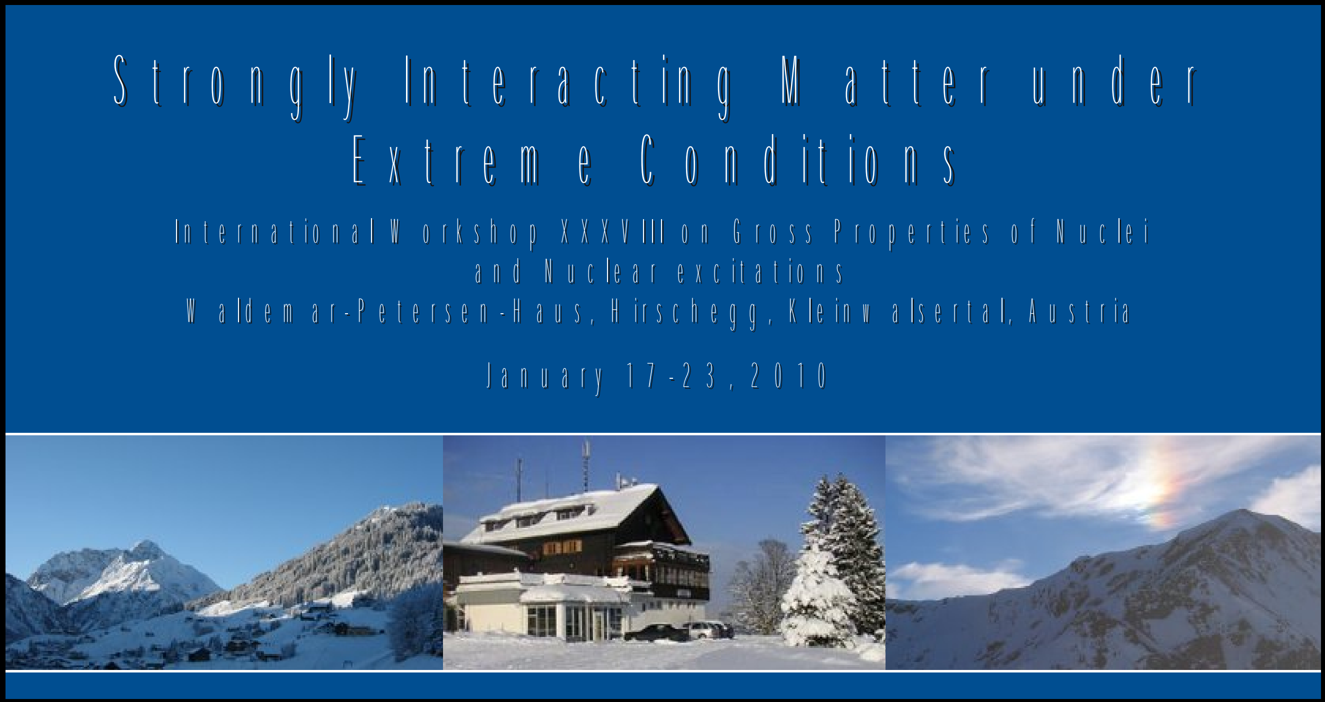
Elliptic Flow: lessons from RHIC

expectations for the LHC

Raimond Snellings
Nikhef, Amsterdam



- Our current understanding of the observables used to estimate anisotropic flow

- Comparing elliptic flow measurements with theory

- Outlook for elliptic flow at the LHC for pp and AA

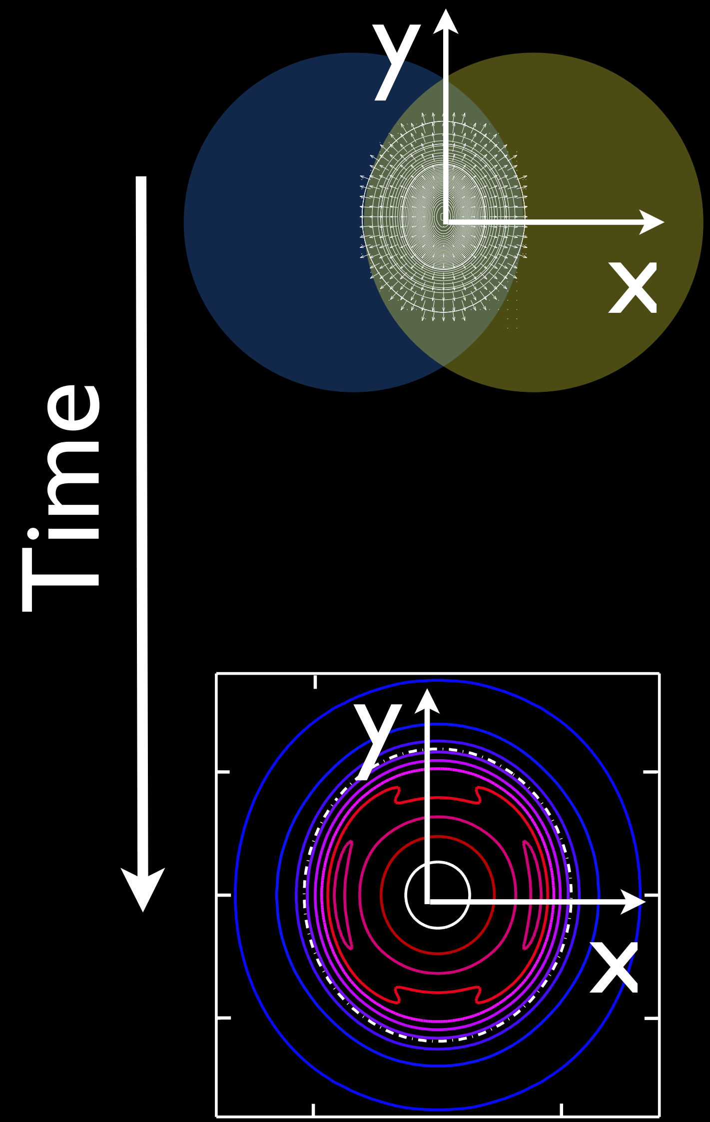
Elliptic Flow

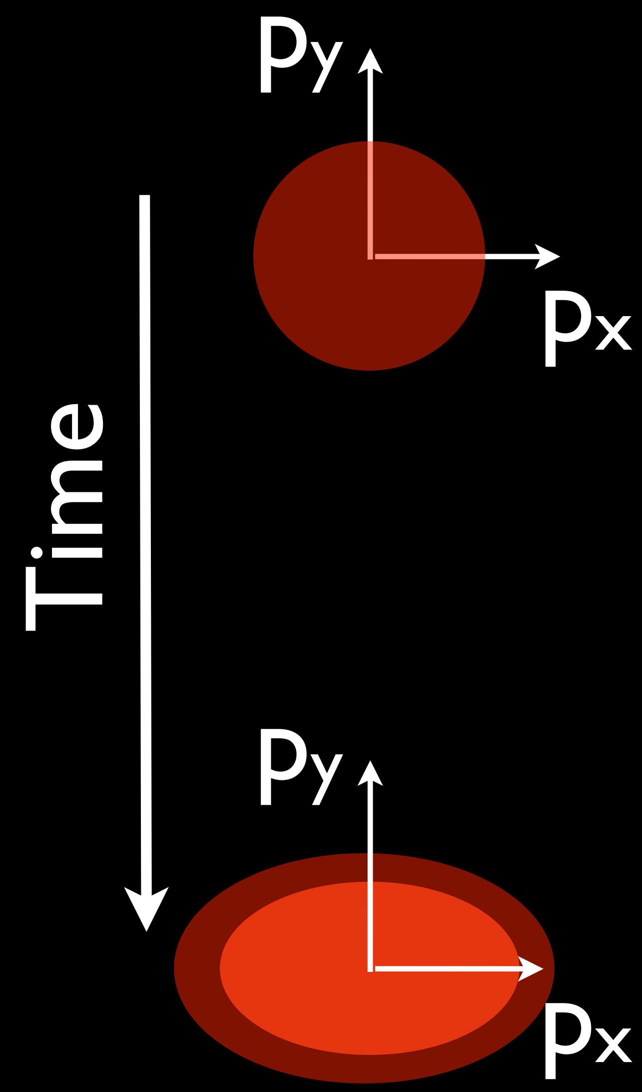
J.Y. Ollitrault, PRD 46, 229 (1992)

- in non central collisions coordinate space configuration is anisotropic (almond shape). However, initial momentum distribution isotropic (spherically symmetric)
- interactions among constituents generate a pressure gradient which transforms the initial coordinate space anisotropy into the observed momentum space anisotropy → anisotropic flow
- self-quenching → sensitive to early stage

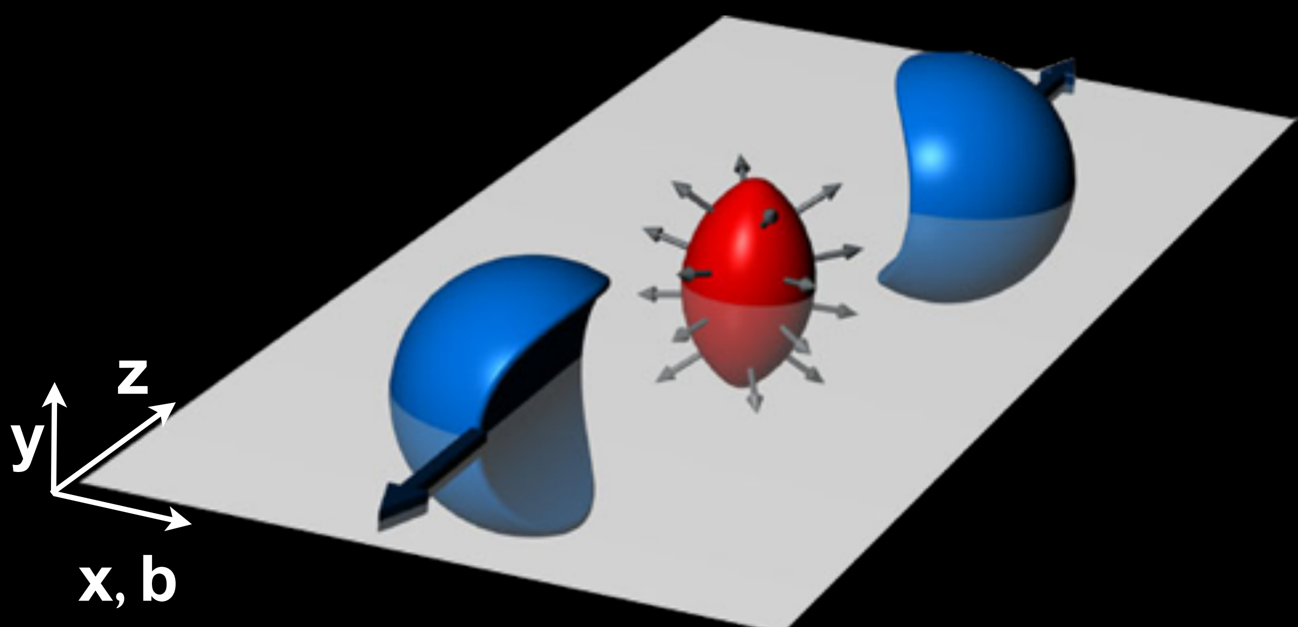
- a unique hadronic probe of the early stage

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} \qquad v_2 = \langle \cos 2\phi \rangle$$





Anisotropic Flow



Azimuthal distributions of particles measured with respect to the reaction plane (spanned by impact parameter vector and beam axis) are not isotropic.

$$E\frac{d^3N}{d^3\vec{p}} = \frac{1}{2\pi}\frac{d^2N}{p_T\,dp_T\,dy}\left(1+\sum_{n=1}^{\infty}2v_n\cos\left(n\left(\phi-\Psi_{\rm RP}\right)\right)\right)$$

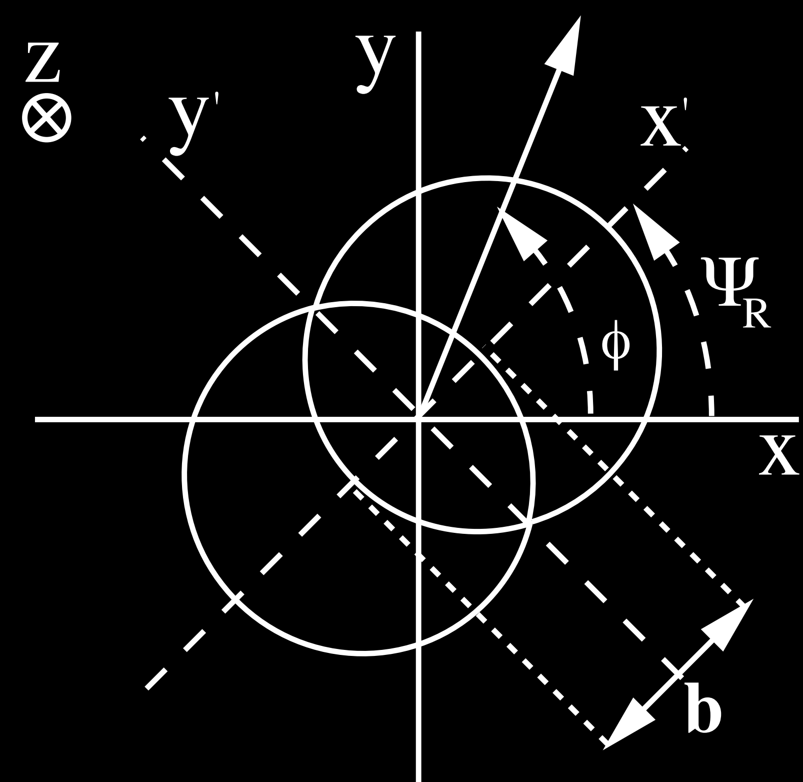
$$v_n = \langle \cos n(\phi - \Psi_{\rm RP})\rangle$$

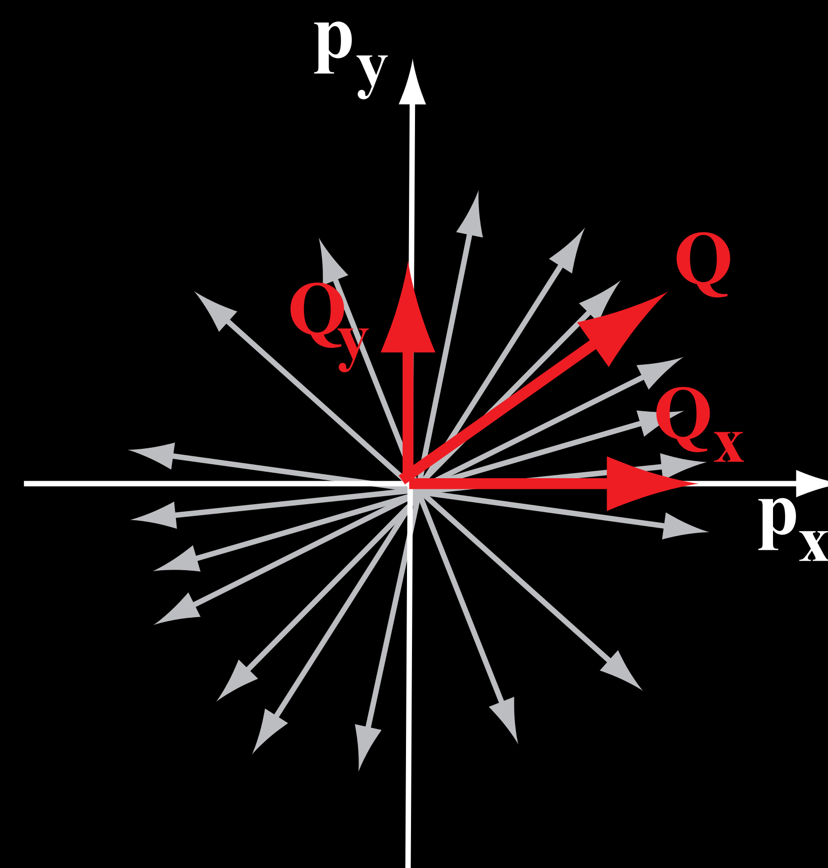
harmonics v_n quantify anisotropic flow

S. Voloshin and Y. Zhang (1996)

Event Plane Method

the event plane is an experimental estimate of the reaction plane





$$Q_{nx} = \sum_i w_i \cos\left(n\phi_i\right)$$

$$\Psi_n^{EP} = \frac{1}{n} \tan^{-1}\left(\frac{Q_{ny}}{Q_{nx}}\right)$$

$$Q_{ny} = \sum_i w_i \sin\left(n\phi_i\right)$$

$$v_n^{\mathrm{obs}} = \left\langle \cos n \left(\phi_i - \Psi_n^{EP}\right)\right\rangle$$

resolution and subevents

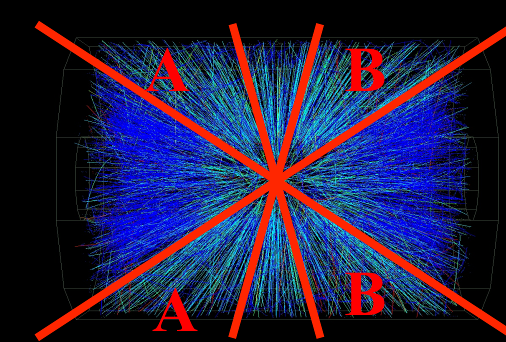
- due to the finite number of detected particles there is a limited resolution in the event plane angle

$$v_n^{\mathrm{obs}} = \left\langle \cos n \left(\phi_i - \Psi_n^{EP} \right) \right\rangle$$

$$v_n = \frac{v_n^{\mathrm{obs}}}{\langle \cos n \left(\Psi_n^{EP} - \Psi_R \right) \rangle}$$

- one can correct for that with subevents

$$\left\langle \cos n \left(\Psi_n^{EP} - \Psi_R \right) \right\rangle = \mathrm{C} \times \sqrt{\langle \cos n \left(\Psi_n^a - \Psi_n^b \right) \rangle}$$



measure anisotropic flow

- since reaction plane cannot be measured event-by-event, consider quantities which do not depend on it's orientation: multi-particle azimuthal correlations

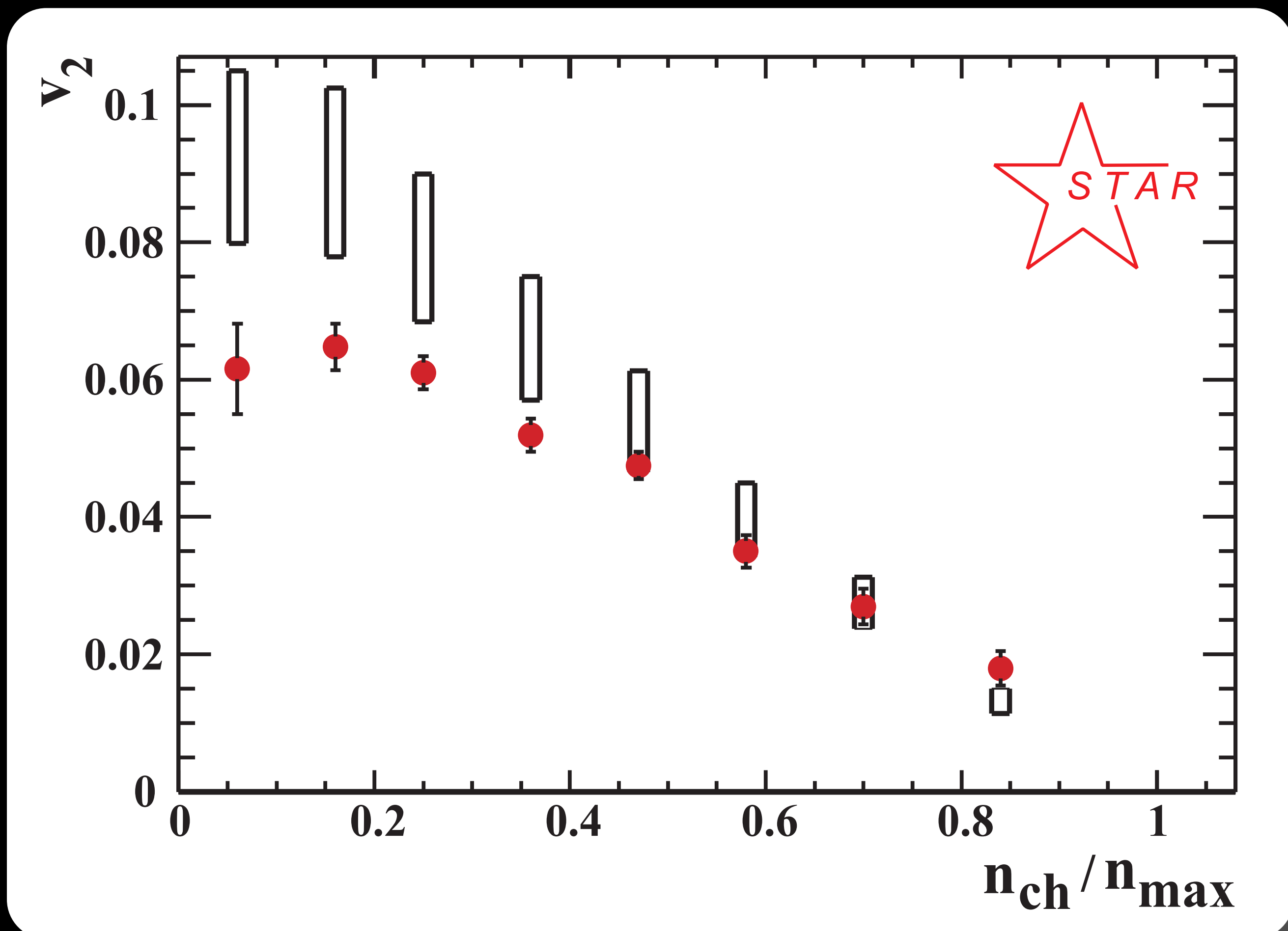
$$\left\langle e^{in(\phi_1-\phi_2)}\right\rangle = \left\langle e^{in\phi_1}\right\rangle \left\langle e^{-in\phi_2}\right\rangle + \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_{\text{corr}}$$

zero for symmetric detector when averaged over many events

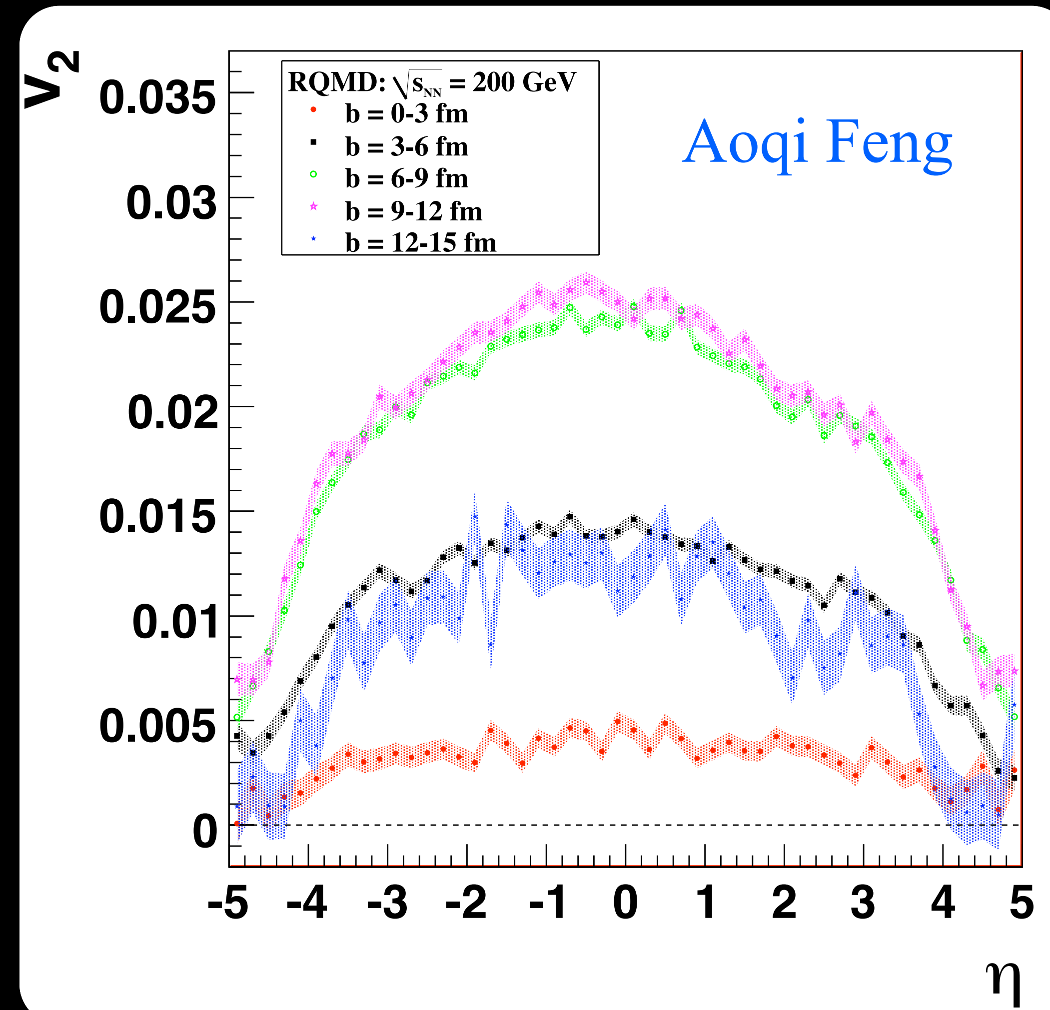
$$\begin{aligned}\langle\langle e^{in(\phi_1-\phi_2)}\rangle\rangle &= \langle\langle e^{in(\phi_1-\Psi_R-(\phi_2-\Psi_R))}\rangle\rangle \\ &= \langle\langle e^{in(\phi_1-\Psi_R)}\rangle\langle e^{-in(\phi_2-\Psi_R)}\rangle\rangle \\ &= \langle v_2^2\rangle\end{aligned}$$

- assuming that only correlations with the reaction plane are present

Elliptic Flow at RHIC



STAR Phys. Rev. Lett. 86, 402–407 (2001)

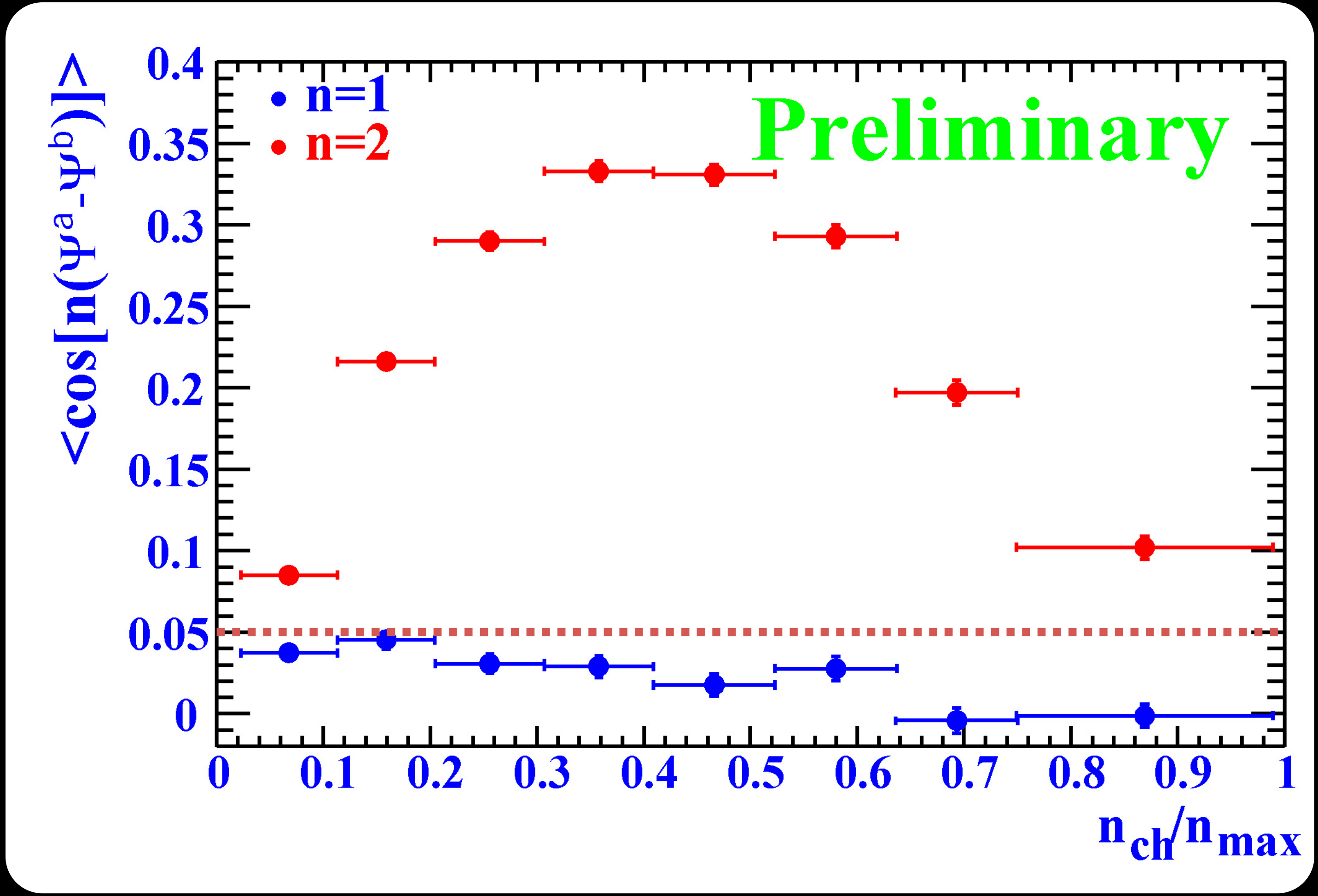


Ideal hydro gets the magnitude for more central collisions

Hadron cascade calculations are factors 2-3 off

What about nonflow?

- when dominated by flow the event plane resolution scales with $M^{1/2}$ x v_2 (when not too close to 1)
- gives very characteristic dependence on centrality
- nonflow will scale very different: the red line was first STAR estimate of nonflow



STAR, PRL 86, (2001) 402, Nucl. Phys. A698 (2002) 193

Estimate is not well defined, requires assumptions on the nature of the nonflow.
How to estimate nonflow as function of transverse momentum?

Can we do better?

- build cumulants using multi-particle correlations

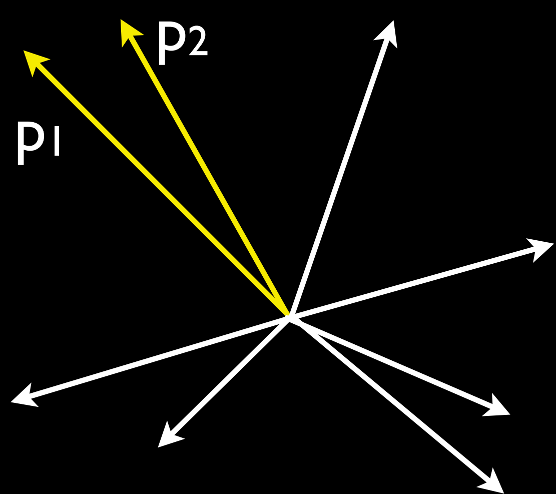
  Borghini, Dinh and Ollitrault (2001)
- for detectors with uniform acceptance 2nd and 4th cumulant are given by:

$$
\begin{aligned}
c_n\{2\} &\equiv \left\langle\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\right\rangle = v_n^2 + \delta_2 \\
c_n\{4\} &\equiv \left\langle\!\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle\!\right\rangle - 2\left\langle\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\right\rangle^2 \\
&= v_n^4 + 4v_n^2\delta_2 + 2\delta_2^2 - 2(v_n^2+\delta_2)^2 + \delta_4 \\
&= -v_n^4 + \delta_4
\end{aligned}
$$

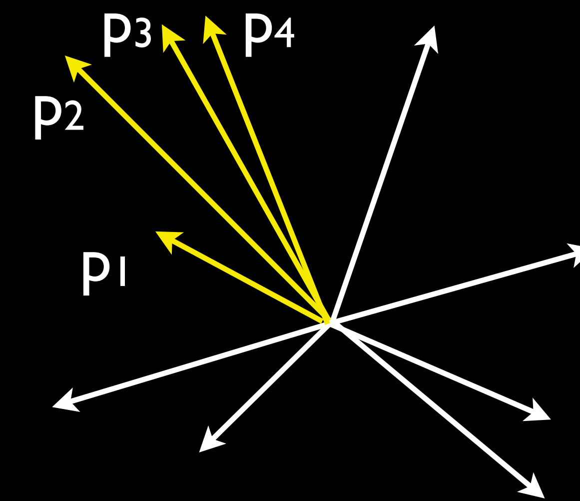
we got rid of two particle nonflow correlations!

we can remove nonflow order by order

Are we doing better?



$$\delta_2 \sim 1/M, \qquad \delta_4 \sim 1/M^3$$



- therefore to reliably measure flow: for M=200

$$v_n^2 \gg 1/M \quad \Rightarrow \quad v_n \gg 1/M^{1/2}$$ >> 0.07

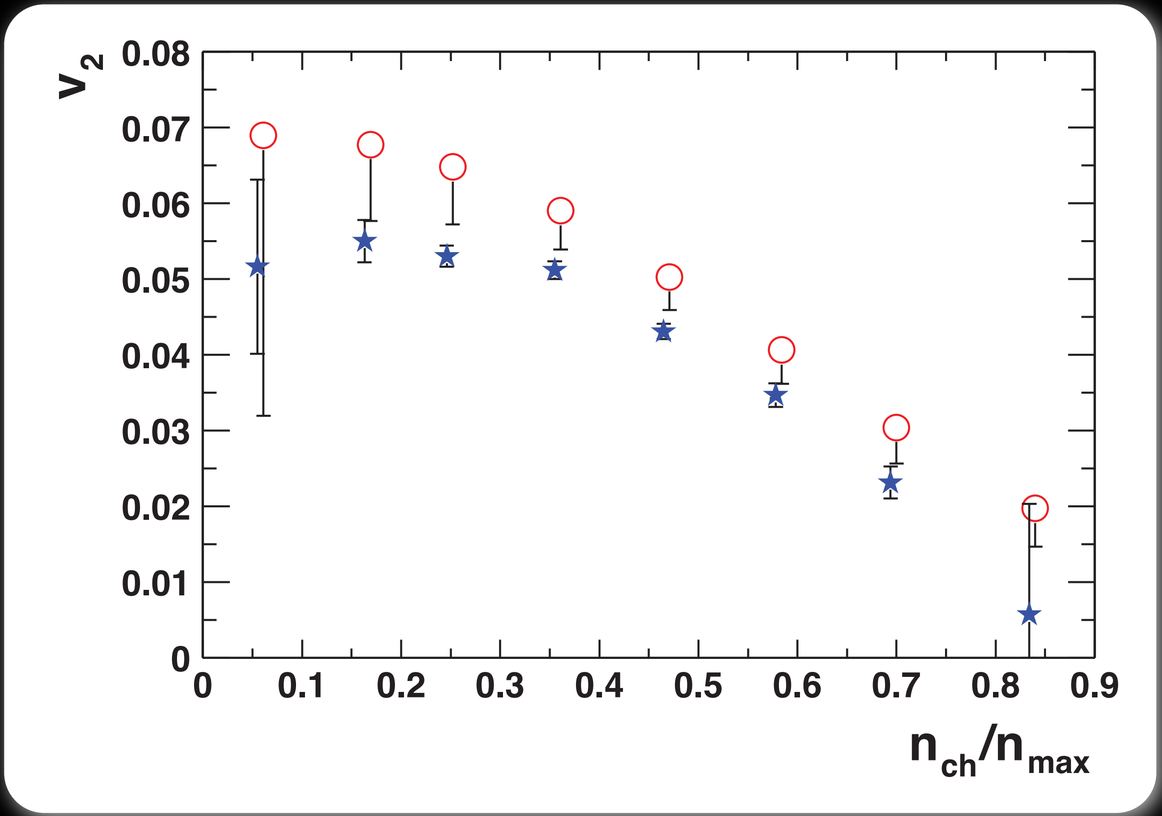
$$v_n^4 \gg 1/M^3 \quad \Rightarrow \quad v_n \gg 1/M^{3/4}$$ >> 0.019

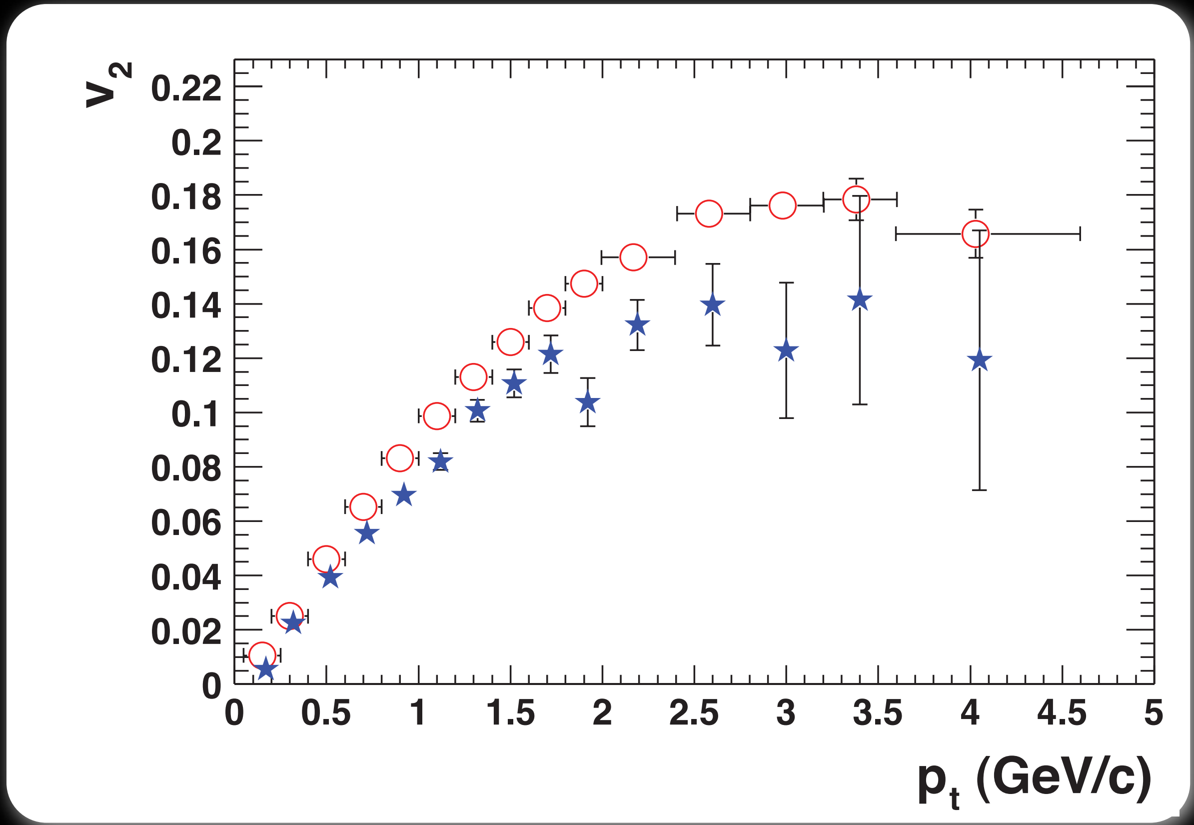
- for Lee Yang Zeroes:

$$v_n \gg 1/M$$ >> 0.005

First cumulant results

○ $v_2\{2\}$ ★ $v_2\{4\}$





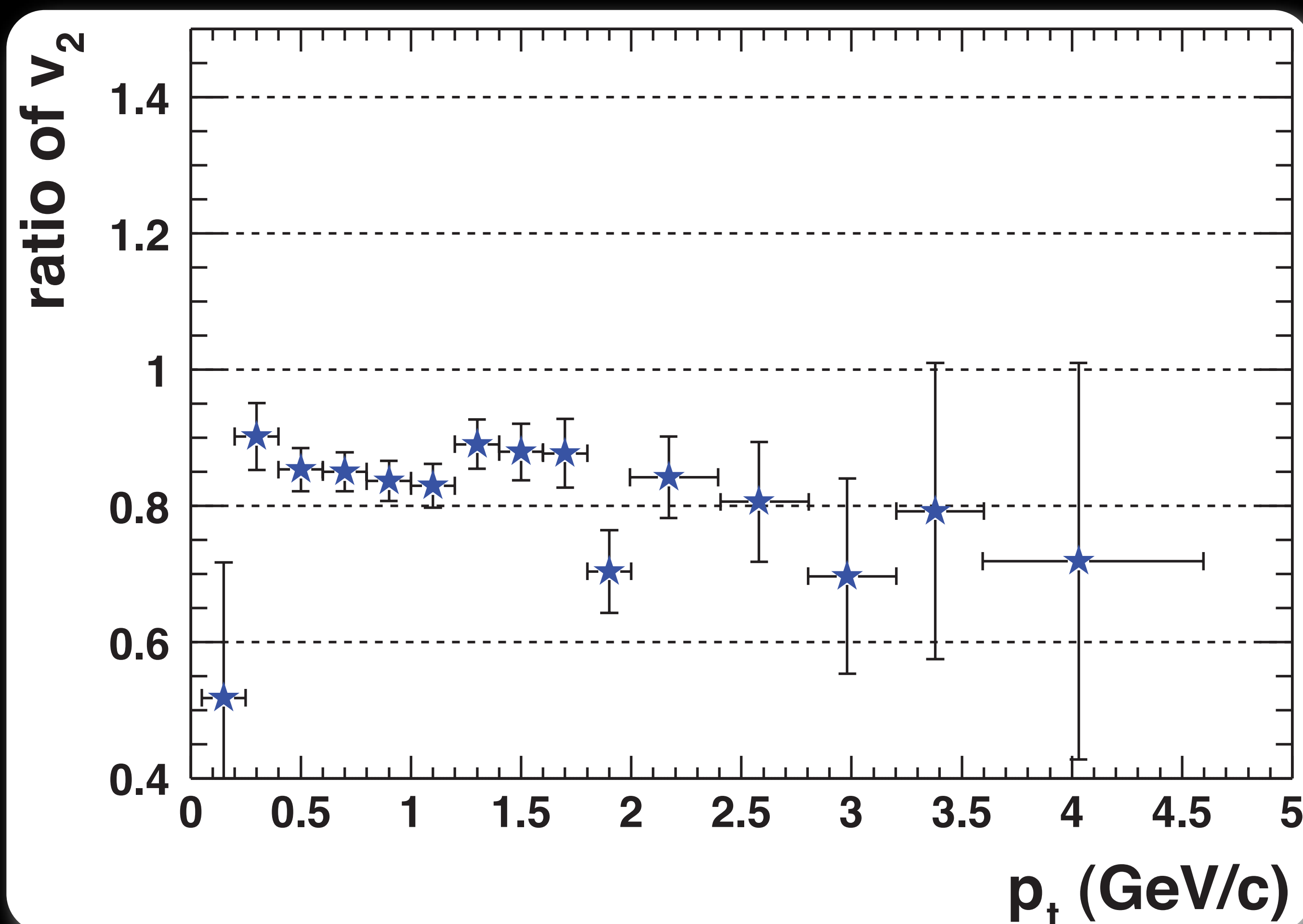
STAR, PRC (nucl-ex/0206001)

observed “nonflow” corrections are significant

corrections larger than earlier estimates

First Surprises/Questions

ratio of $v_2\{4\}/v_2\{2\}$ as function of p_t is rather flat!



STAR, PRC (nucl-ex/0206001

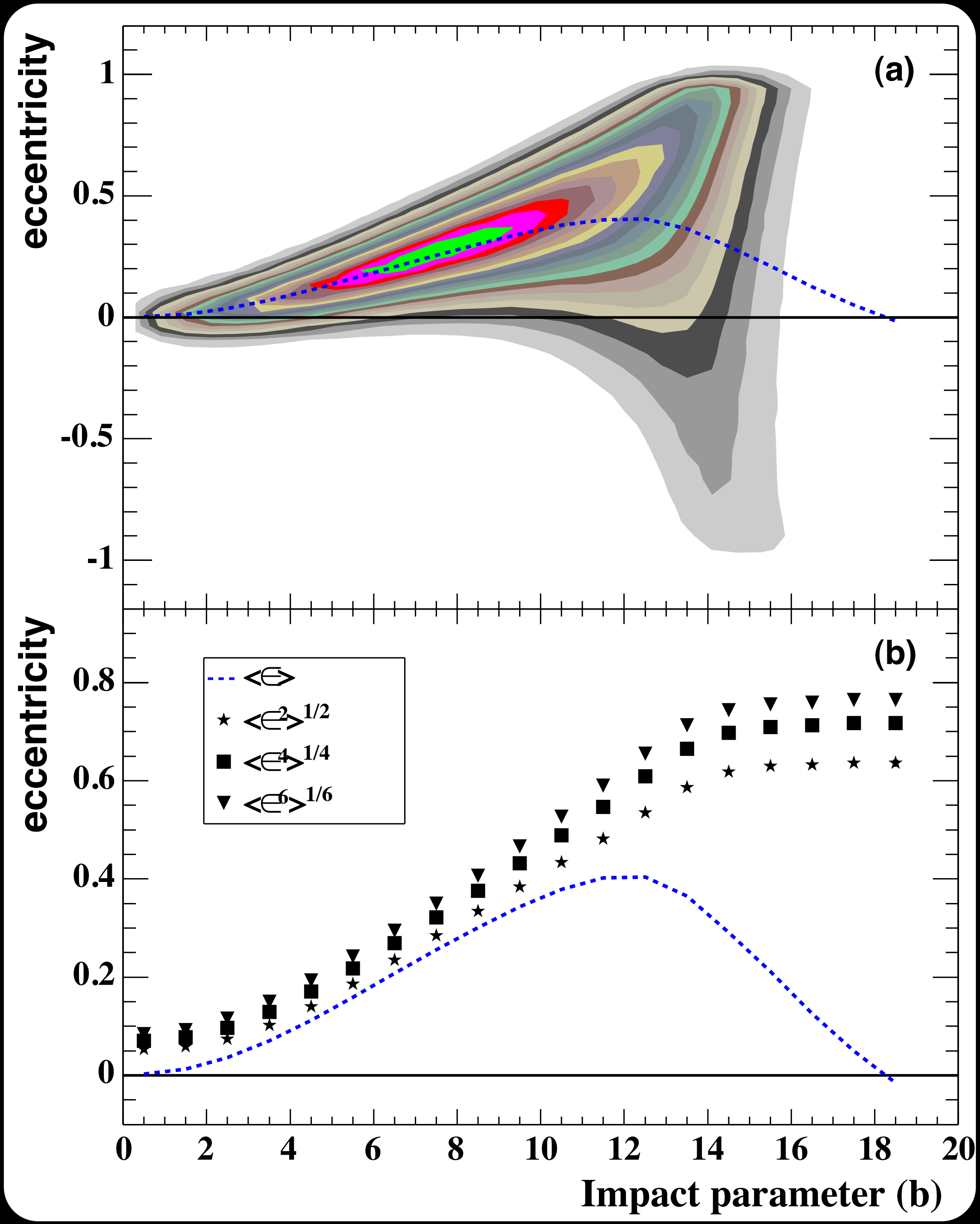
Figure 18 presents the p_t-dependence of the correction factor for non-flow. Within errors, the relative non-flow effect is seen to be about the same or increasing very weakly from low p_t through $p_t \sim 4$ GeV/c — a somewhat surprising result, given the presumption that the processes responsible for non-flow are different at low and high p_t.

V. ELLIPTIC FLOW FLUCTUATIONS

High precision results presented in this publication become sensitive to another effect usually neglected in flow analysis, namely, event-by-event flow fluctuations. The latter can have two different origins: "real" flow fluctuations — fluctuations at fixed impact parameter and fixed multiplicity (see, for example [40]) — and impact parameter variations among events from the same centrality bin in a case where flow does not fluctuate at fixed impact parameter. These effects, in principle, are present in any kind of analysis, including the "standard" one based on pair correlations.

corrections for "trivial" fluctuations were applied for integrated flow (both $v_2\{2\}$ and $v_2\{4\}$)

v_2 fluctuations



M. Miller and RS, arXiv:nucl-ex/0312008

- measured: $v_2\{2\} = \sqrt{(\langle v_2\rangle^2 + \sigma_v^2 + \delta)}$
- using: $v_2 \propto \varepsilon$
- If the eccentricity fluctuates

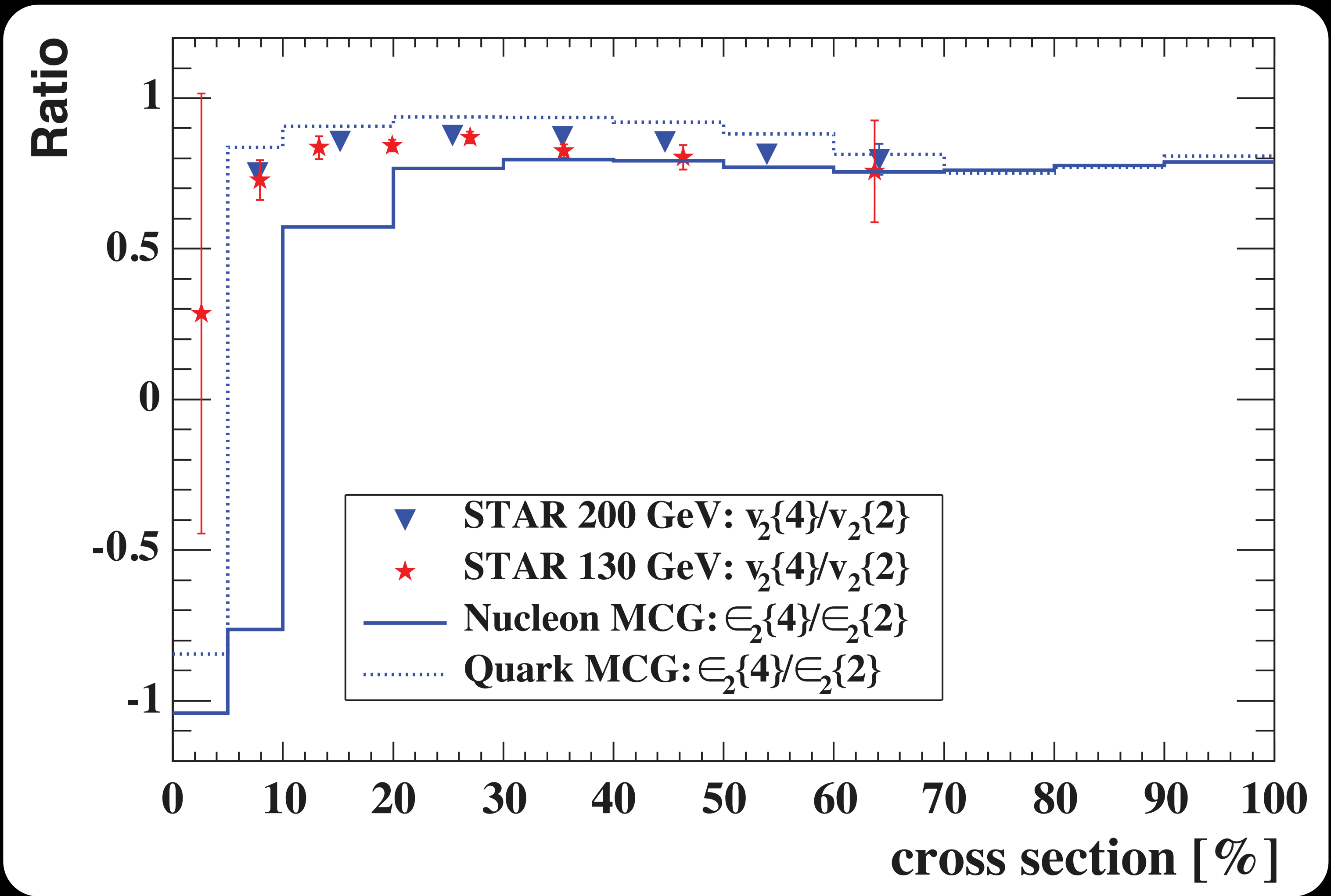
$$\langle \varepsilon^2 \rangle - \langle \varepsilon \rangle^2 \neq 0$$

$$\langle v_2 \rangle \neq \sqrt{\langle (v_2)^2 \rangle}$$

- fluctuations change v_2 estimate significantly!

v_2 fluctuations

Eccentricity fluctuations and its possible effect on elliptic flow measurements



$$v\{2\} = \langle v\rangle + \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$

$$v\{4\} = \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$

$$v\{6\} = \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$

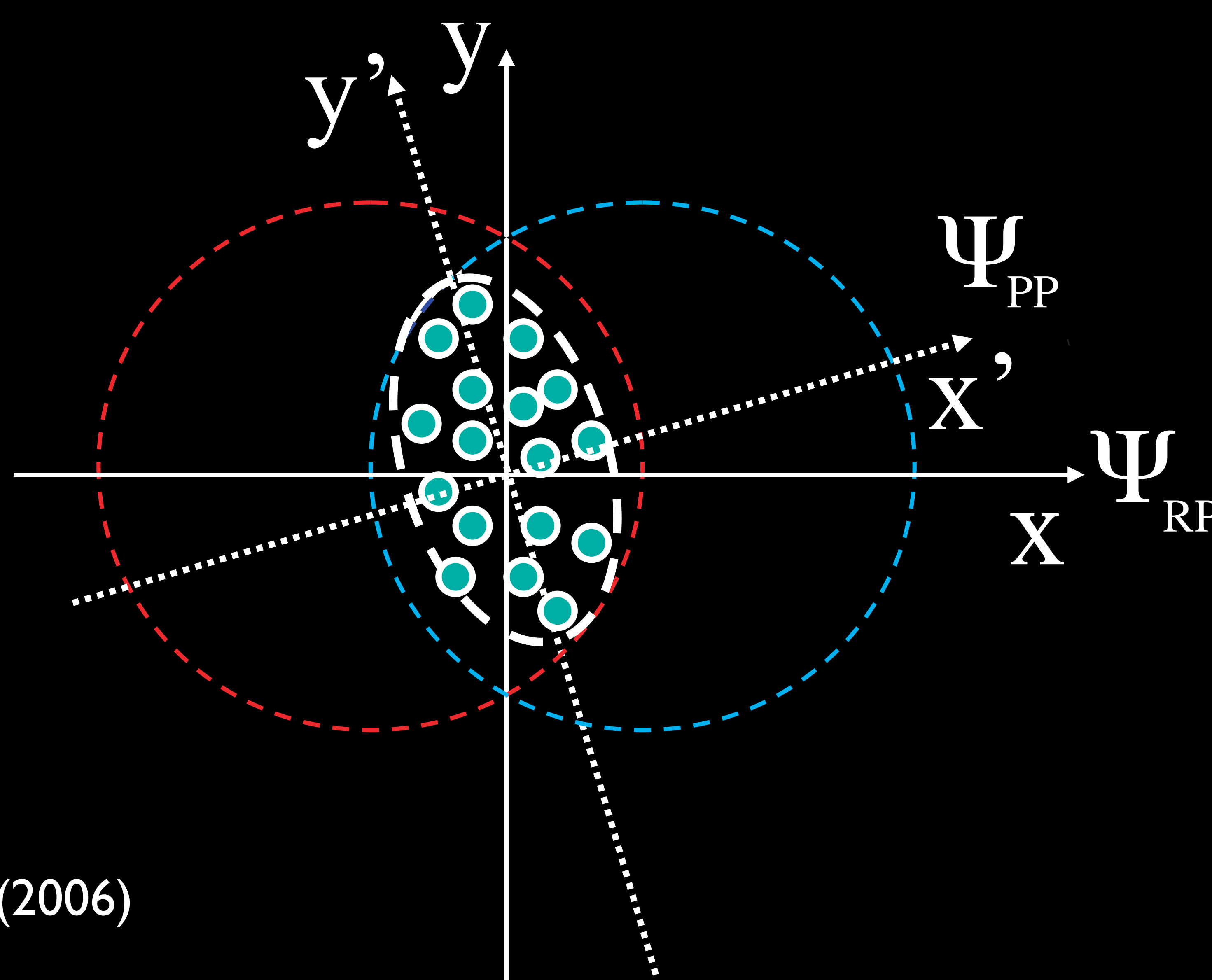
$$v\{8\} = \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$

M. Miller and RS, arXiv:nucl-ex/0312008

eccentricity fluctuations explains much of the observed differences in the cumulants!

Fluctuations and Planes

- RP the reaction plane, defined by the impact parameter
- PP the participant plane, defined by the major axis of the created system



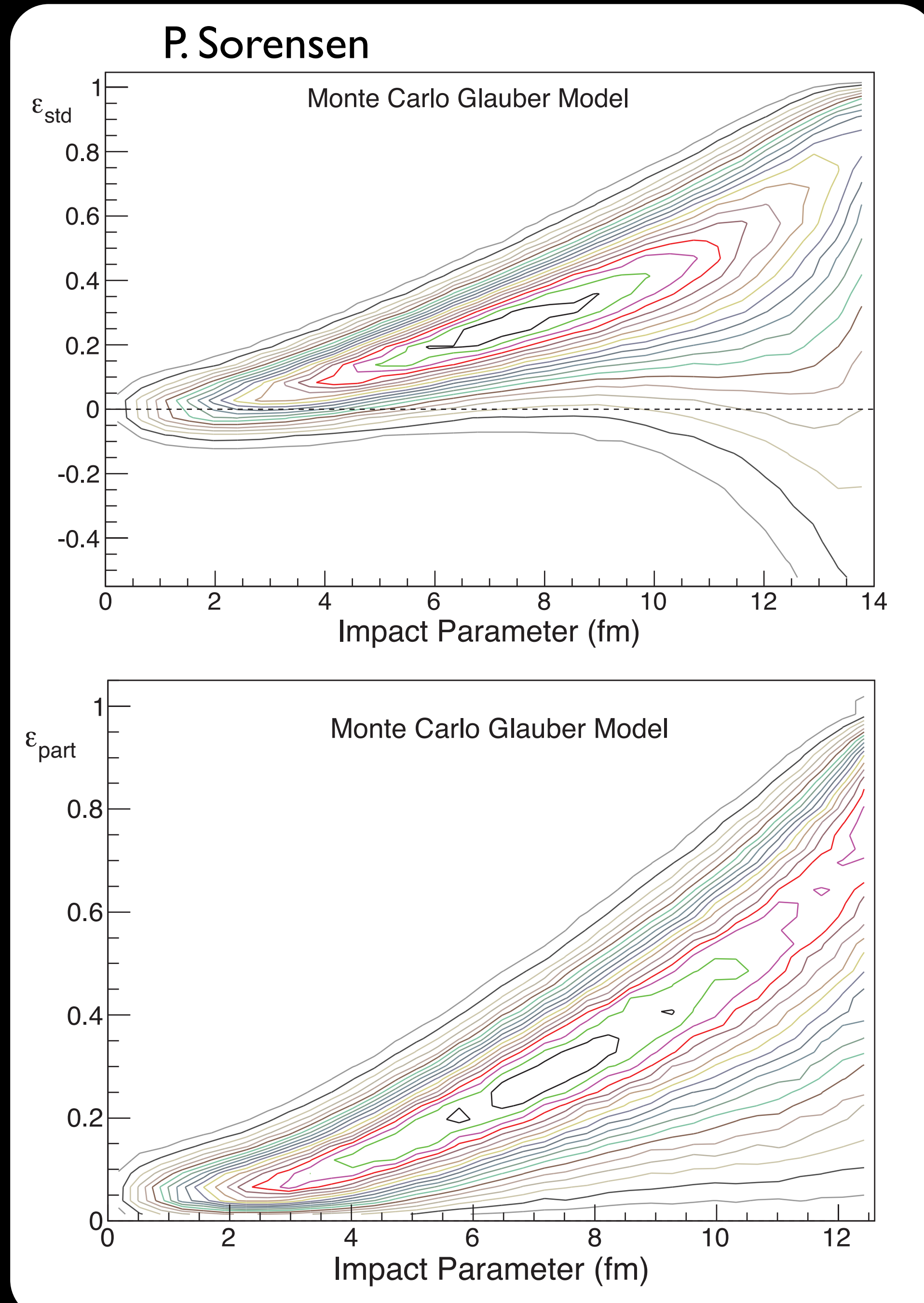
PHOBOS QM2005: Nucl. Phys. A774: 523 (2006)

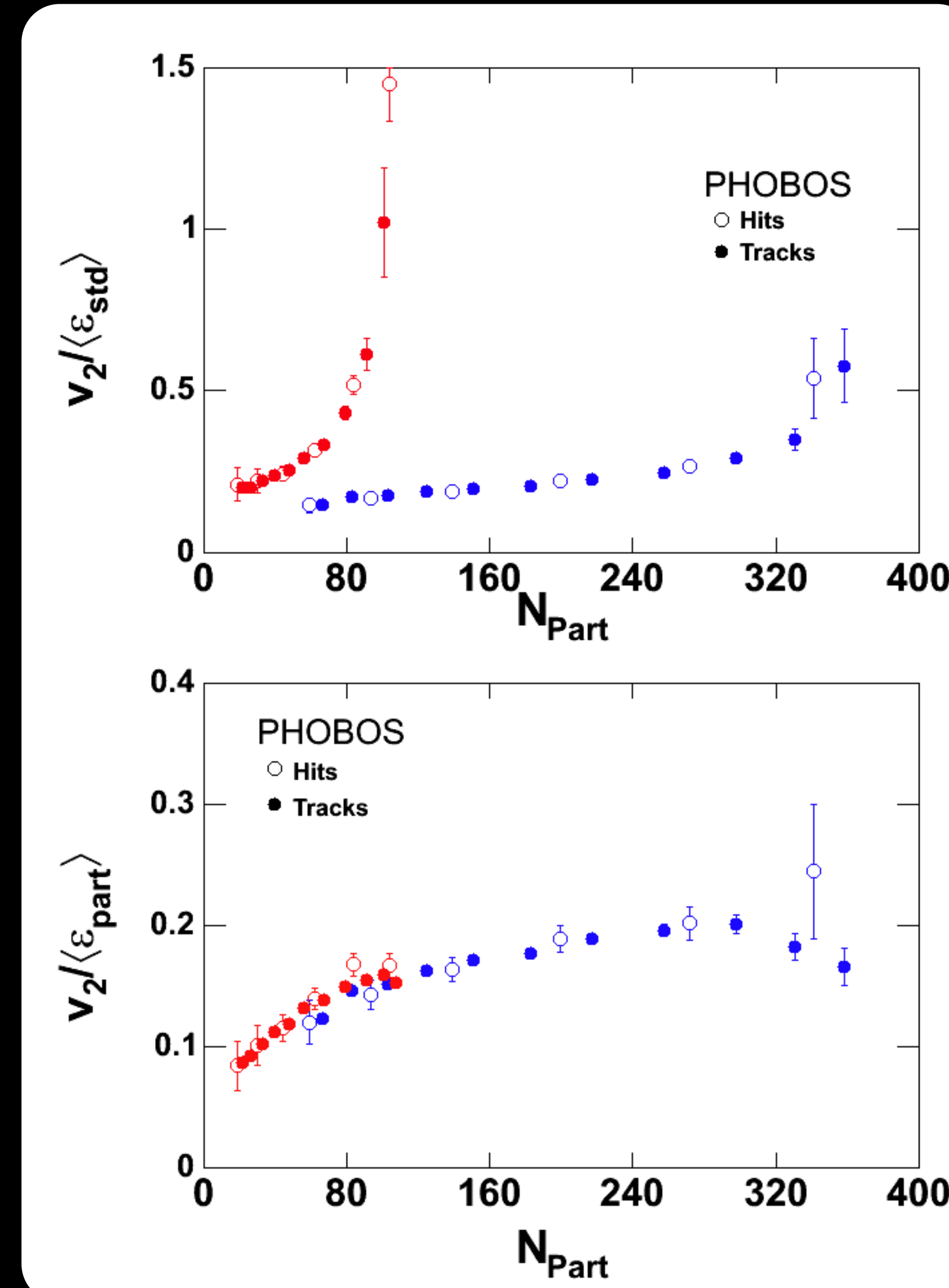
fluctuations in the eccentricity change the angle of the symmetry plane

PHOBOS CuCu results

PHOBOS QM2005: Nucl. Phys. A774: 523 (2006)

correcting the eccentricity for the fluctuations restores the scaling between CuCu and AuAu



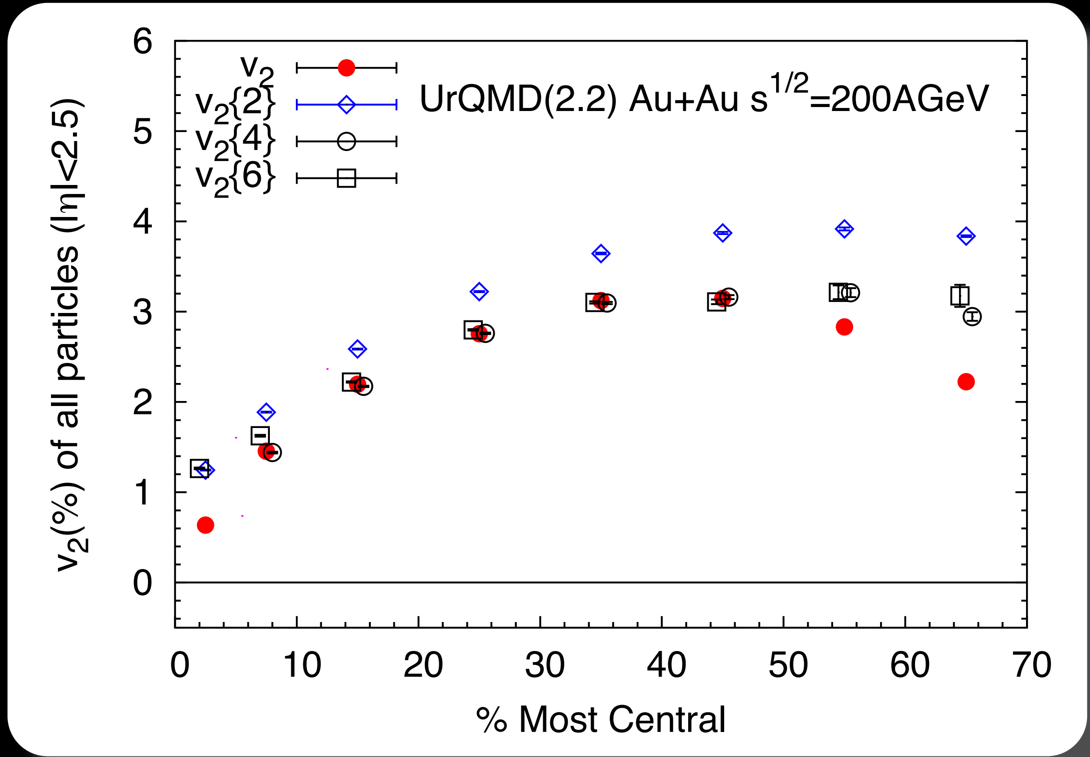


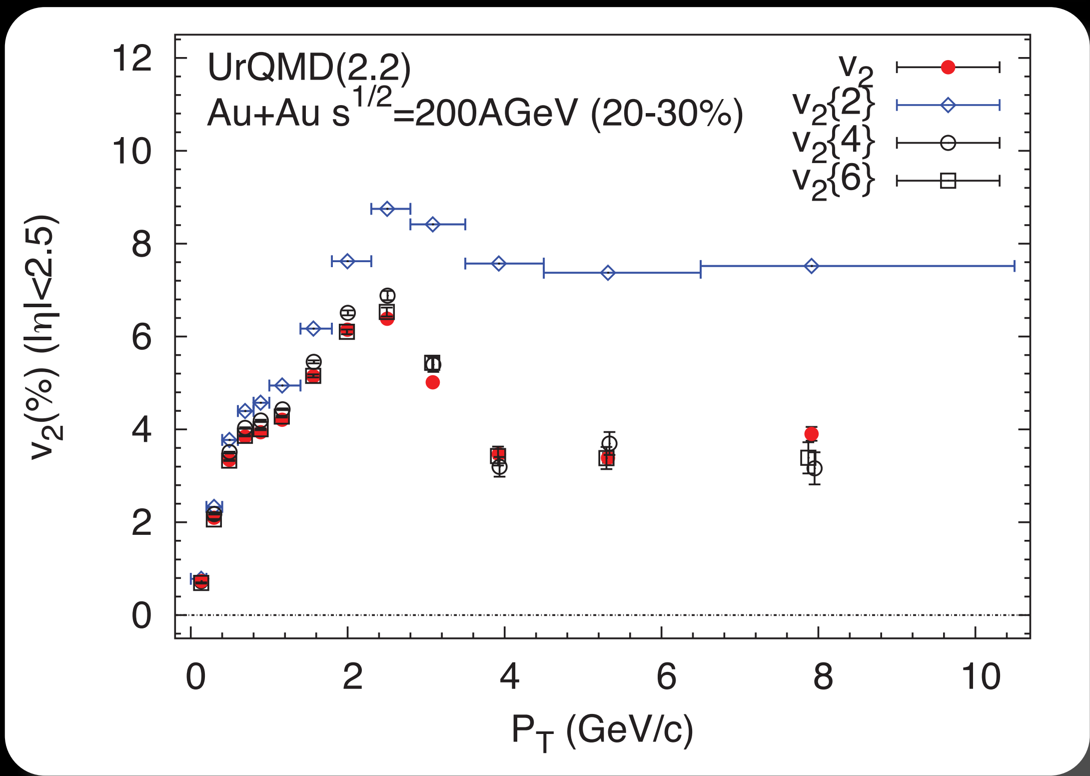
Strong experimental evidence for eccentricity fluctuations

v_2, nonflow and fluctuations

- two-particle correlation methods ($v_2\{EP\}$, $v_2\{2\}$) measure flow in participant plane (+ nonflow)
- multi-particle methods ($v_2\{4\}$ and higher, $v_2\{LYZ\}$) and methods using the directed flow of the spectators (ZDC) measure flow in the reaction plane and in addition remove the nonflow

R.S. Bhalerao , J-Y. Ollitrault
Phys.Lett.B641:260-264,2006
S.A. Voloshin, A.M. Poskanzer, A. Tang, G. Wang
Phys.Lett.B659:537-541,2008

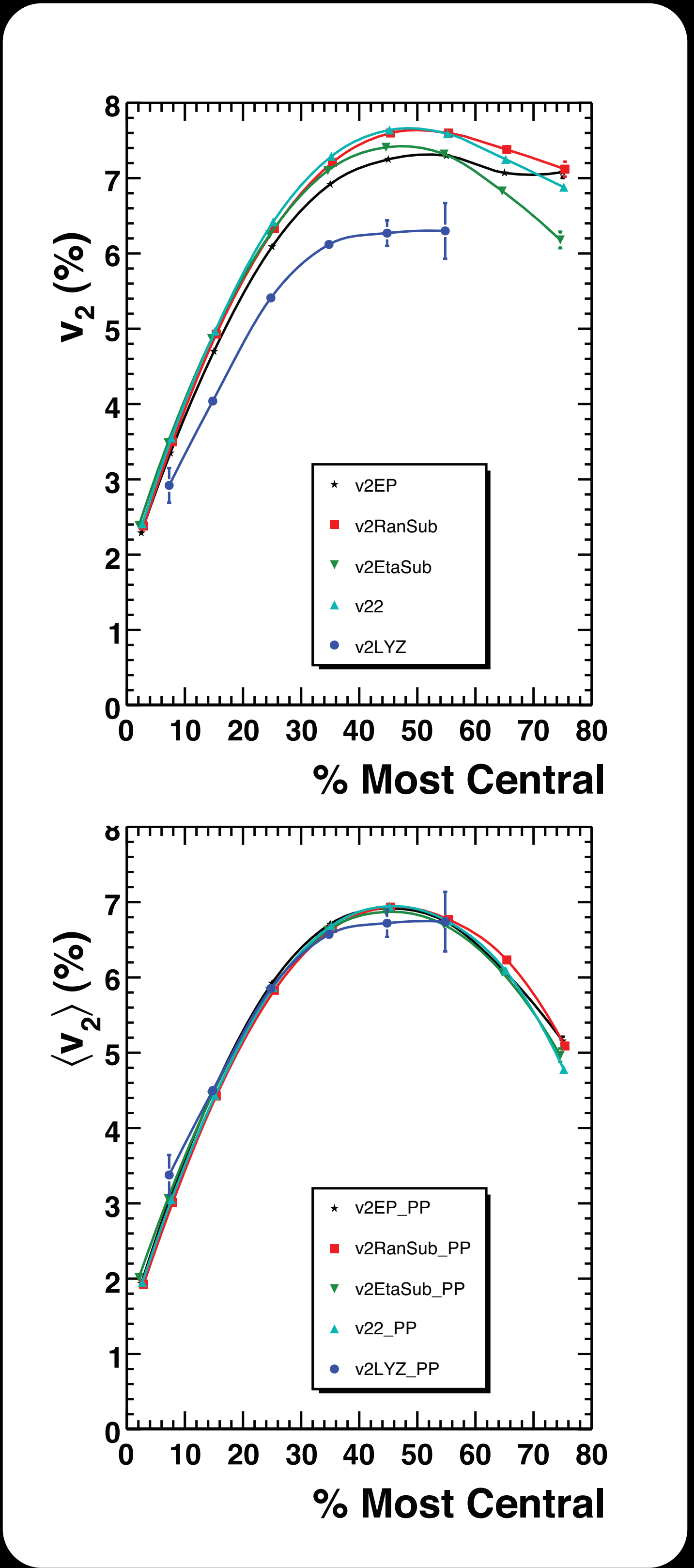




X Zhu, M. Bleicher, H. Stoeker, Phys.Rev.C72:064911(2005)

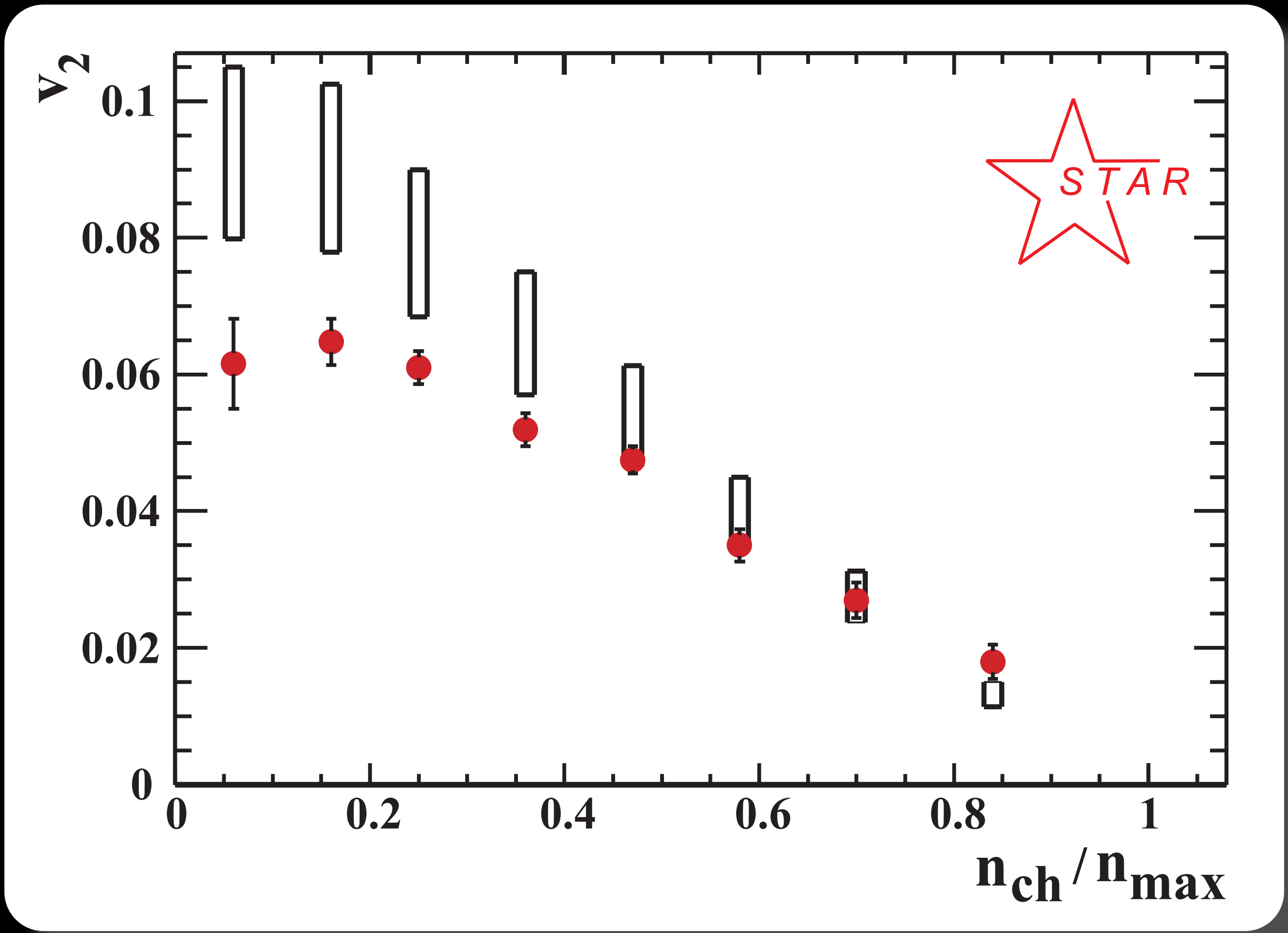
Summary Observables

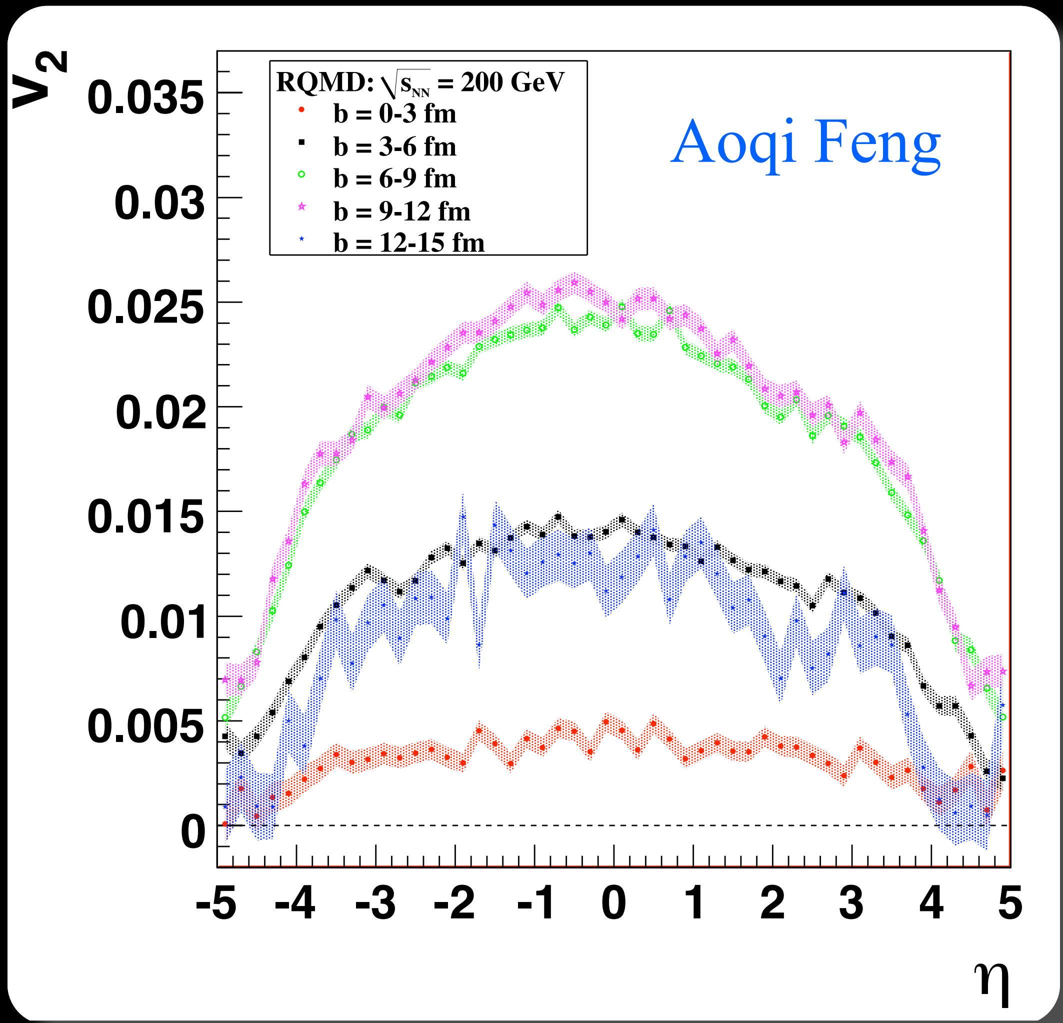
- To compare to theory apples to apples comparisons need to be made
  - particular if we want to do better than 20% (e.g. to constrain viscous corrections)
- Ideally calculate the observables $v_2\{2\}$, $v_2\{4\}$, $v_2\{LYZ\}$ directly in the model



Jean-Yves Ollitrault, Arthur M. Poskanzer and Sergei A. Voloshin: arXiv:0904.2315v3

Compare to Theory





STAR Phys. Rev. Lett. 86, 402–407 (2001)

Ideal hydro gets the magnitude for more central collisions

Hadron cascade calculations are factors 2-3 off

Beyond Ideal Hydro

NA49: C. Alt et al., Phys. Rev. C68, 034903 (2003)

for ideal hydrodynamics:

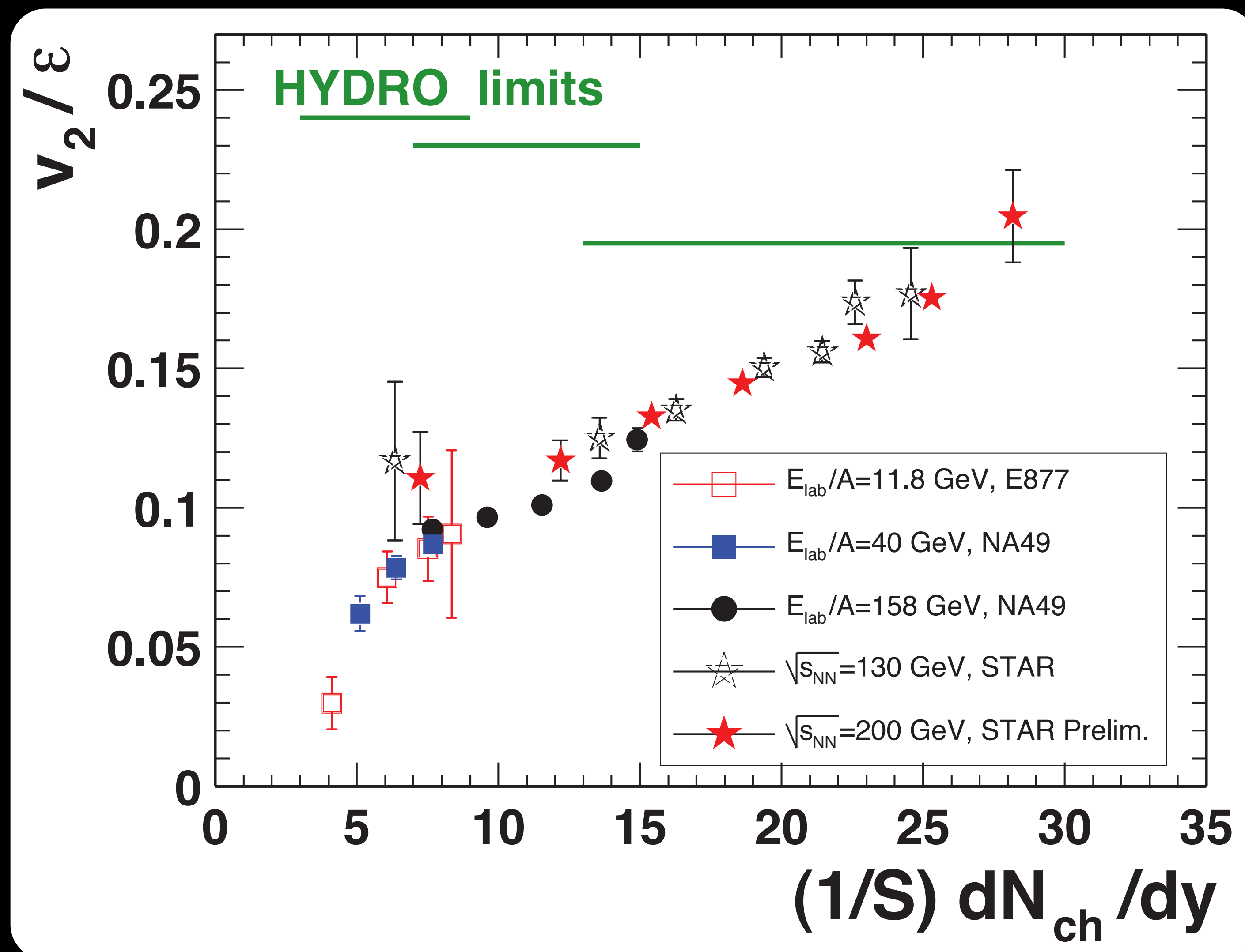
$$\frac{v_2}{\epsilon} = h$$

in the Low Density Limit (LDL):

$$\frac{v_2}{\epsilon} \propto \sigma \frac{1}{S} \frac{dN}{dy}$$

H. Heiselberg and A. M. Levy, Phys. Rev. C 59, 2716 (1999)

S. A. Voloshin and A. M. Poskanzer, Phys. Lett. B 474, 27 (2000)

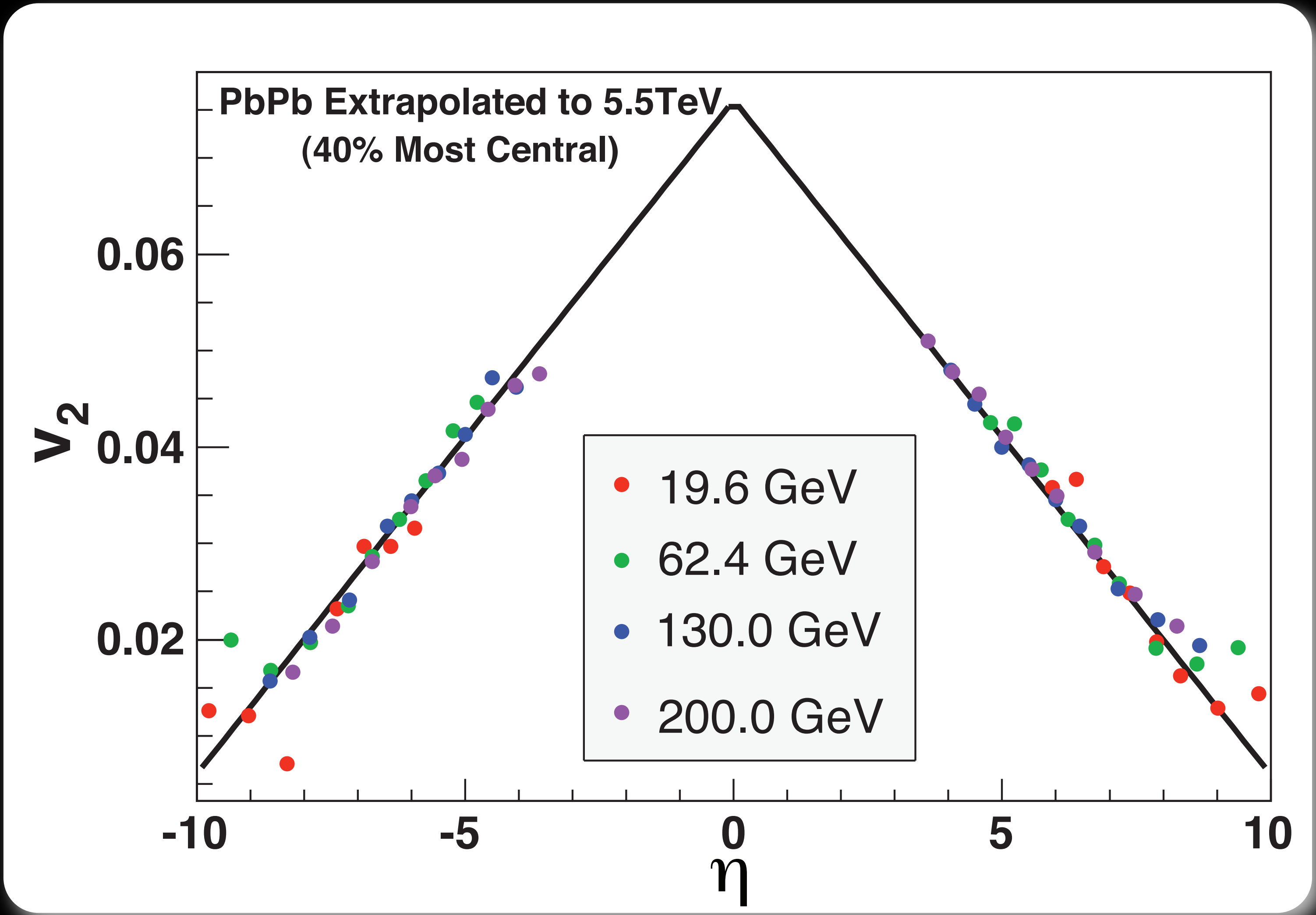


Hydro limits from P.F. Kolb, J. Sollfrank, U.W. Heinz; Phys.Rev.C62:054909,2000.

This figure is <u>not</u> understood in ideal hydrodynamics!
viscous corrections needed: parton cascade, viscous hydro, hadron cascade ...... hybrid models do get the dN/dy dependence

Beyond Ideal Hydro

PHOBOS, W. Busza



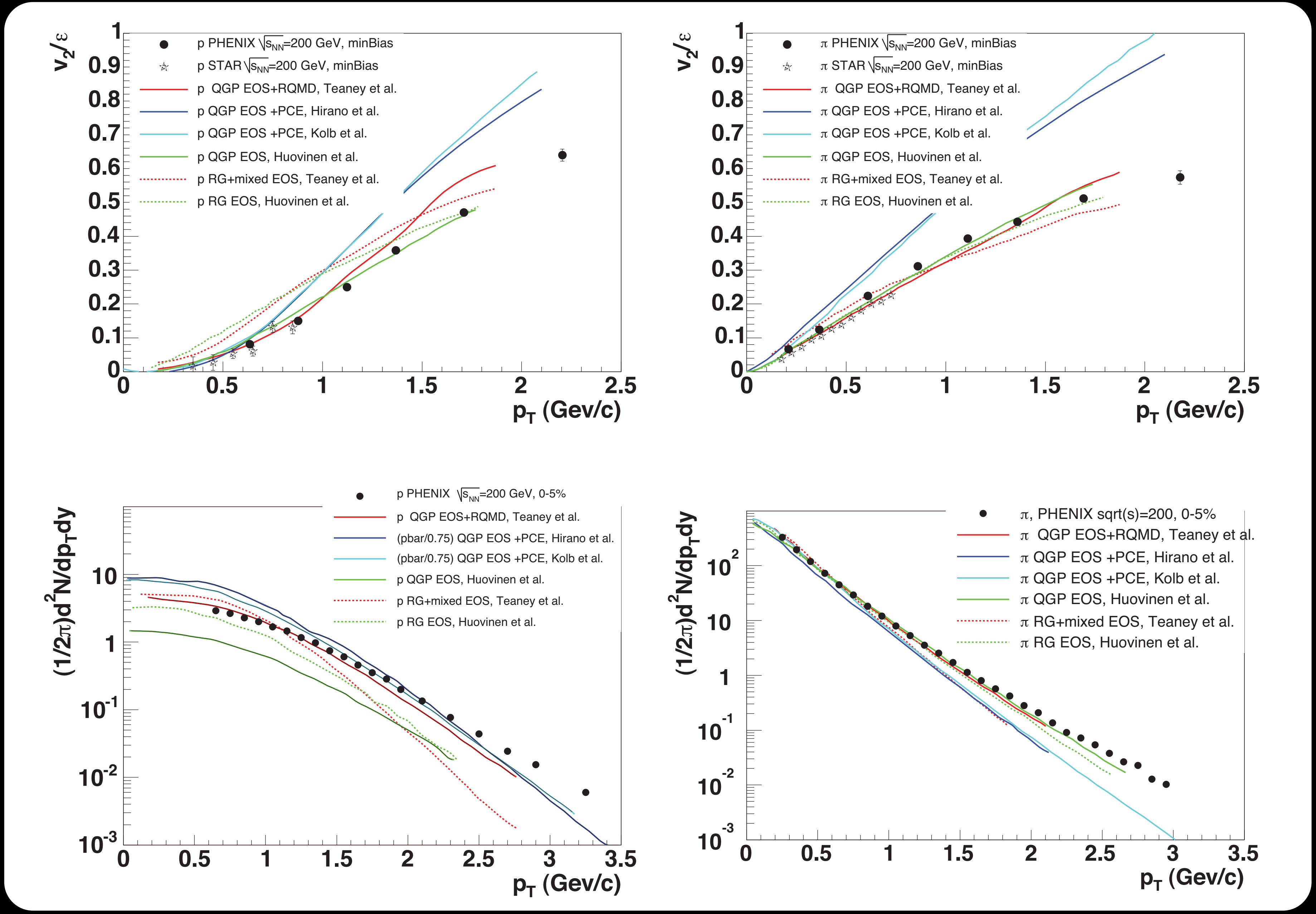
in the Low Density Limit (LDL):

$$\frac{v_2}{\epsilon} \propto \sigma \frac{1}{S} \frac{\mathrm{d}N}{\mathrm{d}y}$$

H. Heiselberg and A. M. Levy, Phys. Rev. C 59, 2716 (1999)

Not only the energy dependence but also rapidity dependence seems to scale with dN/dy
In hybrid models tuning of initial conditions required

spectra and v_2



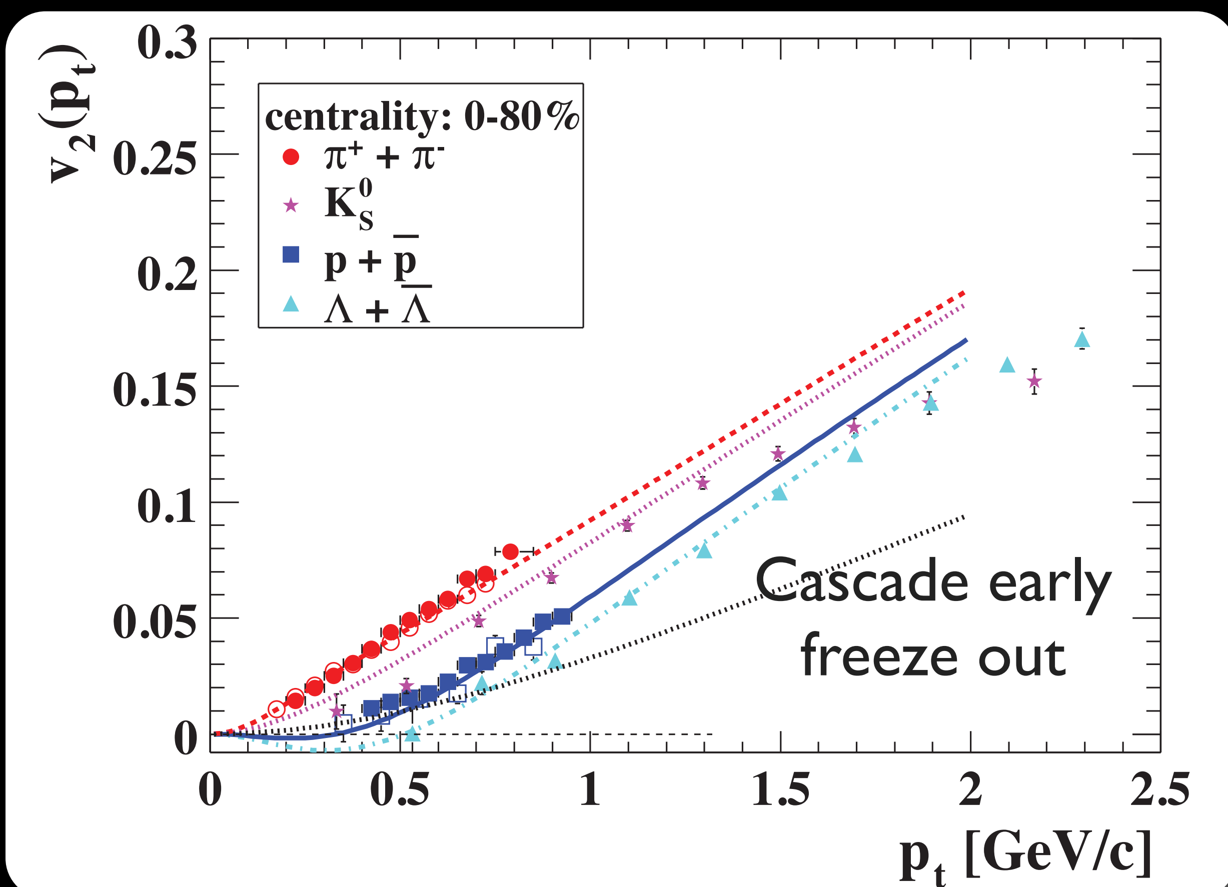
PHENIX, whitepaper

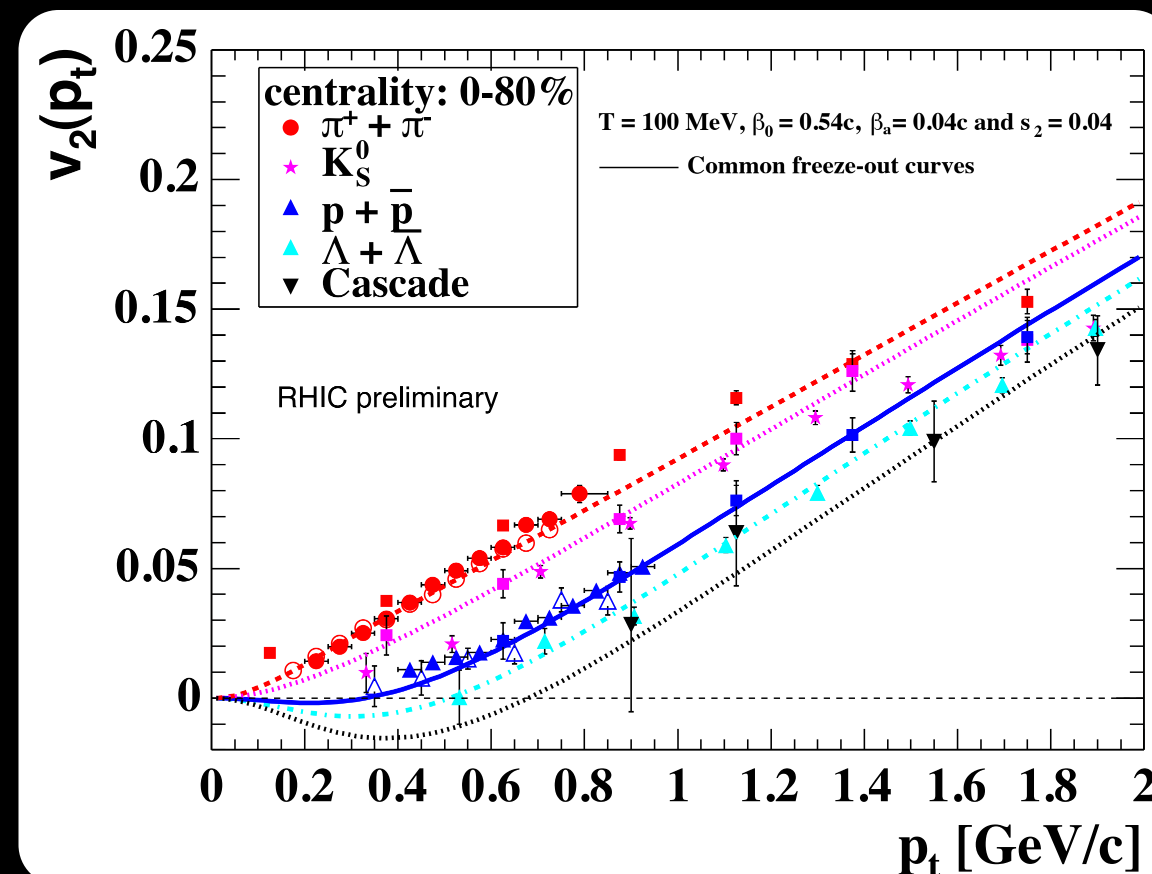
The best simultaneous description of the spectra and v_2 is obtained by the hybrid models

v_2 mass dependence

What happens when a particle freezes-out early?

Fits from STAR Phys. Rev. Lett. 87, 182301 (2001)



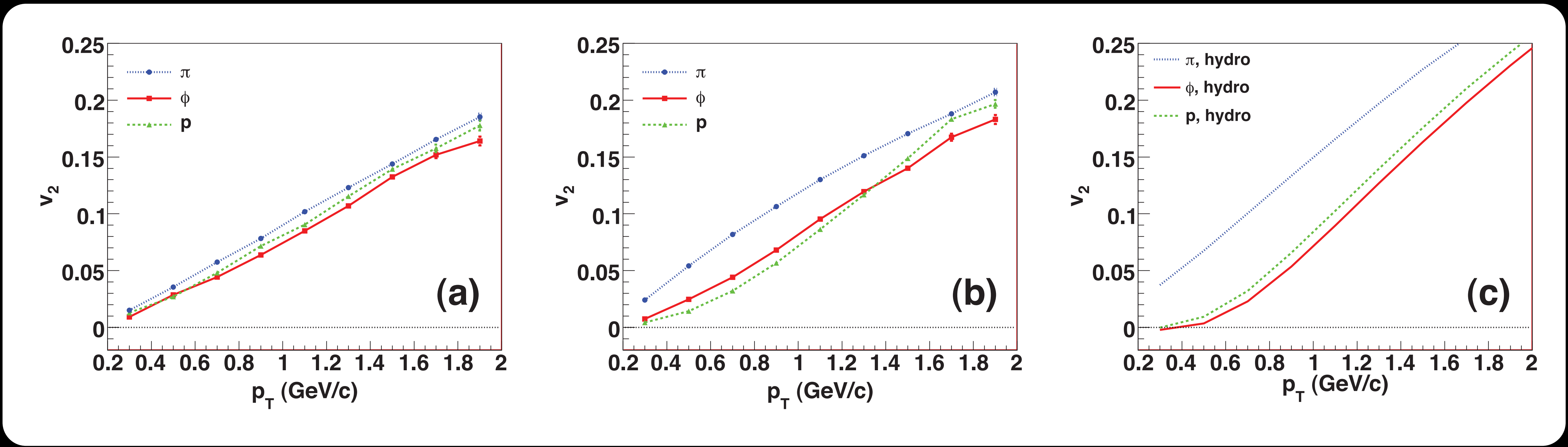


Look for the breaking of the mass scaling

v_2 mass dependence

What happens when a particle freezes-out early?

T. Hirano et al, arXiv:0710.5795

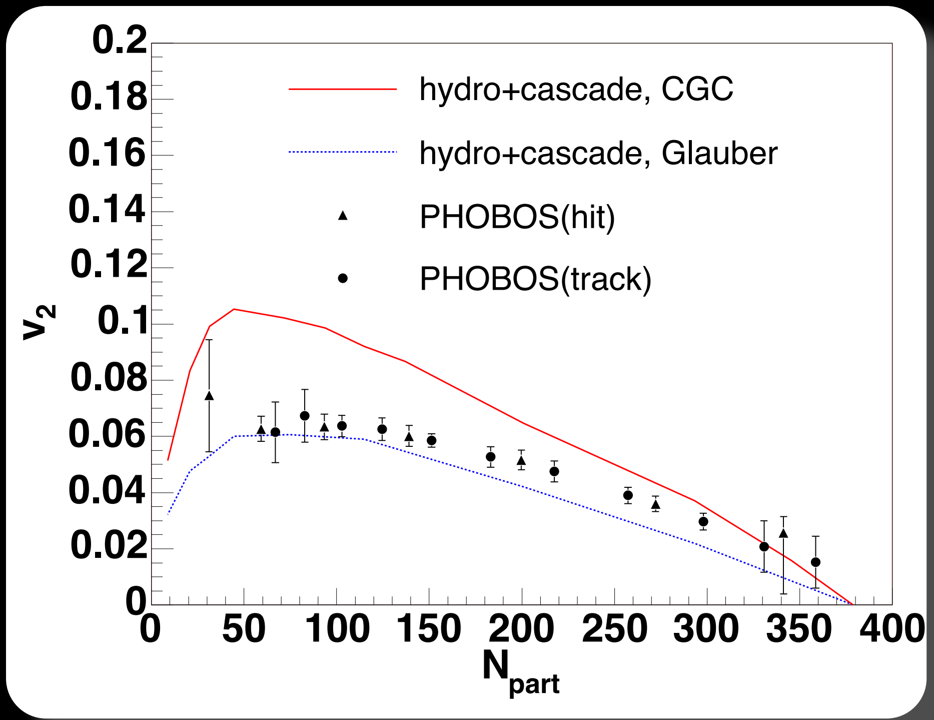


Hydro until T_c | Hybrid | Hydro until T_k

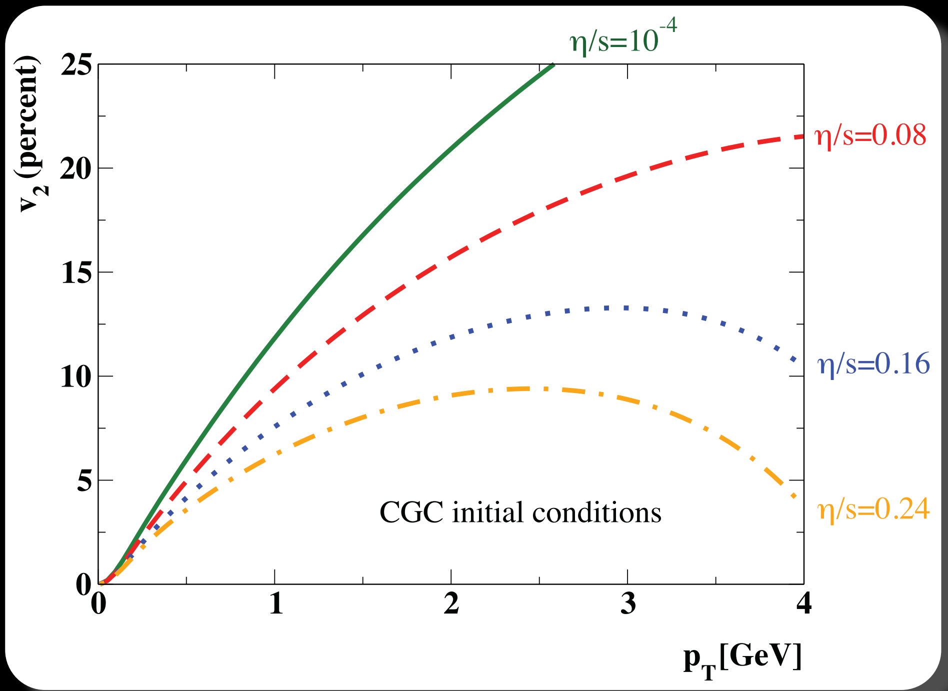
Also in more complete model calculations breaking of the mass scaling

So far not observed in the data

Comparing Theory and Data



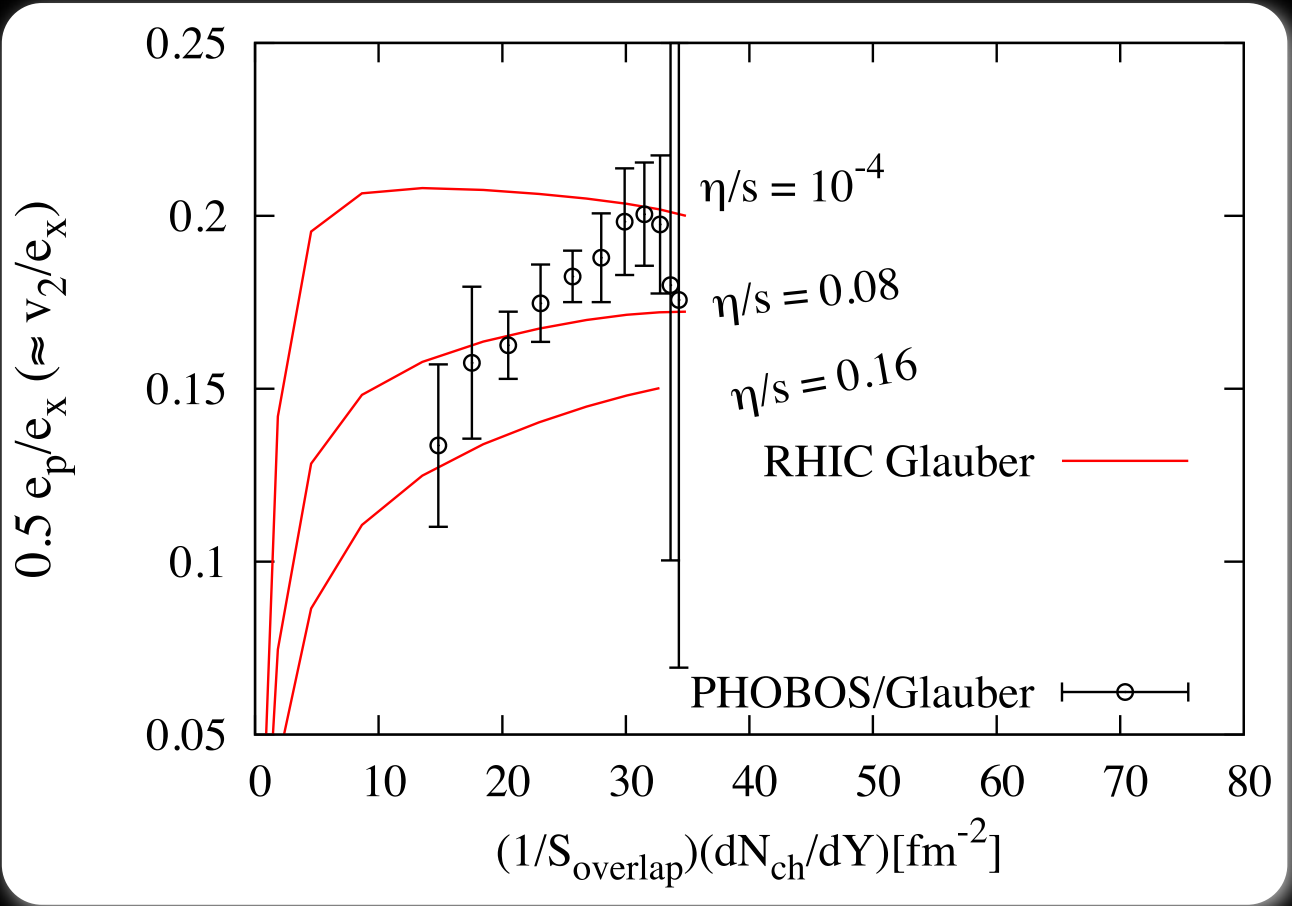
T. Hirano et al., Phys. Lett. B 636 299 (2006)
T. Hirano et al., J.Phys.G34:S879-882,2007

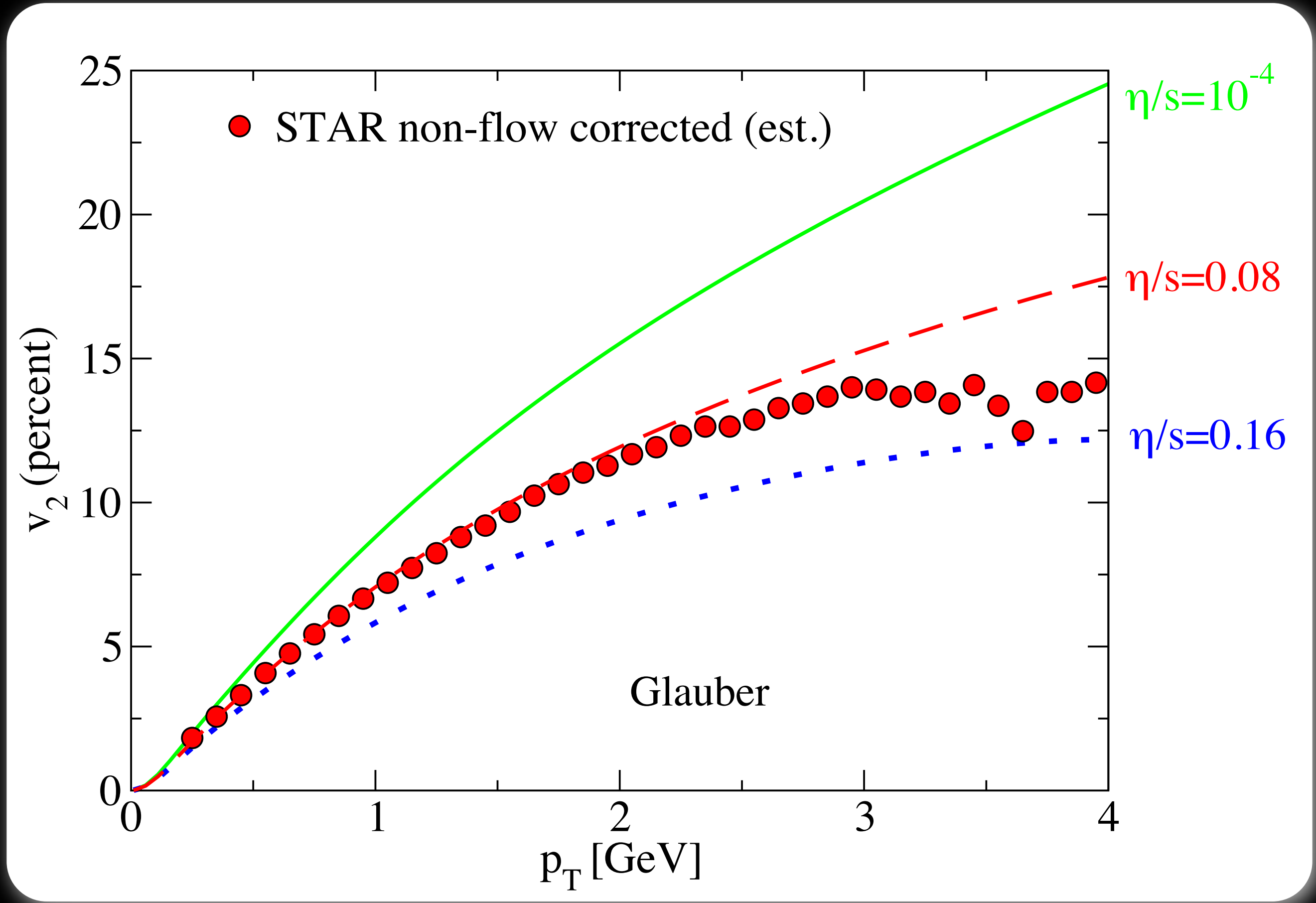


Matthew Luzum, Paul Romatschke arXiv:0804.4015

- hybrid models (ideal hydro + hadron cascade) do after some tuning a fair job but leave some puzzles
- we have learned that some that some contributions are more important than previously thought
    - ε fluctuations and | ε |
    - more realistic EoS
    - η/s
    - core/corona
- requires a new round of hybrid model calculations

Viscous Hydro Glauber



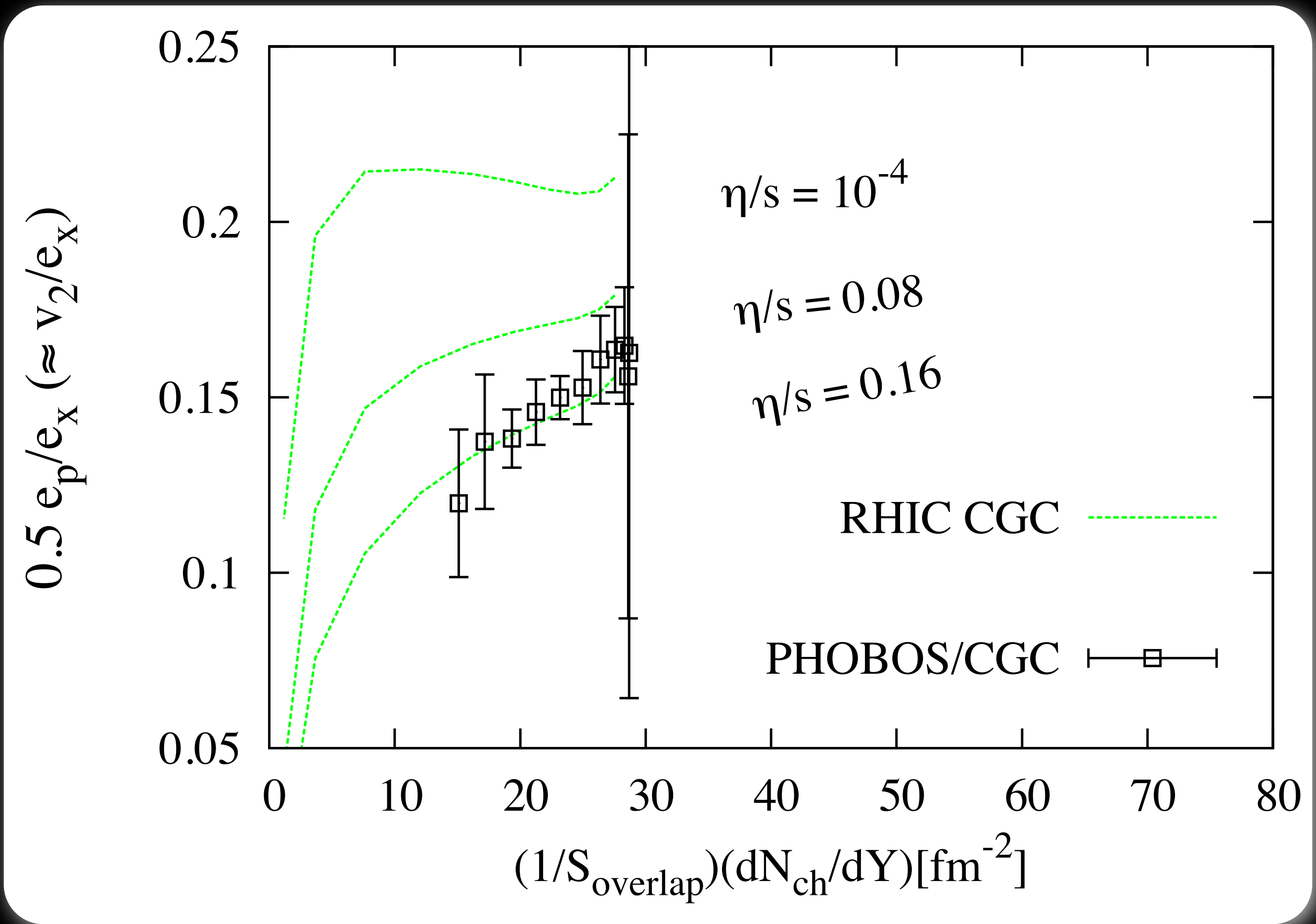


Matthew Luzum and Paul Romatschke; arXiv: 0901.4588 [nucl-th]

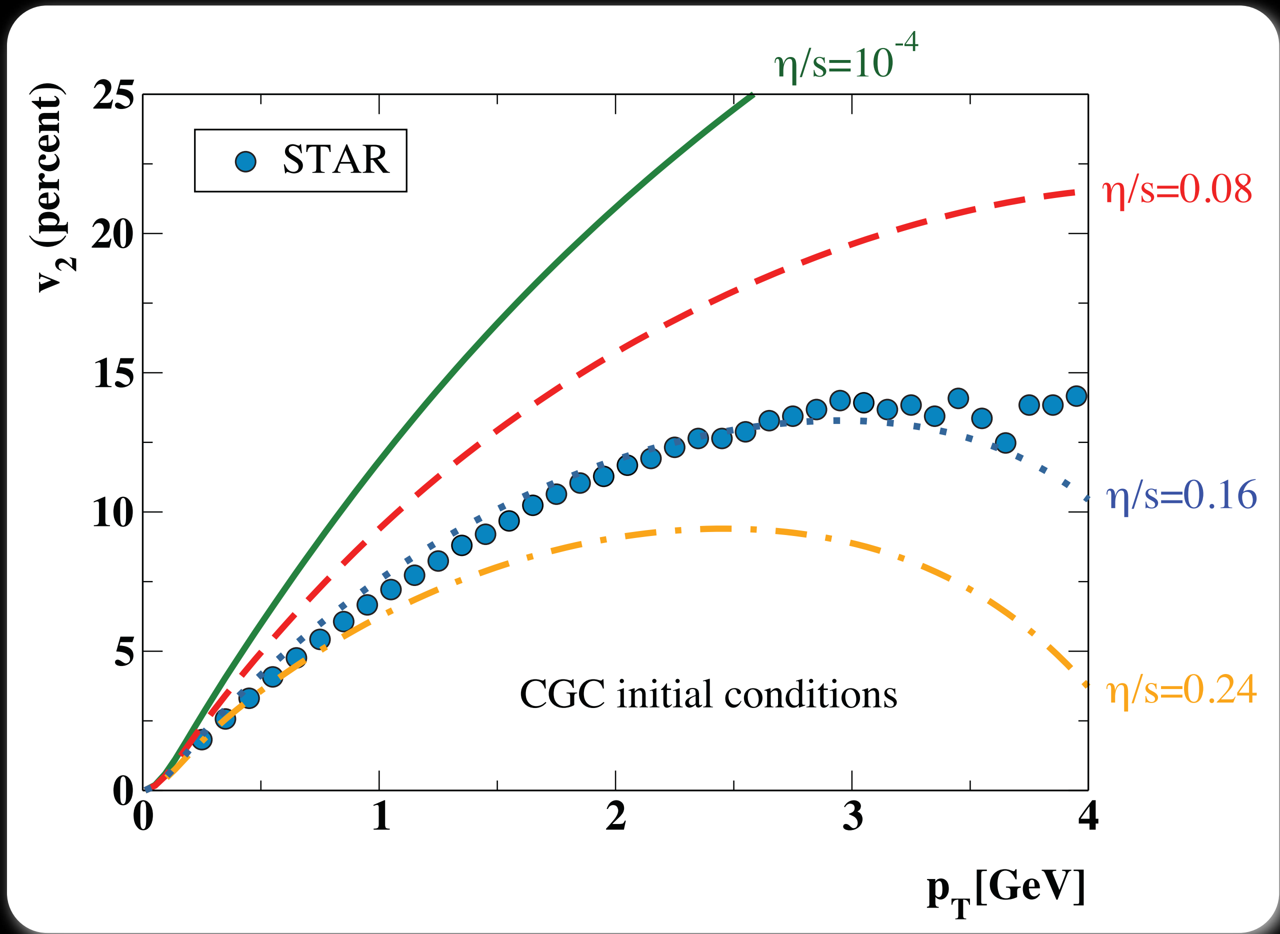
Matthew Luzum, Paul Romatschke arXiv:0804.4015

Viscous hydro calculations using ~ soft EoS and Glauber ε do not describe the measured centrality dependence with a single η/s (η/s varies between 0 and 2/4π)!

Viscous Hydro CGC



Matthew Luzum and Paul Romatschke; arXiv: 0901.4588 [nucl-th]



Matthew Luzum, Paul Romatschke arXiv:0804.4015

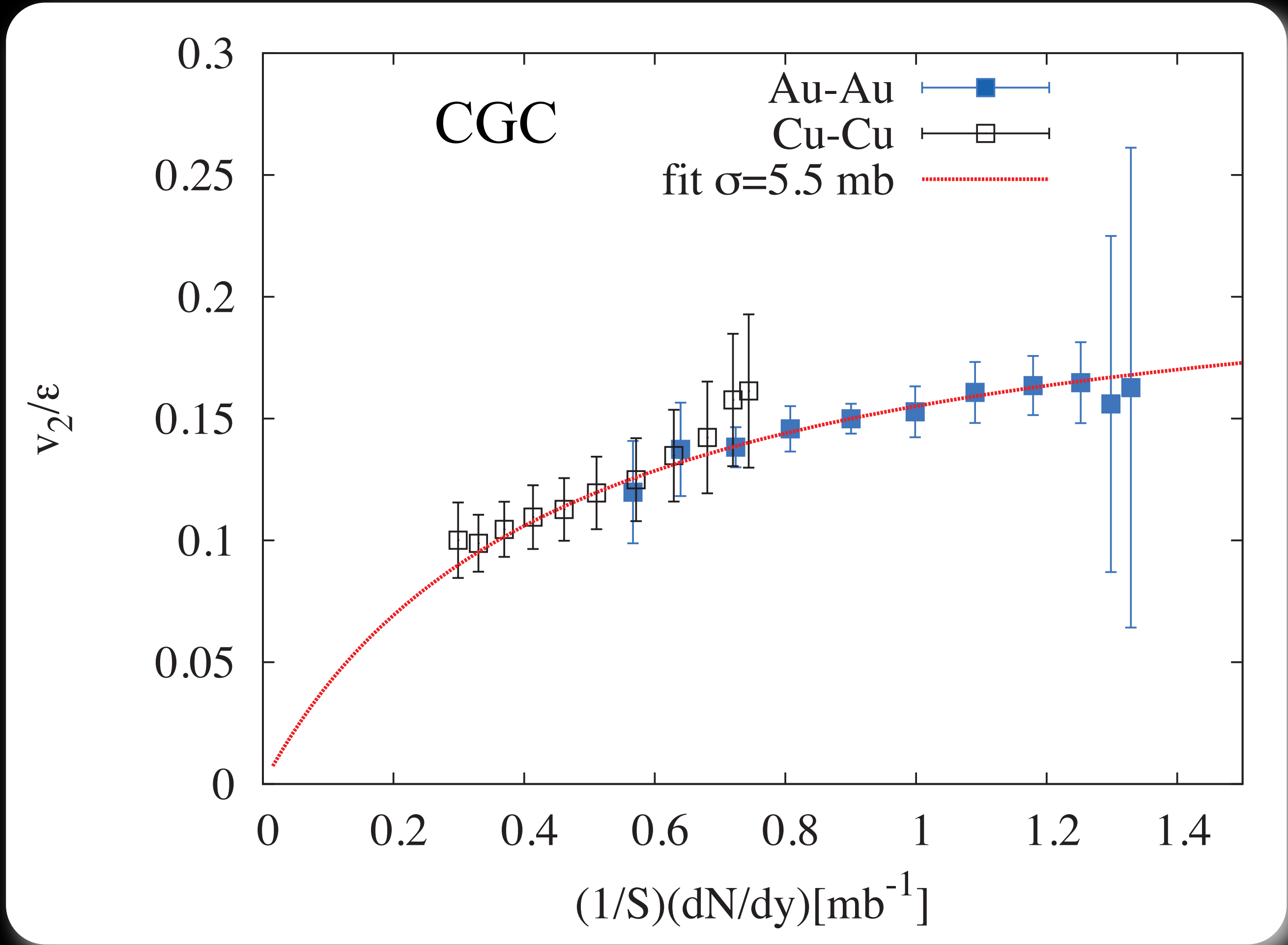
Viscous hydro calculations using ~ soft EoS and CGC ε describe the centrality dependence and p_t dependence using $\eta/s = 2/4\pi$ (doing to well?)

From LDL to viscous Hydro

$$\frac{v_2}{\varepsilon} = \frac{h}{1 + 1.4\mathrm{Kn}}$$

R.S. Bhalerao, J-P. Blaizot, N. Borghini and J-Y. Ollitrault; Phys. Lett. B 627:49-54, 2005

$v_2/\varepsilon = h/(1+1.4Kn)$
h: hydro limit
Knudsen number: $Kn = \lambda/R$
The number of collisions per particle:
$1/Kn = (\sigma/S)(dN/dy)c_s$
σ = partonic cross section
c_s = sound velocity



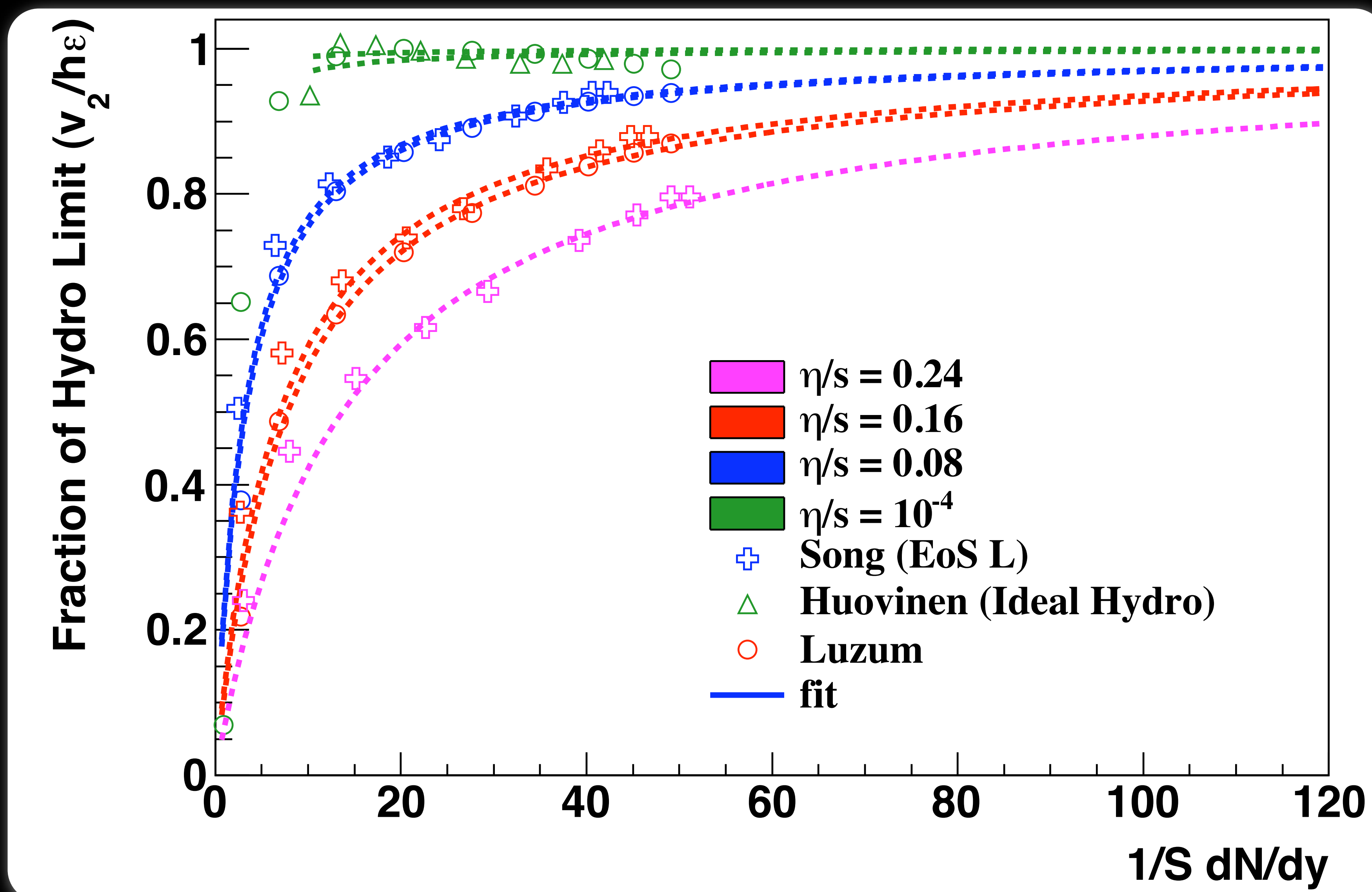
H-J. Drescher, A. Dumitru, C. Gombeaud and J-Y. Ollitrault; Phys.Rev.C76:024905,2007.

PHOBOS v_2 data, eccentricity ε is "corrected" for fluctuations

data reaches 70% of ideal hydro limit and can be describes using CGC ε with soft EoS (ideal hydro $v_2/\varepsilon \sim 0.22$) or Glauber ε with hard EoS (ideal hydro $v_2/\varepsilon \sim 0.3$)

can we get a consistent picture from calculations of different theorists? ☺

slightly different EoS
different codes
different freeze-out
different ε
etc



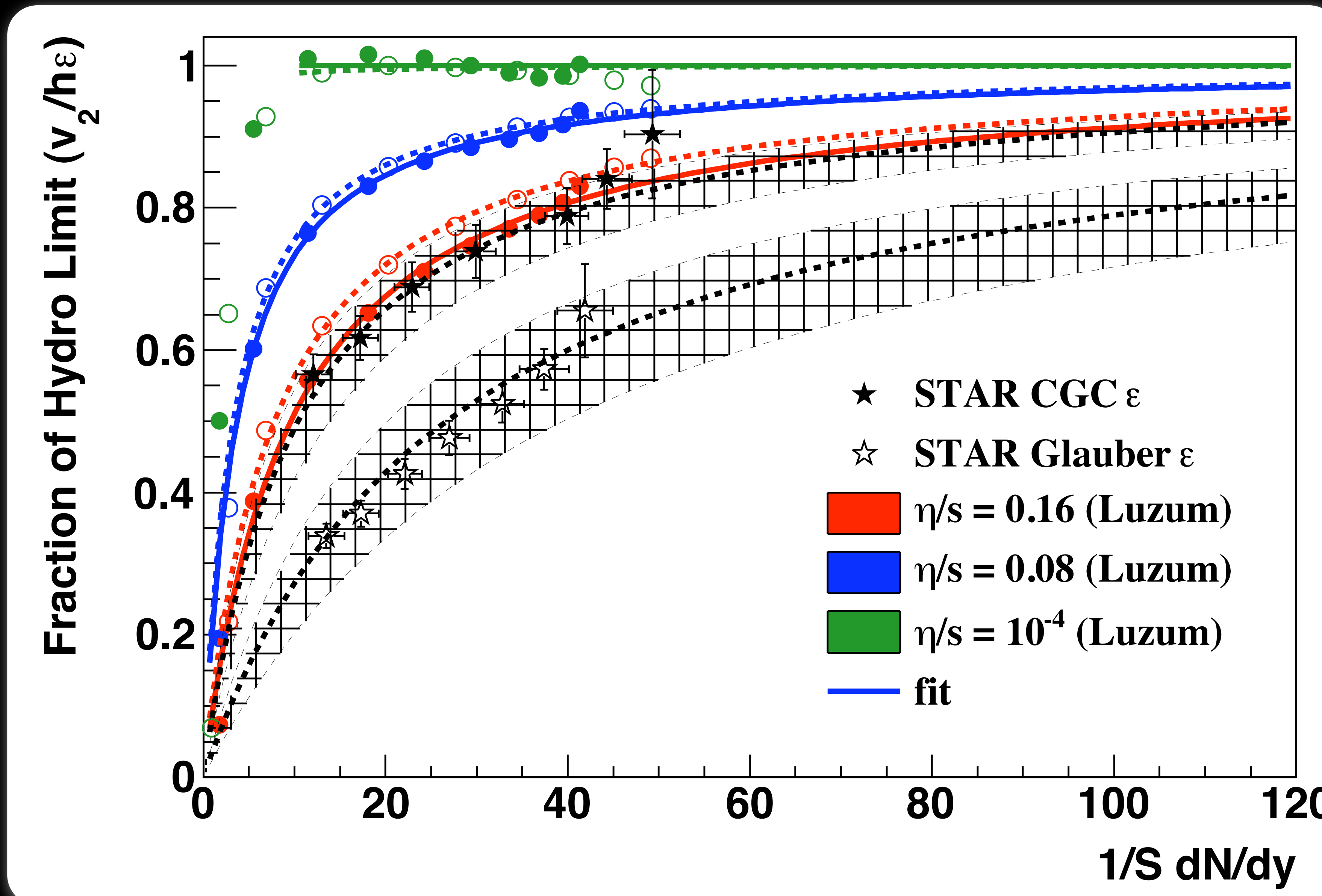
Data from Matthew Luzum and Paul Romatschke; arXiv:0901.4588 [nucl-th]
Huichao Song and Ulrich Heinz arXiv:0805.1756v2 [nucl-th]
Pasi Huovinen

very small 1/S dN/dy finite lifetime effects are important (freeze-out ~ 100 MeV)!

yes we can!

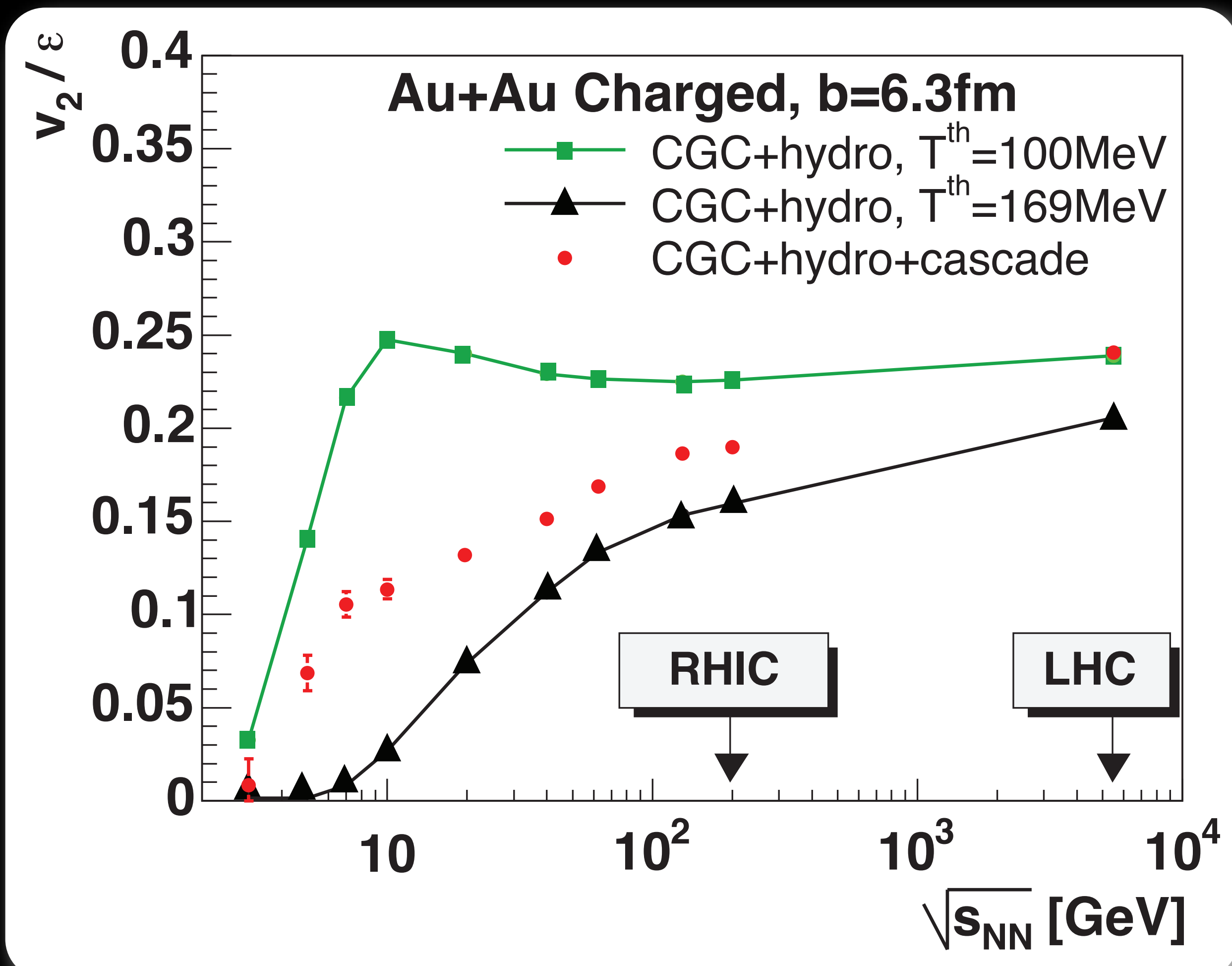
Viscous Hydro and Data

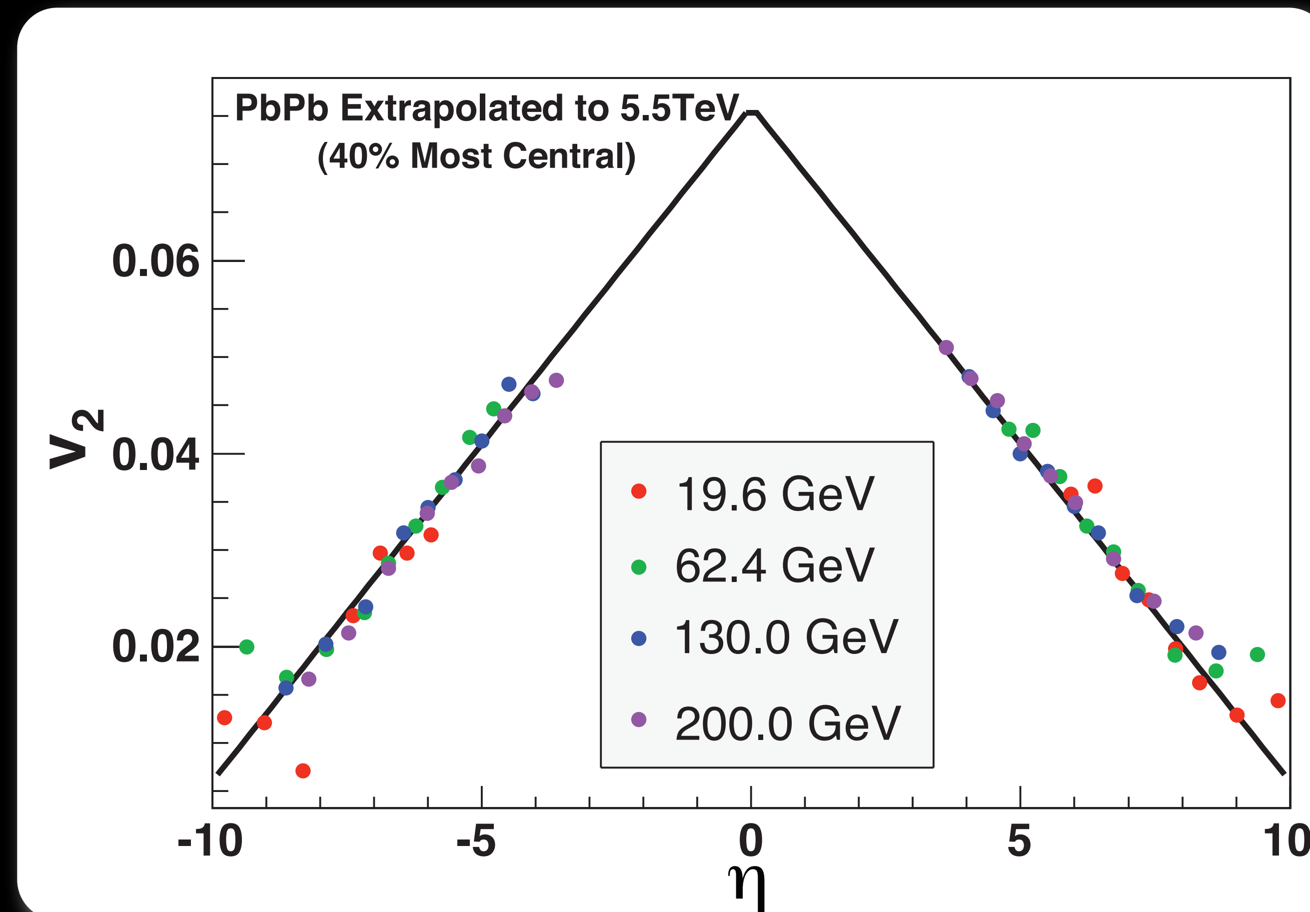
compare directly to viscous hydro calculations.
STAR data well described using a CGC ε with soft EoS and η/s ~ 2/4π or Glauber ε with hard EoS and η/s ~ 4 x 1/4π



estimates of η/s are < 4x the conjectured lower bound from AdS/CFT for a significant fraction of the lifetime of the system

Flow at the LHC



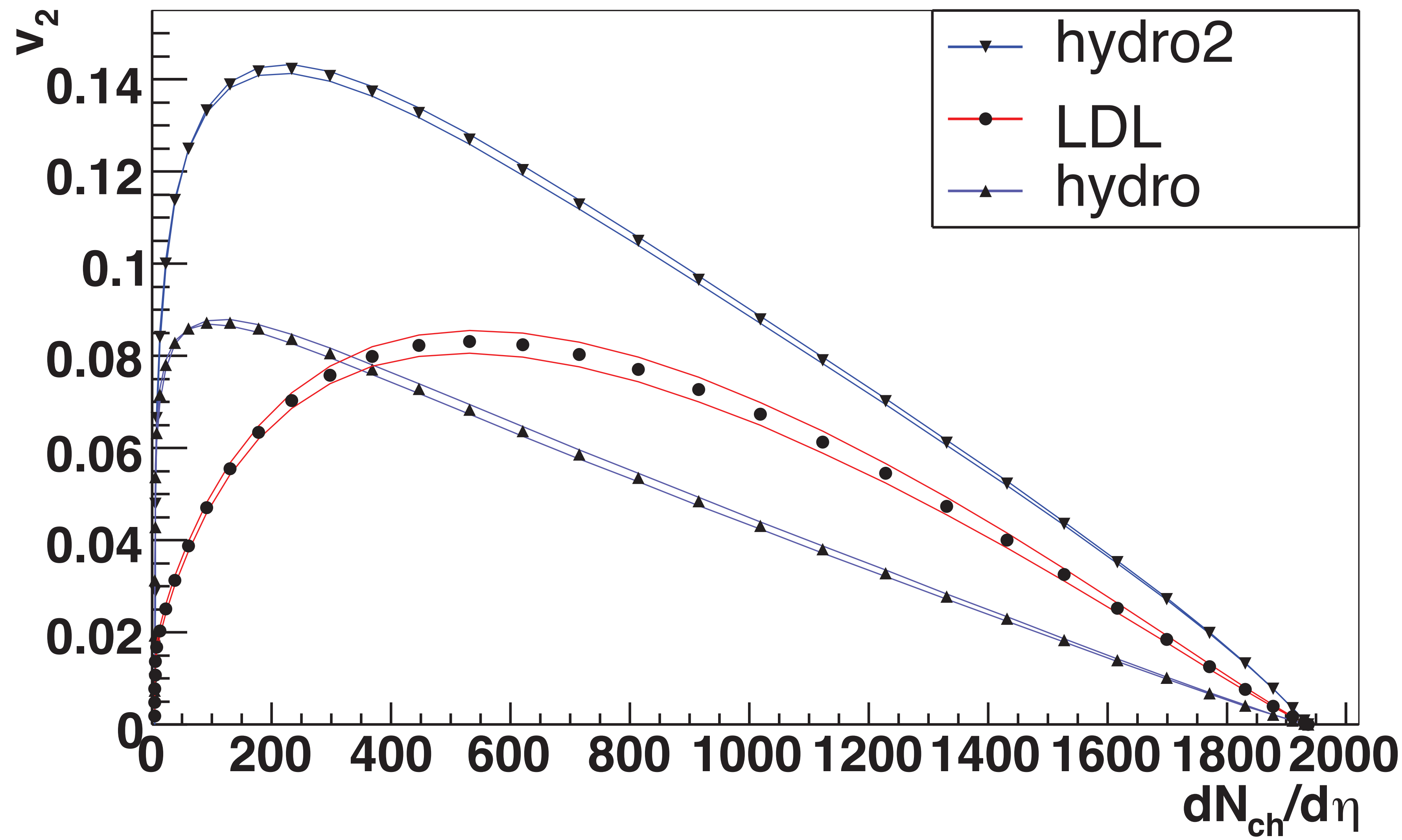


most models predict larger flow, largest values based on “scaling”

Flow at the LHC

E. Simili, Thesis Utrecht (2008)

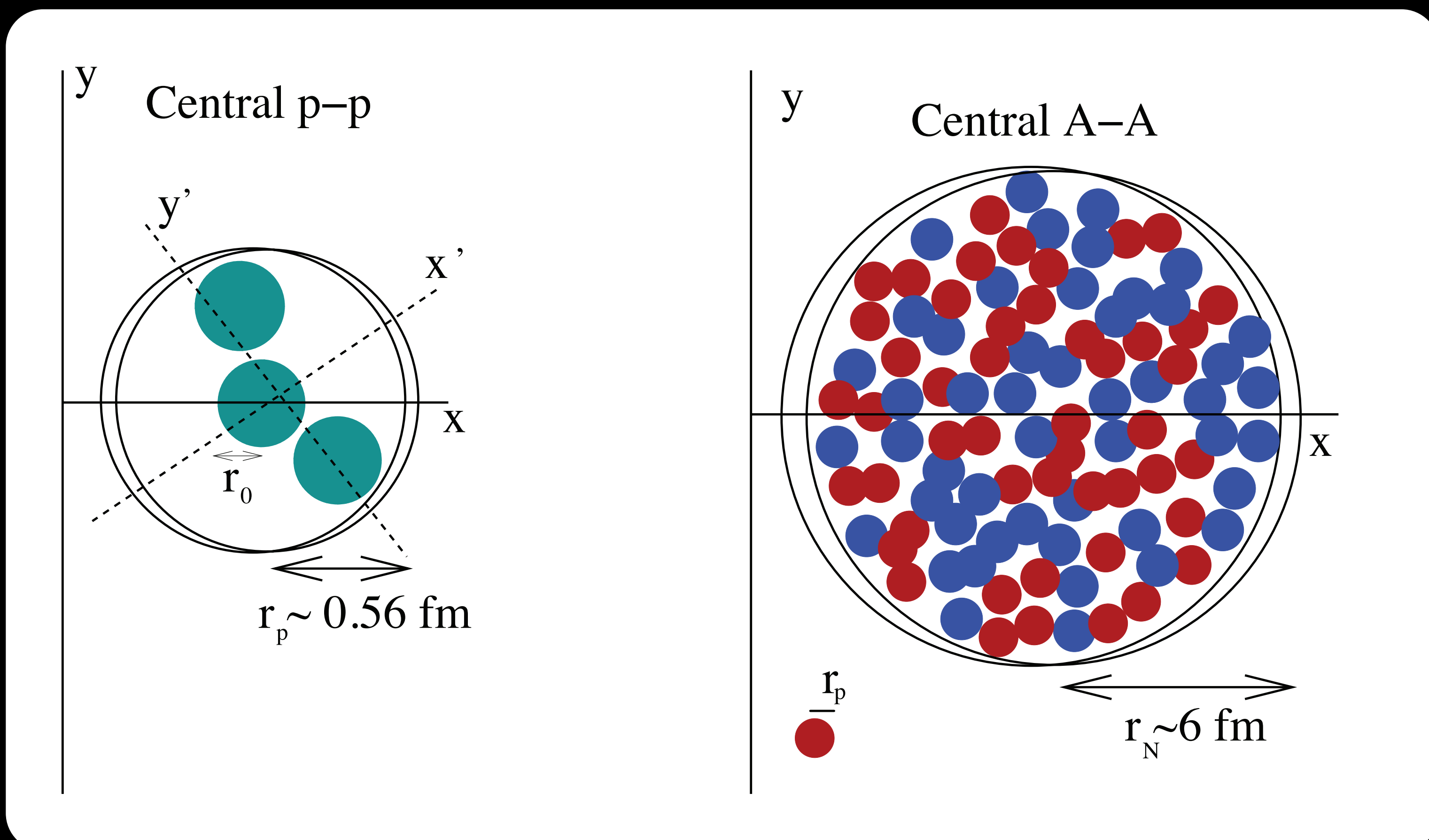
hydro2: Glauber ε, hard EoS
hydro: Glauber ε, soft EoS

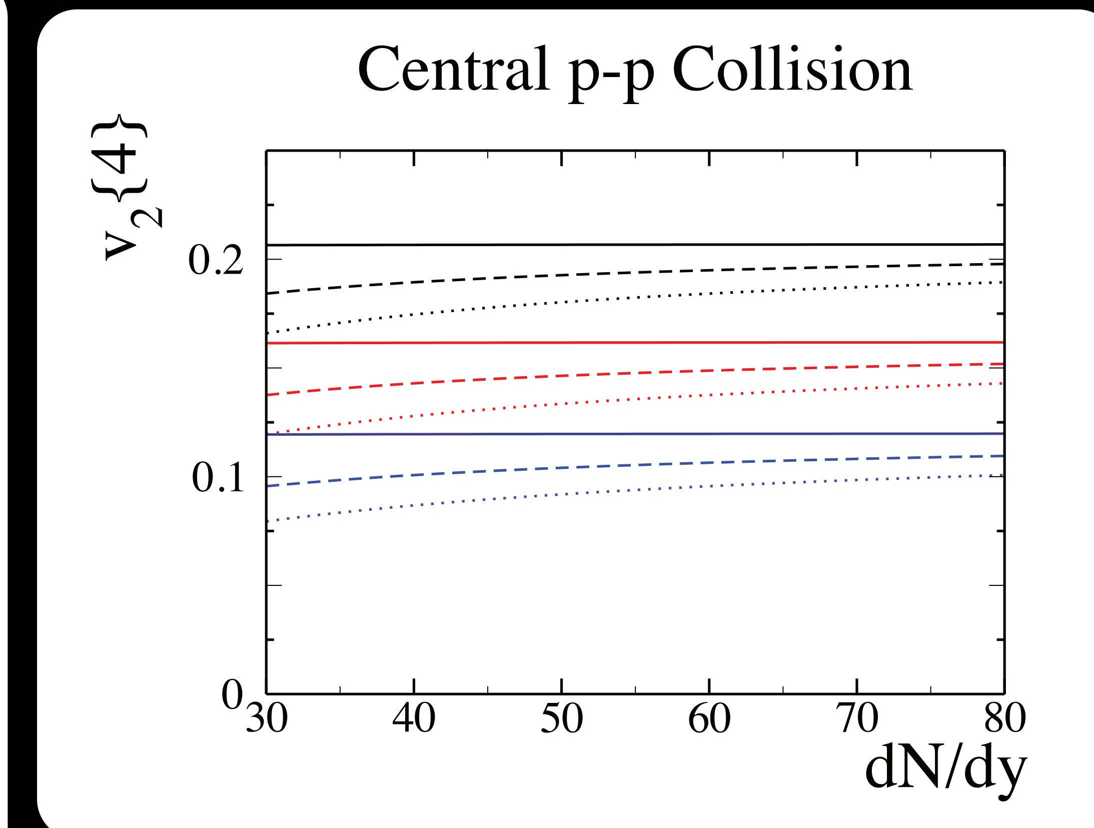


can easily differentiate between LDL and soft EoS + Glauber ε

Flow in pp at the LHC

J. Casalderrey-Solana and U. Wiedemann: arXiv:0911.4400





due to fluctuations large eccentricity in pp might allow for v_2 measurement in central pp

Conclusions

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions
April 18, 2005

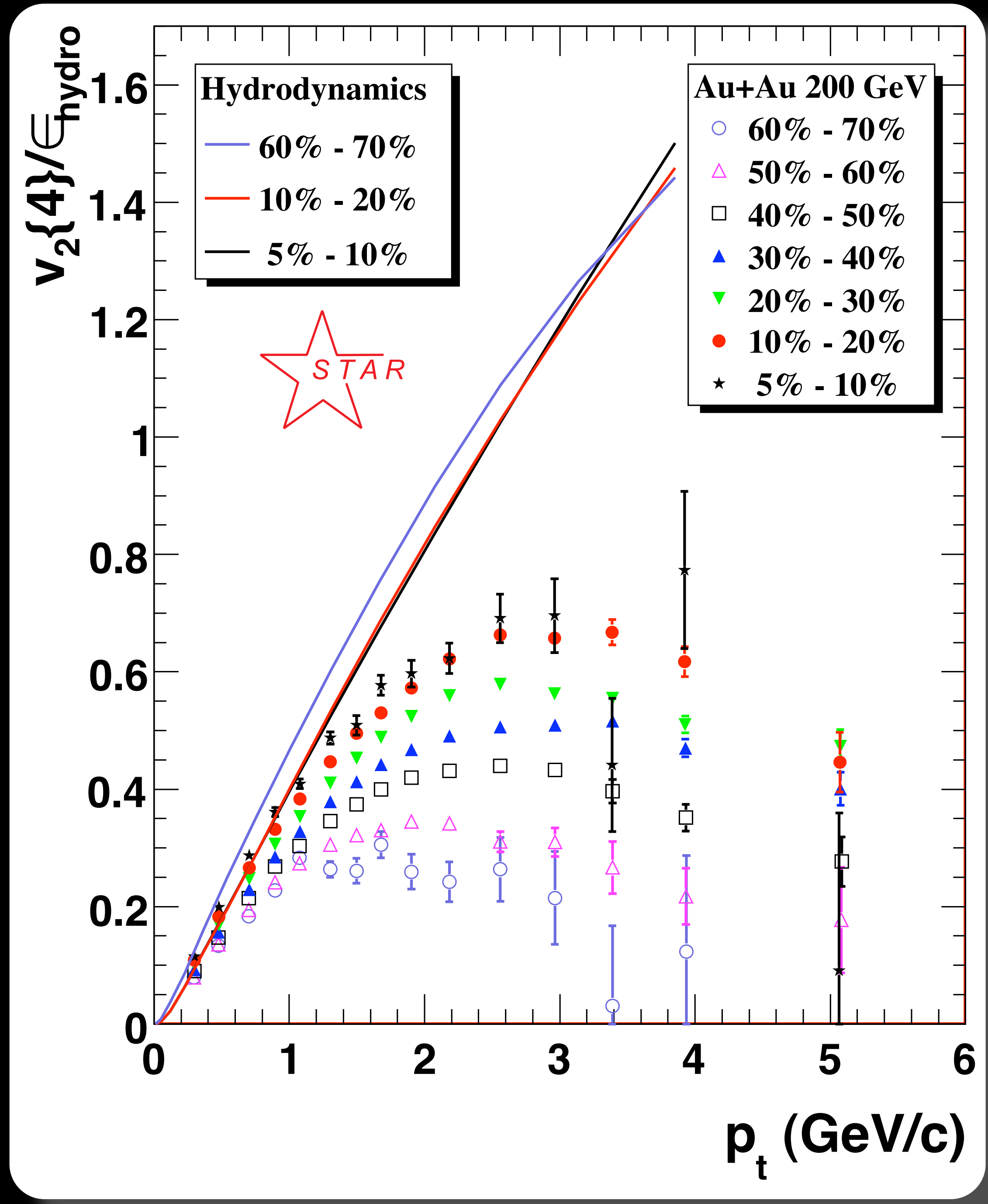
- The observables have been much better understood
  - elliptic flow, nonflow and fluctuations contributions are rather well constrained
- non ideal hydro contributions are important
  - important question is what contribution the hadronic stage gives (Φ, Ξ, Ω)
- initial conditions (e.g. ε, initial flow fields) not sufficiently constrained!
- estimates of $\eta/s < 4$x the conjectured lower bound from AdS/CFT for a significant part of the system evolution
- v_2 at the LHC immediately will put strong constraints on these uncertainties
- correlations with the reaction plane is a much broader topic and can be fully exploited at the LHC

Thanks!

Transverse Momentum?

viscous corrections $\frac{\eta}{s}/(\bar{R}T)$

D. Teaney PRC68 034913 (2003)



Yuting Bai, Nikhef PhD thesis

- In the data the peak position is also shifting as function of centrality → just R?
- caveat: magnitude hard contribution at higher p_t

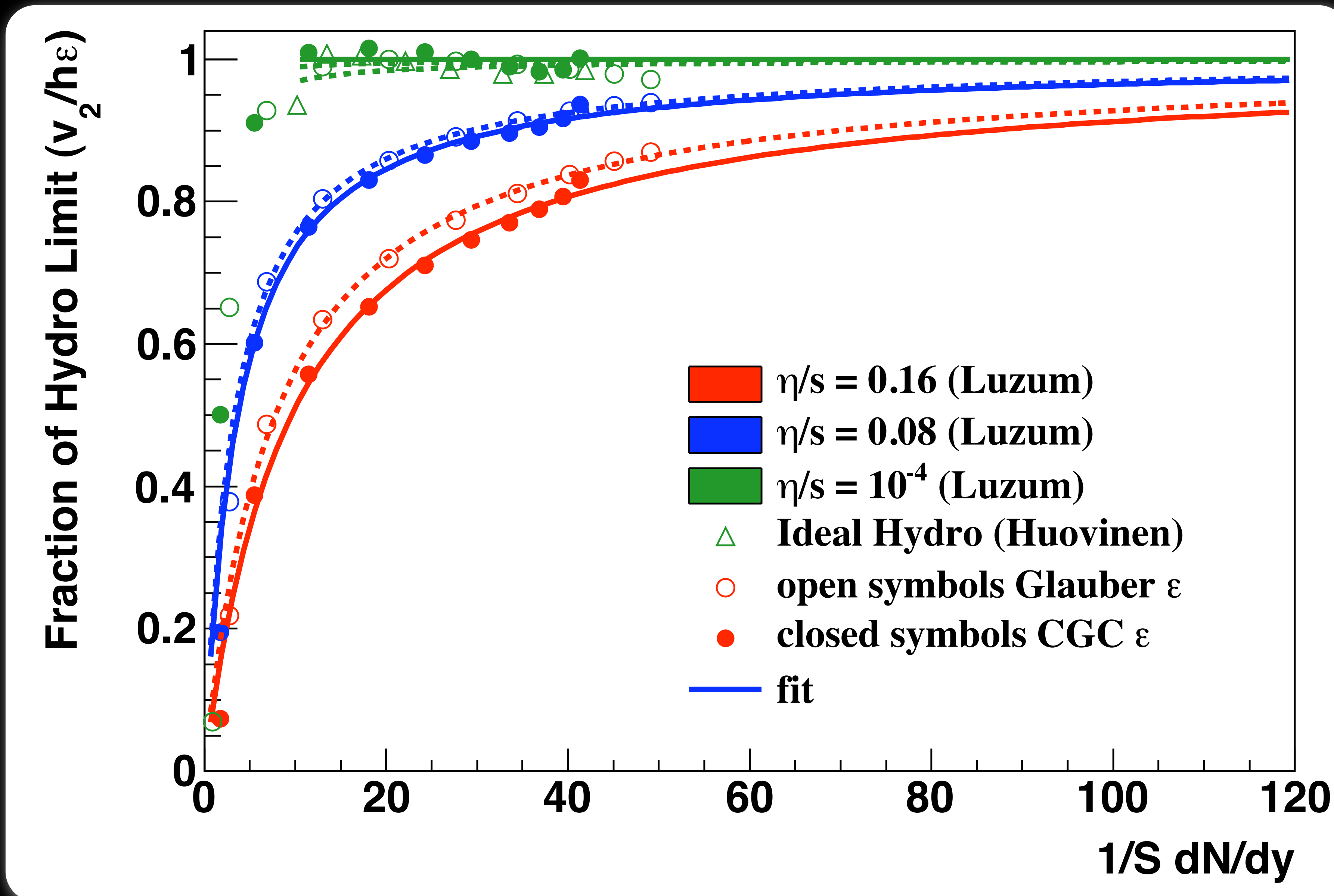
Use shift of peak to determine η/s

B sensitive to η/s?

$$\frac{v_2}{h\varepsilon} = \frac{1}{1 + B / \left(\frac{1}{S} \frac{dN}{dy} \right)}$$

- parameterization works well!
- found h is the same for all curves (ideal hydro v_2/ε)

h= 0.20 ± 0.02
B = 0.70 ± 0.05
B = 0.33 ± 0.03
B = 10^{-7}

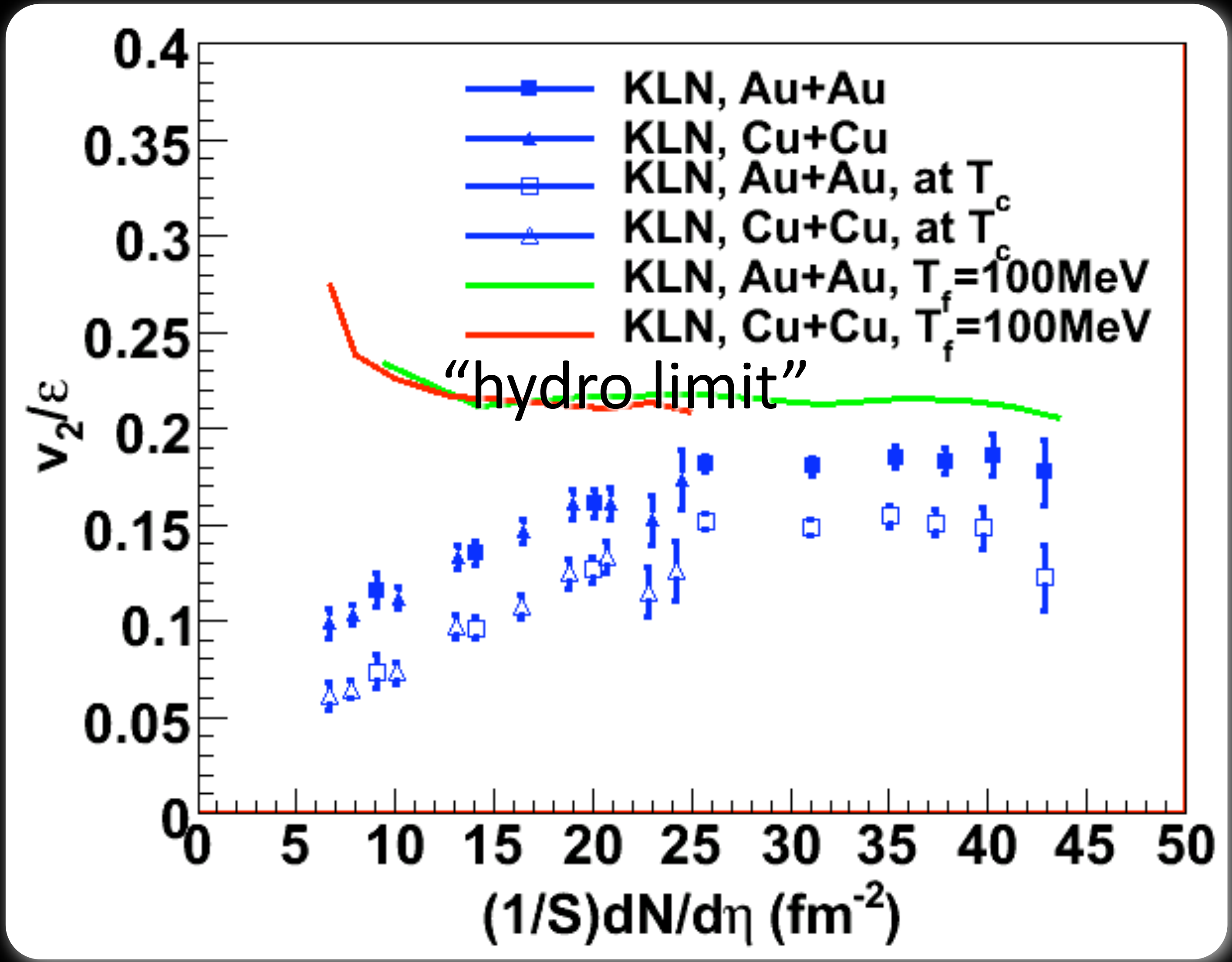


Data from Matthew Luzum and Paul Romatschke; arXiv:0901.4588 [nucl-th]

B scales with η/s

An upper limit for η/s

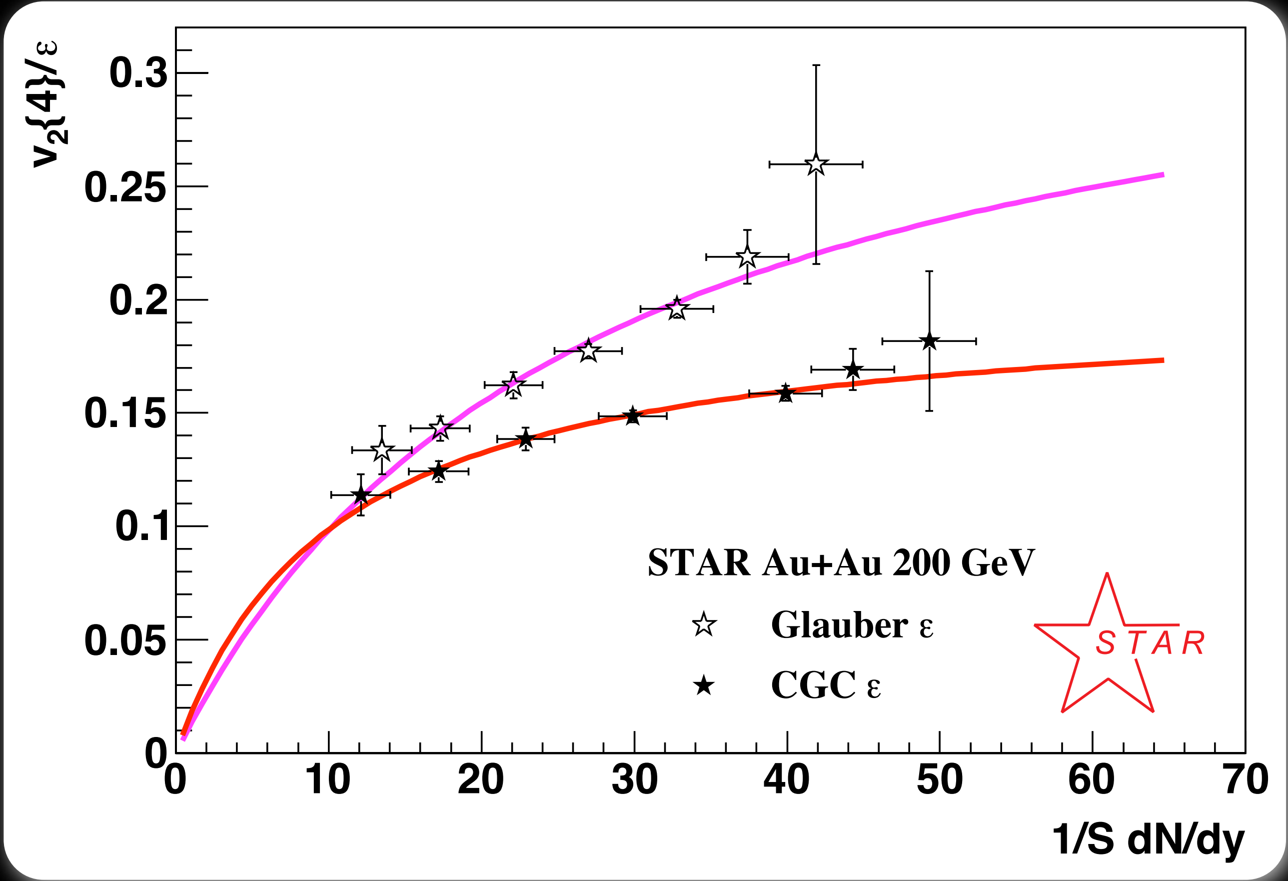
- Describing the data with ideal hydro + cascade the centrality dependence of v_2/ε is due to cutting of the ideal hydro phase at T_C (the flow is not developed completely, just a lifetime effect!)
- not sensitivite to η/s in the hydro phase, estimate is only upper limit

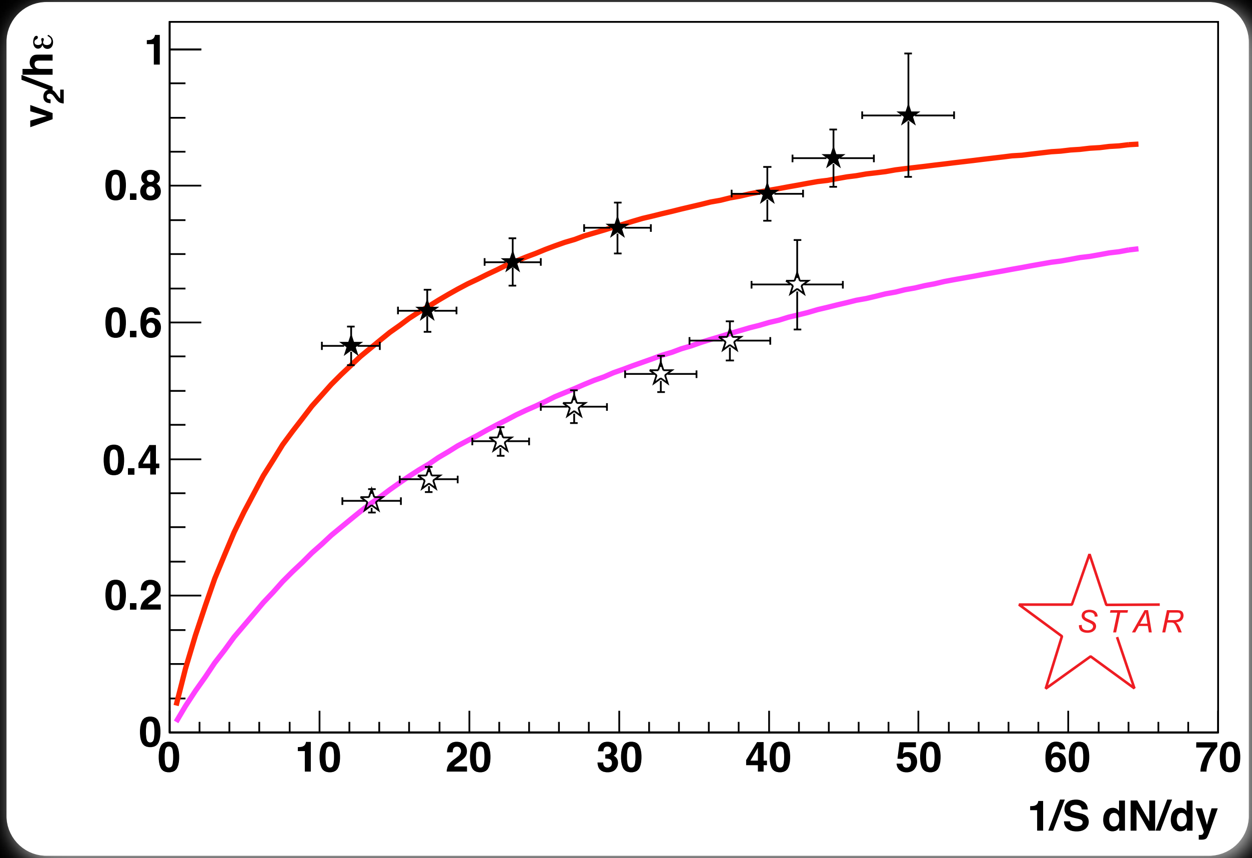


Hirano QM2009

Knudsen Parameterization

$v_2/\varepsilon = h/(1+1.4Kn)$





STAR $v_2\{4\}$ data, eccentricity ε needs no correction for fluctuations

CGC ε → softer EoS ~ Lattice

Glauber ε → harder EoS ~ ideal gas

CGC ε → lower η/s ~ 2/4π

Glauber ε → larger η/s ~ 1/π

data reaches ~60-80% of ideal hydro limit

same conclusions drawn in:

H-J. Drescher, A. Dumitru, C. Gombeaud and J-Y. Ollitrault; Phys.Rev.C76:024905, 2007

Viscous Corrections

viscous corrections to ideal hydro:

$\frac{h}{s} / (\bar{R}T)$

proportional to Kn or $\frac{1}{Re}$

$$\frac{v_2}{e} = \frac{h}{1 + 1.4\,Kn}$$

R.S. Bhalerao, J-P. Blaizot, N. Borghini and J-Y. Ollitrault; Phys. Lett. B 627:49-54, 2005

- density ρ(τ) goes like (dN/dy)/τS
- strength v_2 defined at τ=R/c_s (at approximate constant density and mean free path versus centrality!)
- η/s constant → centrality dependence controlled by system size R

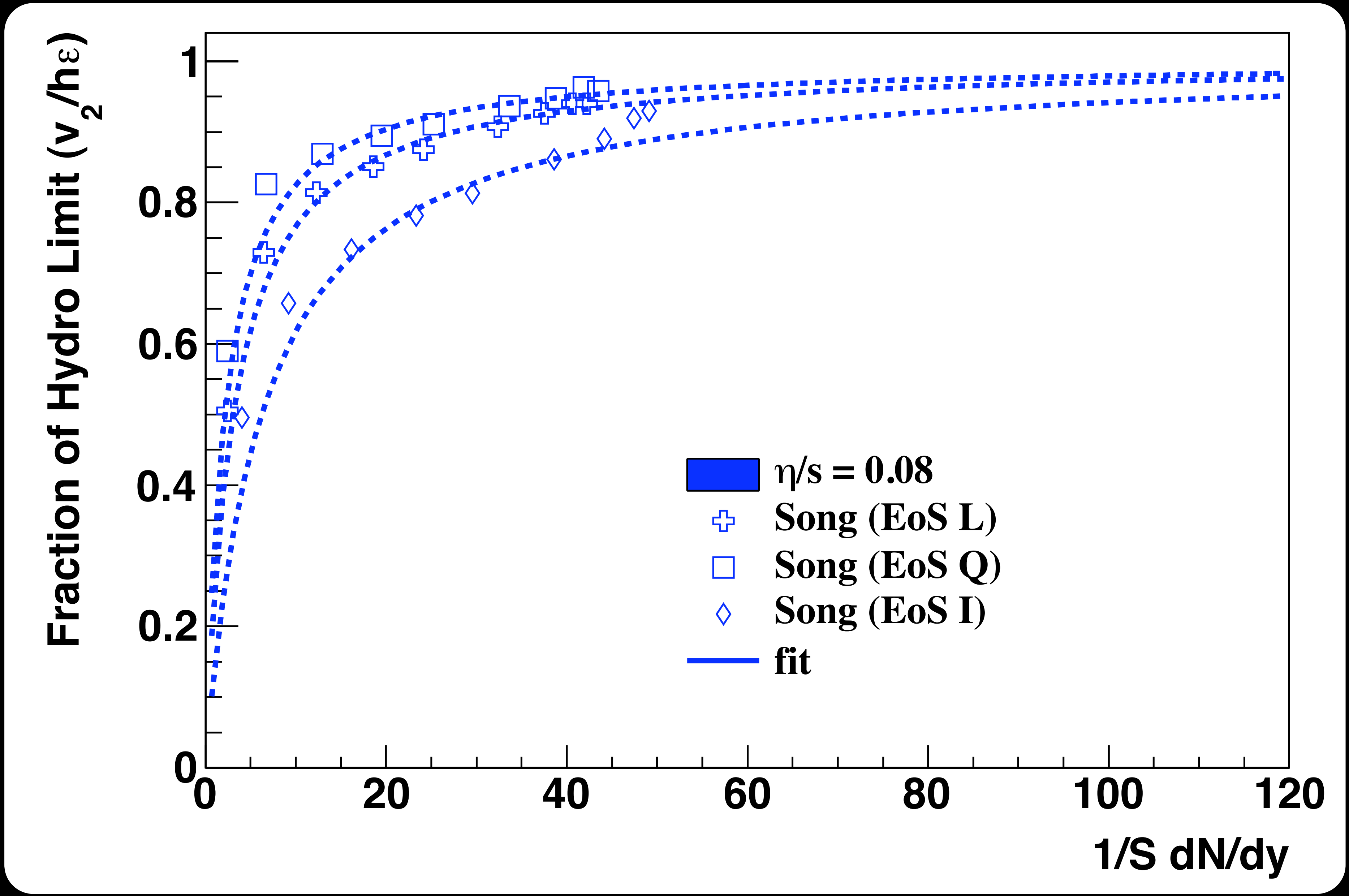
$$\frac{1}{Kn} = \frac{R}{l} = R s r(t) = \frac{R s}{t S} \frac{dN}{dy}$$

$$t_{\langle v_2 \rangle} = R / c_s$$

$$\frac{1}{Kn} = c_s \frac{s}{S} \frac{dN}{dy}$$

Centrality dependence of v_2/ε gives acces to ideal hydro limit v_2/ε (h) related to EoS and η/s!

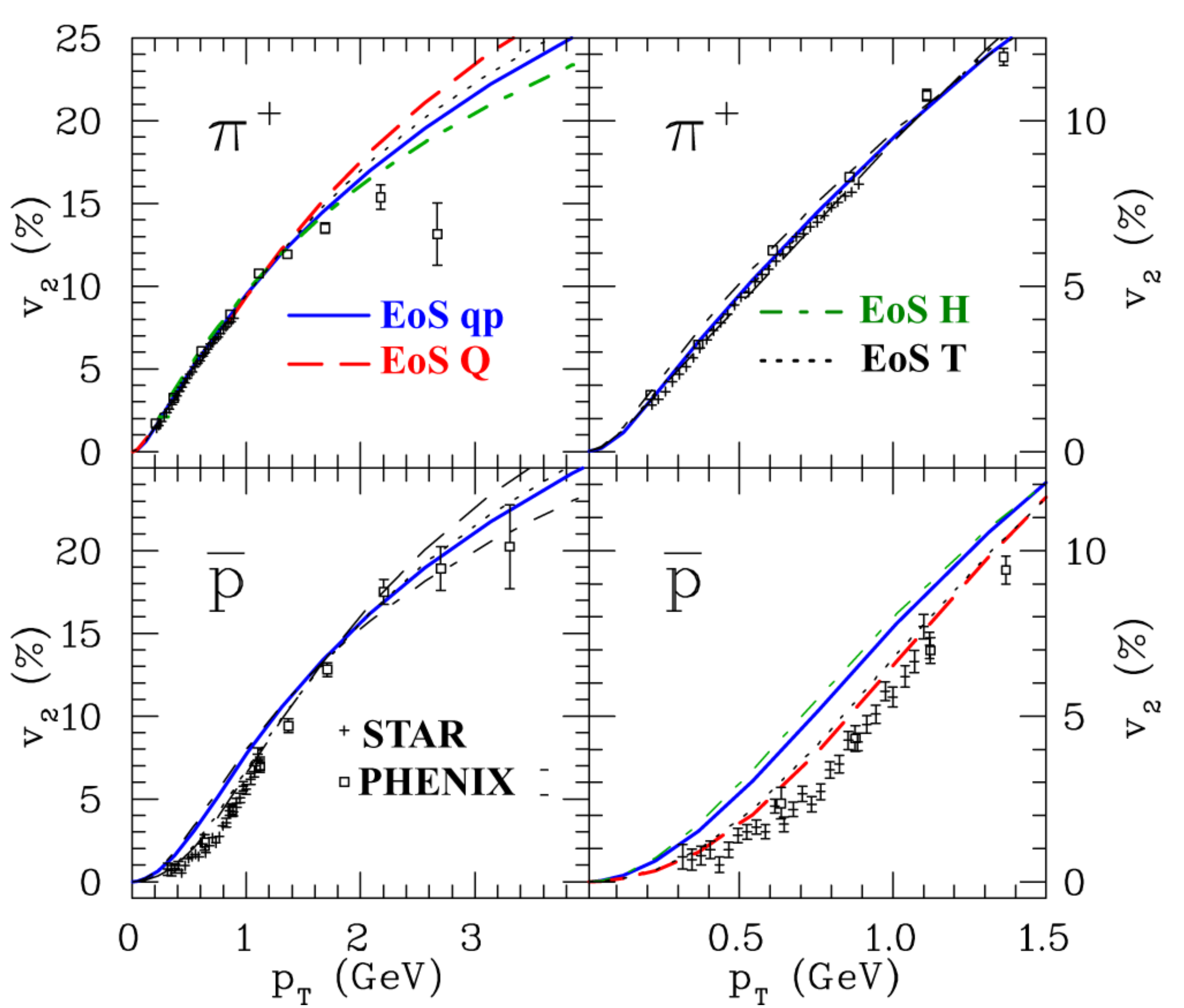
is the B parameter only sensitive to η/s?



Data from Huichao Song and Ulrich Heinz arXiv:0805.1756v2 [nucl-th]

Unfortunately (fortunately?), EoS does not completely factor out of the B parameter → constrain EoS with additional observables (e.g. p_t spectra)

Uncertainties in the EoS



from Pasi Huovinen (arXiv:nucl-th/0505036) Nucl. Phys. A761: 296, 2005 we know it matters, particularly at RHIC