Unitarized Chiral Perturbation Theory in large N_c and Fock space expansion of σ meson

Felipe J. Llanes-Estrada
In collaboration with
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Universidad Complutense de Madrid

Hirschegg, 21/01/2011

Criterion to consider yourself young

Can you still count your Hirschegg workshops?

Outline

1. Are mesons $q\bar{q}$ states?
2. The σ meson
3. Fock space decomposition
4. Unitarized Chiral Perturbation Theory

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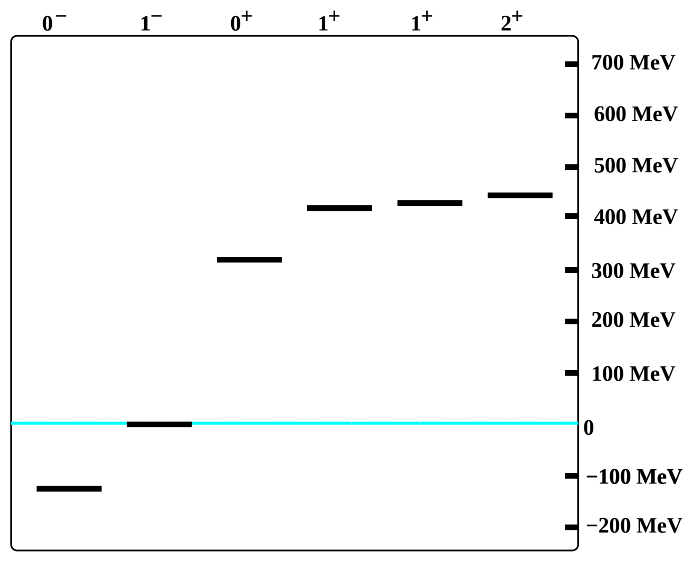
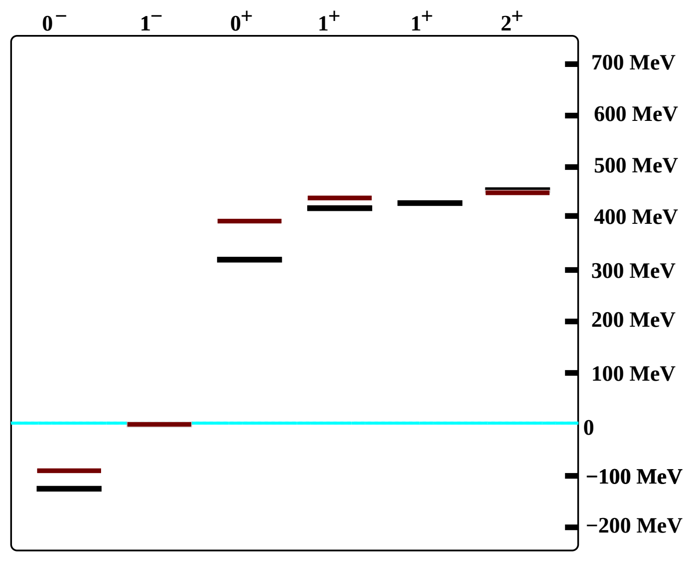
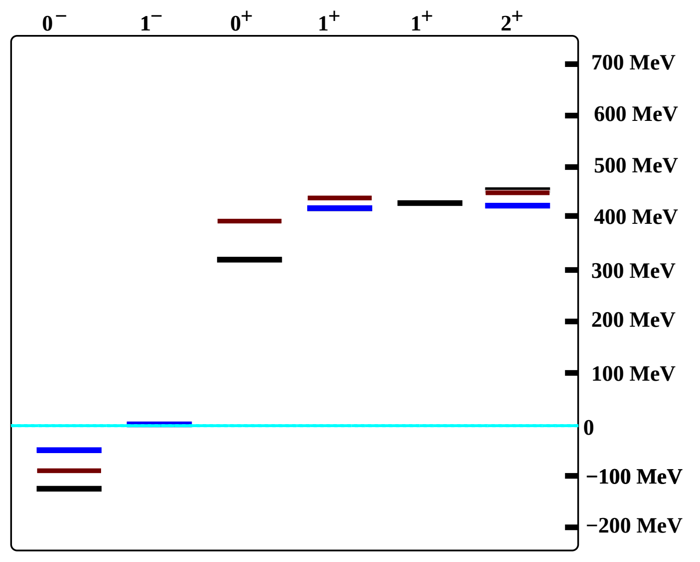
The $c\bar{c}$ charmonium spectrum



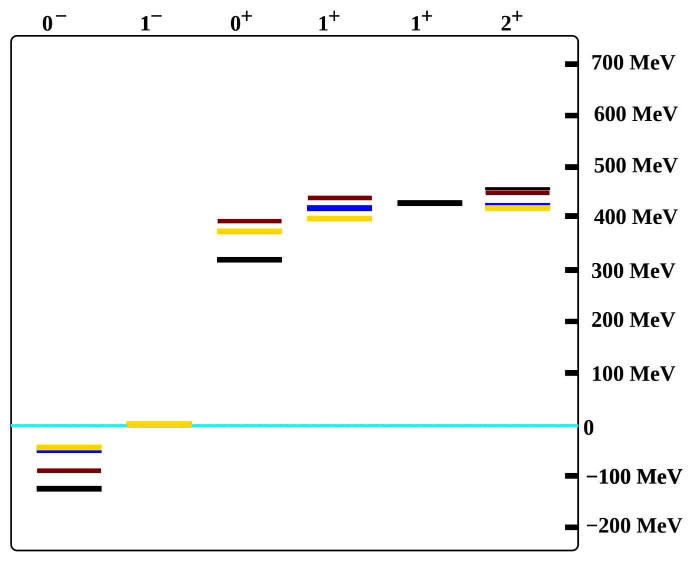
A cross-check at our disposal: $b\bar{b}$



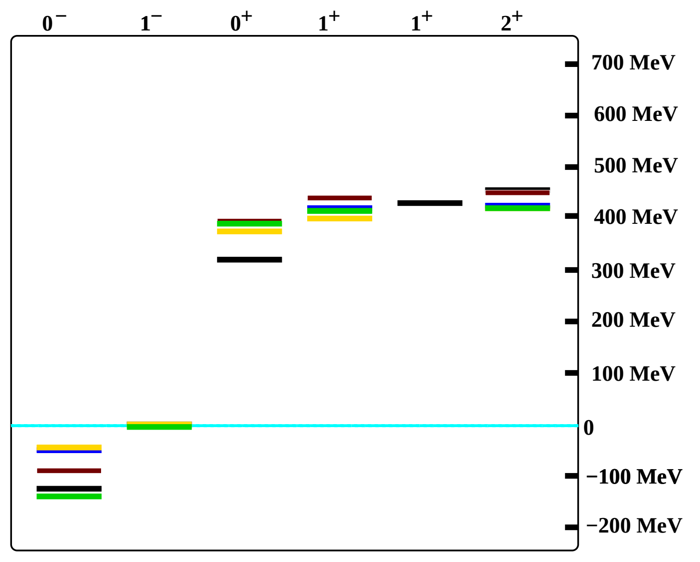
To convince the skeptic, B_s mesons



To convince the theorists, B mesons



To convince you, D mesons



5 copies of the same pattern!

- Well, maybe 6 as the B_c family is unexplored
- The trouble starts as we proceed towards lighter mesons...

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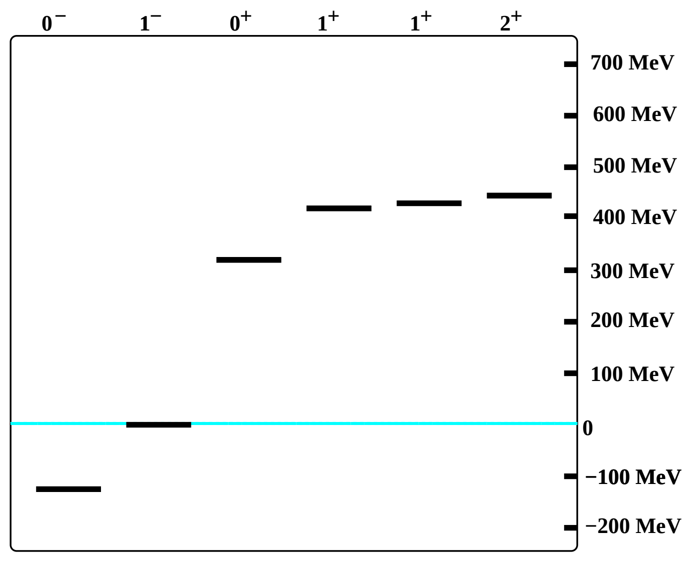
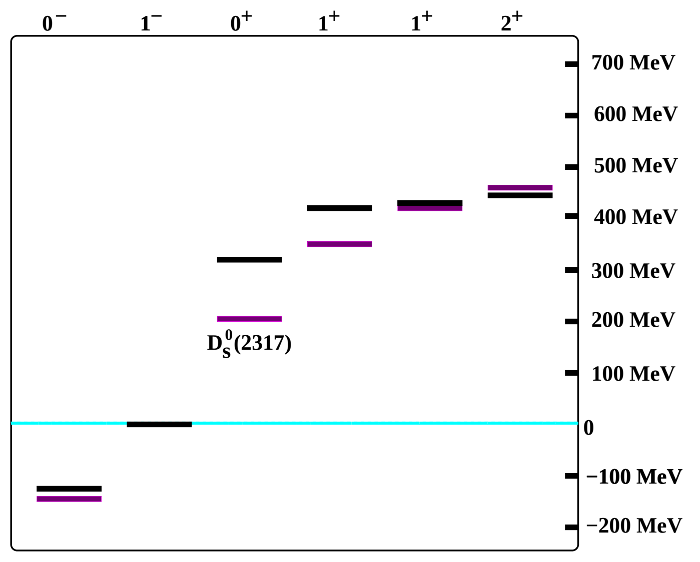
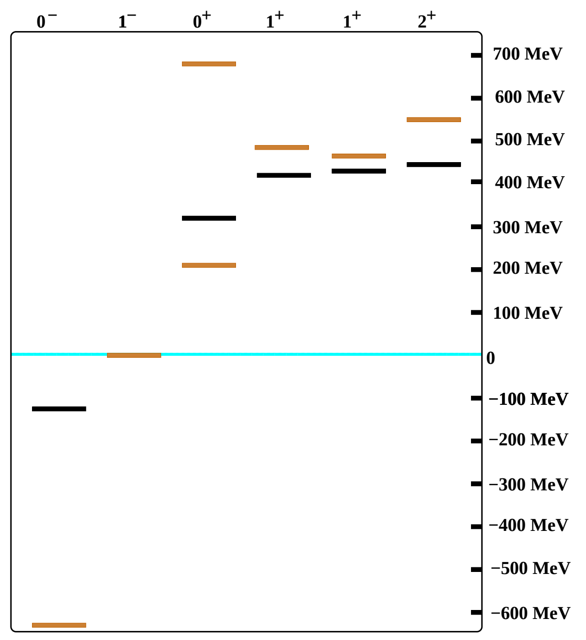
Back to charmonium spectrum



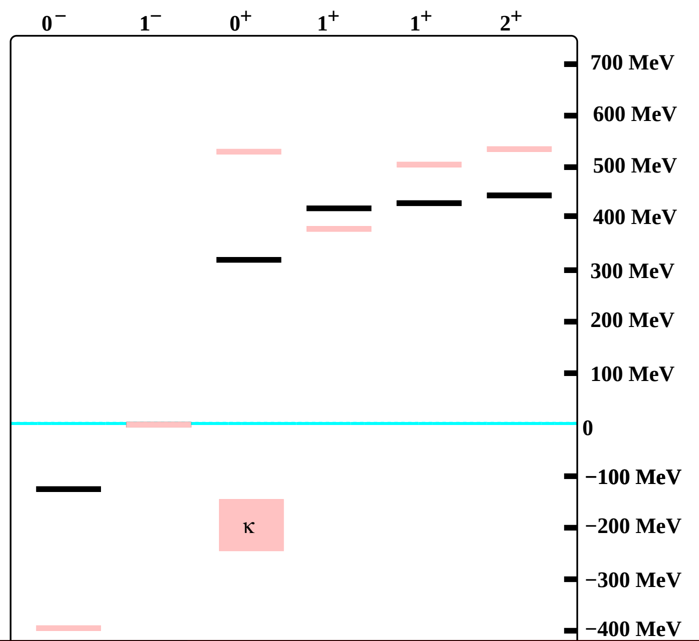
Look now at the D_s family



$I = 1$ light mesons

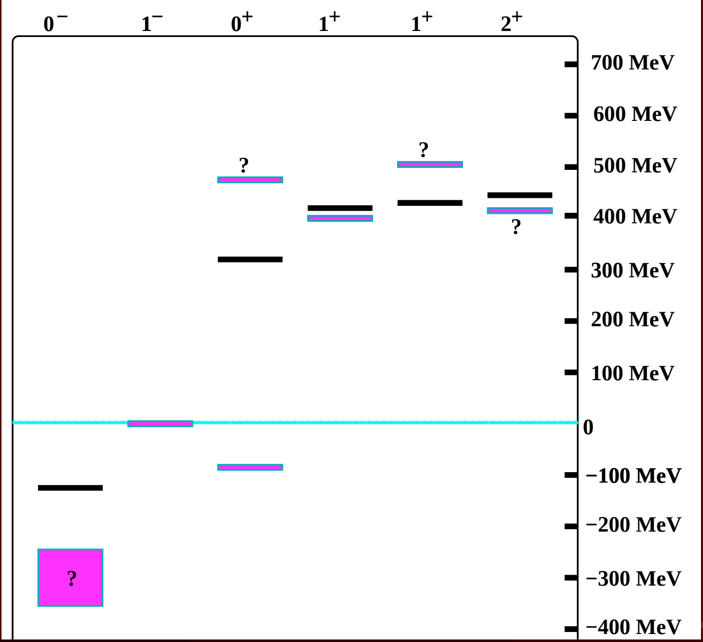


K family

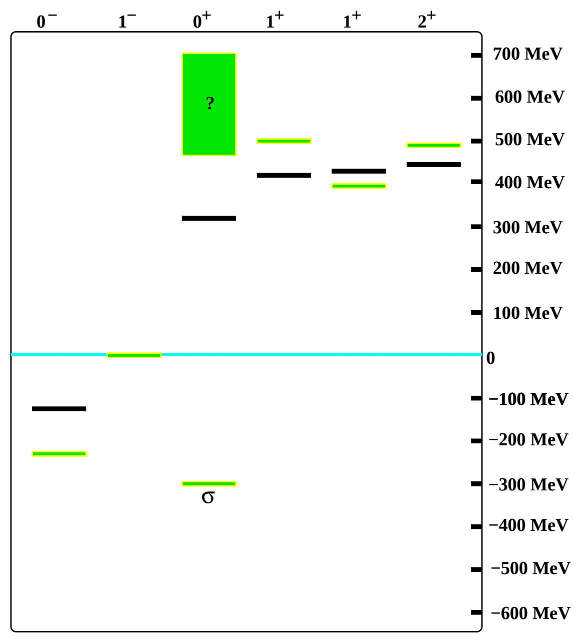


0⁻
1⁻
0⁺
1⁺
1⁺
2⁺
700 MeV
600 MeV
500 MeV
400 MeV
300 MeV
200 MeV
100 MeV
0
−100 MeV
−200 MeV
−300 MeV
−400 MeV
κ

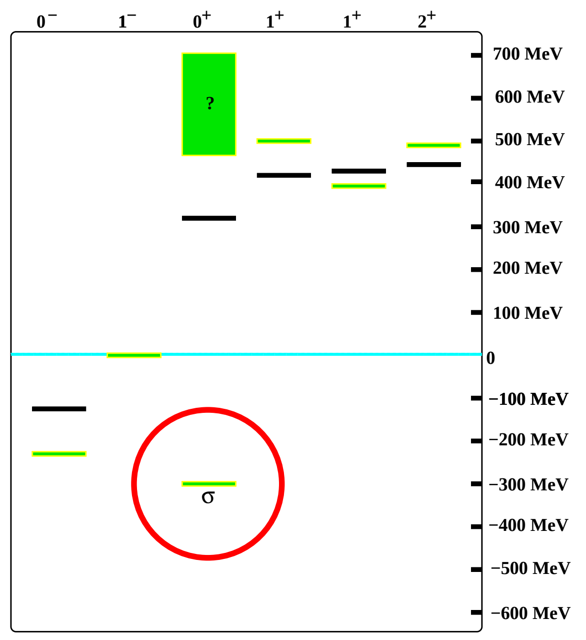
$s\bar{s}$ (ϕ-like) family



$I = 0$ light mesons



A salient non $q\bar{q}$ structure



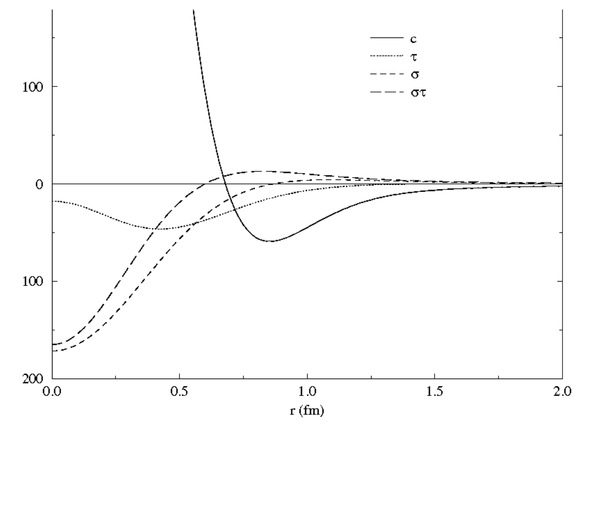
Pure $q\bar{q}$: mass too low

- Non chiral approaches put light scalar meson at/above 1 GeV
- $\gamma^\mu\gamma_\mu$ kernels exaggerate spin-orbit splittings
  - $\simeq 800$ MeV (Ll-E and Cotanch, 2002)
  - $\simeq 650$ MeV (Maris and Tandy, 2002)

Outline

Central attraction in nuclear potential

Johnson and Teller'55:

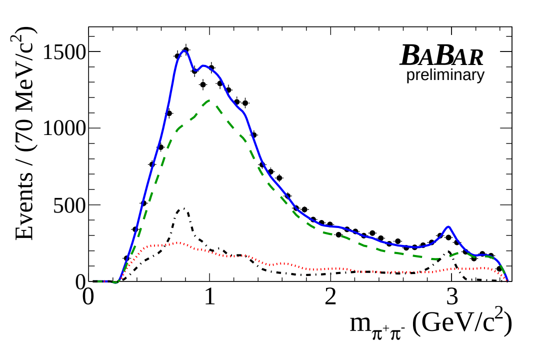


(Wiringa et al. 1995 AV18)

Often missed in $\pi\pi$

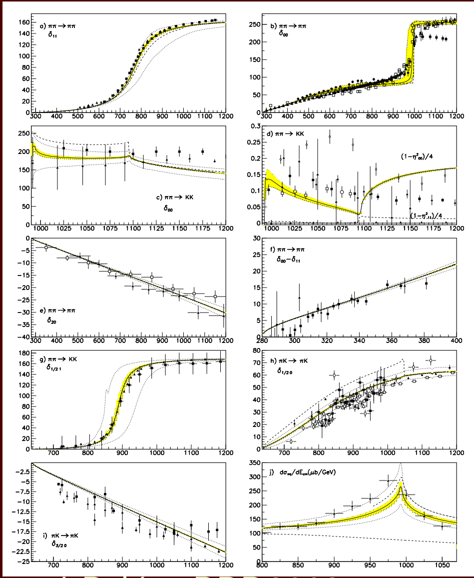
$$B \longrightarrow \bar{D}\pi\pi$$

Babar, 1007.4464 [hep-ex]



The ρ peak dominates the low-mass dipion spectrum

Vague phase motion in a phase-shift analysis

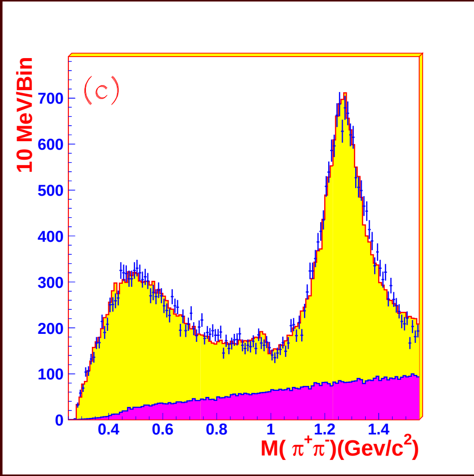


Gómez Nicola and Peláez PRD2002

Long time a theorist's game

- Beveren, Ribeiro, Rupp
- Dobado, Peláez
- Oller, Oset
- Schechter, Black, Sannino
- ...

BES II, PLB 2004



$$J/\psi \rightarrow \omega + \pi^+\pi^- \qquad 1^{--} \rightarrow 1^{--} + J^{++}$$

Pole position precisely known (for $N_c = 3$)

Group	$M_\sigma (MeV)$
Mennessier, Narison and Wang'10	$452(12) - i260(15)$
G.-Martín, Kaminsky, Peláez, Yndurain'08	$458(15) - i262(15)$
Caprini, Colangelo, Leutwyler'06	$441^{+16}_{-8} - i272^{+9}_{-13}$
PDG	$(400:1200) - i800(200)$

What is its Fock space decomposition?

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The problem: Expansion of hadron state in terms of q, g

$$|H\rangle = \alpha_{q\bar{q}}|q\bar{q}\rangle + \alpha_{gg}|g\bar{g}\rangle + \alpha_{qq\bar{q}\bar{q}}|qq\bar{q}\bar{q}\rangle$$

Is this question well posed?

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Is this question well posed?

Fixed frame and gauge: Coulomb-gauge QCD

Only quarks and transverse gluons!

$$H = \int \left[\frac{1}{2}\mathbf{E}_a^{tr}\mathbf{E}^{a\,tr} + \frac{1}{2}\mathbf{B}^a\mathbf{B}_a\right]$$

$$+ \int \overline{\Psi}[\gamma\cdot(\nabla - ig_0 T^a \mathbf{A}_a) + m]\Psi d\mathbf{x}$$

$$+\frac{1}{2}g_0^2 \int\int \rho^a(\mathbf{x})v(|\mathbf{x}-\mathbf{y}|)_{aa'}\rho^{a'}(\mathbf{y})d\mathbf{x}d\mathbf{y}$$

Color density:

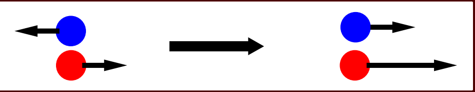
$$\rho^a(\mathbf{x}) \;=\; \Psi^\dagger(\mathbf{x})T^a\Psi(\mathbf{x}) + f^{abc}\mathbf{A}^b(\mathbf{x})\cdot\mathbf{\Pi}^c(\mathbf{x})$$

(Schwinger 1964; Christ and Lee 1980)

Boost operators

The Fock space expansion of a state is *frame dependent*

$$\phi \to \eta' \gamma \quad \text{(KLOE)}$$
$$\psi \to \eta' \gamma \quad \text{(BESIII)}$$



$$|P\rangle = e^{i\zeta(P)\hat{\vec{K}}_{\rm QCD}}|0\rangle$$

Gómez-Rocha, Ll-E, Schütte, Villalba-Chávez EPJA 2010.

Boost operators

$$
\begin{aligned}
\hat{\vec{K}}_{\rm QCD} \;=\; & -\int d^3x \left\{ \frac{1}{2}\mathcal{J}^{-1}\hat{\vec{\Pi}}^a \mathcal{J}\vec{x}\hat{\vec{\Pi}}^a + \frac{1}{2}\hat{\vec{B}}^a\vec{x}\hat{\vec{B}}^a \right. \\
& - \frac{1}{2}g^2\mathcal{J}^{-1}\hat{\varrho}^b\mathcal{J}\hat{\mathrm{M}}^{ba}(\partial_k\vec{x}\partial_k)\hat{\mathrm{M}}^{ac}\hat{\varrho}^c \\
& \left. + \frac{1}{2}g\mathcal{J}^{-1}\hat{\vec{\Pi}}^a\mathcal{J}\hat{\mathrm{M}}^{ab}\hat{\varrho}^b + \frac{1}{2}g\mathcal{J}^{-1}\hat{\varrho}^b\mathcal{J}\hat{\mathrm{M}}^{ba}\hat{\vec{\Pi}}^a \right\} \\
& - \int d^3x\vec{x}\left\{ \hat{\mathfrak{q}}^{\ell\dagger}(\vec{x})\left(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m\right)\hat{\mathfrak{q}}^{\ell}(\vec{x}) \right. \\
& \left. + g\hat{\mathfrak{q}}^{\ell\dagger}(\vec{x})\frac{\lambda^a}{2}\vec{\alpha}\cdot\hat{\vec{A}}^a(\vec{x})\hat{\mathfrak{q}}^{\ell}(\vec{x}) \right\} \\
& + \int d^3x\hat{\mathfrak{q}}^{\ell\dagger}(\vec{x})\left(\frac{i}{2}\vec{\alpha}\right)\hat{\mathfrak{q}}^{\ell}(\vec{x})
\end{aligned}
$$

Large N_c behavior: Gauge,Frame independence

State	M	$\Gamma_{\pi\pi}$
$\pi\pi$, $q\bar{q}q\bar{q}$	$O(1)$	$\mathrm{O}(1)$
$q\bar{q}$	$O(1)$	$O(1/N_c)$
gg	$O(1)$	$O(1/N_c^2)$
$(N_c-1)(q\bar{q})$	$O(N_c)$	$O(e^{-N_c})$

Different behavior of width may help with expansion

But where from do we get $\Gamma_\sigma(N_c)$?

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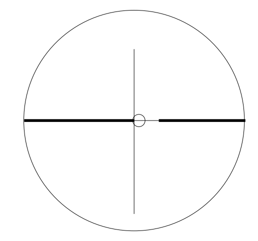
Outline

Write dispersion relation for $G \equiv \frac{(t^{(2)})^2}{t}$

With ChPT notation for $\pi\pi$ scattering $t = t^{(2)} + t^{(4)} + t^{(6)} \dots$

$$G(s) = LC(G) + PC + G(0) + G'(0)s + \frac{1}{2}G''(0)s^2 + \frac{s^3}{\pi}\int_{RC} ds' \frac{\text{Im } G(s')}{s'^3(s'-s)}$$



Inverse Amplitude Method

We now simplify this formula near the right cut

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- In the elastic region $2m_\pi < E < 2m_K$, $\mathrm{Im}\, G = -\mathrm{Im}\, t_4$ is *exact*

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- The contributions of the poles (Adler zeroes of t) and left cut are very small; aprox. left cut in ChPT
- In the elastic region $2m_\pi < E < 2m_K$, $\text{Im}\, G = -\text{Im}\, t_4$ is *exact*
- Use Chiral Perturbation Theory for the subtraction constants

Inverse Amplitude Method

(Truong, Dobado, Herrero, Peláez...)

- ChPT to $O(p^4)$:

$$t(s) \simeq \frac{t_2^2(s)}{t_2(s) - t_4(s)}$$

- ChPT to $O(p^6)$:

$$t \simeq \frac{t_2^2}{(t_2 - t_4 + t_4^2/t_2 - t_6)}$$

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Poles in the complex s-plane

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- Zeroes of denominator $\rightarrow$ elastic $\pi\pi$ resonance
- Position of resonance controlled by ChPT
- Hence, know color scaling $f_\pi \rightarrow f_\pi \sqrt{\frac{N_c}{3}}$, $l_i \rightarrow l_i \frac{N_c}{3}$

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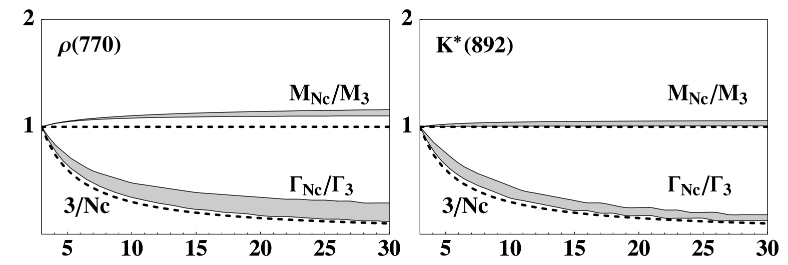
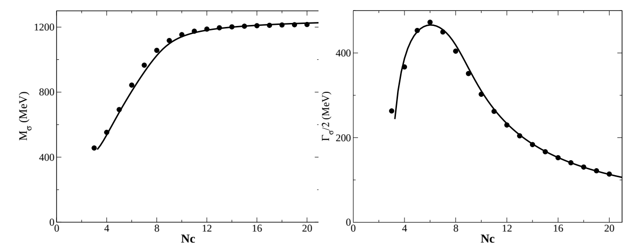
$q\bar{q}$-like mesons

Vector ρ, K^* mesons with three-flavor ChPT



Peláez and Ríos 2006

σ mass and width in $SU(2)$ UChPT at $O(p^6)$



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Simple mixing model: the world in 3 states

Feshbach decomposition of Fock space

- P-space: $|q\bar{q}\rangle$, $|gg\rangle$, $|qq\bar{q}\bar{q}\rangle$
- Q-space: $\pi\pi$ continuum

$$H = \begin{pmatrix} H_{PP} & H_{PQ} \\ H_{QP} & H_{QQ} \end{pmatrix}$$

Simple mixing model: the world in 3 states

3×3 Hamiltonian restricted to the PP sector:

$$H_{eff} = H_{PP} + H_{PQ} \frac{1}{E - H_{QQ} + i\epsilon} H_{QP}$$

- Symmetric (CP) but not Hermitian (decay to Q space)
- Leading-N_c behavior known, coefficients fitted to UChPT
- Diagonalize and obtain M_σ, Γ_σ

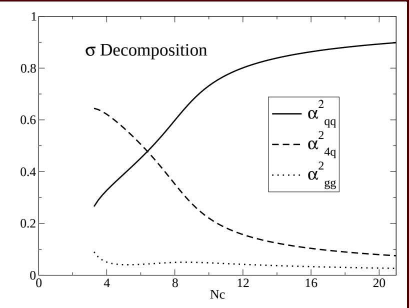
Procedure

1. Experimental data → Roy equations → σ mass and width
2. Pole → UChPT → Large N_c
3. Fit UChPT with 3-state model

Fit with precoefficients of order 1 (naturalness)

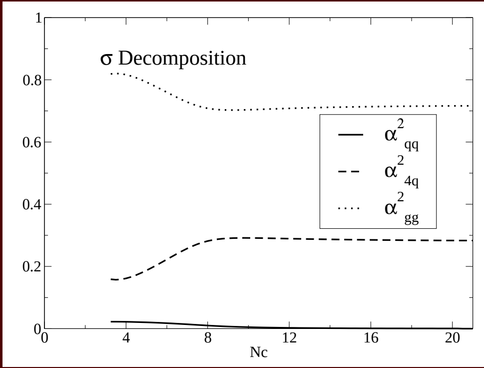
Define *natural* as the coefficient of N_c^α being of order 1,

$$H_{ij} = h_{ij} \times N_c^{\alpha_{ij}} \quad h_{ij} \in \left(\frac{1}{\sqrt{N_c}}, \sqrt{N_c}\right)$$



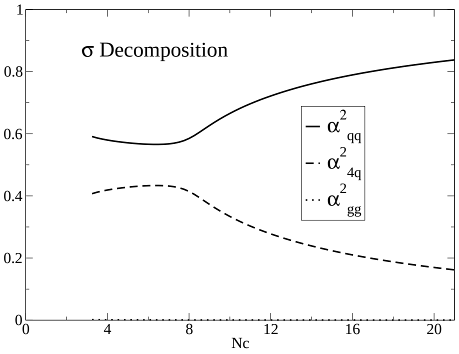
- At $N_c = 3$: mostly molecule
- At $N_c \simeq 10$: mostly $q\bar{q}$
- *gg*-like component never bigger than 10%
- Only two parameters "unnatural"

Can one make the intrinsic component sizeable?



If *gg* dominates, 4 parameters are unnatural (two of them are outright nonsense)

Can one make the intrinsic component sizeable?



If $q\bar{q}$ dominates, 6 parameters are unnatural (3 are nonsense)

Summary

- Many mesons might seem $q\bar{q}$: σ is not one
- Fock expansion is frame, gauge dependent
- Resort to N_c counting
- Unitarized chiral perturbation theory disfavors σ as a $q\bar{q}$, gg...
- Simple model quantifies meson-meson molecular component in σ about 50% at $N_c = 3$

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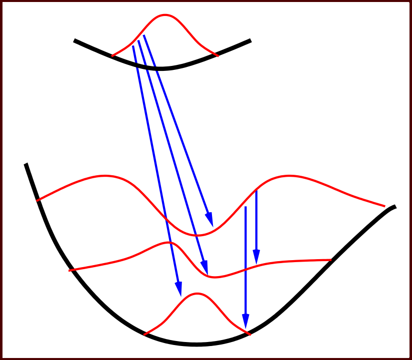
Hirschegg, 21/01/2011

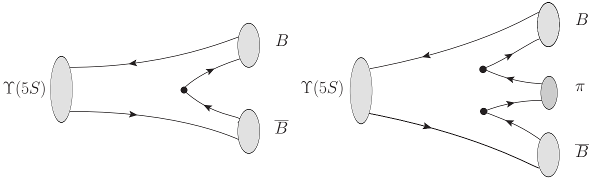
An idea presented at Hirschegg 2007...

Franck-Condon principle in molecular physics



Heavy Quark Fluorescence

- "The velocity distribution of heavy mesons containing one heavy quark each equals the velocity distribution of those heavy quarks inside the parent hadron"



- Learn about confined quarks from hadron distributions
- Leading HQET, NRQCD Lagrangians encompass the principle (quark velocity does not change).

FLLE, Cotanch, General, Wang 2007

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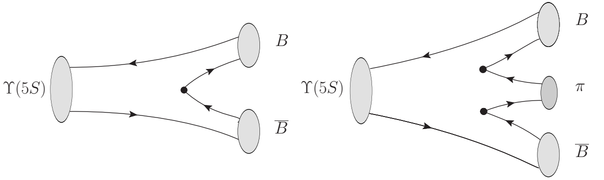

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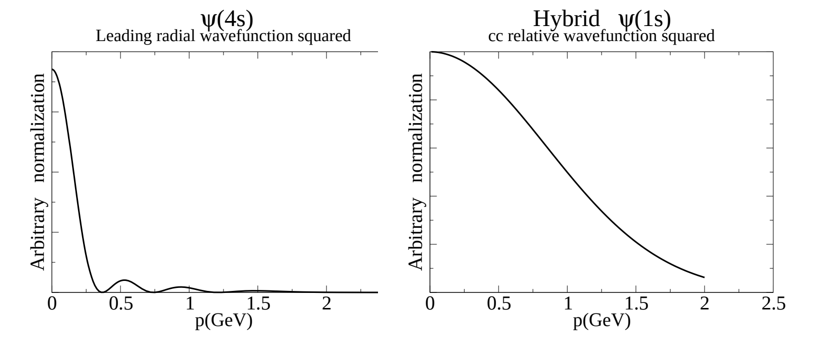
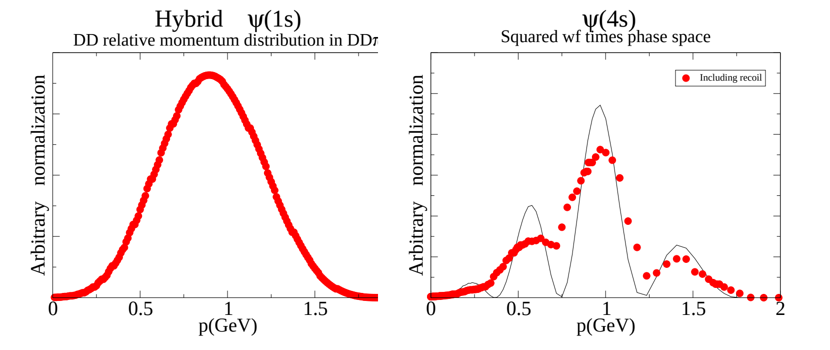
FLLE, Cotanch, General, Wang 2007

XYZ... Distinguishing $q\bar{q}$ mesons from hybrids

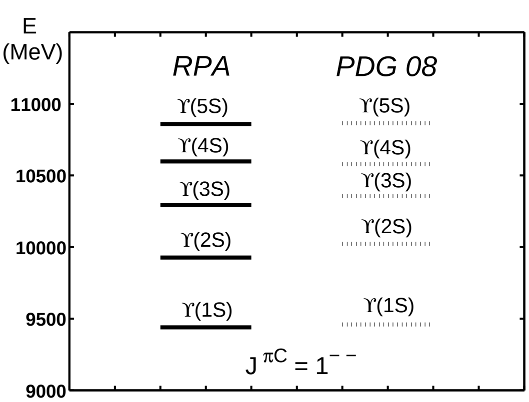
Relative $q\bar{q}$ wavefunction
in an excited $q\bar{q}$ meson or a $q\bar{q}g$ hybrid



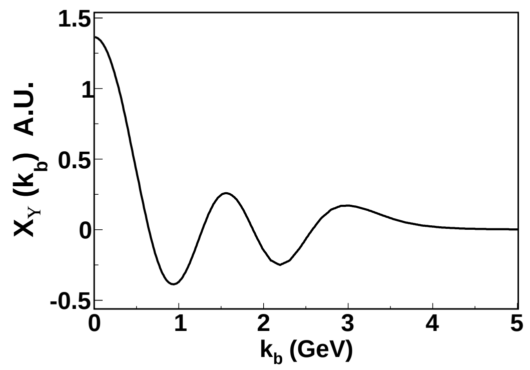
$q\bar{q}$ meson or hybrid?



BELLE data at $\Upsilon(10860)$



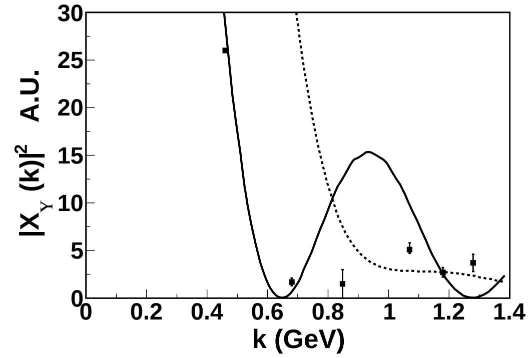
Wavefunction of Upsilon(5s) in Coulomb gauge model



(Not all accessible due to phase space)

2-Body data

First puzzle we explain: $\Gamma_{B_s^* B_s^*} = 12\Gamma_{B_s B_s^*}$ instead of $7/4$ (from spin counting)



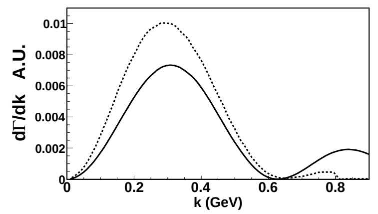
(LL-E and Torres-Rincón 2010)

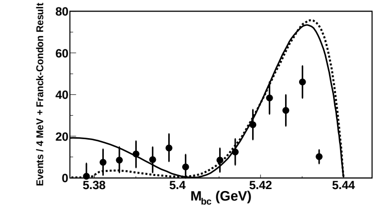
3-Body decays $B\bar{B}\pi$ (or $D\bar{D}\pi$)

- The pion recoils and the heavy meson momentum swipes the entire (open) phase space.
- The Franck-Condon principle is not frustrated: copious decays

For the $Y(10860)$, $b\bar{b}$ ok

Instead of k, experimentals like $M_{bc} \equiv \sqrt{(M_\Upsilon/2)^2 - k^2}$





(LL-E and Torres-Rincón, PRL 2010; expt. data from Belle)