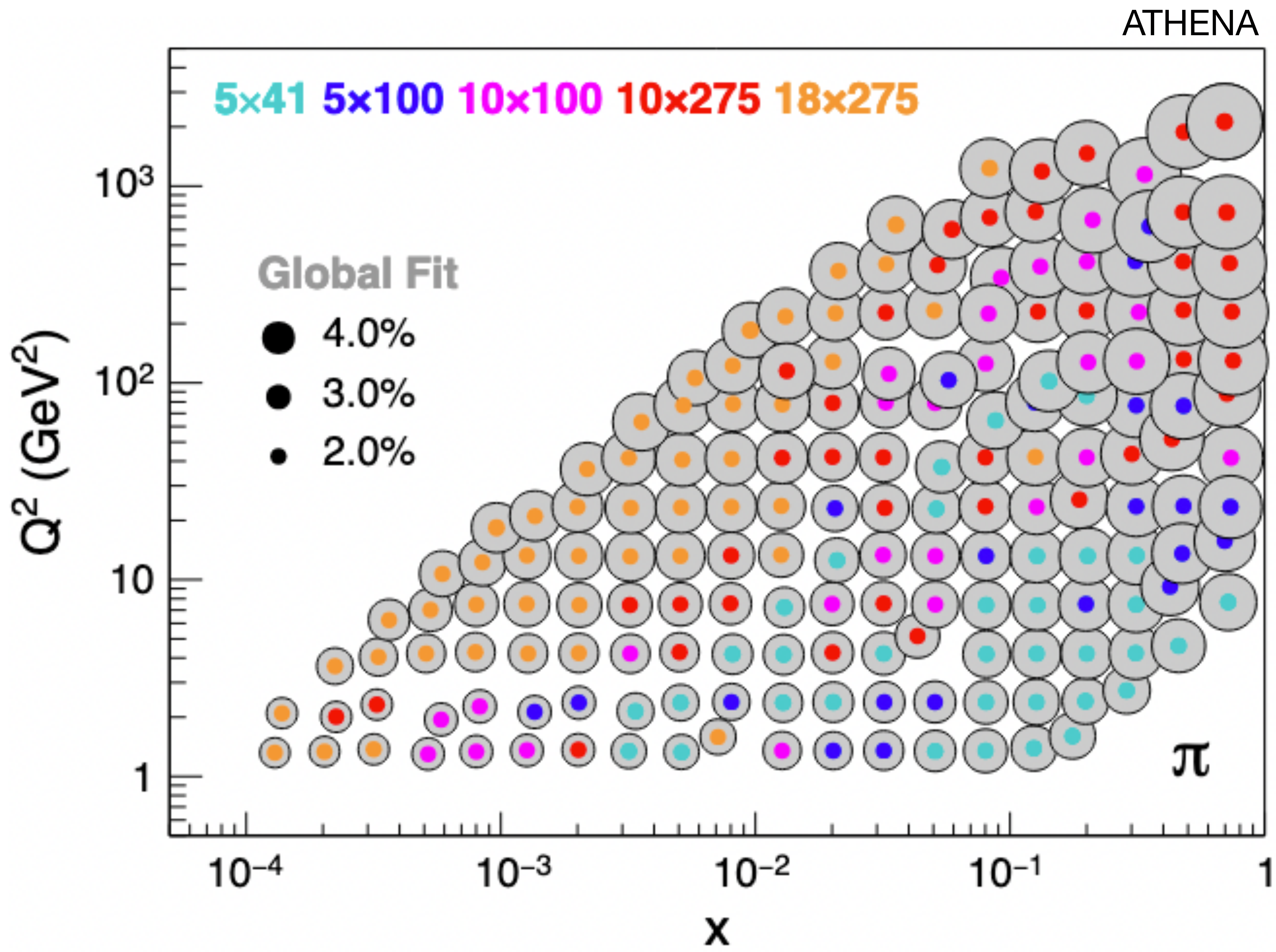
Fit:
A. Bacchetta et al.,
JHEP 06 (2017) 081,
JHEP 06 (2019) 051 (erratum)

EIC uncertainties dominated by assumed
3% point-to-point uncorrelated uncertainty
3% scale uncertainty

Theory uncertainties dominated by TMD evolution.

Spin-independent TMD PDFs at EIC

Large lever-arm in Q^2 over large x range
→ Q^2 evolution of TMD PDF



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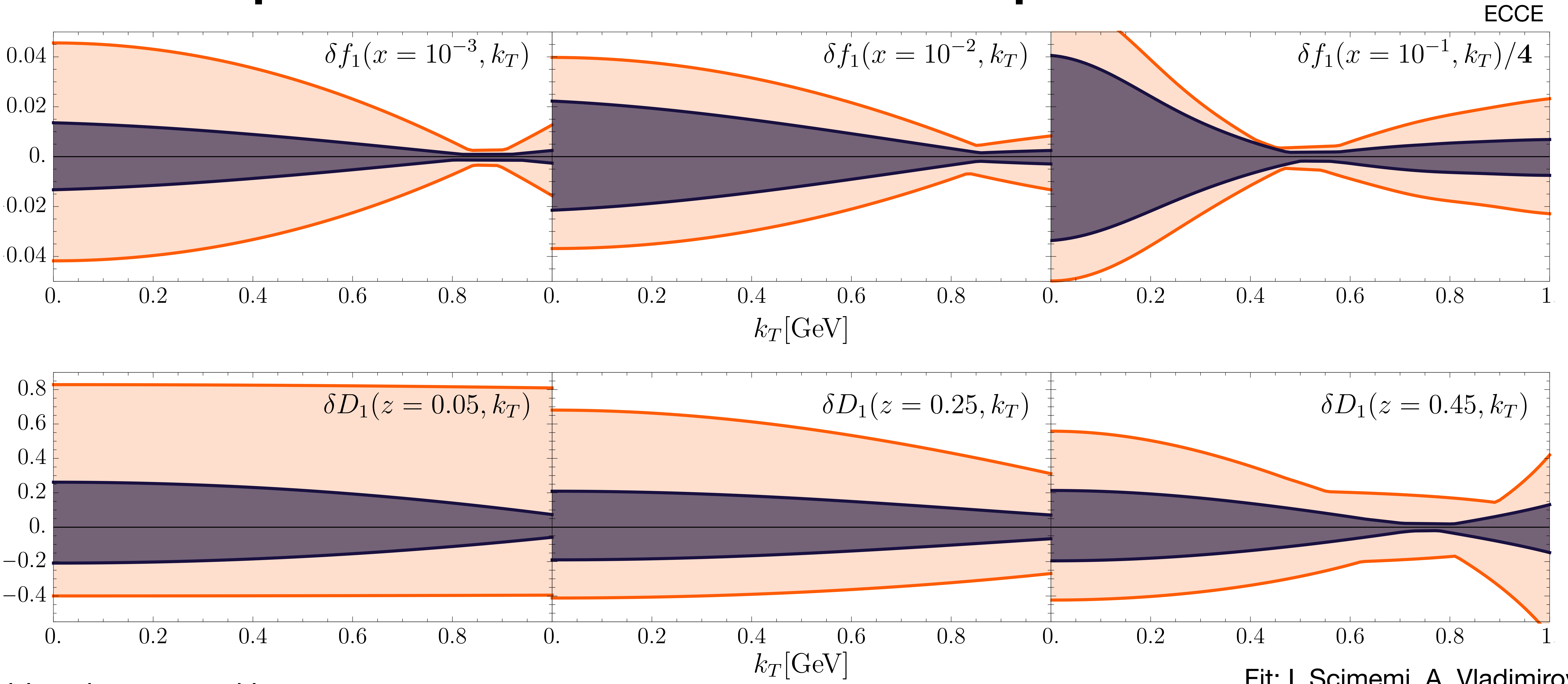
Theory uncertainties dominated by TMD evolution.

Spin-independent TMD PDF: impact of EIC



Fit: I. Scimemi, A. Vladimirov
JHEP, 06:137, 2020

DIS variables via scattered lepton

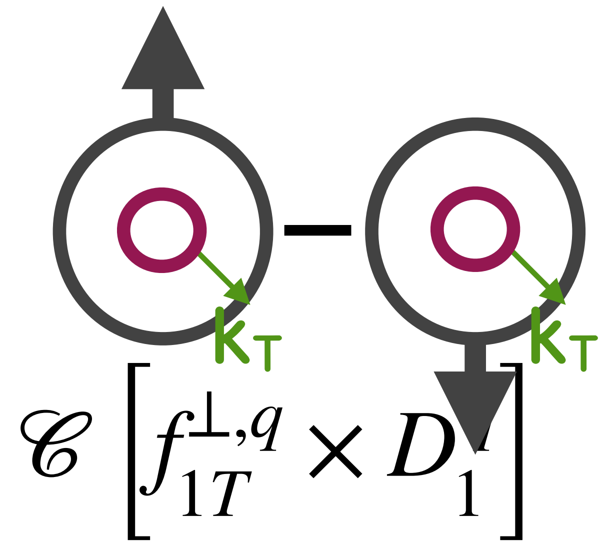
$$Q^2 > 1 \text{ GeV}^2$$
$$0.01 < y < 0.95$$
$$W^2 > 10 \text{ GeV}^2$$

5×41 GeV^2
10×100 GeV^2
18×100 GeV^2
18×275 GeV^2

systematic uncertainty= |generated - reconstructed|

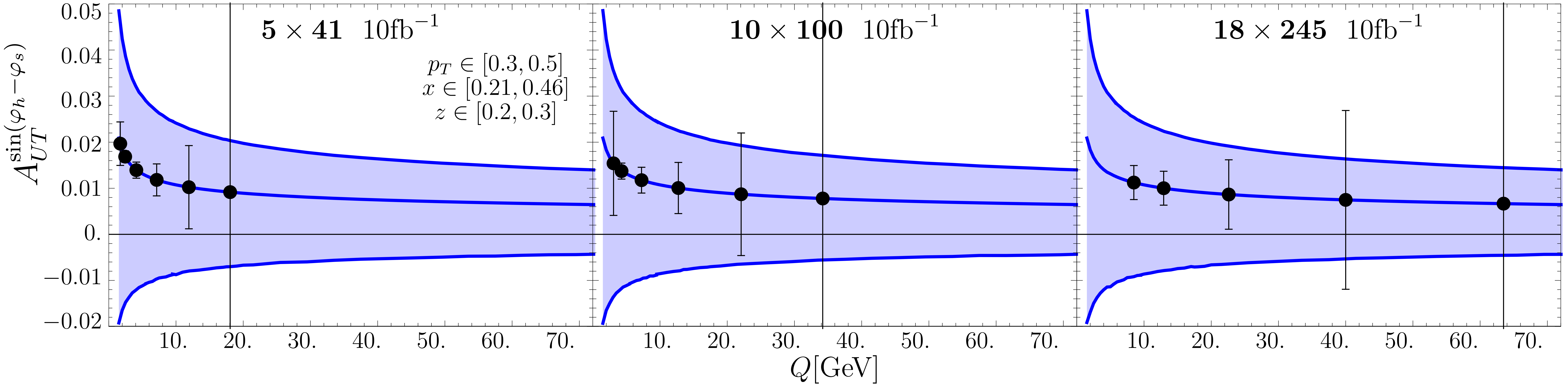
$\mathcal{L} = 10 \text{ fb}^{-1}$ for each collision energy

Sivers TMD PDF: TMD evolution



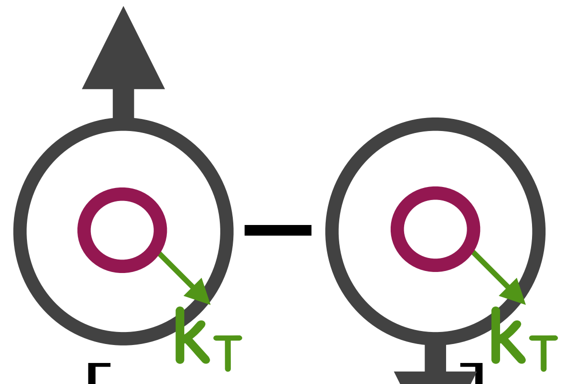
Sivers asymmetry

ECCE
Parametrisation: M. Bury et al., JHEP, 05:151, 2021



Decrease of asymmetry with increasing Q^2 → need high precision (<1%) to measure asymmetry at high Q^2

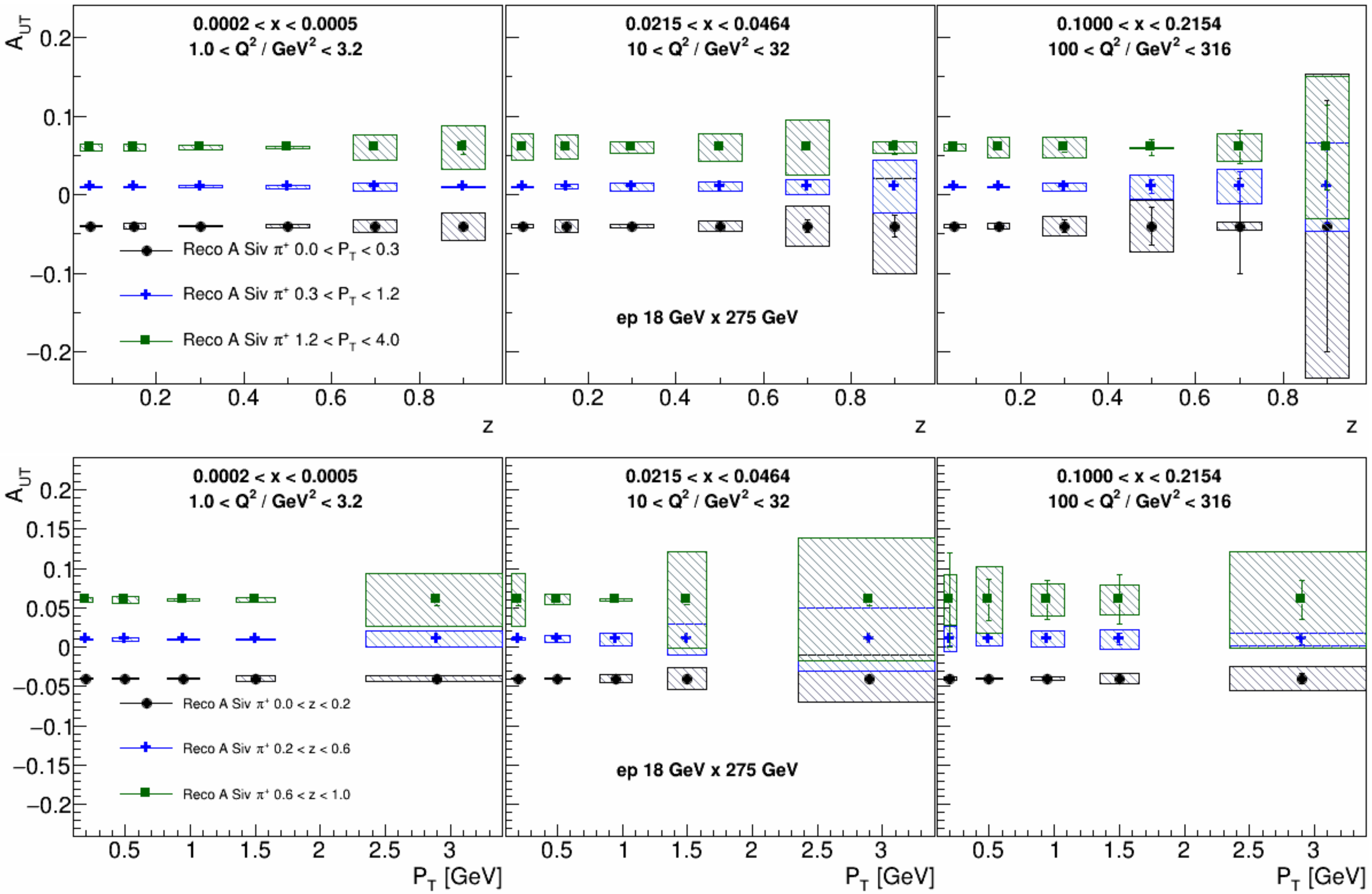
Uncertainties Sivers asymmetry at EIC



$\mathscr{C}\left[f_{1T}^{\perp,q} \times D_1\right]$

Sivers asymmetry

ECCE



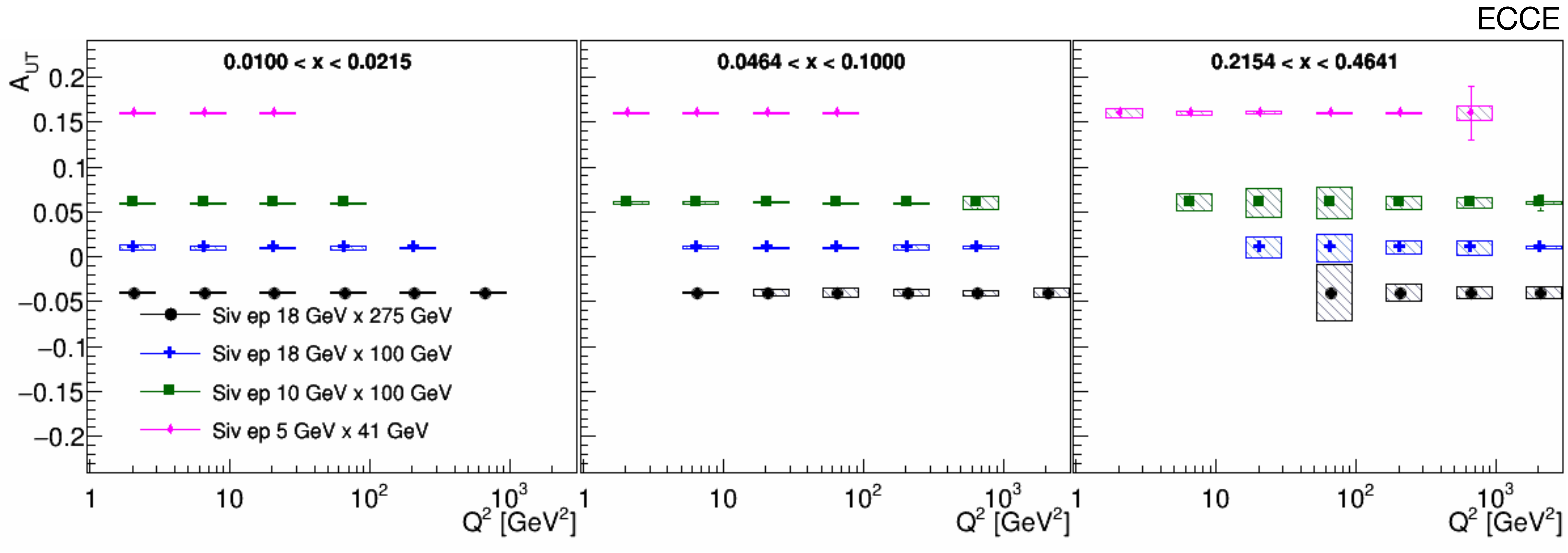
Beam polarisations assumed to be 70%.

systematic uncertainty= |generated - reconstructed|

Additionally: 3% scale uncertainty

- Low x and Q^2: small statistical uncertainty. High precision is needed since asymmetry at low x and Q^2 well below 1%.
- For not too large z and P_T, statistical uncertainty well below 1%.
- Systematic uncertainties increase with z and P_T: likely because of higher smearing effects.

Q^2 dependence of the Sivers asymmetry at EIC



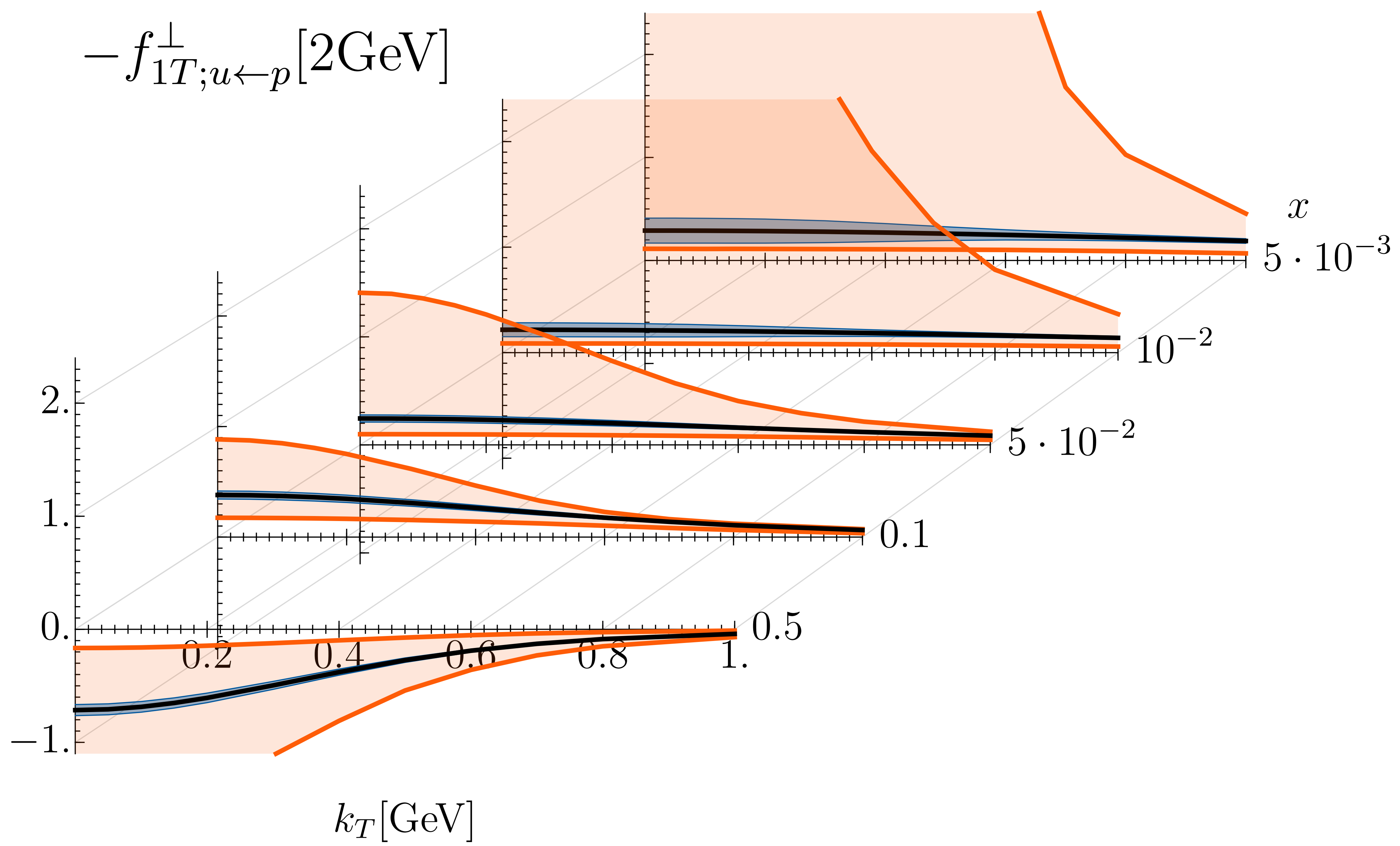
Intermediate and high x: good coverage in Q^2,
with complementarity in coverage at different COM energies.

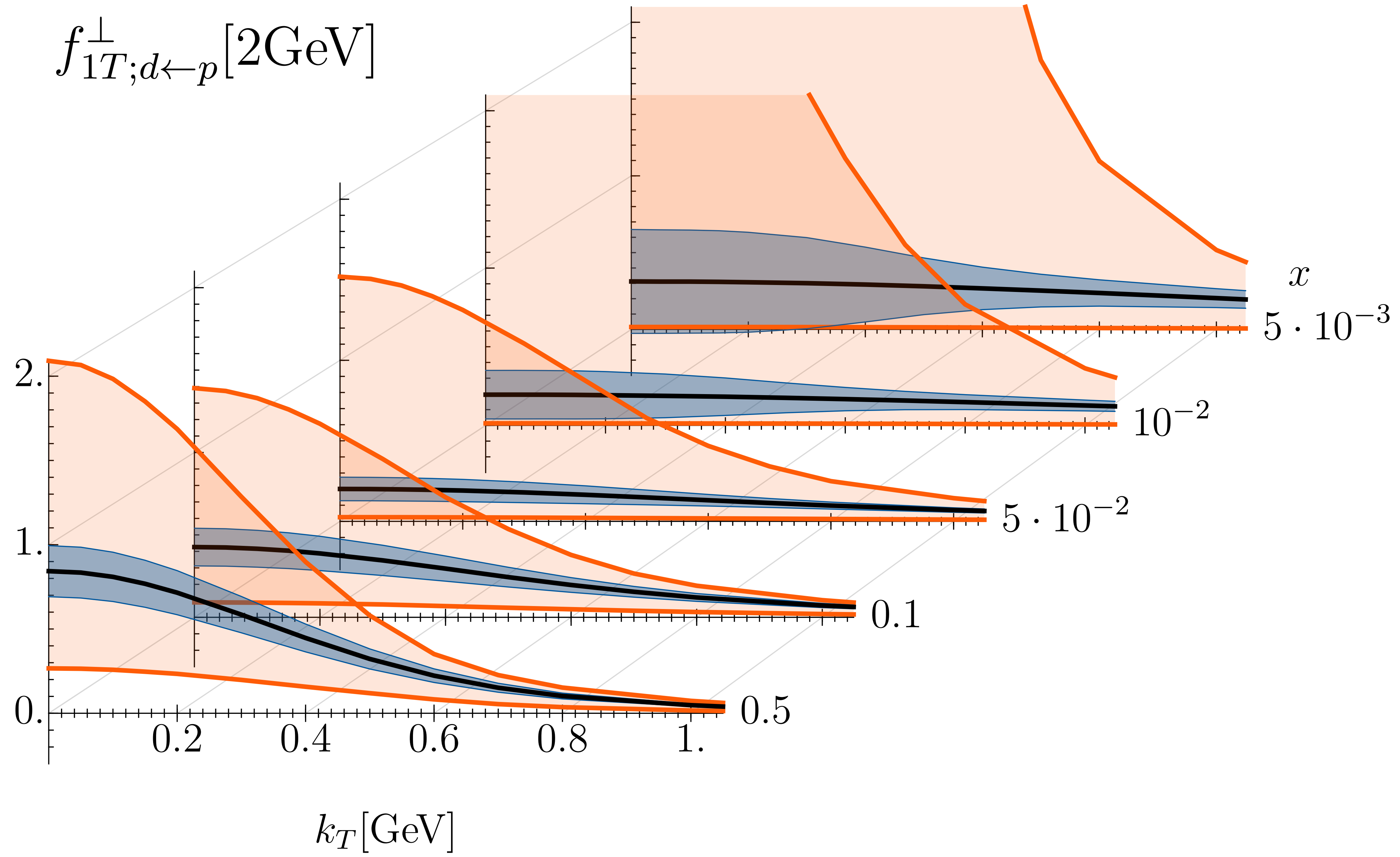
Sivers TMD PDF: impact of EIC

Parametrisation from
M. Bury et al., JHEP, 05:151, 2021

Q=2 GeV

ECCE





DIS variables via scattered lepton

$$Q^2 > 1\ \text{GeV}^2$$
$$0.01 < y < 0.95$$
$$W^2 > 10\ \text{GeV}^2$$

$$5 \times 41\ \text{GeV}^2$$
$$10 \times 100\ \text{GeV}^2$$
$$18 \times 100\ \text{GeV}^2$$
$$18 \times 275\ \text{GeV}^2$$

$\mathcal{L} = 10\ \text{fb}^{-1}$ for each collision energy

Summary

- Transverse momentum dependent hadron structure and hadron formation: rich field of physics, with sensitivity to correlations between quark and hadron spin and transverse momentum.

- Pioneering fixed-target experiments at HERMES, COMPASS, JLab 6 GeV: quark distributions

- Entering era of precision measurements:
  - JLab 12 GeV: unique precision in the valence region
  - EIC: extending down to $x=10^{-4}$
  - LHC measurements can provide additional, invaluable high energy input
  - need to extend measurements with sensitivity to gluons