Evgeny Epelbaum, Ruhr University Bochum

International School of Nuclear Physics, 45th Course
Erice, September 17-22, 2024

Chiral EFT using Gradient Flow

based on work done in collaboration with Hermann Krebs, e-Prints: 2311.10893; 2312.13932
(both papers accepted for publication in PRC)

…opens an avenue for accurate χEFT calculations beyond the 2N system

- Introduction & state-of-the-art
- Statement of the problem
- Solution: Gradient Flow
- Summary & outlook

European Research Council
Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
Ministerium für Kultur und Wissenschaft des Landes Nordrhein-Westfalen
ERATO Exploratory Research for Advanced Technology

Deuteron as a bound state of quarks and gluons

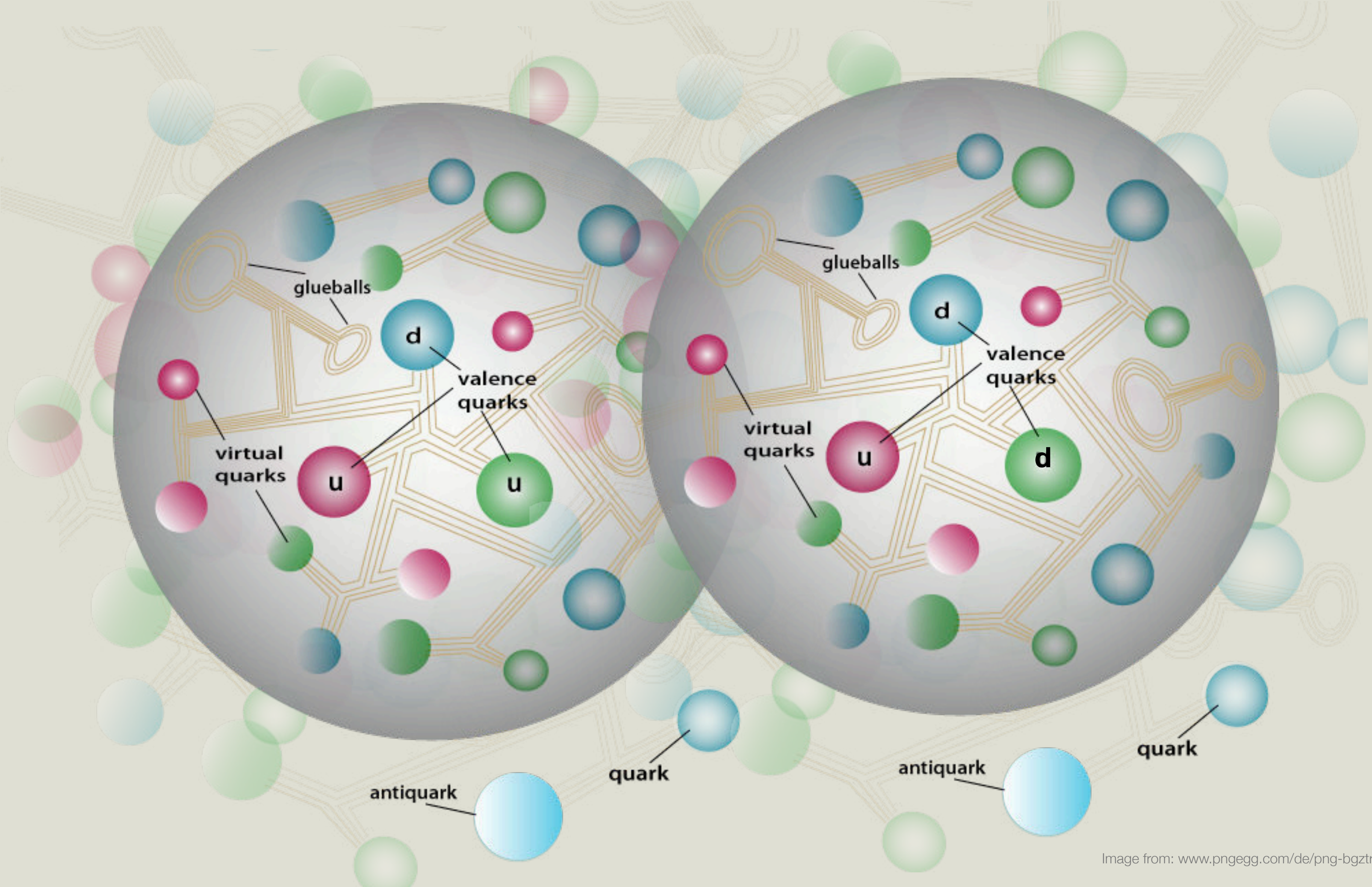
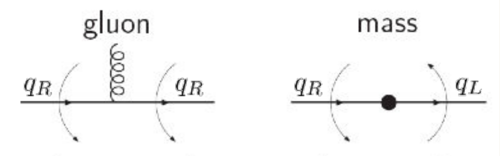


Image from: www.pngegg.com/de/png-bgztr

Is there a way to simplify the picture (without losing connection to QCD)?

Chiral effective field theory

Chiral perturbation theory
Weinberg '79; Gasser, Leutwyler '84,'85



$$\mathcal{L}_{\rm QCD} = \underbrace{-\, q_L \mathcal{M} q_R - q_R \mathcal{M} q_L}_{\text{small for } \rm N_f = 2, (3)} + \underbrace{\bar q_L i D q_L + \bar q_R i D q_R}_{\rm SU(N_f)_L \times SU(N_f)_R \text{ invariant}} - \frac{1}{4} G_a^{\mu\nu} G_{a,\mu\nu}$$

small for $\rm N_f = 2, (3)$ $\rm SU(N_f)_L \times SU(N_f)_R$ invariant $\rightarrow$ SSB to $\rm SU(N_f)_V \leq SU(N_f)_L \times SU(N_f)_R \;\Rightarrow\; N_f{}^2 - 1$ GBs

$$\int [DG_\mu][Dq][D\bar q]\, e^{i\int d^4x \mathcal{L}(q,\bar q, G_{\mu\nu}; v,a,s,p)}\Big|_{\rm low\ energy} = \int \underbrace{[DU]}\, e^{i\int d^4x \mathcal{L}_{\rm eff}(U; v,a,s,p)} \xrightarrow{\rm ChPT} \text{Observables}$$

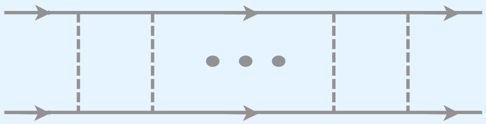
SU(2) matrix collecting pions: $U \rightarrow RUL^\dagger$

(perturbative expansion in p/Λ_χ, M_π/Λ_χ)

Generalization to the single-nucleon sector is straightforward Bernard, Kaiser, Meißner, …

Chiral EFT for nuclear systems
Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Machleidt, …

— non-perturbative re-summation of ladder diagrams



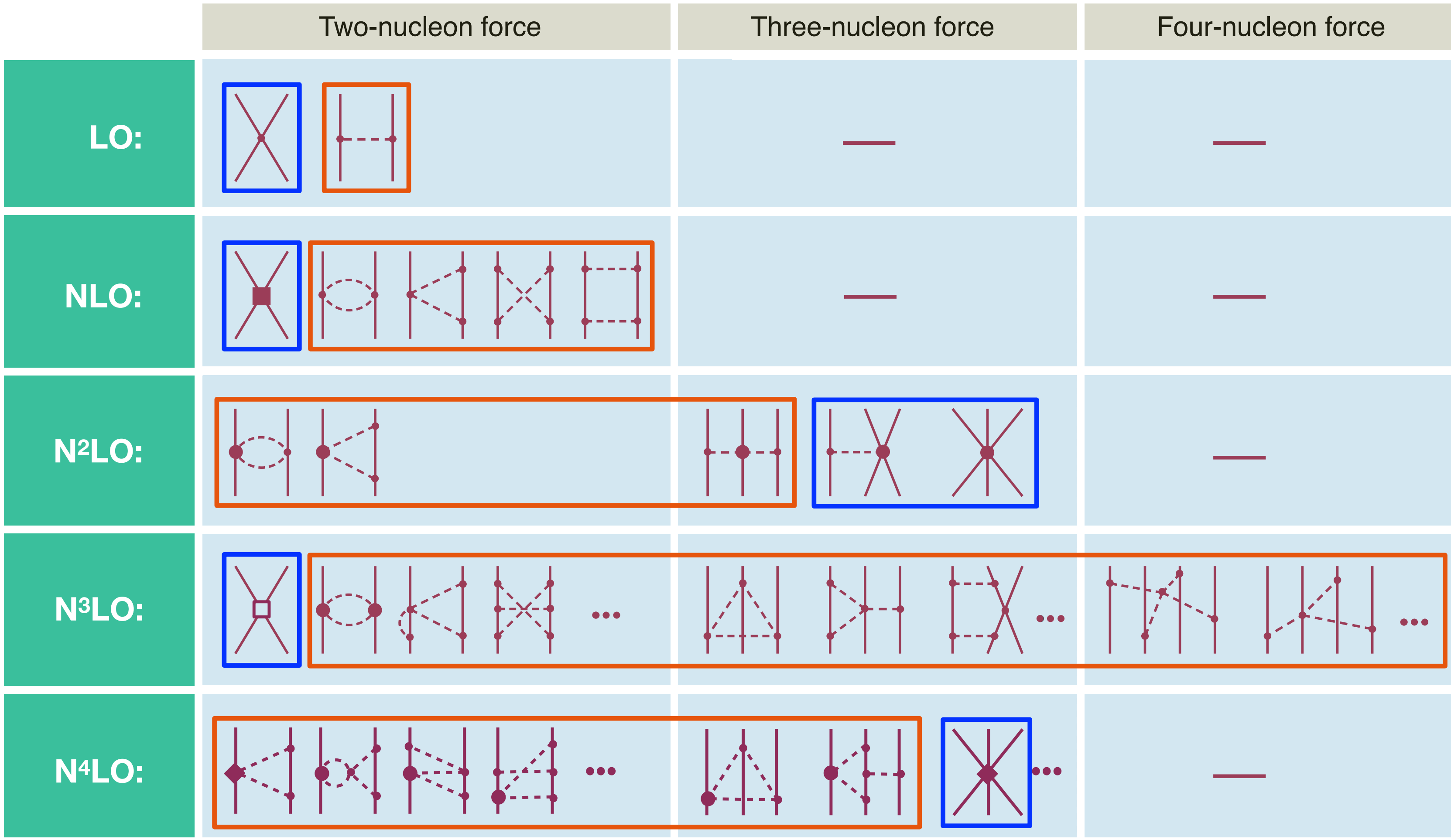
$$\left[\left(\sum_{i=1}^{A} \frac{-\vec\nabla_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

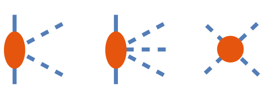
— analytic results for (scheme-dependent!) nuclear forces & currents derived from $\mathscr{L}_{\rm eff}$

— πN LECs from matching to Roy-Steiner eq. Hoferichter et al.'15 $\Rightarrow$ predict large-r interactions

— finite cutoff is required to regularize the Schrödinger equation Lepage, EE, Meißner, Gasparyan, Gegelia
[renormalizability rigorously proven to NLO Ashot Gasparyan, EE, PRC 105 (2022); PRC 107 (2023)]

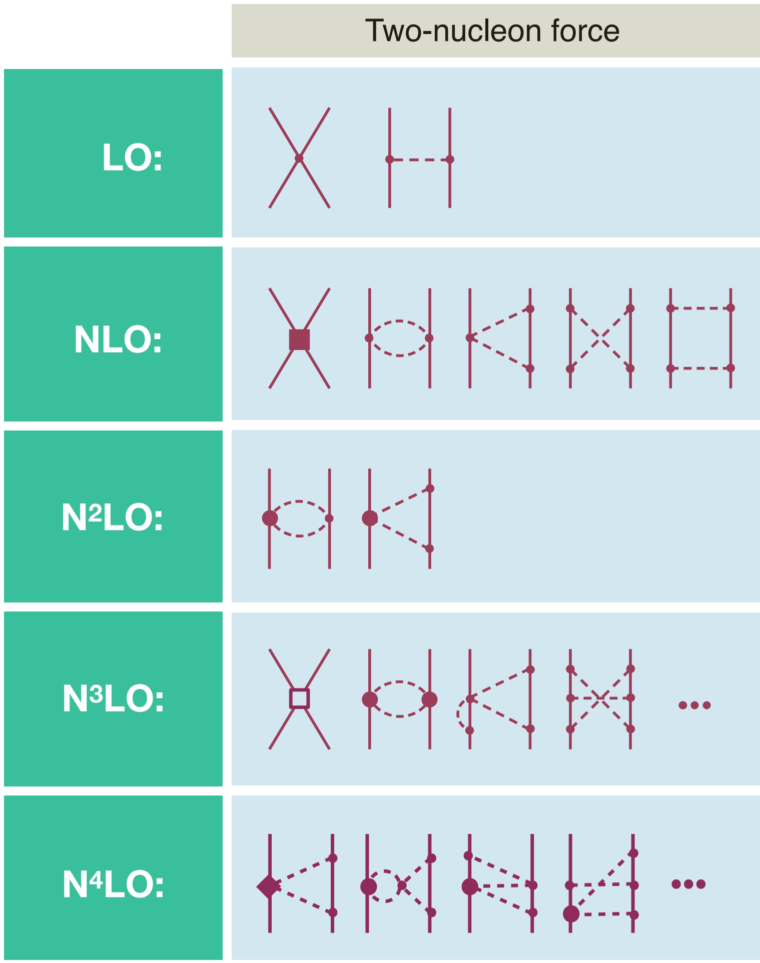
Chiral expansion of nuclear forces



Chiral dynamics: Long-range interactions are predicted in terms of on-shell amplitudes 

Short-range few-N interactions are tuned to experimental data

Chiral expansion of nuclear forces



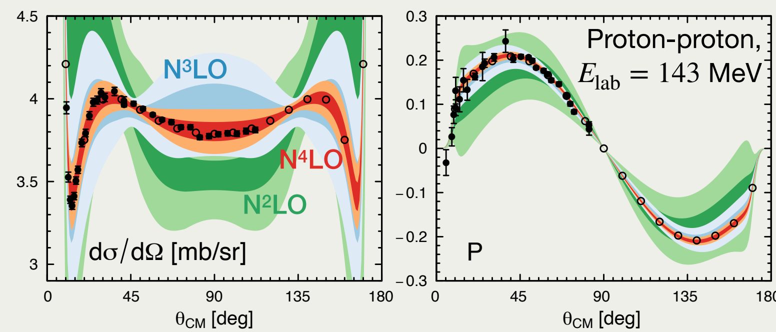
χEFT as a precision tool in the 2N sector

— N^{4}LO$^+$: currently most accurate and precise NN interactions on the market

— clear evidence of the TPEP from NN data

— almost no residual cutoff dependence

— Bayesian truncation-error estimation



— Precision calculations for 2 nucleons:

$g_{\pi NN} = 13.24 \pm 0.04$ Reinert, Krebs, EE '20

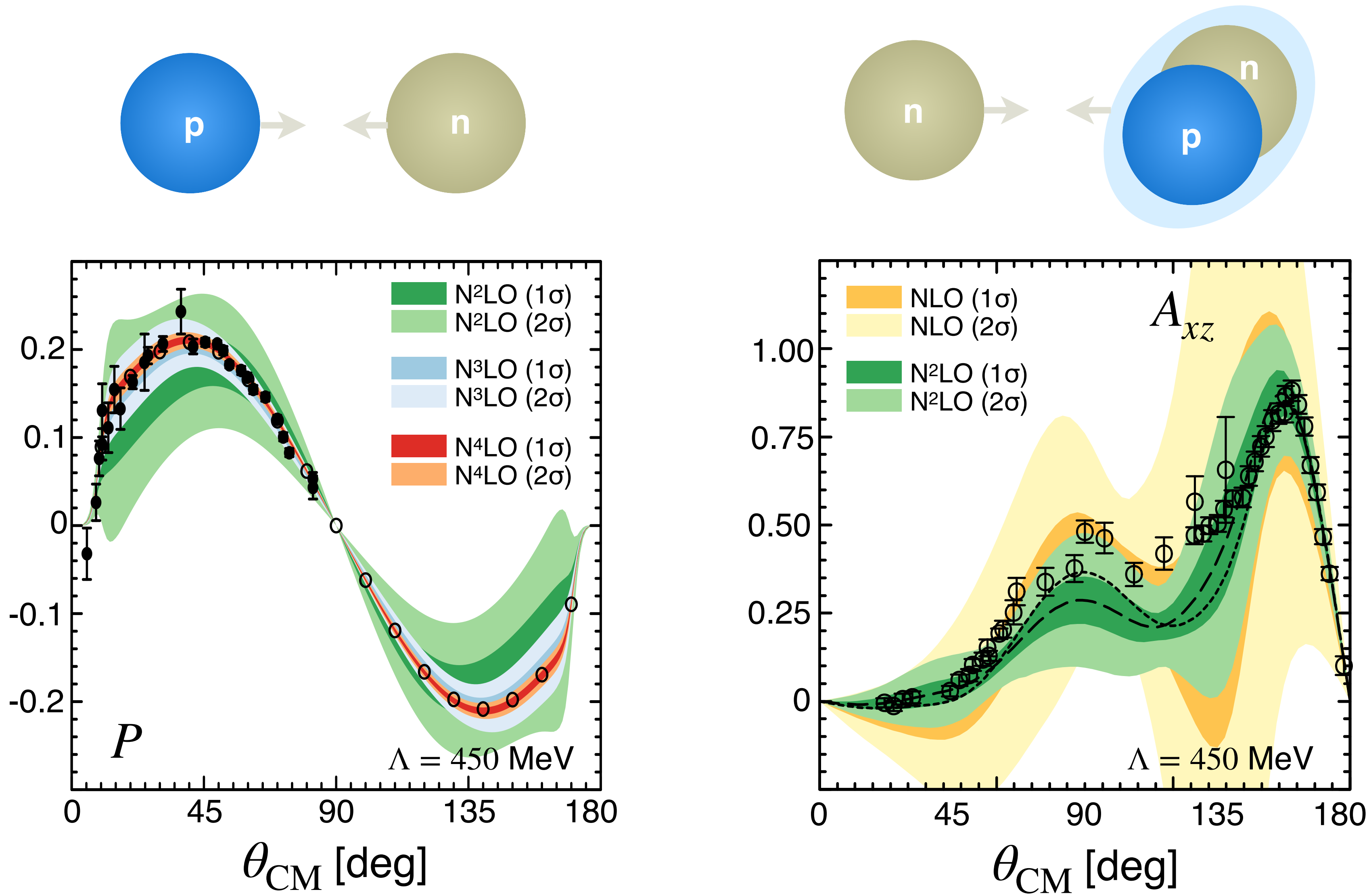
$r_{\rm str}^{^2{\rm H}} = 1.9729^{+0.0015}_{-0.0012}$ fm Filin et al., '21

Semi-local regularization in momentum space Reinert, Krebs, EE, EPJA 54 (2018) 86; PRL 126 (2021) 092501

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^2 + M_\pi^2}\, e^{-\frac{\vec{q}^2 + M_\pi^2}{\Lambda^2}} + \text{subtraction,} \qquad V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu\,\frac{\rho(\mu)}{\vec{q}^2+\mu^2}\, e^{-\frac{\vec{q}^2+\mu^2}{2\Lambda^2}} + \text{subtractions}$$

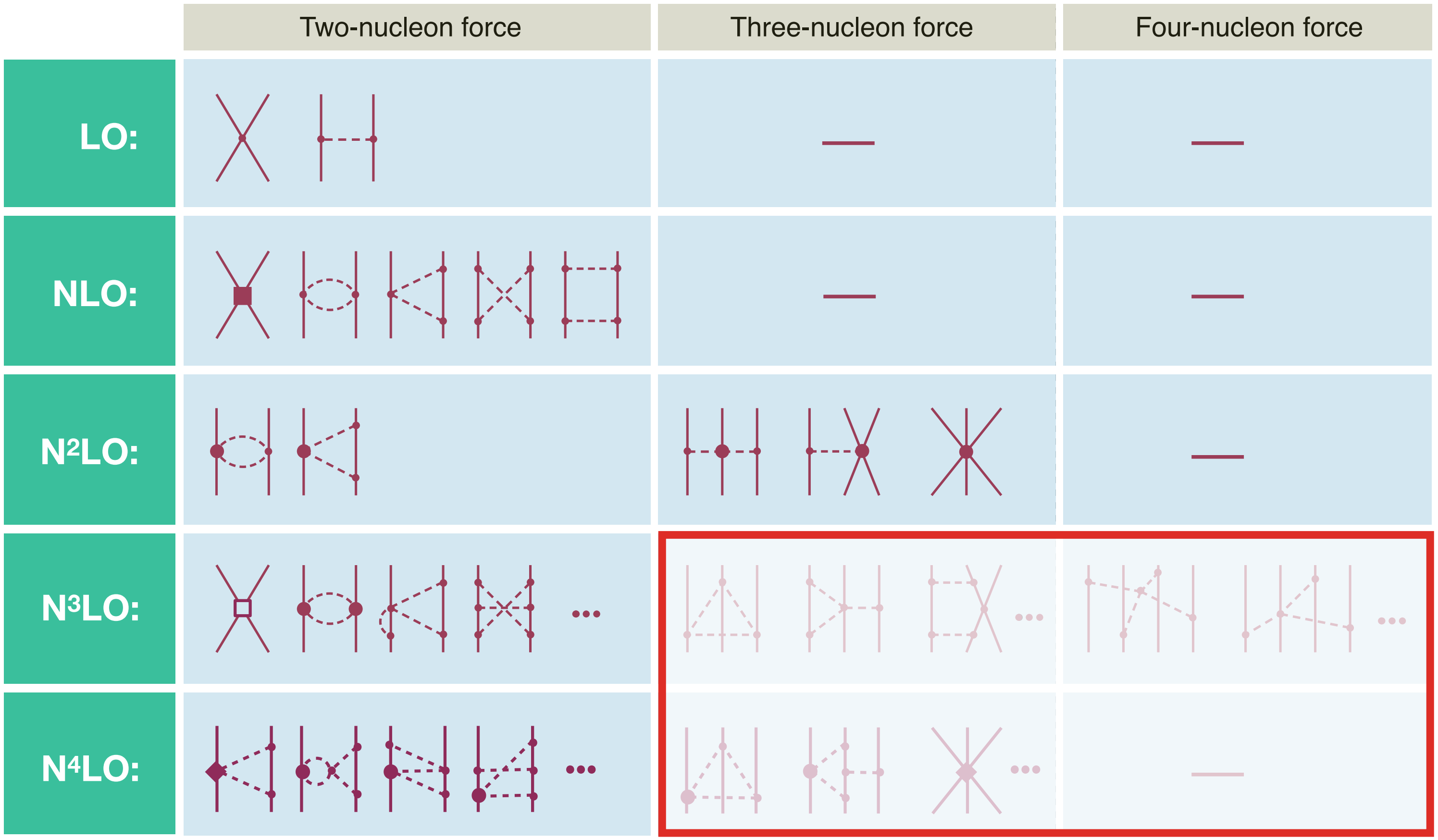
+ nonlocal (Gaussian) cutoff for contacts

Precision physics beyond 2N?



Where are calculations beyond N^2LO?

Chiral expansion of nuclear forces



have been worked out using dimensional regularization

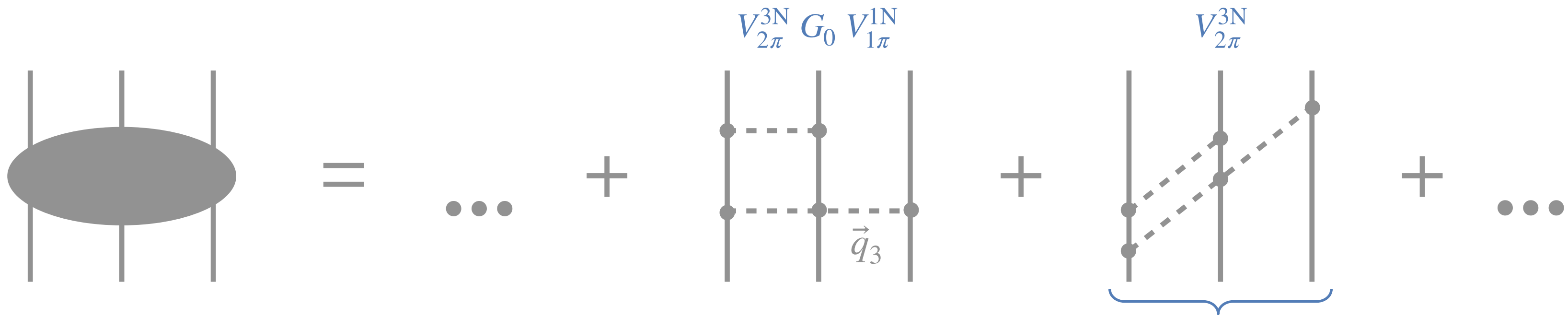
mixing DimReg with Cutoff violates χ-symmetry (also for current operators)

$\Rightarrow$ need to be re-derived using invariant cutoff regulator

DANGER: momentum cutoff for pions breaks chiral symmetry!

Essence of the problem

Faddeev equation for 3N scattering:



derived using DimReg Bernard et al. '08

$$-\Lambda \frac{g_A^4}{96\sqrt{2\pi^3} F_\pi^6} \Big[\underbrace{\tau_1 \cdot \tau_3\, (\vec{q}_3 \cdot \vec{\sigma}_1)}_{\text{absorbable into } c_D} - \underbrace{\frac{4}{3} (\tau_2 \cdot \tau_3 - \tau_1 \cdot \tau_3)(\vec{q}_2 \cdot \vec{\sigma}_3)}_{\text{violates chiral symmetry}} \Big] \frac{\vec{q}_3 \cdot \vec{\sigma}_3}{q_3^3 + M_\pi^2} + \ldots$$

absorbable into c_D: ╳--| violates chiral symmetry

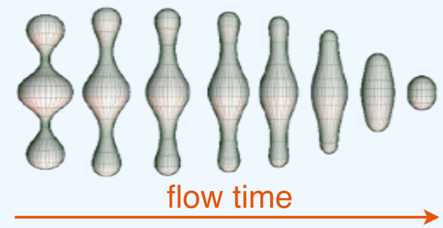
⇒ mixing DimReg with Cutoff regularization breaks chiral symmetry EE, Krebs, Reinert '19

⇒ 3NF, 4NF & MECs beyond N²LO have to be re-derived using Cutoff Reg (2NF ok at fixed M_π)

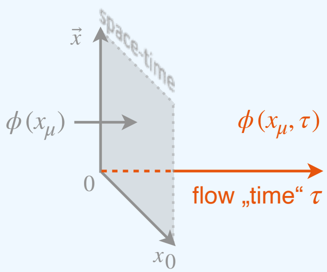
Gradient flow as a **symmetry-preserving** regulator as suggested by David Kaplan

Gradient flow

Gradient flows: methods for smoothing manifolds
(e.g., Ricci flow used in the proof of the Poincaré conjecture)



Gradient flow as a regulator in field theory



Flow equation: $$\frac{\partial}{\partial \tau}\phi(x,\tau) = -\frac{\delta S[\phi]}{\delta \phi(x)}\bigg|_{\phi(x)\to\phi(x,\tau)}$$

subject to the boundary condition $\phi(x,0) = \phi(x)$

Free scalar field:

$$\left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right]\phi(x,\tau) = 0 \quad \Rightarrow \quad \phi(x,\tau) = \int d^4y \, \overbrace{G(x-y,\tau)}^{G(x,\tau) = \frac{\theta(\tau)}{16\pi^2\tau^2}e^{-\frac{x^2+4M^2\tau^2}{4\tau}}}\phi(y) \quad \Rightarrow \quad \tilde{\phi}(q,\tau) = e^{-\tau(q^2+M^2)}\tilde{\phi}(q)$$

(heat kernel: $G(x-y,\tau)$)

YM gradient flow Narayanan, Neuberger '06, Lüscher, Weisz '11: $\partial_\tau A_\mu(x,\tau) = D_\nu G_{\nu\mu}(x,\tau)$ ← extensively used in LQCD

Chiral gradient flow Krebs, EE, 2312.13932, to appear in PRC

Generalize $U(x)$, $U(x) \to RU(x)L^\dagger$ to $W(x,\tau)$: $$\partial_\tau W = -i \underbrace{w}_{\sqrt{W}} \overbrace{\mathrm{EOM}(\tau)}^{[D_\mu, w_\mu] + \frac{i}{2}\chi_-(\tau) - \frac{i}{4}\mathrm{Tr}\,\chi_-(\tau)} w, \quad W(x,0) = U(x)$$

We have proven $\forall\tau$: $W(x,\tau) \in \mathrm{SU}(2)$, $W(x,\tau) \to RW(x,\tau)L^\dagger$

Solving the chiral gradient flow equation

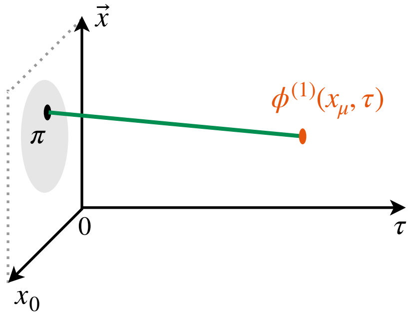
Generalized pion field $\underbrace{\phi(x,\tau)}$: $W = 1 + i\boldsymbol{\tau}\cdot\boldsymbol{\phi}(1-\alpha\,\phi^2) - \frac{\phi^2}{2}\left[1+\left(\frac{1}{4}-2\alpha\right)\phi^2\right] + \ldots$

calculated by recursively solving the GF equation using $\phi = \sum_{n=0}^{\infty}\frac{\phi^{(n)}}{F^n}$

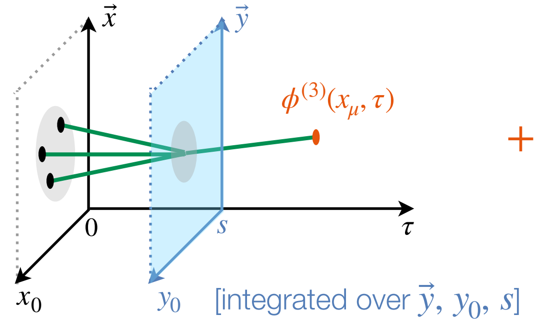
In the absence of external sources, one finds:

Solving the chiral gradient flow equation

$\phi(x_\mu, \tau) \quad =$



$+$



$+ \quad \ldots$

$$\left.\begin{aligned} \left[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)\right] \boldsymbol{\phi}^{(1)}(x,\tau) &= 0 \\ \boldsymbol{\phi}^{(1)}(x,0) &= \boldsymbol{\pi}(x) \end{aligned}\right\} \Rightarrow \quad \boldsymbol{\phi}^{(1)}(x,\tau) = \int d^4y \, \overbrace{G(x-y,\tau)}^{G(x,\tau) = \frac{\theta(\tau)}{16\pi^2\tau^2} e^{-\frac{x^2+4M^2\tau^2}{4\tau}}} \boldsymbol{\pi}(y) \quad \Rightarrow \quad \tilde{\boldsymbol{\phi}}^{(1)}(q,\tau) = e^{-\tau(q^2+M^2)} \tilde{\boldsymbol{\pi}}(q)$$

SMS regulator for $\tau = 1/(2\Lambda^2)$

$$\left[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)\right] \phi_b^{(3)}(x,\tau) = \overbrace{(1-2\alpha)\partial_\mu \boldsymbol{\phi}^{(1)} \cdot \partial_\mu \boldsymbol{\phi}^{(1)} \phi_b^{(1)} - 4\alpha \, \partial_\mu \boldsymbol{\phi}^{(1)} \cdot \boldsymbol{\phi}^{(1)} \partial_\mu \phi_b^{(1)} + \frac{M^2}{2}(1-4\alpha) \boldsymbol{\phi}^{(1)} \cdot \boldsymbol{\phi}^{(1)} \phi_b^{(1)}}^{\equiv \mathrm{RHS}_b(x,\tau)}$$

$$\phi_b^{(3)}(x,0) = 0$$

$$\Rightarrow \quad \phi_b^{(3)}(x,\tau) = \int_0^\tau ds \int d^4y \, G(x-y,\tau-s) \, \mathrm{RHS}_b(y,s)$$

Gradient flow regularization of chiral EFT

— The pion Lagrangian $\mathscr{L}_{\pi}^{\mathrm{E}}$ is left unchanged

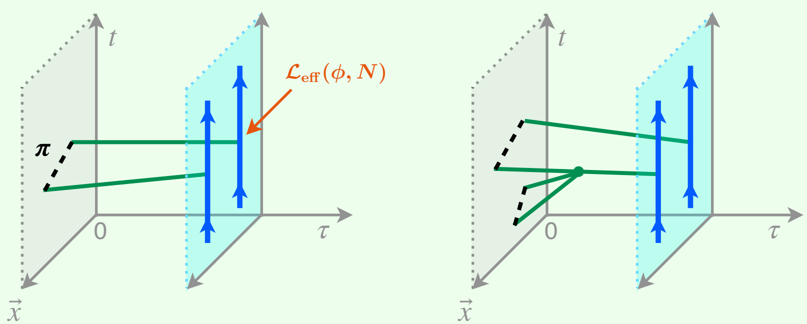
— Nucleons are ***defined*** to „live“ at a fixed $\tau > 0$:

chiral symmetry manifest since $W \to RWL^{\dagger}$

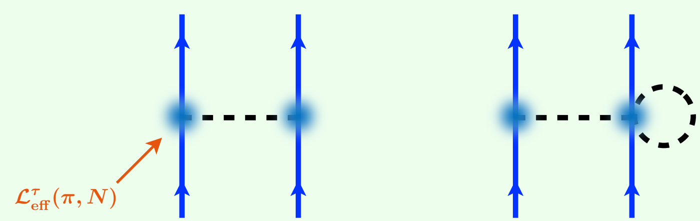
$$\mathscr{L}_{\mathrm{eff}}^{\mathrm{E}} = \mathscr{L}_{\pi}^{\mathrm{E}} + \mathscr{L}_{\pi \mathrm{N}}^{\mathrm{E}} \xrightarrow{\text{regularization}} \mathscr{L}_{\mathrm{eff}}^{\mathrm{E}} = \mathscr{L}_{\pi}^{\mathrm{E}} + \mathscr{L}_{\phi \mathrm{N}}^{\mathrm{E}}(\tau) \quad \text{where} \quad \mathscr{L}_{\phi \mathrm{N}}^{\mathrm{E}}(\tau) = \mathscr{L}_{\pi \mathrm{N}}^{\mathrm{E}} \Big|_{U \to W(\tau)}$$

local in 5d space, but non-local (smeared) if expressed in $\boldsymbol{\pi}$*’s*

Local field theory in 5d

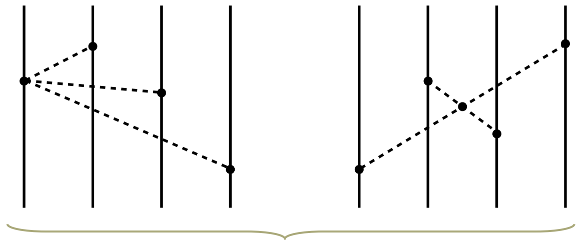


Smeared (non-local) theory in 4d



Consistency check: The 4N force

unregularized



*Both diagrams depend on the parametrization of U (arbitrary α), but the sum **must** be α-independent*

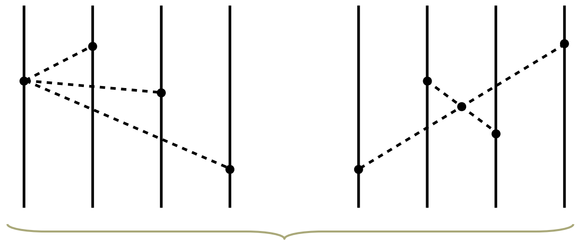
Unregularized expression for this 4NF EE, EPJA 34 (2007):

$$\hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_4 \, \vec{\sigma}_2 \cdot \vec{q}_2 \, \vec{\sigma}_3 \cdot \vec{q}_3 \, \vec{\sigma}_4 \cdot \vec{q}_4$$

$$V^{4N} = -\frac{g^4}{64F^6} \frac{\hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]}}{(\vec{q}_2^{\,2} + M^2)(\vec{q}_3^{\,2} + M^2)(\vec{q}_4^{\,2} + M^2)} \vec{\sigma}_1 \cdot \vec{q}_{12} + \frac{g^4}{128F^6} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]}}{(\vec{q}_1^{\,2} + M^2)(\vec{q}_2^{\,2} + M^2)(\vec{q}_3^{\,2} + M^2)(\vec{q}_4^{\,2} + M^2)} \big(M^2 + \vec{q}_{12}^{\,2}\big) + 23 \text{ perm.}$$

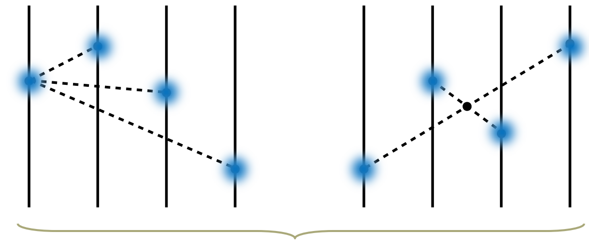
Consistency check: The 4N force

unregularized



Both diagrams depend on the parametrization of U (arbitrary α),
but the sum **must** be α-independent

regularized



the sum **must** be α-independent

$$
\begin{aligned}
V_\Lambda^{4N} = {} & \frac{g^4}{64F^6}\frac{\hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]}}{(\vec{q}_2^{\,2}+M^2)(\vec{q}_3^{\,2}+M^2)(\vec{q}_4^{\,2}+M^2)}\Big[\vec{\sigma}_1\cdot\vec{q}_1\big(2g_\Lambda-4f_\Lambda^{123}+2f_\Lambda^{134}-f_\Lambda^{234}\big)-\vec{\sigma}_1\cdot\vec{q}_2 f_\Lambda^{234} \\
& +2\vec{\sigma}_1\cdot\vec{q}_1\big(5M^2+\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+\vec{q}_3^{\,2}+\vec{q}_4^{\,2}+\vec{q}_{34}^{\,2}\big)\frac{g_\Lambda-f_\Lambda^{134}}{2M^2+\vec{q}_1^{\,2}+\vec{q}_3^{\,2}+\vec{q}_4^{\,2}-\vec{q}_2^{\,2}} \\
& -4\vec{\sigma}_1\cdot\vec{q}_1\big(3M^2+\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+\vec{q}_3^{\,2}+\vec{q}_4^{\,2}-\vec{q}_{34}^{\,2}\big)\frac{g_\Lambda-f_\Lambda^{124}}{2M^2+\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+\vec{q}_4^{\,2}-\vec{q}_3^{\,2}}\Big] \\
& + \frac{g^4}{128F^6}\frac{\vec{\sigma}_1\cdot\vec{q}_1\,\hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]}}{(\vec{q}_1^{\,2}+M^2)(\vec{q}_2^{\,2}+M^2)(\vec{q}_3^{\,2}+M^2)(\vec{q}_4^{\,2}+M^2)}\big(M^2+\vec{q}_{12}^{\,2}\big)\big(4f_\Lambda^{123}-3g_\Lambda\big)+23\text{ perm.},
\end{aligned}
$$

$f_\Lambda^{ijk} = e^{-\frac{\vec{q}_i^{\,2}+M^2}{\Lambda^2}}e^{-\frac{\vec{q}_j^{\,2}+M^2}{\Lambda^2}}e^{-\frac{\vec{q}_k^{\,2}+M^2}{\Lambda^2}}$

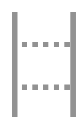
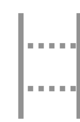
$e^{-\frac{\vec{q}_1^{\,2}+M^2}{2\Lambda^2}}e^{-\frac{\vec{q}_2^{\,2}+M^2}{2\Lambda^2}}e^{-\frac{\vec{q}_3^{\,2}+M^2}{2\Lambda^2}}e^{-\frac{\vec{q}_4^{\,2}+M^2}{2\Lambda^2}}$

(reduces to the unregularized result in the $\Lambda\to\infty$ limit)

Nuclear interactions from path integral

Hermann Krebs, EE, e-Print: 2311.10893, to appear in PRC

The considered 4NFs were calculated using Feynman diagrams. But more generally,

Potential  $\neq$ Feynman diagram 

$\mathcal{L}_{\pi N}$ —*canonical quan.*→ $\mathcal{H}_{\pi N}$ —→ $V_{2N},\ V_{3N},\ \ldots$; $J^\mu_{2N},\ J^\mu_{3N},\ \ldots$; $A^\mu_{2N},\ A^\mu_{3N},\ \ldots$

- **TOPT** Weinberg, van Kolck, Pastore, Schiavilla, …
- **Method of UT** EE, Glöckle, Meißner, Krebs, Bernard, …

$\Rightarrow$ impractical for ***regularized*** Lagrangians, which involve $e^{-\tau(-\partial_x^2+M^2)}\boldsymbol{\pi}(x)$

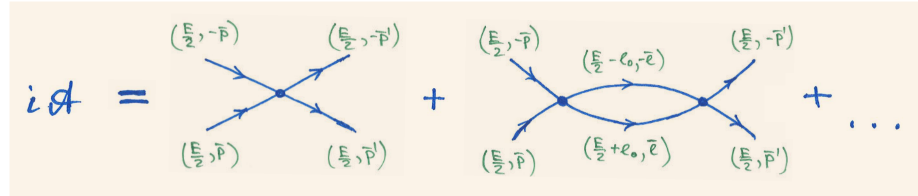
$\Rightarrow$ new method to derive nuclear interactions using the path integral approach Krebs, EE, 2311.10893

Idea of the method

Pion-less EFT:

$$\mathcal{L} = N^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2m_N} \right] N - \frac{C_S}{2} (N^\dagger N)^2 + \ldots$$

$$\Rightarrow \quad \mathcal{A}_{\rm tree} = [C_0 + C_2(\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + \ldots]$$

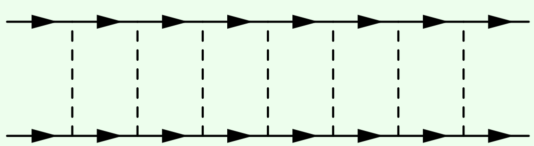


Scattering amplitude to 1 loop:

$$-i\mathcal{A}_{1-\rm loop} = \int \frac{d^4 l}{(2\pi)^4} [C_0 + C_2(\vec{p}^{\,2} + \vec{l}^{\,2}) + \ldots] \frac{1}{\left(\frac{E}{2} + l_0 - \frac{\vec{l}^{\,2}}{2m_N} + i\epsilon\right)\left(\frac{E}{2} - l_0 - \frac{\vec{l}^{\,2}}{2m_N} + i\epsilon\right)} [C_0 + \ldots]$$

$$= -i \int \frac{d^3 l}{(2\pi)^3} [C_0 + C_2(\vec{p}^{\,2} + \vec{l}^{\,2}) + \ldots] \frac{1}{E - \frac{\vec{l}^{\,2}}{m_N} + i\epsilon} [C_0 + (\vec{l}^{\,2} + \vec{p}^{\,\prime 2}) \ldots]$$

All l_0-integrals factorize $\Rightarrow$ Lippmann-Schwinger eq. $\mathcal{A} = \mathcal{V} + \mathcal{V}\, G_0\, \mathcal{A}$ with $\mathcal{V} = -\mathcal{L}_{\rm int}$



But l_0-integrals do not factorize for pions due to l_0-dependence of π-propagators…

Idea: $Z[\eta^\dagger, \eta] = A \int \mathscr{D}N^\dagger\, \mathscr{D}N\, \mathscr{D}\pi \exp\left(iS_{\rm eff}^\Lambda + i \int d^4x [\eta^\dagger N + N^\dagger \eta] \right)$

Hermann Krebs, EE, 2311.10893

nonlocal redefinitions of $N, N^\dagger$ → loops from functional determinant

$$A \int \mathscr{D}\tilde{N}^\dagger\, \mathscr{D}\tilde{N} \exp\left(iS_{\rm eff,\,N}^\Lambda + i \int d^4x [\eta^\dagger \tilde{N} + \tilde{N}^\dagger \eta] \right)$$

instantaneous (→ $S_{\rm eff,\,N}^\Lambda$)

Summary

New formulation of nuclear chiral EFT:

— gradient flow regularized version of chiral EFT

— path integral method to reduce QFT to QM via non-local field redefinitions

⇒ regularized 3N, 4N forces and currents, which are consistent with the SMS NN potentials & respect chiral and gauge symmetries Hermann Krebs, EE, in progress

Already done:

— NN at N^{2}LO, long-range 3NF (still needs to be implemented…) and 4NF at N^{3}LO

Ongoing:

— πN scattering inside the Mandelstam triangle (LECs), 3N scattering to N^{3}LO

Thank you for your attention