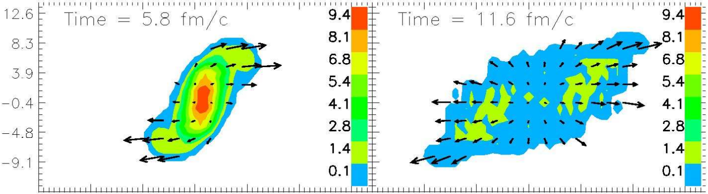
# Continuous Decoupling of Dynamically Expanding Systems

J. Knoll




3 fluid hydro: Ivanov, Russkikh & Toneev

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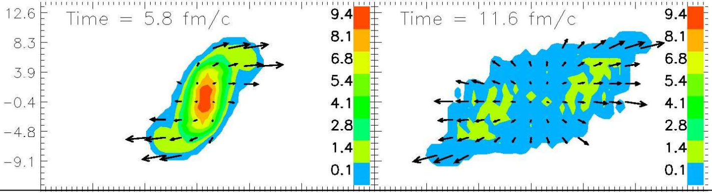



| **Au + Au at SPS** | decoupl. period | volume growth |
|---|---|---|
| phase transition: | 6 - 10 fm/c | $> 5$ |
| chemical freeze-out: | $> 5$ fm/c | $> 4$ |
| kinetic freeze-out: | $> 8$ fm/c | $> 6$ |
| **CMB early universe** | $Z = [1300 - 800]$ | $(13/8)^3 = 4.3$ |

# Exact Detector yields



**inelastic scattering:** $(H_{\rm int} = V_{\rm el} + V_{\rm res})$

$$\frac{E_f\, d\sigma}{d^3 P_f} \propto \langle \chi_f | \underbrace{V_{\rm res.} |\Psi_i\rangle \langle \Psi_i| V_{\rm res.}}_{\langle J^\dagger(x) J(y)\rangle} |\chi_f\rangle \, \delta(E_i - E_f), \qquad \overbrace{\langle \Psi_i| V_{\rm res.} |\chi_f\rangle}^{T\text{-matrix}}$$

**inclusive single particle yield:**

$$(2\pi)^3 \frac{2\omega_f\, dN}{d^3 p_f} = \int d^4x\, d^4y \; \underbrace{\overbrace{\langle J^\dagger(x) J(y)\rangle_{\rm irred.}}^{\text{source}}}_{\Pi^{\rm gain}} \; \underbrace{\overbrace{\chi_f^\dagger(x)\chi_f(y)}^{\text{dist. waves}}}_{\Pi^{\rm R}}$$

$$= \langle \chi_f | \, \Pi^{\rm gain} \, | \chi_f \rangle$$ Gyulassy '78, Danielewicz '92

**dist. waves $\chi_F$ are the optical devices by which the detector views the source!**

# Semi-classical picture

* **The detector views the source by a bundle of classical paths $(x(t), p(t))$ which build up the dist. waves $\chi_f$;**
* **classical paths: determined by $\mathrm{Re}\Pi^{\mathrm{R}}(x,p)$ (WKB/ Hamilton-Jacobi); they are locally on mass shell;**
* **$\mathrm{Im}\Pi^{\mathrm{R}}(x,p)$ determines the opaqueness (damping $\Gamma(x,p)$) defining the escape probability:** $P_{\mathrm{escape}}(x,p) \approx \exp(-\int_t^\infty dt' \Gamma(x(t'), p(t')))$

**inclusive single particle yield:**

**local on-shell momentum**

$$\boxed{(2\pi)^3 \frac{2\omega_f dN}{d^3 p_f} = \int d^4x \, \Pi^{\mathrm{gain}}(x, \underset{\Downarrow}{p(x)}) \, P_{\mathrm{escape}}(x, \underset{\Downarrow}{p(x)}) \, \frac{\partial(\underset{\Downarrow}{p(x)})}{\partial(p_f)}}$$

**local decoupling rate:**

$$\boxed{(2\pi)^4 \frac{dN_a(x,p)}{d^3x dt \; d^4p} \approx \Pi_a^{\mathrm{gain}}(x,p) \; A(x,p) \; P_{\mathrm{escape}}(x,p)}$$

# Conserving scheme

**conserving scheme:**

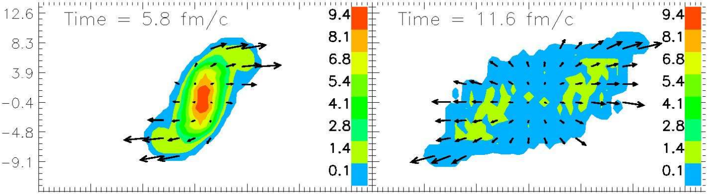
**local rate:** source $\times$ width $\times$ attenuation

$$(2\pi)^4 \frac{dN_a(x,p)}{d^3x dt\ d^4p} \approx \Pi_a^{\mathrm{gain}}(x,p)\ A_a(x,p)\ P_{\mathrm{escape}}(x,p)$$

**drain terms:**

$$\partial_\mu j^\mu_{\alpha,\mathrm{fluid}}(x) = -\sum_a e_{a\alpha} \int d^4p\ \frac{dN_a(x,p)}{d^4p dt d^3x}$$

$$\partial_\mu T^{\mu\nu}_{\mathrm{fluid}} = -\sum_a \int d^4p\ p^\nu \frac{dN_a(x,p)}{d^4p dt d^3x}$$



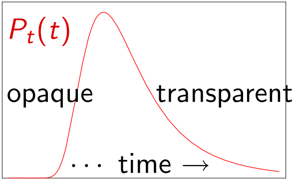

3 fluid hydro: Ivanov, Russkikh & Toneev

# Local Equilibrium

**local equilibrium:**

$$\boxed{\Pi^{\mathrm{gain}}(x,p) \;=\; \underbrace{f_{\mathrm{th}}(p^0)\; 2p^0\; \Gamma(x,p)}_{\text{thermal weight} \times \text{damping}}}$$

$$\int_{-\infty}^{\infty} dt \; \underbrace{\Gamma(t)\exp\{-\int_t^{\infty} dt'\Gamma(t')\}}_{P_t(t)} \;=\; 1.$$




**generic features:**

maximum at: $\left[\dot{\Gamma}(t) + \Gamma^2(t)\right]_{t_{max}} = 0,$ with $P_t(t_{max}) \approx \Gamma(t_{max})/\mathrm{e}$

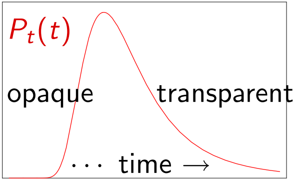
uncertainty relation: $\boxed{\Delta t_{\mathrm{dec}} \approx \dfrac{\mathrm{e}}{\Gamma(t_{max})}}$

$$\boxed{\begin{array}{c} \Gamma_i \;\;/\Gamma_{max}/\;\; \Gamma_f \\ \approx \mathrm{e}^{\mathrm{e}/2}/\;\; 1 \;\;/\mathrm{e}^{-\mathrm{e}/2} \end{array}}$$

# Cooper-Frye (Planck) limit



$$\int_{-\infty}^{\infty} dt\ \underbrace{\Gamma(t)\exp\{-\int_t^{\infty} dt'\Gamma(t')\}}_{P_t(t)} = 1.$$




$$(2\pi)^4\frac{dN(p)}{d^3p} = 2\int p^0dp^0\,\underbrace{d^3\sigma_\mu dx^\mu}_{=d^4x}\ f_{\rm th}(p^0)\ A_{\rm QF}(x,p)\ \Gamma(x,p)\mathrm{e}^{-\int_t^{\infty}\Gamma\ dt'}$$

**Cooper-Frye limit:**

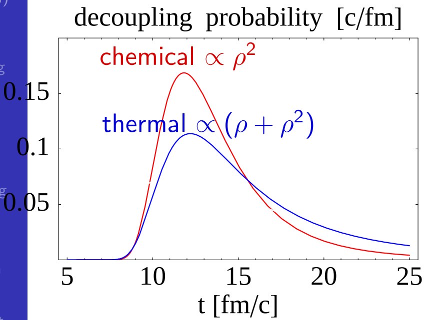
$$\underset{\text{instaneous limit}}{\Longrightarrow}\int_{\sigma_{\rm CFP}} dp^0 d^3\sigma_\mu\ 2p^\mu\ f_{\rm th}(p^0)\ A_{\rm QF}(x,p)\ \Theta(d\sigma_\mu p^\mu > 0)$$

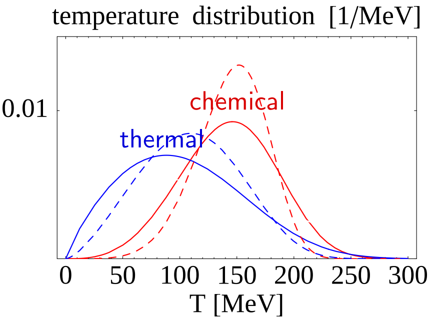
(Cooper-Frye-Planck)

# Expansion model



**schematic expansion model:**




**Input:**

$V \propto t^3, \quad v_{\rm flow} = 0.5$ c

$R_{\rm freeze} = 6$ fm, $T_{\rm freeze} = 160$ Mev

$$\Rightarrow \quad \Gamma_{\rm chem} = 100 \text{ MeV}$$
$$\Delta t_{\rm chem} \approx 5 \text{ fm/c}$$
$$\Delta t_{\rm th} \approx 7 \text{ fm/c}$$

$\Gamma_i : \Gamma_{max} : \Gamma_f$=380 : 100 : 24 MeV

dashed: $\kappa = 4/3$ (mass-less)

solid: $\kappa = 1.5$ $(m \approx T)$

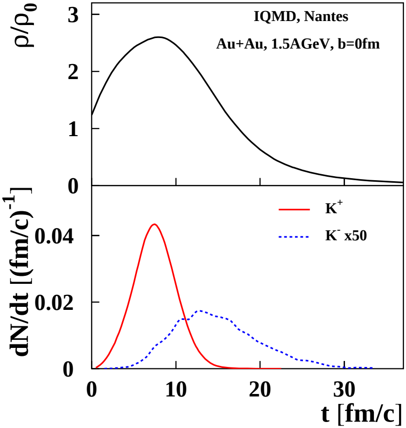
using: $TV^{\kappa - 1}$ =const.

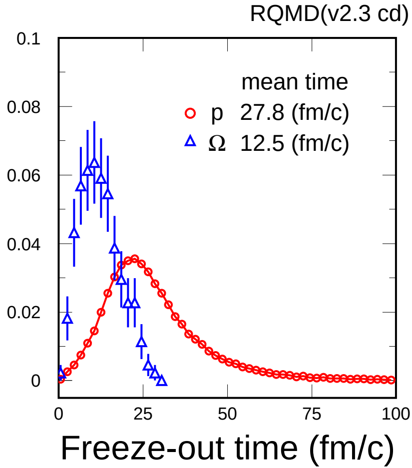
**temperature distributions:**

$$P_{\rm dec}(T) = P_{\rm dec}(t) \left( \frac{dT(t)}{dt} \right)^{-1}$$

# Freeze-out Events (weak versus strong coupling)






IQMD calc. of $K^+$ & $K^-$;
Hartnack et al. 2007

$\Delta t_{\rm dec} \approx 10$ fm/c
$\rho_i/\rho_f \approx 5$

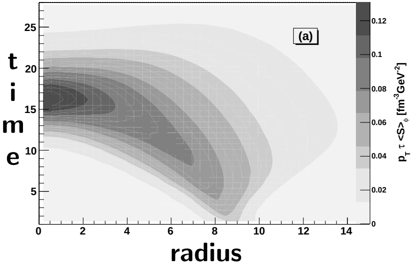
RQMD calc. of $\Omega$ & $P$;
van Hecke, Sorge, Xu '98

$\Delta t_{\rm dec} \approx 25$ fm/c
$\rho_i/\rho_f \approx 8$

# Freeze-out Events (momentum dependence)

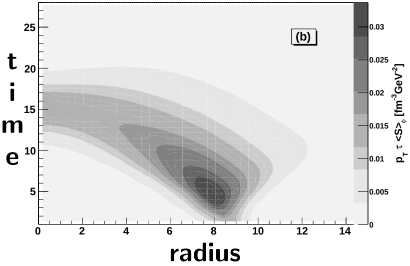


**hybrid model: Hydro + kinetic Transport (Y. Sinyukov et al.)**




**pion momentum = 300 MeV/c**

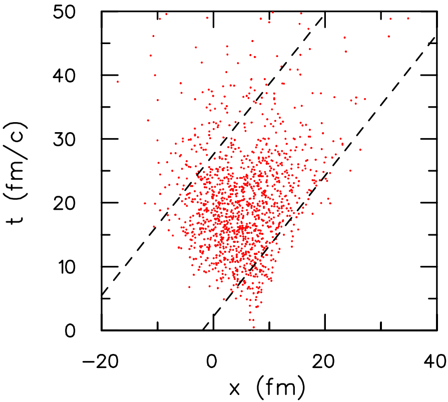
**(volume decoupling)**




**pion momentum = 700 MeV/c**

**(surface decoupling)**

**hybrid model: Hydro + kinetic Transport (S. Pratt)**




**HBT radii:**

$R^2_{\mathrm{out}} = \langle (x - vt)^2 \rangle$

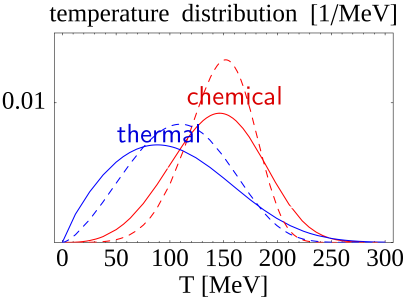
$R^2_{\mathrm{out}} \neq \langle x^2 \rangle + v^2 \langle t^2 \rangle$

$R^2_{\mathrm{side}} = \langle y^2 \rangle$

pion momentum = 300 MeV/c

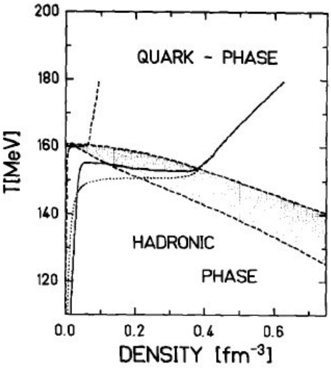
**HBT-radii compatible with RHIC events:** $R_{\mathrm{out}}/R_{\mathrm{side}} \approx 1.2$

**temperature distributions:** $P_{\mathrm{dec}}(T) = P_{\mathrm{dec}}(t)\frac{dt}{dT}$

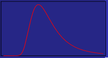



dashed: $\kappa = 4/3$
· (massless ideal gas)
solid: $\kappa = 1.5$
· (half massive gas $m \approx T$)

using: $TV^{\kappa-1}$ =const.

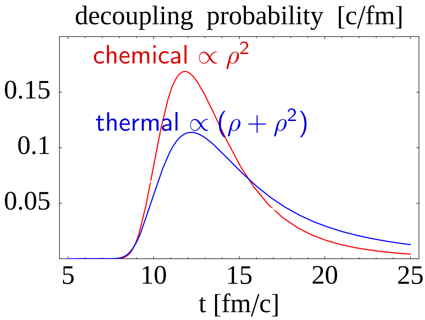



(Flavour kinetics:
Barz et al. (1988))

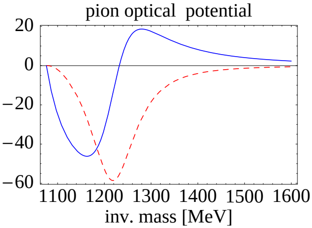


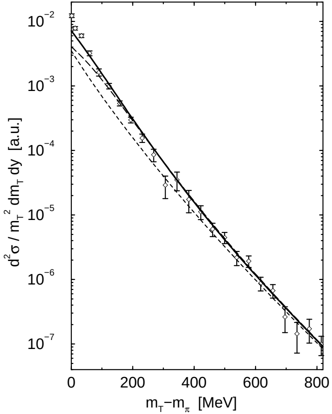
# Finger prints of short lived resonances






**pions from $\Delta$ resonance:**




TAPS $\pi^0$ data

Theory: Weinhold - Friman

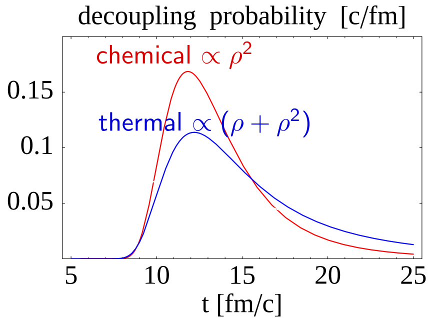
thermal pions

$+$ Delta decay

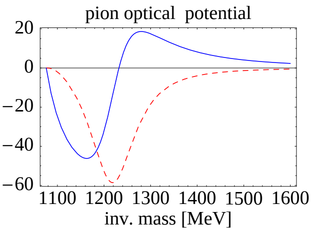
$+$ $\pi N$ correlations

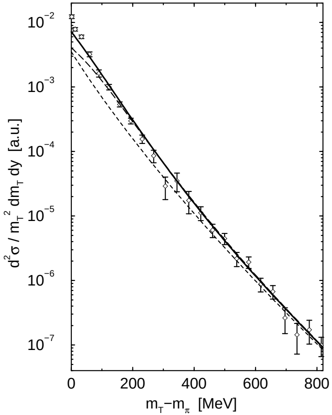
# Finger prints of short lived resonances






**pions from $\Delta$ resonance:**




TAPS $\pi^0$ data
Theory: Weinhold - Friman
thermal pions
~~+ Delta decay~~
$+\ \pi N$ correlations
$\Rightarrow$ two slopes from $\Delta T$?

# Summary

| **nuclear collisions:** | decoupling time | volume growth |
|---|---|---|
| phase transition: | 6 - 10 fm/c | $> 5$ |
| chemical freeze-out: | $> 5$ fm/c | $> 4$ |
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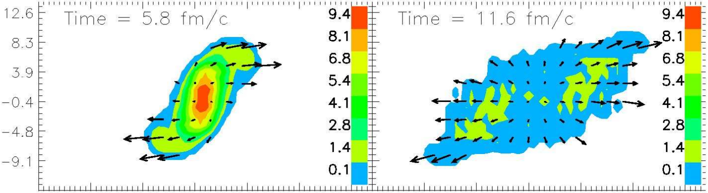
| **CMB early universe:** | $Z = [1300 - 800]$ | $(13/8)^3 = 4.3$ |




3 fluid hydro: Ivanov, Russkikh & Toneev

* stable particles insensitive to in-medium spectral fnct.
* finger prints of short lived resonances not visible; (two slope behaviour: signal for spread in $T$?)
* why is $T_{\mathrm{chem}}$ so sharply determined? $\Rightarrow$ signal for latent heat, phase transition?
* HBT: the method determines the active emission zone
* squeezed particle-antiparticle correlations disappear!