Quark Susceptibilities from the Polyakov-loop Nambu-Jona-Lasinio (PNJL) model

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with M. Cristoforetti,
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Overview

- Lattice results for quark number susceptibilities
- Polyakov-loop extended NJL- model
- Zero-mode fluctuations in finite volume
- Conclusions

QCD at finite *T* and μ

- chiral symmetry restoration transition: chiral condensate
- de-confinement phase transition: Polyakov loop
- crossover transition at small μ and large small *T*
- first order transition at large μ and small *T*
- existence of a critical point?

Methods for the investigation

- lattice simulations of QCD: non-perturbative,

  but fermion sign problem, finite volume, large pion mass

- Dyson-Schwinger Equations

  [C. Fischer, R. Williams *et al.,* R. Alkofer *et al.*, ...]

- functional Renormalization Group methods

  [J.M. Pawlowski, D. Litim, H. Gies, J. Braun, *et al.*, B-J. Schaefer, J. Wambach *et al.* B. Friman, B. Stokic, *et al.*, ...]

- model calculations:
  - Nambu-Jona-Lasinio (NJL) model [M. Buballa, V. Kleinhaus, D. Nickel, J. Wambach *et al.*, ...]
  - quark-meson model [J.M. Pawlowski, B-J. Schaefer, J. Wambach *et al.* B. Friman, B. Stokic, *et al.*, ...]
  - Polyakov-loop extended NJL (PNJL) model

    [K. Fukushima, *et al.*; M. Thaler, C. Ratti, S. Rößner, T. Hell, N. Bratovic, M. Cristoforetti, W. Weise; J.M. Pawlowski, B-J. Schaefer, J. Wambach *et al.*, ...]

- model calculations help to identify mechanisms!

Lattice QCD: Taylor expansion techniques

- fermion sign problem: no simulations at finite baryon chemical potential possible!
- can be overcome by indirect methods
  - imaginary chemical potential [Ph. de Forcrand, O. Philipsen *et al.*]
  - Taylor expansion around μ = 0

[C. R. Allton *et al.*, F. Karsch, *et al.* Brookhaven-Bielefeld collaboration; Z. Fodor *et al.*, Wuppertal-Budpest collaboration; R. V. Gavai, S. Gupta *et al.*]

Taylor expansion of the thermodynamic potential:

$$\Omega(T,\mu_u,\mu_d) = \frac{1}{VT^3}\ln\mathcal{Z} = \sum_{i,j=0}^{\infty}\chi_{ij}(T)\left(\frac{\mu_u}{T}\right)^i\left(\frac{\mu_d}{T}\right)^j,$$

expansion coefficients: quark number susceptibilities

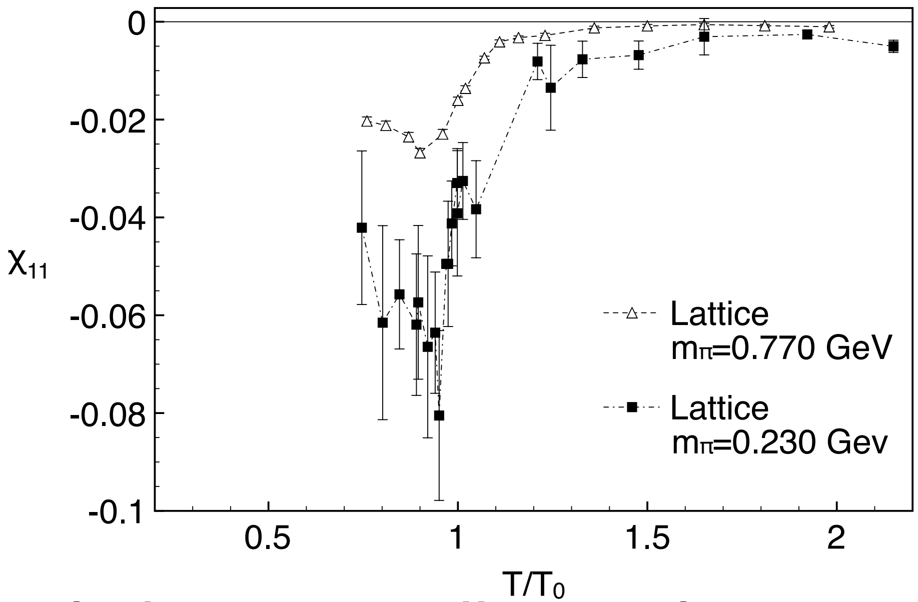
$$\chi_{ij}(T) = \frac{1}{i!j!}\left.\frac{\partial^{i+j}\Omega}{\partial(\mu_u/T)^i\partial(\mu_d/T)^j}\right|_{\mu_{u,d}=0}$$

$$\chi_{ud}(T) = \chi_{11}(T) \qquad \sim\frac{\partial n_u}{\partial\mu_d}$$

u- and *d*- quarks communicate via *A* and π

Lattice: quark number susceptibility

- Results for the quark number susceptibility from lattice simulations with m_π = 230 MeV and m_π = 770 MeV



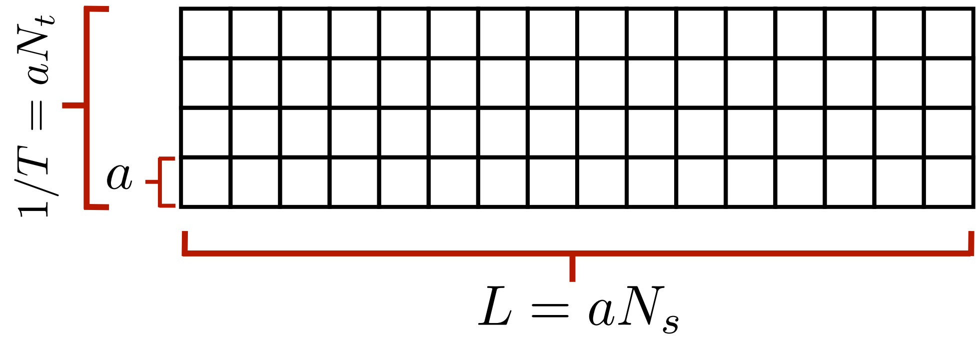
[C.R. Allton et al. PRD 71, 054508 (2005).]

[R.V. Gavai and S. Gupta PRD 78, 114503 (2008).]

- model calculations: non-diagonal susceptibility vanishes in mean-field with saddle-point approximation $\langle\pi\rangle = 0$
- sensitive to pion fluctuations, can we understand this?

Finite volume

- lattice simulations performed in finite volumes $L^3\,(1/T)$
- lattice volumes characterized by aspect ratios $LT=N_s/N_t$ (number of lattice sites in spatial / temporal directions)



- gauge coupling controls lattice spacing a

$$a = \frac{1}{N_t T} \quad \rightarrow \quad V = N_s^3 a^3 = \frac{N_s^3}{N_t^3}\frac{1}{T^3} = k\frac{1}{T^3}$$

- both volume $V = L^3$ and temperature T change with a
- typical lattice ratio: $N_s/N_t = 4 \;\rightarrow\; k = 64$

The PNJL model

Nambu-Jona-Lasinio model: $\langle\bar{\psi}\psi\rangle$

- dynamical breaking of chiral symmetry
- description of chiral phase transition
- drawback: free constituent quarks at low energy

Polyakov loop: $\Phi = \frac{1}{N_c}\langle \mathrm{tr}_c L(\vec{x}\,)\rangle \quad \text{with } L(\vec{x}\,) = \mathcal{P}\exp\left(i\int_0^\beta \mathrm{d}\tau A_4(\tau,\vec{x}\,)\right)$

- order parameter of confinement - deconfinement in pure gauge theory
- description of deconfinement phase transition
- use lattice results to obtain potential for use in phenomenological models

The PNJL model: action

[K. Fukushima, Phys. Rev. D68, 045004 (2003); C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D73, 014019 (2006)].

- action involves N_f =2 quark fields and traced Polyakov loop

$$\mathcal{S}_E[\psi,\bar{\psi},\phi] = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \Big\{ \bar{\psi}(iD + \gamma_0\tilde{\mu} - \mathbf{m})\psi + G\Big[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\Big]\Big\}$$
$$-\beta \int \mathrm{d}^3x\, \mathcal{U}(\phi,\beta)$$

- depends on temperature, chemical potentials, quark mass

$$\beta = 1/T \qquad \tilde{\mu} = \begin{pmatrix} \mu_u & 0 \\ 0 & \mu_d \end{pmatrix} \qquad \mathbf{m} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

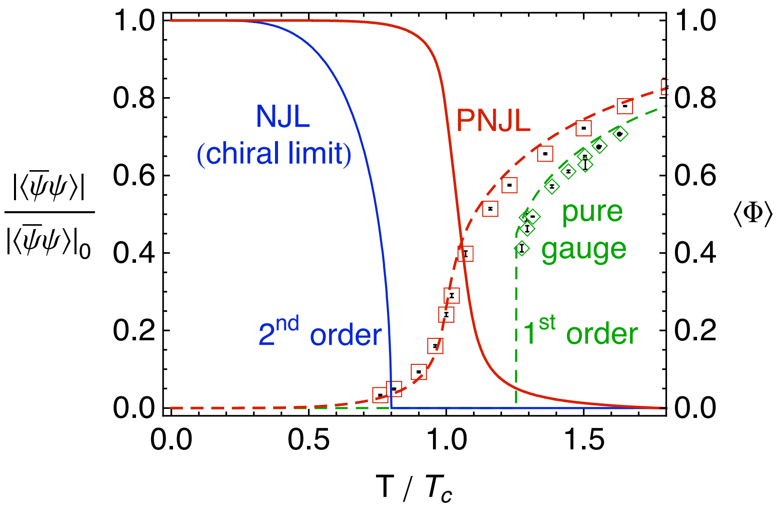
- Polyakov loop potential (adjusted to lattice results)

$$\frac{\mathcal{U}(\Phi,\Phi^*,T)}{T^4} = -\frac{1}{2}a(T)\Phi^*\Phi + b(T)\ln[1 - 6\Phi^*\Phi + 4(\Phi^{*3} + \Phi^3) - 3(\Phi^*\Phi)^2]$$

[S. Roessner, C. Ratti, and W. Weise, Phys. Rev. D75, 034007 (2007);C. Ratti, S. Roessner, M. A. Thaler, and W. Weise, Eur. Phys. J. C49, 213 (2007).]

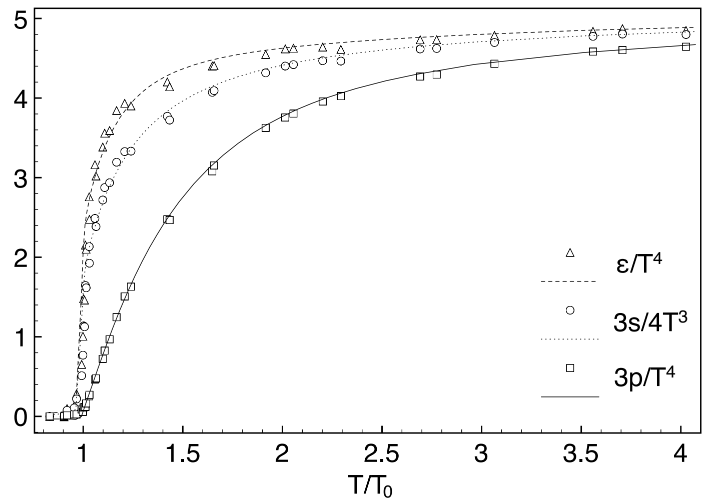
Thermodynamics from the PNJL model

- phase transitions



[S. Rößner, T. Hell, C. Ratti and W. Weise, Nucl. Phys. **A814**, 118 (2008).]

- thermodynamic quantities



[lattice results: G. Boyd et al., Nucl. Phys. B469, 419 (1996).]

Susceptibilities from the PNJL model

- partition function to evaluate after bosonization:

$$\mathcal{Z} = \int \mathcal{D}\phi \mathcal{D}\sigma \mathcal{D}\vec{\pi} \exp\left[\frac{V}{T}\Big(\frac{1}{2}\sum_n \sum_{\vec{p}} \mathrm{Tr}\ln[S^{-1}(T,\mu_u,\mu_d,\sigma,\vec{\pi},\phi)] - \mathcal{U}(\phi,T) - \frac{\sigma^2+\vec{\pi}^2}{2G}\Big)\right]$$

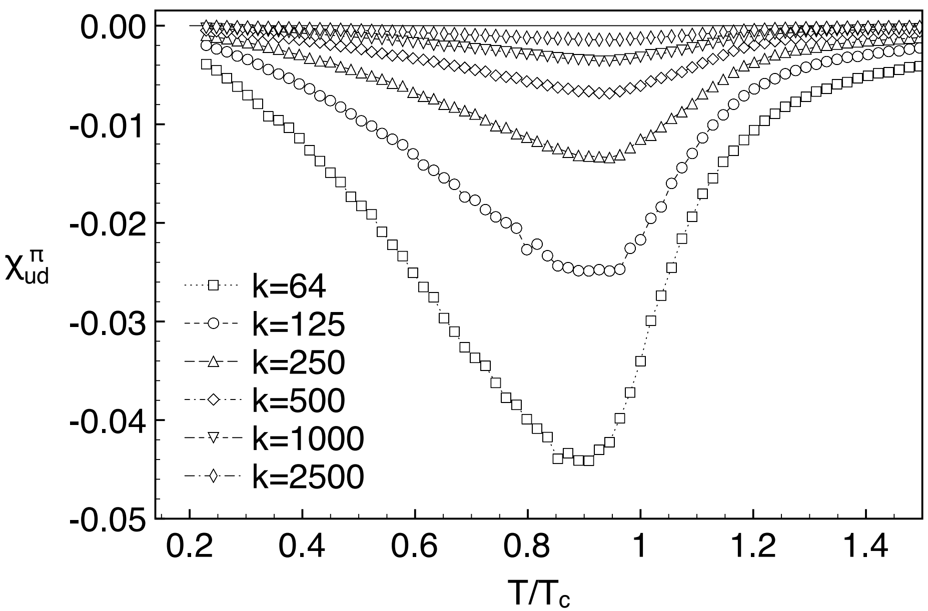
- quark number susceptibility to evaluate:

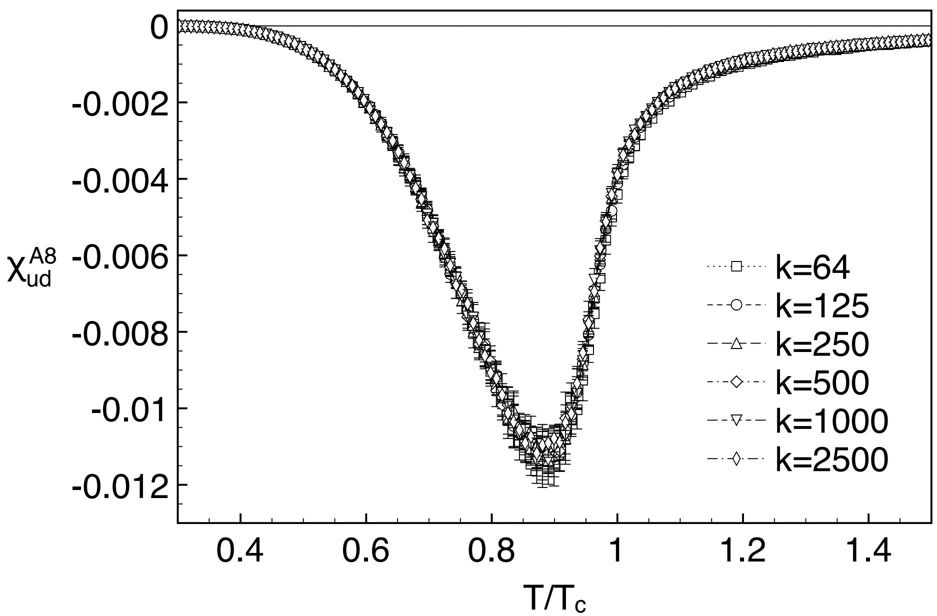
$$\chi_{ud} = \frac{T^2}{VT^3}\frac{\partial^2}{\partial\mu_u\partial\mu_d}\ln\mathcal{Z} = \frac{T^2}{VT^3}\Bigg\{\frac{V}{T}\left\langle\frac{\partial^2}{\partial\mu_u\partial\mu_d}\ln\det S^{-1}(T,\mu_u,\mu_d,\sigma,\vec{\pi},\phi)\right\rangle + \left(\frac{V}{T}\right)^2\left\langle\left(\frac{\partial}{\partial\mu_u}\ln\det S^{-1}(T,\mu_u,\mu_d,\sigma,\vec{\pi},\phi)\right)^2\right\rangle - \left(\frac{V}{T}\right)^2\left\langle\frac{\partial}{\partial\mu_u}\ln\det S^{-1}(T,\mu_u,\mu_d,\sigma,\vec{\pi},\phi)\right\rangle^2\Bigg\}$$

- ordinary multi-dimensional integral of σ, π, ϕ in mean-field
- evaluate by Monte-Carlo calculation for convenience

Flavor non-diagonal quark susceptibility

- Pion and Polyakov-loop contributions to the susceptibility





- Pion zero-mode contribution strongly volume dependent!

Chiral effective Lagrangian

- pions do not couple to baryon chemical potential, but do couple to isospin chemical potential $\mu_I = \mu_u - \mu_d$
- obtain chiral Lagrangian with isospin chemical potential from symmetries of QCD Lagrangian
- calculate pion contribution to χ_{ud} from chiral Lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}\left[\nabla_\nu \Sigma \nabla_\nu \Sigma^\dagger\right] - \frac{m\bar{\Sigma}}{4} \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right]$$

$$\nabla_0 \Sigma = \partial_0 \Sigma + \mu_I \left[\tau_3 \Sigma - \Sigma \tau_3\right], \quad \tau_3 = \mathrm{diag}(1, -1)$$

$$\nabla_i \Sigma = \partial_i \Sigma, \quad i = 1, 2, 3, \quad \Sigma = \exp\left(i \tau^a \pi^a / f_\pi\right) \in SU(2)$$

[J. B. Kogut, M. A. Stephanov, and D. Toublan, Phys. Lett. B464, 183 (1999); D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 86, 592 (2001).]

Chiral effective Lagrangian: static part

- include only pion zero-modes to calculate mean-field fluctuations in a finite volume
- static part of chiral Lagrangian in the partition function:

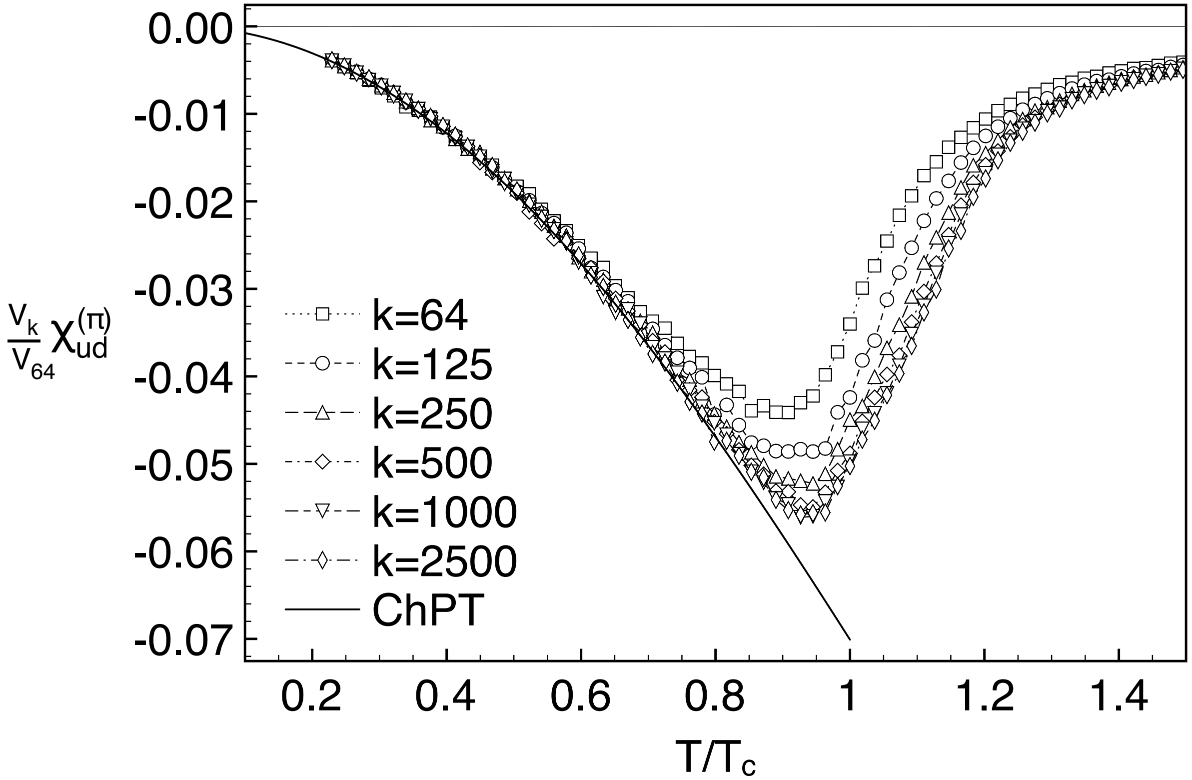
$$\mathcal{Z}^{(\pi)}_{\text{static}} = \int \prod_{a=1}^{3} \mathrm{d}\pi^a \exp - \frac{V}{T}\left[\frac{1}{2} m_\pi^2 \pi^a \pi^a - 4\mu_I^2 (\pi^+ \pi^-)\right]$$

- calculate flavor non-diagonal quark susceptibility:

$$\chi^{(\pi)}_{ud} = \frac{T}{V}\frac{1}{T^2}\frac{\partial^2}{\partial\mu_u \partial\mu_d} \ln \mathcal{Z}^{(\pi)}_{\text{static}}\Big|_{\mu_u=\mu_d=0} = -\frac{2\,T^2}{k}\frac{1}{m_\pi^2}, \quad V = k/T^3$$

Volume-scaled pion contribution

- below T_c, volume-scaled results can be described by static ChPT result with temperature-dependent pion mass



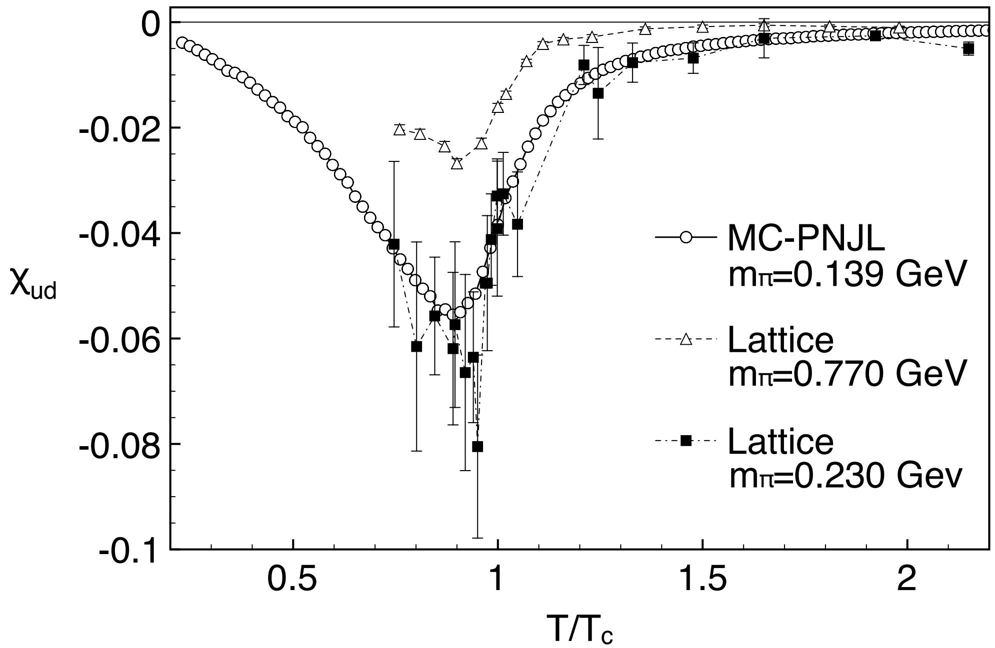
$$\sim -\frac{2T^2}{m_\pi^2(T)}$$

with $m_\pi(T)$ from 1-loop ChPT

→ valid below T_c

Comparison of full results to lattice QCD

- PNJL result at physical pion mass (m_π = 139 MeV)
- lattice results at larger pion mass (m_π = 230 MeV)
- both systems at same volume size! ($k = 64 = 4^3$)



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Conclusions

- Polyakov-loop extended Nambu-Jona-Lasinio model in a finite volume
- results for quark number susceptibilities obtained from this model, taking mean-field fluctuations into account
- Polyakov-loop contributions to these susceptibilities appear volume independent
- significant contributions from zero-mode fluctuations from the pion fields to flavor off-diagonal susceptibility χ_{ud}
- expect *decrease* of this susceptibility in lattice simulations in larger volumes
- non-zero momentum modes not yet included