Two photon physics in the timelike and spacelike regions

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“Abstract”

- Comments on two-photon physics
  - Discuss new muonic Lamb shift meaurements
    - Relativistically, example of two-photon physics
    - Done before, but revisited with Marc Vanderhaeghen
  - Review and critique of old two-photon results

Proton charge radius

- Strikingly interesting because of new experiment measuring proton charge radius from finite proton size effects on the Lamb shift in muonic hydrogen
- Two "old" methods
- Electron scattering: get charge radius from derivative of form factor,

$$R_E^2 \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0} = \left(\, 0.879(8) \text{ fm } \right)^2$$

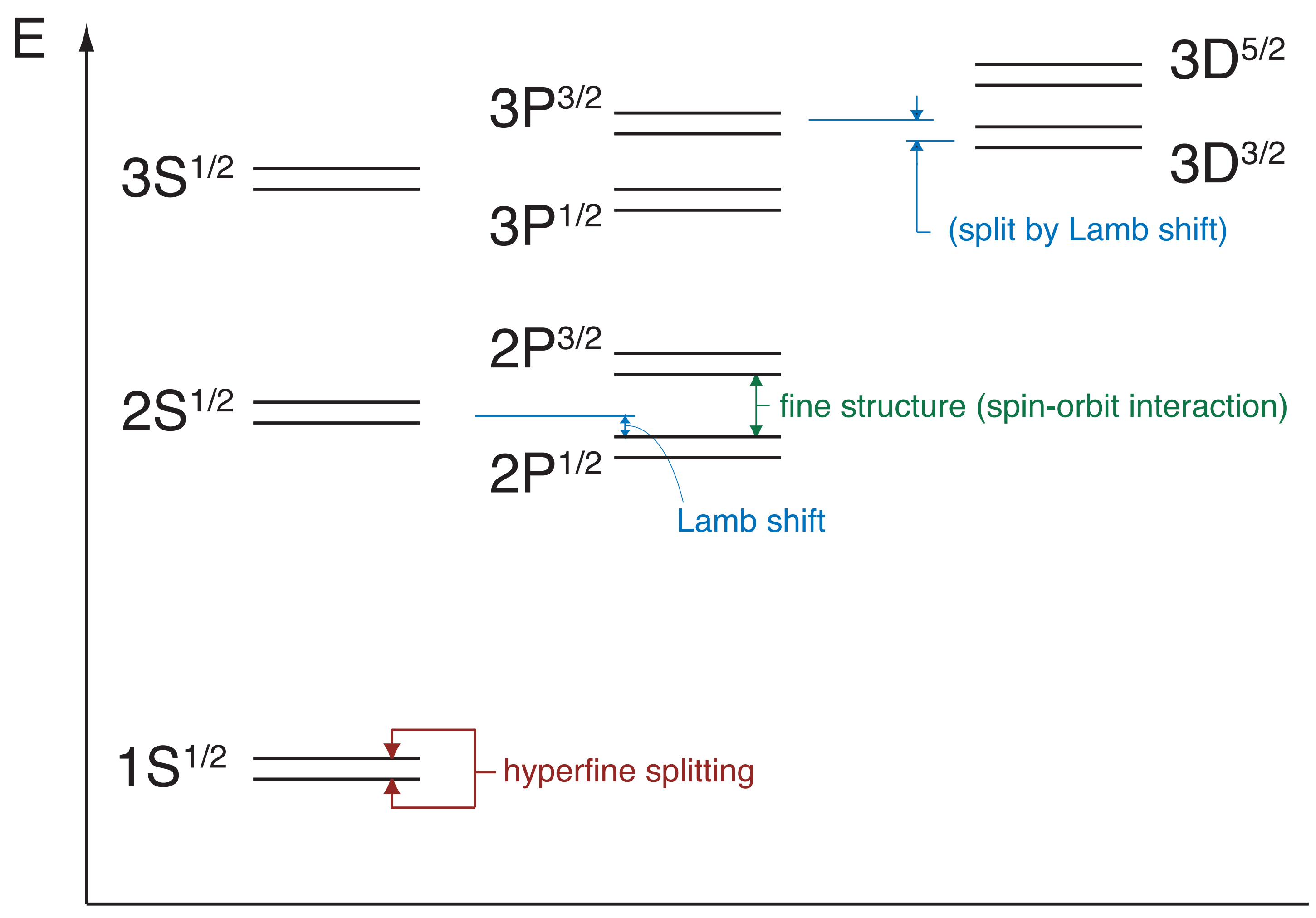
from Bernauer et al. (2010).

- Atomic physics: measure energy levels (Lamb shift, etc.) and isolate proton size dependent terms. Gives

$$R_E = 0.8768\,(69) \text{ fm}$$

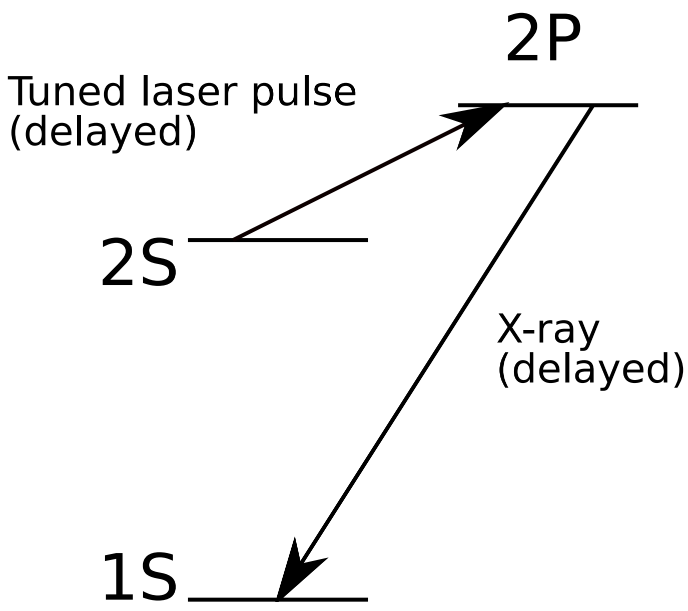
using average from CODATA (2006).

Spectrum of electronic Hydrogen (not to scale)



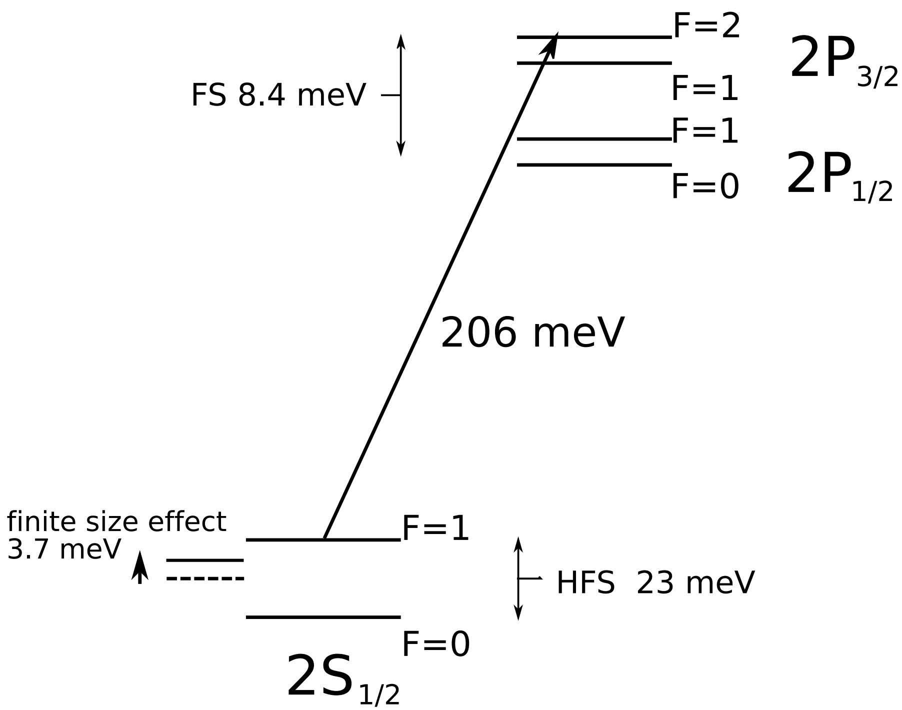
Charge radius from Lamb shift

- Lamb shift, muonic hydrogen. Muon about 200 times closer than electron, proton size effects magnified. Long anticipated 0.1% measurement of charge radius.
- Obtained $R_E = 0.84184\,(67)\,\mathrm{fm}$
- 4% or 5 (old) σ lower than CODATA value



- Method: Induce 2S→2P with tuned and well calibrated laser. 2S metastable; delay laser pulse until most other muons cascaded down to 1S state. Success in tuning laser signaled by X-ray from 2P to 1S transition.

- More on n=2 for muonic hydrogen:
- Lamb shift dominated by vacuum polarization, drops 2S state by a lot



- Experiment measures 2S F=1 to $2P_{3/2}$ F=2 level (F is total angular momentum)

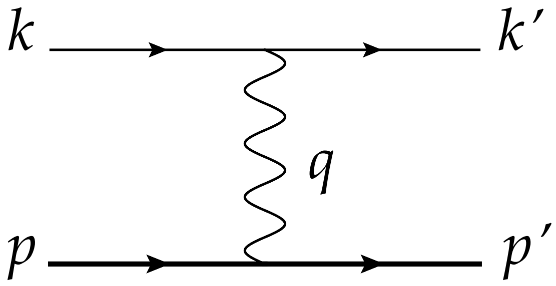
- Calculation = Measurement ⇒ stated R_E

$$209.9779(49) - \left(5.2262 R_E^2 - 0.0347 R_E^3\right) = 206.2949(32)\,\mathrm{meV}$$

- Experiment says corrections are ≈ 300μeV below expectation
  Either have smaller radius or must find -300 μeV further corr.
  Note: R_E^3 term is about -27 μeV

LO calculation (Relativistic)

- One photon exchange in momentum space
- One photon exchange in perturbation theory won't give a bound state, but corrections to one-photon exchange can be treated perturbatively.



$$\mathcal{M} = -\frac{e^2}{q^2}\,\bar{u}(k')\gamma_\mu u(k)\;\;\bar{u}(p')\left[\gamma^\mu(F_1-1)+\frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2\right]u(p)$$

- General: $$\Delta E = -\frac{\mathcal{M}}{state\ normalization}\,\phi_n^2(0)$$

- O.k. to use: $$\bar{u}(k')\gamma_\mu u(k) \rightarrow \bar{u}(k)\gamma_\mu u(k) = 2m\, g_{\mu 0}$$

LO calculation (Relativistic)

- Get

$$\Delta E = -4\pi\alpha \left(F_1'(0) - \frac{1}{4M^2} F_2(0) \right) \phi_n^2(0)$$

- Remember

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

- to get

$$\boxed{\Delta E = -4\pi\alpha\, G_E'(0)\, \phi_n^2(0) = \frac{2\pi\alpha}{3}\, R_E^2\, \phi_n^2(0)}$$

- Really is charge radius.
  Original NR calculation Karplus, Klein, Schwinger (1952)
- $O(\alpha^4)$ correction overall, since $\phi_n^2(0) = m_r^3 \alpha^3 / (\pi n^3)$

Next order calculation

- The experimenters also kept an $O(\alpha^5)$ proton structure-dependent correction, found by Friar (1979)

$$\Delta E = \frac{2\pi\alpha}{3}\phi(0)^2 \left(R_p^2 - \frac{1}{2} m_r \alpha R_{(2)}^3 \right)$$

where nonrelativisticaly

$$R_{(2)}^3 = \int d^3r_1\, d^3r_2\, |r_1 - r_2|^3 \rho_E(r_1) \rho_E(r_2)$$

- reminiscent of integral found by Zemach in a related context; Friar called it the 3rd Zemach moment.

Next order calculation (NR)

- Note for later:
- NR,

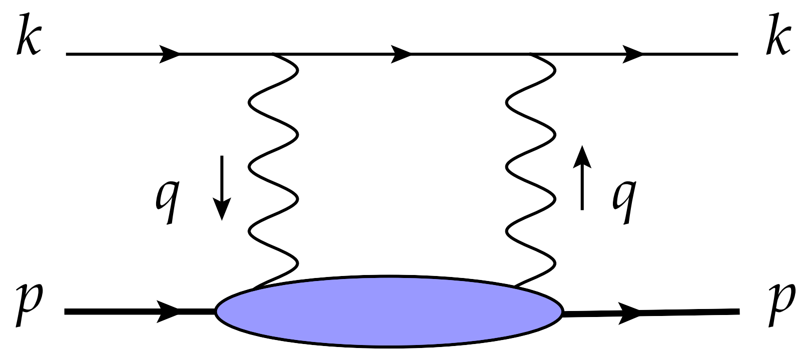
$$\rho_E(r) = \int (d^3q)\, e^{i\vec{q}\cdot\vec{r}} G_E(q^2)$$

- Then,

$$R^3_{(2)} = \frac{48}{\pi} \int_0^\infty \frac{dq}{q^4} \left[G_E^2(q) - 1 - 2q^2 G_E(0) \frac{dG_E}{dq^2}(0) \right]$$

HO corrections

- Modern: HO corrections come from two-photon exchange, and calculation done using field theory



- Result involves Compton tensor,

$$T^{\mu\nu}(p,q) = \frac{i}{8\pi M}\int d^4x\, e^{iqx}\langle p|Tj^\mu(x)j^\nu(0)|p\rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2)$$

$$+\frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right) T_2(\nu, Q^2)$$

$$(q^2 = -Q^2, \quad \nu = p\cdot q/M)$$

HO corrections

- Straightforwardly get energy shift in terms of T_1, T_2,

$$\Delta E = \frac{8\alpha^2 m}{\pi}\phi_n^2(0)\int d^4Q$$

$$\times \frac{(Q^2+2Q_0^2)T_1(iQ_0,Q^2)-(Q^2-Q_0^2)T_2(iQ_0,Q^2)}{Q^4(Q^4+4m^2Q_0^2)}$$

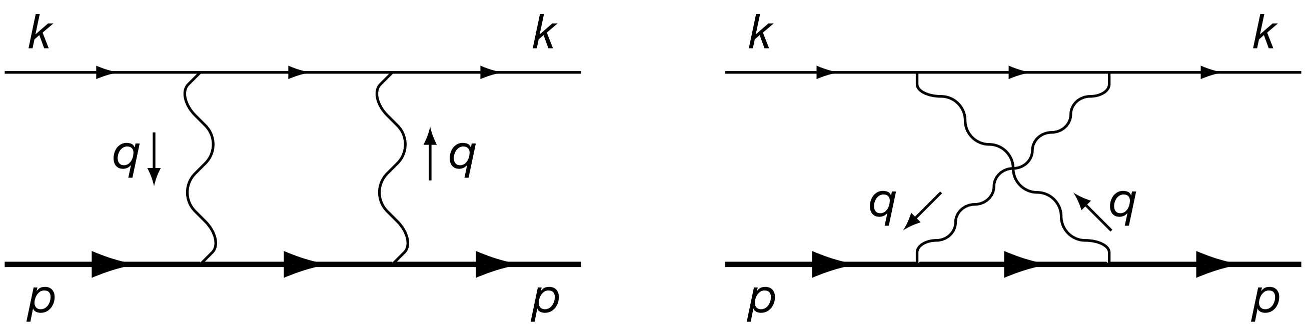
(with Wick rotation $q_0 = iQ_0, \quad \vec{Q} = \vec{q}$)

- Minor problem: We don't know what T_1 and T_2 are. But we know their imaginary parts, because they are the structure functions measured in DIS at Measured at SLAC, DESY, Bonn, JLab, Mainz, ...

$$\begin{aligned} \mathrm{Im}\, T_1(\nu,Q^2) &= \tfrac{1}{4M} F_1(\nu,Q^2) \\ \mathrm{Im}\, T_2(\nu,Q^2) &= \tfrac{1}{4\nu} F_2(\nu,Q^2) \end{aligned}$$

HO corrections

- Record of Born contributions



$$\Gamma^\mu = \gamma^\mu F_1(Q^2) + (i/2M)\sigma^{\mu\nu} q_\nu F_2(Q^2)$$

$$T_1^B = \frac{1}{4\pi M}\left\{\frac{Q^4 G_M^2}{(Q^2 - i\epsilon)^2 - 4M^2 q_0^2} - F_1^2(Q^2)\right\}$$

$$T_2^B = \frac{1}{\pi}\frac{MQ^2}{(Q^2 - i\epsilon)^2 - 4M^2 q_0^2}\frac{G_E^2(Q^2) + \tau_p G_M^2(Q^2)}{1+\tau_p}$$

HO corrections

- Separate out pole terms, and write dispersion relations for T_1 and T_2 to obtain their real parts. T_1 requires a subtraction.

$$T_1(q_0, Q^2) = T_1^{pole} + \overline{T}_1$$

(The dispersion relation does not pick up the non-pole part of T_1.)

$$T_1(q_0, Q^2) = T_1^{pole}(q_0, Q^2) + \overline{T}_1(0, Q^2) + \frac{q_0^2}{2\pi M} \int_{\nu_{th}}^{\infty} d\nu \frac{F_1(\nu, Q^2)}{\nu(\nu^2 - q_0^2)}$$

$$T_2(q_0, Q^2) \quad = T_2^B(q_0, Q^2) + \frac{1}{2\pi} \int_{\nu_{th}}^{\infty} d\nu \frac{F_2(\nu, Q^2)}{\nu^2 - q_0^2}$$

HO corrections

- Insert the DR result into the energy formula, do what integrals can be done, and obtain

$$\Delta E = \Delta E^{subt} + \Delta E^{inel} + \Delta E^{el}$$

- ΔE^{el}: shown on demand. Will note NR limit (M $\to \infty$, m and proton size held fixed) gives NR result.
- ΔE^{inel}: shown on demand.
- ΔE^{subt} deserves some commentary

$$\Delta E^{subt} = \frac{4\pi\alpha^2}{m}\phi_n^2(0)\int_0^\infty \frac{dQ^2}{Q^2}\frac{\gamma_1(\tau_\ell)}{\sqrt{\tau_\ell}}\overline{T}_1(0,Q^2)$$

$$\tau_\ell = Q^2/(4m^2)$$

$$\gamma_1(\tau_\ell) = (1-2\tau_\ell)\left((1+\tau_\ell)^{1/2} - \tau_\ell^{1/2}\right) + \tau_\ell^{1/2}$$

HO corrections

- Subtraction function due to excitations of proton, codified at low energy by effective Hamiltonian

$$\mathcal{H} = -\frac{1}{2}4\pi\alpha_E \vec{E}^2 - \frac{1}{2}4\pi\beta_M \vec{B}^2$$

$$\lim_{\nu^2, Q^2 \to 0} \overline{T}_1(\nu, Q^2) = \frac{\nu^2}{e^2}(\alpha_E + \beta_M) + \frac{Q^2}{e^2}\beta_M$$

Use here
$$\overline{T}_1(0, Q^2) = \frac{\beta_M}{4\pi\alpha} Q^2 F_\pi^2(Q^2)$$

Starting point: PDG quotes $\beta_M = (1.9 \pm 0.5) \times 10^{-4}\ \mathrm{fm}^3$

But other analyses give

$$\beta_M = \begin{cases} (4.0 \pm 0.7) \times 10^{-4}\ \mathrm{fm}^3 & \text{Lensky and Pascalutsa (2009)} \\ (3.4 \pm 1.2) \times 10^{-4}\ \mathrm{fm}^3 & \text{Beane et al. (2005)} \end{cases}$$

Get
$$\Delta E^{subt} = 4.8\ \mu\mathrm{eV} \times \frac{\beta_M}{(3.4 \times 10^{-4}\ \mathrm{fm}^3)}$$

HO corrections

- Results for O(α5) proton structure-dependent terms.

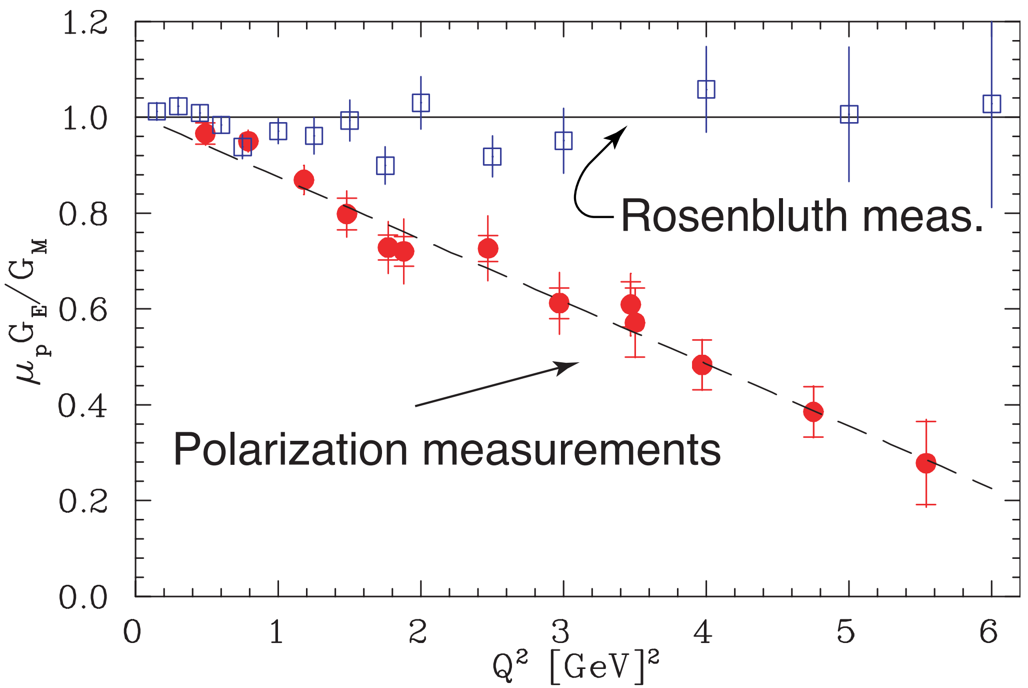
  Energy units are μeV.

(μeV)	here	Pachucki (1999)	Martynenko (2005)
ΔE^{subt}	4.8	1.8	2.3
ΔE^{inel}	-12.7	-13.9	-13.8
ΔE^{el}	-29.5	-23.0	-23.0
ΔE	-37.4	-35.1	-34.5

- Different, but not 300 μeV different.

Two-photon section

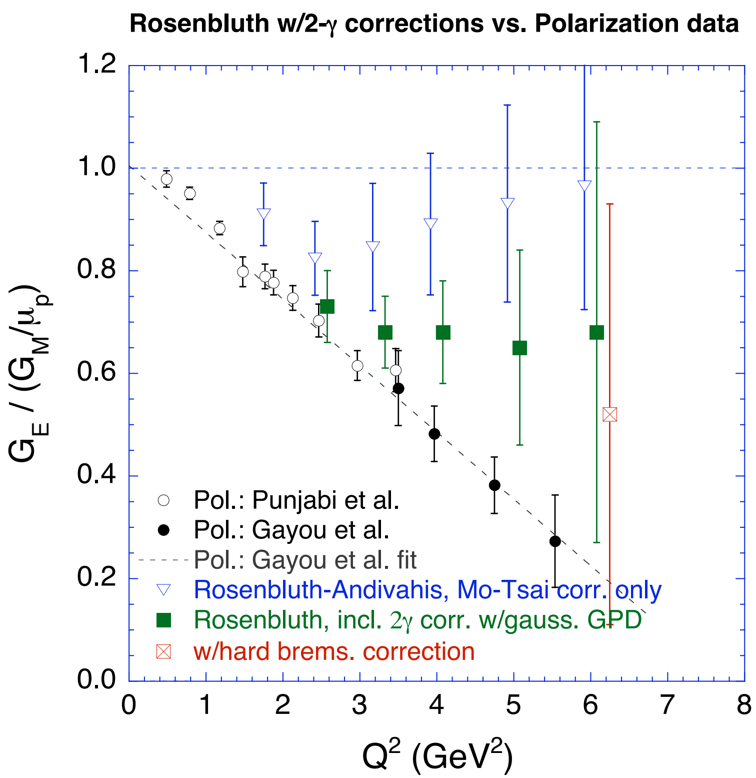
- "Once upon a time" (2003) the elastic form factor ratio G_{Ep}/G_{Mp} appeared to depend whether it was measured using the Rosenbluth method or using polarization transfer.



(figure from Arrington, PRC 2003)

Two-photon section

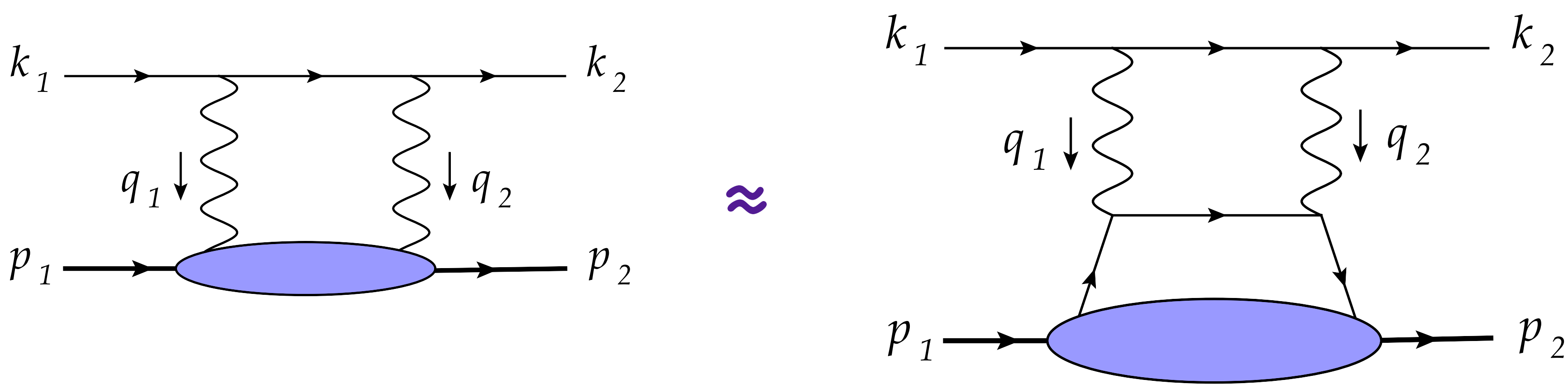
- Putative solution: there were until then uncalculated two-photon corrections. They were not strikingly large, but had a big effect because the G_{Ep} contributions to the cross sections that the Rosenbluth measurements relied on were also not big.



- Some early papers:
  Guichon, Vanderhaeghen (2003)
  Blunden, Melnitchouk, Tjon (2003)
  Chen et al. (2004)
  Afanasev et al. (2005)

Two-photon section

- At higher Q^2, calculate in terms of GPDs



- Lower part of diagram is GPD, upper part is electron-quark (same as electron-muon, mutatis mutando) elastic scattering
- General expression for LO + two-photon amplitude is

$$
\begin{aligned}
\mathcal{M}_{h,\lambda'_N\lambda_N} &= \frac{e^2}{Q^2}\,\bar{u}(k',h)\gamma_\mu u(k,h) \\
&\times \quad \bar{u}(p',\lambda'_N)\left(\tilde{G}_M\,\gamma^\mu - \tilde{F}_2\frac{P^\mu}{M} + \tilde{F}_3\frac{\gamma\cdot K P^\mu}{M^2}\right)u(p,\lambda_N)
\end{aligned}
$$

Two-photon section

- Three form factors:

$$\begin{aligned}\tilde{G}_M &= G_M(Q^2) + \delta\tilde{G}_M(\varepsilon, Q^2)\\ \tilde{G}_E &= G_E(Q^2) + \delta\tilde{G}_E(\varepsilon, Q^2)\\ \tilde{F}_3 &= 0 \quad + \delta\tilde{F}_3(\varepsilon, Q^2)\end{aligned}$$

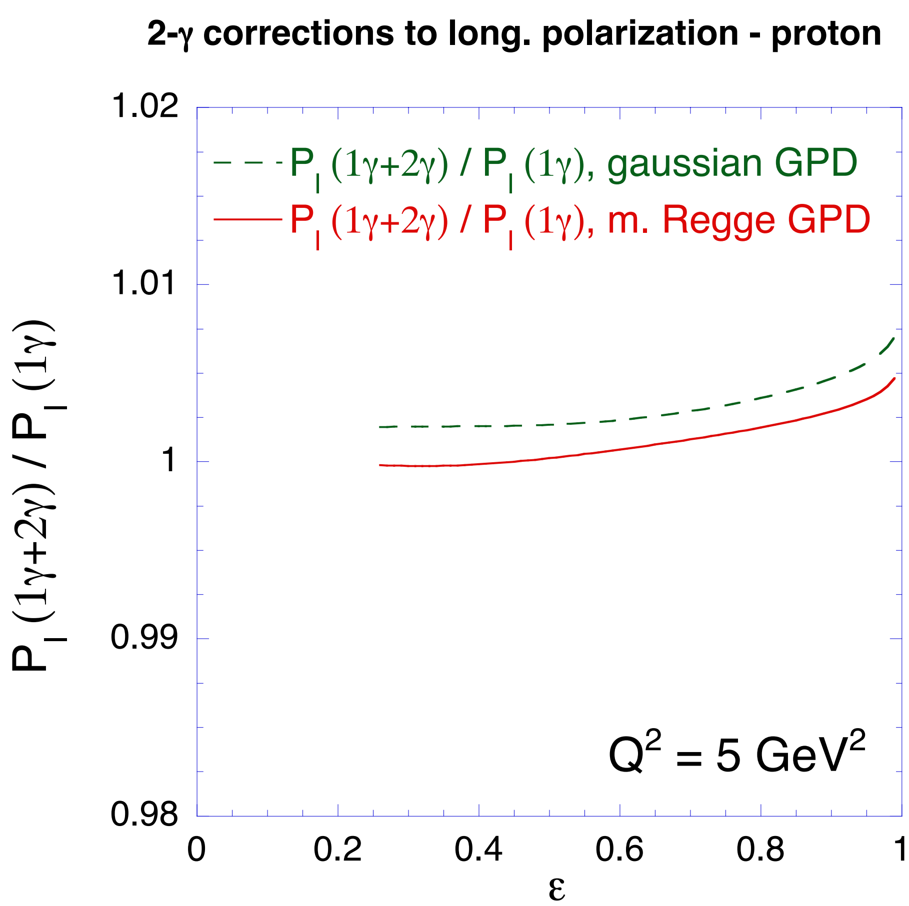
$$\upuparrows \qquad\qquad \upuparrows$$

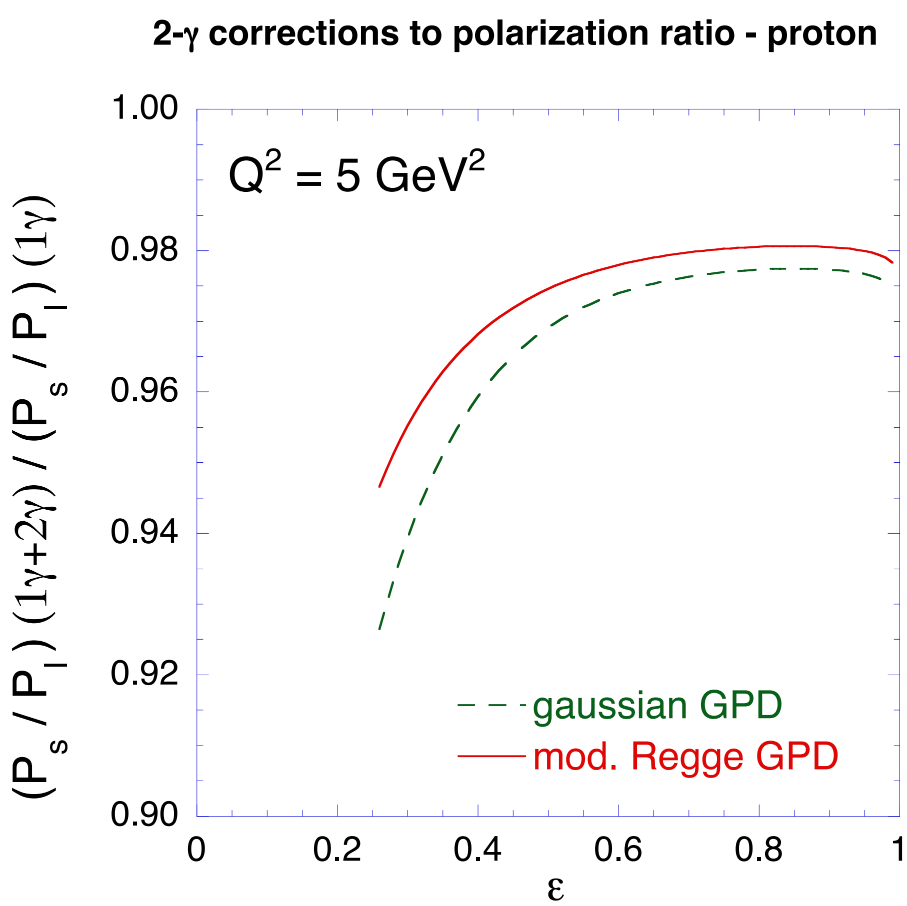
ordinary FF TPE

- The TPE corrections are not functions of one variable and in general are complex
- Calculated using GPD models available in 2004/2005.
- Calculated cross sections for Rosenbluth, but also had predictions for dependences of polarizations upon ε.

Two-photon section

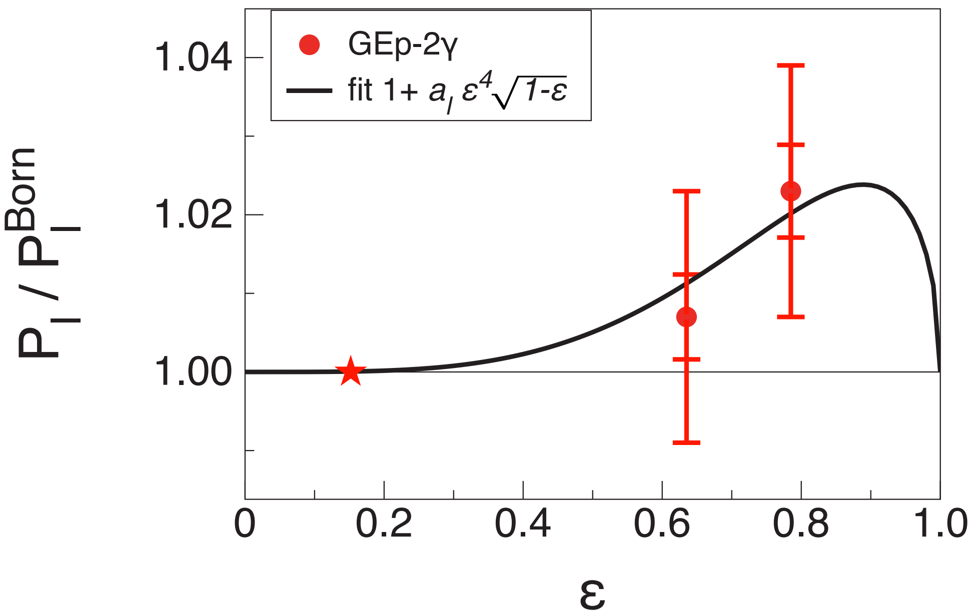
- Examples of polarization predictions

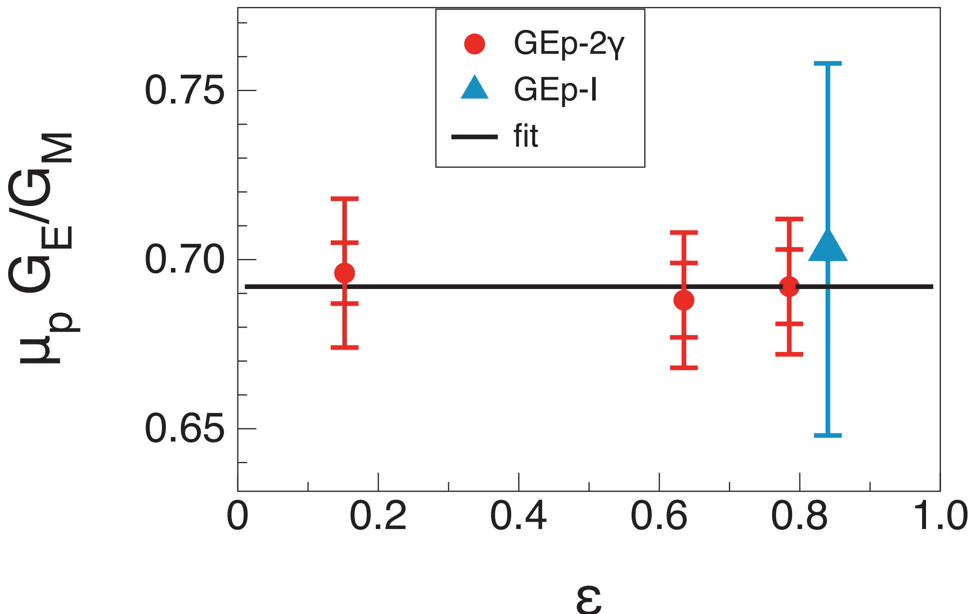




Two-photon section

- Now there are real data (GE-2γ experiment, M. Meziane et al, PRL, submitted), at $Q^2 = 2.64\ \text{GeV}^2$.





(Above same as P_t/P_l)

$$\frac{P_s}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}}\,\frac{G_E(Q^2)}{G_M(Q^2)}$$

- Idea of Guttmann, Kivel, Meziane, & Vanderhaeghen (1012.0564): Take above data, plus Rosenbluth data, and reverse formulas to find actual δG_M, δG_E, and δF_3. See how we did!

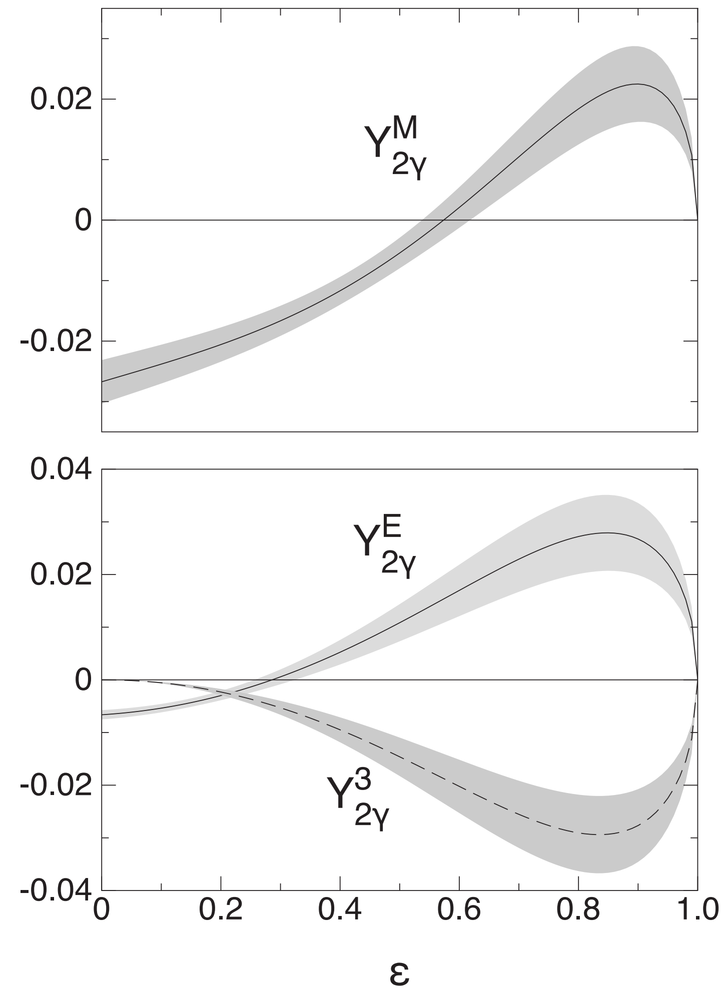
Two-photon section

- Results given in terms of ratios (all for $Q^2 = 2.64\ \mathrm{GeV}^2$)

$$Y_{2\gamma}^{M} = \mathrm{Re}\,\frac{\delta\tilde{G}_M}{G_M}$$

$$Y_{2\gamma}^{E} = \mathrm{Re}\,\frac{\delta\tilde{G}_E}{G_M}$$

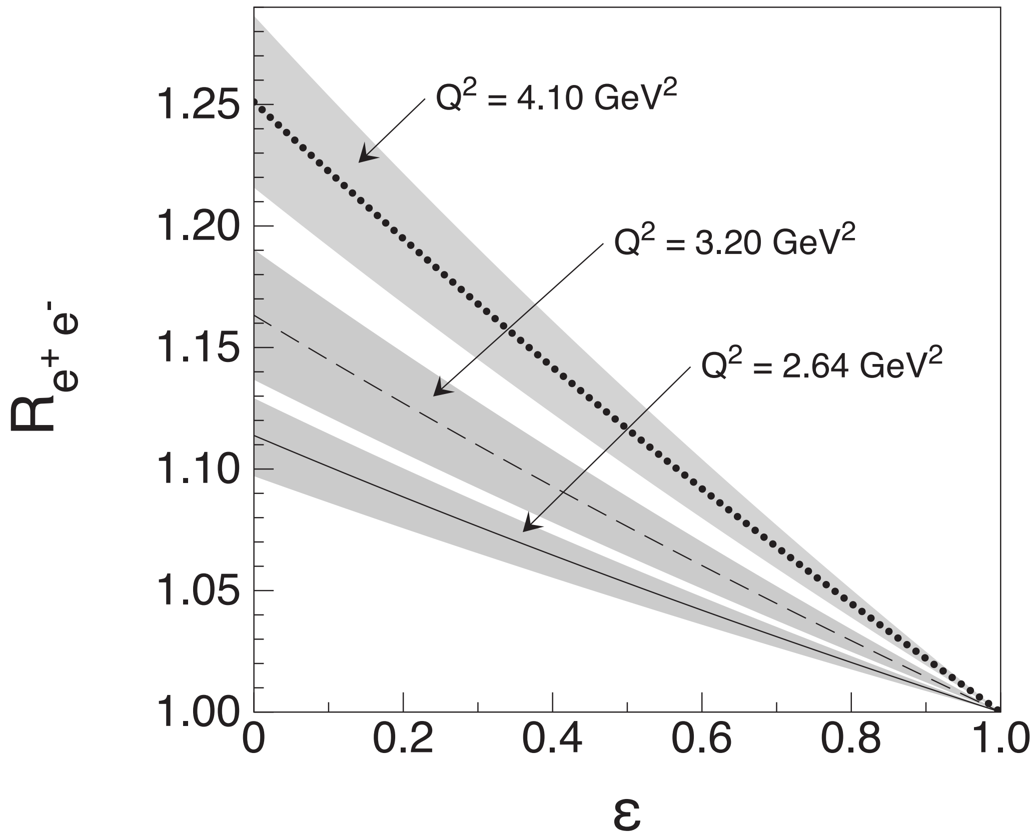
$$Y_{2\gamma}^{3} = \frac{s-u}{M^2}\,\mathrm{Re}\,\frac{\delta\tilde{F}_3}{G_M}$$



- BTW: Old calculations qualitatively o.k. for G_M and F_3, though too small, and flat for G_E.

Two-photon section

- New prediction for σ(e+p)/σ(e-p) considerably larger in magnitude.



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Final remarks

- The proton charge radius puzzle is a major problem
- Did not mention during talk: $(g\text{-}2)_\mu$ experiment makes "new physics" explanations hard. New particles that might affect the Lamb shift also affect $(g\text{-}2)_\mu$, and the accuracy of theory and experiment for $(g\text{-}2)_\mu$ is very good.
- Have here reported redoing the modern analysis of $O(\alpha^5)$ or two-photon exchange corrections, with up to date form factors fits and inelastic structure function fits.
- Problem remains.
- On electron scattering two-photon side, new experiments remind us that old used model GPDs. Now enough data to extract 2-photon terms from data. One part of future: better GPDs.

Extra

Timelike 2γ

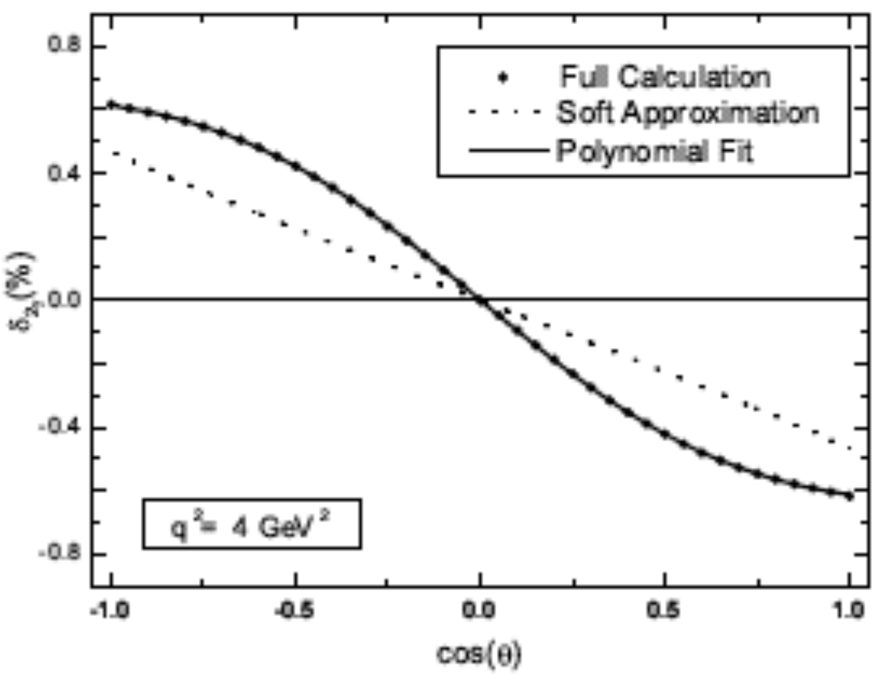
- p p-bar → e^+e^- with 1γ exchange is forward-backward symmetric, i.e., even in cos θ. With 2γ exchange, interference terms give an asymmetry.
- Feasibility study by Sudol+16, EPJA 44, 373 (2010)
- Calculations by Chen, Zhou, & Dong, PRC 78, 045208 (2008) show few percent effects. Used only protons intermediate states (like Blunden et al. (2003))

$$\frac{d\sigma}{d\Omega} \propto |\mathcal{M}(1\gamma)|^2 \left(1 + \delta_{2\gamma}\right)$$

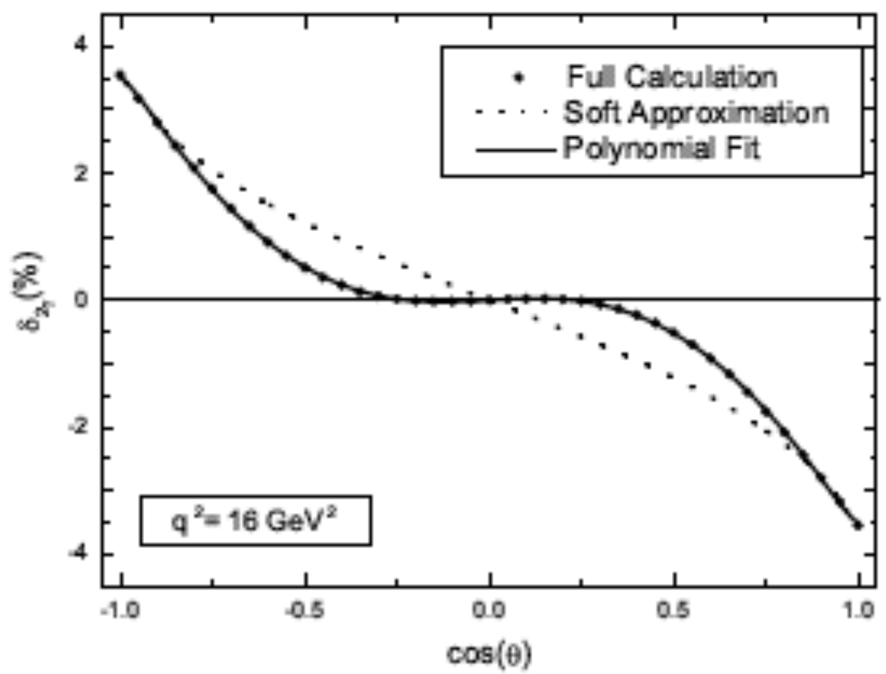
Timelike 2γ

- from Chen, Zhou, & Dong,

$$\frac{d\sigma}{d\Omega} \propto |\mathcal{M}(1\gamma)|^2 \left(1 + \delta_{2\gamma}\right)$$



q^2 = 4 GeV^2



q^2 = 16 GeV^2

Box contributions to energy shift

- Elastic terms

$$\Delta E^{el} = -\frac{\alpha^2 m}{M(M^2 - m^2)} \phi_n^2(0) \int_0^\infty \frac{dQ^2}{Q^2}$$

$$\times \left\{ \left(\frac{\gamma_2(\tau_p)}{\sqrt{\tau_p}} - \frac{\gamma_2(\tau_\ell)}{\sqrt{\tau_\ell}} \right) \frac{G_E^2 + \tau_p G_M^2}{\tau_p(1+\tau_p)} - \left(\frac{\gamma_1(\tau_p)}{\sqrt{\tau_p}} - \frac{\gamma_1(\tau_\ell)}{\sqrt{\tau_\ell}} \right) G_M^2 \right\}$$

$$\tau_p = Q^2/(4M^2), \quad \tau_\ell = Q^2/(4m^2)$$

$$\gamma_1(\tau) = (1-2\tau)\Big((1+\tau)^{1/2} - \tau^{1/2}\Big) + \tau^{1/2}$$

$$\gamma_2(\tau) = (1+\tau)^{3/2} - \tau^{3/2} - \frac{3}{2}\tau^{1/2}$$

Box contributions to energy shift

- Inelastic term

$$\Delta E^{inel} = -\frac{2\alpha^2}{mM}\phi_n^2(0)\int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_{th}}^\infty d\nu\left[\frac{\widetilde{\gamma}_1(\tau,\tau_\ell)F_1(\nu,Q^2)}{\nu} + \frac{\widetilde{\gamma}_2(\tau,\tau_\ell)F_2(\nu,Q^2)}{Q^2/M}\right]$$

$$\tau = \nu^2/Q^2, \quad \tau_\ell = Q^2/(4m^2)$$

$$\begin{aligned}\widetilde{\gamma}_1(\tau,\tau_\ell) &= \frac{1}{\tau_\ell - \tau}\left(\sqrt{\tau_\ell}\gamma_1(\tau_\ell) - \sqrt{\tau}\gamma_1(\tau)\right)\\ \widetilde{\gamma}_2(\tau,\tau_\ell) &= \frac{1}{\tau_\ell - \tau}\left(\frac{\gamma_2(\tau)}{\sqrt{\tau}} - \frac{\gamma_2(\tau_\ell)}{\sqrt{\tau_\ell}}\right)\end{aligned}$$

De Rújula idea

- De Rújula has form factor (PLB 693, 555 (2010))

$$G_E p(Q^2) = \frac{1}{M^2 \cos^2\theta + m^2 \sin^2\theta} \left[\frac{M^2 \cos^2\theta}{1 + Q^2/M^2} + \frac{m^2 \sin^2\theta}{(1 + Q^2/m^2)^2} \right]$$

$$\sin^2\theta = 0.3, \quad M = 750\,\text{MeV}, \quad m = 18\,\text{MeV}$$

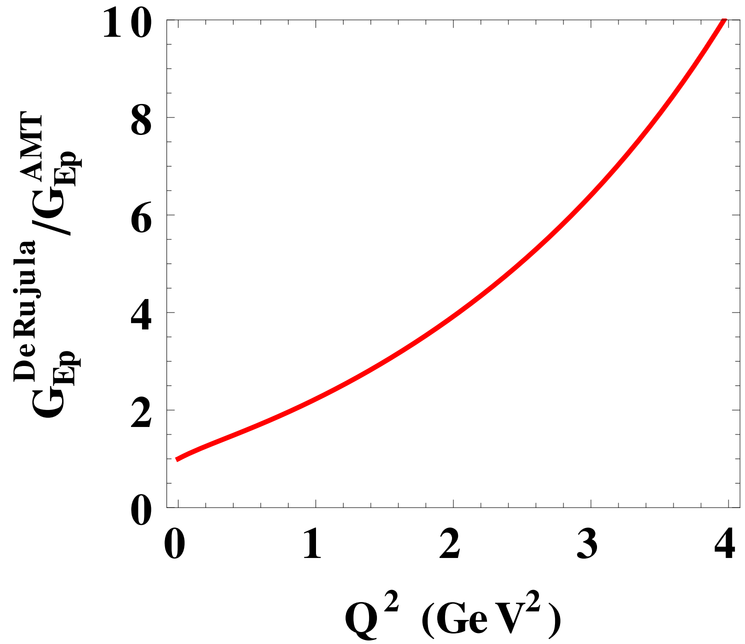
- Gives CODATA value for charge radius
- Gives "Friar radius" or 3rd Zemach moment

$$\left(R^3_{(2)}\right)^{1/3} = 3.27\text{fm}$$

or about 2.35 times AMT result (1.39 fm, with 1.41 fm for new Mainz FF), or about 13 times bigger in cube. "Explains" the muonic hydrogen proton radius measurement.

De Rújula idea

- Fits two numbers, but has trouble globally.
  Compare to Arrington, Melnitchouk, Tjon form factor,



- For regular Zemach radius, which goes into HFS, gives 0.95 fm, versus 1.08 fm for AMT (using AMT G_M in both cases). Leads to 5 ppm excess in HFS, a big number in what is otherwise accurate to 1 ppm.