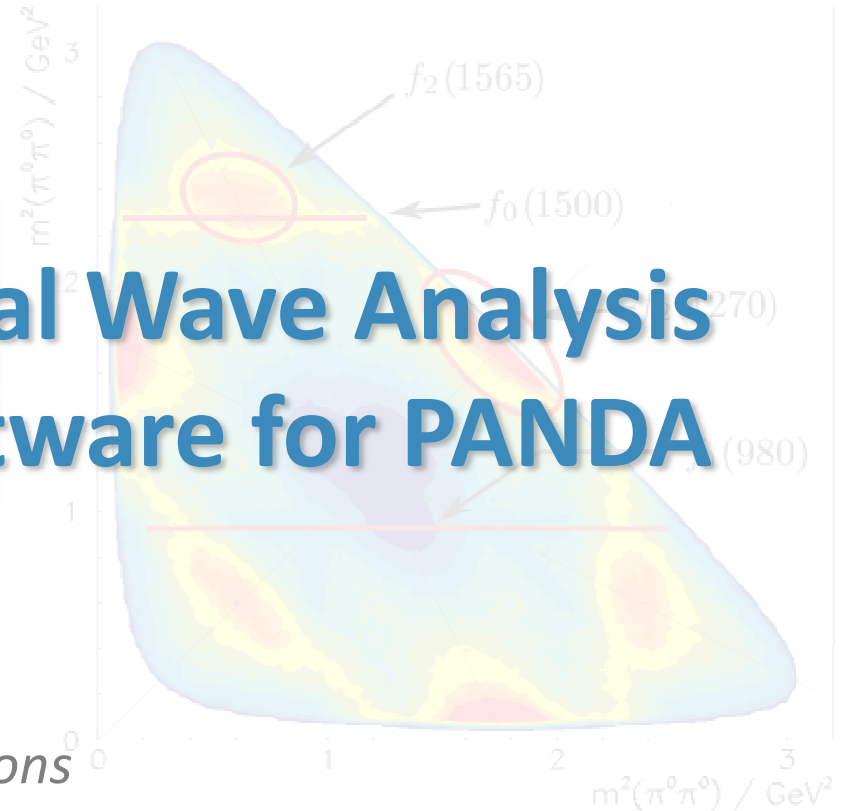


# Generalized Partial Wave Analysis Software for PANDA



*39. International Workshop on the  
Gross Properties of Nuclei and Nuclear Excitations*

*The Structure and Dynamics of Hadrons*

*Hirschegg, January 2011*

**Klaus Götzen**

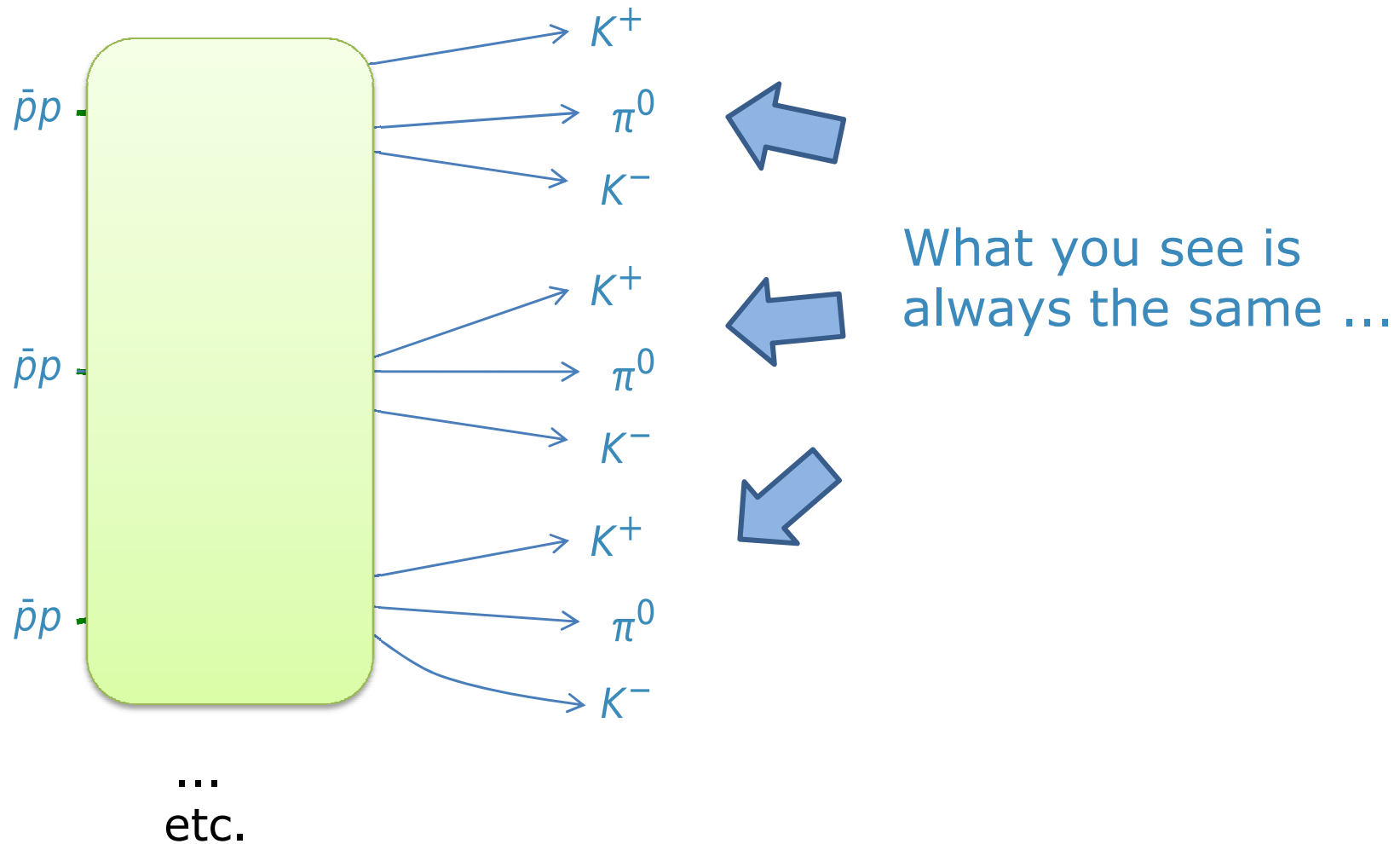
GSI Darmstadt

# Outline

- The Need for Partial Wave Analysis
- Challenges & Requirements for PANDA
- General Software Concept
- Status of Project

# The Need for Partial Wave Analysis

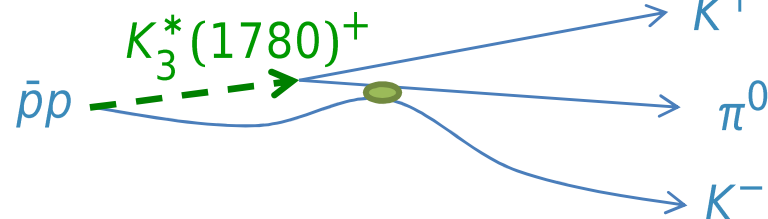
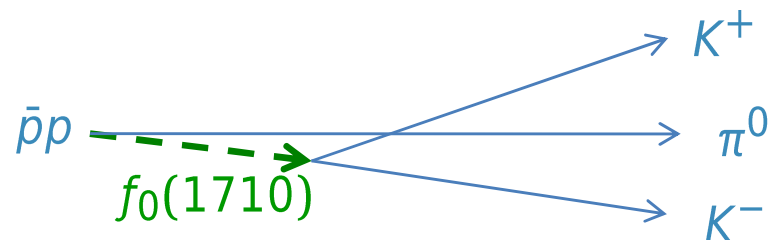
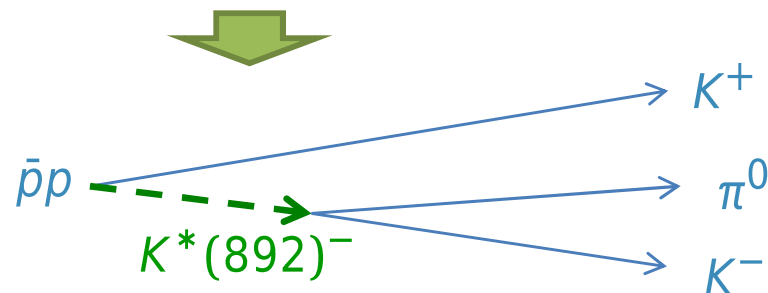
- *Example:* Consider reaction  $\bar{p}p \rightarrow K^+K^-\pi^0$



# The Need for Partial Wave Analysis

- *Example:* Consider reaction  $\bar{p}p \rightarrow K^+K^-\pi^0$

What *really* happened...



...  
etc.

What you see is  
always the same ...

PWA = technique to find  
out what happens in between

## **Primary goal:** Learn about intermediate states

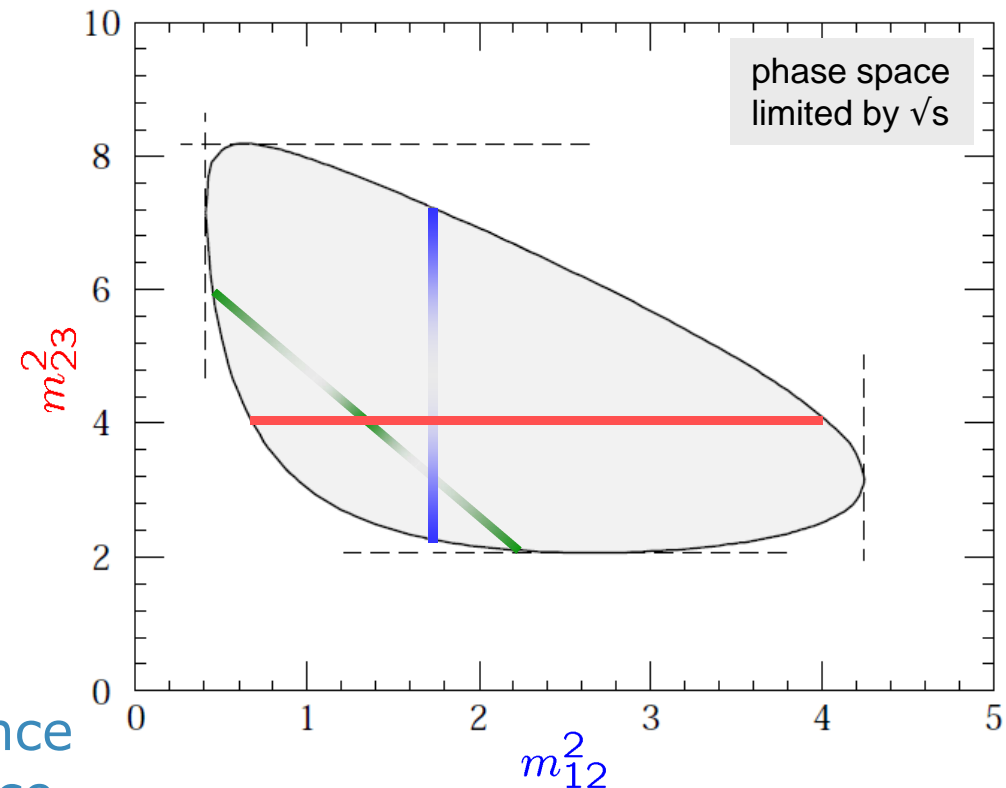
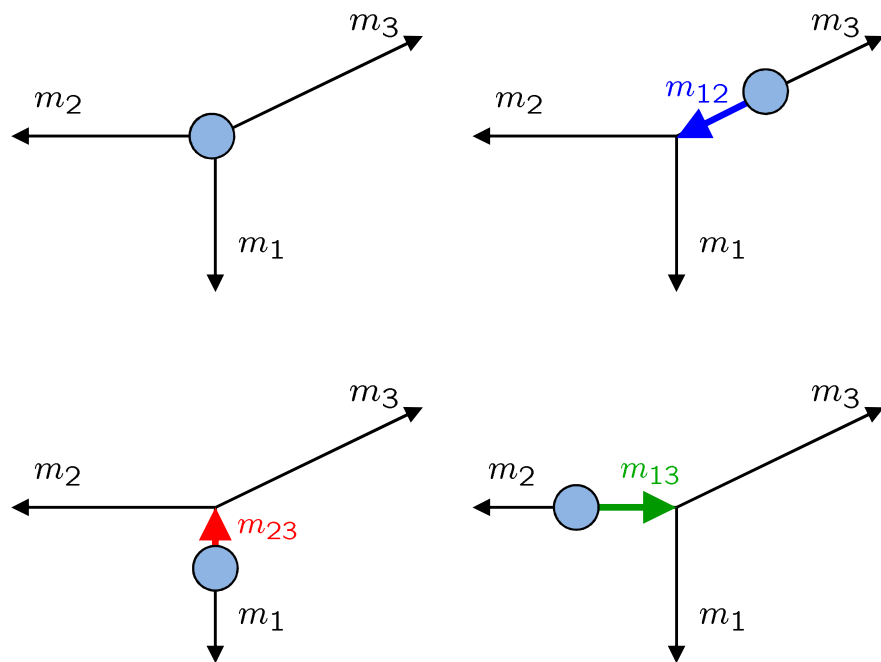
- Choose **final state**, so that
  - Resonances of interest have high probability to appear
- **Discovery of new resonances!**
- Precise determination of **resonance properties** like
  - Mass
  - Width
  - Spin-Parity
  - Relativ production strength
  - Relativ phases

# 3-Body Case: Dalitz Plot Analysis

- 3-body-decay: *Dalitz-Plot-Analysis* for  $\bar{p}p \rightarrow m_1 m_2 m_3$
- Dynamics fully described by two quantities:

$$m_{12}^2 \text{ vs. } m_{23}^2$$

$$m_{ij}^2 = (E_i + E_j)^2 - (\vec{p}_i + \vec{p}_j)^2$$



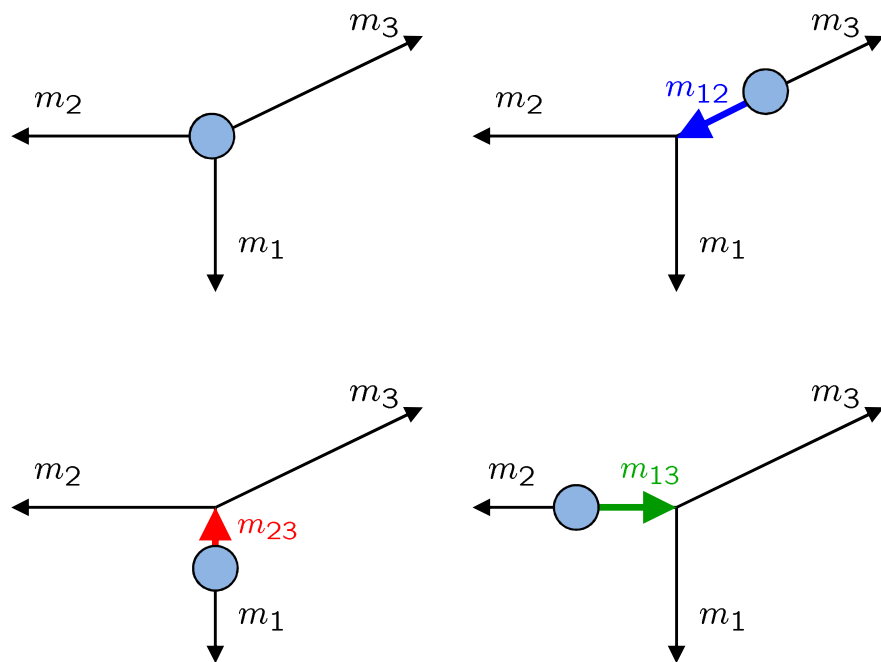
Location of Band → Mass of Resonance  
 Density in Band → Spin of Resonance

# 3-Body Case: Dalitz Plot Analysis

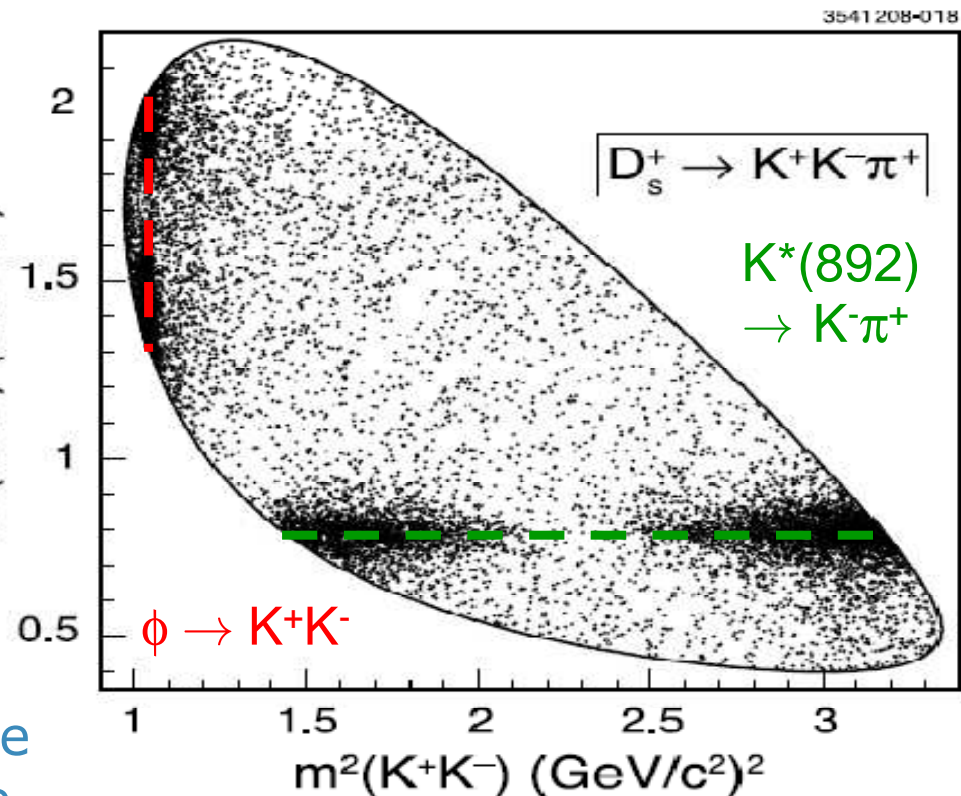
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$m^2(K^-\pi^+) \text{ (GeV}/c^2)^2$



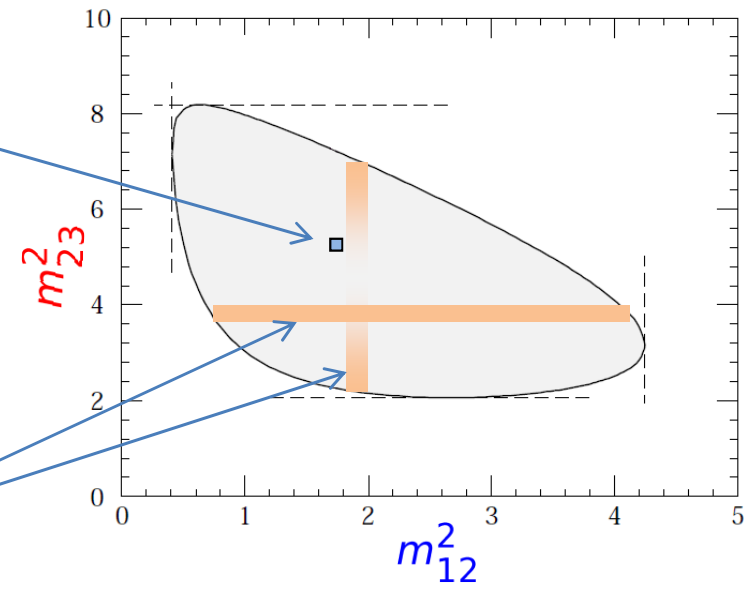
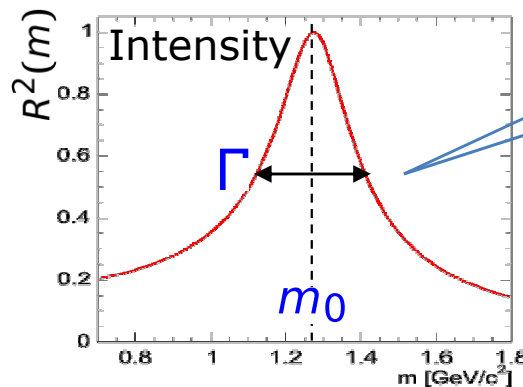
Location of Band  $\rightarrow$  Mass of Resonance  
 Density in Band  $\rightarrow$  Spin of Resonance

# 3-Body Case: Dalitz Plot Analysis

- What is intensity at  $(m_{12}^2, m_{23}^2)$ ?

- Resonances: Complex Functions  
e.g. Breit-Wigner

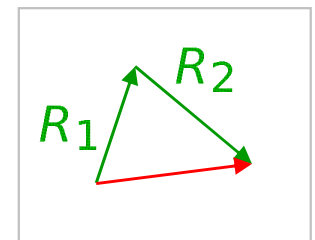
$$R(m) = \frac{\frac{\Gamma}{2}}{m_0 - m - i\frac{\Gamma}{2}}$$



- Total intensity:

$$I_{tot} \sim \left| \sum_n a_n \cdot R_n(m) \cdot B(p, q) \cdot Z(L) \right|^2$$

← sum over resonances  
← kinematic factor     ← angular distribution  
↑ complex coefficient     ↑ complex dynamic function



Sum of **complex** numbers = Interference Pattern!



# PWA – Simple Recipe

In principle simple straightforward strategy:

1. Reconstruct/measure the channel of interest experimentally
2. Create an appropriate fit model
  - choice of formalism
  - the contributing resonances
  - the according dynamic functions
3. Fit the model to the data
  - Maximum-Likelihood or binned approach
4. Extract the physical parameters of interest
  - Masses, widths, spin-parities of resonances
  - fit fractions

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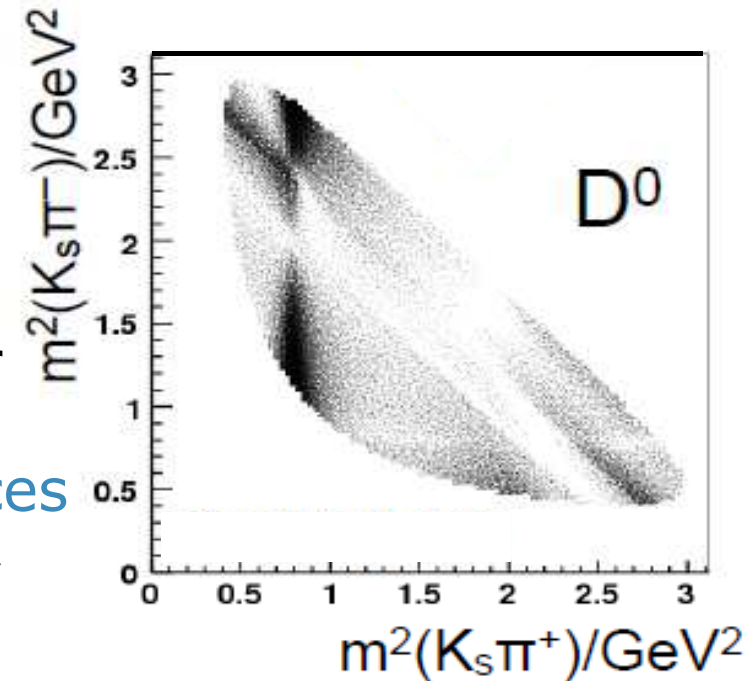
*... but, the devil is in the details!*



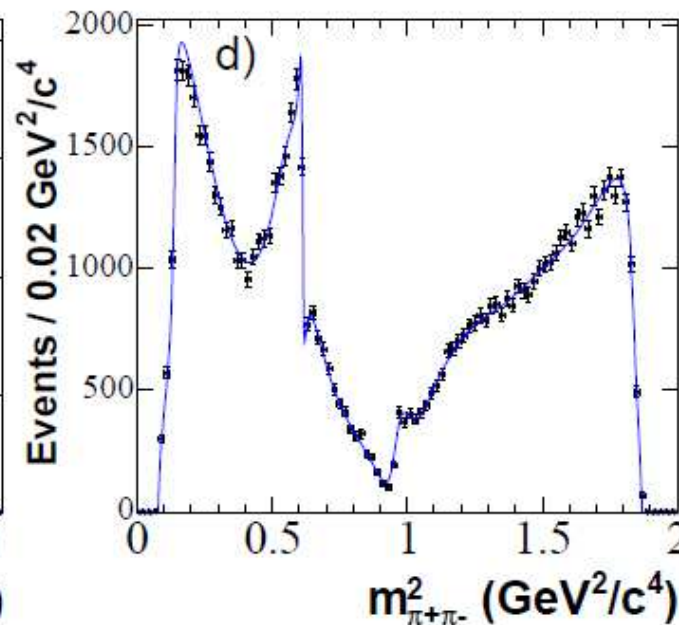
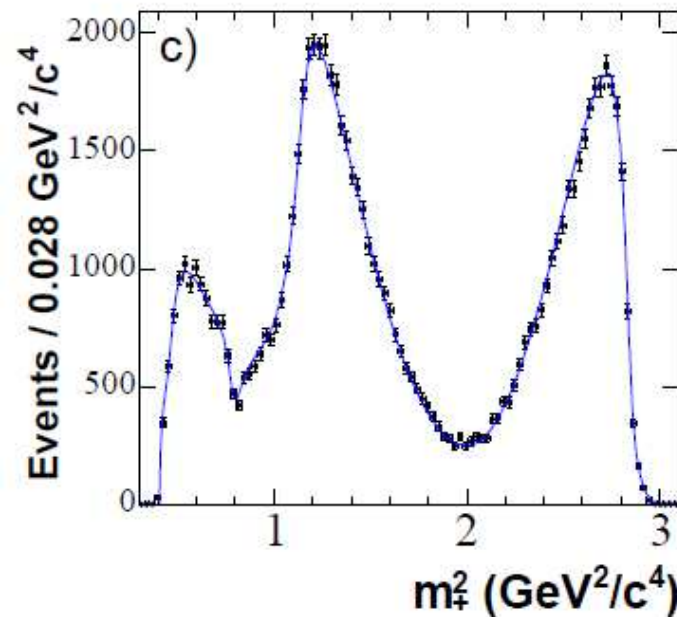
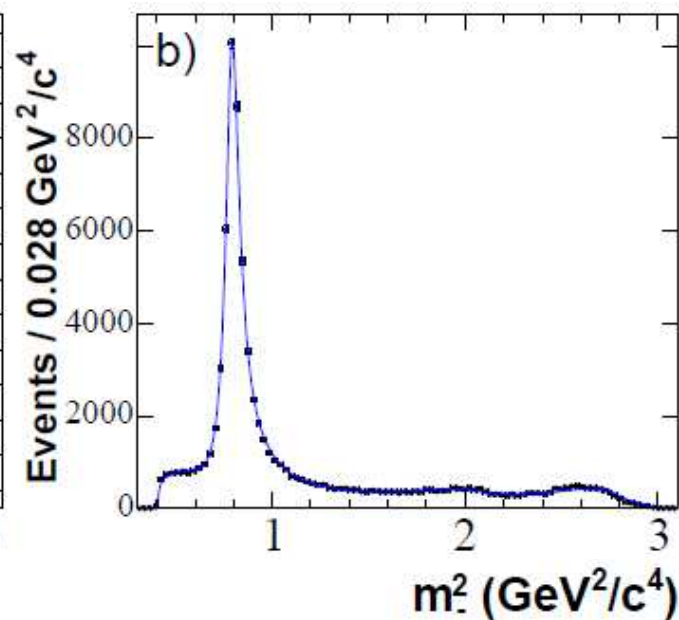
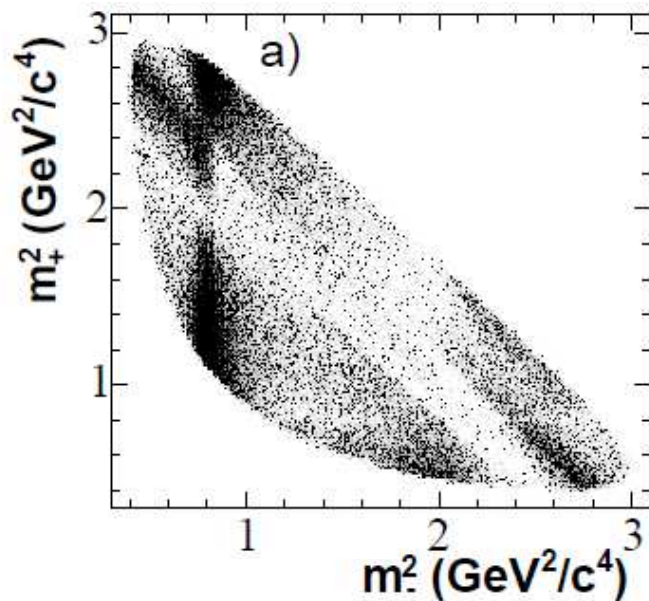
# Appropriate Fit Model

## Challenges

- Setup of the Amplitude
  - what is appropriate formalism?  
→ helicity, canonical, covariant tensor
- Educated guess of contributing resonances
  - can be a hard job – need to try many combinations
  - initial state might produce restrictions to final states or vice versa
- Appropriate choice of dynamic functions
  - myriads of Breit-Wigner like functions exist
  - complicated things like e.g. K-Matrix or Flatté approach taking into account coupled channels or thresholds

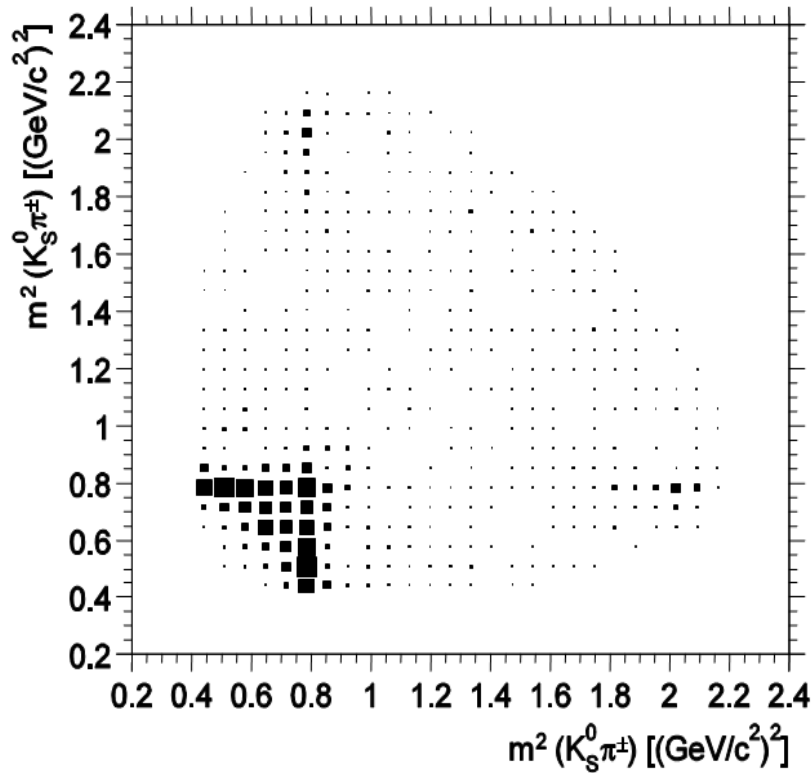


# Example: $D_s \rightarrow K_s \pi^+ \pi^-$



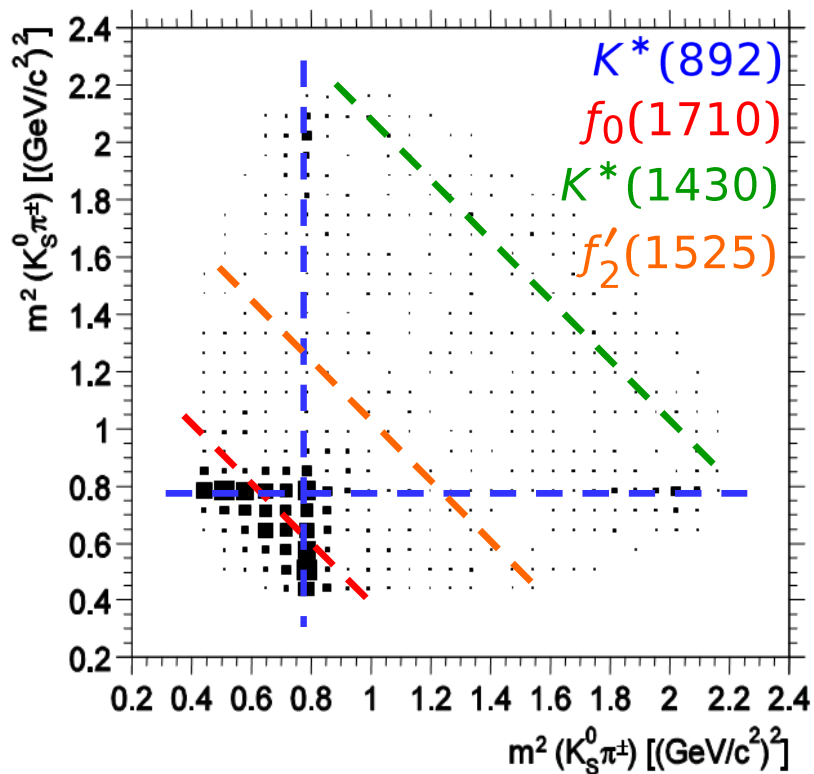
Resonance	Fit fraction
$K^*(892)^-$	0.586
$K_0^*(1430)^-$	0.083
$K_2^*(1430)^-$	0.027
$K^*(1410)^-$	0.004
$K^*(1680)^-$	0.003
$K^*(892)^+$	0.006
$K_0^*(1430)^+$	0.002
$K_2^*(1430)^+$	0.000
$\rho(770)$	0.224
$\omega(782)$	0.006
$f_0(980)$	0.061
$f_0(1370)$	0.032
$f_2(1270)$	0.030
$\rho(1450)$	0.002
$\sigma$	0.093
$\sigma'$	0.013
Non Resonant	0.073

# Fit Model Example: $D_s^\pm \rightarrow K_S K_S \pi^\pm$

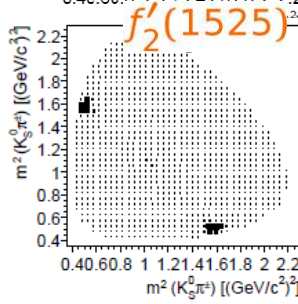
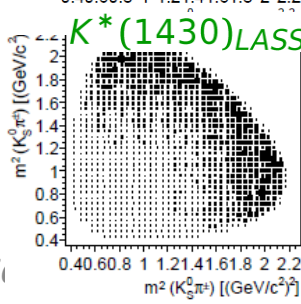
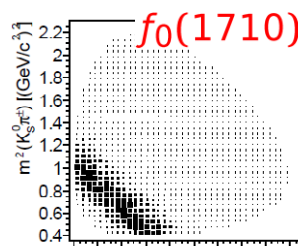
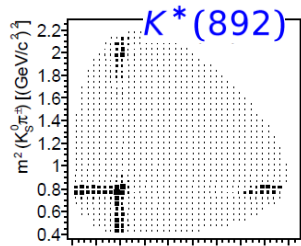


Hypothese	- ln L + 3037.8					Investigated hypotheses
	Massen und Breiten:					
	Alle frei (free-A)	Alle fixiert (fix-A)	Nur $f_J(17xx)$ frei (free-f)	Nur $f_J(17xx)$ $K^*(892)$ frei (free-fk)	Nur $f_J(17xx)$ $f_0(980)$ frei (free-ff)	
H-1	1845.3	2380.8	1845.3	1845.3	1845.3	$f_2(1710)$
H-2	827.5	1386.7	827.5	827.5	827.5	$f_0(1710)$
H-3	439.9	510.7	510.7	439.9	510.7	$K^*(892)$
H-4	281.1	422.6	395.3	281.1	395.3	$K^*(892) f_2(1710)$
H-5	—	330.5	311.1	231.9	189.9	$K^*(892) f_2(1710) f_{0,Flatte}(980)$
H-6	—	377.6	369.4	273.5	236.2	$K^*(892) f_2(1710) f_{0,BW}(980)$
H-7	—	416.2	400.6	272.8	400.6	$K^*(892) f_2(1710) f_2(1525)$
H-8	—	406.2	367.9	242.1	367.9	$K^*(892) f_2(1710) f_0(1500)$
H-9	58.8	202.2	175.3	139.7	175.3	$K^*(892) f_2(1710) K_{0,LASS}^*(1430)$
H-10	10.4	136.7	111.2	99.5	7.5	$K^*(892) f_2(1710) K_{0,LASS}^*(1430) f_{0,Flatte}(980)$
H-11	1.7	182.0	159.2	130.5	116.6	$K^*(892) f_2(1710) K_{0,LASS}^*(1430) f_{0,BW}(980)$
H-12	—	175.9	135.9	109.5	135.9	$K^*(892) f_2(1710) K_{0,LASS}^*(1430) f_2(1525)$
H-13	—	120.4	112.9	90.1	112.9	$K^*(892) f_2(1710) K_{0,LASS}^*(1430) f_0(1500)$
H-14	-14.5	52.9	0.0	-2.8	0.0	$K^*(892) f_2(1710) K_{0,LASS}^*(1430) f_0(1710)$
H-15	—	166.4	146.7	116.7	146.7	$K^*(892) f_2(1710) K_{0,LASS}^*(1430) K_2^*(1430)$
H-16	—	410.6	353.3	244.4	353.3	$K^*(892) f_2(1710) K_2^*(1430)$
H-17	—	199.9	186.4	115.4	186.4	$K^*(892) f_2(1710) K_{0,BW}^*(1430)$
H-18	71.3	237.7	88.3	71.3	88.3	$K^*(892) f_0(1710)$
H-19	—	142.6	41.4	40.5	34.1	$K^*(892) f_0(1710) f_{0,Flatte}(980)$
H-20	—	189.8	68.9	61.2	35.0	$K^*(892) f_0(1710) f_{0,BW}(980)$
H-21	—	203.5	75.9	57.0	75.9	$K^*(892) f_0(1710) f_2(1525)$
H-22	—	226.2	68.5	62.5	68.5	$K^*(892) f_0(1710) f_0(1500)$
H-23	68.0	186.9	69.3	68.0	69.3	$K^*(892) f_0(1710) f_2(1710)$
H-24	3.5	89.8	22.2	17.1	22.2	$K^*(892) f_0(1710) K_{0,LASS}^*(1430)$
H-25	-7.1	51.3	14.7	11.2	6.5	$K^*(892) f_0(1710) K_{0,LASS}^*(1430) f_{0,Flatte}(980)$
H-26	-3.8	79.7	21.6	15.6	4.2	$K^*(892) f_0(1710) K_{0,LASS}^*(1430) f_{0,BW}(980)$
H-27	—	79.7	4.0	1.1	4.0	$K^*(892) f_0(1710) K_{0,LASS}^*(1430) f_2(1525)$
H-28	—	60.6	21.4	16.3	21.4	$K^*(892) f_0(1710) K_{0,LASS}^*(1430) f_0(1500)$
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H-30	—	80.6	15.6	9.9	15.6	$K^*(892) f_0(1710) K_{0,LASS}^*(1430) K_2^*(1430)$
H-31	—	217.3	88.3	71.2	88.3	$K^*(892) f_0(1710) K_2^*(1430)$
H-32	15.8	90.8	37.6	33.3	37.6	$K^*(892) f_0(1710) K_{0,BW}^*(1430)$
H-33	-9.5	68.6	21.9	13.6	21.9	$K^*(892) f_0(1710) K_{0,BW}^*(1430) f_2(1710)$
H-34	83.4	267.7	267.7	215.5	267.7	$K^*(892) K_{0,LASS}^*(1430)$
H-35	—	508.0	508.0	438.8	508.0	$K^*(892) K_2^*(1430)$
H-36	—	297.7	297.7	232.6	297.7	$K^*(892) K_{0,BW}^*(1430)$

# Fit Model Example: $D_s^\pm \rightarrow K_S K_S \pi^\pm$



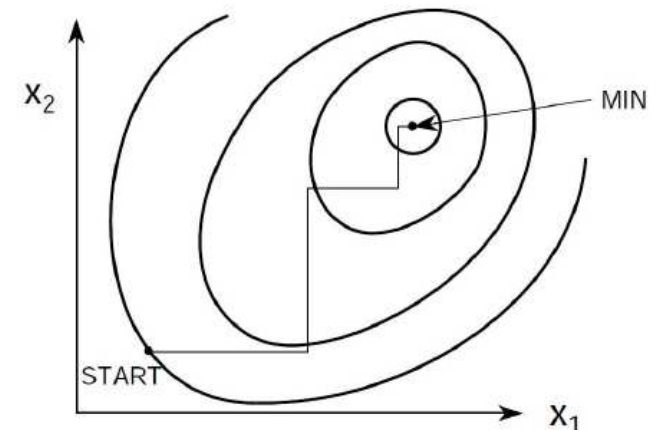
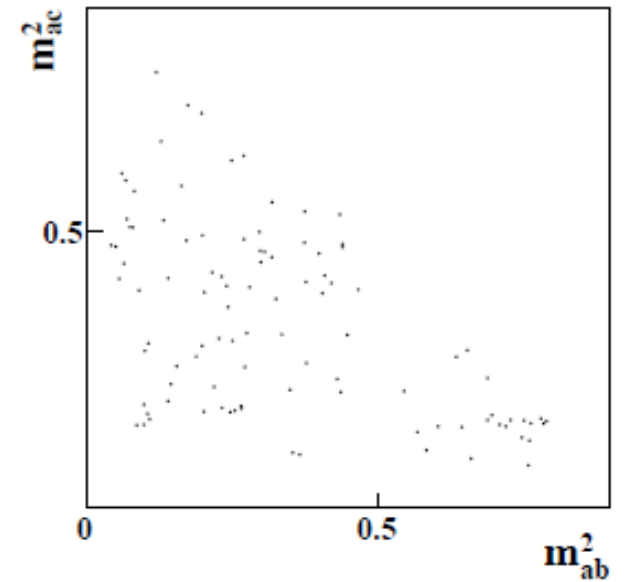
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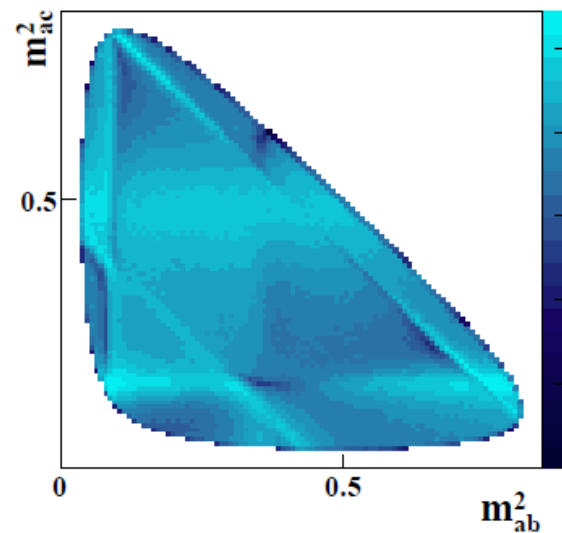
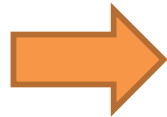
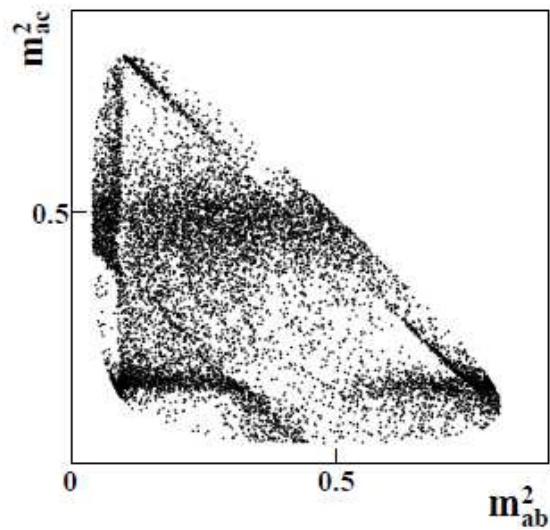
# Fit Model to Data

## Challenges

- Data
  - Low statistics
  - Inhomogeneous efficiency distribution
  - Finite resolution effects
    - how to treat shifts in phase space?
- Parameter space (typical  $>50$  parameters)
  - Problem: getting stuck in local extrema
  - How to achieve fast convergence?
- Goodness of fit
  - Significance of parameters
  - Sensitivity to noise effects
  - Sensitivity of model composition
- Demand in Computing
  - Many MC validation fits necessary



# Statistics and Goodness of Fit Validation



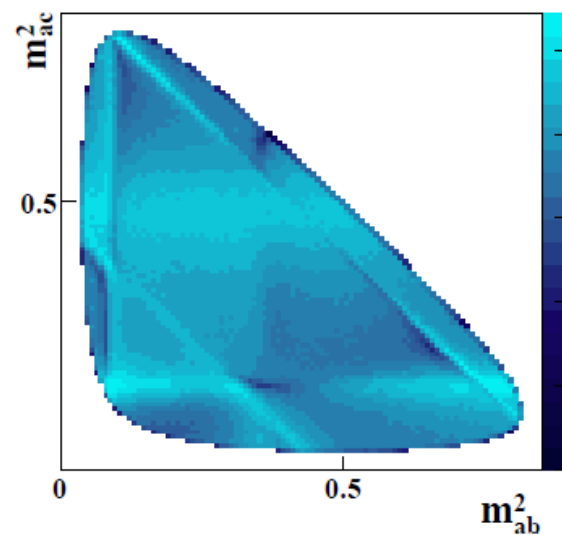
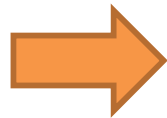
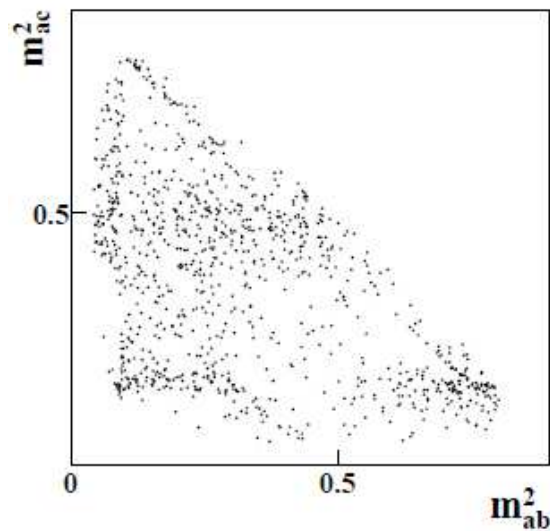
Daughters	$J^P$	Mass	Width	Fit Fraction
$a, b$	$0^+$	0.3	0.025	6%
$a, b$	$2^+$	0.6	0.05	2%
$a, c$	$1^-$	0.4	0.04	18%
$a, c$	$0^+$	0.7	0.1	43%
$b, c$	$1^-$	0.35	0.01	10%
$b, c$	$0^+$	0.75	0.02	17%
$a, b, c$	non-resonant			1%

## How reliable is the fit result?

- $\chi^2$  method for binned case inappropriate for low statistics.
- Reliable goodness-of-fit method for unbinned case?
- Need to do many validation fits on MC generated data  
→ Fluctuations in fit parameters tell about significance



# Statistics and Goodness of Fit Validation

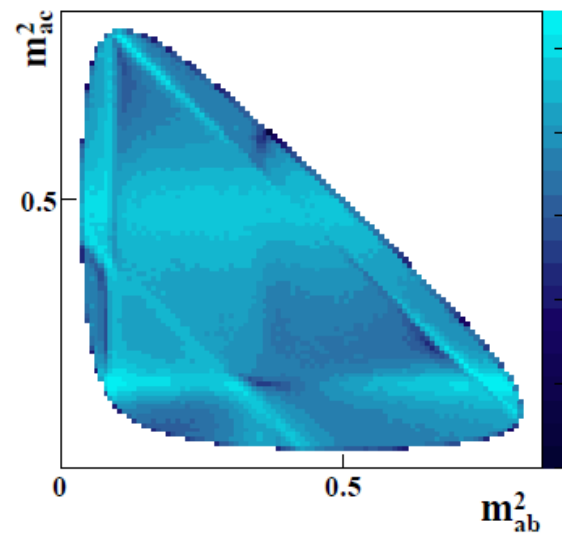
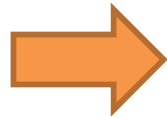
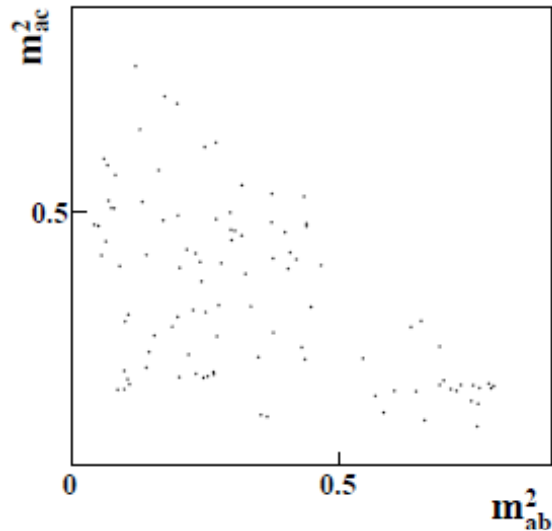


Daughters	$J^P$	Mass	Width	Fit Fraction
$a, b$	$0^+$	0.3	0.025	6%
$a, b$	$2^+$	0.6	0.05	2%
$a, c$	$1^-$	0.4	0.04	18%
$a, c$	$0^+$	0.7	0.1	43%
$b, c$	$1^-$	0.35	0.01	10%
$b, c$	$0^+$	0.75	0.02	17%
$a, b, c$	non-resonant			1%

## How reliable is the fit result?

- $\chi^2$  method for binned case inappropriate for low statistics.
- Reliable goodness-of-fit method for unbinned case?
- Need to do many validation fits on MC generated data
  - Fluctuations in fit parameters tell about significance

# Statistics and Goodness of Fit Validation

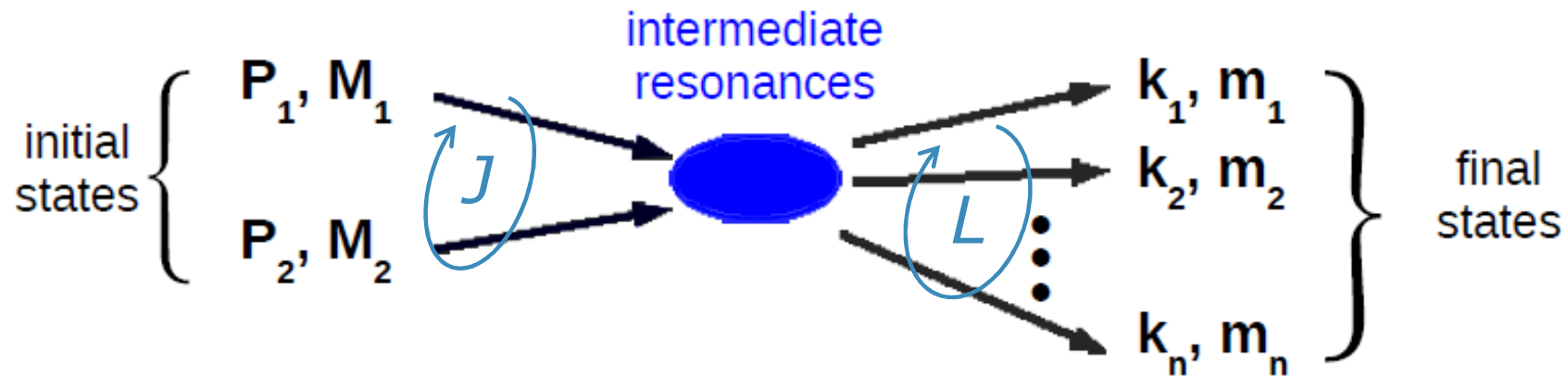


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  - Fluctuations in fit parameters tell about significance

# PWA Challenges for PANDA



- $L_{max}$  depends on available phase space  $p_{cms}$
- LEAR @ 1.94 GeV/c:  $p_{cms, \bar{p}p} \approx 1 \text{ GeV}/c$   
 $\rightarrow L_{max} \approx p_{cms}/200 \text{ MeV}/c \approx 5$
- HESR @ 15 GeV/c:  $L_{max} \approx 13$  for  $\bar{p}p$   
 $L_{max} \approx 10$  for  $D^* \bar{D}^*$   
 $L_{max} \approx 5$  for  $\tilde{\eta}_{c1} \eta$
- High angular momenta
  - $\rightarrow$  many waves can contribute
  - $\rightarrow$  dramatic increase of number of fit parameters!

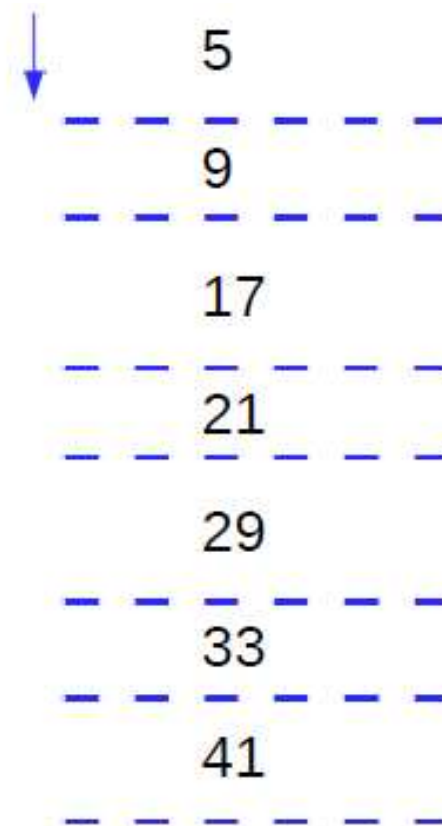
# Example channel: $\bar{p}p \rightarrow \omega\pi^0$

- Example analysis: Highest initial  $J^{PC}$  in channel

$$\bar{p}p \rightarrow \omega\pi^0, \omega \rightarrow \pi^0\gamma$$

$J^{PC}$	$\lambda$ ( $\bar{p}p$ )	$L(\omega\pi^0)$
$1^-$	-1, 0, +1	1
$1^+$	0	0, 2
$2^-$	-1, +1	1, 3
$3^-$	-1, 0, +1	3
$3^+$	0	2, 4
$4^-$	-1, +1	3, 5
$5^-$	-1, 0, +1	5
$5^+$	0	4, 6
$6^-$	-1, +1	5, 7
$7^-$	-1, 0, +1	7
$7^+$	0	6, 8

*Number of parameters increases very quickly!*



# PWA Challenges for PANDA

- Number of final state particles @ PANDA

e.g.  $\bar{p}p \rightarrow D^{*+}D^{*-} \rightarrow D^0\pi^+\bar{D}^0\pi^- \rightarrow 2K^\pm 8\pi^\pm$

has 10 particles in final state



Need reliable reco.  
at high multiplicities

- Statistics @ PANDA

Channels of interest have low cross-section (pb ... nb), and low branching ratios involved

*Example:* Charmed hybrid candidate  $\tilde{\eta}_{c1}$  in

$$\bar{p}p \rightarrow \tilde{\eta}_{c1}\eta \rightarrow DD^*\eta \rightarrow 2K^\pm 2\pi^\pm 8\gamma$$

– Estimate:  $\sigma \cdot BR \approx 0.06$  pb

$$\Rightarrow \dot{N} < 1 \text{ event/day}$$



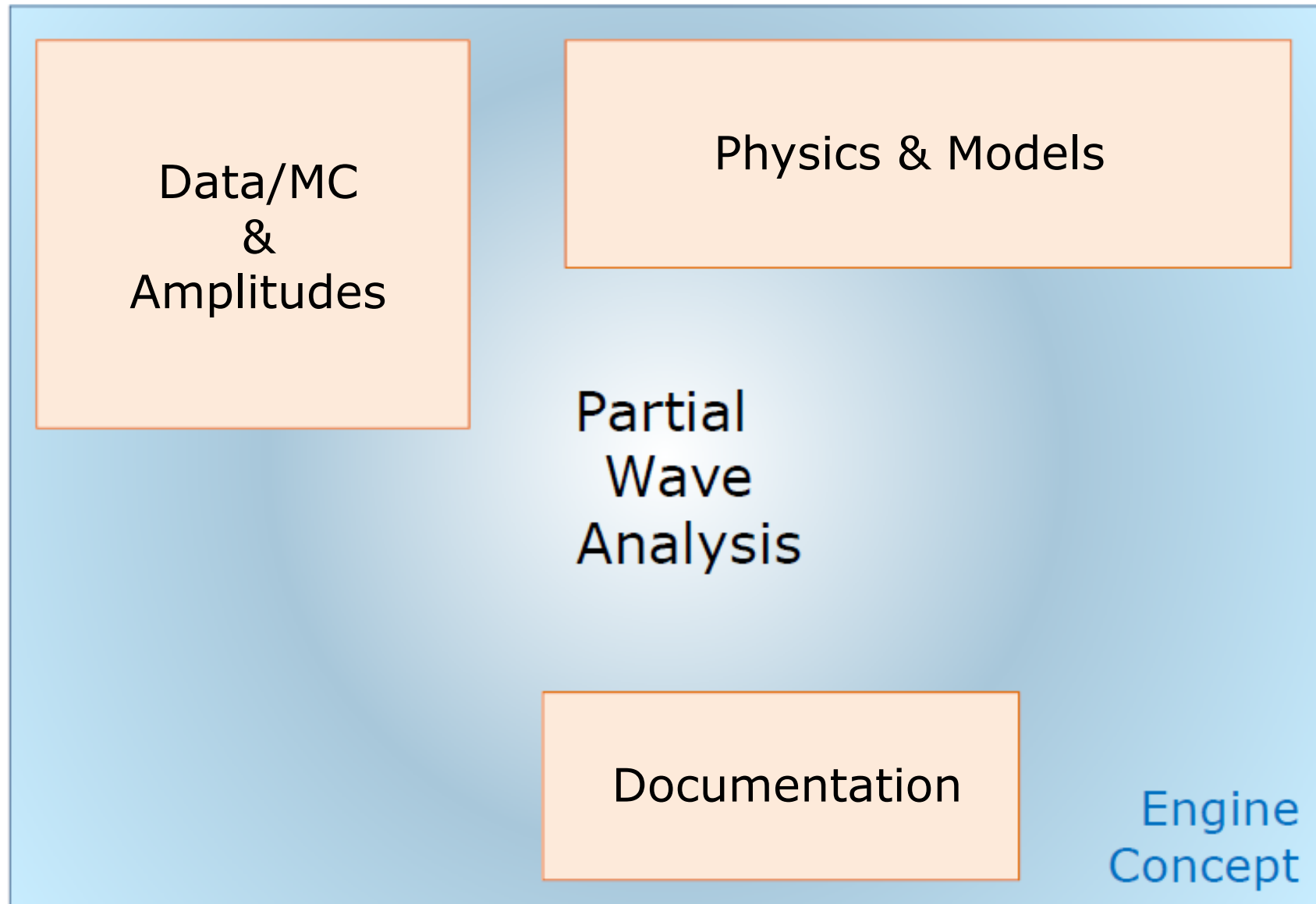
Need sensitivity also  
with low statistics

# Partial Wave Analysis Software Package

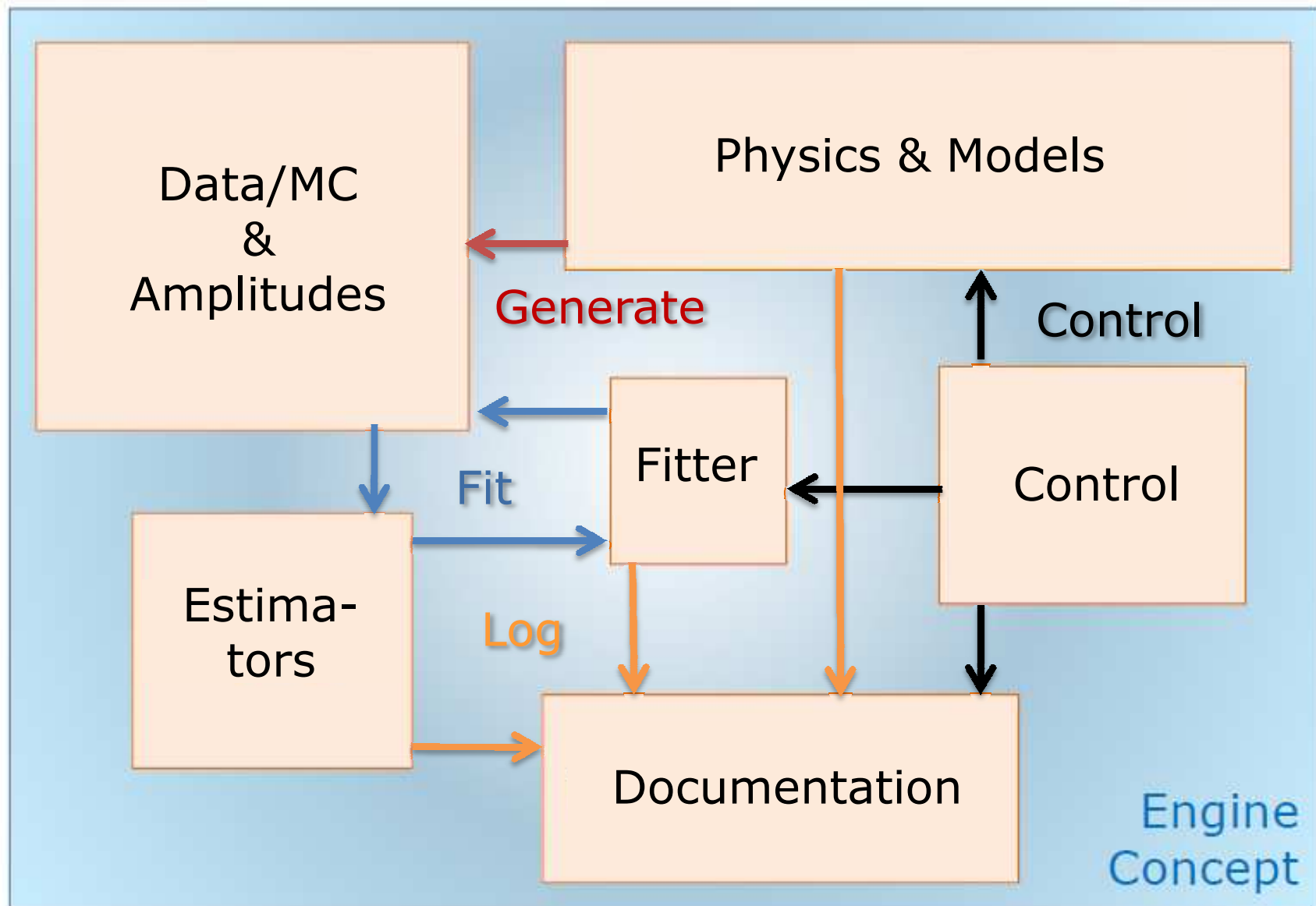
## *Wish list for Software*

- Experiment independent (as far as possible)
- Modular design
  - Generators, fitters, dynamic function lib., estimators
- Simultaneous treatment of multiple datasets
  - Coupled channel analysis
  - Simultaneous treatment of data from different experiments
- Performant algorithms
  - Parallel (GPU/CPU)
  - Caching techniques
- Automatic documentation
  - Histograms, fit hypothesis etc.

# PWA-Framework Concept

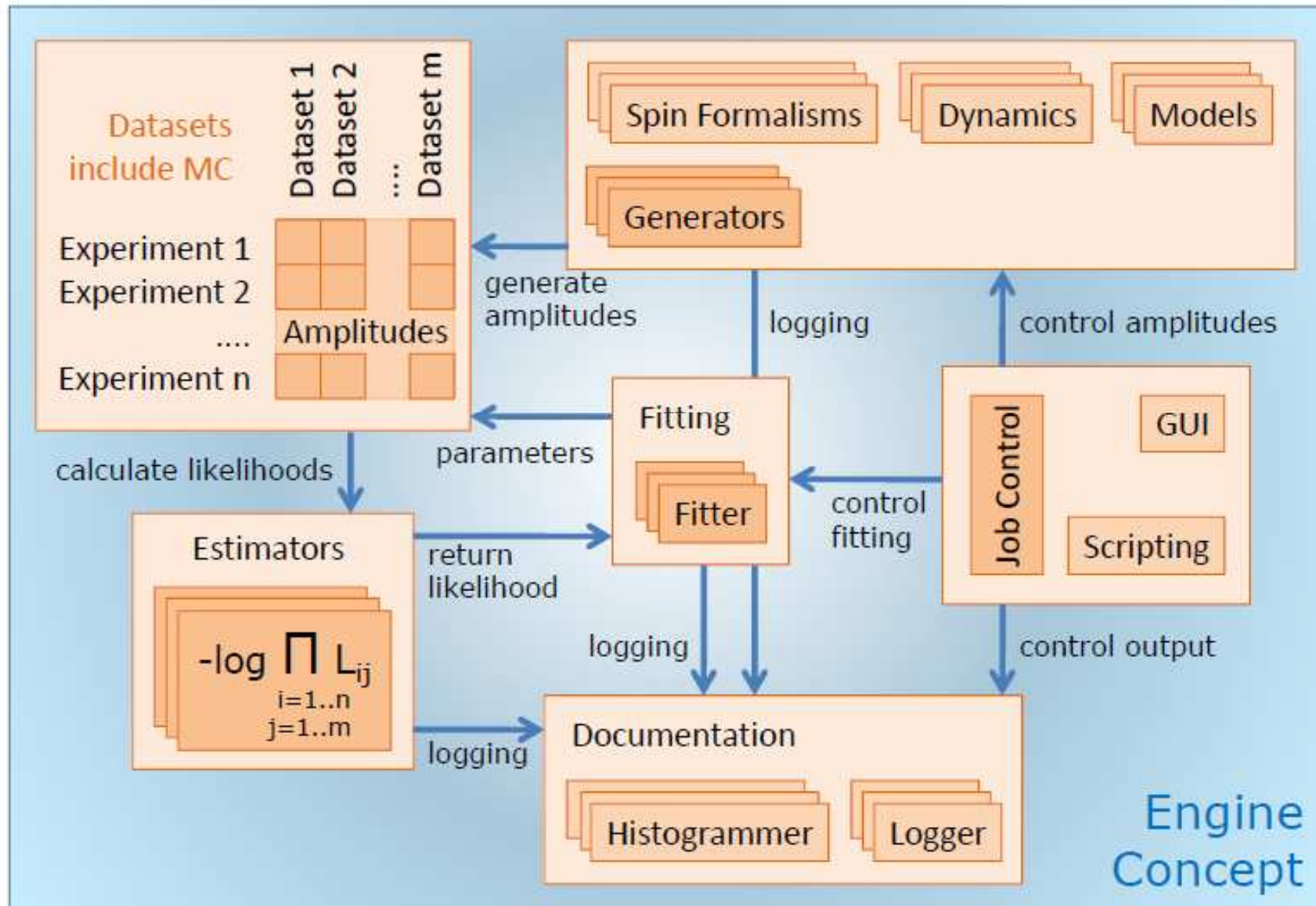


# PWA-Framework Concept





# PWA-Framework Concept



# Status of the Software

Software Project has been initiated by PANDA groups from Bochum, GSI and Mainz

- Computation of Amplitudes & Intensities
  - qft++ package (Quantum Field Theory in C++)
- Minimization
  - MINUIT2 (gradient descent)
  - GenEvA (genetic & evolutionary algorithms)
- Miscellaneous Tools
  - Particle Database
  - Data reader interface
- Wiki Page for Documentation

*... and a bit analysis (BES3 data)*

# qft++ Package

- qft++ = Numerical Object Oriented Quantum Field Theory (by Mike Williams, Carnegie Mellon Univ.)
- Calculation of the matrices, tensors, spinors, angular momentum tensors etc. with C++ classes

qft++ Class	Symbol	Concept
Matrix<T>	$a_{ij}$	matrices of any dimension
Tensor<T>	$x_\mu$	tensors of any rank
MetricTensor	$g_{\mu\nu}$	Minkowski metric
LeviCivitaTensor	$\epsilon_{\mu\nu\alpha\beta}$	totally anti-symmetric Levi-Civita tensor
DiracSpinor	$u_{\mu_1 \dots \mu_{J-1/2}}(p, m)$	half-integral spin wave functions
DiracAntiSpinor	$v(p, m)$	spin-1/2 anti-particle wave functions
DiracGamma	$\gamma^\mu$	Dirac matrices
DiracGamma5	$\gamma^5$	
DiracSigma	$\sigma^{\mu\nu}$	
PolVector	$\epsilon_{\mu_1 \dots \mu_J}(p, m)$	integral spin wave functions
OrbitalTensor	$L_{\mu_1 \dots \mu_\ell}^{(\ell)}$	orbital angular momentum tensors

# qft++ Package

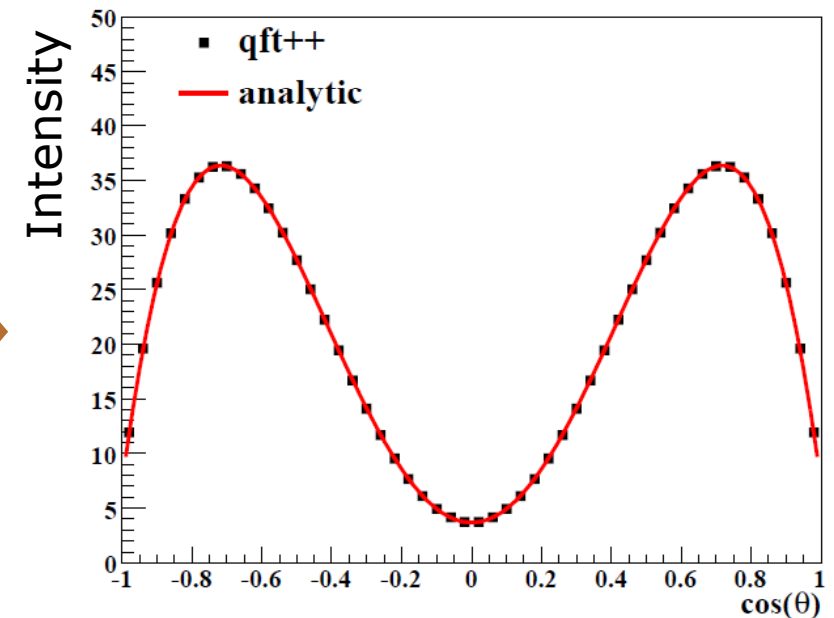
- *Example:*  $X(2^-) \rightarrow \omega K \rightarrow \pi^+ \pi^- \pi^0 K$
- Amplitude and Intensity given by

$$A \propto \epsilon_{\mu}^*(p_{\omega}, m_{\omega}) L^{(3)\mu\nu\alpha}(p_{\omega K}) \epsilon_{\nu\alpha}(P, M) \quad \text{and} \quad \mathcal{I} \propto \sum_{M=\pm 1} \sum_{m_{\omega}=\pm 1,0} |A|^2$$

- **qft++: Declaration and Calculation**

```
PolVector epso; // omega
PolVector epsx(2); // X
OrbitalTensor orb3(3); // L^3
Tensor<complex<double>> amp;
Vector4<double> p4o,p4k,p4x;

double intensity = 0.;
for(Spin m = -1; m <= 1; m+=2){
    for(Spin mo = -1; mo <= 1; mo++){
        amp = conj(epso(mo))*orb3|epsx(m);
        intensity += norm(amp());
    }
}
```



Angular distribution of  
 $X \rightarrow \omega K$

# qft++ Package

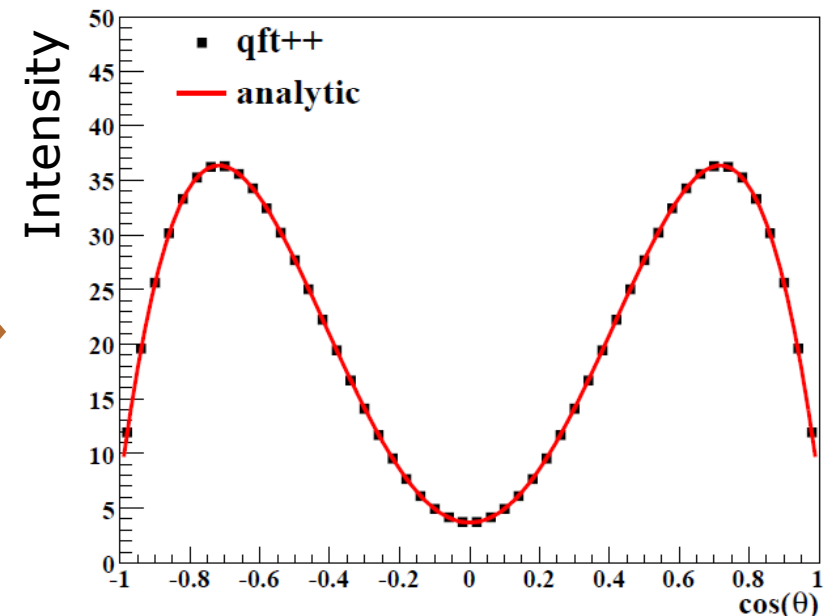
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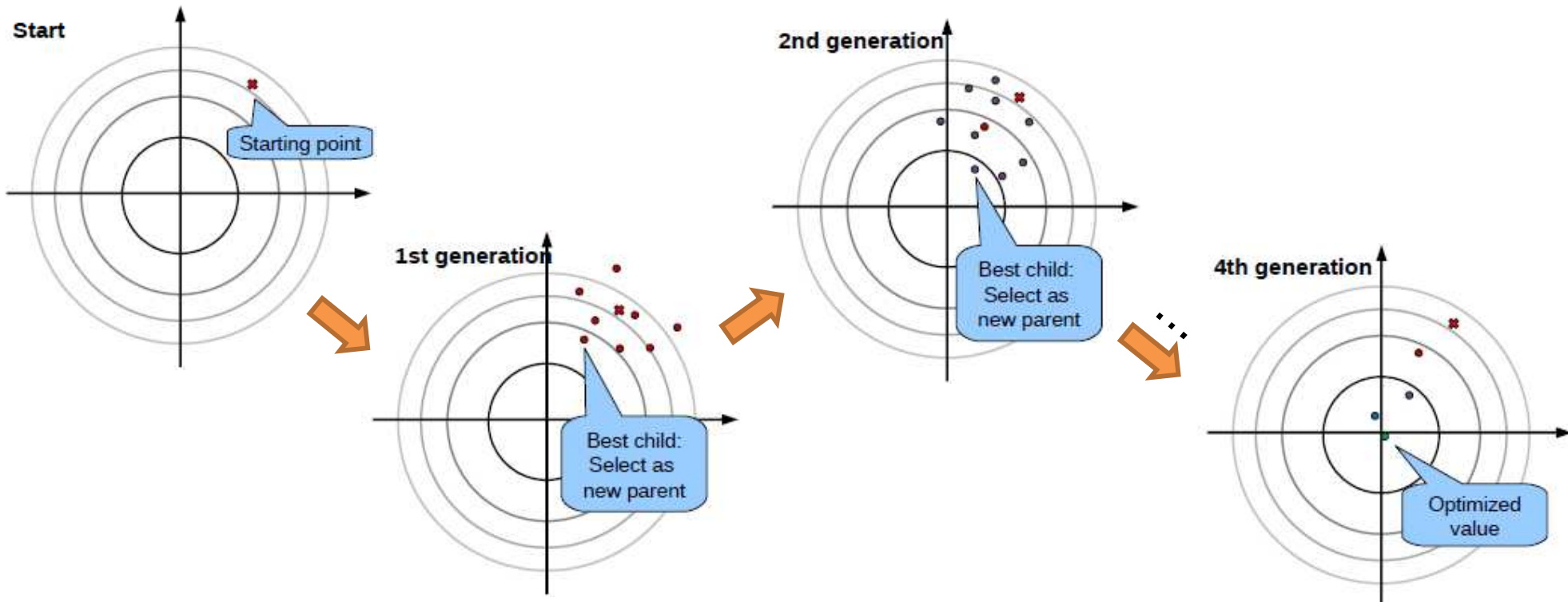
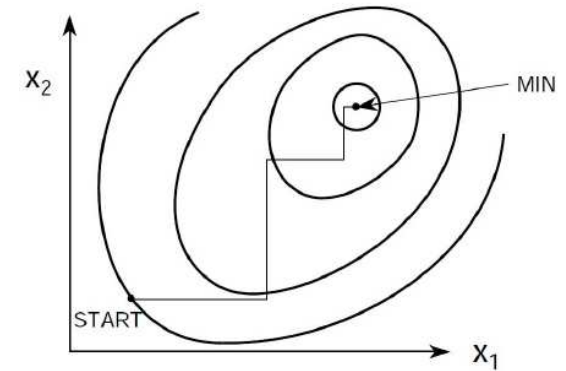
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    intensity += norm(amp());
  }
}
```



Angular distribution of  
 $X \rightarrow \omega K$

# Minimization

- MINUIT2 = classical gradient descent
- Sometimes gets stuck in local minima
- Alternative: Evolutionary Strategy (GenEvA)  
→ new solutions created from previous ones (offspring)

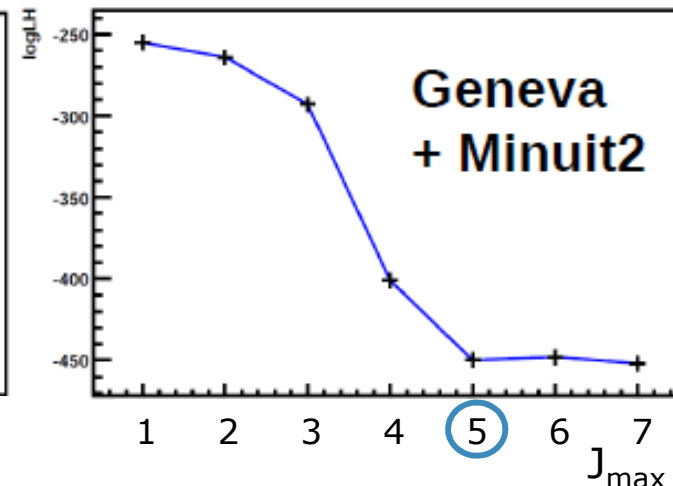
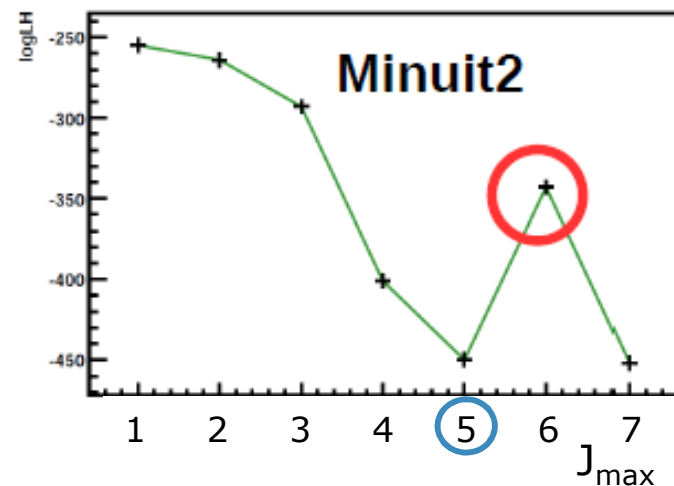
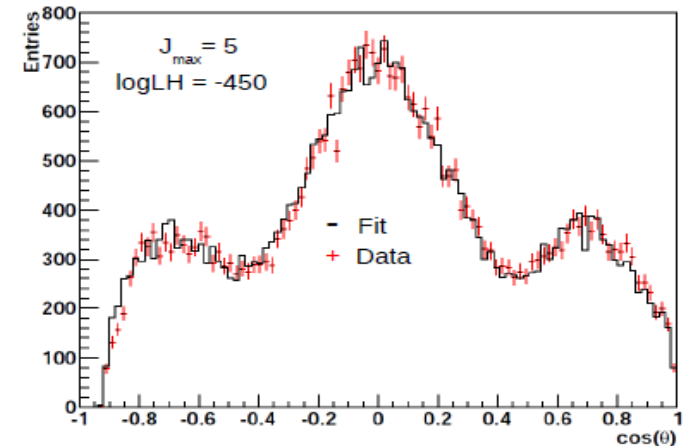
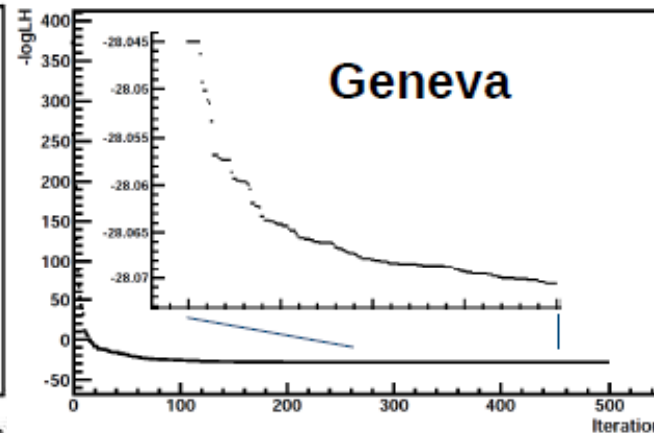
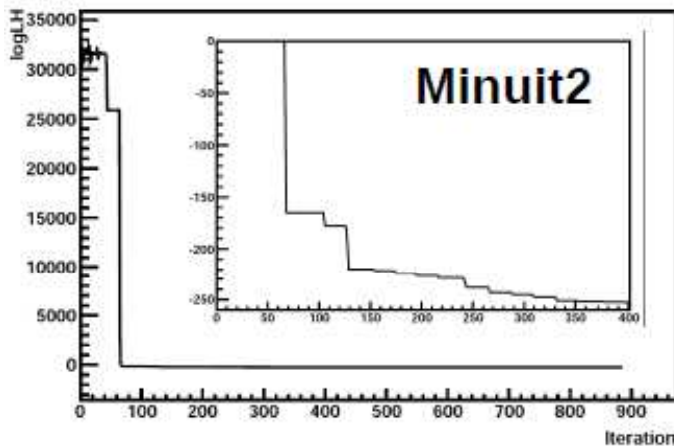


# GenEvA Example

Example: Angular distribution + maximum spin of  $\bar{p}p \rightarrow \omega\pi^0$ ,  $\omega \rightarrow \pi^0\gamma$  @ 1940 MeV/c (LEAR data)

Convergence behaviour of minimizing  $\log(\text{LH})$

Result:  $J_{\max} = 5$



Less probability to get stuck in local minima!

# Documentation – PWA Wiki Page



Jump:

PWA

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[PWA WebHome](#) r1.6 - 21 Apr 2010 - 17:13 - [KlausGoetzen](#) [topic\\_end](#)

## PANDA PWA

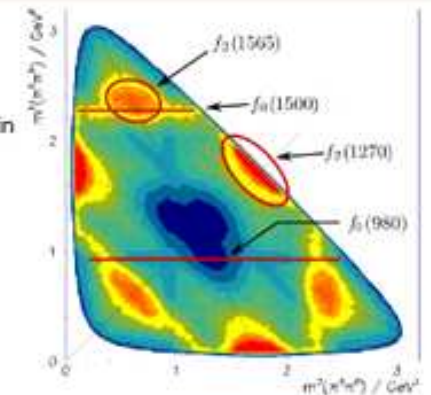
- + PANDA PWA
  - ↓ Introduction
  - ↓ Contact
  - ↓ Physics Resources
  - ↓ Software Documentation
  - ↓ Meetings
  - ↓ Site Tools of the PWA Web

### Introduction

In the 90s of the last century high statistics experiments with fully equipped 4n detectors have lead to a better insight in the spectrum of hadrons.

In particular the finding of crypto-exotic and JPC exotic states tremendously improved the experimental situation in meson spectroscopy. All this was possible only with sophisticated analysis methods like the decomposition of measured phase-space distribution into partial waves and to express the partial waves in terms of complicated dynamical functions.

This Wiki page represents the documentation of the PANDA Partial Wave Analysis software project aiming to provide a flexible and experiment independent framework for performing all kinds of amplitude analysis.



### Contact

The contact person for the Panda PWA software project is [Bertram Kopf](#), Ruhr-Universität Bochum.

#### PWA Web

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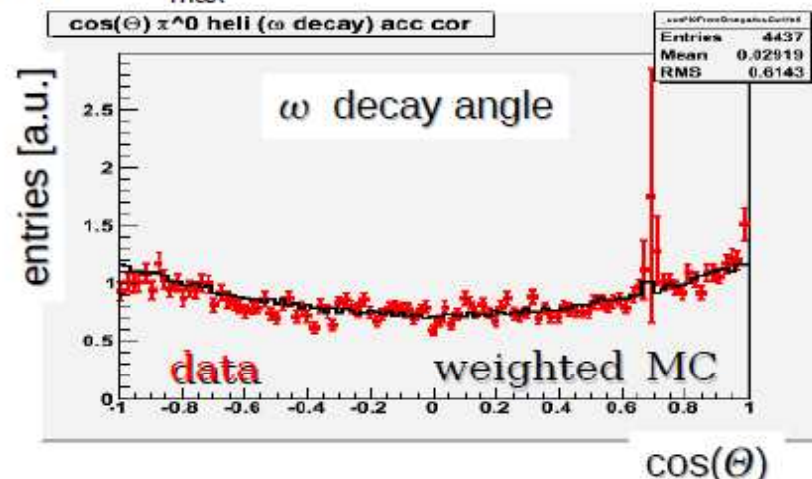
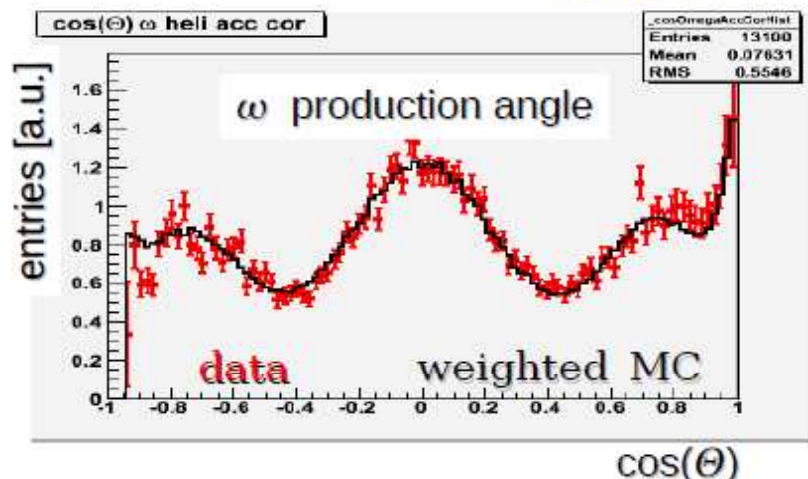


# Crystal Barrel Data: $\bar{p}p \rightarrow \omega\pi^0$

- Highest J in channel  $\bar{p}p \rightarrow \omega\pi^0$ ,  $\omega \rightarrow \pi^0\gamma$  at various energies

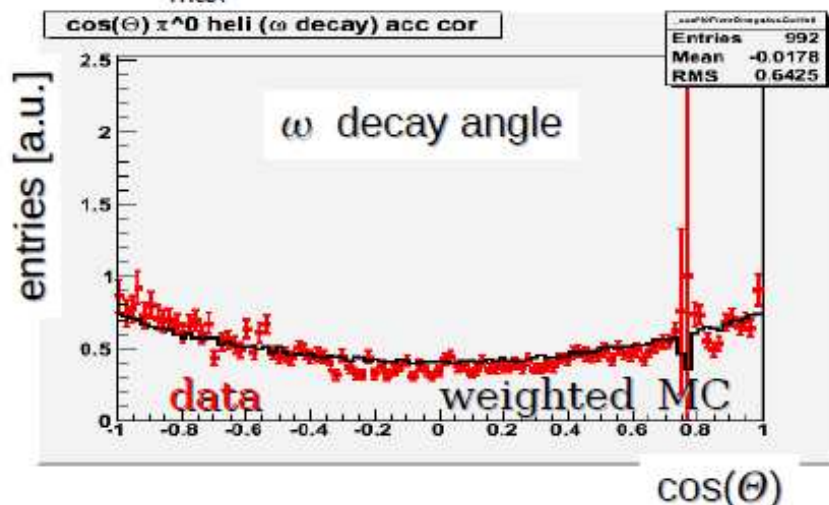
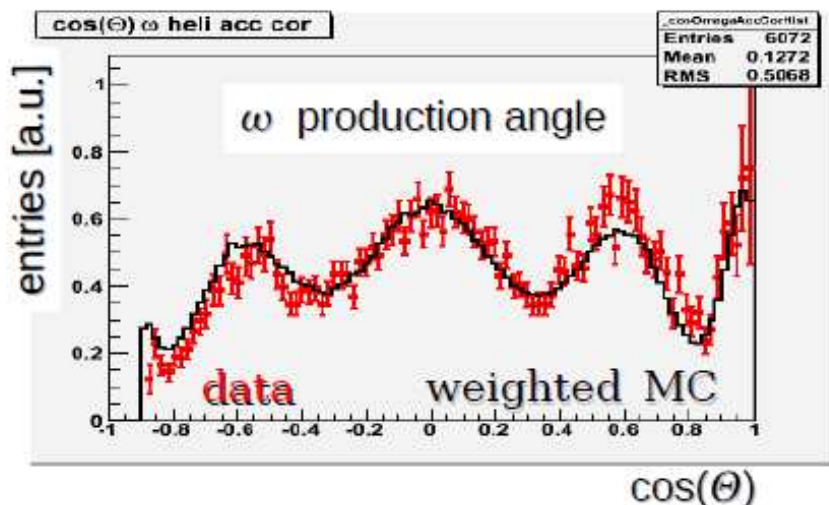
1200 MeV/c

Angular Distribution for  $J_{\max} = 4$



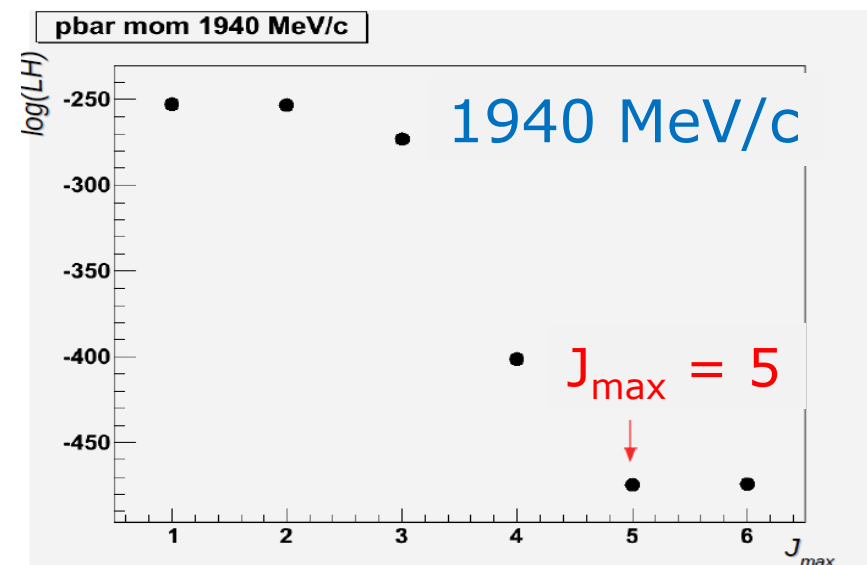
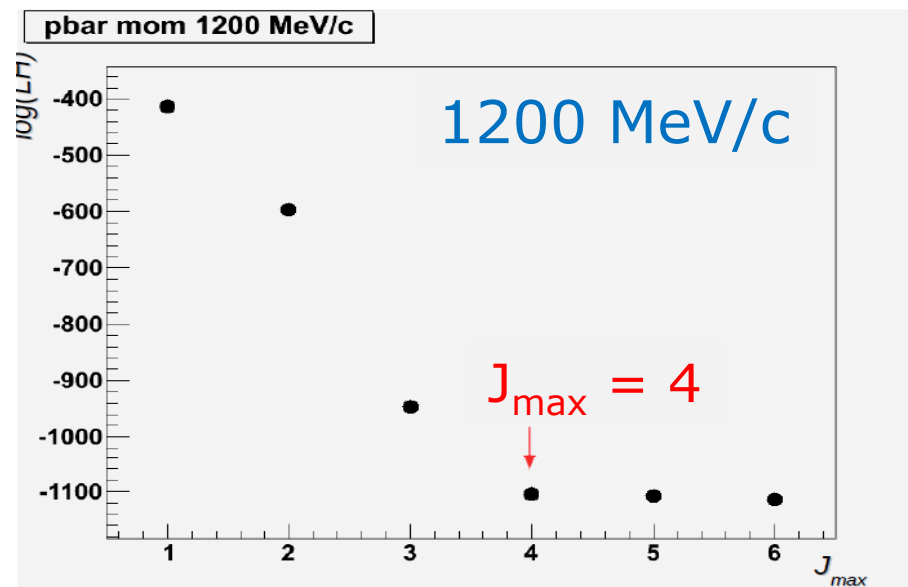
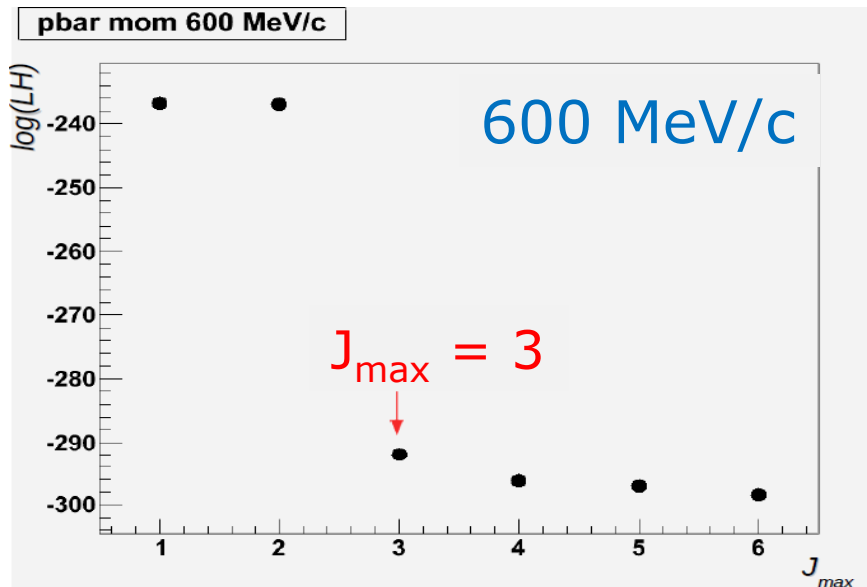
1940 MeV/c

Angular Distribution for  $J_{\max} = 5$



# Crystal Barrel Data: $\bar{p}p \rightarrow \omega\pi^0$

- Highest  $J$  in channel  $\bar{p}p \rightarrow \omega\pi^0$ ,  $\omega \rightarrow \pi^0\gamma$  at various  $p_{\text{beam}}$



$J_{\text{max}} = 3$  @ 600, 900 MeV/c

$J_{\text{max}} = 4$  @ 1050, 1200, 1350 MeV/c

$J_{\text{max}} = 5$  @ 1525, 1642, 1800, 1940 MeV/c

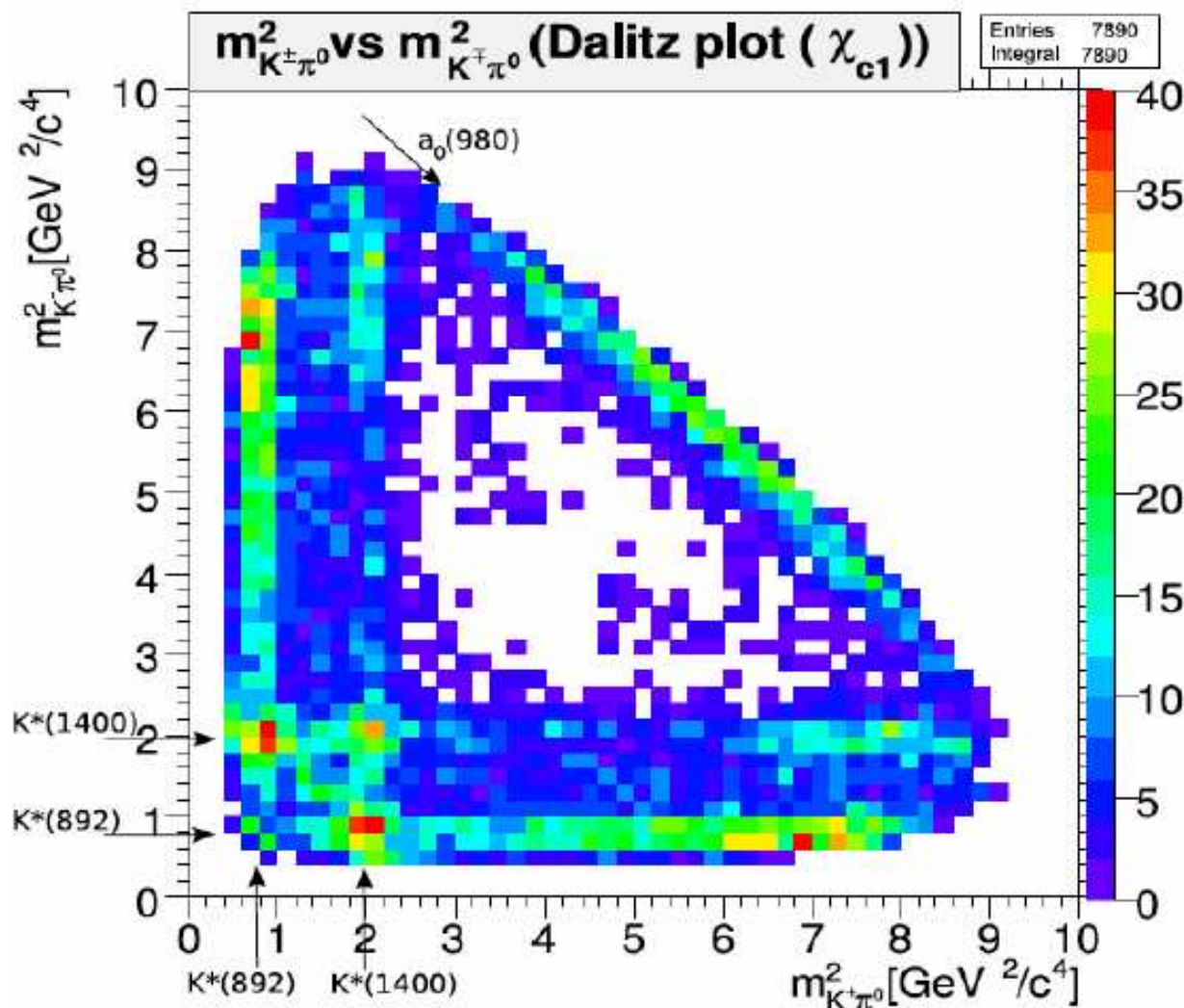
$$J_{\text{max}} \approx p_{\text{cms}} / (171 \pm 24) \text{ MeV/c}$$

Studies concerning spin-density matrix are ongoing.

# BES3 Analysis: $\psi(2S) \rightarrow \chi_{c1} \gamma \rightarrow (K^+ K^- \pi^0) \gamma$

**PRELIMINARY**

(Patrick Friedel, Bochum)



- Visible contributions

$$K^{*\pm}(892) \rightarrow K^\pm \pi^0$$

$$K_J^{*\pm}(1430) \rightarrow K^\pm \pi^0$$

$$a_0(980) \rightarrow K^+ K^-$$



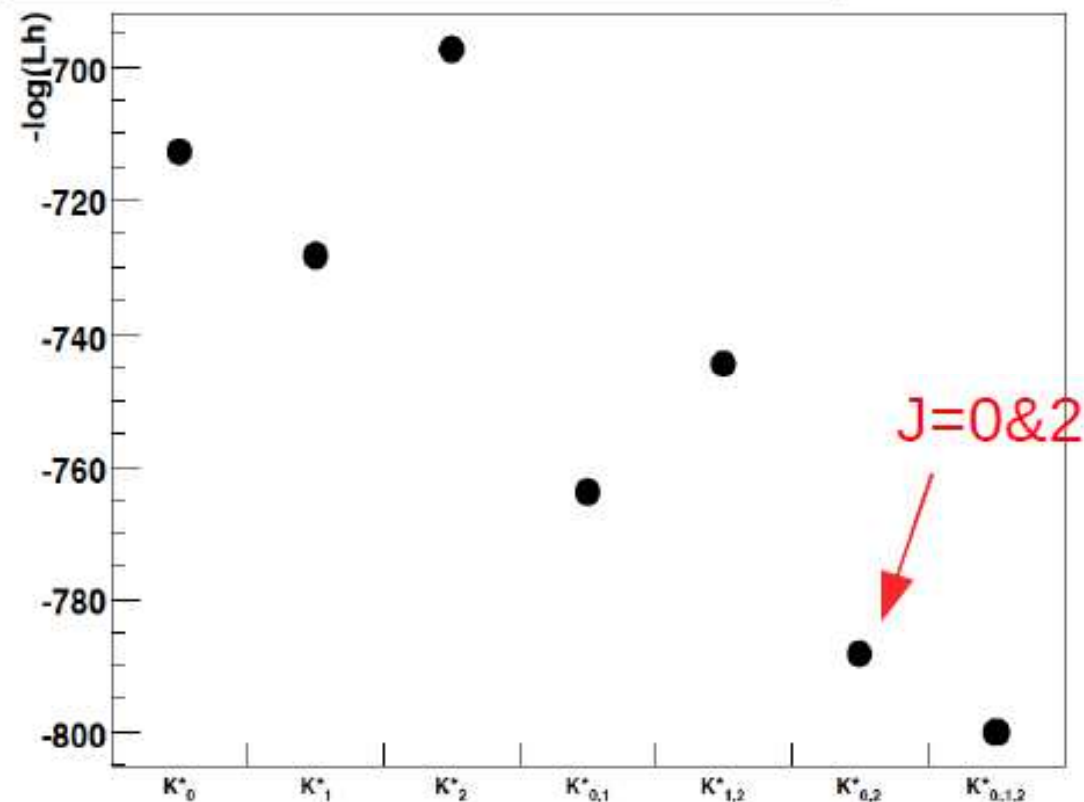
Determine J of  $K^*$

# BES3 Analysis: $\psi(2S) \rightarrow \chi_{c1} \gamma \rightarrow (K^+ K^- \pi^0) \gamma$

**PRELIMINARY**

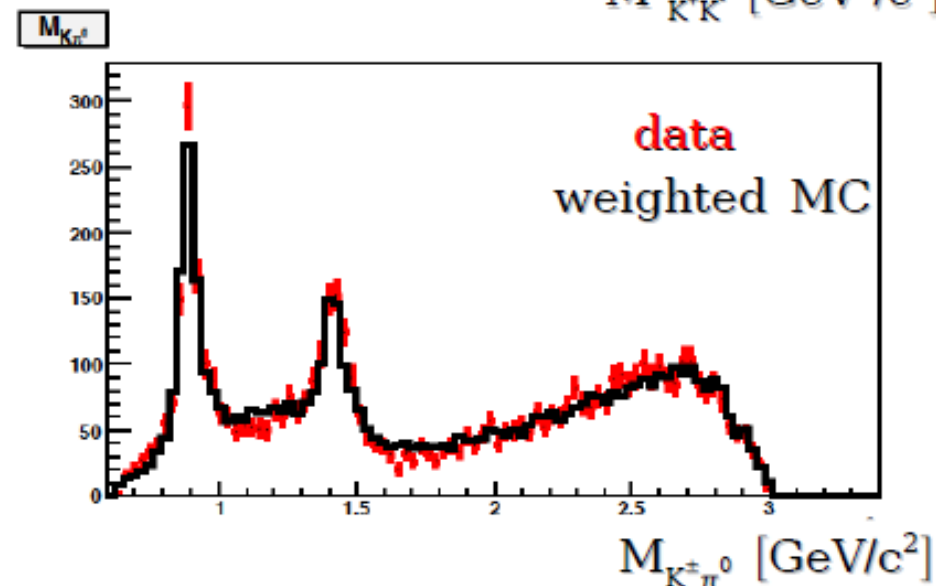
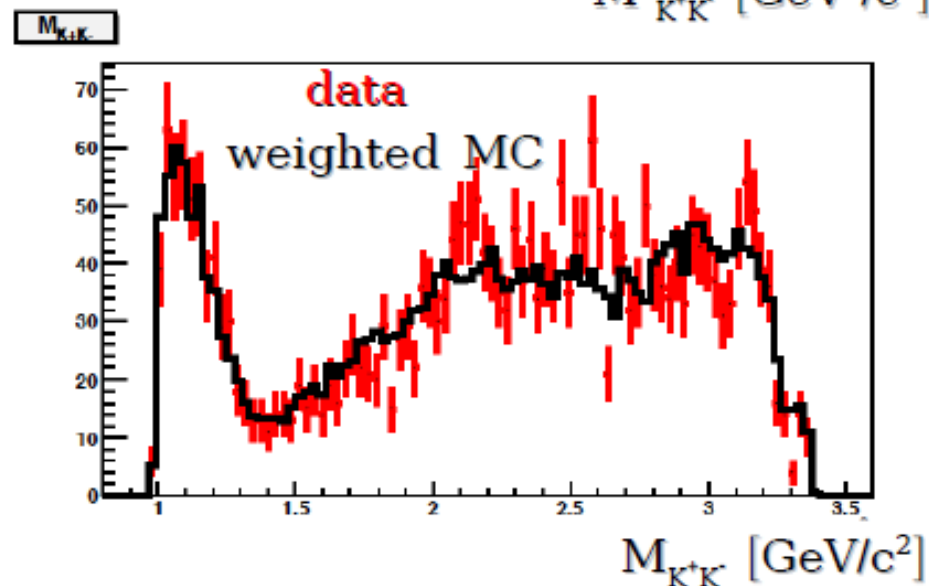
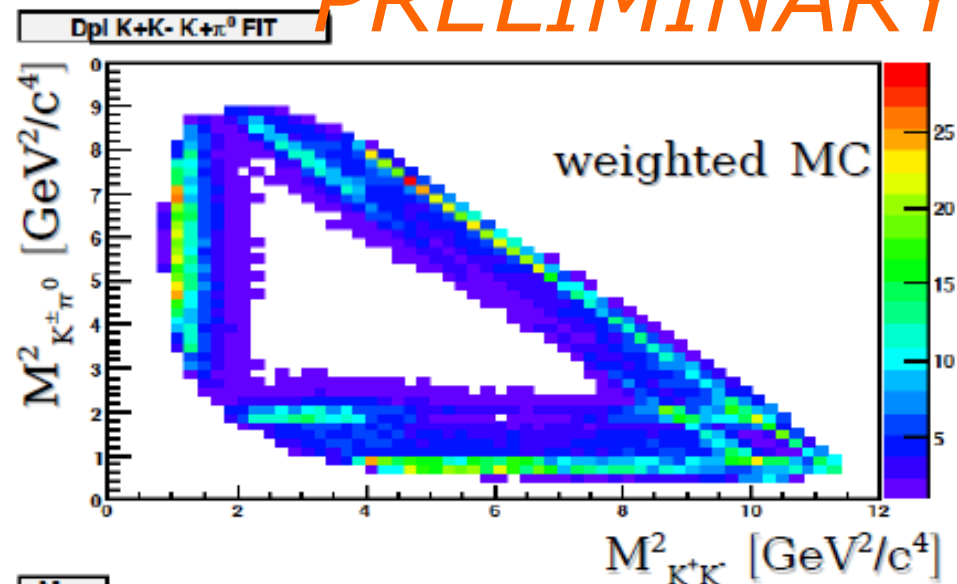
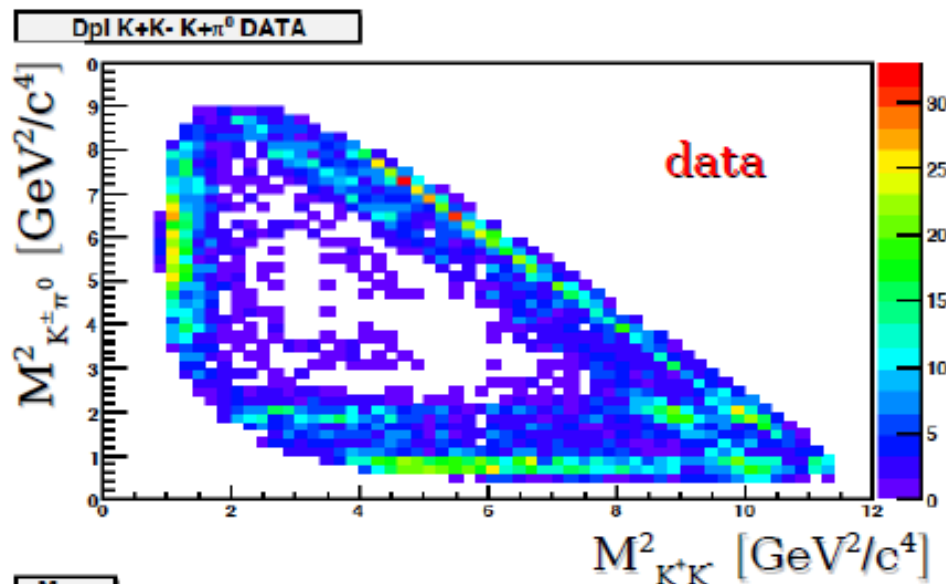
- 2<sup>nd</sup> step: PWA with four resonances
  - $a_0(980)\pi^0$
  - $K^{*\pm}(892)K^\mp$
  - two resonances for  $K_J^{*\pm}(1430)K^\mp$  with  $J=0&1, 0&2, 1&2$
- Best result with combination **J=0&2**
  - $K_0^{*\pm}(1430)K^\mp$
  - $K_2^{*\pm}(1430)K^\mp$
- No significant improvement for combination **J=0,1&2**

logLh der verschiedenen  $K_J^*(1430)$  Hypothesen



# BES3 Analysis: $\psi(2S) \rightarrow \chi_{c1} \gamma \rightarrow (K^+ K^- \pi^0) \gamma$

**PRELIMINARY**



# Summary

- Versatile **Partial Wave Analysis Software** mandatory for Hadron Spectroscopy @ PANDA
- **Many challenges** – experimental, mathematical, computational have to be faced
- **Highly Modular Software Concept** for a generalized software package
- Software project has **successfully been initiated** within PANDA Collaboration

# BACKUP

# PANDA Physics Programme

- **Charmonium/Open Charm Physics**
  - Precise Spectroscopy
  - Investigation of Confinement Potential
  - X, Y, Z, D<sub>SJ</sub> States up to 5.5 GeV
  - D-Mixing & CP-Violation
- **Exotic Matter**
  - Search for Glueballs and Hybrids
  - Spectroscopy of light Mesons
- **Hadrons in Media**
  - In-medium Modification of Hadrons
- **Nucleon-Structure**
  - Generalized Parton Distribution
  - Timelike Form Factor of the Proton
  - Drell-Yan Processes
- **Hypernuclear Physics**

