Exotic hadrons with heavy quarks

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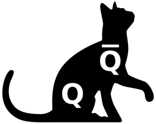
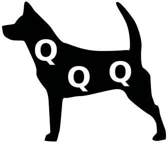
Ordinary hadrons



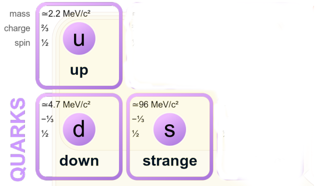


Quark model: The structure of hadrons

1964 — Quark model by Gell-Mann & Zweig $\Longrightarrow$ $SU(3)$ multiplets

"Ordinary" hadrons*:

- Meson consists of quark and antiquark
- Baryon consists of 3 quarks



* Compact "exotic" hadrons anticipated

All hadrons understood $\Longrightarrow$ No "exotic" states

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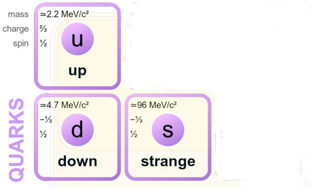

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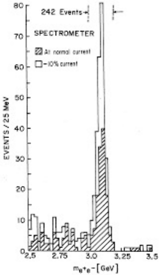
Prediction of the fourth quark:

- Glashow & Bjorken (1964)
- Glashow, Iliopoulos & Maiani (1970)

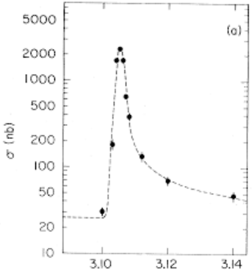
November revolution 1974: Discovery of charm

BNL ($p + Be \to e^+e^- X$)



$m_J = 3.1$ GeV
$\Gamma_J \approx 0$

SLAC ($e^+e^- \to$ hadrons)



$m_\psi = 3.105 \pm 0.003$ GeV
$\Gamma_\psi \leqslant 1$ MeV

Narrow resonance J/ψ with mass around 3.1 GeV

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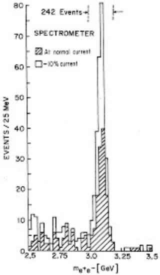

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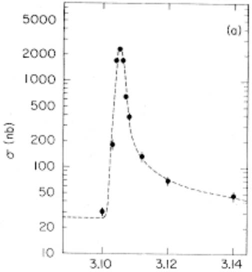


$m_\psi = 3.105 \pm 0.003$ GeV
$\Gamma_\psi \leqslant 1$ MeV

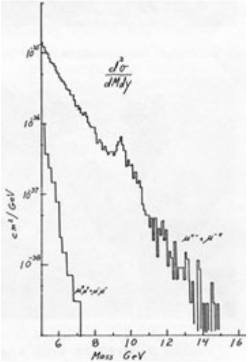
Narrow resonance J/ψ with mass around 3.1 GeV

5 years later $\Longrightarrow$ 10 charmonia states!

Bottomonia

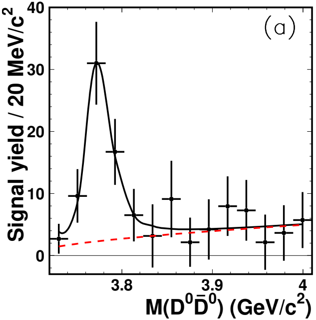
- 1977 — L.Lederman (Fermilab): discovery $\Upsilon(1S)$ with mass 9.54 GeV

$p + (Cu, Pt) \to \mu^+ + \mu^- +$ anything



- 1978 — DESY (Germany): discovery of $\Upsilon(2S)$
- 1980 — CESR (USA): discovery of $\Upsilon(3S)$ and $\Upsilon(4S)$

Breit-Wigner parametrisation: Mass, Width, Poles



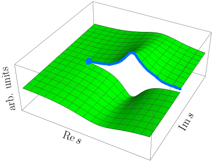
$$\mathcal{A} = \mathcal{A}_{\rm bg} + \mathcal{A}_{\rm BW}$$

$$\mathcal{A}_{\rm BW} \propto \frac{1}{M^2 - M_0{}^2 + iM\Gamma_0}$$

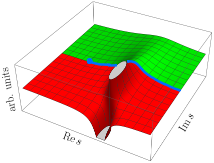
$$\Gamma_0 = \Gamma(R \to H_1 H_2)$$

$$M_0 > M_{H_1} + M_{H_2}$$

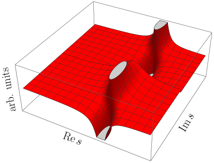
Pole positions: $\begin{cases} M_{\rm pole} \approx M_0 - \frac{i}{2}\Gamma_0 \\ M^*_{\rm pole} \approx M_0 + \frac{i}{2}\Gamma_0 \end{cases}$



first Riemann sheet

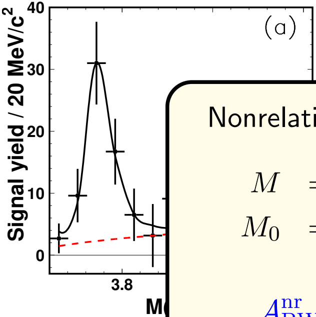


transition from first to second Riemann sheet



second Riemann sheet

Breit-Wigner parametrisation: Mass, Width, Poles



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$$\mathcal{A}_{\rm BW} \propto \frac{1}{\ldots M_0^2 + iM\Gamma_0}$$

$\ldots \to H_1 H_2)$

$\ldots + M_{H_2}$

$\ldots$pole $\approx M_0 - \frac{i}{2}\Gamma_0$

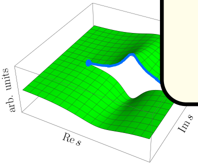
$\ldots^{*}$pole $\approx M_0 + \frac{i}{2}\Gamma_0$

Nonrelativistic expansion:

$$M = M_{H_1} + M_{H_2} + E$$
$$M_0 = M_{H_1} + M_{H_2} + E_0$$

$$\mathcal{A}^{\rm nr}_{\rm BW} \propto \frac{1}{E - E_0 + \frac{i}{2}\Gamma_0}$$

$$|\Psi|^2 \sim \left|e^{-iE_0 t - \frac{1}{2}\Gamma_0 t}\right|^2 \sim e^{-\Gamma_0 t}$$



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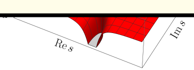
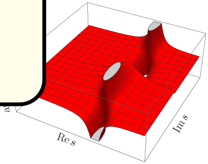
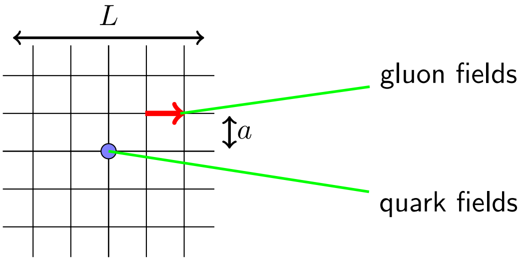


transition from first to second Riemann sheet



second Riemann sheet

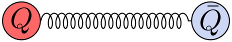
Lattice simulations



$$C_{ij}(t) = \langle 0|O_i(t)O_j(0)|0\rangle = \sum_n \frac{e^{-E_n t}}{2E_n}\langle 0|O_i(0)|n\rangle\langle n|O_j^\dagger(0)|0\rangle$$

- Continuum limit $\Longrightarrow$ $a \to 0$
- Infinite box $\Longrightarrow$ $L \to \infty$
- Unphysical light quark mass $\Longrightarrow$ Chiral extrapolation

Quark model: Adding dynamics



$$\boxed{\hat{H}_0\psi = E_0\psi}$$

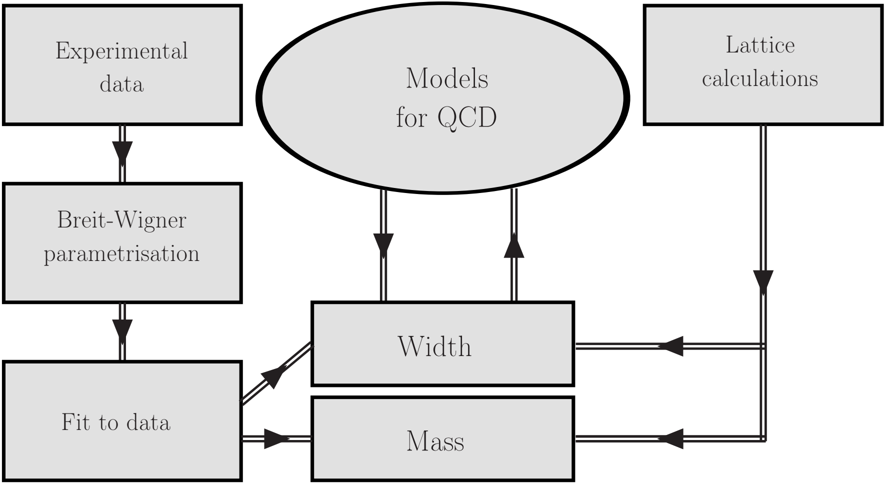
$$\hat{H}_0 = \frac{p^2}{m_Q} + V_0(r) + V_{\rm SD}(r)$$

$$V_0(r) = \sigma r - \frac{\frac{4}{3}\alpha_s}{r} + C_0 \qquad \text{(Cornell potential)}$$

$$V_{SD}(r) = \underbrace{V_{LS}(r)(\boldsymbol{L}\cdot(\boldsymbol{S}_Q+\boldsymbol{S}_{\bar{Q}}))}_{\text{fine structure}} + \underbrace{V_{SS}(r)(\boldsymbol{S}_Q\cdot\boldsymbol{S}_{\bar{Q}})}_{\text{hyperfine structure}}$$

$$+\underbrace{V_{ST}(r)\Big((\boldsymbol{S}_Q\cdot\boldsymbol{S}_{\bar{Q}}) - 3(\boldsymbol{S}_Q\cdot\boldsymbol{n})(\boldsymbol{S}_{\bar{Q}}\cdot\boldsymbol{n})\Big)}_{\text{spin−tensor force}} \quad \propto \quad \frac{1}{m_Q^2}$$

Approach to ordinary states



Hadronic physics: Consensus before 2003

- Quark model provides a decent description of low-lying hadrons
- Quark model works surprisingly well even for light flavours
- Heavy flavours (c and b) comply with nonrelativistic theory
- Relativistic corrections improve the description
- Experiment gradually fills "missing states"
- Lattice provides additional/alternative source of information

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General conclusion: Hadronic physics is well understood

Exotic states with heavy quarks

"Exotic animal is more unusual and rare than normal domesticated pets like cats or dogs"

Revolution of 2003: Enfant terrible $X(3872)$

- $I = 0$, $J^{PC} = 1^{++}$, contains $c\bar{c}$
- Too light compared with Quark Model prediction

$$M_{\chi_{c1}(2P)}^{\rm QM} - M_X^{\rm exp} \sim 100 \text{ MeV}$$

- Strongly attracted to $D\bar{D}^*$ threshold

$$M_X^{\rm exp} - (M_{D^0} + M_{\bar{D}^{*0}}) \ \sim \ 0$$

- Large ($\sim 40\%$) probability of the decay into $D\bar{D}^*$
- Strong isospin violation

$$Br(X \to \pi^+\pi^-\pi^0 J/\psi) \approx Br(X \to \pi^+\pi^- J/\psi)$$

Revolution of 2003: Enfant terrible $X(3872)$

- $I=0$, $J^{PC}=1^{++}$, contains $c\bar{c}$

> - $\sim$2500 citations (the most cited paper by Belle)
> - $J^{PC}=1^{++}$ unambiguously established by LHCb in 2013
> - Nature of $X(3872)$ still under debate
> - New name by PDG — $\chi_{c1}(3872)$

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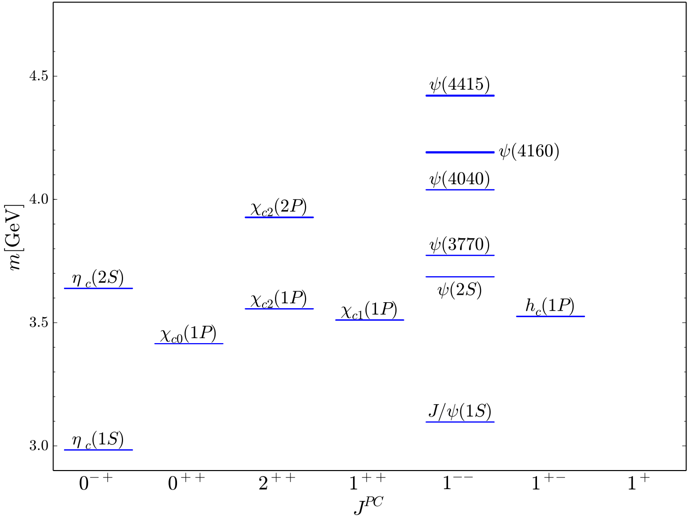
$$Br(X\to\pi^+\pi^-\pi^0 J/\psi)\approx Br(X\to\pi^+\pi^- J/\psi)$$

Spectrum of charmonium



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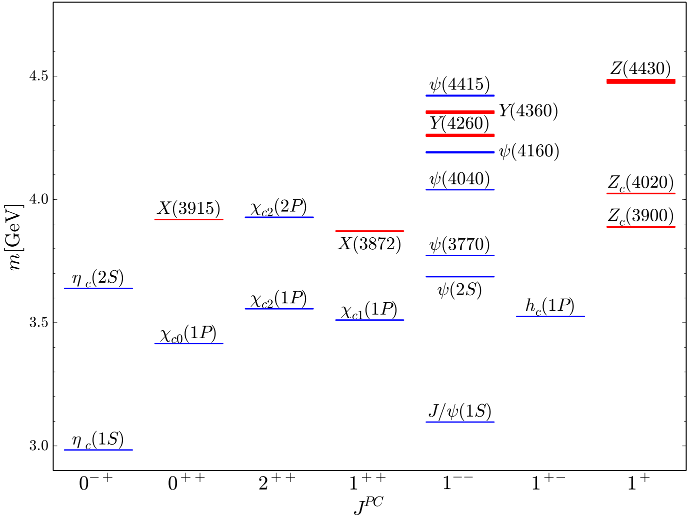

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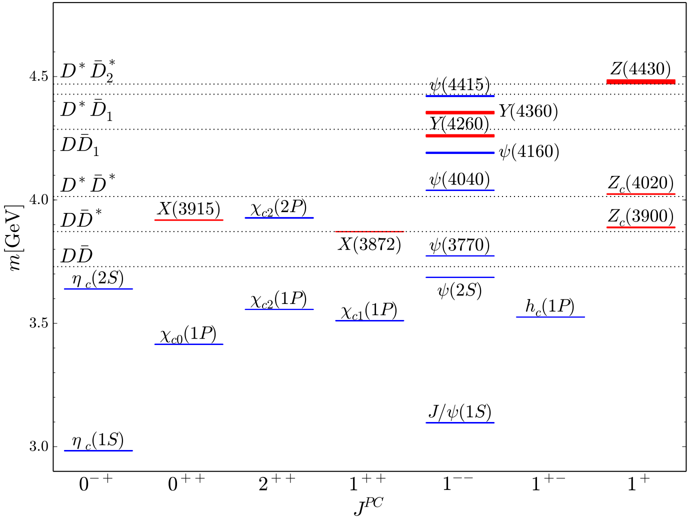

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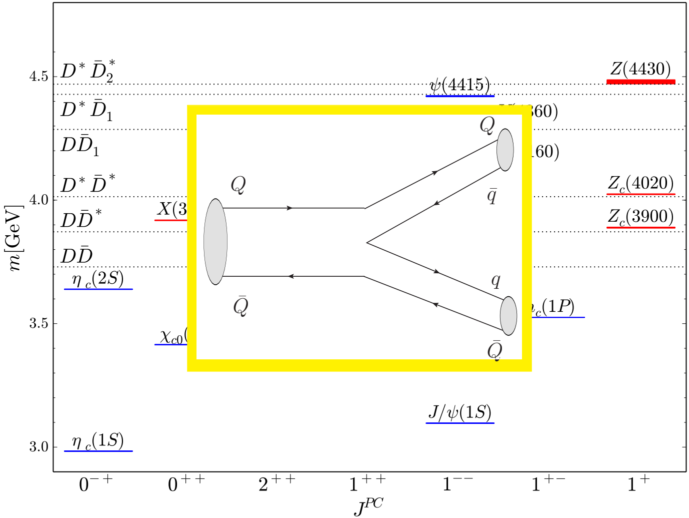
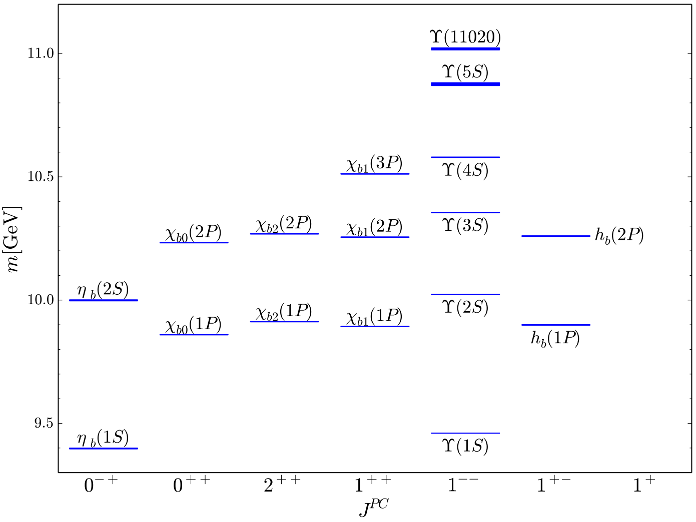


Spectrum of bottomonium



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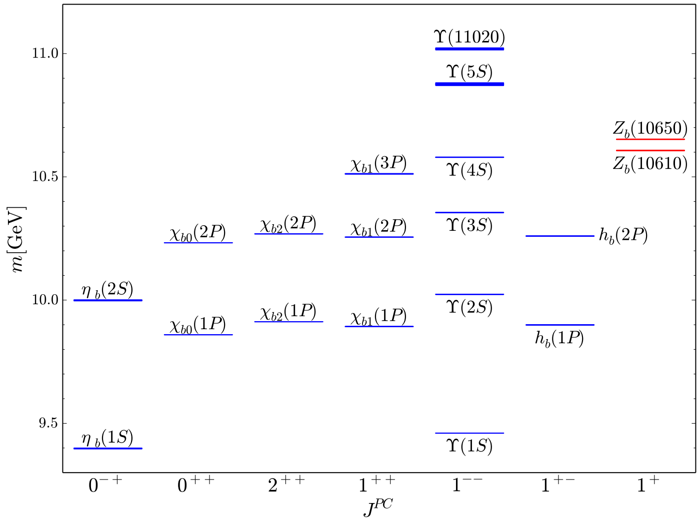

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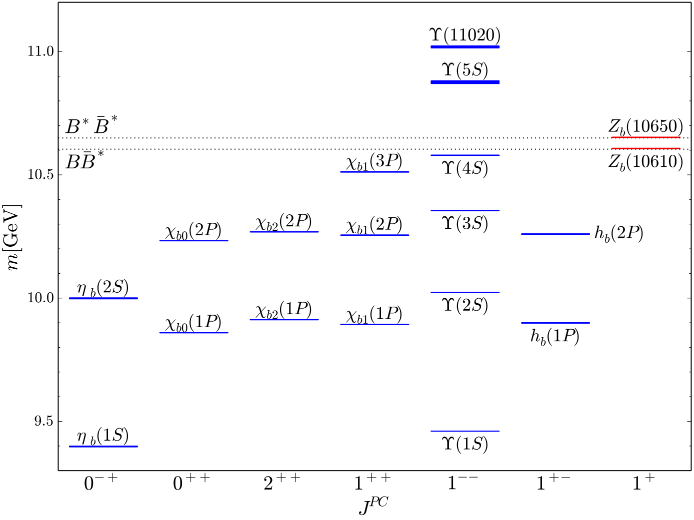

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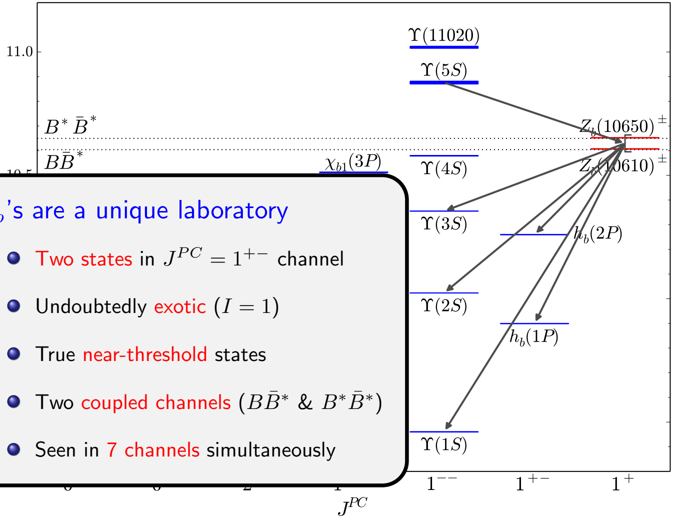
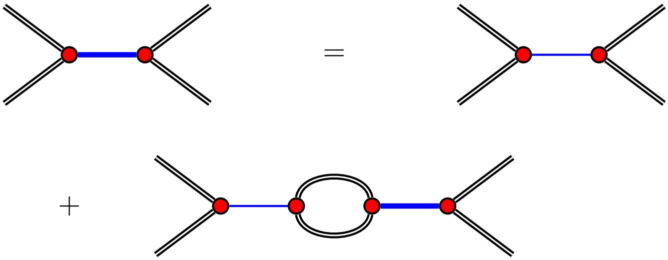


Z_b's are a unique laboratory

- Two states in $J^{PC} = 1^{+-}$ channel
- Undoubtedly exotic $(I = 1)$
- True near-threshold states
- Two coupled channels $(B\bar{B}^*$ & $B^*\bar{B}^*)$
- Seen in 7 channels simultaneously

Effect of hadronic loops

$$|\Psi\rangle = \begin{pmatrix} \sqrt{Z}|\psi_0\rangle \\ \chi(\boldsymbol{k})|H_1H_2\rangle_{L=0} \end{pmatrix}$$



$$\frac{1}{E-E_0+\frac{i}{2}\Gamma_0} \Longrightarrow \frac{1}{E-E_f+\frac{i}{2}(gk+\Gamma_0)} \qquad k=\sqrt{2\mu E}$$

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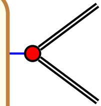
Flatté parametrisation:

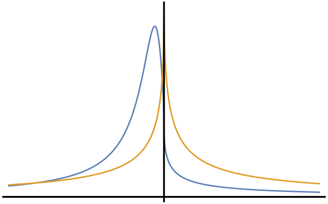
- $+$ Simple and physically transparent
- $+$ Accounts for threshold phenomena
- $-$ Difficult multichannel generalisation
- $-$ Obscure effect of particle exchanges
- $-$ Not systematically improvable



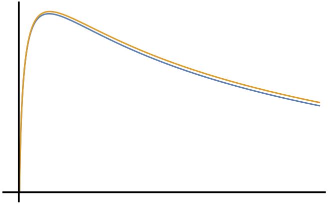
$$\frac{1}{E-E_0+\frac{i}{2}\Gamma_0} \Longrightarrow \frac{1}{E-E_f+\frac{i}{2}(gk+\Gamma_0)} \qquad k=\sqrt{2\mu E}$$

Examples of line shapes

$$\frac{\Gamma_0}{\left|E - E_f + \frac{i}{2}(gk + \Gamma_0)\right|^2}$$



$$\frac{gk}{\left|E - E_f + \frac{i}{2}(gk + \Gamma_0)\right|^2}$$

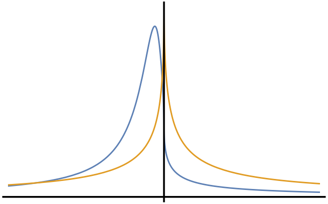


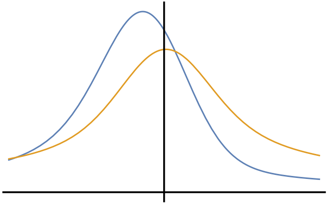
- Bound state ($E_f < 0$) — blue curve
- Virtual state ($E_f > 0$) — yellow curve

Pole resides on real axis below threshold on RS-I or RS-II

Effect of experimental resolution

$$\frac{\Gamma_0}{\left|E-E_f+\frac{i}{2}(gk+\Gamma_0)\right|^2} \qquad \int \frac{\Gamma_0 f_{\rm res}(E'-E)dE'}{\left|E'-E_f+\frac{i}{2}(gk+\Gamma_0)\right|^2}$$



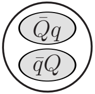


- Left plot — before convolution with resolution
- Right plot — after convolution with resolution

Sharp structures turn to broad humps

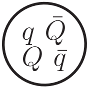
Models for exotic states

- Hadronic Molecule



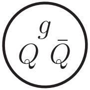
Extended object made of $(\bar{Q}q)$ and $(\bar{q}Q)$

- Compact Tetraquark



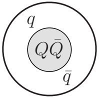
Compact object made of $(Q\bar{Q}q\bar{q})$

- Hybrid



Compact object made of $(Q\bar{Q})$ + gluon(s)

- Hadro-Quarkonium



$(Q\bar{Q})$ surrounded by light quarks

Models for exotic states

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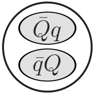


Extended object made of $(\bar{Q}q)$ and $(\bar{q}Q)$

- $^3S_1$ NN system with $I = 0$:

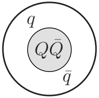
  Pole on RS-I with $E_B = 2.23$ MeV $\Longrightarrow$ deuteron

- $^1S_0$ NN system with $I = 1$:

  Pole on RS-II with $E_B = 0.067$ MeV $\Longrightarrow$ virtual state

Compact object made of $(Q\bar{Q})$ + gluon(s)

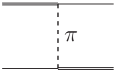
- Hadro-Quarkonium



$(Q\bar{Q})$ surrounded by light quarks

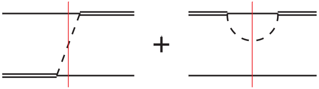
Some generalities

Pion exchange

$V_{\text{OPE}} =$ 

$\sim \dfrac{q_i q_j}{q^2 - m_\pi^2} \underset{\text{S-wave, \sout{recoil}}}{\Longrightarrow} \dfrac{1}{3}\delta_{ij}\Bigg(-1 + \overbrace{\dfrac{\mu_\pi^2}{\boldsymbol{q}^2 + \underbrace{[m_\pi^2 - (M_* - M)^2]}_{\text{Effective mass } \mu_\pi^2}}}^{\text{Long-range OPE}}\Bigg)$

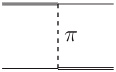
- Deuteron $(m_\pi \gg M_n - M_p \implies \mu_\pi = m_\pi) \implies V_{\text{OPE}}^{\text{long-range}} \sim \frac{1}{r} e^{-m_\pi r}$
- Charmonium system $(m_\pi < M_{D^*} - M_D \implies \mu_\pi^2 < 0 \;\&\; |\mu_\pi| \ll m_\pi)$:

3-body unitarity: 

- Bottomonium system $(m_\pi > M_{B^*} - M_B \implies \mu_\pi^2 > 0 \;\&\; \mu_\pi < m_\pi)$:

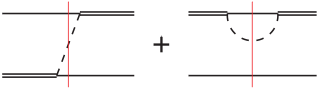
$$\int d\Omega_{\hat{\boldsymbol{k}}\hat{\boldsymbol{k}}'} V_{\text{OPE}}(\boldsymbol{k} - \boldsymbol{k}') \sim \log \frac{\mu_\pi^2 + (k + k')^2}{\mu_\pi^2 + (k - k')^2} \underset{k' = k}{\Longrightarrow} \text{left-hand cut at } k^2 < -\tfrac{1}{4}\mu_\pi^2$$

Pion exchange

$V_{\text{OPE}} =$  $\sim \dfrac{q_i q_j}{q^2 - m_\pi^2}$ $\underset{\text{S-wave, }\cancel{\text{recoil}}}{\Longrightarrow}$ $\dfrac{1}{3}\delta_{ij}\Big(-1 + \overbrace{\dfrac{\mu_\pi^2}{\boldsymbol{q}^2 + \underbrace{[m_\pi^2 - (M_* - M)^2]}_{\text{Effective mass } \mu_\pi^2}}}^{\text{Long-range OPE}}\Big)$

> If $m_\pi^{\text{lat}} > m_\pi^{\text{phys}} \Longrightarrow$ Interpretation of lattice results may be nontrivial

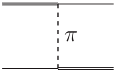
- Charmonium system ($m_\pi < M_{D^*} - M_D \Longrightarrow \mu_\pi^2 < 0$ & $|\mu_\pi| \ll m_\pi$):

3-body unitarity: 

- Bottomonium system ($m_\pi > M_{B^*} - M_B \Longrightarrow \mu_\pi^2 > 0$ & $\mu_\pi < m_\pi$):

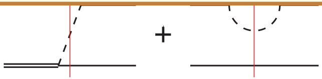
$$\int d\Omega_{\hat{\boldsymbol{k}}\hat{\boldsymbol{k}}'} V_{\text{OPE}}(\boldsymbol{k} - \boldsymbol{k}') \sim \log \frac{\mu_\pi^2 + (k + k')^2}{\mu_\pi^2 + (k - k')^2} \underset{k' = k}{\Longrightarrow}$$ left-hand cut at $k^2 < -\frac{1}{4}\mu_\pi^2$

Pion exchange

$V_{\text{OPE}} =$  $\sim \dfrac{q_i q_j}{q^2 - m_\pi^2} \underset{\text{S-wave, \sout{recoil}}}{\Longrightarrow} \dfrac{1}{3}\delta_{ij}\left(\overbrace{-1 + \dfrac{\mu_\pi^2}{\boldsymbol{q}^2 + \underbrace{[m_\pi^2 - (M_* - M)^2]}_{\text{Effective mass } \mu_\pi^2}}}^{\text{Long-range OPE}}\right)$

If $m_\pi^{\text{lat}} > m_\pi^{\text{phys}} \Longrightarrow$ Interpretation of lattice results may be nontrivial

For data in a broad energy range, D waves from OPE are important

3-body unitarity: 

- Bottomonium system ($m_\pi > M_{B^*} - M_B \Longrightarrow \mu_\pi^2 > 0$ & $\mu_\pi < m_\pi$):

$$\int d\Omega_{\hat{\boldsymbol{k}}\hat{\boldsymbol{k}}'} V_{\text{OPE}}(\boldsymbol{k} - \boldsymbol{k}') \sim \log \frac{\mu_\pi^2 + (k+k')^2}{\mu_\pi^2 + (k-k')^2} \underset{k'=k}{\Longrightarrow} \text{left-hand cut at } k^2 < -\tfrac{1}{4}\mu_\pi^2$$

Heavy-quark spin symmetry

- Exotic states contain heavy quarks (HQ)

- In the limit $m_Q \to \infty$ $(m_Q \gg \Lambda_{\rm QCD})$ spin of HQ decouples

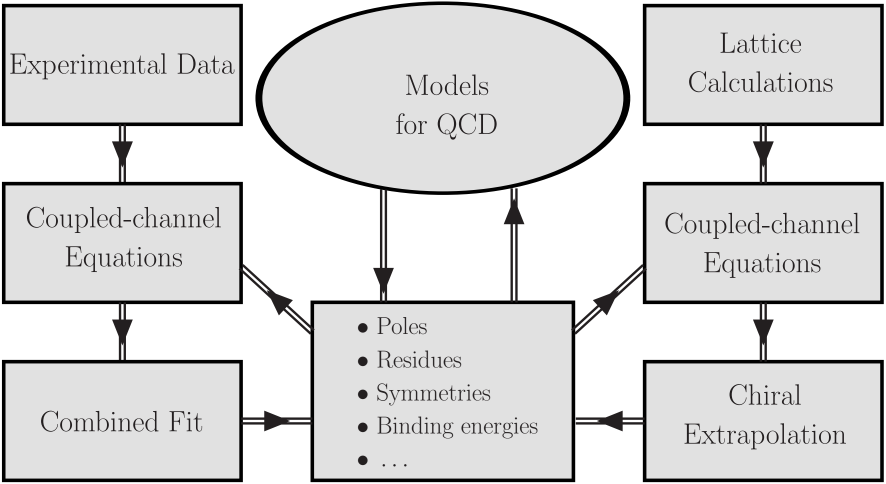
  $\Longrightarrow$ Heavy Quark Spin Symmetry (HQSS)

- For realistic m_Q's HQSS is approximate but accurate symmetry of QCD

- HQSS $=$ tool to relate properties of states with different HQ spin orientation

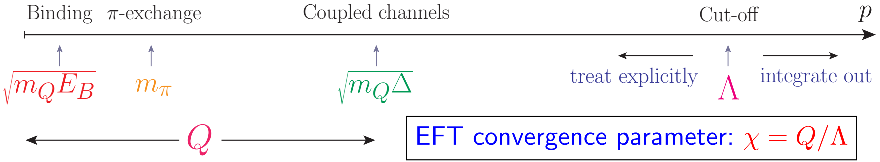
  $\Longrightarrow$ Spin partners

Approach to exotic states



Effective Field Theory for Hadronic Molecules

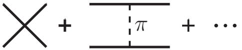
Effective field theory for hadronic molecules



EFT convergence parameter: $\chi = Q/\Lambda$

Interaction potential between heavy hadrons:

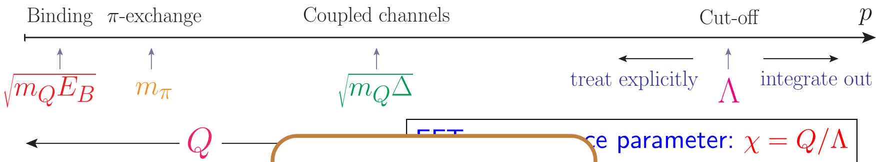
- Includes all relevant interactions



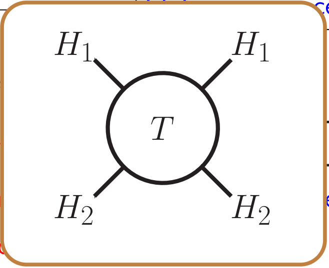
- Complies with relevant symmetries (chiral, HQSS, etc)
- Incorporates coupled-channel dynamics
- Expanded in powers of p^2/Λ^2 and truncated at necessary order (LO, NLO...)
- Iterated to all orders via (multichannel) Lippmann-Schwinger equation

$$T = V - VGT$$

Effective field theory for hadronic molecules



EFT … ce parameter: $\chi = Q/\Lambda$



Interaction potential betwe …

+ ⋯

- Includes all relevant ir …
- Complies with relevan … etc)
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- Expanded in powers of p^2/Λ^2 and truncated at necessary order (LO, NLO...)
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$$T = V - VGT$$

Effective field theory for hadronic molecules

Free parameters:

- Low-energy constants
- (Bare) couplings to hadronic channels

Input (combined analysis):

- Line shapes (Dalitz plots)
- Partial branchings

Output:

- Pole position M_0 ("mass"$=\mathrm{Re}(M_0)$, "width"$= 2\times\mathrm{Im}(M_0)$)
- Residues at the poles (dressed couplings)

Predictions:

- New properties of state: line shapes, partial widths,...
- Spin partners: poles, line shapes, partial widths,...
- Chiral extrapolations

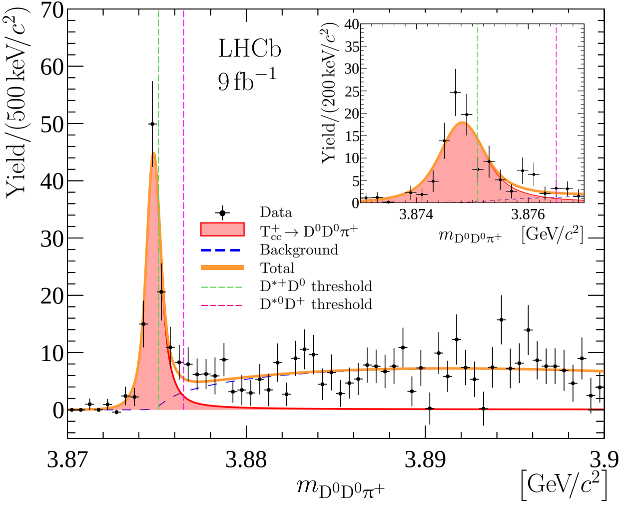
Double-charm state T_{cc}^{+}

$$I = 0 \quad J^P = 1^+$$

Minimal quark content: $cc\bar{u}\bar{d}$

$$T_{cc}^{+} \to D^0 D^0 \pi^+$$

T_{cc}^+ @ LHCb (Nature Phys. 18 (2022) 7, 751)



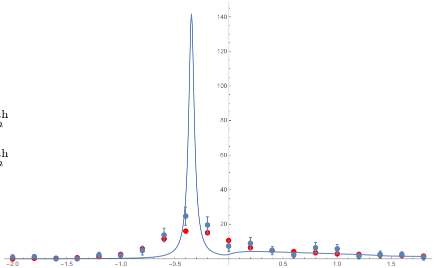
$\delta m_{\rm BW} = -273 \pm 61 \pm 5^{+11}_{-14}$ keV $\quad \Gamma_{\rm BW} = 410 \pm 165 \pm 43^{+18}_{-38}$ keV

Simple Flatté fit $(\chi^2/N_{\rm dof} \approx 1)$

$$\mathcal{A} = \frac{\sqrt{\mathcal{N}}}{E - E_f + \frac{i}{2}\left[g(\tilde{k}_1 + \tilde{k}_2) + \Gamma_0\right]}$$

$$\tilde{k}_n = \begin{cases} \sqrt{\mu_n\left(\sqrt{(E-E_n^{\rm th})^2 + \frac{1}{4}\Gamma_{D^*}^2} + (E-E_n^{\rm th})\right)}, & E > E_n^{\rm th} \\ -i\sqrt{\mu_n\left(\sqrt{(E-E_n^{\rm th})^2 + \frac{1}{4}\Gamma_{D^*}^2} - (E-E_n^{\rm th})\right)}, & E < E_n^{\rm th} \end{cases}$$

$\Gamma_0^{\rm fit} = 0 \implies$ No compact component



Pole position:

$$E_{\rm pole} = (-347 - i31) \text{ keV}$$

In neglect of D^* width

$$X_1 = \frac{\sqrt{E_B + \Delta}}{\sqrt{E_B} + \sqrt{E_B + \Delta}} \qquad X_2 = \frac{\sqrt{E_B}}{\sqrt{E_B} + \sqrt{E_B + \Delta}}$$

For $E_B = 347$ keV and $\Delta = 1.41$ MeV

$$X_1 = 0.7 \quad X_2 = 0.3$$

EFT approach to T_{cc}^{+}

$$\left.\begin{aligned} \gamma_B &= \sqrt{m_D E_B} \simeq 25 \text{ MeV} \\ p_{\text{data}}^{\text{max}} &= \sqrt{m_D\, \Delta E_{\text{data}}} \simeq 100 \text{ MeV} \\ p_{\text{coupl.ch.}} &= \sqrt{m_D (m_{D^*} - m_D)} \simeq 500 \text{ MeV} \end{aligned}\right\} \Longrightarrow$$

$\Lambda = 500$ MeV
Potential at LO
OPE included
No couple channels

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$\Lambda = 500$ MeV
Potential at LO
OPE included
No couple channels

- Lippmann-Schwinger equation for scattering amplitude (1 free parameter)

$$T(M,p,p') = V(M,p,p') - \int \frac{d^3q}{(2\pi)^3} V(M,p,q) G(M,q) T(M,q,p')$$

$$V(M,p,p') = v_0 + V_{\text{OPE}}$$

- Production amplitude (1 additional free parameter: P = point-like source)

$$U(M,p) = P - \int \frac{d^3q}{(2\pi)^3} T(M,p,q) G(M,q) P$$

EFT approach to T_{cc}^+

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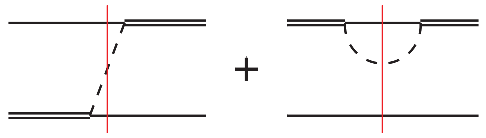
$\Lambda = 500$ MeV
Potential at LO
OPE included
No couple channels

- Lippmann-S … parameter)

$T(M,$ … $T(M, q, p')$

3-body effects:

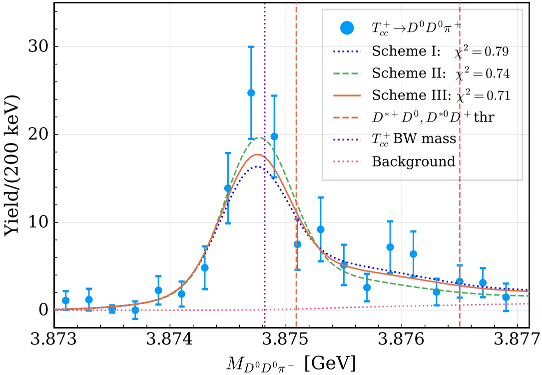


- Production amplitude (1 additional free parameter: P = point-like source)

$$U(M,p) = P - \int \frac{d^3q}{(2\pi)^3} T(M,p,q)G(M,q)P$$

Fitting schemes, results, and conclusions

	Γ_{D^*}=const, ~~OPE~~	$\Gamma_{D^*}(p,M)$, ~~OPE~~	$\Gamma_{D^*}(p,M)$, OPE
χ^2/d.o.f.	0.79	0.74	0.71
v_0 [GeV^{-2}]	-23.34 ± 0.08	$-22.88^{+0.08}_{-0.06}$	$-5.04^{+0.10}_{-0.08}$
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$



- (Quasi)bound state just below $D^{*+}D^0$ threshold
- Compositeness: 70% & 30%
- Spin partner T_{cc}^{*+} near $D^{*+}D^{*0}$ threshold is likely to exist but predictions uncertain

Fitting schemes, results, and conclusions

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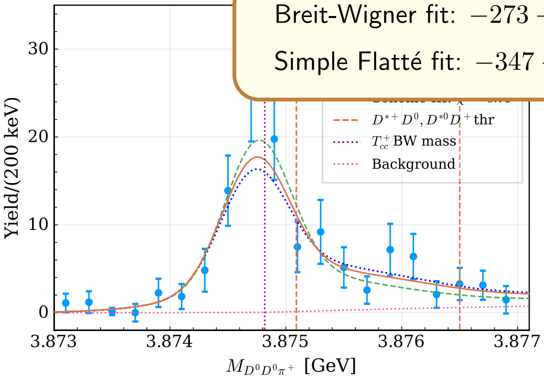
Simple Flatté fit: $-347 - i31$ keV



...nd state just below ...reshold

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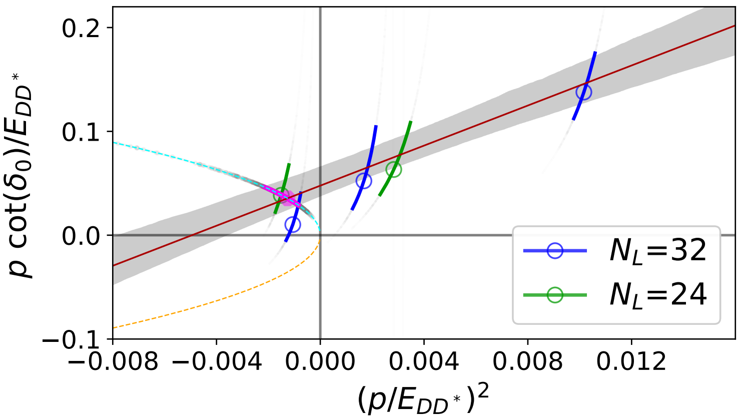
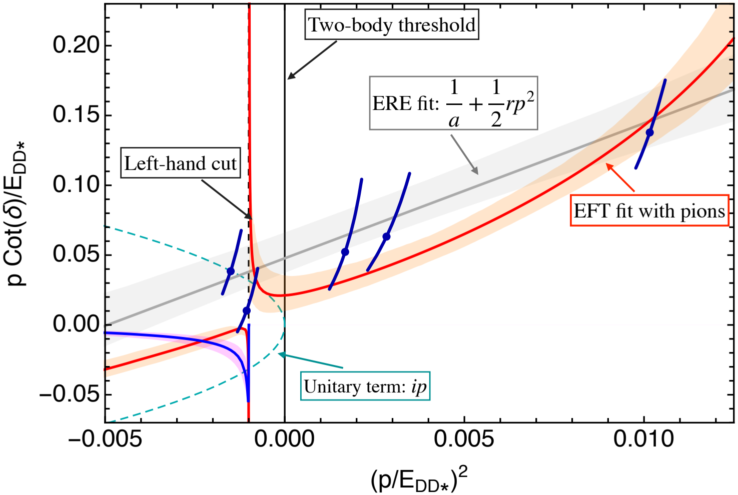
Comment on lattice studies of T_{cc}^+

Padmanath & Prelovsek, Phys.Rev.Lett. 129 (2022), 032002



$E_B = 9.9^{+3.6}_{-7.1}$ MeV

Comment on lattice studies of T_{cc}^+



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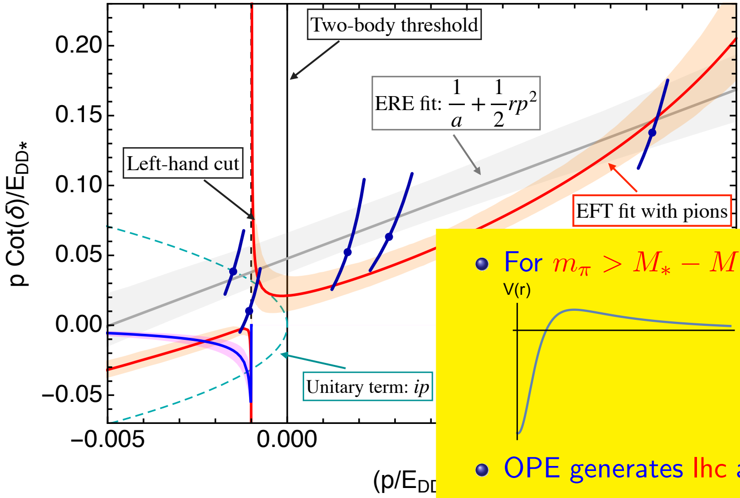
Comment on lattice studies of T_{cc}^+



Lattice data: Padmanath & Prelovsek,

- For $m_\pi > M_* - M$ OPE is repulsive

- OPE generates lhc and pole in $p\cot\delta$
- ERE $\frac{1}{a} + \frac{1}{2}rp^2$ is not reliable near lhc
- lhc to be included in data extraction

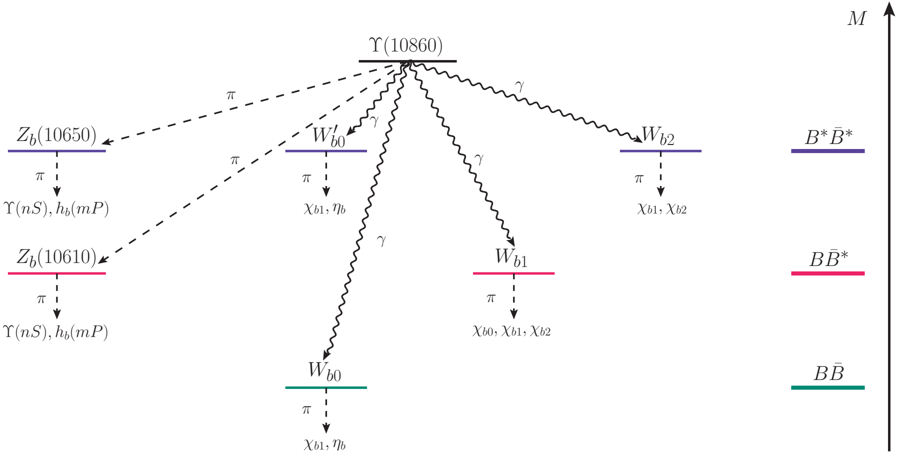
Twins $Z_b(10610)$ & $Z_b(10650)$

$$I = 1 \quad J^{PC} = 1^{+-}$$

Minimal quark content: $\bar{b}b\bar{q}q$

$$\Upsilon(10860) \to \pi Z_b^{(\prime)} \to \pi\left[B\bar{B}^{(*)}\right]$$
$$\Upsilon(10860) \to \pi Z_b^{(\prime)} \to \pi\left[\pi h_b(1,2P)\right]$$
$$\Upsilon(10860) \to \pi Z_b^{(\prime)} \to \pi\left[\pi\Upsilon(1,2,3S)\right]$$

Z_b's ($J^{PC}=1^{+-}$) and W_{bJ}'s ($J^{PC}=J^{++}$) in decays of $\Upsilon(10860)$

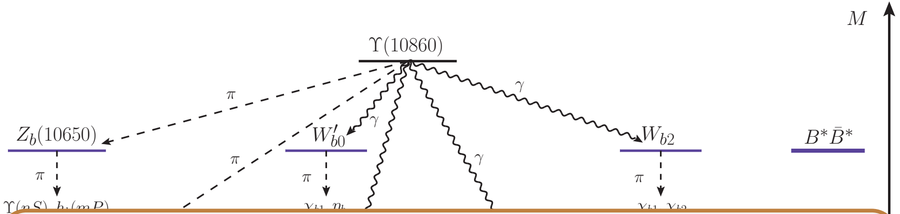


$$Z_b(10610) \sim B\bar{B}^* \sim 0^-_{\bar{q}b} \otimes 1^-_{\bar{b}q} \sim 1^-_{\bar{b}b} \otimes 0^-_{\bar{q}q} + 0^-_{\bar{b}b} \otimes 1^-_{\bar{q}q}$$

$$Z'_b(10650) \sim B^*\bar{B}^* \sim 1^-_{\bar{q}b} \otimes 1^-_{\bar{b}q} \sim 1^-_{\bar{b}b} \otimes 0^-_{\bar{q}q} - 0^-_{\bar{b}b} \otimes 1^-_{\bar{q}q}$$

(Bondar et al'2011,Voloshin'2011,...)

Z_b's ($J^{PC}=1^{+-}$) and W_{bJ}'s ($J^{PC}=J^{++}$) in decays of $\Upsilon(10860)$



$\Longrightarrow$ Constructive interference between Z_b & Z_b' in $\pi\pi\Upsilon$ channels

$\Longrightarrow$ Destructive interference between Z_b & Z_b' in $\pi\pi h_b$ channels

$\Longrightarrow$ Relevant (HQSS breaking!) parameter $r=(m_{z'}-m_z)/\Gamma_z$ ($r_{\rm phys}\approx 3$)

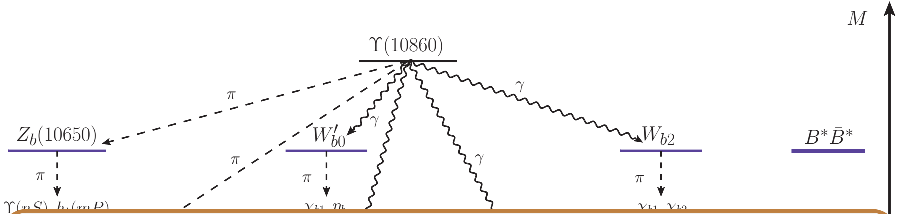
$\Longrightarrow$ Br$(\pi\pi h_b)[r_{\rm phys}]/Br(\pi\pi\Upsilon)[r_{\rm phys}]\sim 1$

$$Z_b(10610) \sim B\bar{B}^* \sim 0^-_{\bar{q}b}\otimes 1^-_{\bar{b}q} \sim 1^-_{\bar{b}b}\otimes 0^-_{\bar{q}q} + 0^-_{\bar{b}b}\otimes 1^-_{\bar{q}q}$$

$$Z_b'(10650) \sim B^*\bar{B}^* \sim 1^-_{\bar{q}b}\otimes 1^-_{\bar{b}q} \sim 1^-_{\bar{b}b}\otimes 0^-_{\bar{q}q} - 0^-_{\bar{b}b}\otimes 1^-_{\bar{q}q}$$

(Bondar et al'2011,Voloshin'2011,...)

Z_b's ($J^{PC}=1^{+-}$) and W_{bJ}'s ($J^{PC}=J^{++}$) in decays of $\Upsilon(10860)$



- $\Longrightarrow$ Constructive interference between Z_b & Z_b' in $\pi\pi\Upsilon$ channels
- $\Longrightarrow$ Destructive interference between Z_b & Z_b' in $\pi\pi h_b$ channels
- $\Longrightarrow$ Relevant (HQSS breaking!) parameter $r=(m_{z'}-m_z)/\Gamma_z$ ($r_{\rm phys}\approx 3$)
- $\Longrightarrow$ $\mathrm{Br}(\pi\pi h_b)[r_{\rm phys}]/\mathrm{Br}(\pi\pi\Upsilon)[r_{\rm phys}]\sim 1$

$p_{\text{coupl.ch.}}=\sqrt{m_B(m_{B^*}-m_B)}\approx 500$ MeV $\Longrightarrow$
- $\Lambda\simeq 1$ GeV
- Potential at NLO
- OPE included (D waves!)

Z_b's in EFT approach

$B^{(*)}\bar{B}^*$ potential:

$$V = V_{\rm CT}(\text{to order } O(p^0))$$

Coupled channels:

$$1^{+-} : B\bar{B}^*({}^3S_1, -), B^*\bar{B}^*({}^3S_1)$$
$$0^{++} : B\bar{B}({}^1S_0), B^*\bar{B}^*({}^1S_0)$$
$$1^{++} : B\bar{B}^*({}^3S_1, +)$$
$$2^{++} : B^*\bar{B}^*({}^5S_2)$$

Z_b's in EFT approach

$B^{(*)}\bar{B}^*$ potential:

$$V = V_{\rm CT}(\text{to order } O(p^2)) + V_\pi$$

Coupled channels:

$$1^{+-} : B\bar{B}^*({}^3S_1,-), B^*\bar{B}^*({}^3S_1), B\bar{B}^*({}^3D_1,-), B^*\bar{B}^*({}^3D_1)$$
$$0^{++} : B\bar{B}({}^1S_0), B^*\bar{B}^*({}^1S_0), B^*\bar{B}^*({}^5D_0)$$
$$1^{++} : B\bar{B}^*({}^3S_1,+), B\bar{B}^*({}^3D_1,+), B^*\bar{B}^*({}^5D_1)$$
$$2^{++} : B^*\bar{B}^*({}^5S_2), B\bar{B}({}^1D_2), B\bar{B}^*({}^3D_2), B^*\bar{B}^*({}^1D_2), B^*\bar{B}^*({}^5D_2), \cancel{B^*\bar{B}^*({}^5G_2)}$$

Lippmann-Schwinger equation ($\alpha,\beta,\gamma = (B\bar{B}^*, B^*\bar{B}^*) \otimes (L=0, L=2)$):

$$T_{\alpha\beta}(M,\boldsymbol{p},\boldsymbol{p}') = V^{\rm eff}_{\alpha\beta}(\boldsymbol{p},\boldsymbol{p}') - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V^{\rm eff}_{\alpha\gamma}(\boldsymbol{p},\boldsymbol{q}) G_\gamma(M,\boldsymbol{q}) T_{\gamma\beta}(M,\boldsymbol{q},\boldsymbol{p}')$$

$B^{(*)}\bar{B}^*$ pot

Coupled ch

1 ... $^3D_1)$

0

1

2 ... $_2)$

Lippmann-S ... $0, L = 2))$:

$T_{\alpha\beta}(M, \boldsymbol{p},$ ... $_\beta(M, \boldsymbol{q}, \boldsymbol{p}')$

Free parameters:

- Contact potentials (4)
- Couplings to hidden-bottom channels (5)
- Overall normalisations (7)
- π-π interaction (6)

Total: 22

$B^{(*)}\bar{B}^*$ pot

Coupled ch

1

0

1

2

$^3D_1)$

$_2)$

Lippmann-S

$), L=2)$):

$T_{\alpha\beta}(M, \boldsymbol{p},$

$_\beta(M, \boldsymbol{q}, \boldsymbol{p}')$

Free parameters:

- Contact potentials (4)
- Couplings to hidden-bottom channels (5)
- Overall normalisations (7)
- π-π interaction (6)

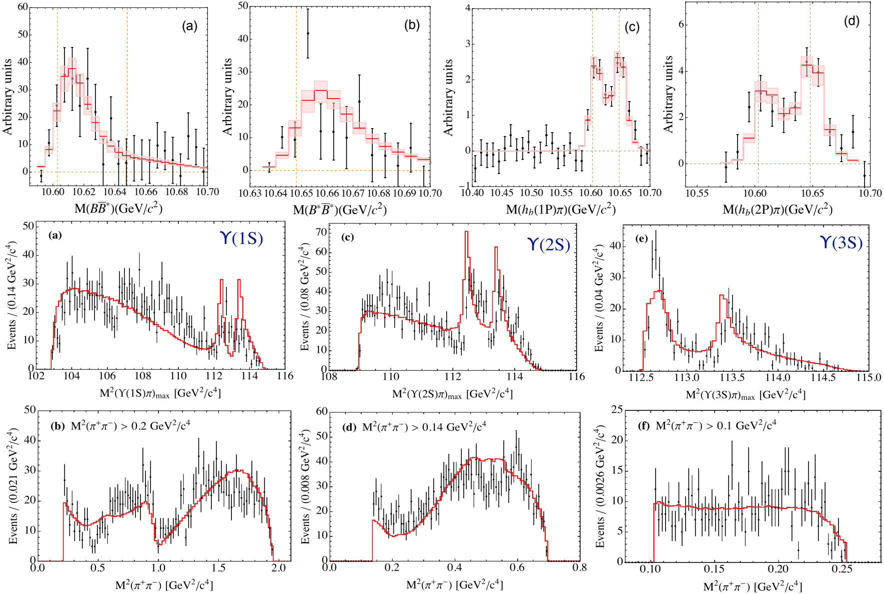
Total: 22

Naive sum of BW's:

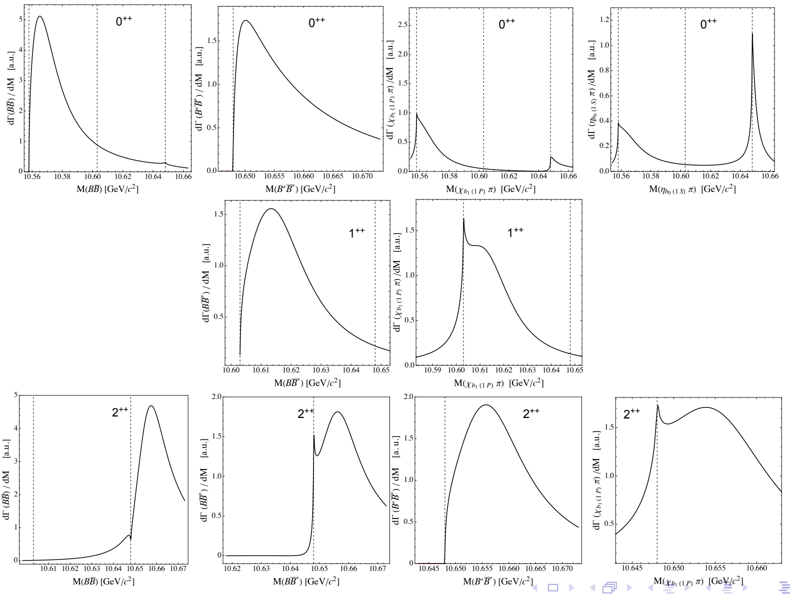
- Masses $(2 \times 7 = 14)$
- Widths $(2 \times 7 = 14)$
- Relative phases (7)
- Overall normalisations (7)
- π-π interaction (?)

Total: > 42

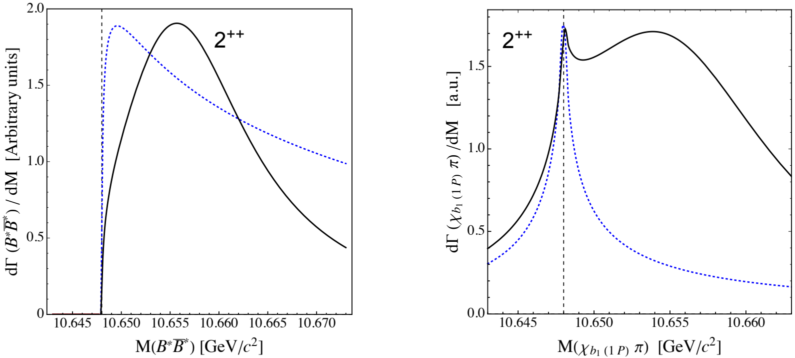
Fitted line shapes for Z_b's



Predicted line shapes for W_{bJ}'s



Role of pions



- Blue dashed line — pionless theory
- Black solid line — full theory with pions

Conclusions

- Collider experiments at energies above open-flavour thresholds started new era in hadronic physics
- Threshold phenomena, coupled channels, pion exchange are important
- Multibody unitarity and analyticity of amplitude need to be preserved
- Line shapes of non-Breit-Wigner form is current reality
- From "mass" and "width" to pole position and residues (couplings)
- EFT can be employed to a success as model-independent, systematically improvable analysis and prediction tool
- Results of EFT analysis to be used as input for QCD-inspired models
- Lattice simulations are important to fill the gap in experimental data and provide numerical experiment in "alternative Universe"