QCD thermodynamics with HISQ action
Alexei Bazavov (University of Arizona, Tucson)
Péter Petreczky (BNL, Upton)

for HotQCD Collaboration

For further details see

• Introduction : cutoff effects and thermodynamics in the low-T region, lattice vs. HRG

• HISQ action and $T=0$ physics

• QCD thermodynamics with HISQ action and comparsion with other improved action calculations : $p4$, asqtad (RBC-BI, HotQCD) and stout (Budapest-Wuppertal)
Cheng et al, arXiv:0911.2215 [hep-lat]
Lattice results for “physical” quark masses

Lattice calculations at the physical quark mass and $N_\tau=8$, Cheng et al, arXiv:0911.2215

- Thermodynamics quantities are quark mass independent for $T>200\text{MeV}$
- The quark mass effect is also small at low temperature and is similar to cutoff effects
- Lattice results are significantly below the Hadron Resonance Gas
Improved staggered calculations at finite temperature

low T region
$T < 200\,\text{MeV}$

high-T region
$T > 200\,\text{MeV}$

$\mathcal{O} (\alpha_s^n (a \Lambda_{QCD})^2)$ errors

$a = 1 / (TN_T)$

N_T = 8

$a > 0.125\,\text{fm}$

hadronic degrees of freedom

improvement of the flavor symmetry is important → fat links

for $#\text{flavors} < 4$

rooting trick

$\delta m_{ps_i}^2 = m_{ps_i}^2 - m_{ps_G}^2 \sim \alpha_s a^2$

N_T = 6

N_T = 8

N_T = 12

quark degrees of freedom

quark dispersion relation

for $#\text{flavors} < 4$

rooting trick

$\text{det} D \rightarrow \left( \text{det} D \right)^{n_f/4}$

N_T

$p4, \text{asqtad}, \text{HISQ}$
The Highly Improved Staggered Quark (HISQ) Action

**HISQ action**

two levels of gauge field smearing with re-unitarization

Follana et al, PRD75 (07) 054502

\[
U_{\mu}(x) = e^{igaA_{\mu}(x)} = U_{\mu}^{fat7} \rightarrow \tilde{U}_{\mu} = \frac{U_{\mu}^{fat7}}{\sqrt{U_{\mu}^{fat7}U_{\mu}^{fat7\dagger}}}
\]

Smearing level 1

3-link (Naik) term to improve the quark dispersion relation + asqtad smearing

projection onto U(3) improves flavor symmetry

Hasefratz,

arXiv:hep-lat/0211007
**T=0 calculations with HISQ action**

\[ T=0 \text{ calculations with HISQ action} \]

\[ 0.2m_{S}, \, N_{\tau}=6 \]

\[
m_{\pi} = 306 - 312 \text{ MeV} \\
m_{K} = 522 - 532 \text{ MeV} \\
m_{\rho} = 850 - 883 \text{ MeV} \\
m_{N} = 1130 - 1183 \text{ MeV}
\]

\[ 0.05m_{S}, \, N_{\tau}=8 \]

\[
m_{\pi} = 156 - 161 \text{ MeV} \\
m_{K} = 493 - 501 \text{ MeV} \\
m_{\rho} = 760 - 808 \text{ MeV} \\
m_{N} = 1014 - 1083 \text{ MeV}
\]

The lattice spacing is set through the calculation of static potential

\[
\left( \frac{dV_{qq}(r)}{dr} \right)_{r=r_{0}} = 1.65, \quad r_{0} = 0.469(7) \text{ fm}
\]
Mass splitting of pseudo-scalar mesons

Only one out of 16 PS mesons has zero mass in the chiral limit, the quadratic mass splitting is the measure of flavor symmetry breaking

PS meson splittings in HISQ calculations are reduced by factor $\sim 2.5$ compared to asqtad at the same lattice spacing and are even smaller than for stout action $\Rightarrow$ discretizations effects for $N_\tau=8$ HISQ calculations are similar to those in $N_\tau=12$ asqtad calculations
Deconfinement transition: the Polyakov loop

\[ L_{\text{ren}} = \exp(-F_Q(T)/T) \]

- Polyakov loops in HISQ and p4 calculations agree reasonably well at high T.
- HISQ results are consistently larger than p4 results at low temperatures and transition.
- If the same procedure for fixing the lattice spacing is used \( (r_0) \) the HISQ and stout calculation agree for the raw data of \( L_{\text{ren}}(T) \).
Deconfinement transition: strangeness susceptibility

\[
\chi_s = \frac{1}{VT^3} \frac{\partial^2 \ln Z(T, \mu_s)}{\partial \mu_s^2}
\]

- Significant enhancement in strangeness fluctuations at low \( T \) compared to \textit{asqtad} and \textit{p4}.
- At high \( T \) HISQ results agree reasonably well with \textit{p4} and \textit{asqtad} results.
- HISQ results are closer to the HRG than \textit{p4} and \textit{asqtad} results but still smaller than HRG.
- HISQ calculations agree with the stout results on the raw data on \( \chi_s \) when \( r_0 \) scale is used.

- At high \( T \) HISQ results agree reasonably well with \textit{p4} and \textit{asqtad} results.
- Significant enhancement in strangeness fluctuations at low \( T \) compared to \textit{asqtad} and \textit{p4}.
- HISQ calculations agree with the stout results on the raw data on \( \chi_s \) when \( r_0 \) scale is used.
- HISQ results are closer to the HRG than \textit{p4} and \textit{asqtad} results but still smaller than HRG.
Deconfinement transition: strangeness susceptibility

\[ \chi_s = \frac{1}{VT^3} \frac{\partial^2 \ln Z(T, \mu_s)}{\partial \mu_s^2} \]

- HISQ calculations on \(N_\tau = 8\) agree reasonably well with asqtad calculations on \(N_\tau = 12\).

- Lattice results agree with the HRG model results, where hadron masses have been corrected for finite lattice spacing effects, Huovinen, P.P., arXiv:0912.2541 [hep-ph].

  => cutoff effects are similar to quark mass effect.

- HISQ results are closer to the HRG than p4 and asqtad results but still smaller than HRG.
Further comparison with HRG model

\[ \chi_B = \frac{1}{VT^3} \frac{\partial^2 \ln Z(T, \mu_B)}{\partial \mu_B^2} \]

- significant enhancement of baryon number fluctuations and trace anomaly at low T compared to asqtad and p4 obtained using \( N_\tau = 8 \) and \( N_\tau = 6 \) lattices
The subtracted chiral condensate

\[ \Delta_{s,l}(T) = \frac{\langle \bar{q} q \rangle_T - \frac{m_q}{m_s} \langle \bar{s} s \rangle_T}{\langle \bar{q} q \rangle_{T=0} - \frac{m_q}{m_s} \langle \bar{s} s \rangle_{T=0}} \]

• The transition region defined by the rapid change in the chiral condensate is shifted to smaller temperature compared to asqtad and p4 action
• HISQ results on the \( \Delta_{l,s} \) raw data agree with stout results if the scale is set by \( r_0 \)
The subtracted chiral condensate

\[
\Delta_{s,l}(T) = \frac{\langle \bar{q}q \rangle_T - \frac{m_q}{m_s} \langle \bar{s}s \rangle_T}{\langle \bar{q}q \rangle_{T=0} - \frac{m_q}{m_s} \langle \bar{s}s \rangle_{T=0}}
\]

- The transition region defined by the rapid change in the chiral condensate is shifted to smaller temperature compared to asqtad and p4 action.
- HISQ results on the $\Delta_{l,s}$ raw data agree with $N_\tau=10$ stout results if the scale is set by $r_0$.
- HISQ calculations on $N_\tau=8$ agree reasonably well with asqtad calculations on $N_\tau=12$. 
Disconnected chiral susceptibility

\[ \chi_{\text{disc}} = N_\sigma^3 N_\tau \left( \langle (\bar{q}q)^2 \rangle - \langle \bar{q}q \rangle^2 \right) \]

- The disconnected chiral susceptibility in HISQ calculations shows broad plateau at low temperatures due to Goldstone effect similar to the p4 action
  
  \[ \Rightarrow \text{difficult to extract} \; T_c \]

- The transition region is shifted to smaller temperature compared to asqtad at the same lattice spacing

- finite lattice spacing effects act like finite mass effect
Summary and outlook

• The use of the HISQ action reduces discretization effects in the hadron spectrum, especially in the pseudo-scalar sector

• Thermodynamic quantities are enhanced in the low $T$ region compared to calculations with p4 and asqtad action at the same lattice spacing, while consistent with the previous calculations in high temperature region => cutoff effects act like quark mass effect

• A consistent picture emerges in the low $T$ region: due to reduced flavor symmetry breaking calculations with HISQ action on $N_{\tau}=8$ lattices agree well with the raw stout results on $L_{\text{ren}}$, $\chi_s$ and subtracted chiral condensate if $r_0$ is used to set the lattice spacing; There is also reasonably good agreement with the $N_{\tau}=12$ asqtad calculations

• Future: detailed calculations of EoS, Taylor expansion coefficients, quark mass dependence of the transition and verification of the $O(N)$ scaling with HISQ action on $N_{\tau}=8$ and $N_{\tau}=12$ lattices
Back-up: Results from improved staggered calculations at $T=0$

$a=0.125\text{fm}, 0.09\text{fm}, 0.06\text{fm}$, chiral and continuum extrapolations

HPQCD, UKQCD, MILC and Fermilab, PRL 92 (04) 022001

Fermilab, HPQCD, MILC PRL 94 (05) 011601 (hep-ph/0408306)
Exp.: Belle, hep-ex/0510003

Back-up: Results from improved staggered calculations at $T=0$

$a=0.125\text{fm}, 0.09\text{fm}, 0.06\text{fm}$, chiral and continuum extrapolations

HPQCD, UKQCD, MILC and Fermilab, PRL 92 (04) 022001

Fermilab, HPQCD, MILC PRL 94 (05) 011601 (hep-ph/0408306)
Exp.: Belle, hep-ex/0510003

also $N$, $\Omega$ and $\phi$ masses

Bernard et al (MILC), PoSLAT2007 (07) 137;
Aoki et al, arXiv:0903.4155v1 [hep-lat]

To obtain these results it was necessary to implement:

1) $O(a^2)$ improvement of quark dispersion relation
2) reduce the flavor symmetry breaking in the staggered fermion formulation

LQCD: $M_{B_c} = (6304 \pm 12 + 18)\text{MeV}$
Fermilab, HPQCD, UKQCD
PRL 94 (05) 172001 [hep-lat/0411027]
Exp: $M_{B_c} = (6285.7 \pm 5.3 \pm 1.2)\text{MeV}$
CDF, PRL 96 (06) 082002 [hep-ex/0505076]