Rescattering effects in meson decays

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Hirschegg, January 19th 2011

Schneider, BK, Ditsche, JHEP 2011

Schneider, BK, work in progress

Rescattering effects in meson decays – Outline

Rescattering effects in 3-meson decays – Outline

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Introduction

Perturbative rescattering: $\boldsymbol{\eta \to 3\pi}$ decays (1)

- Understanding the $\eta \to 3\pi^0$ Dalitz plot parameter α
- Relating charged and neutral Dalitz plot parameters

Non-perturbative rescattering: $\boldsymbol{\eta \to 3\pi}$ decays (2)

- Dispersion relations for three-meson decays
- Transfer to other decay channels

Summary / Future wonders

Introduction

Various reasons why final-state interactions are important

- if rescattering strong, significantly enhances decay probabilities

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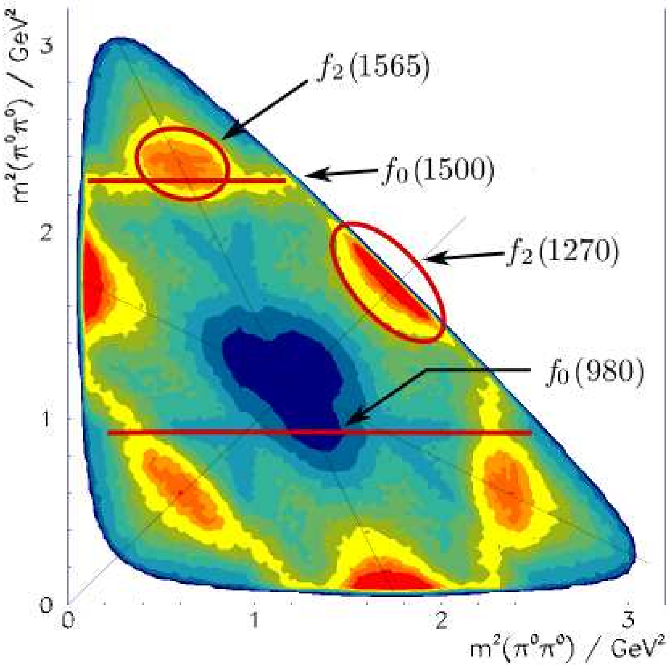

Introduction

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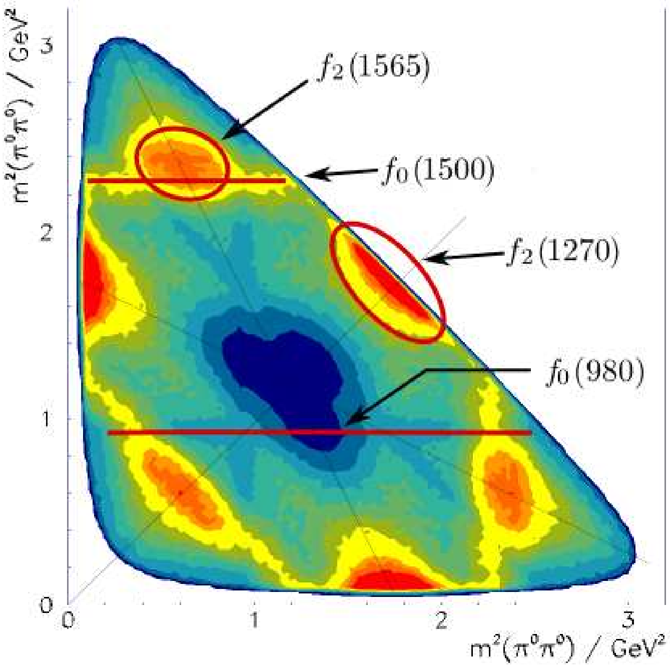
- introduce phases/imaginary parts; need hadronic phases to extract weak (CP-violating) phases from asymmetries



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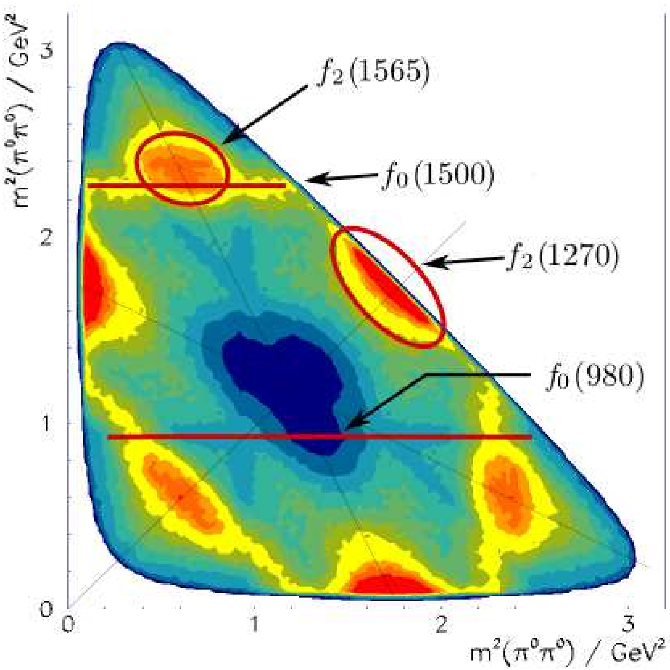
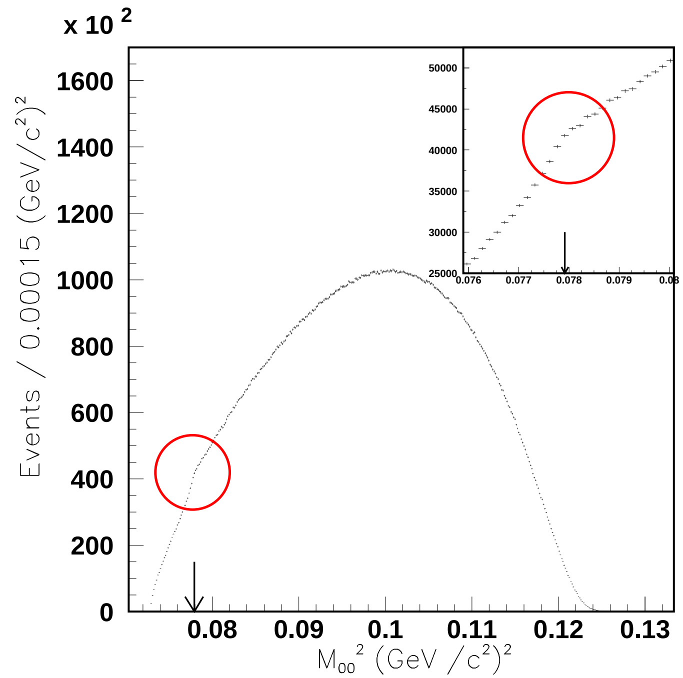
- introduce phases/imaginary parts; need hadronic phases to extract weak (CP-violating) phases from asymmetries
- new analytic features in the Dalitz plot (cusp effect in $K \to 3\pi$)





Enhancement through rescattering: $\eta \longrightarrow 3\pi$ decays

- $\eta \to 3\pi$ isospin violating; two sources in the Standard Model:

$$m_u \neq m_d \qquad\qquad e^2 \neq 0$$

- electromagnetic contribution small

Sutherland 1967

Baur, Kambor, Wyler 1996; Ditsche, BK, Meißner 2009

$$\mathcal{A}_c^{\mathsf{LO}}(s,t,u) \;=\; \frac{B(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left\{1 + \frac{3(s-s_0)}{M_\eta^2 - M_\pi^2}\right\}$$

$$s = (p_\eta - p_{\pi^0})^2,\; t = (p_\eta - p_{\pi^+})^2,\; u = (p_\eta - p_{\pi^-})^2,\; s+t+u = M_\eta^2 + 3M_\pi^2 \doteq 3s_0$$

- $\Delta I = 1$ relation between charged and neutral decay amplitudes:

$$\mathcal{A}_n(s,t,u) \;=\; \mathcal{A}_c(s,t,u) + \mathcal{A}_c(t,u,s) + \mathcal{A}_c(u,s,t)$$

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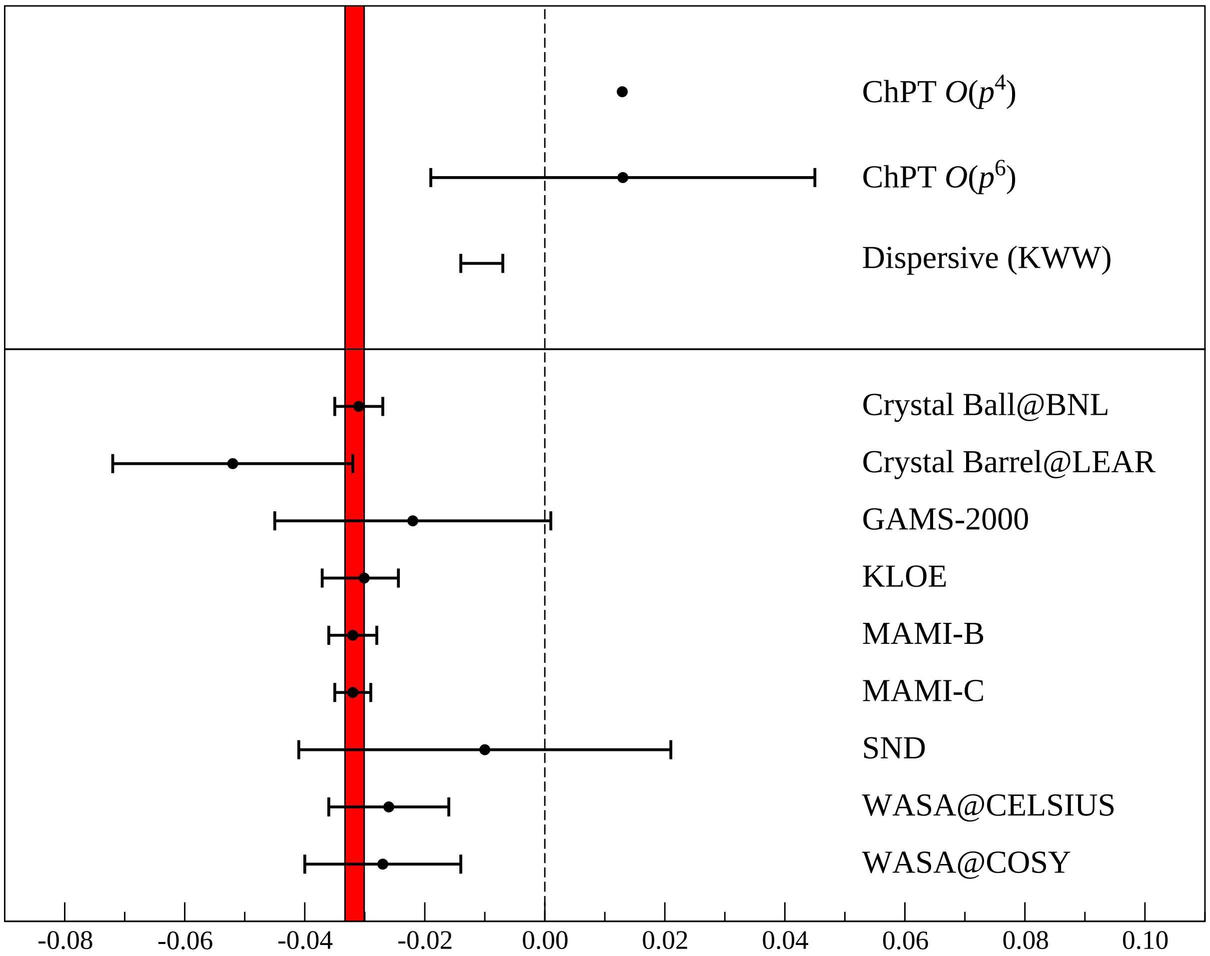
- Relevance: (potentially) clean access to $m_u - m_d$
  but: large higher-order / final-state interactions

- Chiral perturbation theory (ChPT) to 2 loops   Bijnens, Ghorbani 2007
  new dispersion-relation analysis   Colangelo, Lanz, Leutwyler, Passemar

- strong experimental activities   WASA-at-COSY, MAMI-B/-C, KLOE, ELSA

$\eta \longrightarrow 3\pi^0$ Dalitz plot parameter α: a puzzle

$$|\mathcal{A}_n(x,y)|^2 = |\mathcal{N}_n|^2\{1 + 2\,\alpha\, z + \ldots\} \qquad z \propto (s-s_0)^2 + (t-s_0)^2 + (u-s_0)^2$$



$10^3 \times \alpha$
$+13$
$+13 \pm 32$
$-7 \ldots -14$

PDG average:

-31.7 ± 1.6

- puzzle: why isn't two-loop ChPT closer to dispersive result?

Bijnens, Ghorbani 2007

Kambor, Wiesendanger, Wyler 1995

Precision rescattering: “non-relativistic” EFT

- theoretical tool for $\pi\pi$ scattering length extraction from cusp in $K \to 3\pi$: non-relativistic effective field theory

  Colangelo, Gasser, BK, Rusetsky 2006

  - parametrise T directly in terms of scattering lengths etc.
  - no quark-mass expansion of these parameters ($\leftrightarrow$ ChPT)
  - retain recoil corrections $\Leftrightarrow$ correct relativistic kinematics
    only inelasticities (far outside physical region) neglected

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- combined expansion in a ($\pi\pi$ scattering lengths) and non-relativistic parameter $\epsilon \propto |\mathbf{p}_\pi|/\mathcal{M}$

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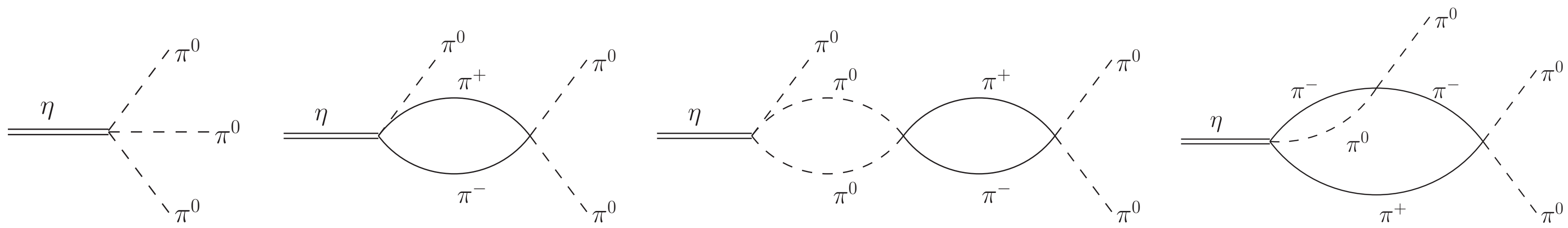
- combined expansion in a ($\pi\pi$ scattering lengths) and non-relativistic parameter $\epsilon \propto |\mathbf{p}_\pi|/\mathcal{M}$

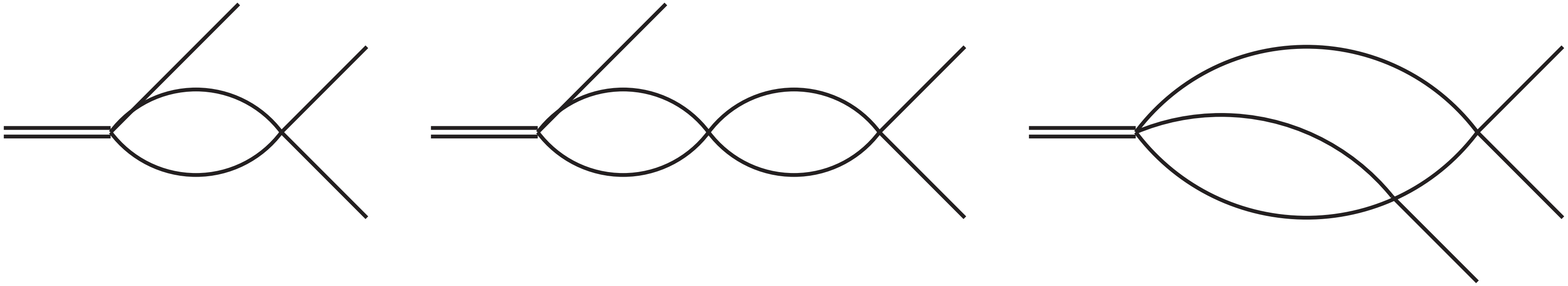
- complete to $\mathcal{O}(\epsilon^4, a\epsilon^5, a^2\epsilon^4)$:



- $\eta \to 3\pi$ tree couplings matched to ChPT $\mathcal{O}(p^4)$
  $\pi\pi$ from phenomenology (including isospin breaking)

Understanding α in NREFT

Schneider, BK, Ditsche 2010



$$\alpha = \Big(\underbrace{+10.7}_{\text{tree}}\ \underbrace{+12.4}_{\text{1-loop}}\ \underbrace{-44.1}_{\text{2-loop}}\ \underbrace{-6.0}_{\text{higher}}\ \underbrace{-0.6}_{\text{isospin}}\Big) \times 10^{-3} = (-24.6 \pm 4.9) \times 10^{-3}$$

- $\eta \to 3\pi$ tree couplings matched to ChPT $\mathcal{O}(p^4)$
- error: (1) $\pi\pi$ scattering Ananthanarayan et al. 2001 vs. Kamiński et al. 2008
  (2) estimate of higher orders ("bubble resummation")
- NREFT power counting tells us:
  - ▷ 1-loop and 2-loop both of same $\mathcal{O}(a_{\pi\pi}^2)$
  - ▷ rescattering enhanced in α: loops $\mathcal{O}(\epsilon^2)$ vs. tree $\mathcal{O}(\epsilon^4)$

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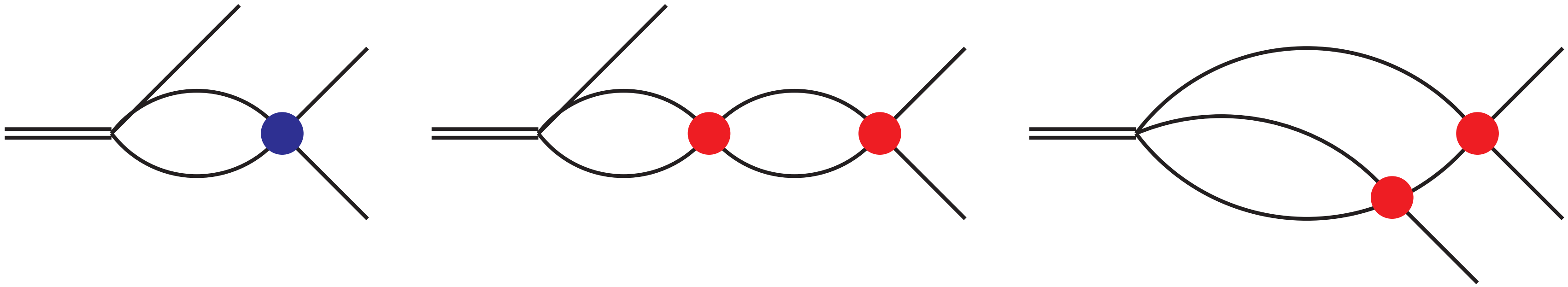

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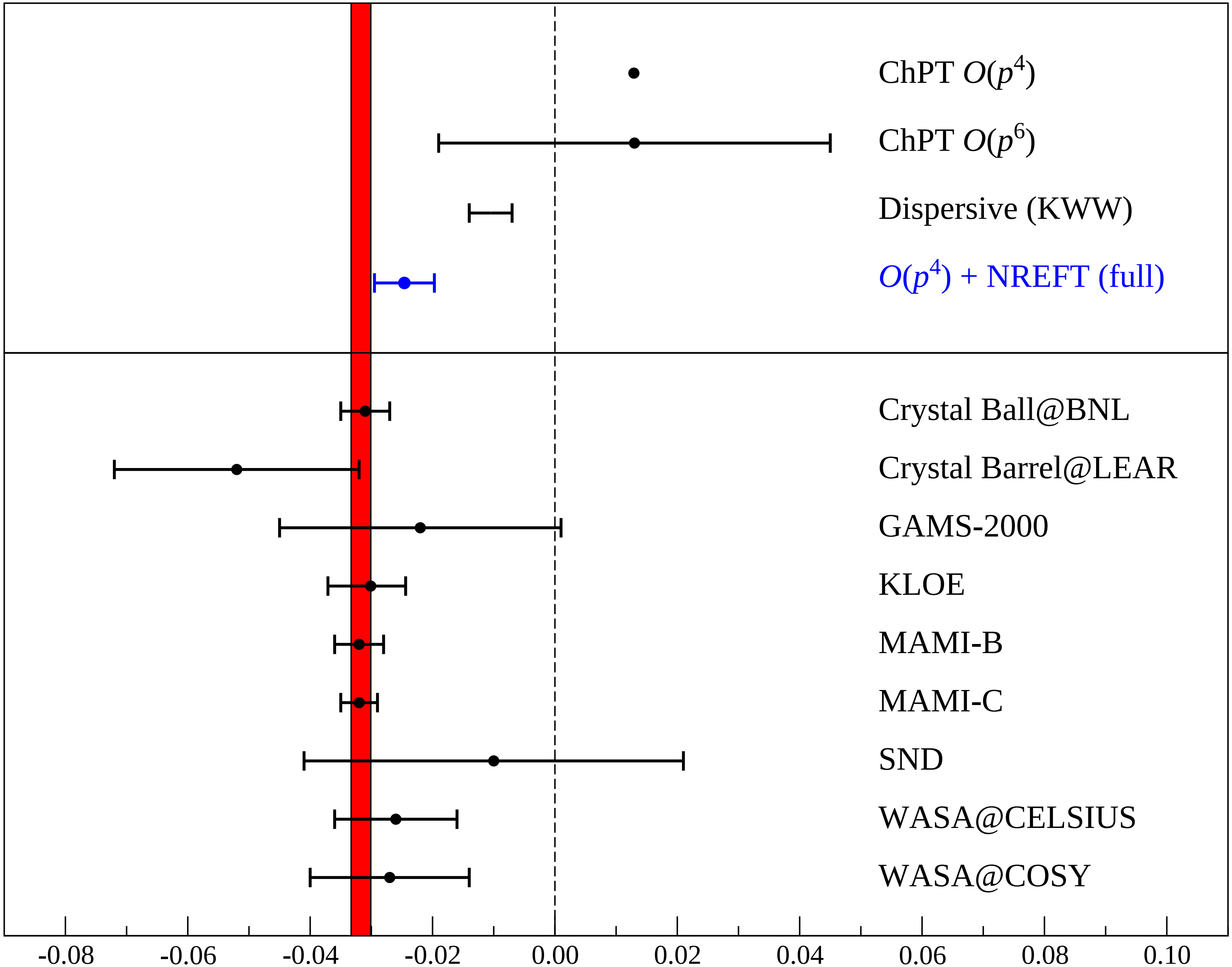
Why is the ChPT $\mathcal{O}(p^6)$ result so different, $\alpha = (+13 \pm 32) \times 10^{-3}$?

Bijnens, Ghorbani 2007

- "emulate" the chiral two-loop result:
  - $\bullet$: rescattering parameters $\mathcal{O}(p^4)$ in 1-loop graphs
  - $\bullet$: rescattering parameters $\mathcal{O}(p^2)$ in 2-loop graphs, e.g. $a_0^0 = 0.16$
- result: find $\alpha = -1.1 \times 10^{-3}$!
  $\Rightarrow$ "weaker" rescattering at 2 loops leads to totally different result

Total result for α



Isospin relation for Dalitz plot parameters

- Dalitz plot vs. amplitude expansion: $x \propto t - u$, $y \propto s - s_0$

$$|\mathcal{A}_c|^2 = |\mathcal{N}_c|^2 \{1 + ay + by^2 + dx^2 + \ldots\} \quad |\mathcal{A}_n|^2 = |\mathcal{N}_n|^2 \{1 + 2\alpha z + \ldots\}$$

$$\mathcal{A}_c = \mathcal{N}_c \{1 + \bar{a}y + \bar{b}y^2 + \bar{d}x^2 + \ldots\} \qquad \mathcal{A}_n = \mathcal{N}_n \{1 + \bar{\alpha} z + \ldots\}$$

$$a = 2\,\mathrm{Re}\,\bar{a}\,, \quad b = |\bar{a}|^2 + 2\,\mathrm{Re}\,\bar{b}\,, \quad d = 2\,\mathrm{Re}\,\bar{d}\,, \quad \alpha = \mathrm{Re}\,\bar{\alpha}$$

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- isospin relation between neutral and charged parameters:

$$\bar{\alpha} = \frac{1}{2}\big(\bar{b} + \bar{d}\big) \;\Rightarrow\; \alpha = \frac{1}{4}\bigg(b + d - \frac{a^2}{4} - (\mathrm{Im}\,\bar{a})^2\bigg) < \frac{1}{4}\bigg(b + d - \frac{a^2}{4}\bigg)$$

Bijnens, Ghorbani 2007

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- isospin relation between neutral and charged parameters:

$$\boxed{\alpha = \frac{1}{4}\Big(b + d - \frac{a^2}{4}\Big) - \zeta_1(1+\zeta_2 a)^2\,,\ \zeta_1 = 0.050 \pm 0.005\,,\ \zeta_2 = 0.225 \pm 0.003}$$

$\zeta_{1/2}$ determined by $\pi\pi$ phases

Schneider, BK, Ditsche 2010

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Schneider, BK, Ditsche 2010

- use precise KLOE data on a, b, d as input:

$$\left.\begin{aligned} \alpha^{\text{theo}}_{\text{KLOE}} &= -0.062 \pm 0.003_{\text{stat}}{}^{+0.004}_{-0.006}{}_{\text{syst}} \pm 0.003_{\pi\pi} \\ \alpha^{\text{exp}}_{\text{KLOE}} &= -0.030 \pm 0.004_{\text{stat}}{}^{+0.002}_{-0.004}{}_{\text{syst}} \end{aligned}\right\}$$

significant tension!

Isospin relation for Dalitz plot parameters

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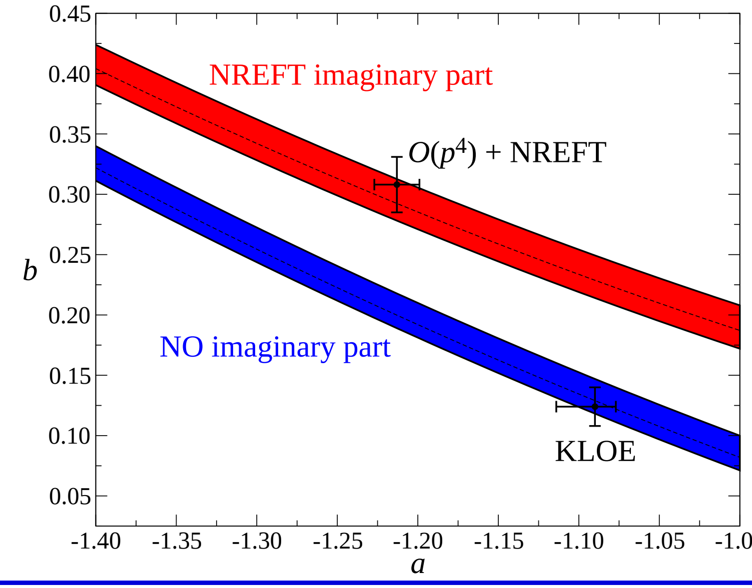
displayed as constraint in $a - b$ plane:



Dispersion relations for $\eta \longrightarrow 3\pi$: essentials

Fundamentals

- aim: resum $\pi\pi$ rescattering to all orders
- use modern high-precision parametrizations of $\pi\pi$ scattering
  Ananthanarayan et al. 2001, Kamiński et al. 2008
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$$\mathcal{M}(s,t,u) = \mathcal{M}_0(s)+(s-t)\mathcal{M}_1(u)+(s-u)\mathcal{M}_1(t)+\mathcal{M}_2(t)+\mathcal{M}_2(u)-\frac{2}{3}\mathcal{M}_2(s)$$

$\mathcal{M}_I(s)$ functions of one variable with only a right-hand cut

Stern, Sazdjian, Fuchs 1993; Anisovich, Leutwyler 1998

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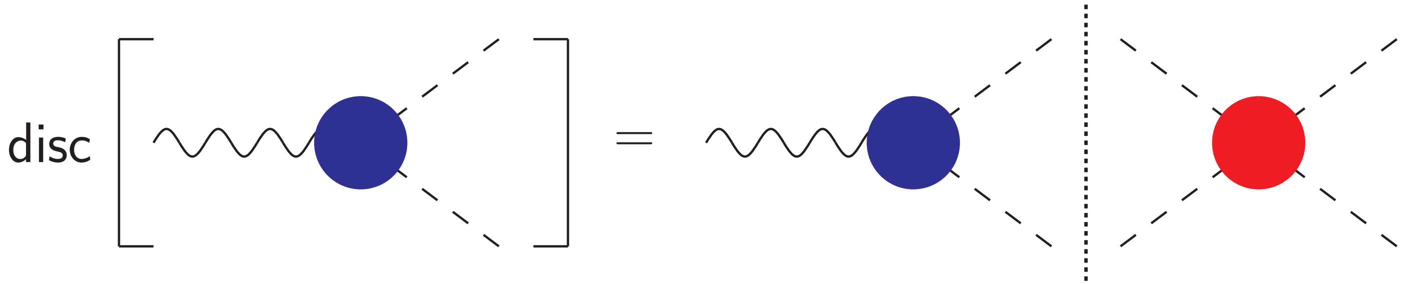
$\mathcal{M}_I(s)$ functions of one variable with only a right-hand cut

Stern, Sazdjian, Fuchs 1993; Anisovich, Leutwyler 1998

- decomposition exact only if $l \geq 2$ partial waves are real
  $\hat{=}$ $\mathcal{O}(p^8)$ or 3 loops in chiral counting

From unitarity to integral equations: form factor

- just two particles in final state (form factor); from unitarity:



$$\text{disc}\, F_I(s) \quad = \quad F_I(s) \times \theta(s - 4\,M_\pi^2) \times \sin\,\delta_I(s)\, e^{i\delta_I(s)}$$

$\Rightarrow$ Watson's final-state theorem Watson 1954

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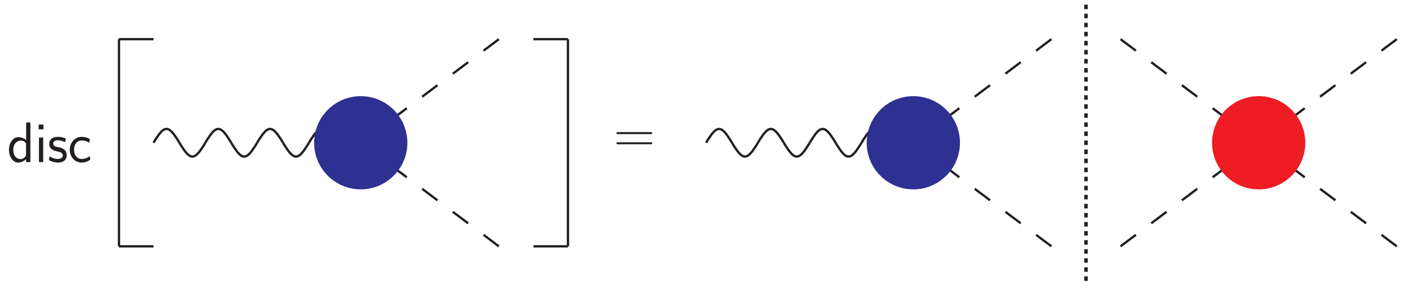

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- solution to this homogeneous integral equation known:

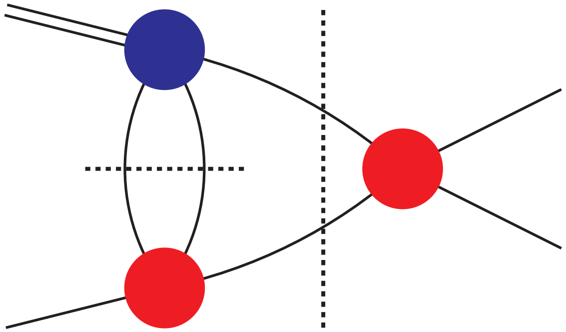
$$F_I(s) = P_I(s)\Omega_I(s)\;, \quad \Omega_I(s) = \exp\left\{ \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\delta_I(s')}{s'(s'-s)} \right\}$$

$P_I(s)$ polynomial, $\Omega_I(s)$ Omnès function — Omnès 1958

completely given in terms of phase shift $\delta_I(s)$

From unitarity to integral equations: inhomogeneities

- more complicated unitarity relation for 4-point functions:



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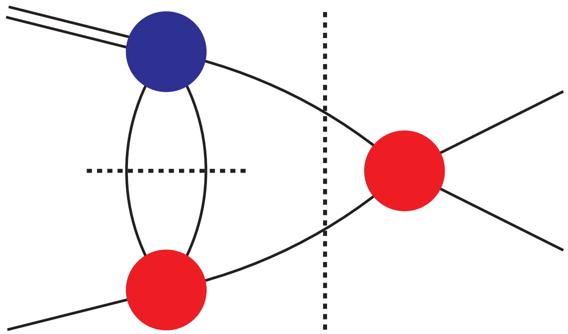

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- inhomogeneities $\hat{\mathcal{M}}_I(s)$: angular averages over the $\mathcal{M}_I(s)$: e.g.

$$\hat{\mathcal{M}}_0 = \tfrac{2}{3}\langle\mathcal{M}_0\rangle + \tfrac{20}{9}\langle\mathcal{M}_2\rangle + 2(s - s_0)\langle\mathcal{M}_1\rangle + \tfrac{2}{3}\kappa\langle z\mathcal{M}_1\rangle$$

$$\langle z^n f\rangle(s) = \tfrac{1}{2}\int_{-1}^{1} dz\, z^n f\left(\tfrac{1}{2}(3s_0 - s + z\kappa(s))\right)$$

$$\kappa(s) = \sqrt{1 - \frac{4\,M_\pi^2}{s}} \times \sqrt{(s - (M_\eta + M_\pi)^2)(s - (M_\eta - M_\pi)^2)}$$

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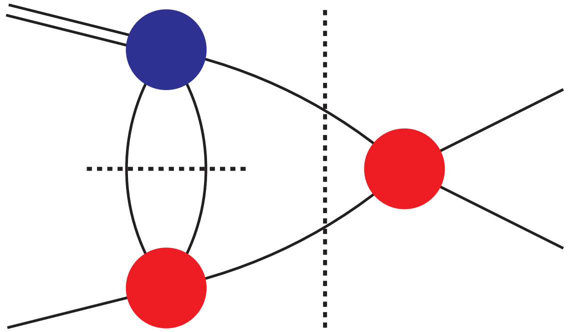

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- allows for cross-channel scattering between s-, t-, and u-channel
- $\kappa(s)$ generates complex analytic structure
  $\Rightarrow$ 3-particle cuts, the η is unstable

From unitarity to integral equations: solution

- integral equations including the inhomogeneities $\hat{\mathcal{M}}_I$:

$$\mathcal{M}_0(s) = \Omega_0(s)\left\{\alpha_0 + \beta_0\, s + \gamma_0\, s^2 + \frac{s^3}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'^3}\frac{\sin\delta_0(s')\hat{\mathcal{M}}_0(s')}{|\Omega_0(s')|(s'-s-i\epsilon)}\right\}$$

$$\mathcal{M}_1(s) = \Omega_1(s)\left\{\beta_1\, s + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'}\frac{\sin\delta_1(s')\hat{\mathcal{M}}_1(s')}{|\Omega_1(s')|(s'-s-i\epsilon)}\right\}$$

$$\mathcal{M}_2(s) = \Omega_2(s)\frac{s^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'^2}\frac{\sin\delta_1(s')\hat{\mathcal{M}}_2(s')}{|\Omega_2(s')|(s'-s-i\epsilon)}$$

Anisovich, Leutwyler 1998

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Anisovich, Leutwyler 1998

- overall 4 subtraction constants that need to be fixed:
  - ▷ matching to $\mathcal{O}(p^4)$-ChPT at the Adler zero (protected against large m_s-corrections)
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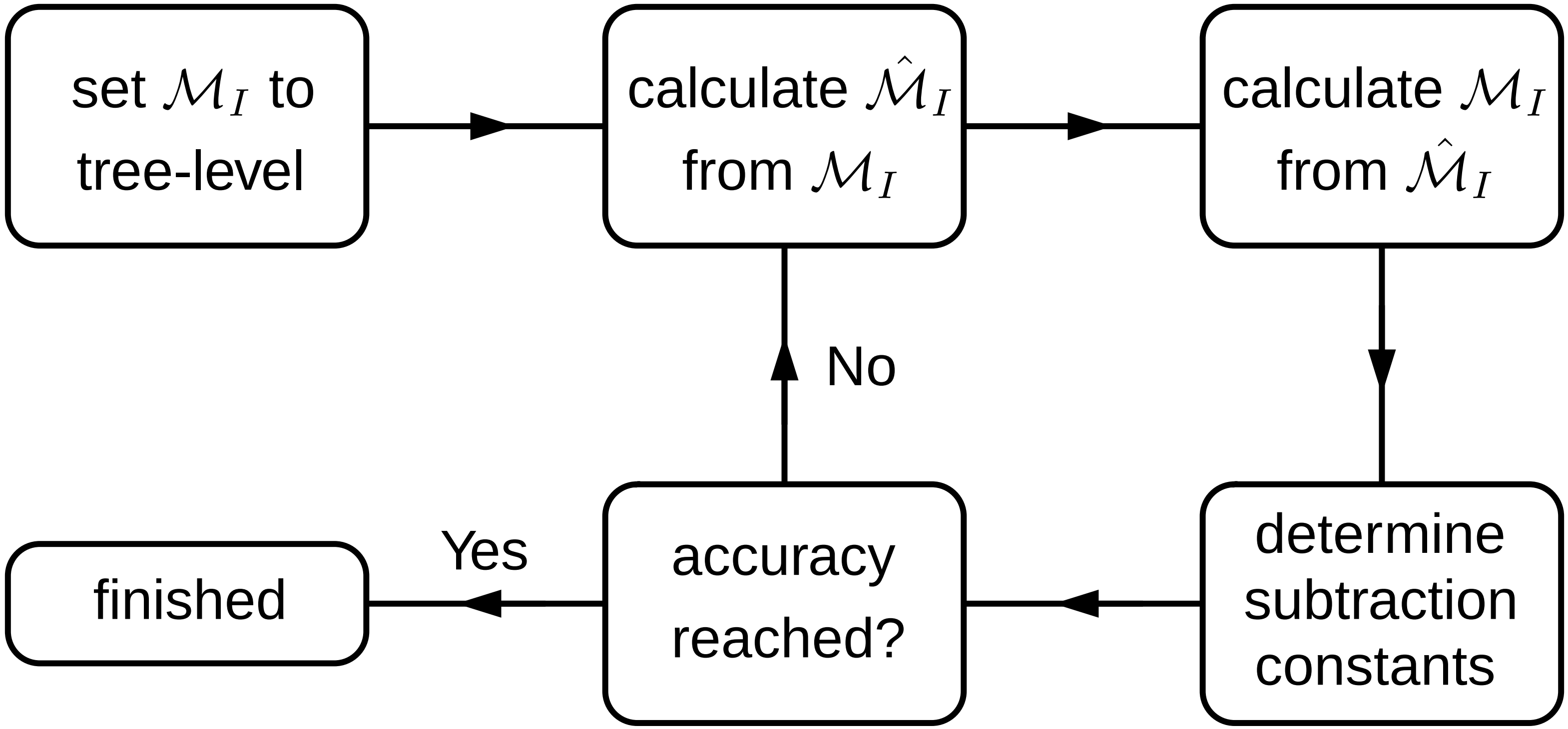
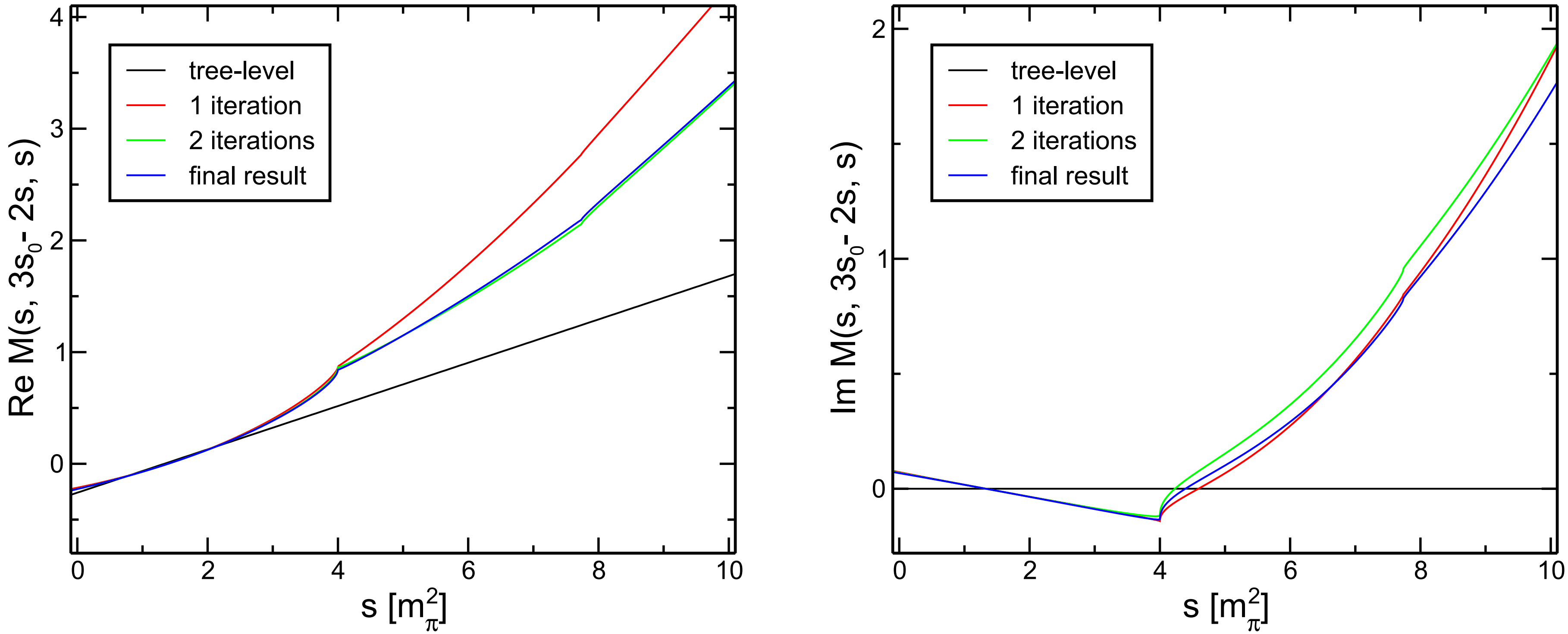
- solve these equations iteratively by a numerical procedure

Iterative procedure for solving integral equations



Numerical results: $\mathcal{M}(s, 3s_0 - 2s, s)$



Colangelo, Lanz, Leutwyler, Passemar 2010 (preliminary)

- fast convergence: real part almost indistinguishable from final result after 2 iterations

Hadronic η' decays

$\eta' \to \eta\pi\pi$

- one of the very few channels where $\pi\eta$ scattering can be studied
- neutral channel shows cusp effect BK, Schneider 2009
- expect short-term increase in statistical data base from BES-III, WASA@COSY, ELSA, CB@MAMI-C
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$\eta' \to 3\pi$

- again isospin-violating decay
- two possible mechanisms: direct isospin breaking $\eta' \to 3\pi$ *and* $\eta' \to \eta\pi\pi$ + isospin-breaking $\eta\pi \to \pi\pi$ rescattering
- larger phase space: ρ resonance in the decay region; inelasticities more important? how important are crossed-channel effects in the context of resonances?

Integral equations for $\eta' \longrightarrow \eta\pi\pi$

- $\eta' \to \eta\pi\pi$ amplitude decomposed into S- and P-waves:

$$\mathcal{M}(s,t,u) = \mathcal{M}_0^{\pi\pi}(s)+\mathcal{M}_0^{\pi\eta}(t)+\big\{(s-u)t+\Delta_{\eta'\pi}\Delta_{\eta\pi}\big\}\mathcal{M}_1^{\pi\eta}(t)+(t\leftrightarrow u)$$

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- may even neglect P-wave discontinuity: "exotic", real to 3 loops
- 3 constants to be fixed; numerical implementation underway

Challenges in $\eta' \longrightarrow \eta\pi\pi$

fixing subtraction constants

- matching to large-N_c ChPT or Resonance Chiral Theory (are there any low-energy theorems $\sim$ Adler zero?)
  Beisert, Borasoy 2002; Escribano, Masjuan, Sant-Cillero 2010
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$\pi\eta$ phase shifts

- $\pi\eta$ scattering not well-known
- inverse amplitude method, other non-perturbative approaches?

  Oller, Oset, Peláez 1998; Oller, Oset, Ramos 2000

- can we learn something about $\pi\eta$ phase-shifts?

Summary

Rigorous methods for ChPT-motivated investigations of hadronic meson decays:

- NREFT for perturbative final-state interactions in $\eta \to 3\pi$
  - ▷ understand the $\eta \to 3\pi^0$ slope parameter α:
    $\alpha_{\text{theo}} = -0.025 \pm 0.005$ vs. $\alpha_{\text{exp}} = -0.0317 \pm 0.0016$
  - ▷ rescattering ($\simeq$ imaginary parts) relate charged and neutral Dalitz parameters $\Rightarrow$ tension in experimental data (KLOE)

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  - ▷ rescattering ($\simeq$ imaginary parts) relate charged and neutral Dalitz parameters $\Rightarrow$ tension in experimental data (KLOE)

- non-perturbative final-state interactions via dispersion relations:
  - ▷ iterative numerical solution
  - ▷ input full $\pi\pi$ phase shifts
  - ▷ more appropriate for precision determination of quark mass ratios from $\eta \to 3\pi$
  - ▷ to be extended to hadronic η' decays

What else can be done? – Outlook

- study CP-violation in Dalitz plots for $D \to 3\pi,\ \pi\pi K,\ \pi K \bar{K}$, also similar B-decays: Les Nabis
  - ▷ larger branching fractions than two-body decays
  - ▷ enhancement of CP-violation in resonance-rich environment
  - ▷ enhancement of CP-violation in parts of the Dalitz plot?

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- challenges:
  - ▷ at higher energies: coupled-channel integral equation
  - ▷ inelasticities certainly not negligible
  - ▷ perturbative treatment of crossed-channel effects reliable?
  - ▷ when are higher partial waves non-negligible?

Spares

Non-relativistic EFT (1): basics

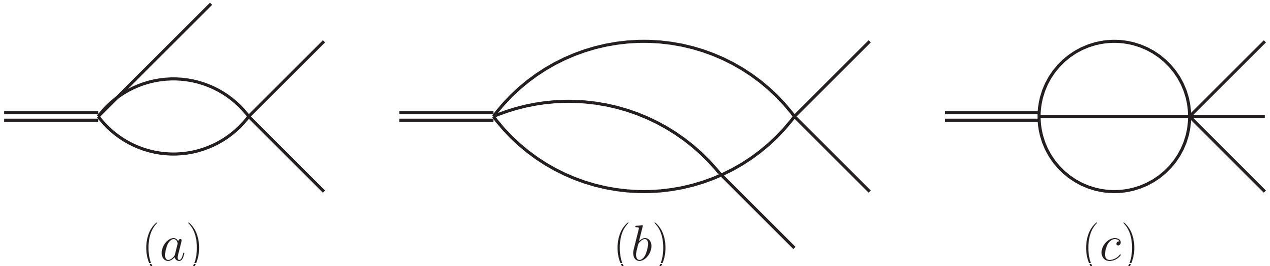
momenta	:	$\lvert\mathbf{p}\rvert/M_\pi = \mathcal{O}(\epsilon)$
kinetic energy	:	$T = \omega(\mathbf{p}) - M_\pi = \mathcal{O}(\epsilon^2)$
in $\eta \to 3\pi$	:	$M_\eta - \sum_i M_i = \sum_i T_i = \mathcal{O}(\epsilon^2)$

where $\omega(\mathbf{p}) = \sqrt{M_\pi^2 + \mathbf{p}^2}$

- non-relativistic region = whole decay region (and slightly beyond)
- two-fold expansion in ϵ and $\pi\pi$ scattering length a
- at given order a, ϵ, only finite number of graphs contribute
  $\Rightarrow$ power counting

Non-relativistic EFT (2): power counting

- organise tree level polynomials in even powers of momenta
  $\Rightarrow \mathcal{O}(\epsilon^0)$, $\mathcal{O}(\epsilon^2) \hat{=} a$, $\mathcal{O}(\epsilon^4) \hat{=} b$, d, α, ...
- loops:



(a) (b) (c)

propagator: $$\frac{1}{\omega(\mathbf{p}) - p^0} = \mathcal{O}(\epsilon^{-2})$$

loop integration: $$d^4p = dp^0 d^3\mathbf{p} = \mathcal{O}(\epsilon^5)$$

- each loop with two-body rescattering $(\epsilon^{-2})^2\epsilon^5 = \mathcal{O}(\epsilon)$ suppressed
  (a) $= \mathcal{O}(a^1\epsilon^1)$
  (b) $= \mathcal{O}(a^2\epsilon^2)$ $\Rightarrow$ correlated expansion in a and ϵ
- loop with three-body rescattering $(\epsilon^{-2})^3(\epsilon^5)^2 = \mathcal{O}(\epsilon^4)$ suppressed
  (c) $= \mathcal{O}(\epsilon^4)$

Non-relativistic EFT (3): Lagrangian

- propagator: $$\underbrace{\frac{1}{M_\pi^2-p^2}}_{\text{relativistic}} = \underbrace{\frac{1}{2\omega(\mathbf{p})}\frac{1}{\omega(\mathbf{p})-p^0}}_{\text{"non-relativistic"}} + \underbrace{\frac{1}{2\omega(\mathbf{p})}\frac{1}{\omega(\mathbf{p})+p^0}}_{\text{antiparticles}}$$

  generated by Lagrangian

  $$\mathcal{L}_{\text{kin}} = \Phi^\dagger(2W)(i\partial_t - W)\Phi \,, \qquad W = \sqrt{M_\pi^2 - \triangle}$$

  Note: non-local $\mathcal{L}_{\text{kin}}$ generates all relativistic corrections; manifestly Lorentz-invariant / frame-independent

- correctly reproduces singularity structure at small momenta $|\mathbf{p}| \ll M_\pi$, subsumes far-away singularities in effective couplings
- interaction terms:

$$\mathcal{L}_{\pi\pi} = C_x\big(\pi_-^\dagger\pi_+^\dagger(\pi_0)^2 + h.c.\big) + \big(\text{derivative terms}\big)$$
$$\mathcal{L}_{\eta 3\pi} = \frac{K_0}{6}\big(\eta^\dagger\pi_0^3 + h.c.\big) + L_0\big(\eta^\dagger\pi_0\pi_+\pi_- + h.c.\big) + \ldots$$

- Lagrangian-based QFT, analyticity + unitarity obeyed

Non-relativistic EFT (4): matching

- match the $\pi\pi$ coupling constants to the effective range expansion of the $\pi\pi$ scattering amplitude:

$$\mathrm{Re}\, T(\pi^+\pi^- \to \pi^0\pi^0) = 2C_x + \mathcal{O}(\epsilon^2)$$

$$2C_x = -\frac{32\pi}{3}(a_0 - a_2)\Big\{1 + \underbrace{\frac{M_{\pi^+}^2 - M_{\pi^0}^2}{3M_\pi^2}}_{\text{ChPT } \mathcal{O}(e^2)} + \ldots\Big\}$$

$$= -\frac{32\pi}{3}\Big\{a_0 - a_2 + \underbrace{(0.61 \pm 0.16) \times 10^{-2}}_{\text{ChPT } \mathcal{O}(e^2p^2)}\Big\}$$

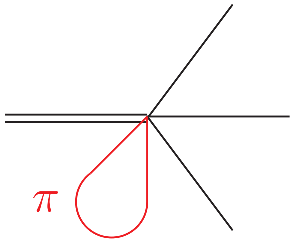
Knecht, Urech 1997; Gasser et al. 2001

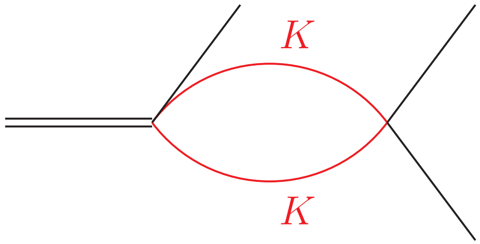
isospin-breaking corrections in matching calculated in ChPT

- parametrise polynomial $\eta \to \pi^+\pi^-\pi^0$ in terms of $L_0, L_1, \ldots$
  $\eta \to 3\pi^0$ in terms of $K_0, K_1, \ldots$
- match decay parameters ($K_0, \ldots, L_0, \ldots$) to ChPT at $\mathcal{O}(p^4)$
  $\Leftrightarrow$ tree Dalitz plot parameters a^{tree}, b^{tree}, α^{tree} …

Comparison (1): NREFT vs. ChPT

– not predictive: a^{tree}, b^{tree}, α^{tree} … input parameters, subsume e.g.





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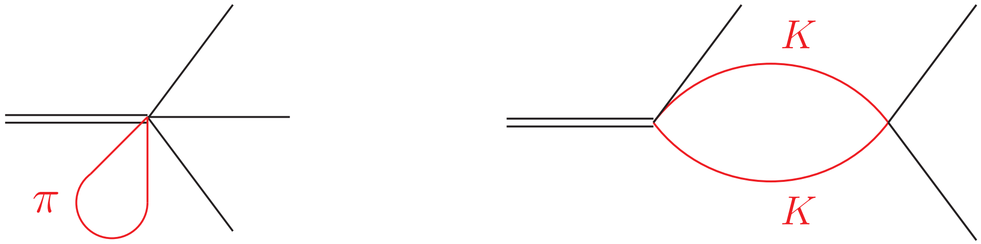

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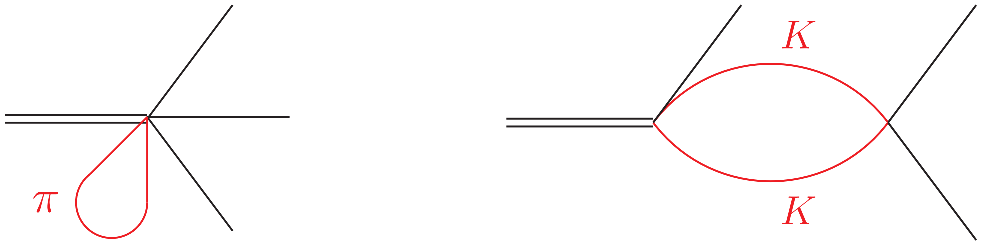

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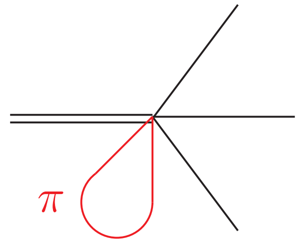

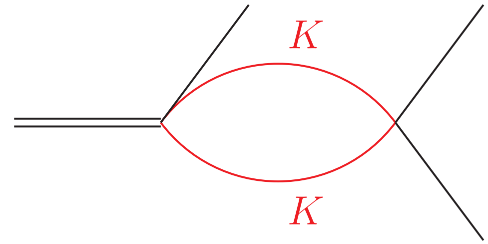

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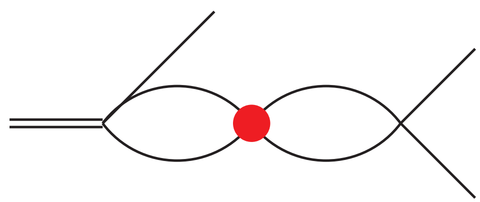

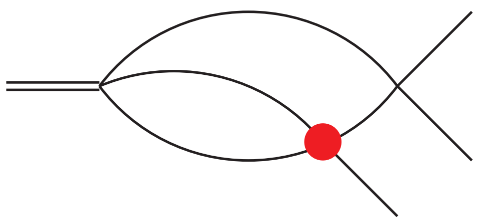

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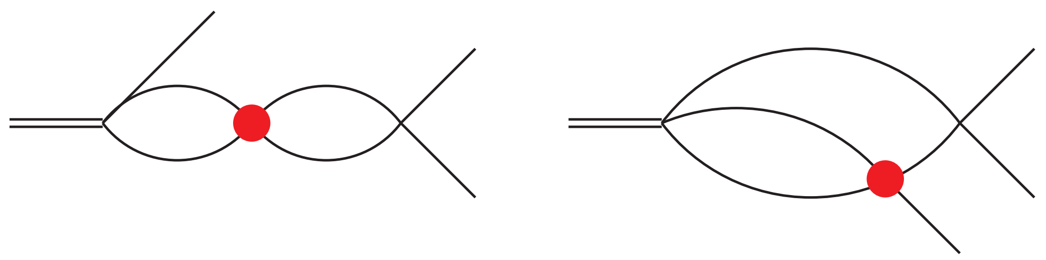

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$+$ more efficient in including isospin breaking
in particular kinematic effects due to $M_{\pi^+} \neq M_{\pi^0}$

Comparsion (2): NREFT vs. dispersion relations (DR)

– only 2 loops, while DR resum higher orders

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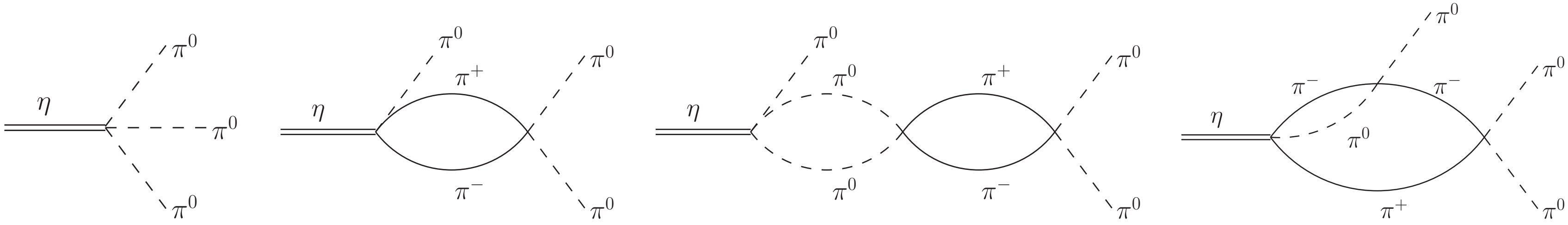
  correct thresholds everywhere

Representation of $\eta \longrightarrow 3\pi$ amplitude up to two loops

- complete representation to $\mathcal{O}(\epsilon^4, a\epsilon^5, a^2\epsilon^4)$, partial $\mathcal{O}(a^2\epsilon^6, a^2\epsilon^8)$:



- only one-loop function purely imaginary (above $\pi\pi$ threshold):

$$J(s) = \frac{i\, v(s)}{16\pi}\,, \qquad v(s) = \sqrt{1 - \frac{4M_\pi^2}{s}}$$

- non-trivial two-loop function purely real (above $\pi\pi$ threshold):

$$F(s) = \frac{v(s)}{256\pi^2}\sqrt{\frac{M_\eta^2 - 9M_\pi^2}{M_\eta^2 - M_\pi^2}} + \mathcal{O}\big(v(s)^3\big)$$

analytical representation in terms of arctan functions available

Bissegger et al. 2008

same goes for "bubble-sum" two-loop graph: $[J(s)]^2$ real

Understanding α in NREFT: power counting

1-loop vs. 2-loop

- power counting in $a_{\pi\pi}$:
  symbolically (in the center of the Dalitz plot):

$$\begin{aligned}
\mathcal{A} &= \mathcal{A}_{\text{tree}} + i\,\mathcal{A}_{1-\text{loop}}\, a_{\pi\pi} + \mathcal{A}_{2-\text{loop}}\, a_{\pi\pi}^2 + \mathcal{O}(i\, a_{\pi\pi}^3) \\
\Rightarrow\quad |\mathcal{A}|^2 &= \mathcal{A}_{\text{tree}}^2 + \big(\mathcal{A}_{1-\text{loop}}^2 + \mathcal{A}_{\text{tree}} \times \mathcal{A}_{2-\text{loop}}\big)\, a_{\pi\pi}^2 + \mathcal{O}(a_{\pi\pi}^4)
\end{aligned}$$

  expect 2-loop effects on slope parameters as large as 1-loop!

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- power counting in $a_{\pi\pi}$:
  symbolically (in the center of the Dalitz plot):

$$\begin{aligned} \mathcal{A} &= \mathcal{A}_{\text{tree}} + i\,\mathcal{A}_{1-\text{loop}}\, a_{\pi\pi} + \mathcal{A}_{2-\text{loop}}\, a_{\pi\pi}^2 + \mathcal{O}(i\, a_{\pi\pi}^3) \\ \Rightarrow \quad |\mathcal{A}|^2 &= \mathcal{A}_{\text{tree}}^2 + \big(\mathcal{A}_{1-\text{loop}}^2 + \mathcal{A}_{\text{tree}} \times \mathcal{A}_{2-\text{loop}}\big)\, a_{\pi\pi}^2 + \mathcal{O}(a_{\pi\pi}^4) \end{aligned}$$

  expect 2-loop effects on slope parameters as large as 1-loop!

importance of rescattering

- power counting in ϵ: $\alpha^{\text{tree}} = \mathcal{O}(\epsilon^4)$
- loops: $\mathcal{A}_{1-\text{loop}} = \mathcal{O}(\epsilon)$, $\mathcal{A}_{2-\text{loop}} = \mathcal{O}(\epsilon^2)$
  contribute to all Dalitz plot parameters at $\mathcal{O}(a_{\pi\pi}^2 \epsilon^2)$
  $\Rightarrow$ heightened importance of loops for higher slope parameters!

Charged Dalitz plot parameters

	a	b	d
KLOE 2008	$-1.090^{+0.009}_{-0.020}$	0.124 ± 0.012	$0.057^{+0.009}_{-0.017}$
Crystal Barrel 1998	-1.22 ± 0.07	0.22 ± 0.11	0.06 ± 0.04
ChPT $\mathcal{O}(p^4)$	-1.34 ± 0.04	0.43 ± 0.02	0.077 ± 0.008
ChPT $\mathcal{O}(p^6)^*$	-1.27 ± 0.08	0.39 ± 0.10	0.055 ± 0.057
dispersive**	-1.16	$0.24 \ldots 0.26$	$0.09 \ldots 0.10$
$\mathcal{O}(p^4)$ + NREFT	-1.21 ± 0.02	0.31 ± 0.02	0.050 ± 0.002

*Bijnens, Ghorbani 2007

**Kambor, Wiesendanger, Wyler 1995

- note significant discrepancy theory vs. experiment (KLOE) for b
  large violation of current-algebra relation $b = a^2/4$
  about to be remeasured WASA-at-COSY

$\pi\pi$ scattering lengths

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- linked to the quark-mass expansion of the pion mass
- large-/small-condensate scenario of chiral symmetry breaking?

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- 2-loop ChPT + Roy equations (dispersion theory):

$$\boxed{\begin{aligned} a_0 &= 0.220 \pm 0.005 \\ a_2 &= -0.0444 \pm 0.0010 \\ a_0 - a_2 &= 0.265 \pm 0.004 \end{aligned}}$$

(for QCD in the isospin limit) $\Rightarrow$ $\simeq 1.5\%$ theoretical precision

Colangelo, Gasser, Leutwyler 2000, 2001

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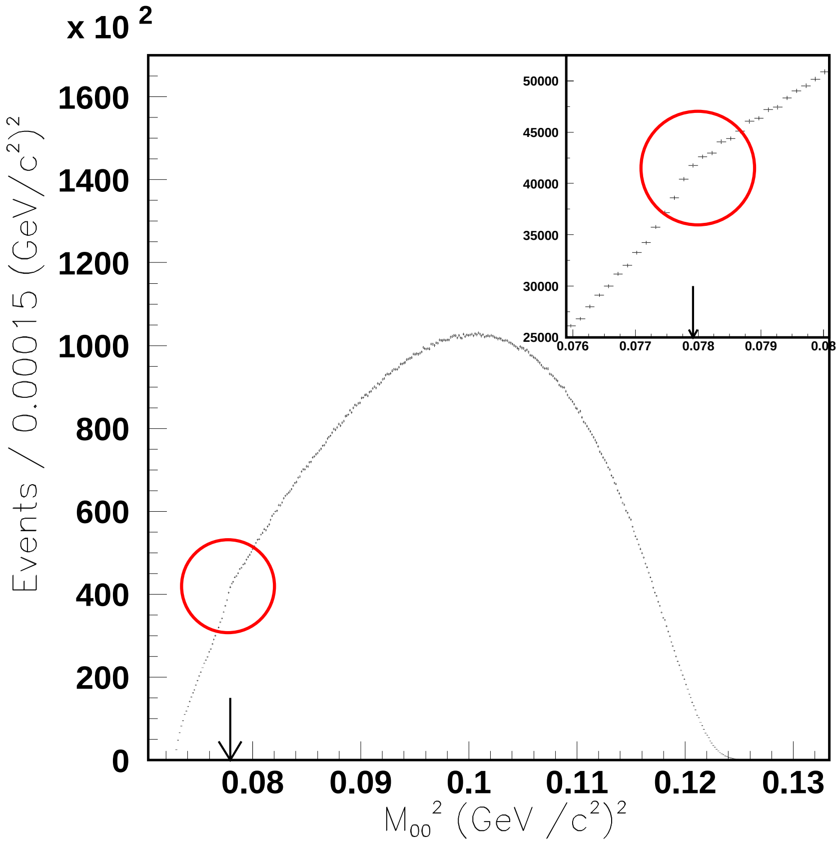
Experiments:

- $K^+ \to \pi^+\pi^- e^+\nu_e$ (K_{e4}) — BNL-865, NA48/2
- pionium (= $\pi^+\pi^-$ atom) lifetime — DIRAC
- cusp in $K^+ \to \pi^0\pi^0\pi^+$ — NA48/2

The cusp effect in $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$



- cusp at $M_{\pi^0\pi^0} = 2M_{\pi^+}$

NA48/2 2006

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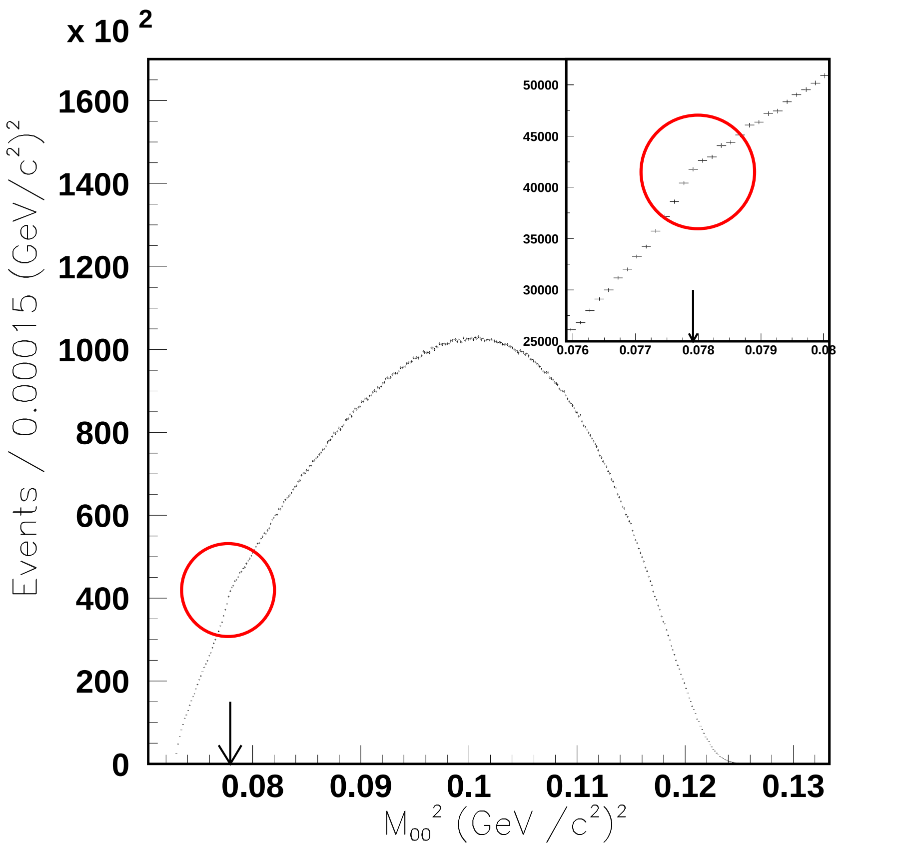

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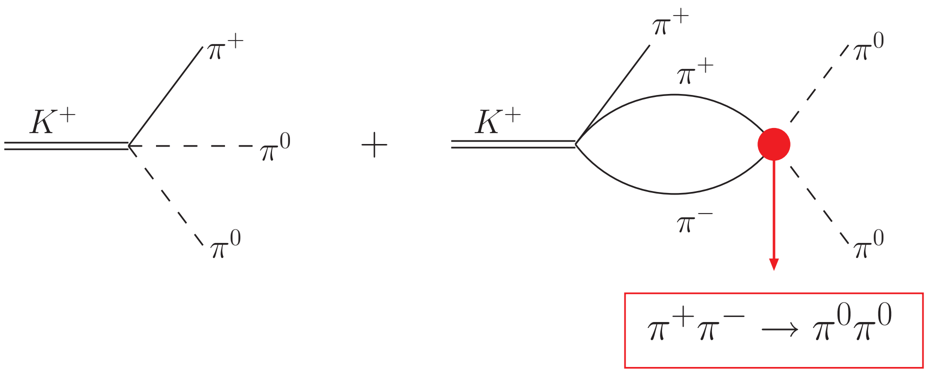

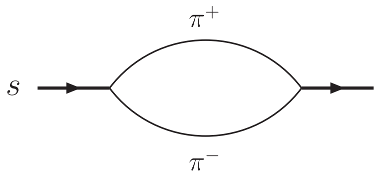

$$= \ldots + \frac{i}{16\pi}\, v_\pm(s)$$

$$v_\pm(s) = \begin{cases} \sqrt{1 - \dfrac{4M_{\pi^+}^2}{s}}\,, & s > 4M_{\pi^+}^2 \\ i\,\sqrt{\dfrac{4M_{\pi^+}^2}{s} - 1}\,, & s < 4M_{\pi^+}^2 \end{cases}$$

- interference tree + 1-loop below $\pi^+\pi^-$ threshold
- square-root behaviour = cusp

Cabibbo 2004