Physics of nuclear thresholds effects

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International School of Nuclear Physics in Erice

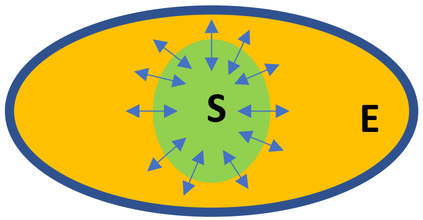
45th Course: «*Nuclei in the Laboratory and in Stars*»

Erice, Sicily, September 16-22, 2024

1. What is the open quantum system
2. Atomic nucleus: the open quantum system
   - Why do we care about the continuum?
2. Shell model for open quantum systems
   - Non-Hermitian vs Hermitian formulations
   - NN interaction in different regimes of binding
3. Configuration mixing in open quantum system
   - Coalescence of resonance wave functions
   - Near-threshold instability of shell model eigenstates
4. Near-threshold states and origin of clustering
   - Astrophysical relevance for α- and proton-capture reactions
5. Mimicry mechanism of clusterization
   - Chameleon nature of resonances
6. Message to take

What is the open quantum system?

An *open* **quantum system** is a quantum system which is found to be in interaction with an external quantum system, the *environment*. The open quantum system can be viewed as a distinguished part of a larger quantum system.



$\rho_S = \mathrm{Tr}_E\{\rho_{full}\}$

Standard techniques developed in the context of open quantum systems have proven powerful in fields such as:

quantum optics, quantum measurement theory, quantum statistical mechanics, quantum information science, quantum cosmology, mesoscopic physics …

Description in terms of the density matrix is not well suited for nuclear physics which deals with the well-defined quantum states → *shell model formulation*

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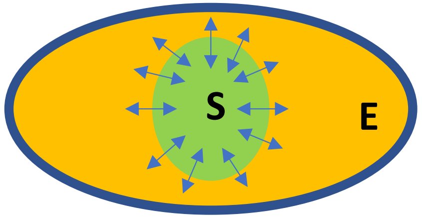

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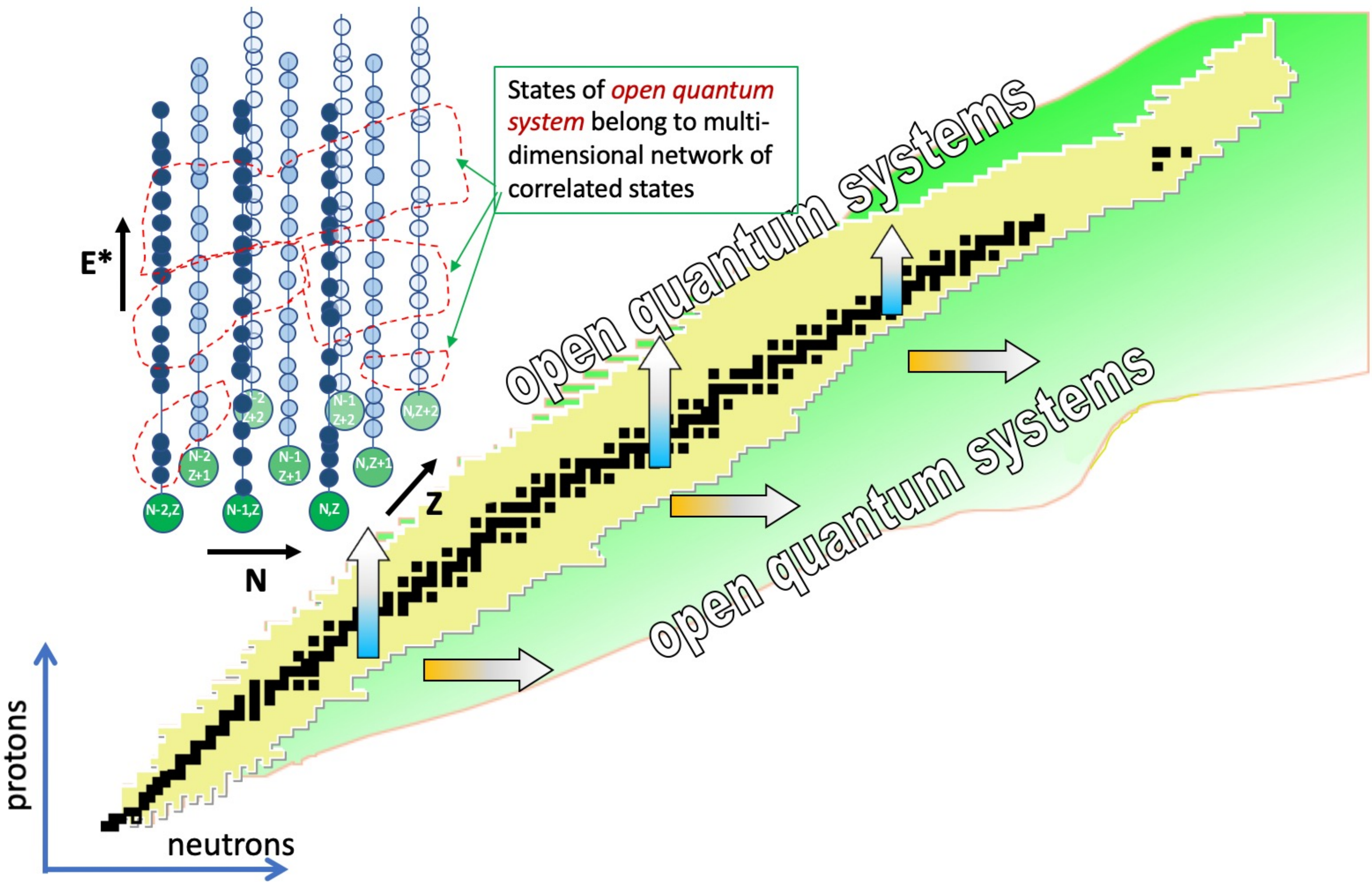
Any formulation of the open quantum system theory should conserve *unitarity* which is the fundamental property of quantum mechanics

'Mainstream' nuclear theory describes nucleus as the *closed* quantum system, i.e. in the unitarity violating scheme

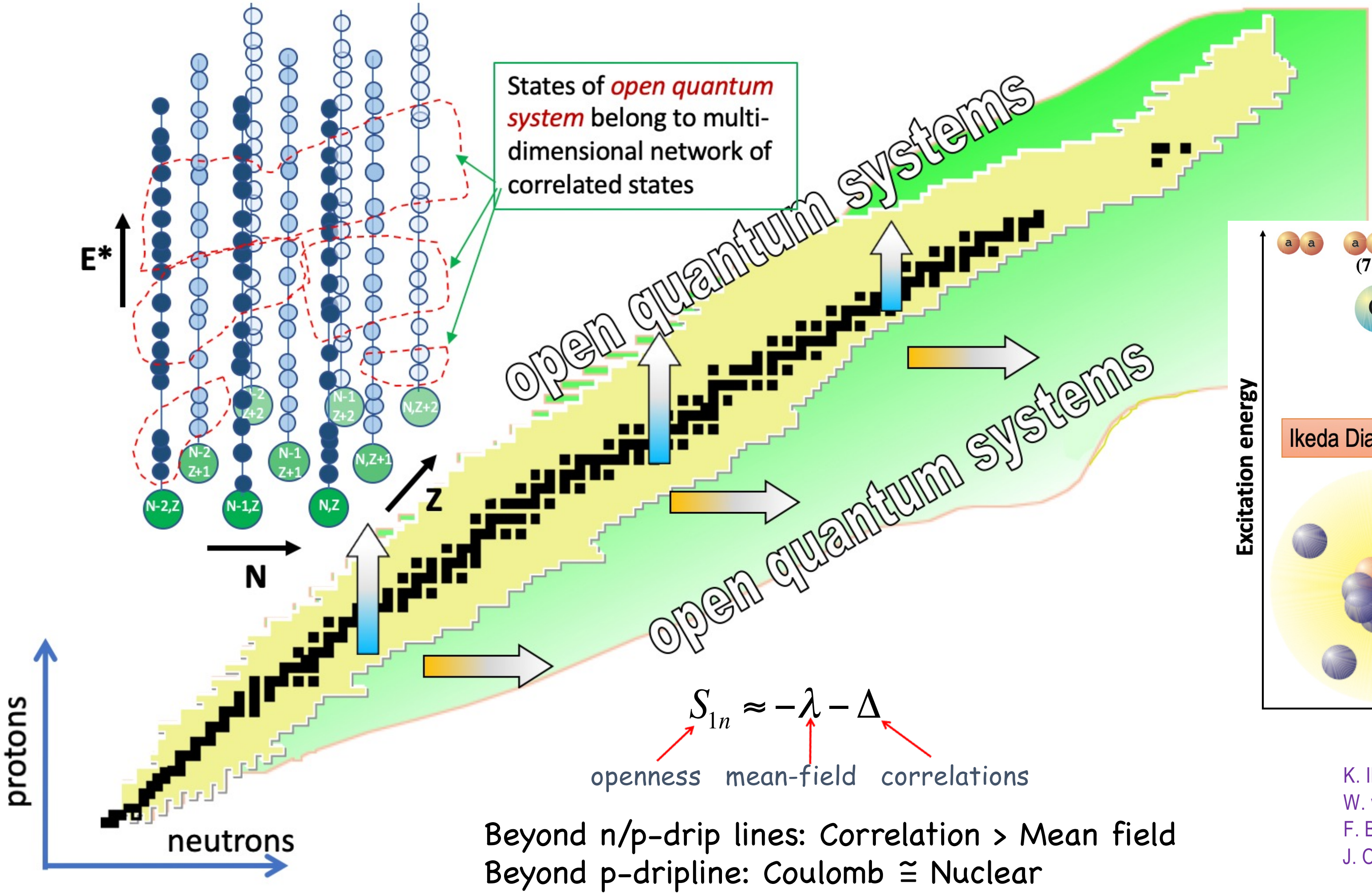
→ *'Unitarity crisis'* in nuclear theory

Atomic nucleus: the open quantum system



Atomic nucleus: the open quantum system

Why do we care about the continuum?

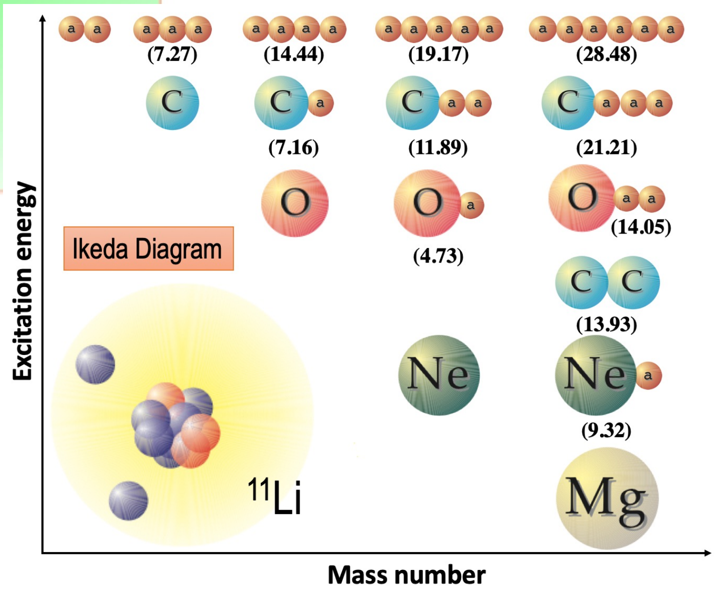


$$S_{1n} \approx -\lambda - \Delta$$

openness mean-field correlations

Beyond n/p-drip lines: Correlation > Mean field
Beyond p-dripline: Coulomb ≅ Nuclear

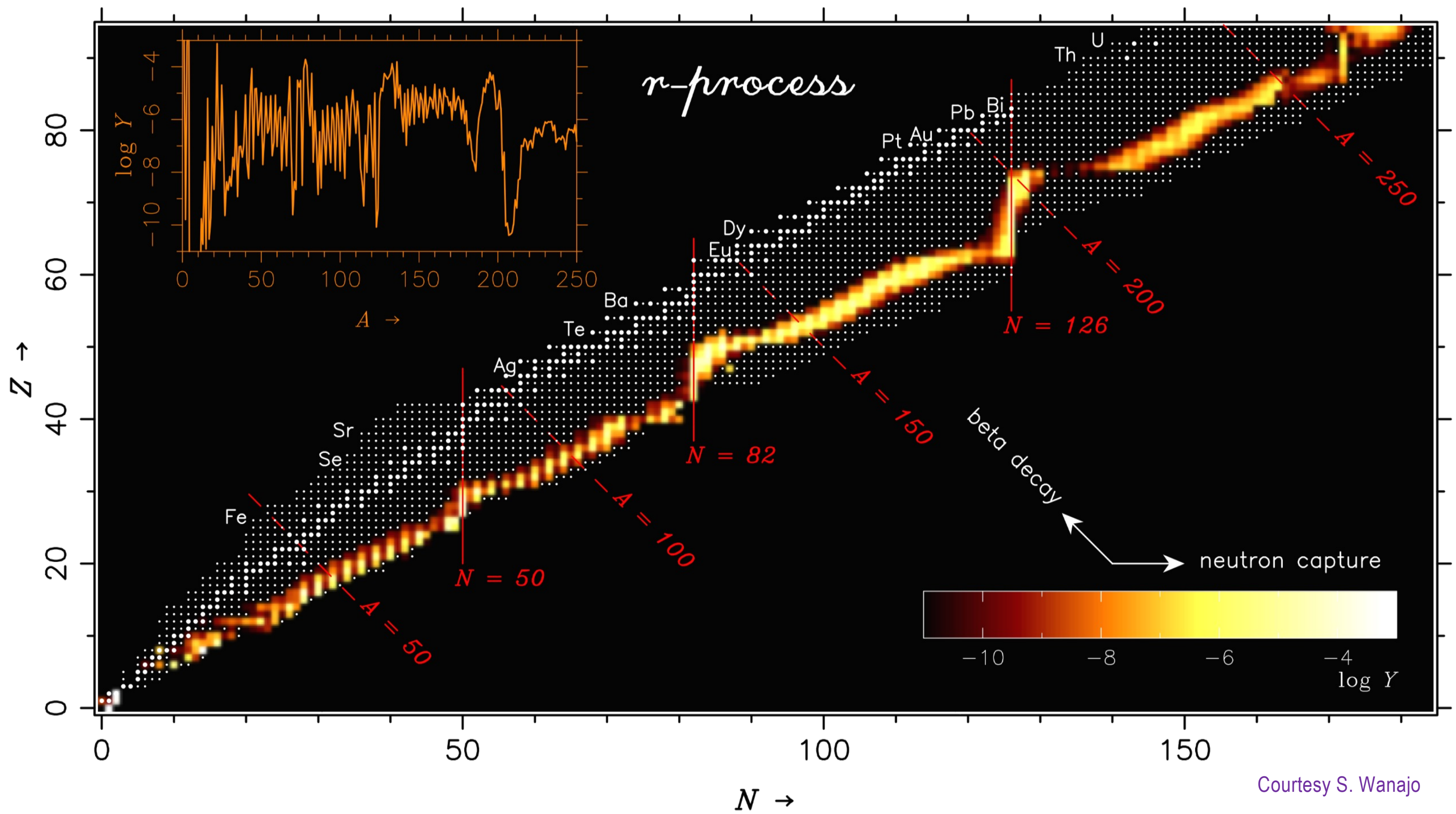
Near-threshold clustering



K. Ikeda, et al., Prog. Theor. Phys. Suppl. E68, 464 (1968)
W. von Oertzen et al., Eur. Phys. J. A 46, 345 (2010)
F. Barker, Proc. Phys. Soc. 84, 681 (1964)
J. Okołowicz et al., Prog. Theor. Phys. Suppl. 196, 230 (2012); Fortschr. Phys. 61, 66 (2013)

Atomic nucleus: the open quantum system

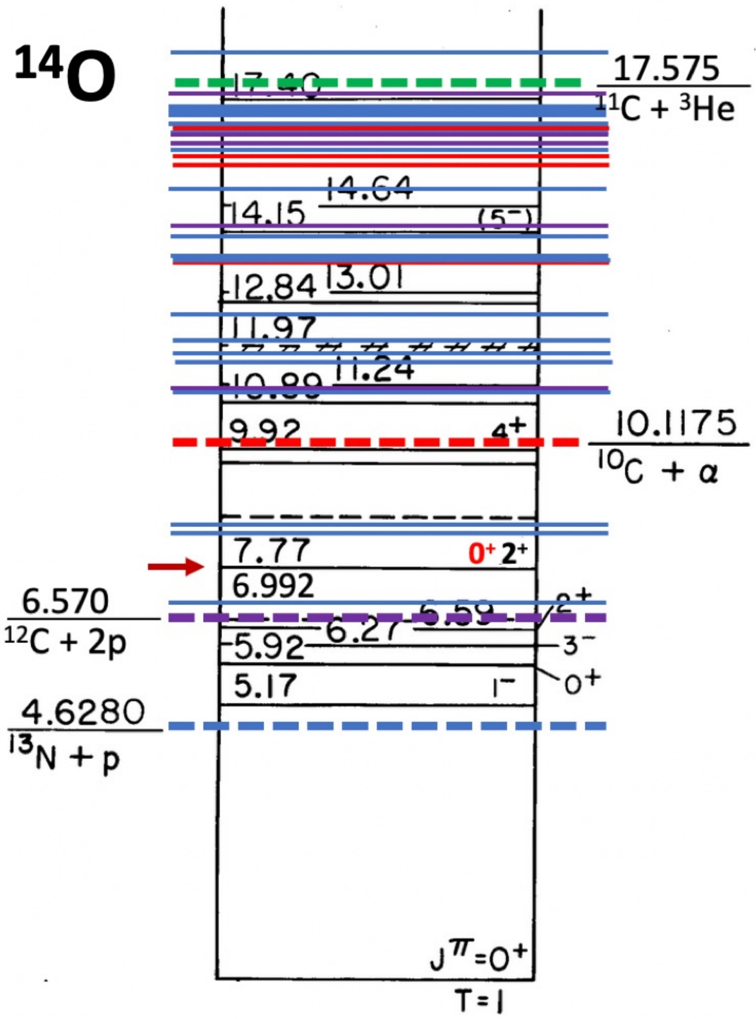
Nucleosynthesis takes place far from the valley of β-stability



Courtesy S. Wanajo

Atomic nucleus: the open quantum system

Why do we care about the continuum?



- Nuclear states are *embedded in the scattering continuum*
- Couplings to various particle emission channels are crucial for the properties of near-threshold states
- Thresholds are **branching points** ➡ *nonanalytic behavior*
  - Wigner threshold law for *elastic and total cross-sections*

    E.P. Wigner, Phys. Rev. 73, 1002 (1948)

$\sigma(i \to j) \sim (k_j)^{2\ell_j+1} \sim (E_j)^{\ell_j+1/2}$ for endoergic reactions: the production of slow neutral particles

$\sigma(i \to j) \sim (k_i)^{2\ell_i-1} \sim (E_i)^{\ell_i-1/2}$ for exoergic reactions: the absorption of slow neutral particles

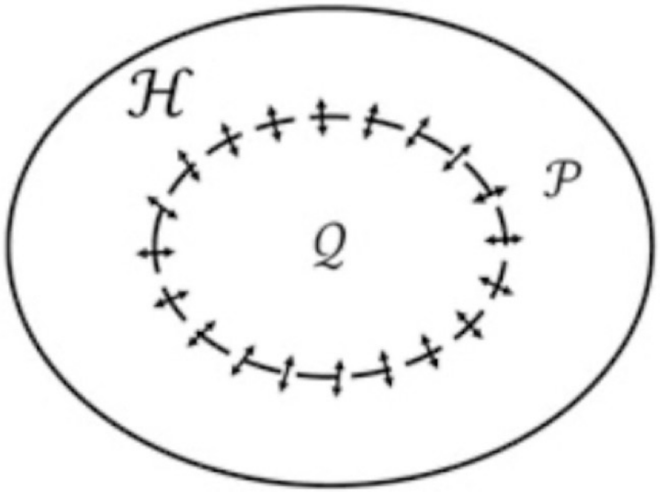
  - Analogous law for *spectroscopic factors*

    N. Michel, W. Nazarewicz., M. P., Phys. Rev. C(R) 75, 031301 (2007)

➡ Shell model for *open* quantum systems

Shell model for open quantum systems

Non-Hermitian QM in Hilbert space



Shell model embedded in the continuum (SMEC)

$$H^{(SM)} \underset{[N\times N]}{} \rightarrow \underset{[N\times N]}{\mathcal{H}^{eff}(E)} = \underset{[N\times N]}{H'(E)} - (i/2)\underset{[N\times k]}{V(E)}\underset{[k\times N]}{V^{T}(E)}$$

$$= \underbrace{H^{(SM)} + u(E)}_{\text{Hermitian}} - \underbrace{(i/2)w(E)}_{\text{anti-Hermitian}}$$

Coupling to the environment (in P) cannot be reduced to refitting the Hamiltonian of the *closed* *QS*

Coupling of 'internal' (in Q) and 'external' (in P) states induces effective A-particle correlations

C. Mahaux, H.A. Weidenmüller,
« *Shell Model Approach to Nuclear Reactions* »
(North-Holland Publishing Company, 1969)

J. Okołowicz, M. P., I. Rotter,
Physics Reports 374, 271 (2003)

Shell model for open quantum systems

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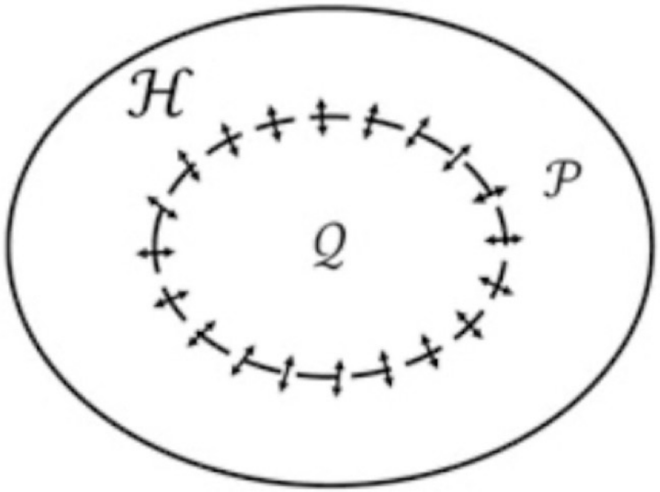

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[N×N] [N×N] [N×N] [N×k] [k×N]

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Coupling to the environment (in P) cannot be reduced to refitting the Hamiltonian of the *closed QS*

Coupling of 'internal' (in Q) and 'external' (in P) states induces effective A-particle correlations

Open QS solution in Q space

$$\mathcal{H}_{QQ}^{eff}|\Psi_{\alpha}\rangle = \mathcal{E}_{\alpha}(E)|\Psi_{\alpha}\rangle$$
$$\langle\Psi_{\tilde{\alpha}}|\mathcal{H}_{QQ}^{eff} = \mathcal{E}_{\alpha}^{*}(E)\langle\Psi_{\tilde{\alpha}}|$$
$\leftarrow$ $\langle\Psi_{\tilde{\alpha}}|\Psi_{\beta}\rangle = \delta_{\alpha\beta}$

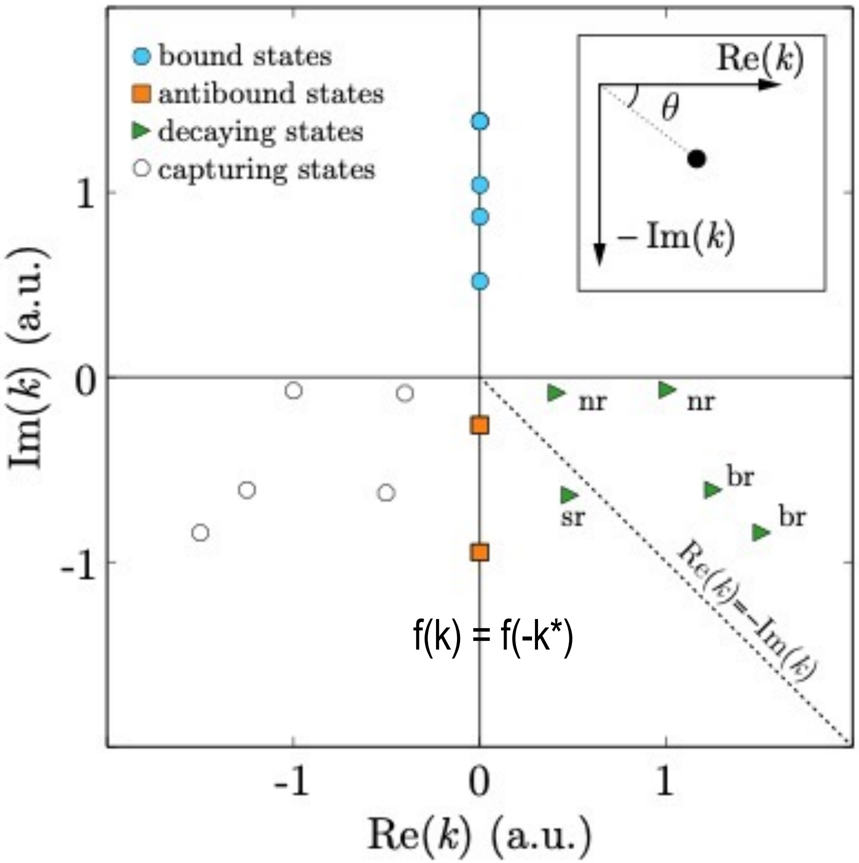
For bound states: $\mathcal{E}_{\alpha}(E)$ is real.
Physical resonances correspond to the poles of the scattering matrix

$$\Psi_{\alpha} = \sum_{i} b_{\alpha i}\Phi_{i}^{(SM)} \Rightarrow \Psi_{E}^{c} \sim \sum_{\alpha} c_{\alpha}\Psi_{\alpha}$$

- Entrance and exit reaction channels defined
  $\Rightarrow$ Shell model and reaction theory reconciled

Shell model for open quantum systems

Quasi-stationary extension in the complex k-plane: Gamow poles



$$i\hbar \frac{\partial}{\partial t}\Phi(r,t) = \hat{H}\Phi(r,t) \ ; \quad \Phi(r,t) = \tau(t)\Psi(r)$$

$$\hat{H}\Psi = \left(e - i\frac{\Gamma}{2}\right)\Psi \quad \Rightarrow \quad \tau(t) = \exp\left(-i\left(e - i\frac{\Gamma}{2}\right)\right)$$

G. Gamow (1928)

$$\Psi(0,k) = 0 \ , \quad \begin{cases} \Psi(\vec{r},k) \underset{r\to\infty}{\rightarrow} O_l(kr) \\ \Psi(\vec{r},k) \underset{r\to\infty}{\rightarrow} I_l(kr) + O_l(kr) \end{cases}$$

Only bound states are integrable!

Euclidean inner product $\quad\Rightarrow\quad$ Rigged Hilbert Space inner product

$$\langle u_n | u_n \rangle = \int_0^\infty dr u_n^*(r) u_n(r) \quad \Rightarrow \quad \langle \tilde{u}_n | u_n \rangle = \int_0^\infty dr \tilde{u}_n^*(r) u_n(r)$$

Rigged Hilbert Space (RHS) is the natural setting of QM in which resonance spectrum, Dirac bra-ket formalism (and Heisenberg uncertainty relations) have place

I.M. Gel'fand and N. J. Vilenkin. Generalized Functions, vol. 4: *Some Applications of Harmonic Analysis. Rigged Hilbert Spaces*, Academic Press, New York, 1964

G. Ludwig, *Foundation of Quantum Mechanics*, Vol. I and II, Springer-Verlag, New York, 1983

Shell model for open quantum systems

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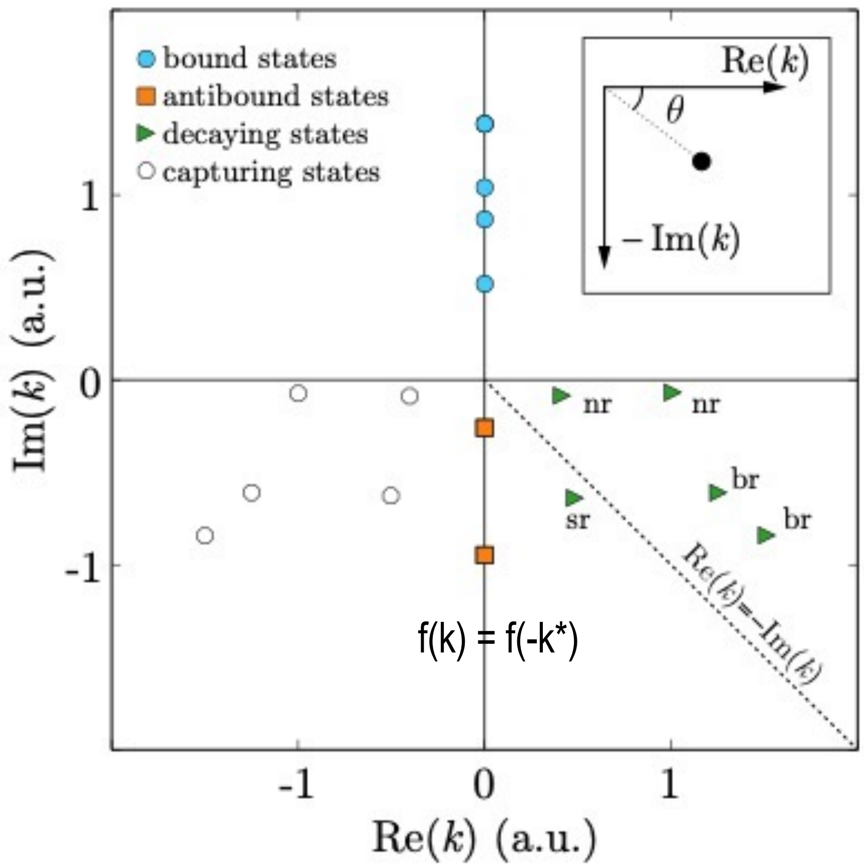
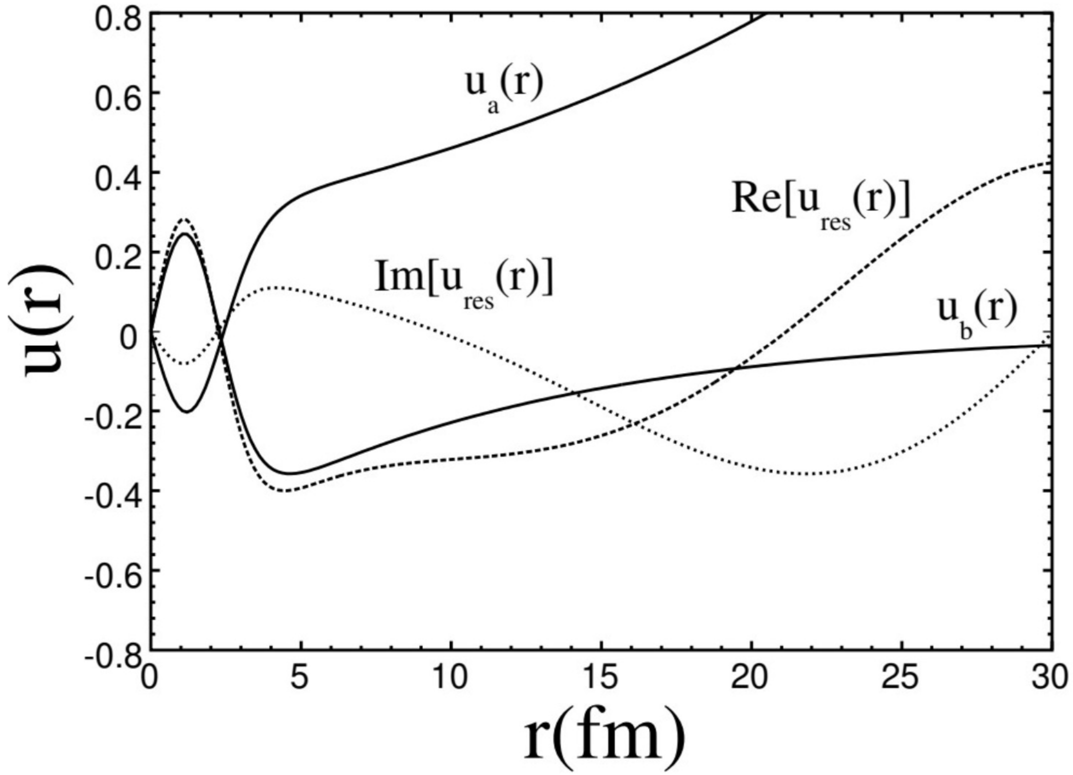
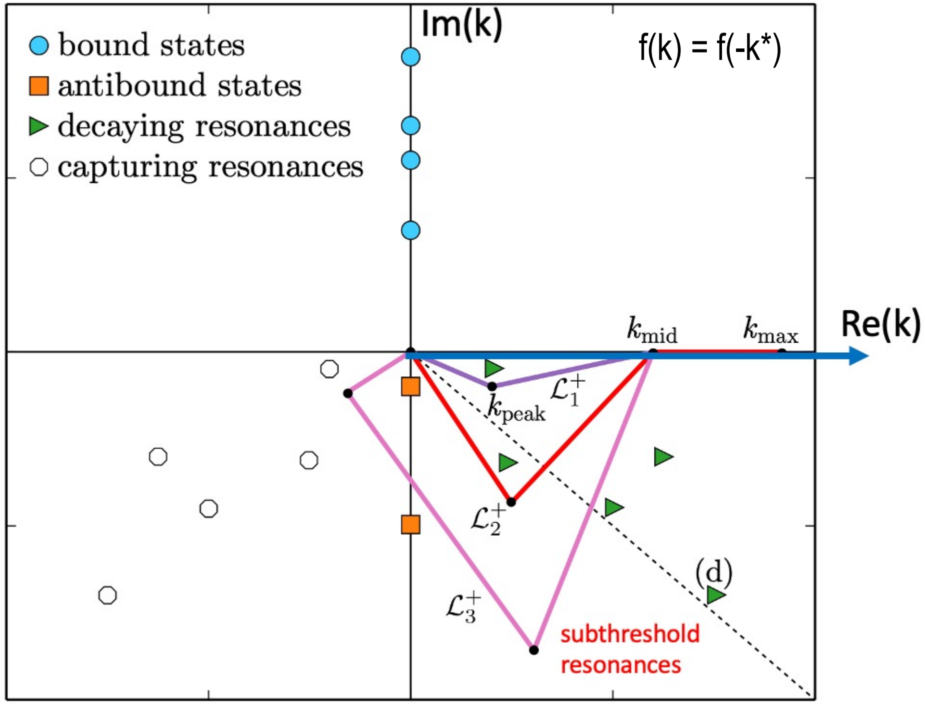




- Bound and resonance states: same object
- Similar behavior inside the nucleus
- Asymptote is different for bound, virtual, and resonance states

Shell model for open quantum systems

Gamow shell model



$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k| dk = 1 \;;\; \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

T. Berggren, Nucl. Phys. A109, 265 (1968)
K. Maurin, Generalized Eigenfunction Expansion, Polish Scientific Publishers, Warsaw (1968)
T. Lind, Phys. Rev. C47, 1903 (1993)

Gamow shell model (GSM)

$$|SD_i\rangle = |u_{i_1} \ldots u_{i_A}\rangle \Rightarrow \sum_k |SD_k\rangle\langle SD_k| \cong 1$$

N. Michel et al, PRL 89, 042502 (2002)
N. Michel, et al, J. Phys. G37, 064042 (2010)

- Calculation in the relative coordinates of core cluster SM coordinates Y. Suzuki, K. Ikeda, PRC 38 (1988) 410
- Center-of-mass handled by recoil term:

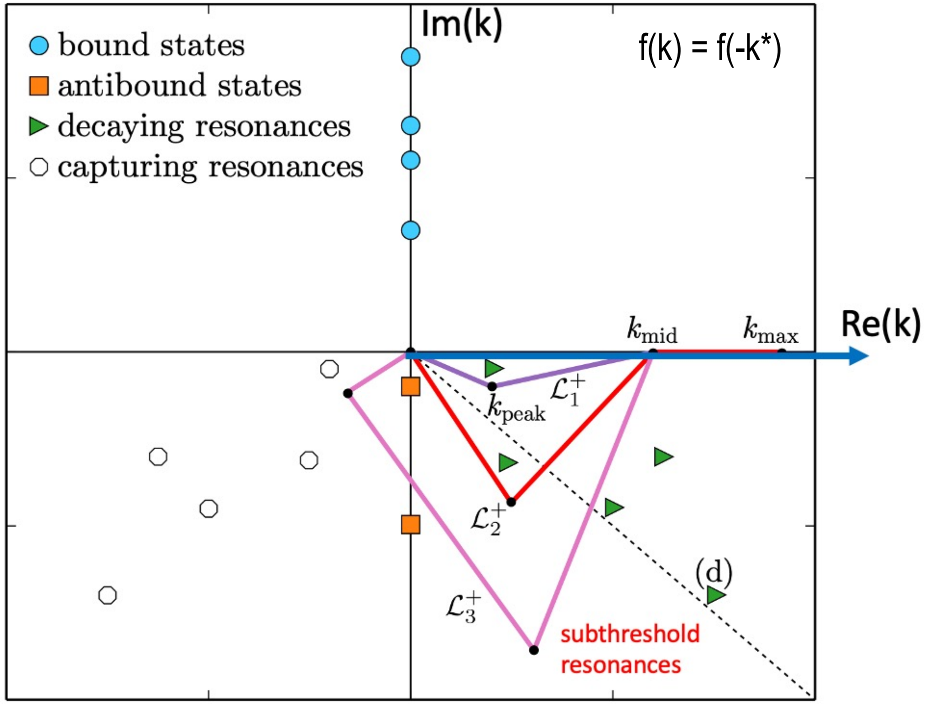
$$H \rightarrow H + \frac{1}{M_{\rm core}} \sum_{(i<j)\in {\rm val}} \mathbf{p}_i \cdot \mathbf{p}_j$$

in the Hamiltonian

- *Unitary formulation* of the nuclear Shell Model
- *No identification of reaction channels*

Shell model for open quantum systems

Gamow shell model



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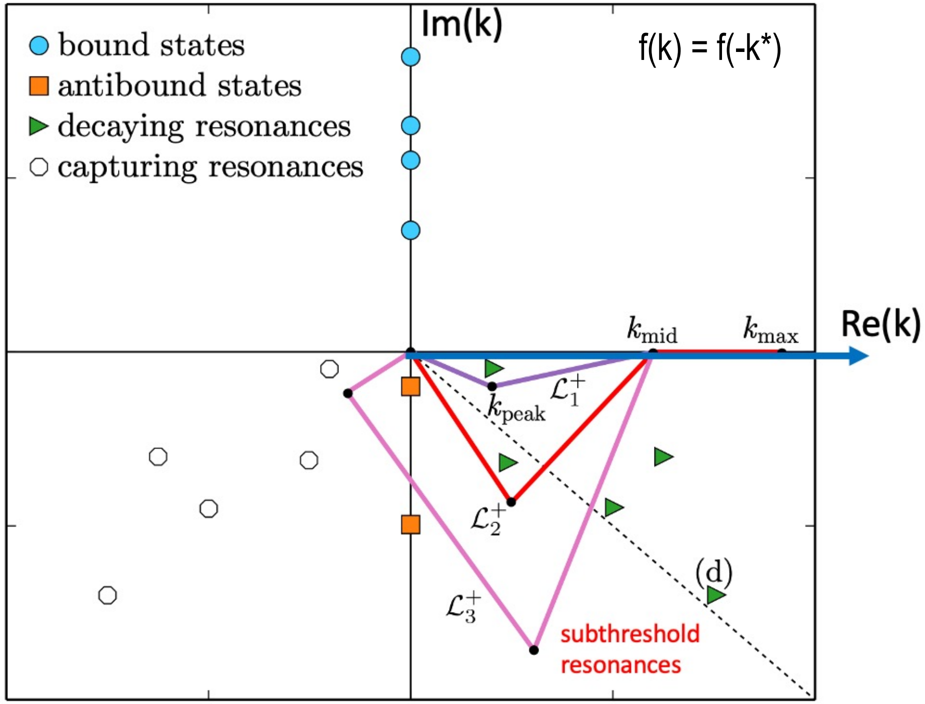
Resonant states of the NN system

- **np** bound state (deuteron): $k=+i0.2315$ fm^{-1} **T=0**
- **np** virtual state (deuteron): $k=-i0.044$ fm^{-1} **T=1**
- **nn** virtual state: $k=-i0.0559(33)$ fm^{-1} **T=1**
  V.A. Babenko, N.M. Petrov, Phys. At. Nucl. 76, 684 (2013)
- **pp** threshold resonant state: $k=(0.0647-i0.0870)$ fm^{-1} **T=1**
  L.P. Kok, Phys. Rev. Lett. 45, 427 (1980)

Shell model for open quantum systems

Gamow shell model in the coupled-channel representation



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$$|\Psi_M^J\rangle = \sum_c \int_0^{+\infty} |(c,r)_M^J\rangle \frac{u_c^{JM}(r)}{r} r^2 \, dr$$

$$|(c,r)\rangle = \hat{\mathcal{A}}[|\Psi_T^{J_T}; N_T, Z_T\rangle \otimes |r \ L_{\rm CM} \ J_{\rm int} \ J_{\rm P}; n, z\rangle]_M^J$$

$$H|\Psi_M^J\rangle = E|\Psi_M^J\rangle \Rightarrow \sum_c \int_0^{\infty} r^2 \left(H_{cc'}(r,r') - E N_{cc'}(r,r')\right) \frac{u_c(r)}{r} = 0$$

$$H_{cc'}(r,r') = \langle (c,r)|\hat{H}|(c',r')\rangle$$

$$N_{cc'}(r,r') = \langle (c,r)|(c',r')\rangle$$

- Entrance and exit reaction channels defined
  ⇒ *Unification of nuclear structure and reactions*
- Reaction channels with different (binary) mass partitions
- Core is arbitrary

Y. Jaganathen et al, PRC 88, 044318 (2014)
K. Fossez et al., PRC 91, 034609 (2015)
A. Mercenne et al., PRC 99, 044606 (2019)

N. Michel, M. P.,
«Gamow Shell Model: The Unified Theory of Nuclear Structure and Reactions »
Lecture Notes in Physics, Vol. 983, (Springer Verlag, 2021

Shell model for open quantum systems

NN interaction in different regimes of binding

★ B(ja,jb)=-10 MeV
★ B(ja,jb)= -1 MeV
★ B(ja,jb)=+1 MeV

ℓ=p,d,f,g,h
Minnesota interaction

$\Re(V_{12})=E_n/\langle E_n\rangle$; $E_n=E-e_a-e_b$

$\langle E_n\rangle=|\Sigma_J(2J+1)(E-e_a-e_b)/\Sigma_J(2J+1)|$

$\Im(V_{12})=\Gamma_n/\langle\Gamma_n\rangle$; $\Gamma_n=\Gamma-\gamma_a-\gamma_b$

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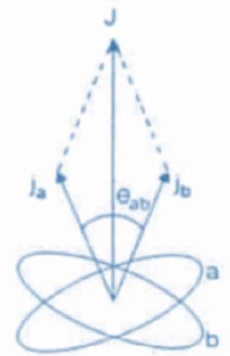
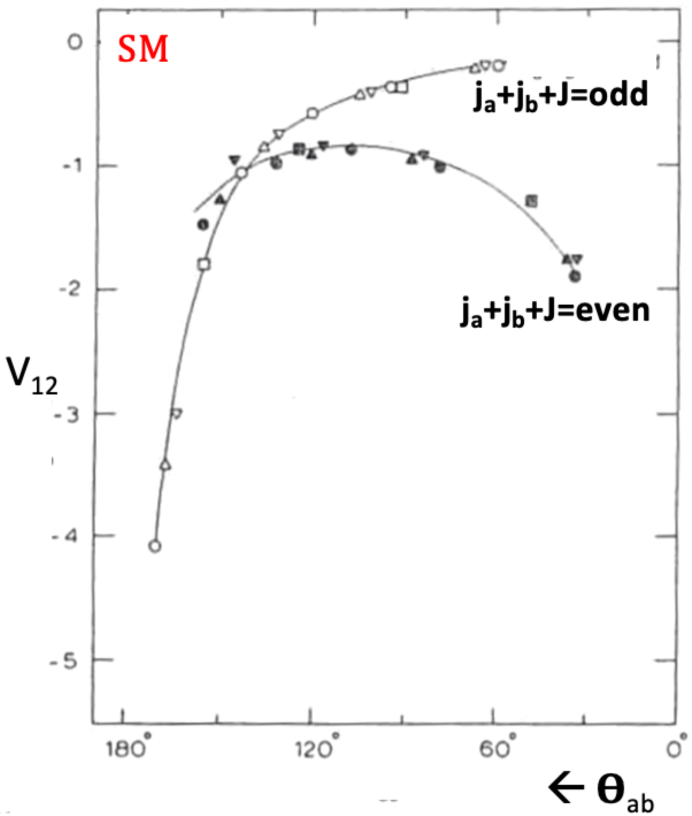
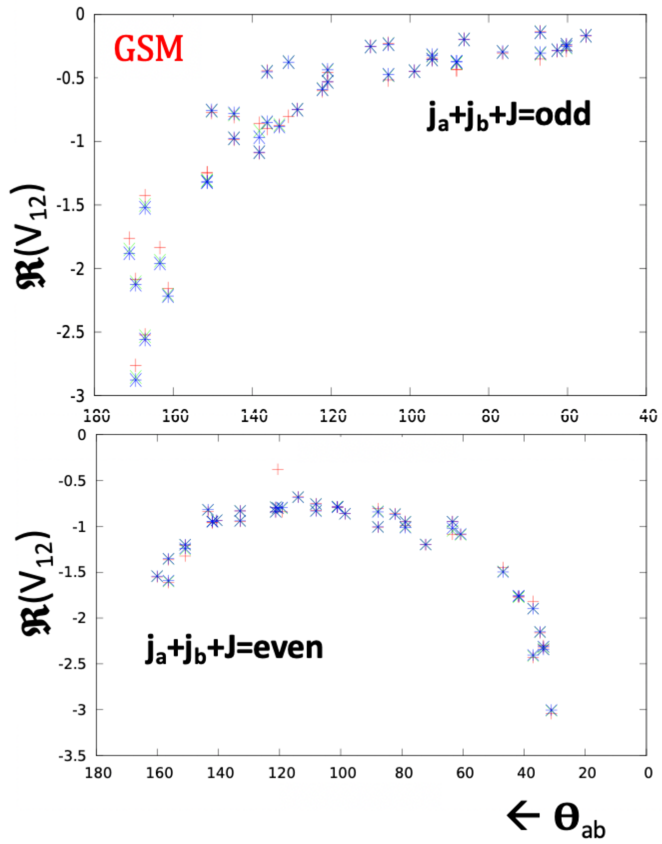


$$\cos(\theta)=\frac{J(J+1)-j_a(j_a+1)-j_b(j_b+1)}{2\sqrt{j_a(j_a+1)j_b(j_b+1)}}$$





N. Michel, M. P.,
«*Gamow Shell Model: The Unified Theory of Nuclear Structure and Reactions* »
Lecture Notes in Physics, Vol. 983, (Springer Verlag, 2021)

- Similar qualitative dependence of the TBMEs on angle θ_{ab} in SM and GSM
- *TBMEs are complex* in weakly bound/unbound nuclei

Shell model for open quantum systems

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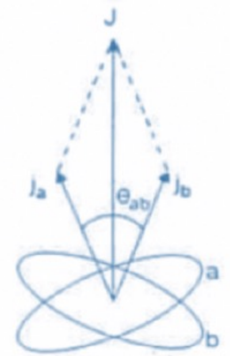

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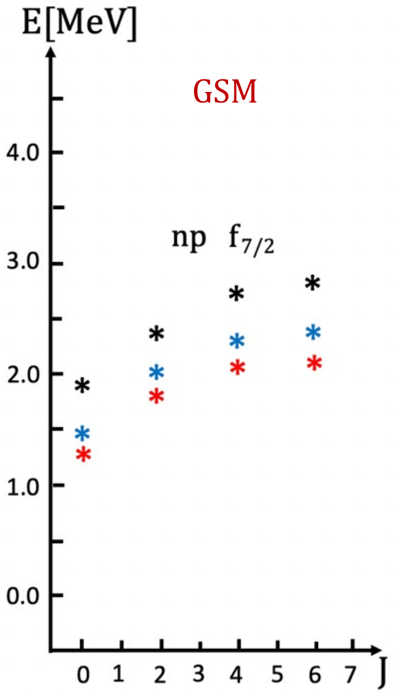
* (B(ja), B(jb)) = (-10, -10) MeV
* (B(ja), B(jb)) = (-1, -10) MeV
* (B(ja), B(jb)) = (+1, -10) MeV



Strong reduction of np interaction
in weakly bound/unbound nuclei:
~50% reduction in *p*-shell

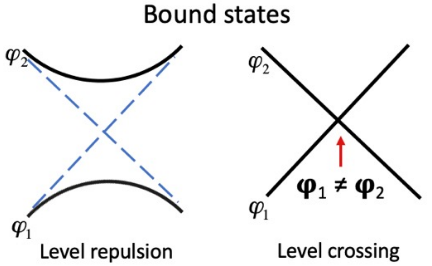
Dependence of V_{nn}/V_{pp} on $S_n - S_p$ asymmetry

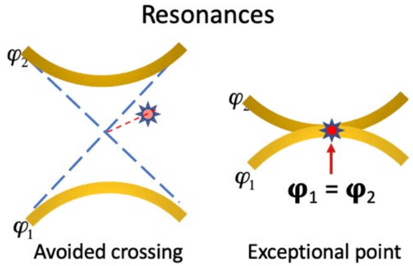
ℓ_j	J^π	S_p[MeV]	S_n[MeV]	V_{nn}/V_{pp}
$P_{1/2}$	2^+	10	-1	0.39
		1	-1	0.58
$d_{5/2}$	2^+	10	-1	0.83
		1	-1	0.835
	4^+	10	-1	0.75
		1	-1	0.84

- Strong asymmetry of V_{nn} and V_{pp} for large $|S_n - S_p|$ and low ℓ_j
- If $S_n \ll S_p$, then $V_{pp} > V_{nn}$, i.e. protons in the neutron-rich environment interact stronger than neutrons
  ➡ Proton SF is reduced with respect to neutron SF (and vice versa) if $S_p << S_n$ ($S_p >> S_n$)

Configuration mixing in open quantum system

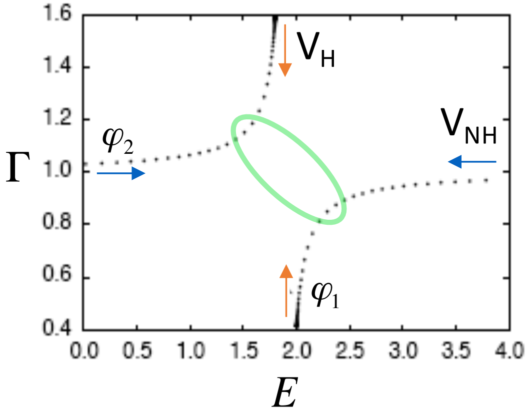
Coalescence of resonance wave functions





Exceptional points (EPs) and avoided crossings are responsible for the configuration mixing in the continuum

V_H	vs	V_{NH}
Level repulsion		Level clustering
Width clustering		Width repulsion



$\omega = V_H + i\,V_{NH}$

$$\mathscr{H} = \begin{pmatrix} \epsilon_1 & \omega \\ \omega & \epsilon_2 \end{pmatrix} \equiv \begin{pmatrix} e_1 - \frac{i}{2}\gamma_1 & 0 \\ 0 & e_2 - \frac{i}{2}\gamma_2 \end{pmatrix} + \begin{pmatrix} 0 & \omega \\ \omega & 0 \end{pmatrix}$$

Resonances coalesce as a result of the interplay between *hermitian* and *non-hermitian* components of the residual interaction

- Bose-Einstein condensation of gases with attractive 1/r – interaction
- Microwave cavity experiments
- Atoms coupled to radiation field
- Atom – cavity quantum composite
- Optical lattices
- Atomic nuclei

M.R. Zirnbauer et al., Nucl. Phys. A411, 161 (1983)
C. Dembowski et al., PRL 86, 787 (2001); PRL 90, 034101 (2003)
J. Okołowicz, et al, PRC 80, 034619 (2009)

Configuration mixing in open quantum system

Instability of SM eigenstates and appearance of the aligned state

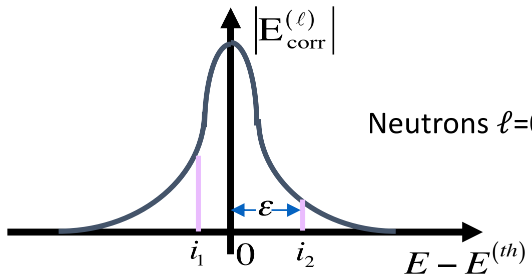
Continuum coupling correlation energy

SMEC: $E_{corr;i}(E) = \langle \Psi_i(E) | \mathcal{H}_{QQ}(E) - H_{QQ} | \Psi_i(E) \rangle$

GSM-CC: $E^{(\mathrm{corr})}_{J^\pi,M} = \langle \tilde{\Psi}^J_M | H | \Psi^J_M \rangle - \langle \tilde{\Phi}^{J;(\alpha)}_M | H | \Phi^{J;(\alpha)}_M \rangle$

$$|\Phi^{J;(\alpha)}_M\rangle = \sum_{\mathrm{c;c}\neq\alpha} \int_0^{+\infty} |(\mathrm{c},r)^J_M\rangle \frac{\bar{u}^{JM}_\mathrm{c}(r)}{r} r^2 \, dr$$

Near-threshold collectivization



Admixture of many-body continuum states
$\Psi^{(SM)}_{[A-1],\mu} \otimes \varphi^{(c)}_{[1],i}$, $\Psi^{(SM)}_{[A-2],\nu} \otimes \varphi^{(c)}_{[2],j}$ with $E > E_{th}$

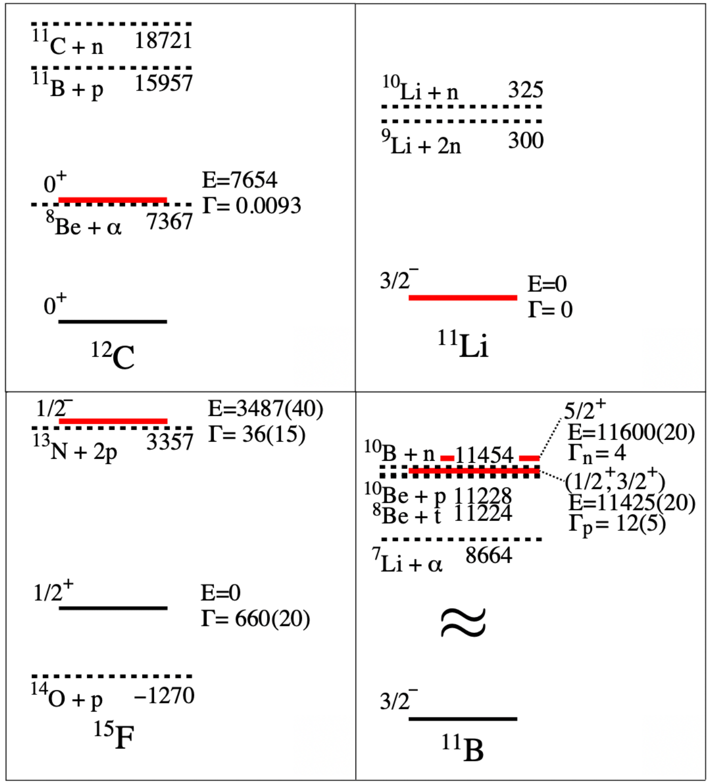
- Emergence of *new energy scale* related to the configuration mixing via decay channel(s)
- Interaction via the continuum leads to a formation of the *collective eigenstate* (*aligned eigenstate*) which couples strongly to the decay channel and carries many of its characteristics
- The *optimal collectivization* is determined by a balance between the Coulomb/centrifugal interactions, and the continuum coupling

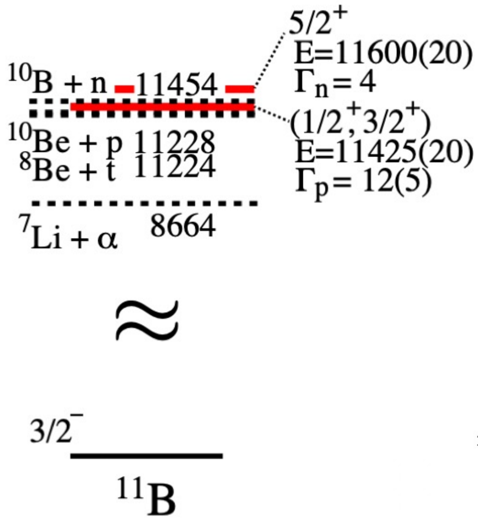
J. Okolowicz, W. Nazarewicz, M.P., Prog. Theor. Phys. Suppl. 196, 230 (2012);
Fortschr. Phys. 61, 66 (2013); Acta Phys. Pol. B45, 331 (2014)

Near-threshold states and origin of clustering

α-clustering "…α-cluster states can be found in the proximity of α-particle decay threshold…" K. Ikeda, N. Takigawa, H. Horiuchi (1968)

But this is only the tip of the iceberg!





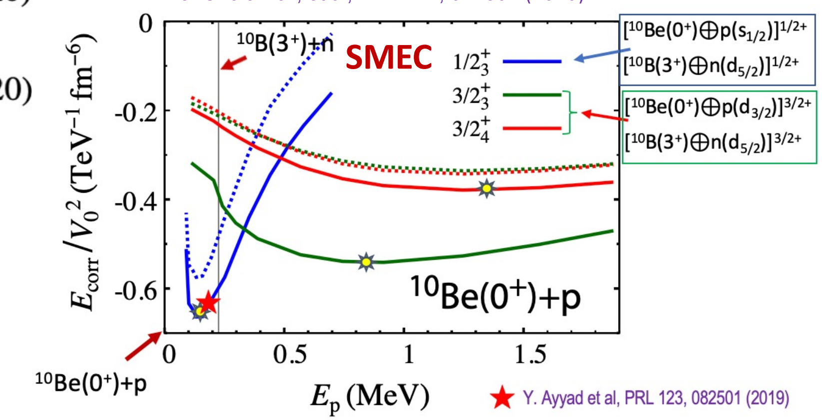
Excitation function of elastic scattering ^{10}Be+p

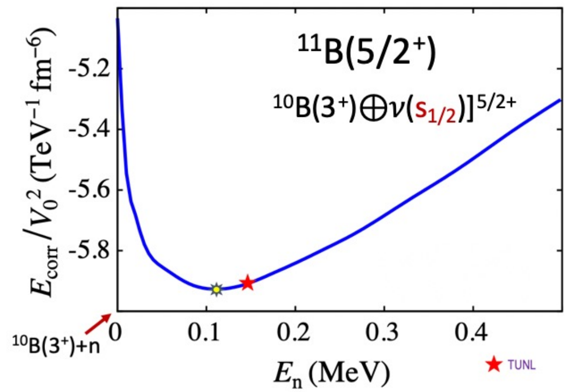
Y. Ayyad et al (MSU Coll.), (2022)

Transfer reaction ^{10}Be(d,n)^{11}B* ⟶ ^{10}Be+p

E. Lopez-Saavedra et al (FSU Coll.), (2022)

J. Okolowicz, et al, PRL 124, 042502 (2020)





- Key resonance in the absorption of thermal neutrons (σ ~ 3800 barn)

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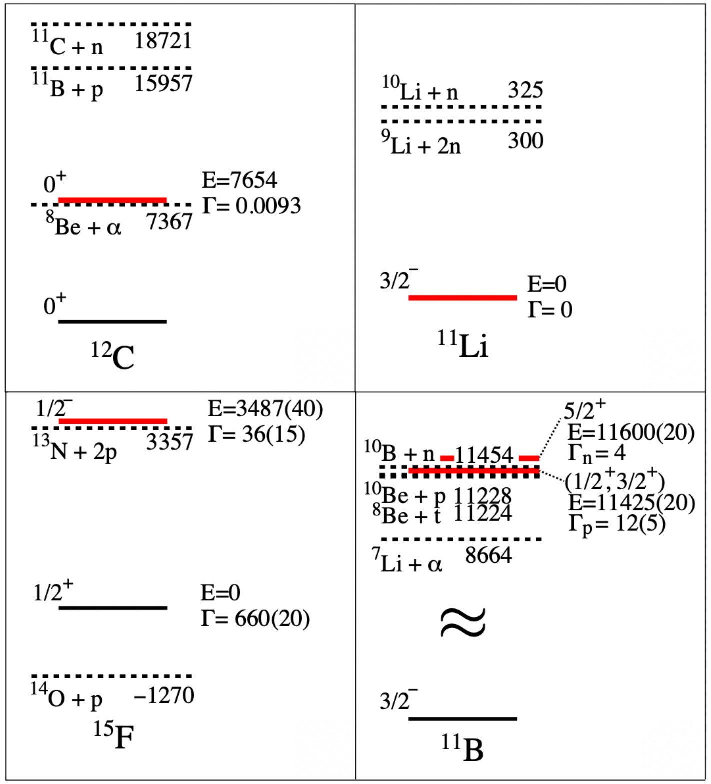

- Other cases: ^{6}He, ^{6}Li, ^{7}Be, ^{7}Li, ^{11}O, ^{11}C, ^{17}O, ^{20}Ne, ^{26}O, ^{24}Mg,…
- *Various clusterings*: ^{2}H, ^{3}He, ^{3}H, 2p, 2n
- *Astrophysical relevance* of near-threshold resonances for α- and proton-capture reactions of nucleosynthesis

- *'Fortuitous'* appearance of correlated states close to open channels?

  ➡ They cannot result from any particular feature of the NN interaction or any dynamical symmetry of the nuclear many-body problem

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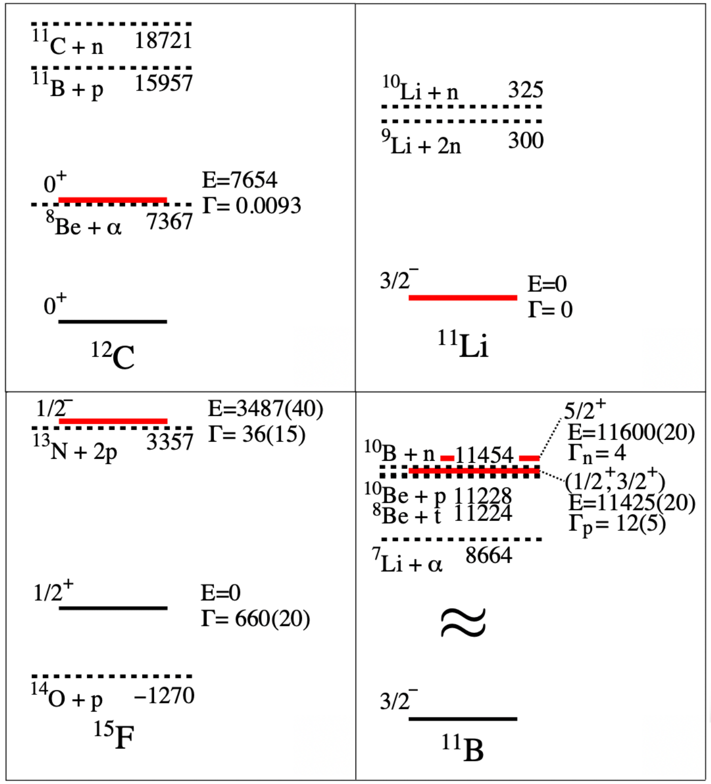

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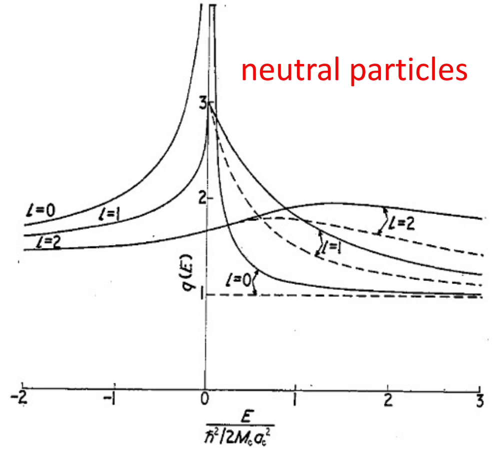

Figure 1. Enhancement factors for neutron channels with orbital angular momenta $l = 0$, 1 and 2 and reduced widths $\gamma_{\lambda c}^2 = \hbar^2/M_c a_c^2$ as functions of channel energy E (in units of $\hbar^2/2M_c a_c^2 \simeq 1$ Mev). Full curves give values of $q(E)$, broken curves values of $q_1(E)$.

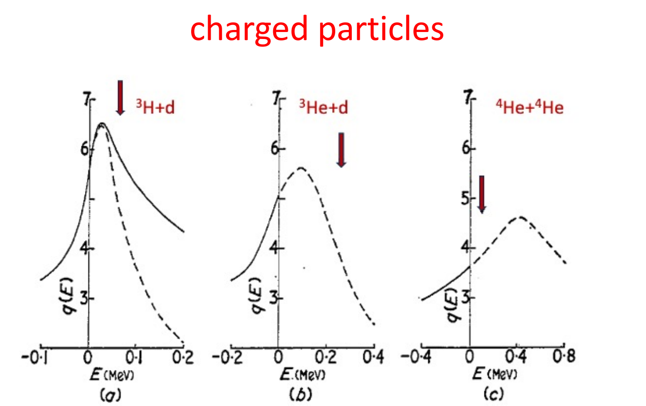

Figure 2. Enhancement factors for channels (a) ^{3}H + d, (b) ^{3}He + d, (c) ^{4}He + ^{4}He, all with $l = 0$ and with values of a_c and $\gamma_{\lambda c}^2$ given in the text. Full curves give values of $q(E)$, broken curves values of $q_1(E)$. Arrows indicate energies of observed levels of ^{5}He, ^{5}Li and ^{8}Be.

F. Barker, Proc. Phys. Soc. 84, 681 (1964)

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- The appearance of near-threshold resonances can be explained in terms of the increased level density: $g_\ell(E) = \frac{1}{\pi}\frac{d\delta_\ell(E)}{dE}$
- The enhancement of the level density is largest for low-barrier potentials

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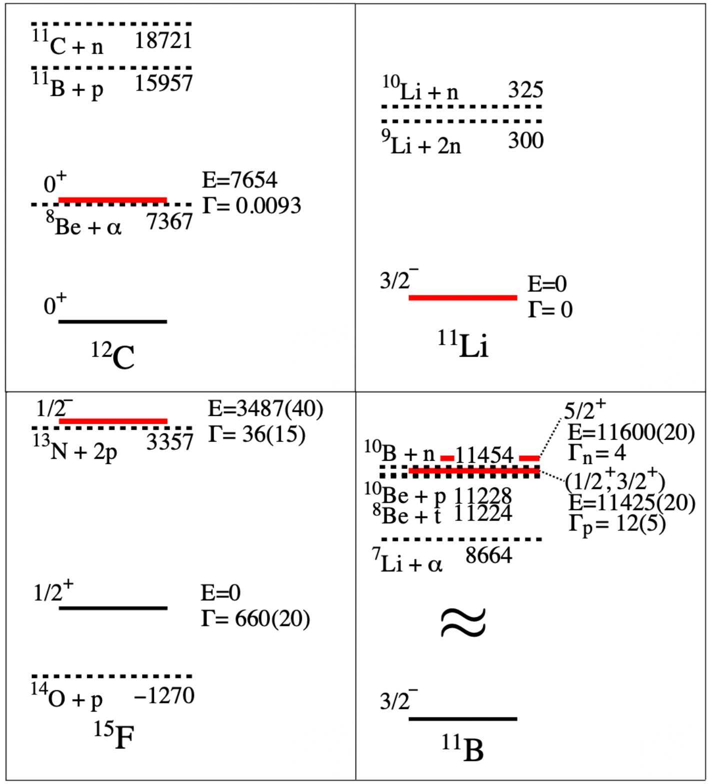

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Continuum shell model perspective

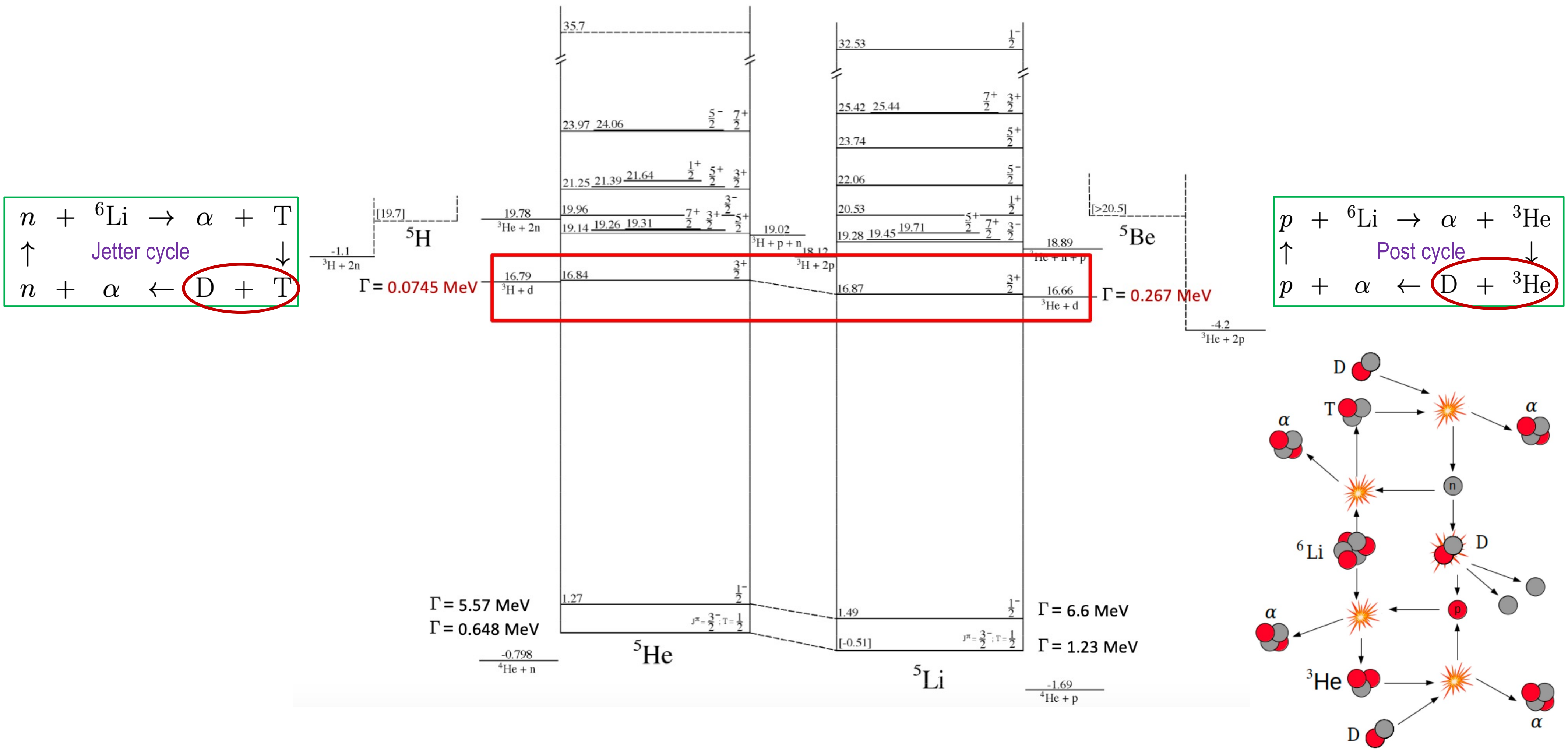
J. Okołowicz, M. P., W. Nazarewicz, Prog. Theor. Phys. Suppl. 196, 230 (2012); Fortschr. Phys. 61, 66 (2013)

- The appearance of correlated (cluster) states close to open channels is the generic *open quantum system phenomenon* related to the collective rearrangement of SM wave functions due to the coupling via the continuum

- Specific aspects:
  - Energetic order of particle emission thresholds depends on (nuclear) Hamiltonian
  - Absence of stable cluster entirely composed of like nucleons

Near-threshold states and origin of clustering

^{5}He, ^{5}Li and fusion

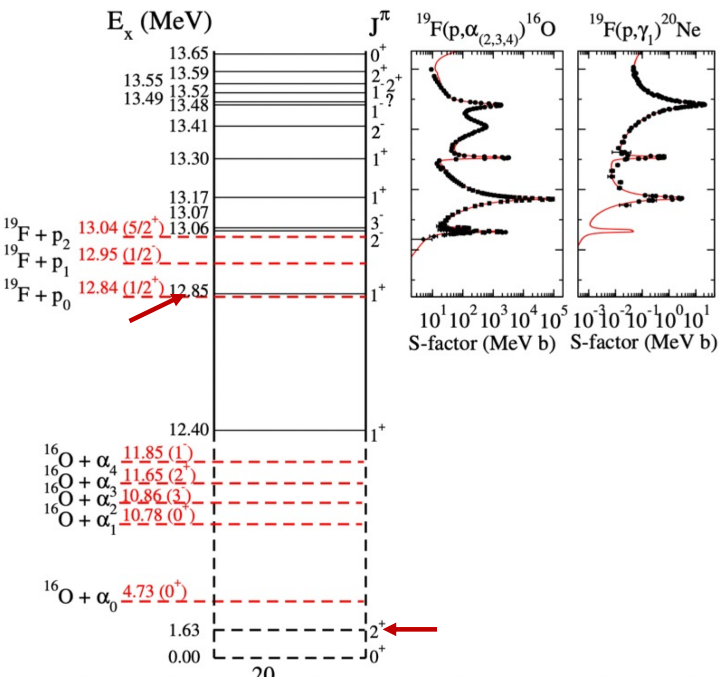


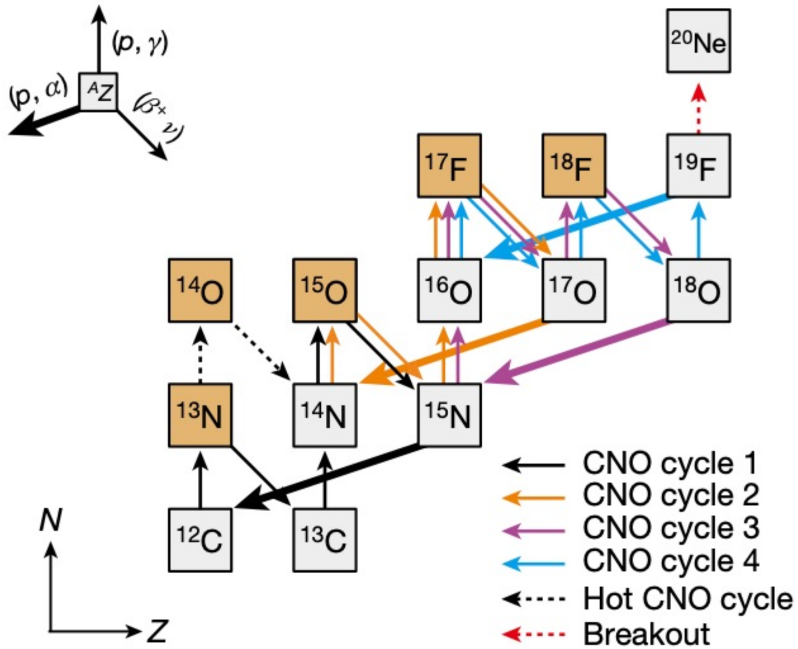
Courtesy A.F. Lopez Loaiza

Near-threshold states and origin of clustering

Astrophysical relevance for α- and proton-capture reactions of nucleosynthesis

R.J. DeBoer et al, Nature 610, 656 (2022)





Liyong Zhang et al., Nature 610, 656 (2022)

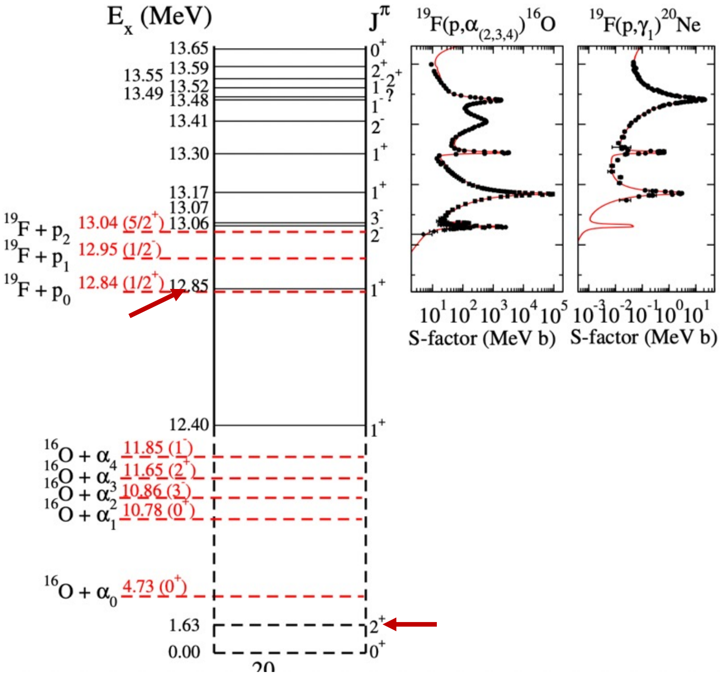
What is the effect of 1$^+$ resonance at ~10 keV above the proton emission threshold on the S-factor?

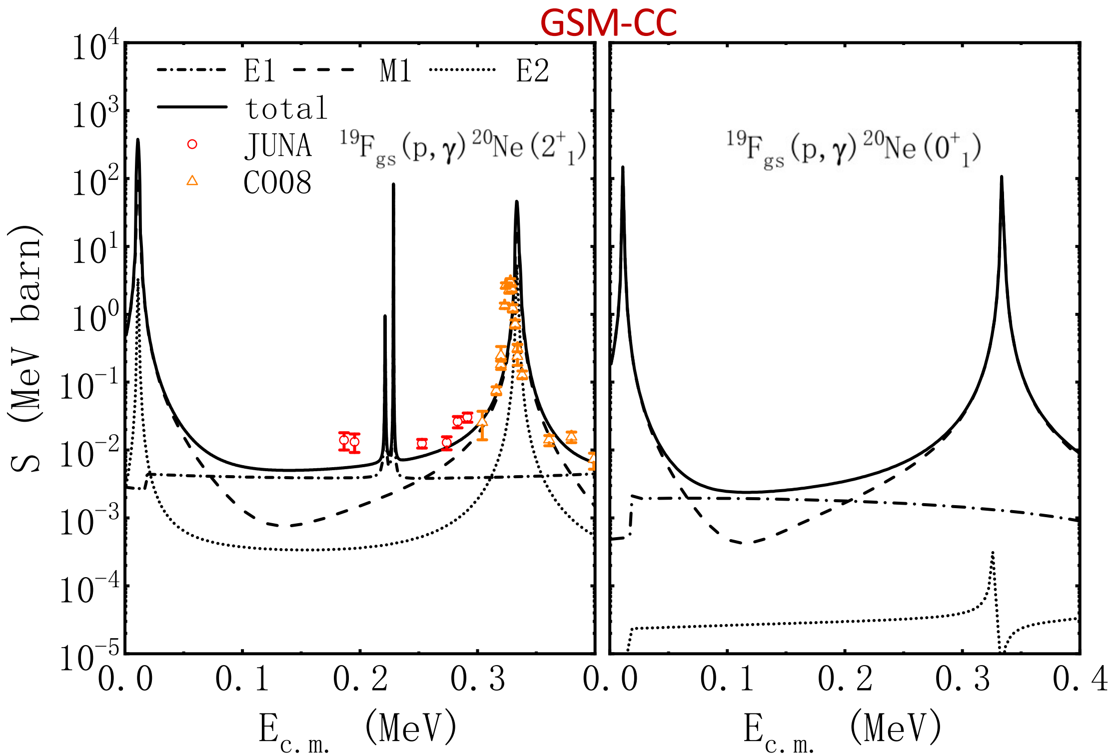
Does ^{19}F(p,γ)^{20}Ne breakout reaction from the CNO cycle overcomes ^{19}F(p,α)^{16}O back-process reaction cross section becoming a source of the Ca abondance in the first generation stars?

Near-threshold states and origin of clustering

Near-threshold resonances in ^{20}Ne and their role for ^{19}F(p,γ)^{20}Ne and and ^{19}F(p,α)^{16}O reaction rates

R.J. DeBoer et al, Nature 610, 656 (2022)





Exp: Liyong Zhang et al., Nature 610, 656 (2022)

What is the effect of 1⁺ resonance at ~10 keV above the proton emission threshold on the S-factor?

- S(0) astrophysical factor increases by more than 2 orders of magnitude!
- The decay to the 2+ first excited state in ^{20}Ne dominates

X.B. Wang, G.X. Dong, N. Michel, M. P. (2024)

Near-threshold states and origin of clustering

Near-threshold resonances in ^{20}Ne and their role for ^{19}F(p,γ)^{20}Ne and and ^{19}F(p,α)^{16}O reaction rates

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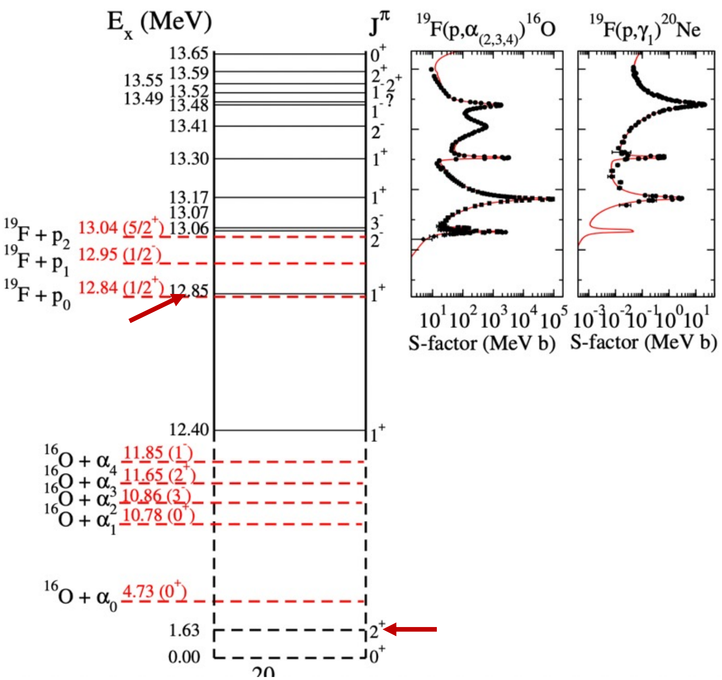

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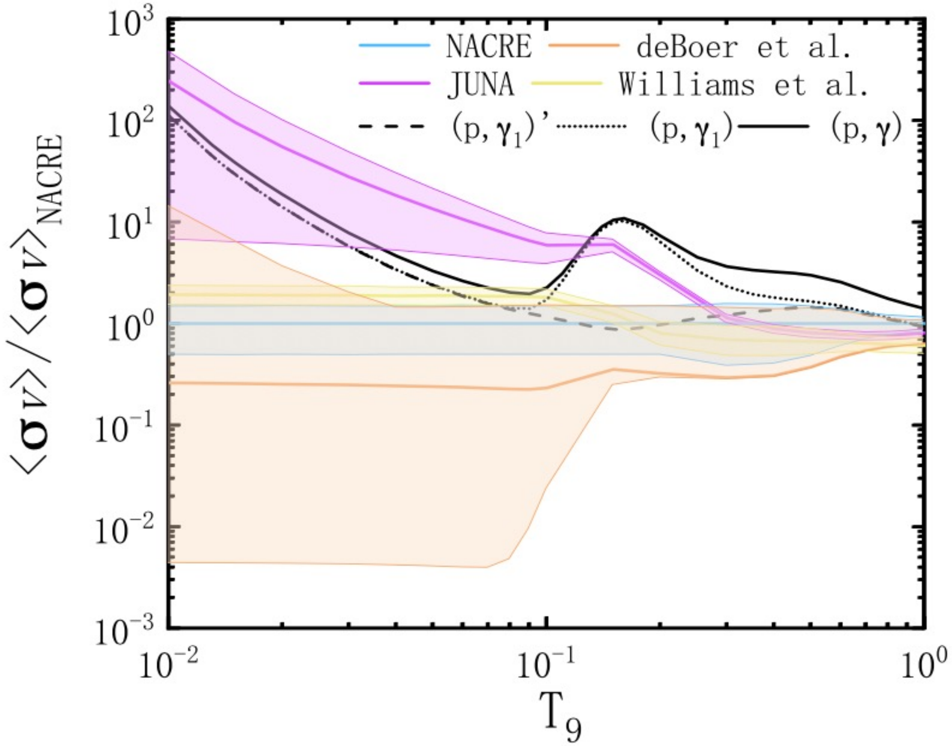
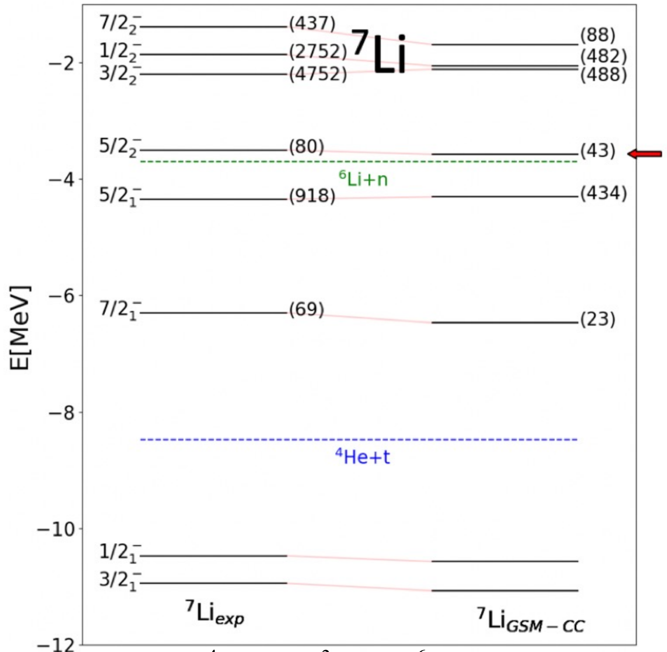


- GSM-CC reaction rates are significantly larger than in NACRE and comparable with JUNA data
- ^{19}F(p,α)^{16}O back-process reaction should be remeasued to verify the hypothesis of breaking from hot-CNO cycle

Mimicry mechanism of clusterization

Chameleon nature of resonances



$[|^4\mathrm{He}\rangle \otimes |^3\mathrm{H}\rangle], [|^6\mathrm{Li}\rangle \otimes |n\rangle]$

- Hamiltonian: 1-body potential, 2-body FHT interaction

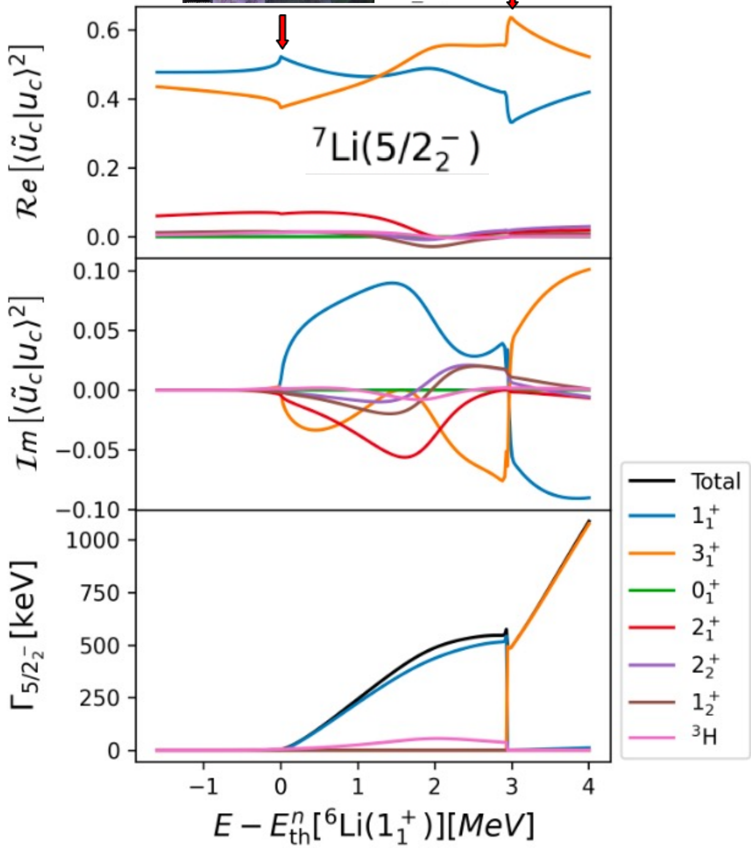
  H. Furutani et al, Prog. Theor. Phys. 62, 981 (1979)

  ^{3}H wave functions calculated using N^{3}LO$_{(2\text{-body})}$ interaction

- Channels: ^{6}Li(K^π): $K^\pi = 1_1^+, 1_2^+, 3_1^+, 0_1^+, 2_1^+, 2_2^+$

  n: $\ell_j = s_{1/2}, p_{1/2}, p_{3/2}, d_{3/2}, d_{5/2}, f_{5/2}, f_{7/2}$

  ^{3}H(L): $L \equiv {}^{2J_{int}+1}[L_{CM}]_{JP} = {}^2S_{1/2}, {}^2P_{1/2}, {}^2P_{3/2}, {}^2D_{3/2}, {}^2D_{5/2}, {}^2F_{5/2}, {}^2F_{7/2}$



- The resonance (*chameleon*) changes its structure (*skin color*) as a result of the alignment (*mimicry*) with the nearby new reaction channel (*changing environment*)

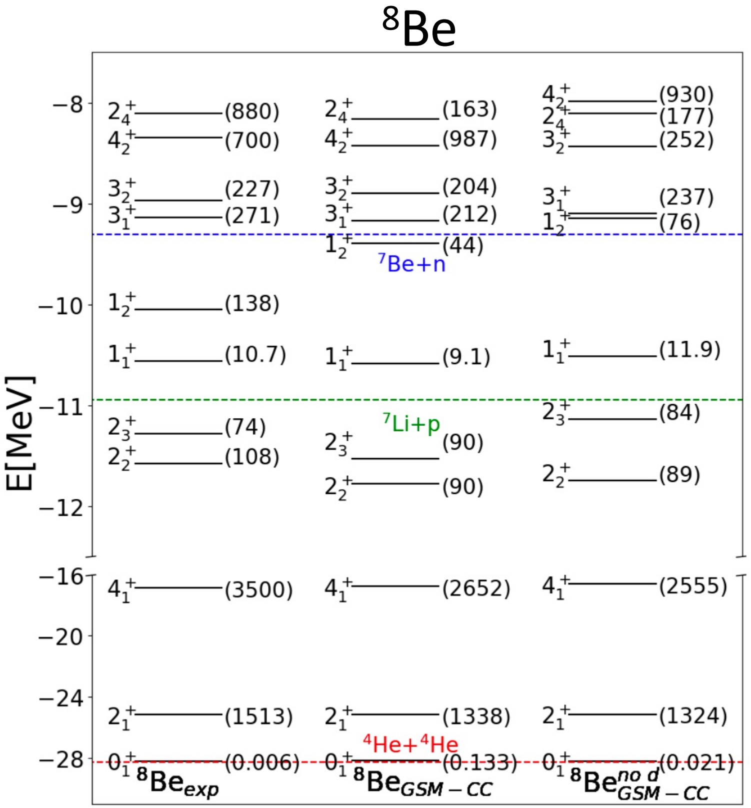
  J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

Mimicry mechanism of clusterization

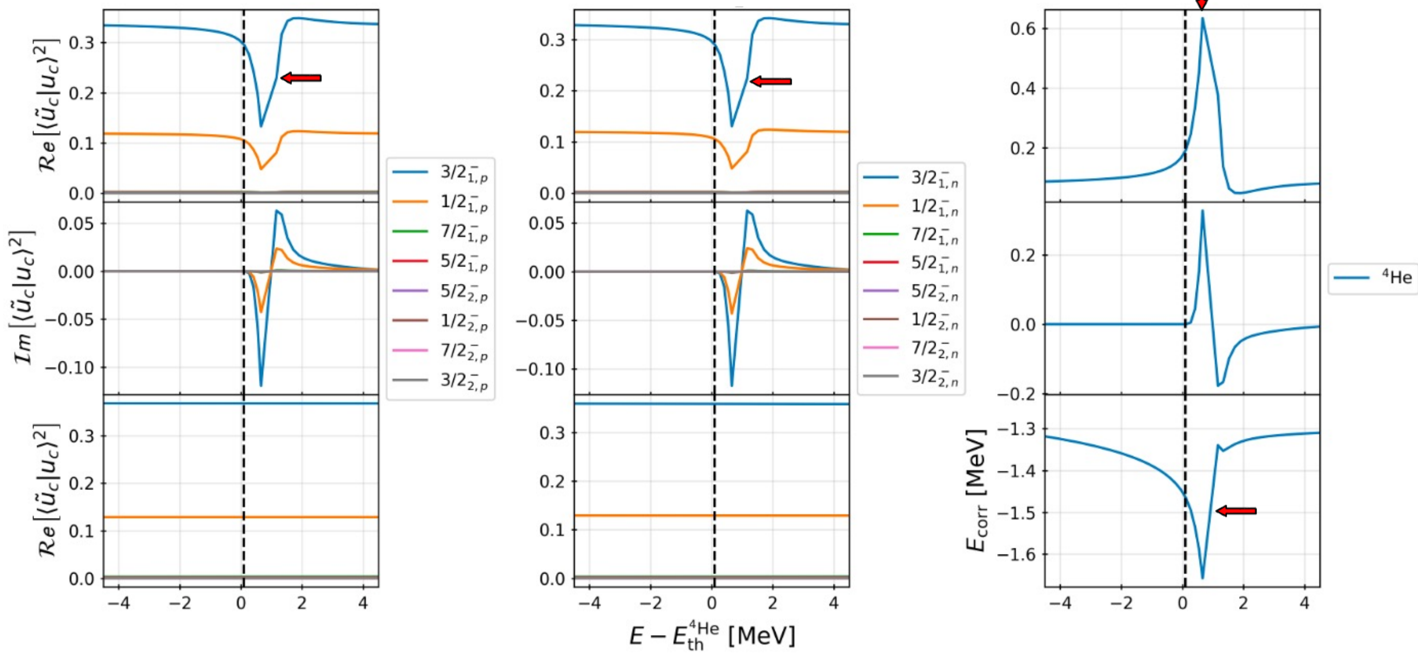
Near-threshold clustering in ^{8}Be

Continuum coupling correlation energy ➡ $E^{(\mathrm{corr})}_{J^\pi,M} = \langle\tilde{\Psi}^J_M|\,H\,|\Psi^J_M\rangle - \langle\tilde{\Phi}^{J;(\alpha)}_M|\,H\,|\Phi^{J;(\alpha)}_M\rangle \equiv \mathcal{E}_{J^\pi,M} - \mathcal{E}^{(\alpha)}_{J^\pi,M}$

$$|\Phi^{J;(\alpha)}_M\rangle = \sum_{\mathrm{c;c}\neq\alpha}\int_0^{+\infty} |(\mathrm{c},r)^J_M\rangle\,\frac{\bar{u}^{JM}_{\mathrm{c}}(r)}{r}r^2\,dr$$



Near-threshold *alignment* of ^{8}Be(0_1^+)



Near-threshold clustering is the *emergent phenomenon* in SM for *open* quantum systems

Mass partitions:

$[|^4\mathrm{He}\rangle\otimes|^4\mathrm{He}\rangle]$, $[|^7\mathrm{Li}\rangle\otimes|\mathrm{p}\rangle]$, $[|^7\mathrm{Be}\rangle\otimes|\mathrm{n}\rangle]$, $[|^6\mathrm{Li}\rangle\otimes|\mathrm{d}\rangle]$

J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

Mimicry mechanism of clusterization

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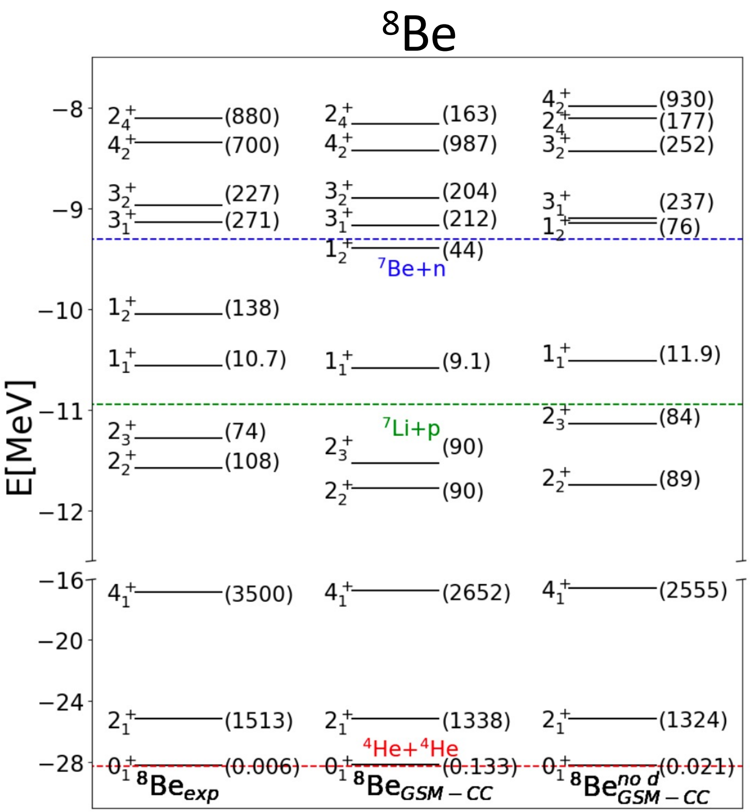

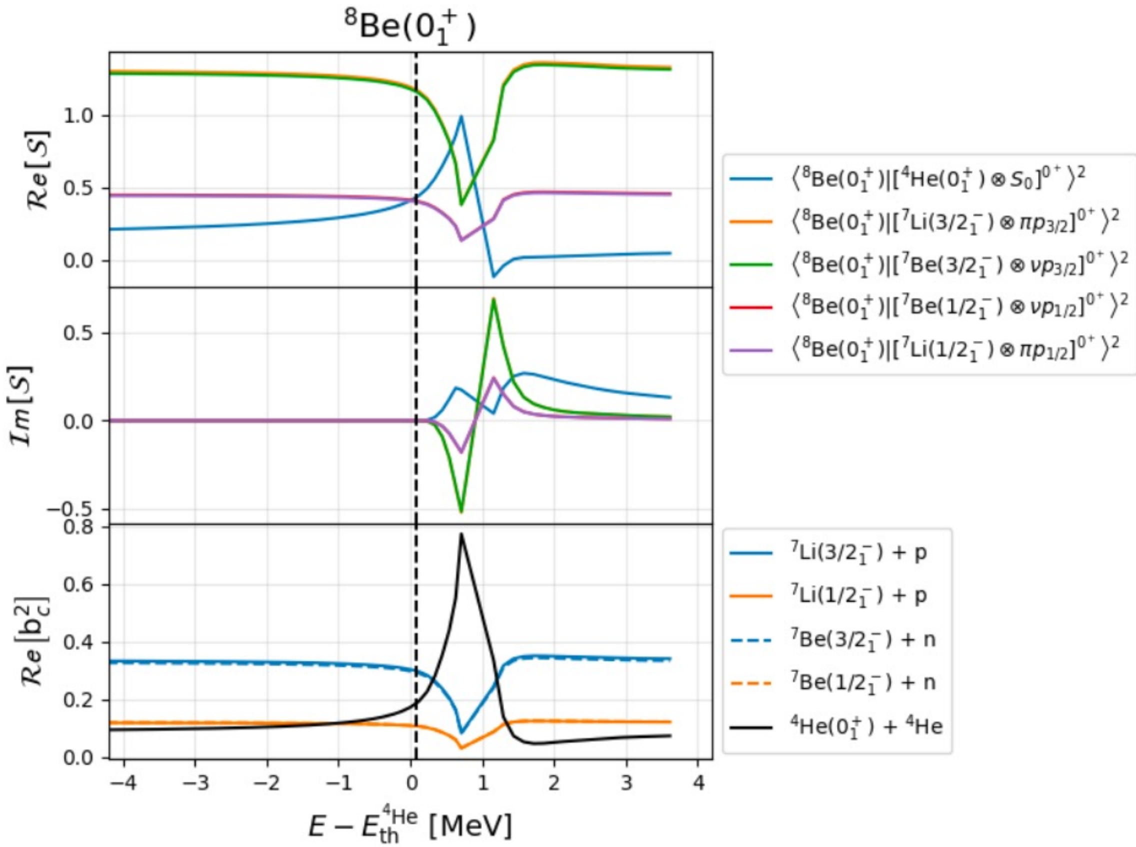

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Near-threshold clustering is the *emergent phenomenon* in SM for *open* quantum systems

J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

Message to take

- Atomic nucleus in the vicinity of a particle emission threshold belongs to the category of *open quantum systems* having unique properties which distinguish them from well-bound *closed quantum systems*

- Proximity of the threshold (branching point) induces the collective mixing of eigenstates resulting in a single *aligned eigenstate* of the open quantum system Hamiltonian (→ *chameleon resonance*)

- The correlated (cluster) states in a vicinity of reaction channel thresholds are the generic manifestations of *openness* of a many-body system related to the *collective rearrangement* of wave functions due to their mutual coupling via the continuum.

  Clustering in the *mimicry mechanism* is the *emergent phenomenon* associated with the branch point singularity at the particle emission threshold.

- Near-threshold phenomena are *terra incognita* of the nuclear physics:
  - *Collectivization* of wave functions due to the coupling to decay channel(s)
  - Formation of clusters/correlations: ^2H, ^3H, ^3He, ^3n, ^4n, … which carry an imprint of nearby decay channel(s)
  - Modification of NN interaction/spectroscopic factors
  - Effects of *coalescing resonances* in nuclear spectroscopy and reactions
  - … …

- The richness of nuclear interaction and the existence of nucleons in four distinct states (proton/neutron, spin-up/spin-down) make studies on the near-threshold phenomena in atomic nucleus unique and exciting!

Thanks to my collaborators:			
	Nicolas	Michel	IMP/CAS Lanzhou/Beijin, China
	Witek	Nazarewicz	MSU/FRIB East Lansing, USA
	Jacek	Okołowicz	INP Kraków, Poland
	Jose Pablo	Linares	LSU Baton Rouge, USA
	Xiaobao	Wang	Huzhou University
	Guoxiang	Dong	Huzhou University

Thank You