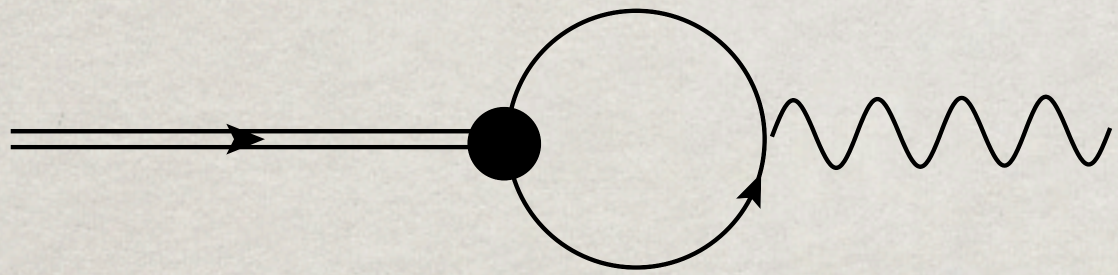
Strong & Electroweak Hadronic Decays

Eric Swanson



Electroweak Transitions

Decay Constants



no approximations other than specific model of the state e+e− width of the ψ(3770). Namely, the large decay constant fψ(3770) = 99 ± 20 MeV can perhaps be explained by mixing with nearby S-wave states. Again, the computed effect due to the tensor interaction is an order of magnitude too small

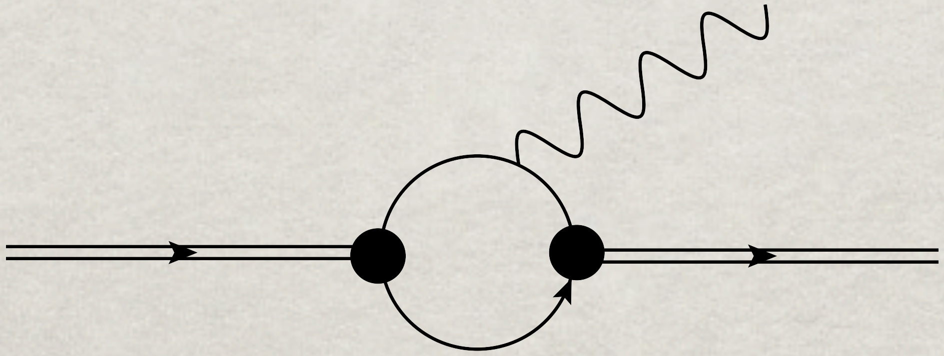
Constants

$$\alpha_C \to \alpha_C(k) = \frac{4\pi}{\beta_0 \log(\mathrm{e}^{-\frac{4\pi}{\beta_0 \alpha_0}} + \frac{k^2}{\Lambda^2})}$$

Meson	BGS NonRel	BGS Rel	BGS log $\Lambda = 0.4$ GeV	BGS log $\Lambda = 0.25$ GeV	lattice	experiment
η_c	795	493	424	402	$429 \pm 4 \pm 25$	335 ± 75
η_c'	477	260	243	240	$56 \pm 21 \pm 3$	
η_c''	400	205	194	193		
J/ψ	615	545	423	393	399 ± 4	411 ± 7
$\psi(2S)$	431	371	306	293	143 ± 81	279 ± 8
$\psi(3S)$	375	318	267	258		174 ± 18
χ_{c1}	239	165	155	149		
χ_{c1}'	262	167	157	152		
χ_{c1}''	273	164	155	151		

Lakhina & Swanson, PRD74, 014012 (06)

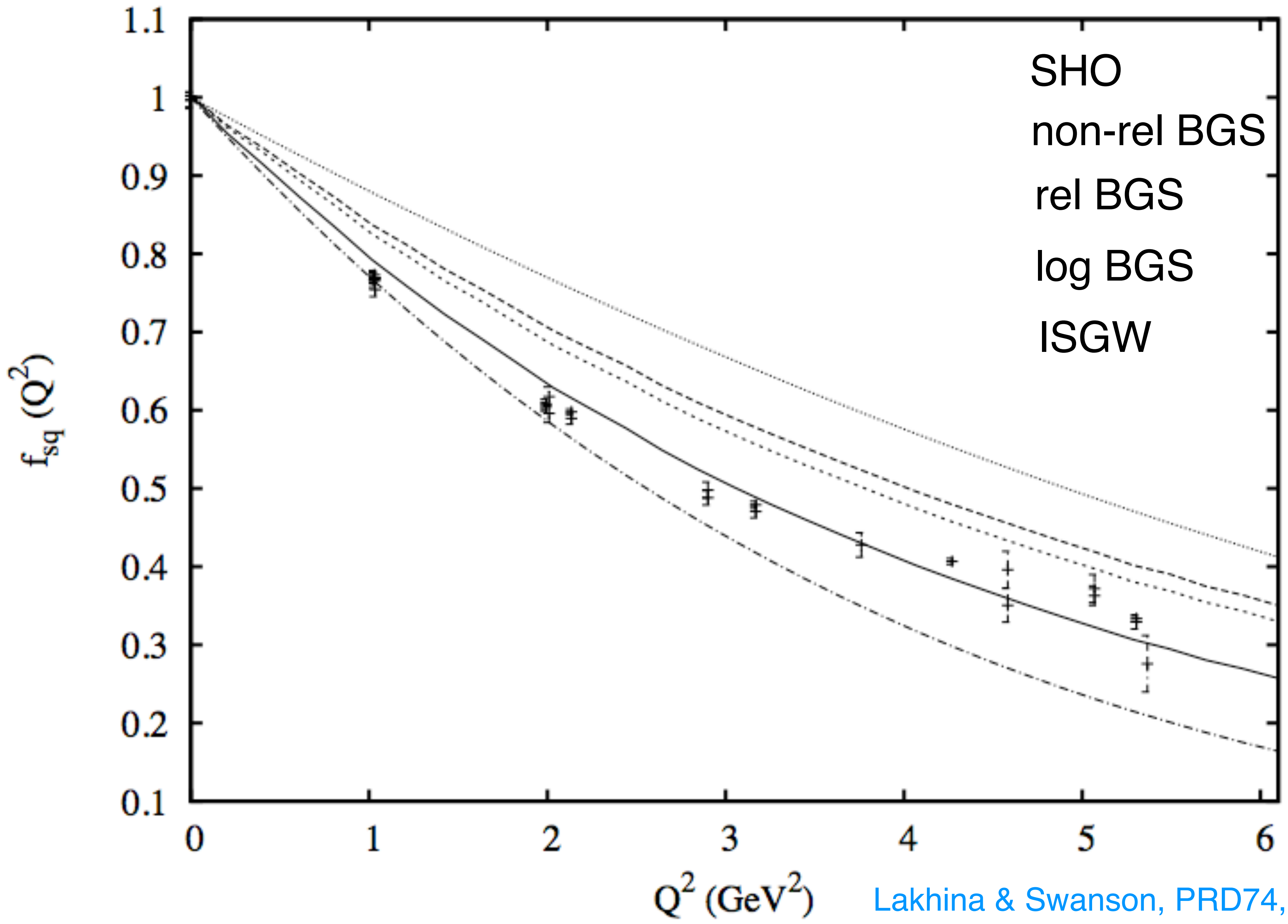
Elastic Photo-transitions



'impulse approximation': neglects vertex corrections and three-body diagrams

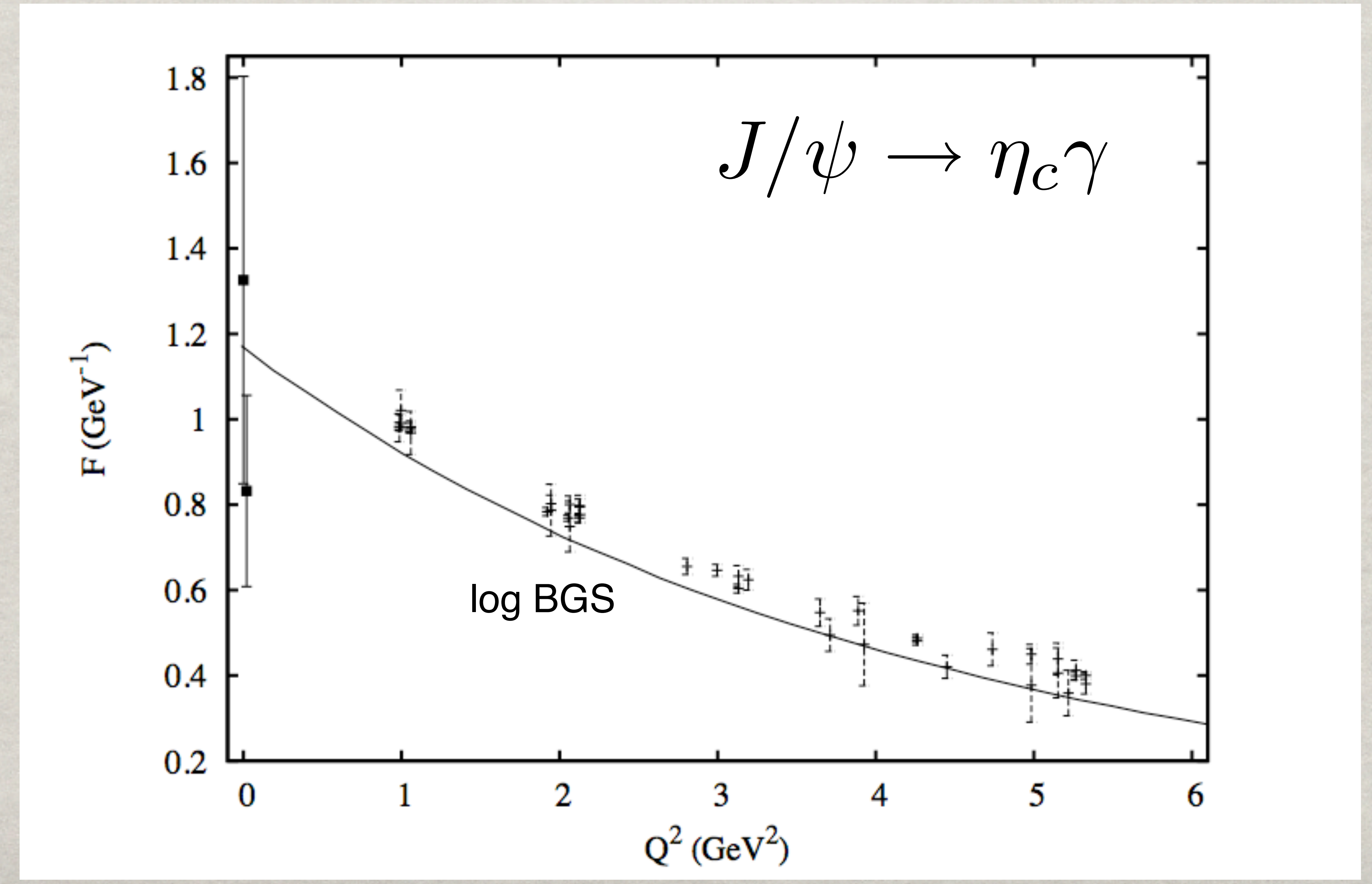
Elastic Photo-transitions

$\eta_c \to \eta_c \gamma$

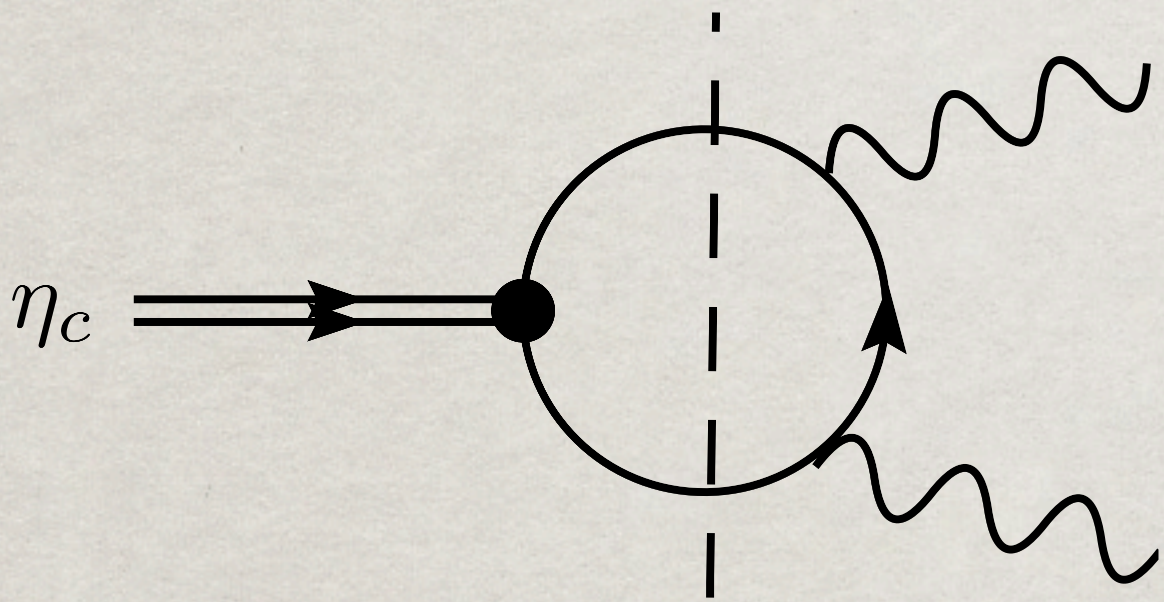


Lakhina & Swanson, PRD74, 014012 (06)

Inelastic Photo-transitions



γγ Decays

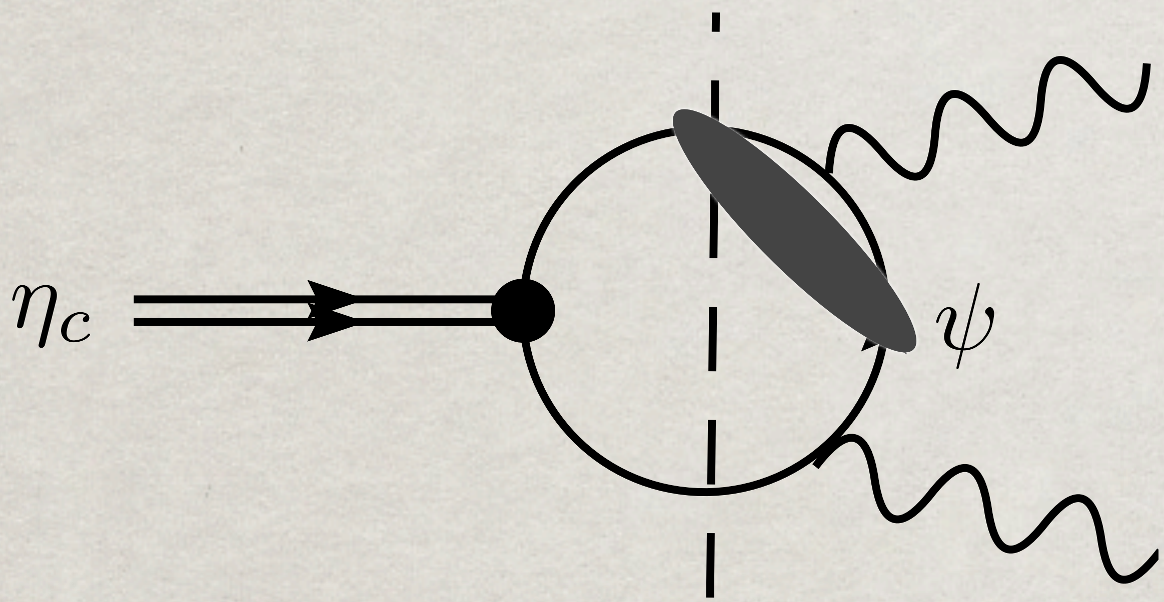


naive factorisation
Low's theorem
m_{Ps}^3
oblique cuts

- $\mathcal{L} = g \int \eta F^{\mu\nu} \tilde{F}_{\mu\nu}$
- $\Gamma(\eta \to \gamma\gamma) \propto g^2 m_\eta^3$

m_η does not enter the quark model

γγ Decays



combines previous two calcs

get m^3 behaviour!

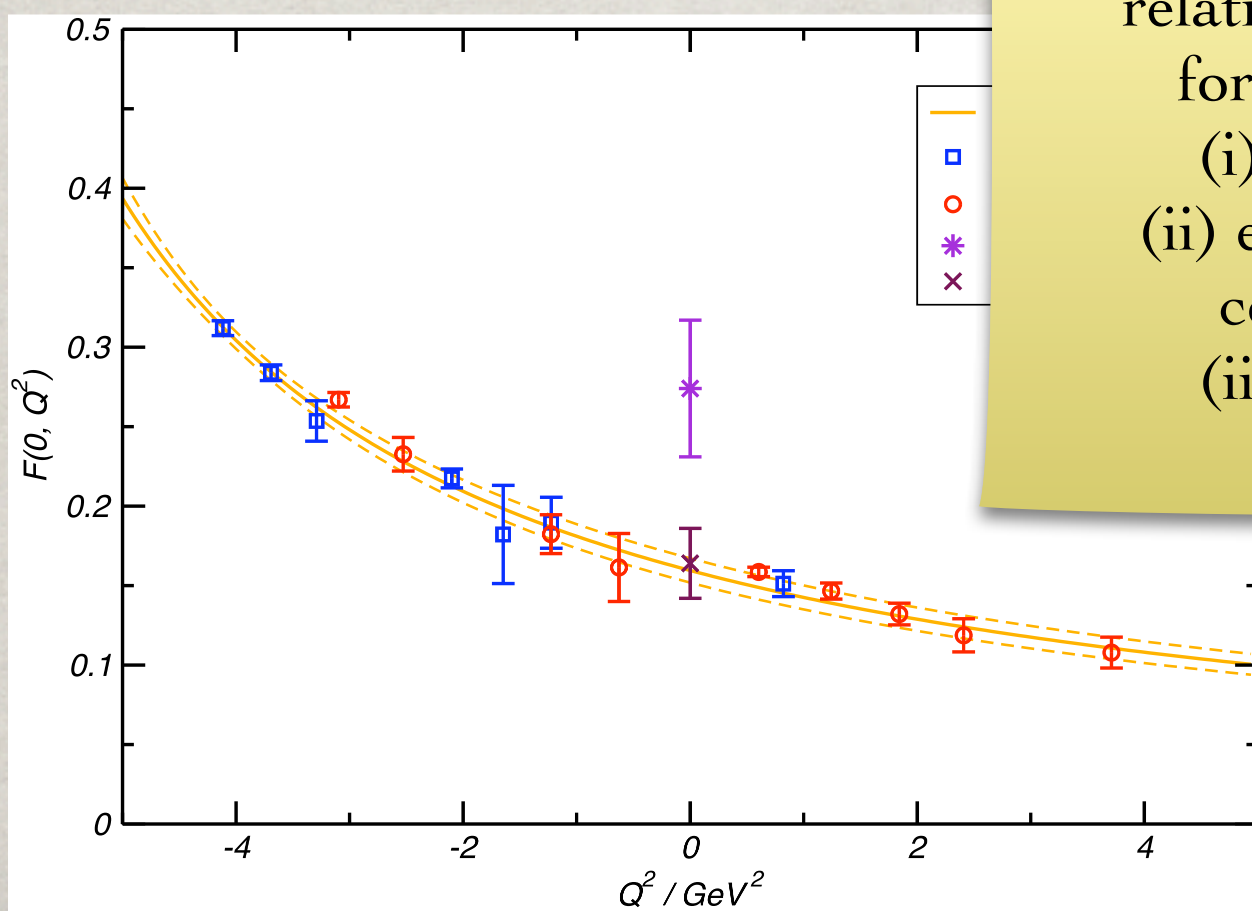
$$\Gamma(\eta \to \gamma\gamma) = \frac{m_\eta^3 Q^4}{16\pi} \left[\sum_\psi \frac{f_\psi F_\psi(0)}{m_\psi} \right]^2$$

γγ Decays

Table 1: Charmonium Two-photon Decay Rates (keV).

process	BGS	BGS log	G&I	HQ	A&B	EFG	Munz	Chao	CWV	PDG
$\eta_c \to \gamma\gamma$	14.2	7.18	6.76	7.46	4.8	5.5	3.5(4)	6-7	6.18	7.44 ± 2.8
$\eta_c' \to \gamma\gamma$	2.59	1.71	4.84	4.1	3.7	1.8	1.4(3)	2	1.95	1.3 ± 0.6
$\eta_c'' \to \gamma\gamma$	1.78	1.21	–	–	–	–	0.94(23)	–	–	–
$\chi_{c0} \to \gamma\gamma$	5.77	3.28	–	–	–	2.9	1.39(16)	–	3.34	2.63 ± 0.5

Lakhina & Swanson, PRD74, 014012 (06)



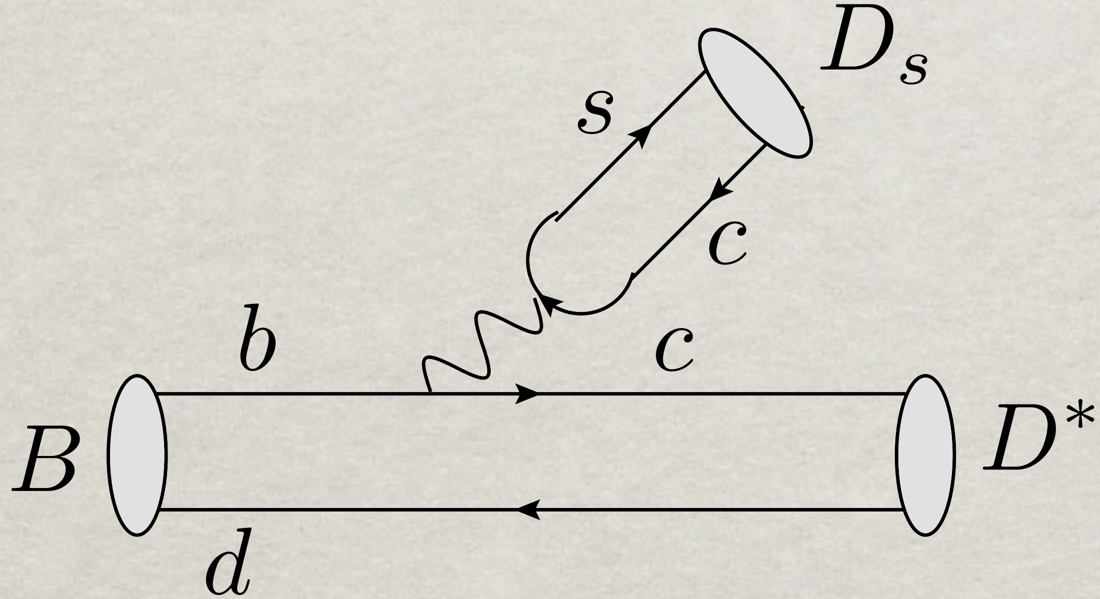
the use of a running coupling, relativistic effects, and bound state formalism obviate the need for
(i) ISGW ‘correction factors’
(ii) energy ‘prescriptions in decay constant calculations (m/E)
(iii) phase space modification

J.J. Dudek, 0711.1600

B Decays-FSIs

a 'colour enhanced' decay

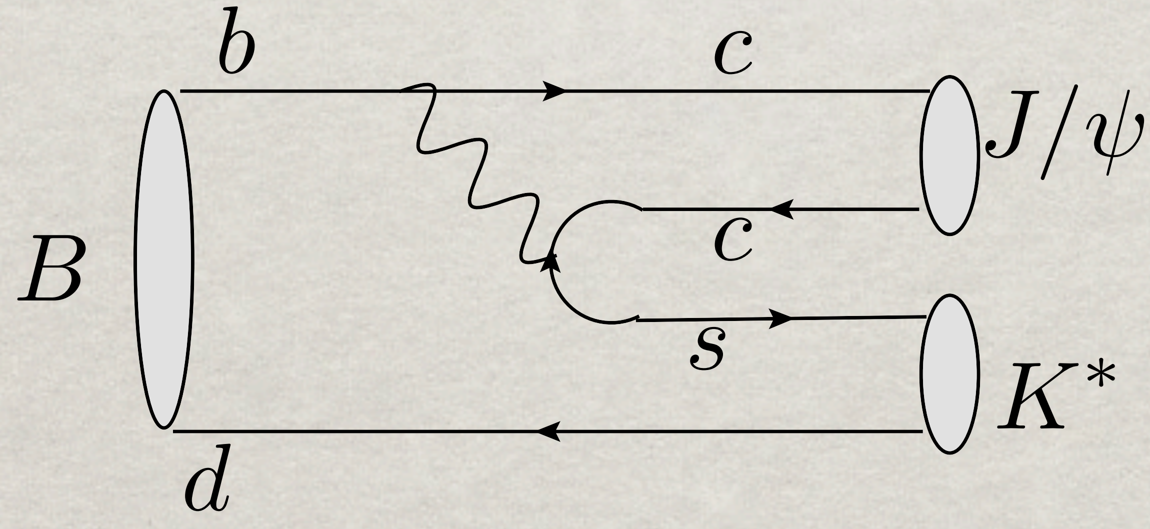
$$\mathcal{A} \sim N_c^2$$



B Decays-FSIs

A ‘colour suppressed’ decay

$$\mathcal{A} \sim N_c$$



B Decays-FSIs

with the factorisation approximation:

$$B \not\to \chi_{c0} K$$

chi0 thru V0, but current conservation implies the decay constant is zero

but experimentally

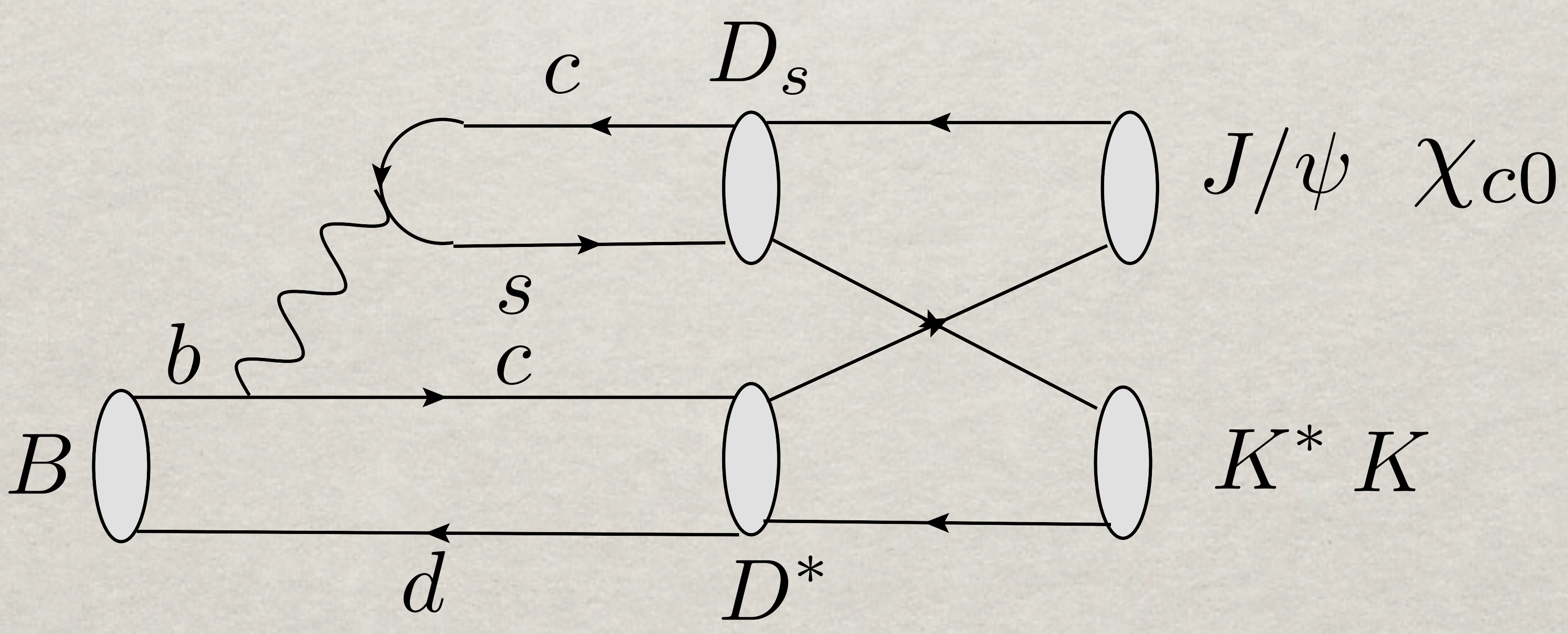
$$B^+ \to \chi_{c0} K^+ = (6.0 \pm 2 \pm 1.1) \cdot 10^{-4} (\text{Belle})$$

$$B^+ \to \chi_{c0} K^+ = (2.7 \pm 2 \pm 0.7) \cdot 10^{-4} (\text{BaBar})$$

What is going on?

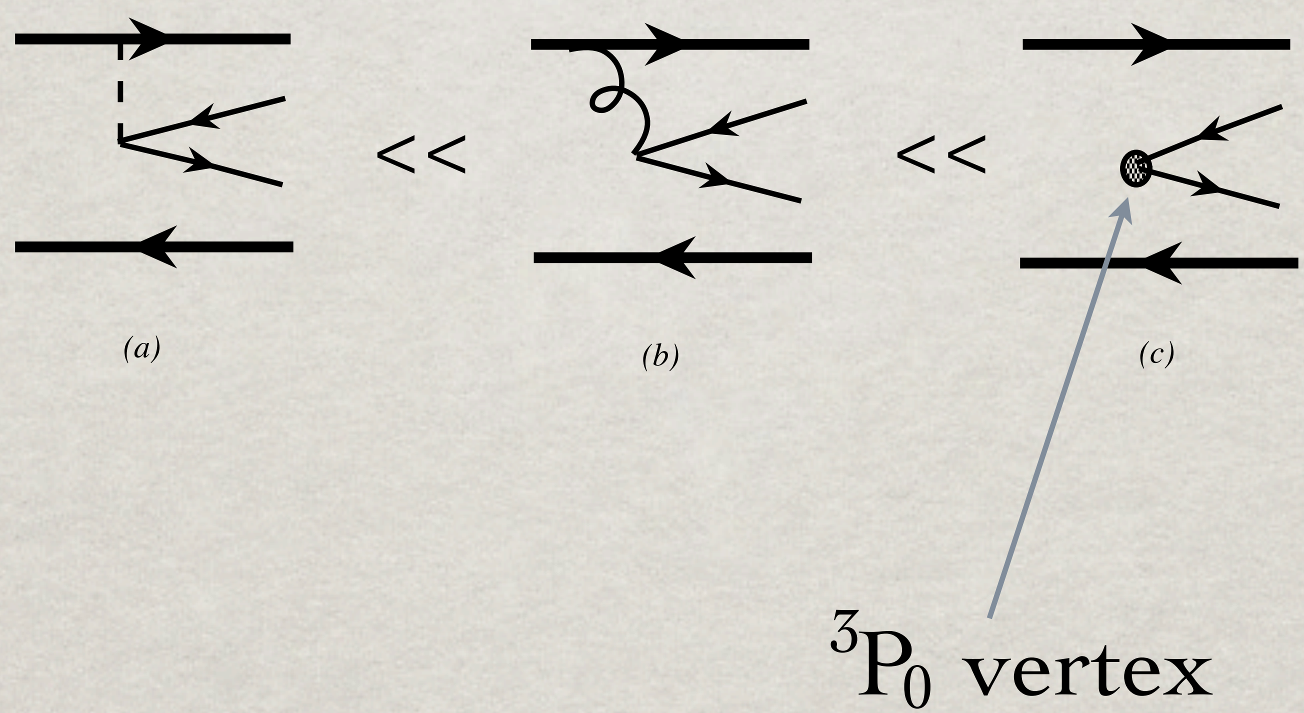
B Decays-FSIs

final state interactions with colour enhanced driving



Strong Decays

Strong Decays

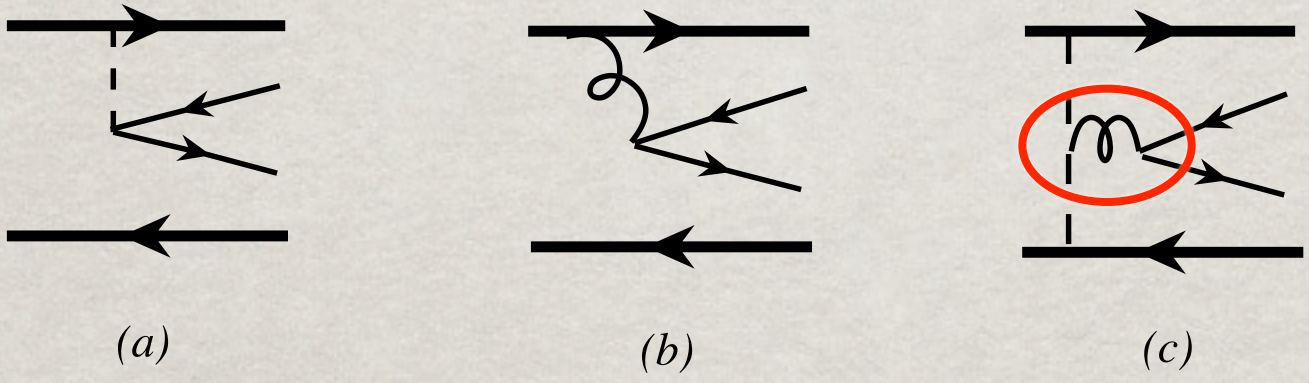


Ackleh, Barnes, & ES, PR**D54,** 6811 (96)

Strong Decays

Szczepaniak & Swanson, PRD65, 025012 (01)

all diagrams to $O(\Lambda_{QCD}/m_g)$



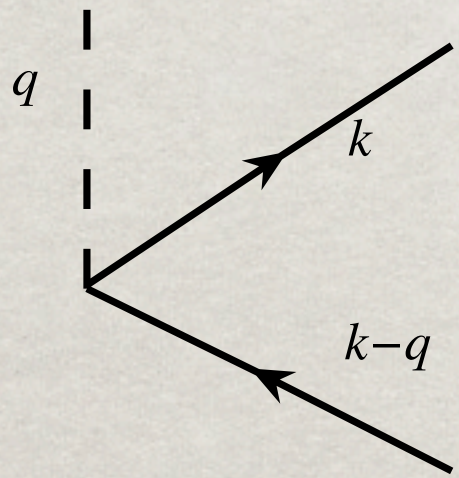
$$H_{qg}K^{(1)} \to \psi^\dagger \alpha \cdot \mathbf{A}\psi f \mathbf{A} \cdot \nabla \to f\psi^\dagger \alpha \cdot \nabla \psi$$

similar to $^3P_0$ model

Strong Decays

ie, the leading (coulomb gauge) term is suppressed

Coulomb vertex



$$K^{(0)}\psi^\dagger\psi \to \frac{b}{q^4}\frac{1}{m}\boldsymbol{\sigma}\cdot\mathbf{q}\, b^\dagger_k d^\dagger_{k-q}$$

$$K^{(0)}\bar{\psi}\psi \to \frac{b}{q^4}\frac{1}{m}\boldsymbol{\sigma}\cdot(2\mathbf{k}-\mathbf{q})\, b^\dagger_k d^\dagger_{k-q}$$

confinement severely damps the integral over *q*

Strong Decays -- IKP Model

quark creation operator

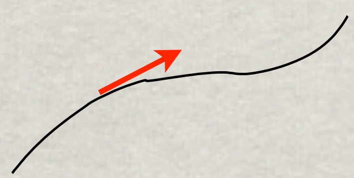
Kokoski & Isgur, PRD35, 907 (87)

Isgur, Kokoski, & Paton, PRL54, 869 (85)

$$H = \frac{g^2}{2a}\sum_{\ell} E^a_\ell E_{a\ell} + \sum_n m\bar{\psi}_n\psi_n + \frac{1}{a}\sum_{n,\mu}\psi^\dagger_n\alpha_\mu U_\mu(n)\psi_{n+\mu} + \frac{1}{ag^2}\sum_P \mathrm{tr}(N - U_P - U^\dagger_P)$$

$$H_{int} \sim \psi^\dagger_n \alpha\cdot\mu\psi_{n+\mu}$$

$$\sim \psi^\dagger_n\alpha\cdot\mu\psi_n + a\psi^\dagger_n\alpha\cdot\mu\,\mu\cdot\nabla\psi_n$$



$\psi^\dagger_n\alpha\cdot\mu\psi_n$ $\psi^\dagger_n\alpha\cdot\nabla\psi_n$

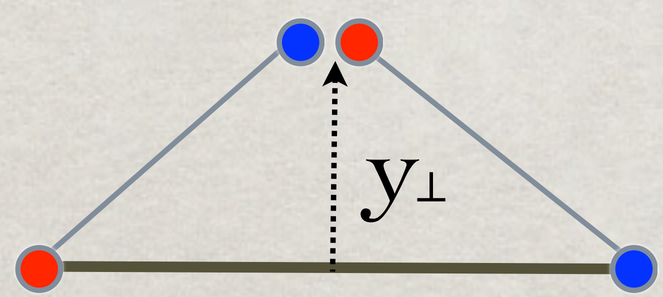
$^3S_1$ $^3P_0$

Strong Decays -- IKP Model

meson decay

$$<\{0...0\}bd;\ \{0...0\}bd|O|\{0...0\}b^\dagger d^\dagger> \quad \sim \quad <bd;\ bd|{}^3P_0|b^\dagger d^\dagger> \ \cdot$$

$$<\{0...0\};\ \{0...0\}|\{0...0\}>$$



$$\downarrow$$

$$e^{-fby_\perp^2}$$

hybrid decay

$$<\{0...0\}bd;\ \{0...0\}bd|O|\{1,0...0\}b^\dagger d^\dagger> \quad \sim \quad <bd;\ bd|{}^3P_0|b^\dagger d^\dagger> \ \cdot$$

$$<\{0...0\};\ \{0...0\}|\{1,0...0\}>$$

$$y_\perp\, e^{-fby_\perp^2} \ \swarrow$$

Strong Decays -- Vector Decay Model

Szczepaniak & Swanson, PRD55, 3987 (97)

Page, Szczepaniak & Swanson, PRD59, 014035 (99)

map chromofields to phonon degrees of freedom

$$E^a_\lambda(n) = \frac{\kappa}{a^3}\left(y^a_\lambda(n+1) - y^a_\lambda(n)\right)$$

$$B^a_\lambda(n) = \frac{-i}{\kappa a}\frac{\partial}{\partial y^a_\lambda(n)} \qquad \kappa = a\sqrt{b_0}$$

$$B^a_\lambda(n) = \frac{-i}{\kappa}\sqrt{\frac{b_0}{r}}\sum_m \sin\frac{m\pi}{N+1} n\sqrt{\omega_m}\left(\alpha^a_{m\lambda}\mathrm{e}^{-i\omega_m t} - \alpha^{a\dagger}_{m\lambda}\mathrm{e}^{i\omega_m t}\right)$$

Strong Decays -- Vector Decay Model

use the same mapping to obtain $\bar{\psi}\alpha \cdot A\psi$

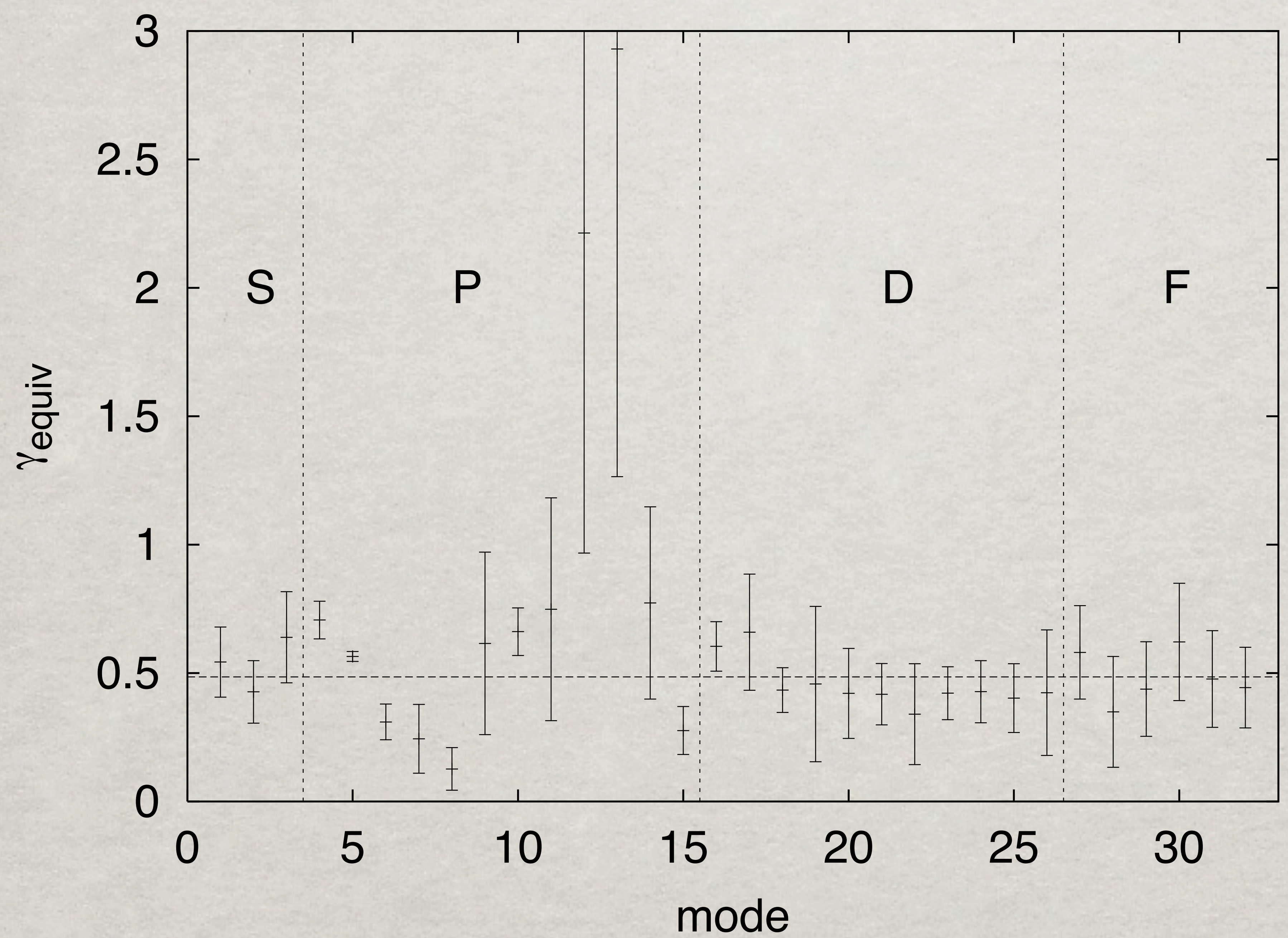
$$H_{int} = \frac{iga^2}{\sqrt{\pi}} \sum_{m,\lambda} \int_0^1 d\xi \cos(\pi\xi) T^a_{ij}\, h_i^\dagger(\xi \mathbf{r}_{Q\bar{Q}})\sigma \cdot \hat{\mathbf{e}}_\lambda(\hat{\mathbf{r}}_{Q\bar{Q}}) \left(\alpha^a_{m\lambda} - \alpha^{a\dagger}_{m\lambda}\right) \chi_j(\xi \mathbf{r}_{Q\bar{Q}})$$

$$\langle H|H_{int}|AB\rangle = i\frac{ga^2}{\sqrt{\pi}}\frac{2}{3}\int_0^1 d\xi \int d\mathbf{r} \cos(\pi\xi)\sqrt{\frac{2L_H+1}{4\pi}} \mathrm{e}^{\frac{i\mathbf{p}\cdot\mathbf{r}}{2}} \varphi_H(r)\varphi_A^*(\xi\mathbf{r})\varphi_B^*((1-\xi)\mathbf{r}) \cdot \left[\mathcal{D}^{L_H *}_{M_L \Lambda}(\phi,\theta,-\phi)\chi^{PC}_{\Lambda,\lambda}\hat{\mathbf{e}}_\lambda(\hat{\mathbf{r}})\cdot\langle\sigma\rangle\right]$$

both models obtain the "S+P" decay selection rule: hybrids cannot decay to two S-wave states with identical spatial wavefunctions.

P.R. Page, PLB401, 313 (97)

$^3P_0$ Strong Decay Model



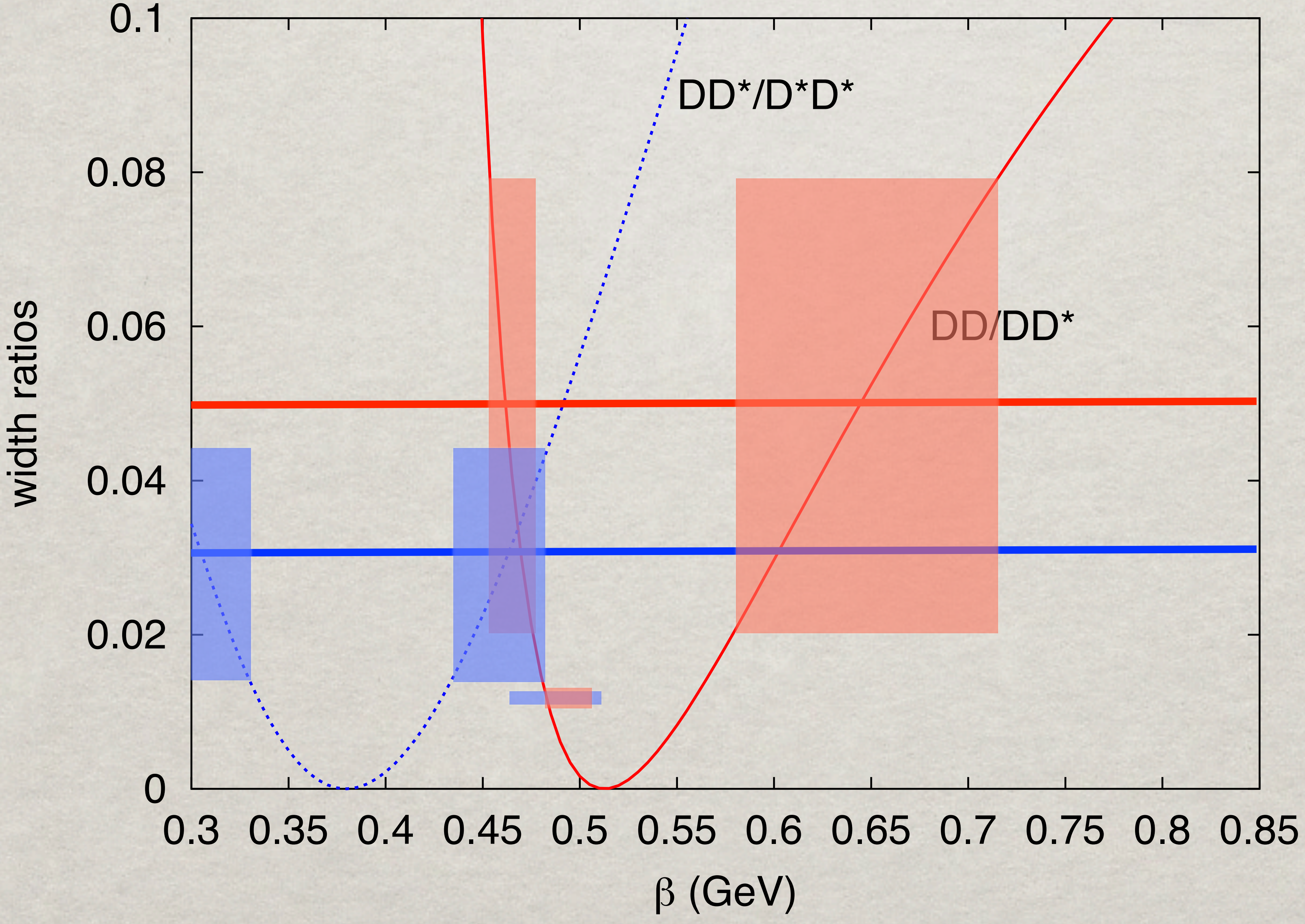
[1] $b_1 \to \omega\pi$
[2] $\pi_2 \to f_2\pi$
[3] $K_0 \to K\pi$
[4] $\rho \to \pi\pi$
[5] $\phi \to K\bar{K}$
[6] $\pi_2 \to \rho\pi$
[7] $\pi_2 \to K^*\bar{K} + cc$
[8] $\pi_2 \to \omega\rho$
[9] $\phi(1680) \to K^*\bar{K} + cc$
[10] $K^* \to K\pi$
[11] $K^{*\prime} \to K\pi$
[12] $K^{*\prime} \to \rho K$
[13] $K^{*\prime} \to K^*\pi$
[14] $D^{*+} \to D^0\pi^+$
[15] $\psi(3770) \to D\bar{D}$
[16] $f_2 \to \pi\pi$
[17] $f_2 \to K\bar{K}$
[18] $a_2 \to \rho\pi$
[19] $a_2 \to \eta\pi$
[20] $a_2 \to K\bar{K}$
[21] $f_2' \to K\bar{K}$
[22] $D_{s2} \to DK + D^*K + D_s\eta$
[23] $K_2 \to K\pi$
[24] $K_2 \to K^*\pi$
[25] $K_2 \to \rho K$
[26] $K_2 \to \omega K$
[27] $\rho_3 \to \pi\pi$
[28] $\rho_3 \to \omega\pi$
[29] $\rho_3 \to K\bar{K}$
[30] $K_3 \to \rho K$
[31] $K_3 \to K^*\pi$
[32] $K_3 \to K\pi$

Close & Swanson, PRD72, 094004 (05)

$^3P_0$ Strong Decay Model

$$\psi(3S) \to D\bar{D},\ D\bar{D}^*,\ D^*\bar{D}^*$$

T. Barnes, S. Godfrey, ESS, PRD72, 054026 (05)



PDG; MarkI, PLB69, 503 (1977)

Lattice Decays

C. McNeile, C. Michael, and P. Pennanen [UKQCD], PRD 65, 094505 (02).

$$1^{-+}(b\bar{b}) \to \eta_b \eta(s\bar{s}) \sim 1 \text{ MeV}$$

$$1^{-+}(b\bar{b}) \to \chi_b \sigma(s\bar{s}) \sim 60 \text{ MeV}$$

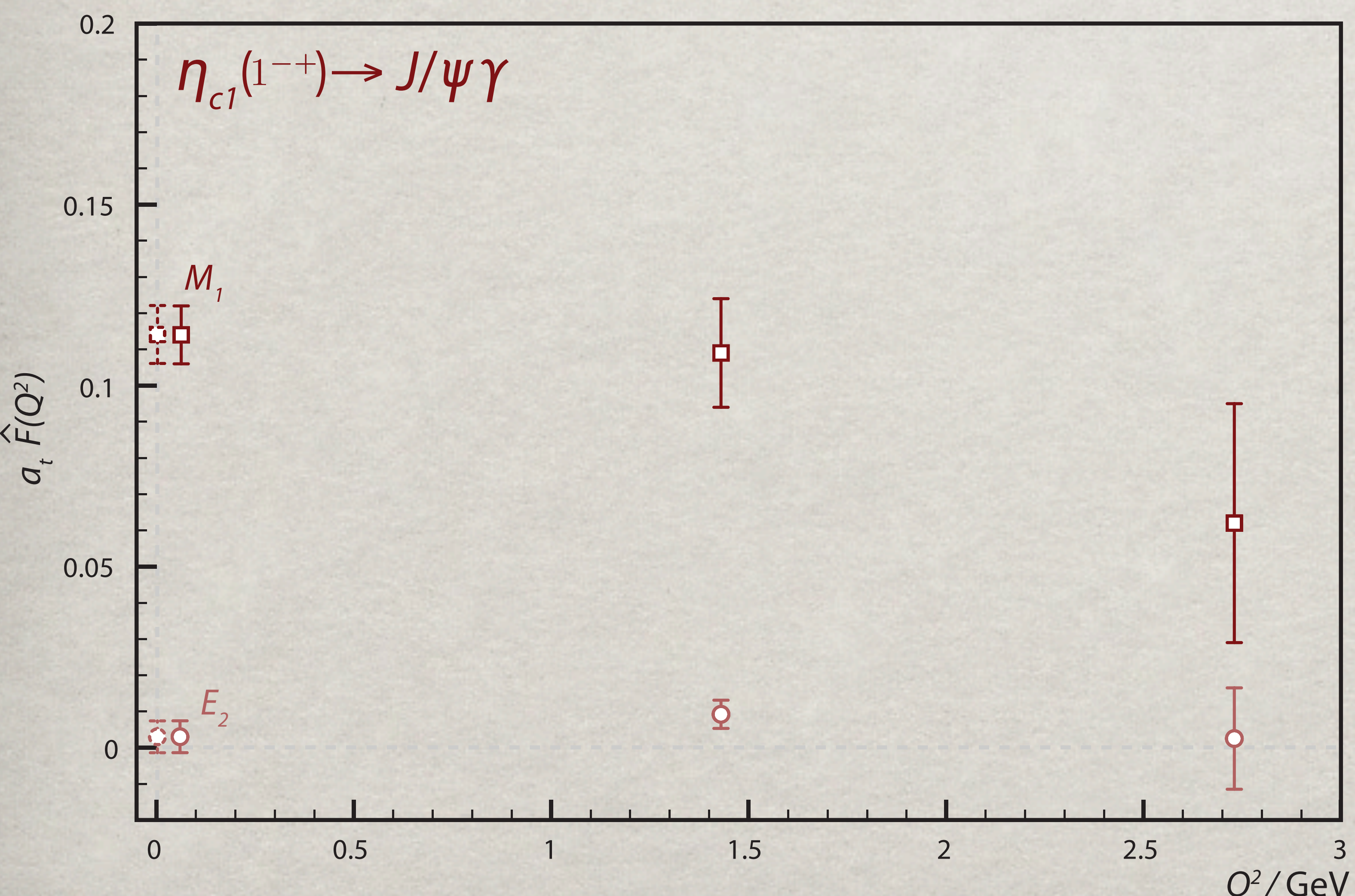
			alt	10.9 GeV hybrid	standard	IKP	reduced
2^{-+}	B^*B	P	.1	0	.5	3	44
1^{-+}	B^*B	P	.1	0	.5	3	44
0^{-+}	B^*B	P	.5	0	2	13	177
1^{--}	B^*B	P	.2	0	1.2	7	88
2^{+-}	B^*B	D	.08	.05	.25	1	22
1^{+-}	B^*B	S	.02	.1	.2	5	13
	B^*B	D	.02	.02	.15	.6	12
1^{++}	B^*B	S	.01	.05	.25	2	7
	B^*B	D	.1	.05	.5	1	24

Lattice Decays

JLab, PRD79, 094504 (09)

$$\Gamma(H(1^{--}) \to \eta_c \gamma) = 42 \pm 18 \text{ keV}$$

$$\Gamma(H(1^{-+}) \to J/\psi \gamma) \approx 100 \text{ keV}$$



this is an M1 decay that is comparable to an E1 decay!

Supports the idea that 1-+ is Sqq = 1 (as in FTM)

flux tube computation finds similar results (30-60 keV for 1-+)

F. Close and J.J. Dudek, PRL91, 142001 (03)

Strong Decays in the Spectrum & Mixing

Hybrid-Nonexotic Mixing

T. Burch and D. Toussaint [MILC], PRD68, 094504 (03)

Compute vector meson - vector hybrid meson mixing in NRQCD on the lattice.

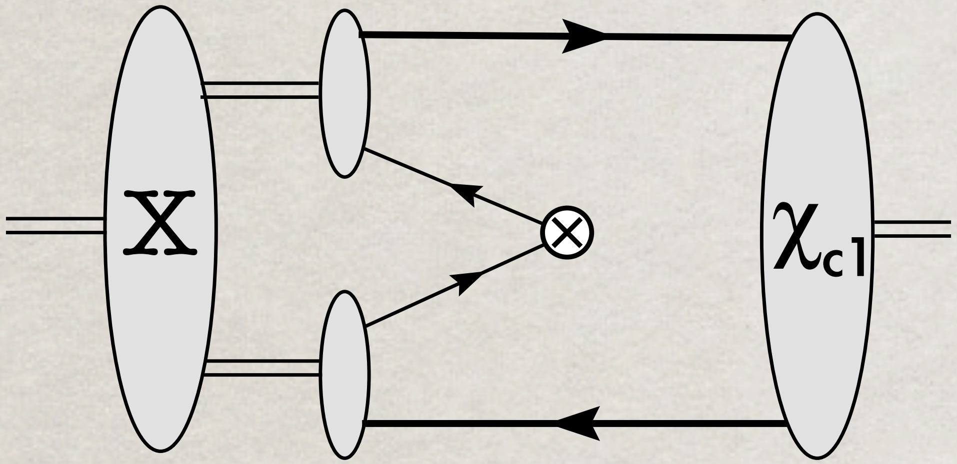
This mixing occurs via $\mathcal{O} = g\frac{\sigma \cdot B}{2M}$

Obtain

$$\Upsilon(H) \approx 0.4\% \qquad \eta_b(H) \approx 1\%$$

$$J/\psi(H) \approx 2.3\% \qquad \eta_c(H) \approx 6\%$$

Exotic Mixing



$$a_\chi = \sqrt{2} Z_{00}^{1/2} \int d^3k \, \psi_X(k) \mathcal{A}(-k)$$

state	E_B (MeV)	a (fm)	Z_{00}	a_χ (MeV)	prob
χ_{c1}	0.1	14.4	93%	94	5%
	0.5	6.4	83%	120	10%
χ_{c1}'	0.1	14.4	93%	60	100%
	0.5	6.4	83%	80	> 100%

Hadronic Mixing

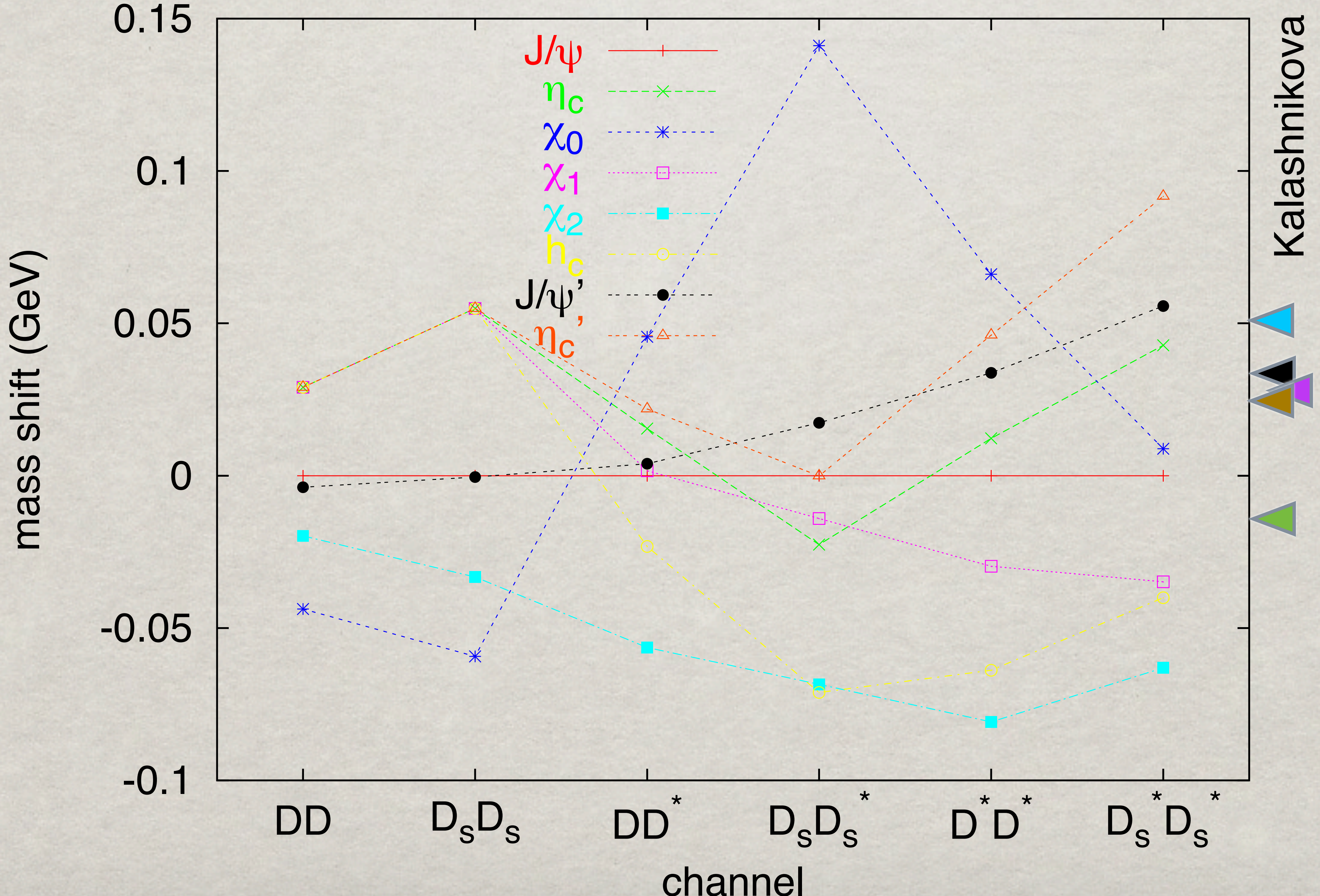
"Oakes-Yang Problem"

R.J. Oakes and C.N. Yang, PRL 11, 174 (63)

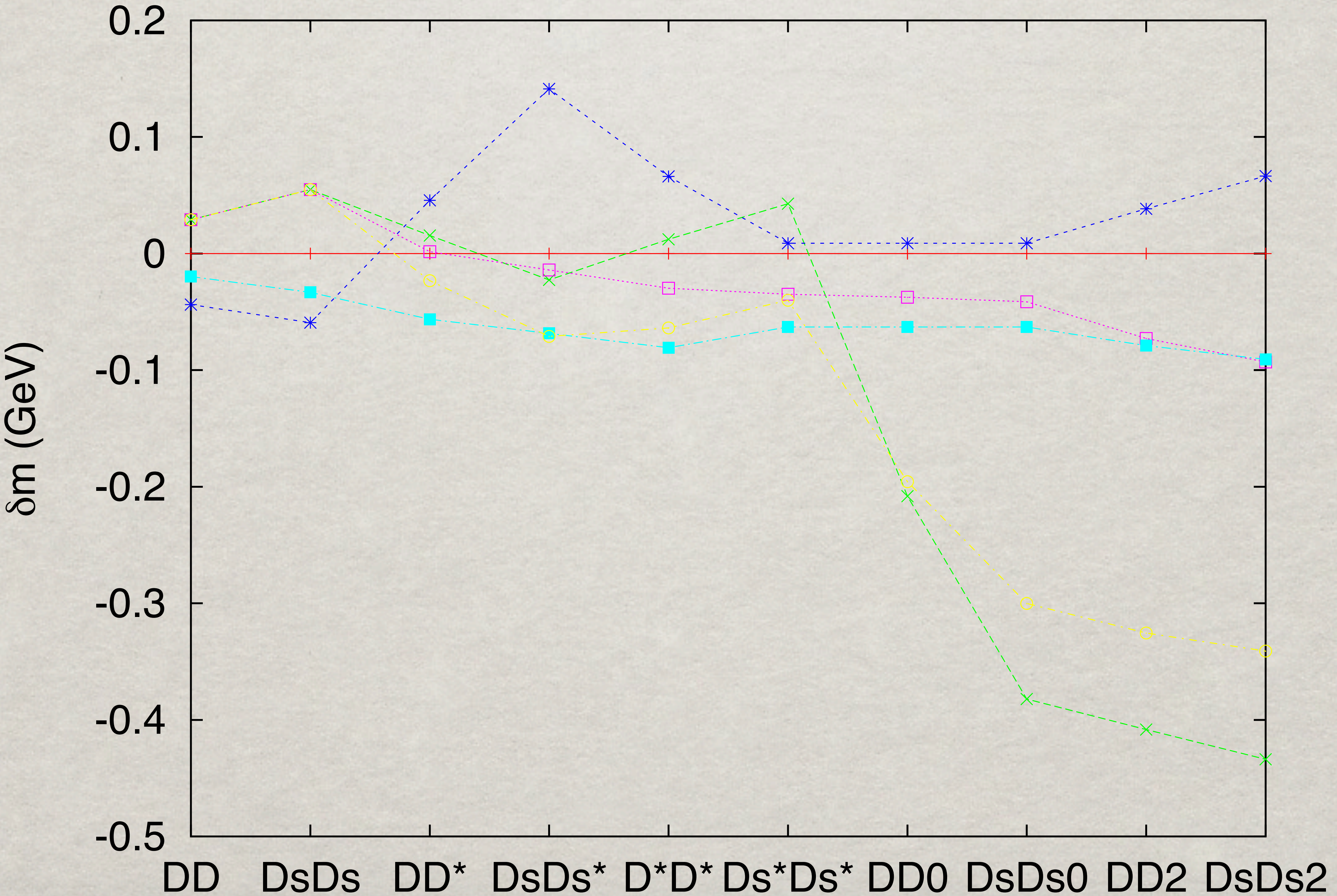
Why does the Gell-Mann-- Okubo mass formula work? Thresholds affect the decuplet states differently!

Hadronic Mixing

T. Barnes and E.S. Swanson, PRC77, 055206 (08)



Hadronic Mixing

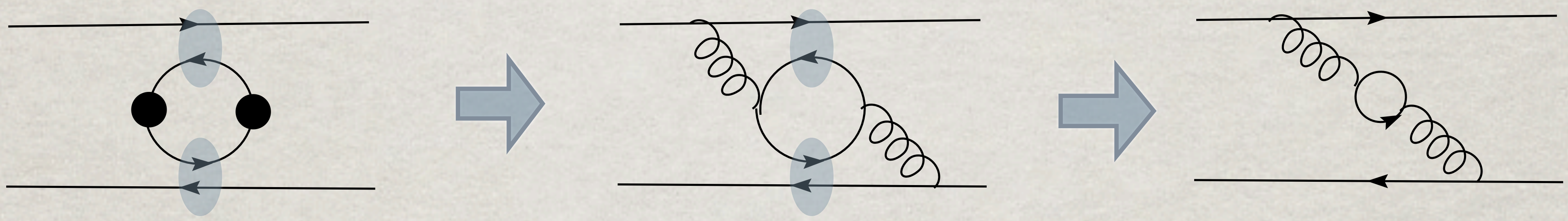


RENORMALISATION-1

of course the 'bare' quark model must have its parameters refit to yield the experimental spectrum

RENORMALISATION-2

summing the continuum



$q^2 < \Lambda^2 \approx 1\text{GeV}^2$

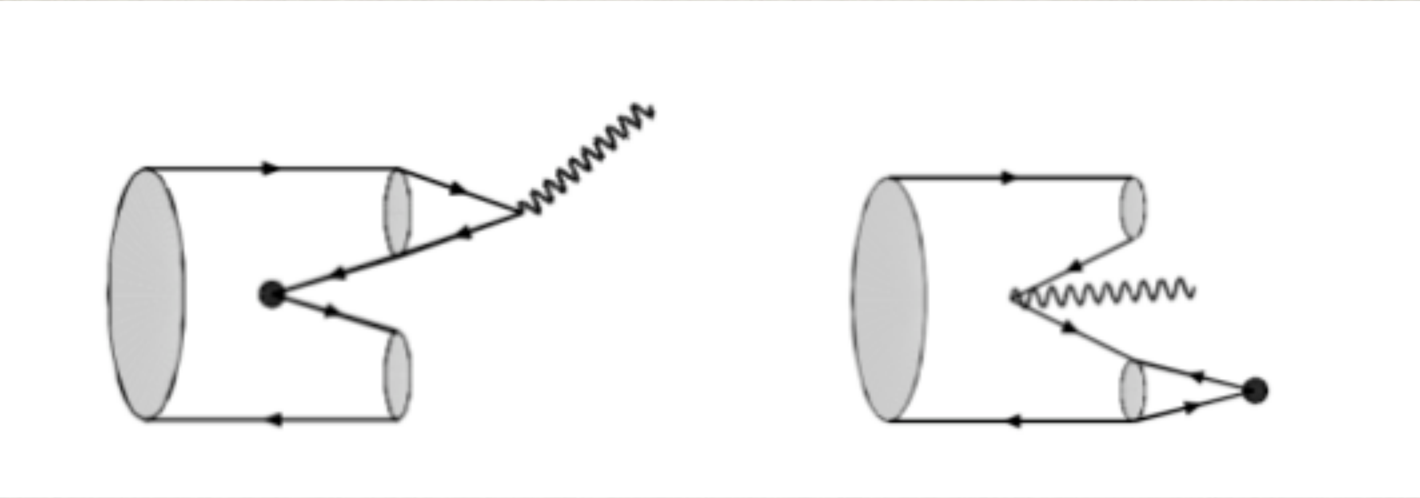
$1\text{GeV}^2 < q^2 < \Lambda^2 \approx 4\text{GeV}^2$

$4\text{GeV}^2 < q^2 < \Lambda^2 \to \infty$

how does one bridge the renormalisation gap between QCD and a model of QCD?

RENORMALISATION-3

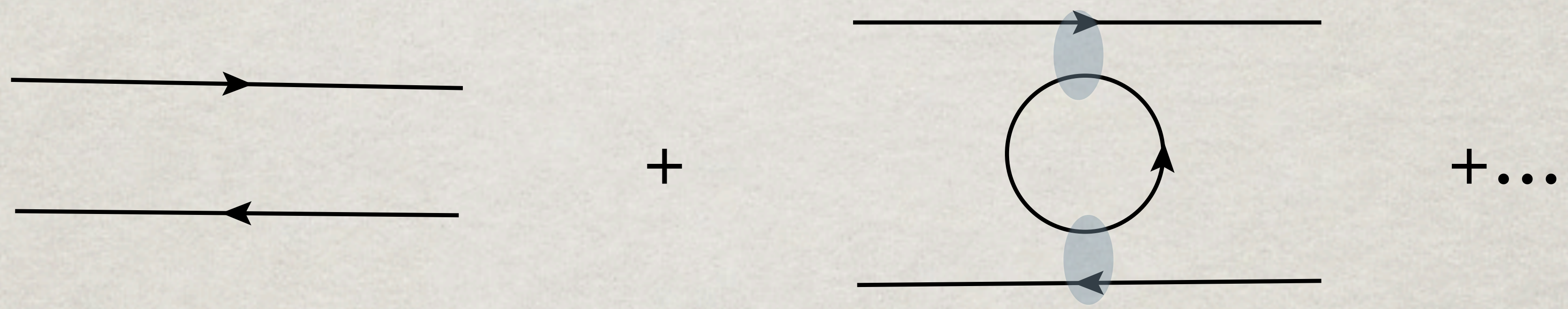
$$J/\psi \to \eta_c \gamma$$



$$A^{(HO)} = A^{(0)}(1 + 0.334 + 0.036) = 0.197 \text{ GeV}$$

$$A^{imp} = |\vec{q}|\sqrt{M_\psi E_\eta}\frac{eQ_q + eQ_{\bar{q}}}{m_q}\mathrm{e}^{-q^2/16\beta^2} \approx 0.095 \text{ GeV}$$

RENORMALISATION-3

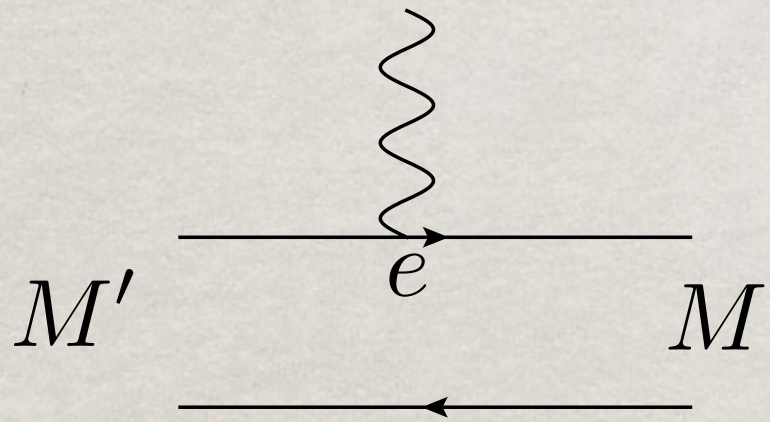


$$Z_{q\bar{q}} < 1 \qquad \Gamma_{ee} = Z_{q\bar{q}}\Gamma_0$$

$\Rightarrow$ a disaster for the quark model

RENORMALISATION-4

what <u>is</u> the quark model?



$$\frac{e^2}{4\pi} = \frac{1}{137}$$

$$m_q = m_N/3$$

<u>the</u> defining characteristic of the quark model

RENORMALISATION-4

- the quark model should be treated as a standard model ... there are no 'external parameters'
- an unquenched quark model is a field theory and needs to be properly renormalised

$$e \to e_R = \frac{e}{\sqrt{Z}} \qquad \frac{e_R^2}{4\pi} = \frac{1}{137}$$

AND WHILE WE'RE AT IT...

- nonperturbative gluodynamics
- multipion intermediate states
- chiral restoration
- emergence of the string regime

Conclusions

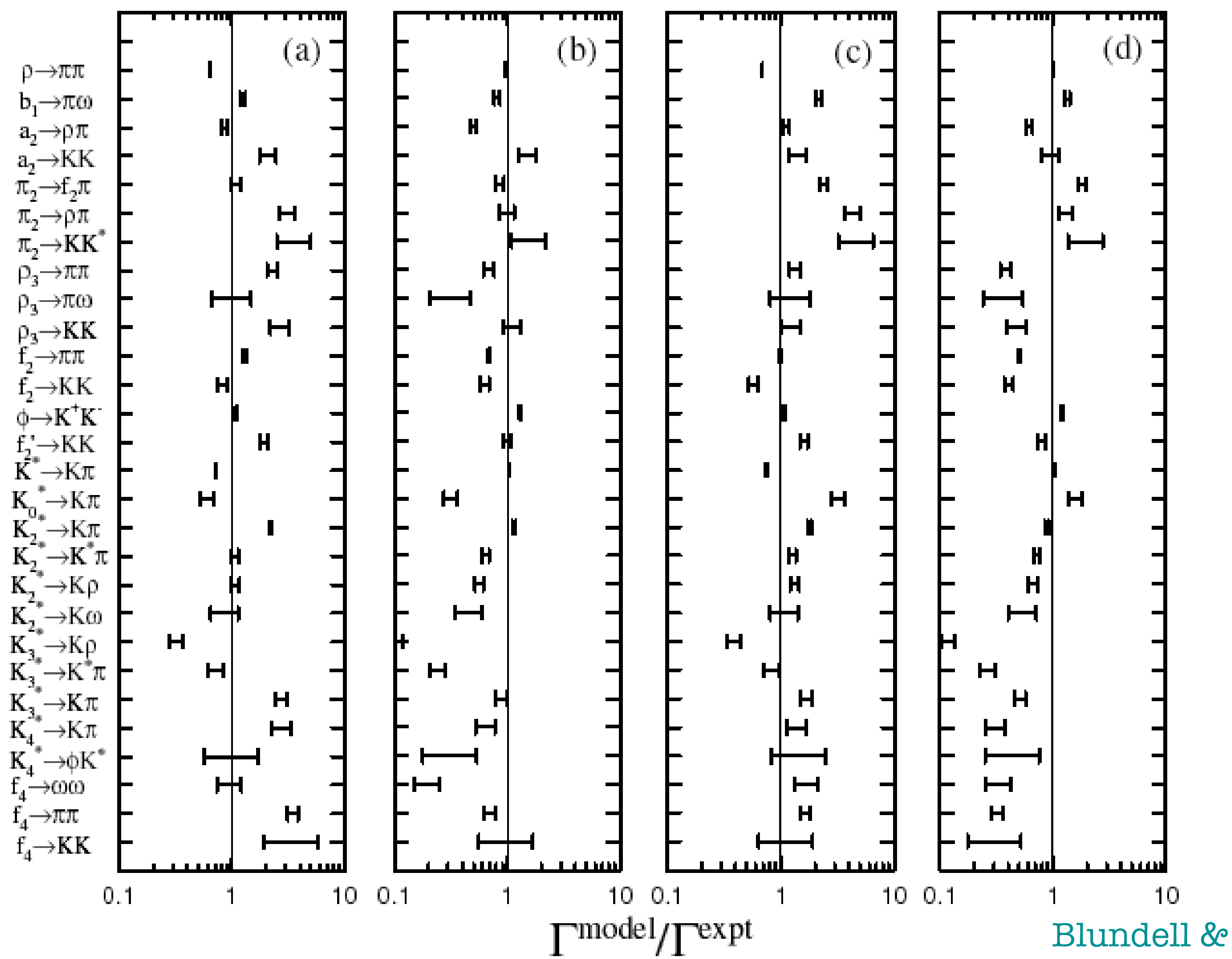
- simple overlaps (decay constants, elastic EM couplings) work well
- dynamical EW transitions are tougher, but work reasonably well for heavy quarks if one employs bound state perturbation theory
- strong interactions are tougher, the 3P0 model is a reasonable, but rough, guide
- fully incorporating strong decays into hadronic dynamics is a difficult problem. Lattice gauge theory may be our only hope, although is in a nascent state.

✢ ÆRIC MEC HEHT GEWYRCAN

NOTES

strong decays

model:	$^3P_0$	$^3P_0$	Flux Tube	Flux Tube
wavefunction:	SHO	SHO	RQM	RQM
phase space:	Rel	K&I	Rel	K&I



Blundell & Godfrey,

strong decays

$N\pi$ decay widths Γ[MeV] | $\Delta\pi$ decay widths Γ[MeV]

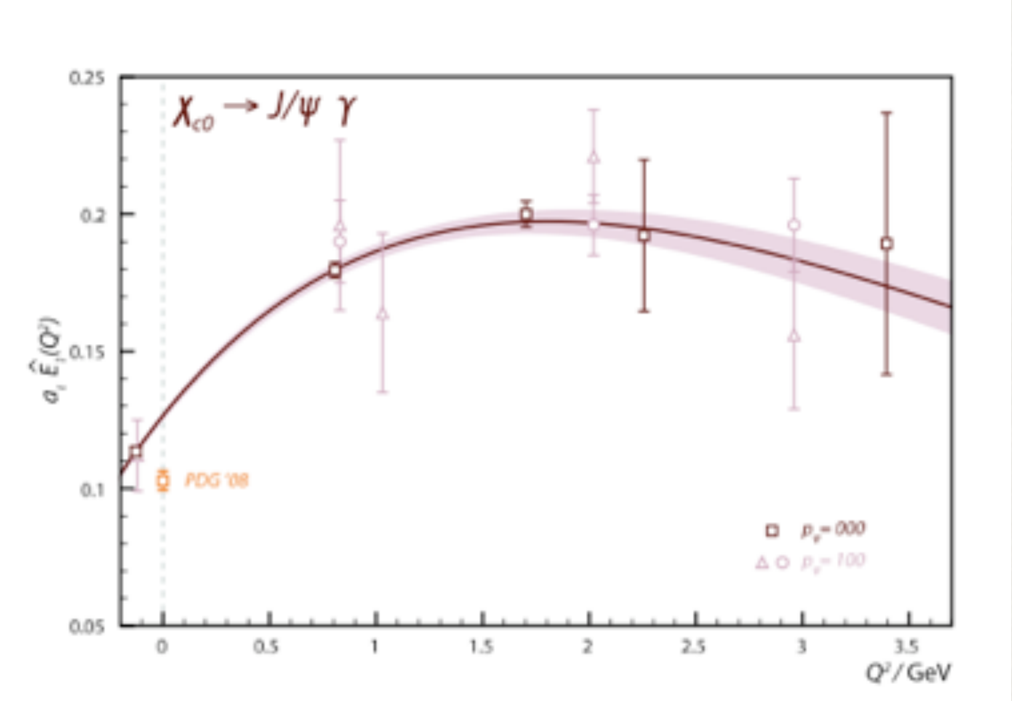
Decay	Calc	$^3P_0$	PDG	Decay	Calc	$^3P_0$	PDG
$S_{11}(1535) \to N\pi$	33	216	$(68 \pm 15)\,^{+45}_{-23}$	$\to \Delta\pi$	1	2	< 2
$S_{11}(1650) \to N\pi$	3	149	$(109 \pm 26)^{+29}_{-4}$	$\to \Delta\pi$	5	13	$(6 \pm 5)\ \ ^{+2}_{0}$
$D_{13}(1520) \to N\pi$	38	74	$(66 \pm 6)\ \ ^{+8}_{-5}$	$\to \Delta\pi$	35	35	$(24 \pm 6)\ \ ^{+3}_{-2}$
$D_{13}(1700) \to N\pi$	0.1	34	$(10 \pm 5)\ \ ^{+5}_{-5}$	$\to \Delta\pi$	88	778	seen
$D_{15}(1675) \to N\pi$	4	28	$(68 \pm 7)\ \ ^{+14}_{-5}$	$\to \Delta\pi$	30	32	$(83 \pm 7)\ \ ^{+17}_{-6}$
$P_{11}(1440) \to N\pi$	38	412	$(228 \pm 18)^{+65}_{-65}$	$\to \Delta\pi$	35	11	$(88 \pm 18)\ ^{+25}_{-25}$
$P_{33}(1232) \to N\pi$	62	108	$(119 \pm 0)\ \ ^{+5}_{-5}$				
$S_{31}(1620) \to N\pi$	4	26	$(38 \pm 7)\ \ ^{+8}_{-8}$	$\to \Delta\pi$	72	18	$(68 \pm 23)\ ^{+14}_{-14}$
$D_{33}(1700) \to N\pi$	2	24	$(45 \pm 15)\ ^{+15}_{-15}$	$\to \Delta\pi$	52	262	$(135 \pm 45)^{+45}_{-45}$

$^3P_0$: S. Capstick, W. Roberts, Phys.Rev. D49 (1994) 4570-4586

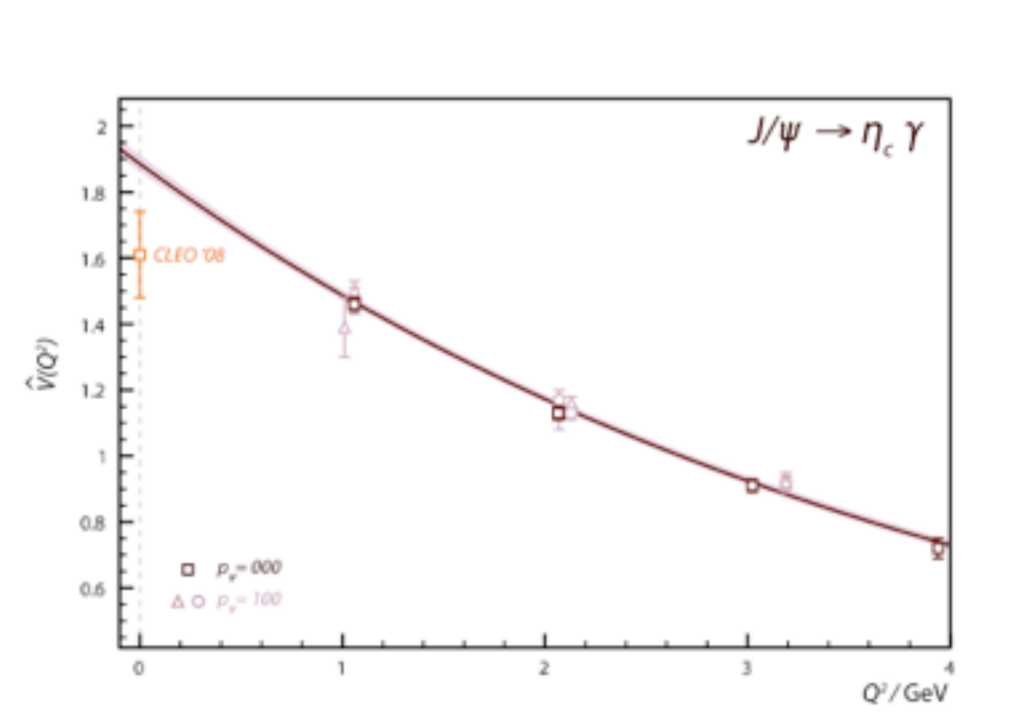
lattice

Dudek, Edwards, Thomas, 0902.2241

sink level	suggested transition	$a_t\hat{E}_1(0)$	β/MeV λ/GeV^{-2}	$\Gamma_{\rm lat}$/keV	$\Gamma_{\rm expt}$/keV
0	$\chi_{c0} \to J/\psi\gamma$	0.127(2)	409(12) 1.14(5)	199(6)	131(14)
1	$\psi' \to \chi_{c0}\gamma$	0.092(19)	164(55) 0[fixed]	26(11)	30(2)
3	$\psi'' \to \chi_{c0}\gamma$	0.265(33)	324(77) 0.58(56)	265(66)	199(26)
5	$Y_{\rm hyb.} \to \chi_{c0}\gamma$	0.00(3)	linear fit	$\lesssim 20$	-



sink level	suggested transition	$\hat{V}(0)$	β/MeV λ/GeV^{-2}	$\Gamma_{\rm lat}$/keV	$\Gamma_{\rm expt}$/keV
0	$J/\psi \to \eta_c\gamma$	1.89(3)	513(7) 0[fixed]	2.51(8)	1.85(29)
1	$\psi' \to \eta_c\gamma$	0.062(64)	530(110) 4(6)	0.4(8)	0.95(16) 1.37(20)
3	$\psi'' \to \eta_c\gamma$	0.27(15)	367(55) −1.25(30)	10(11)	-
5	$Y_{\rm hyb.} \to \eta_c\gamma$	0.28(6)	250(200) 0[fixed]	42(18)	-

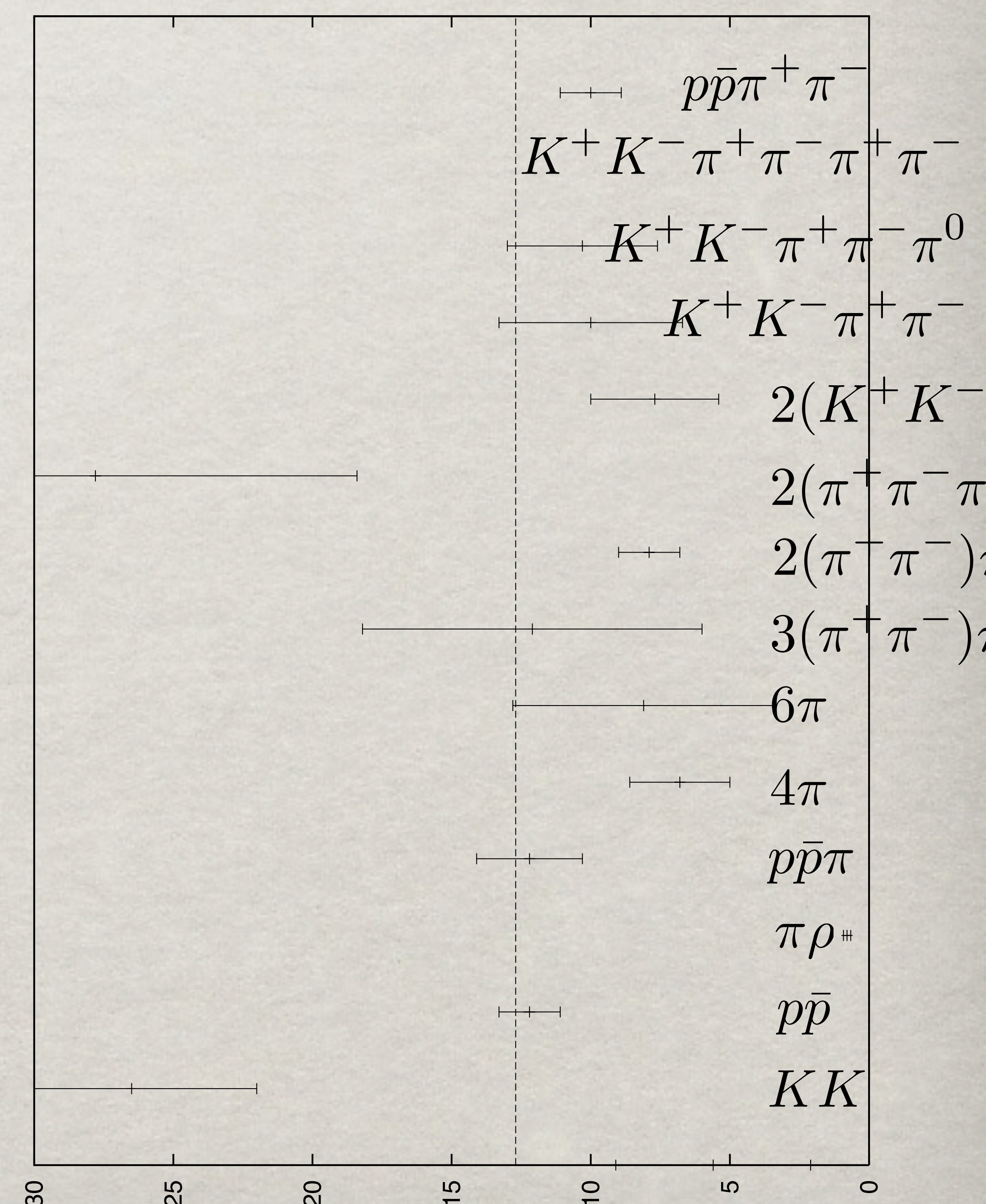


PERTURBATIVE QCD

the pi-rho puzzle

$$Q_h \equiv \frac{Bf(\psi' \to h)}{Bf(J/\psi \to h)} = \frac{Bf(\psi' \to e^+e^-)}{Bf(J/\psi \to e^+e^-)} \approx 12.7\%$$

Appelquist and Politzer, PRL34, 43 (75)

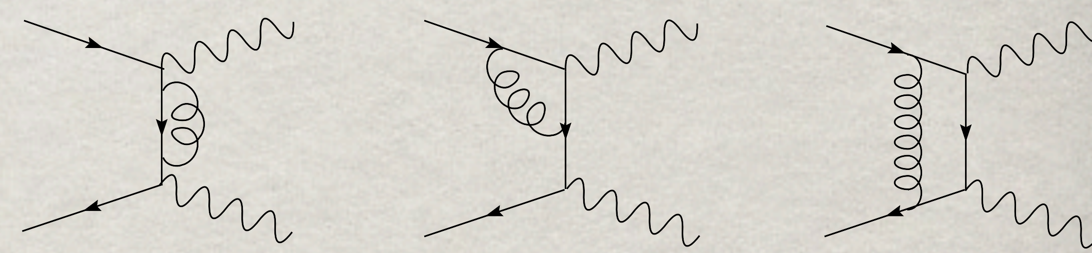


Mo et al., hep-ph/0611214

PERTURBATIVE QCD

$$R = \frac{\Gamma(\chi_{c2} \to \gamma\gamma)}{\Gamma(\chi_{c0} \to \gamma\gamma)} = \frac{4}{15}(1 - 1.76\alpha_s)$$

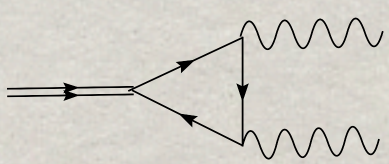
pQCD W. Bardeen et al. PRD18, 3998 (78)



NRQCD Bodwin, Braaten, Lepage, PRD51, 1125 (95)

pNRQCD N. Brambilla et al., hep-ph/0604190

pCQM Ackleh, Barnes, & Close, PRD46, 2257 (92)



bsCQM Lakhina & Swanson, PRD74, 014012 (06)

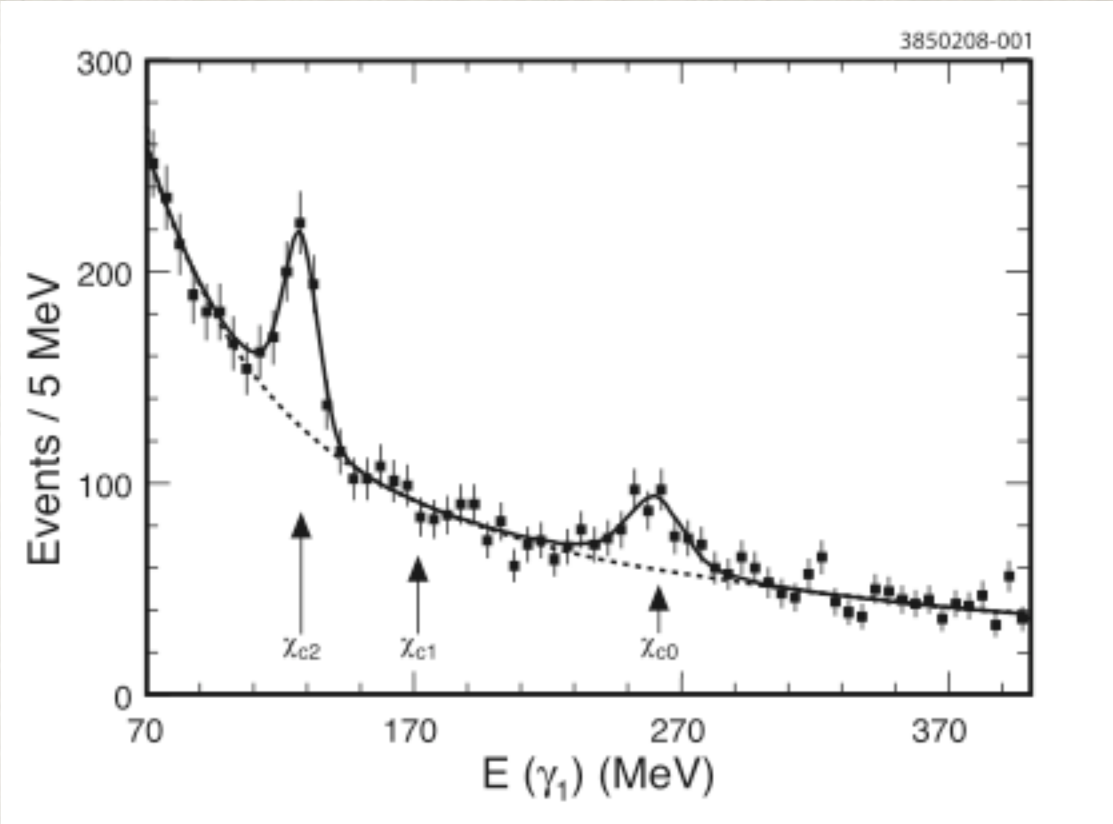
PERTURBATIVE QCD

$$R = \frac{\Gamma(\chi_{c2} \to \gamma\gamma)}{\Gamma(\chi_{c0} \to \gamma\gamma)} = \frac{4}{15}(1 - 1.76\alpha_s) = 0.12\ (\alpha_s = 0.32)$$

W. Bardeen et al. PRD18, 3998 (78)

$$R = \frac{0.66 \pm 0.07 \pm 0.04 \pm 0.05\ \text{keV}}{2.36 \pm 0.35 \pm 0.11 \pm 0.19\ \text{keV}} = 0.278 \pm 0.050 \pm 0.018 \pm 0.031$$

CLEO, PRD78, 091501 (2008



note: 4/15 = 0.27!

PERTURBATIVE QCD

e^+e^- widths

van Royen and Weisskopf

$$\Gamma(^3S_1 \to e^+e^-) = 16\alpha_s^2 Q^2 \frac{|\psi(0)|^2}{M^2}$$

$$\Gamma(^3D_1 \to e^+e^-) = 50\alpha_s^2 Q^2 \frac{|\psi''(0)|^2}{M^2 m_c^4}$$

state	qn	thy (keV)	expt (keV)	
J/ψ	1^3S_1	12	5.40(17)	
ψ'	2^3S_1	5	2.12(12)	} mixing?
$\psi(3770)$	1^3D_1	0.06	0.26(4)	} mixing?
$\psi(4040)$	3^3S_1	3.5	0.75(15)	
$\psi(4159)$	2^3D_1	0.1	0.77(23)	
$\psi(4415)$	4^3S_1	2.6	0.47(10)	

$$\mathcal{A} = \mathcal{M}^{\mu\nu}\epsilon_\mu\epsilon_\nu$$

Lakhina & Swanson, PRD74, 014012 (06)

$$\mathcal{M}_S^{\mu\nu} = M_S(p_1^2, p_2^2, p_1 \cdot p_2)g^{\mu\nu}$$

$$\Gamma(S \to \gamma\gamma) = \frac{|M_S(0,0)|^2}{8\pi m_S}$$

$$M_S = \sum_V Q^2 \sqrt{\frac{m_V}{E_V}} f_V \frac{E_1^{(V)}(q)}{m_S - E_{\gamma V}(g)}$$

$$\Gamma(\chi_{c2} \to \gamma\gamma) = 3.3 \text{ keV}$$

Ackleh, Barnes, & Close, PRD46, 2257 (92)

nonrel result is 2/0 = 15/4