

... after an intense discussion



1 Connection between resonances and QCD

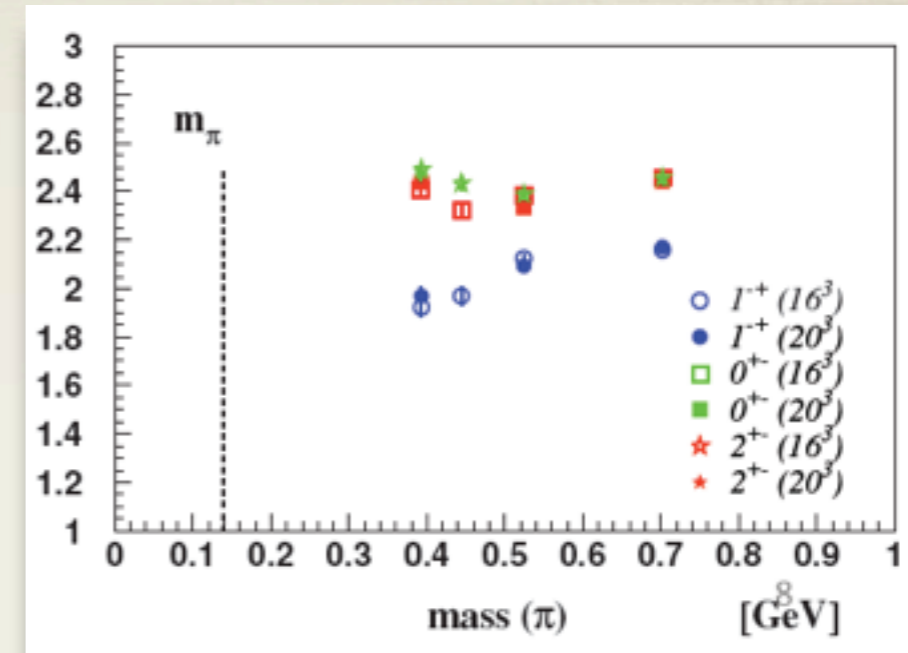
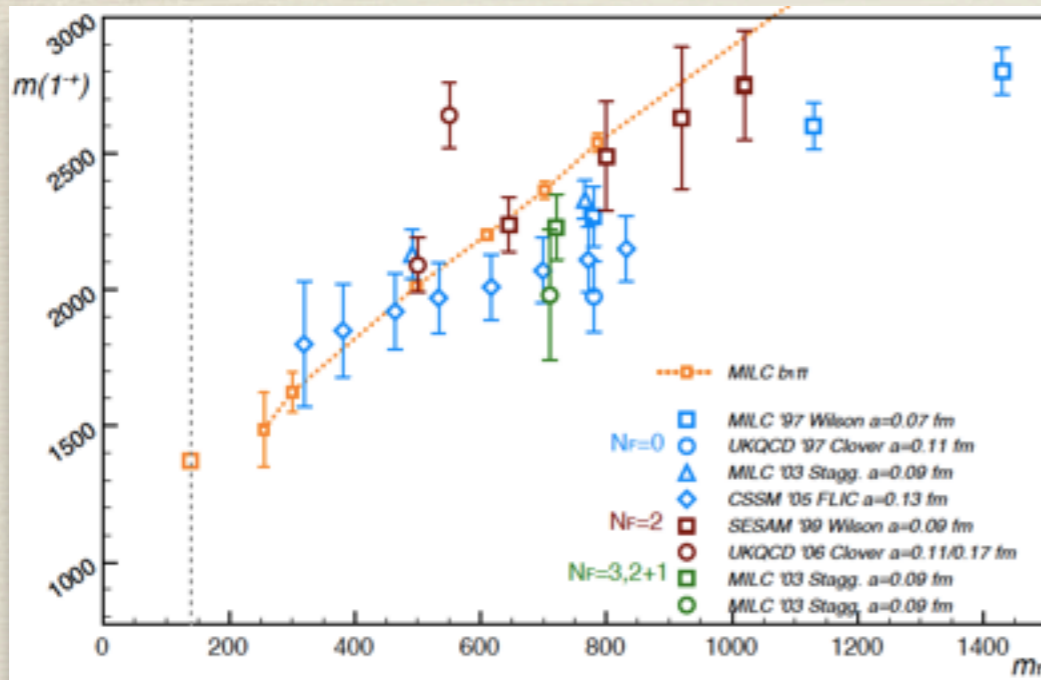
2 Connection between real (data) and imaginary (resonances) worlds

# Hybrid Mesons

Adam Szczepaniak  
Indiana University

- \* (Selected) aspects on theory and phenomenology
  - \* Structure of gluonic excitations
- \* (Selected) aspects of PWA

\*  $1_{S_{Q\bar{Q}}=1}^{-+} = \frac{0^{++}}{2} + \rho \sim 0.8 \text{ GeV} + 0.77 \text{ MeV} \sim 1.6 \text{ GeV}$



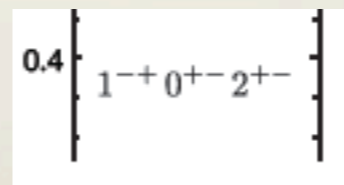
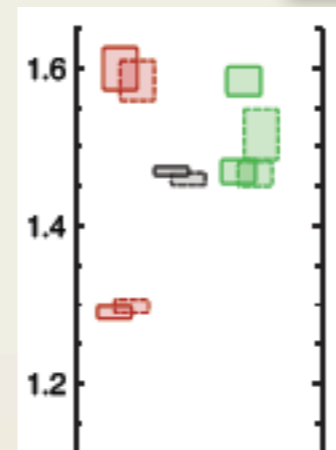
\*  $J^{PC} = 1^{-+}$  lowest state

Higher masses have also been resolved

Chiral extrapolations 100-200 MeV (Thomas,APS)

In large- $N_c$  same as for ordinary mesons  $O(1/N_c)$  (Cohen)

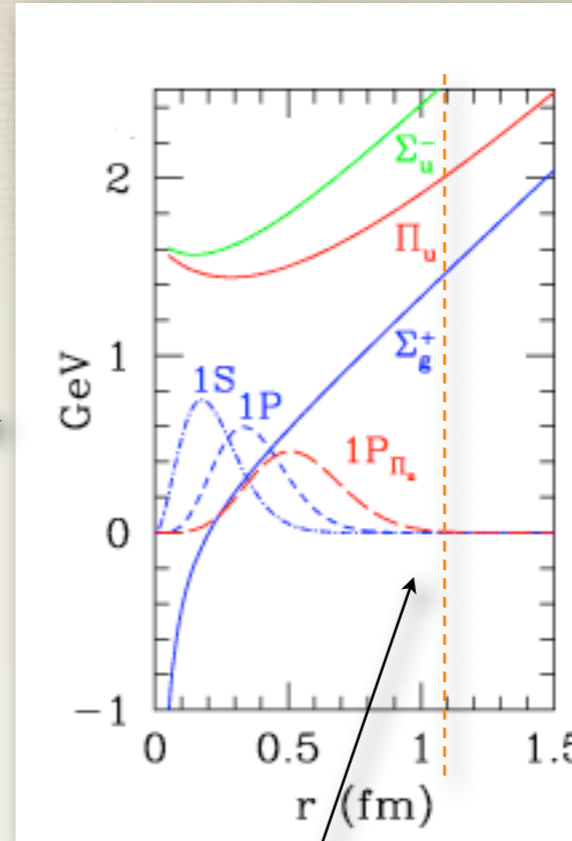
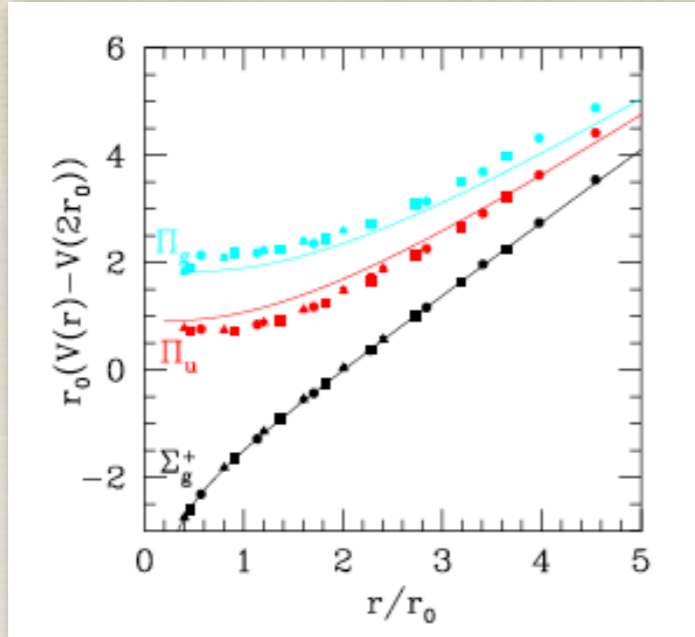
Preliminary (toy) lattice computation of widths agrees with models (Michael,McNeile) (Burns,Close)



Charmonium $1^{-+}$		
Ref.	Method	$\Delta M$ (GeV)
MILC 97	W	1.34(8)(20)
MILC 99	SW	1.22(15)
CP-PACS 99	NR	1.323(13)
JKM 99	LBO	1.19

Excitations in excess of 1GeV

# more on widths



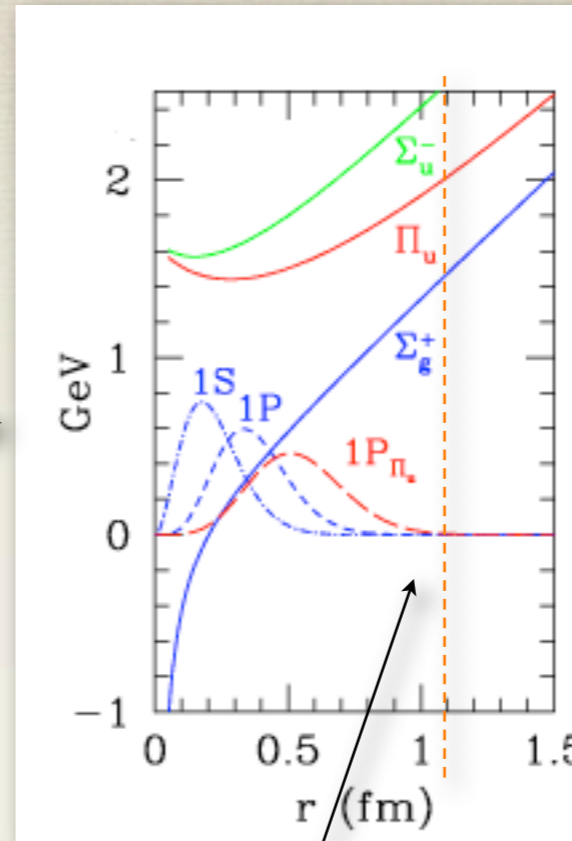
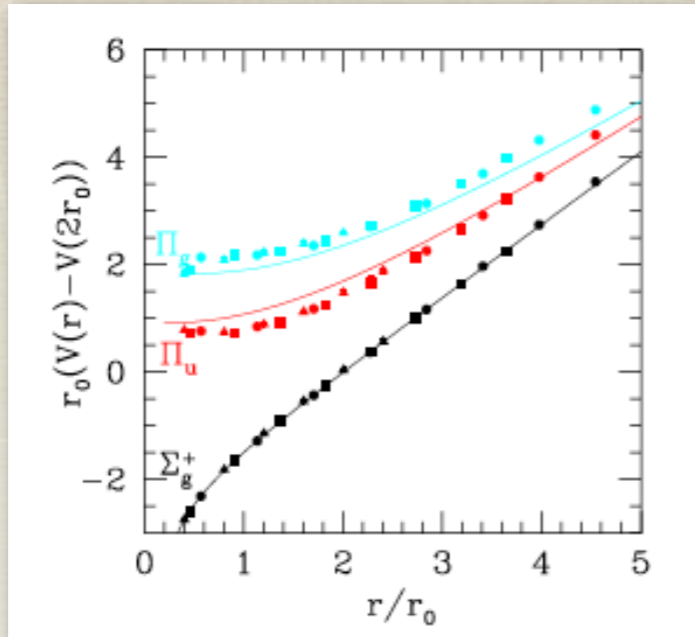
Bali (00)

$1^- (1.8 \text{ GeV})$	$b_1 \pi$	$f_1 \pi$	$\rho \pi$	$\Gamma \text{ MeV}$
PSS	S 73 D 1	S 9 D 0.04	P 13	
IKP	S 51 D 11	S 14 D 7	P 12	

Isgur, Kokosky, Paton (85)  
 Close, Page (95)  
 Page, Swanson, Szczepaniak (99)  
 Close, Dudek (04)

- Low lying states expected below string breaking !
- Unusual decay modes in the flux tube model

# more on widths



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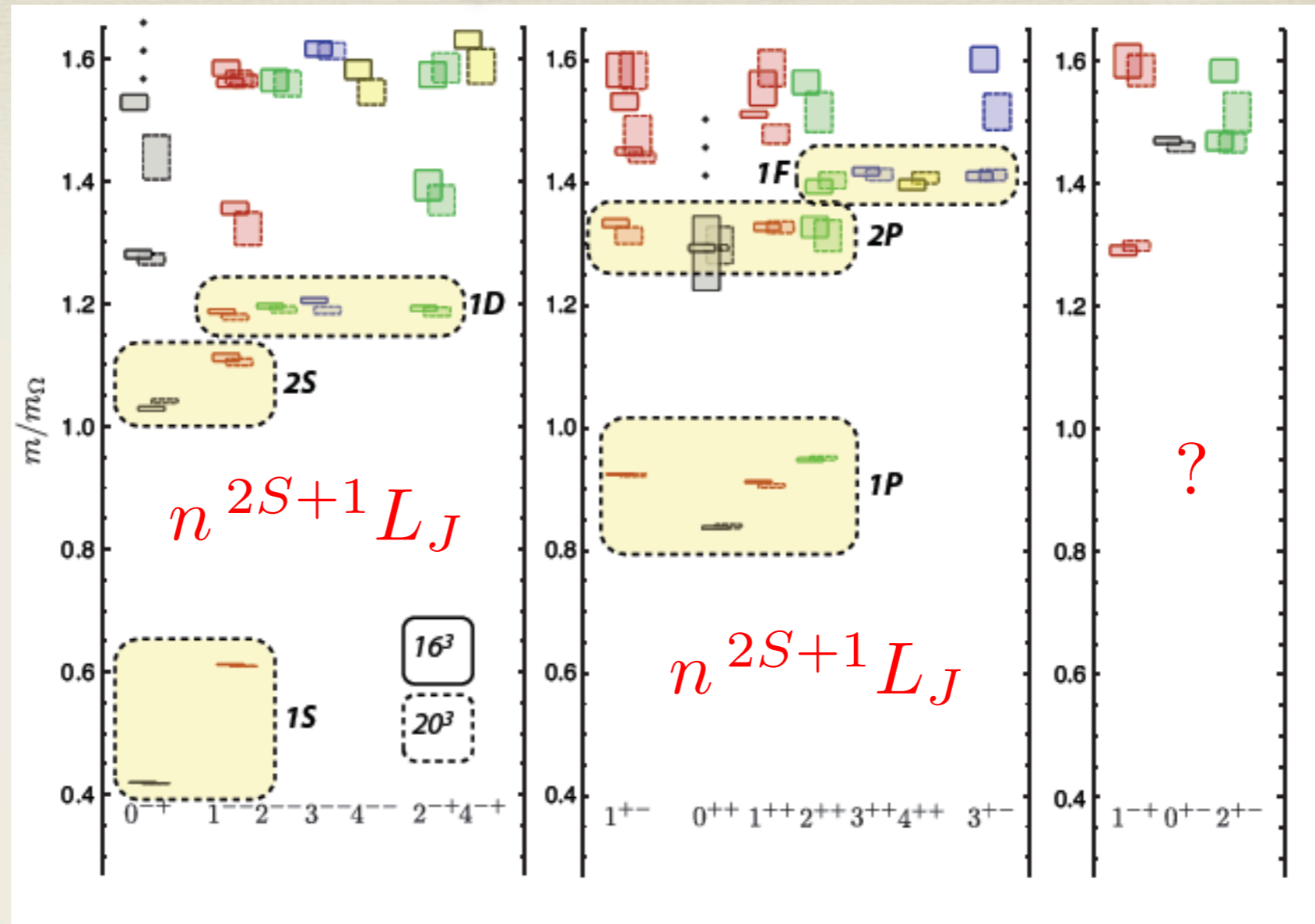
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- Low lying states expected below string breaking !
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# Structure

\* normal meson spectrum seems to be very quark model-like !

J.Dudek et al.  
Phys.Rev.D82:0345  
08,2010

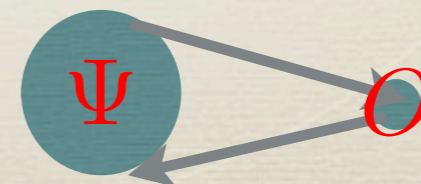


to determine structure study

$$\langle \text{Vacuum} | O[q, g] | \text{Meson} \rangle$$

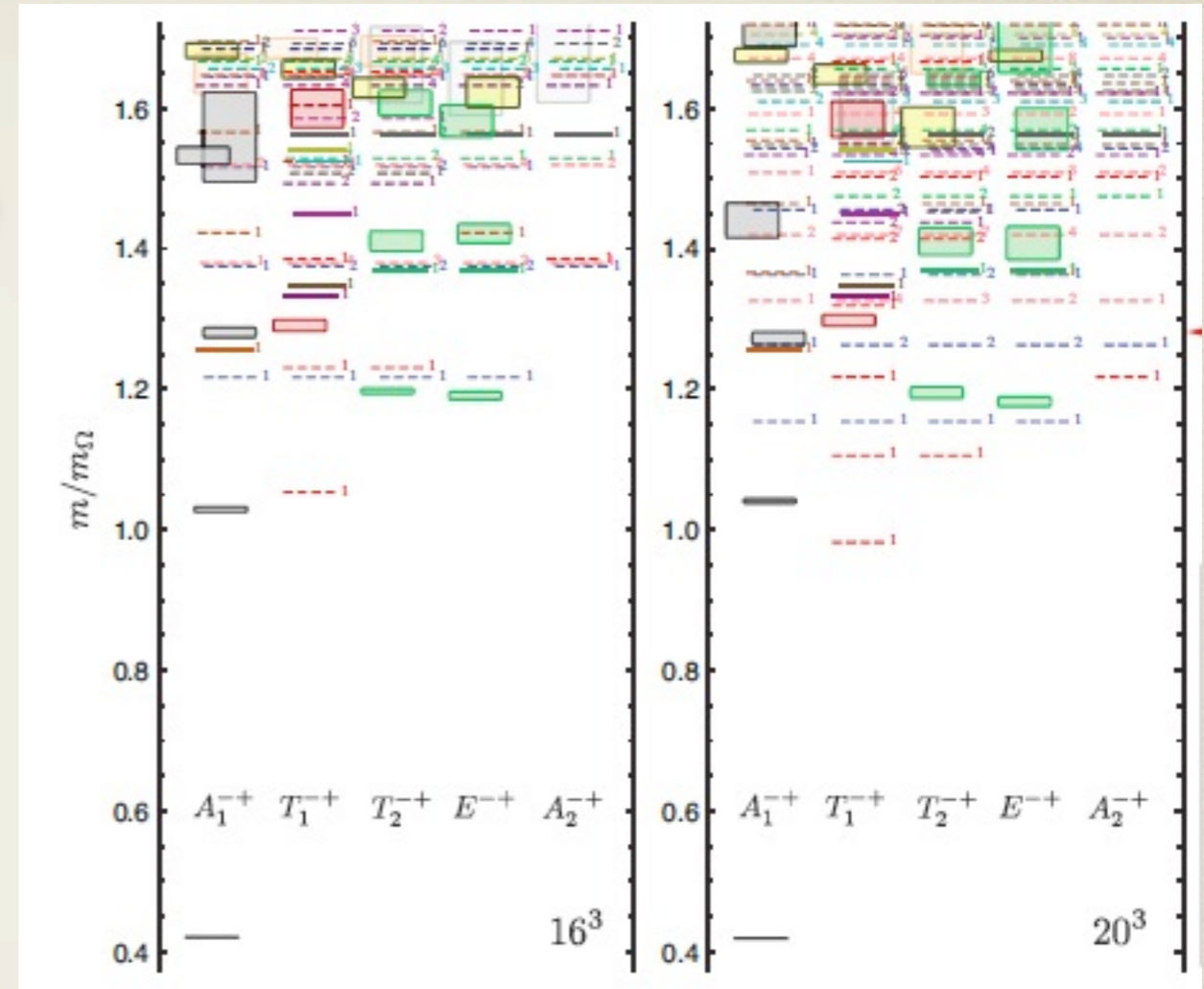
$$\bar{q}(x) \Gamma^i q(x) \sim b^\dagger(k) \sigma^i d(-k)$$

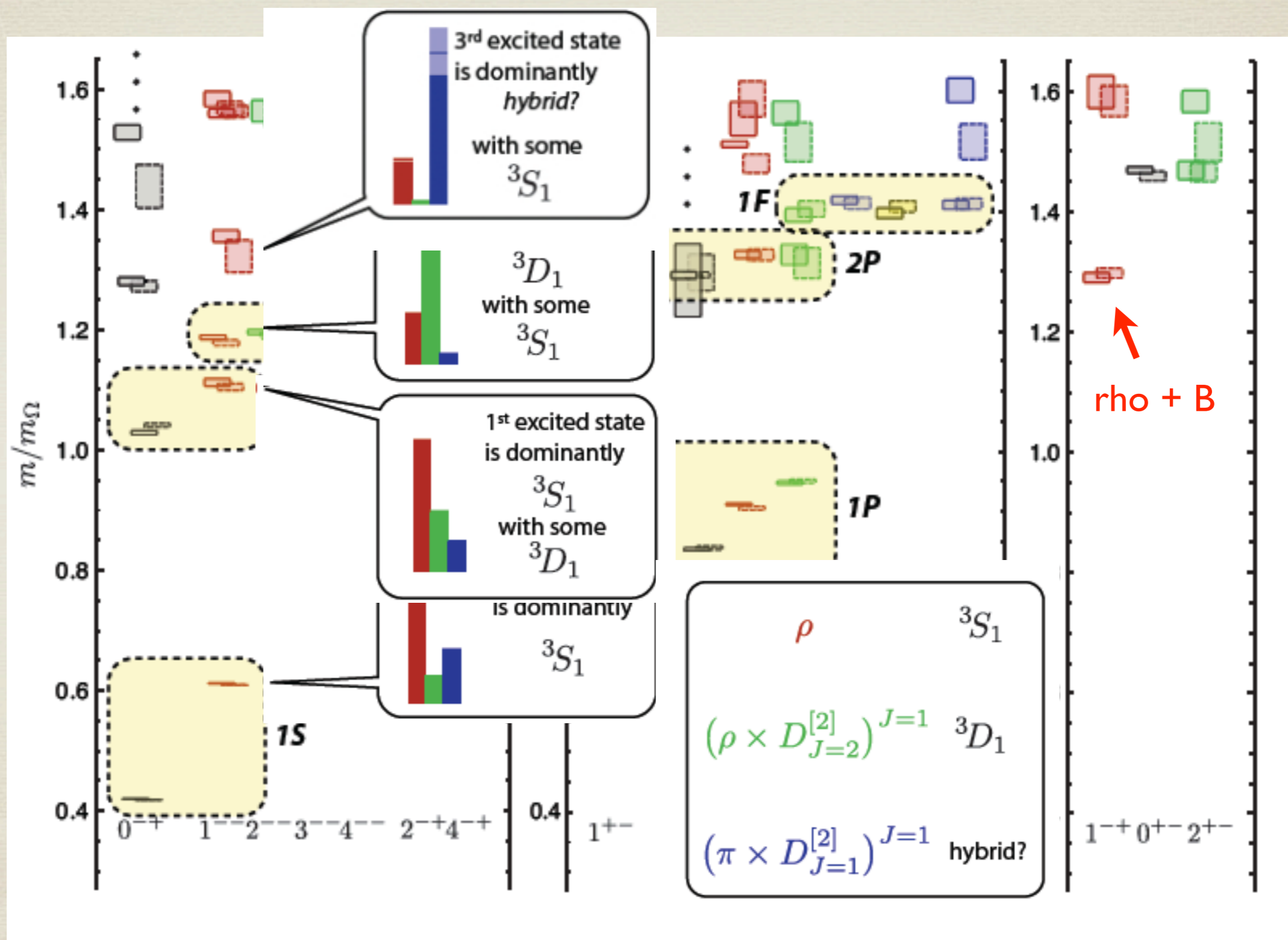
$$\bar{q}(x) F_{ij}(x) q(x) \sim b^\dagger(k) \vec{k} \times \vec{a}(q) d(-q - k)$$



# in unquenched lattice lowest energies correspond to continuum states

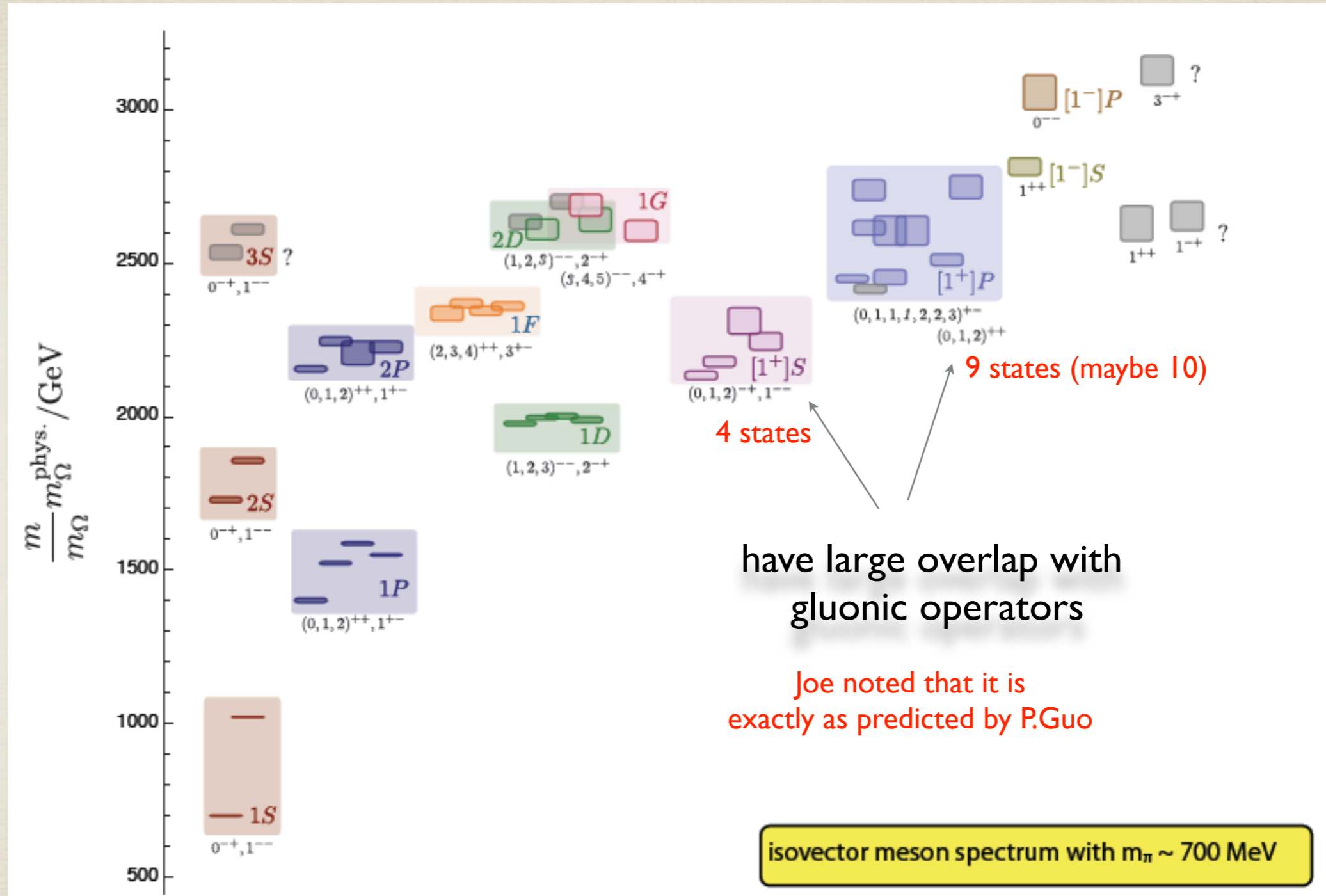
- \* On finite volume multi-meson state and single hadron states are discrete.
- \* If there are single hadron states, use volume dependence to disentangle
- \* Continuum states can have any J,P,C but not single hadron states
- \* The choice of operators minimizes overlap with multi-meson states







\* state of the art full spectrum

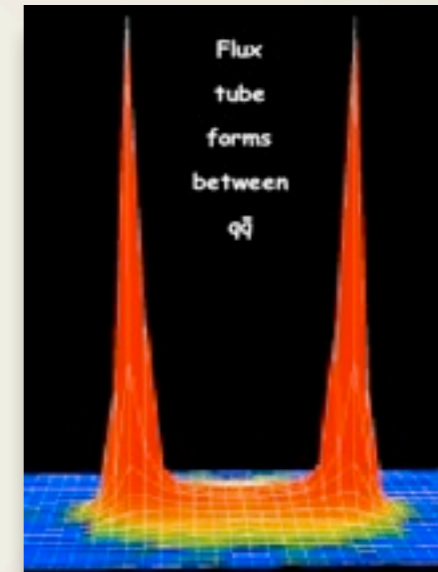


# Gluon structure models



Bag Model

Flux tube model



# Gluon structure models

And The Winer Is !



Quasi-particles





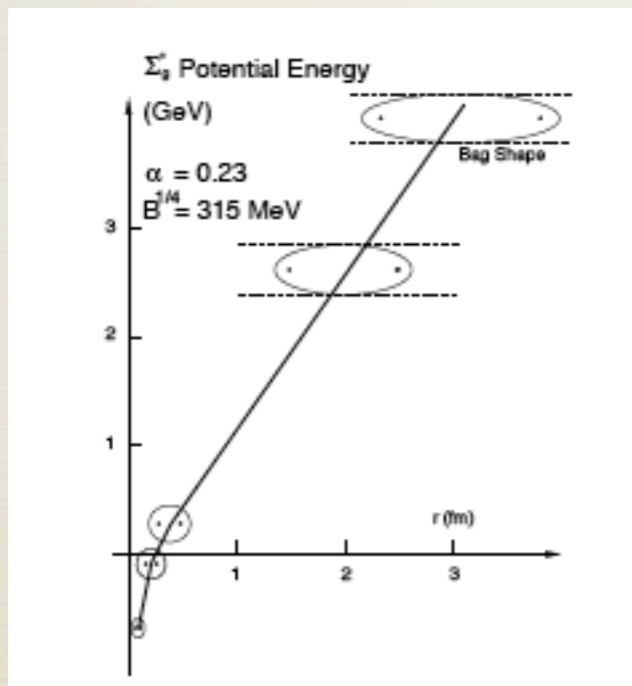
$a = 1 \text{ fm}$	$q\bar{q}(J^P = 1^-)$	$g, J^{PC} = 1^{+-}$	$g, J^{PC} = 1^{--}$
$E$	810 MeV	550 MeV	900 MeV

shifts the energy scale

$E_{q\bar{q}} + E_g + E_{\text{Bag}} + E_{\text{Vac}} + E_{\text{QCD}}$	$\pi_0/\eta_0(0^{-+})$	$\pi_1/\eta_1(1^{+-})$	$\rho_1/\omega_1(1^{--})$	$\pi_2/\eta_2(2^{-+})$
Barnes, Close [GeV]	1.1	1.3		
Chanowitz, Sharp [GeV]	1.4	1.8	1.6	2.0

$\pi(1300)/\eta(1300)$

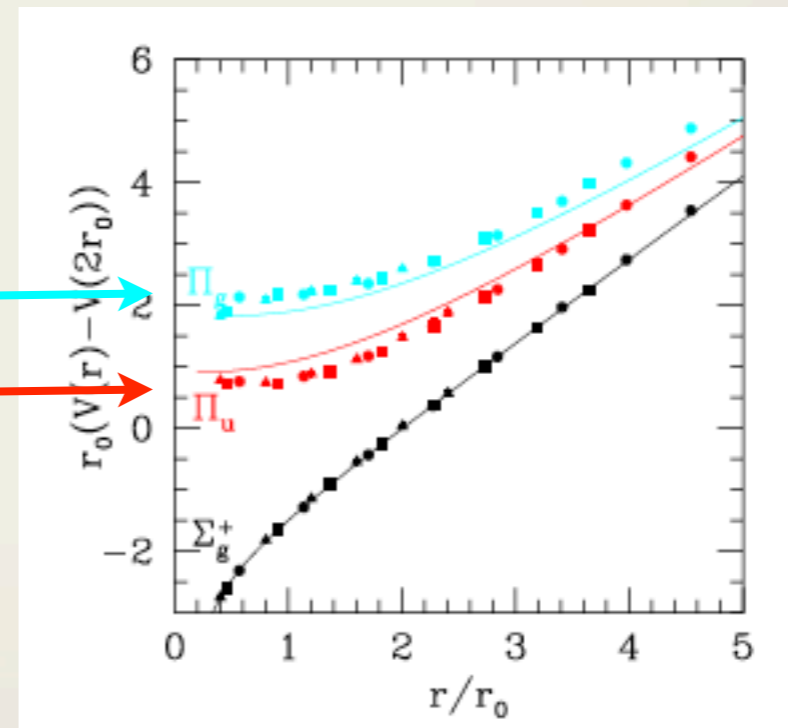
$\rho(1450), \rho(1700)$      $\pi_2(1670)$   
 $\omega(1420), \omega(1650)$      $\pi_2(2100)$



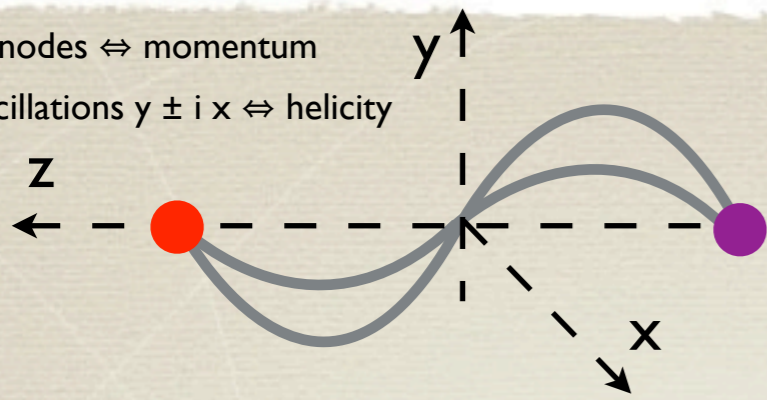
$P \times C = +$

$P \times C = -$

Juge, Kuti,  
Morningstar

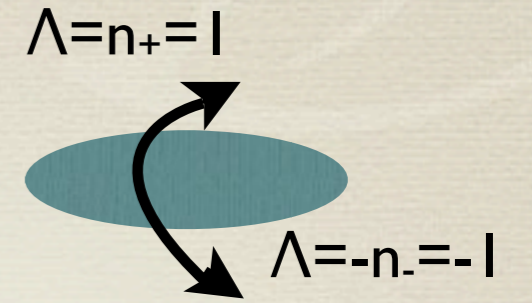


m nodes  $\Leftrightarrow$  momentum  
 oscillations  $y \pm i x \Leftrightarrow$  helicity

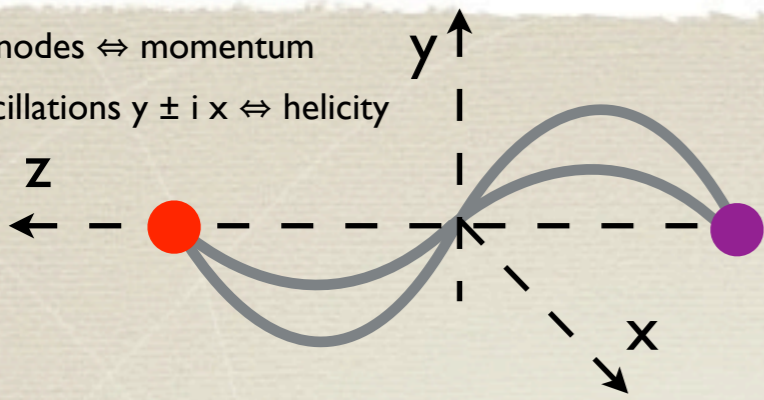


$n_{\pm}(m)$  = number of m-momentum modes of helicity  $\pm$   
 $N = \sum_m m [n_+(m) + n_-(m)]$  total momentum  
 $\Lambda = \sum_m [n_+(m) - n_-(m)]$  spin projection on the z axis

for example the lowest energy mode  $N=1$   $|\Lambda|=1$  (0-nodes)

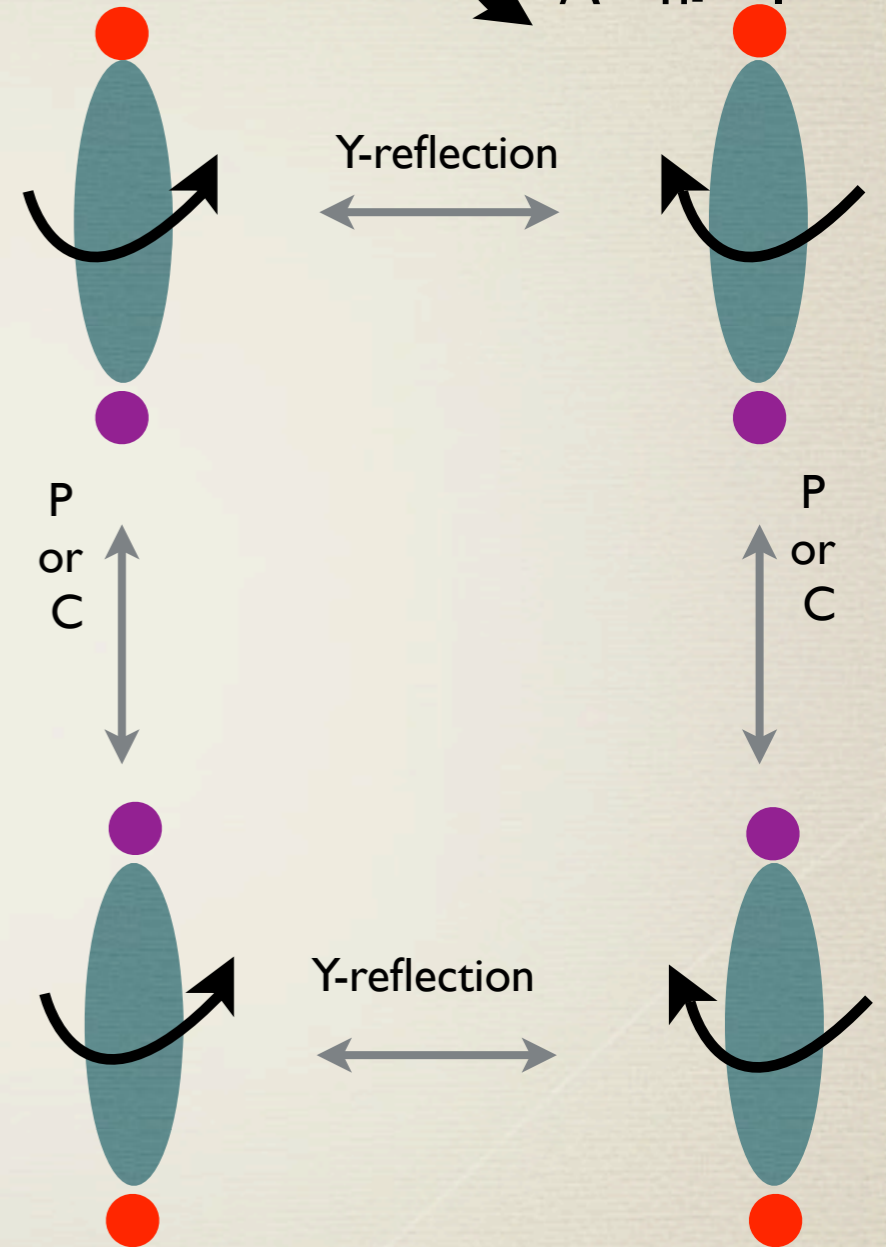
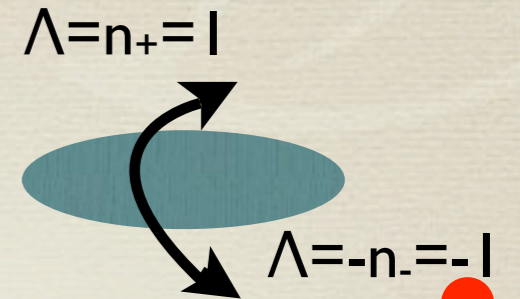


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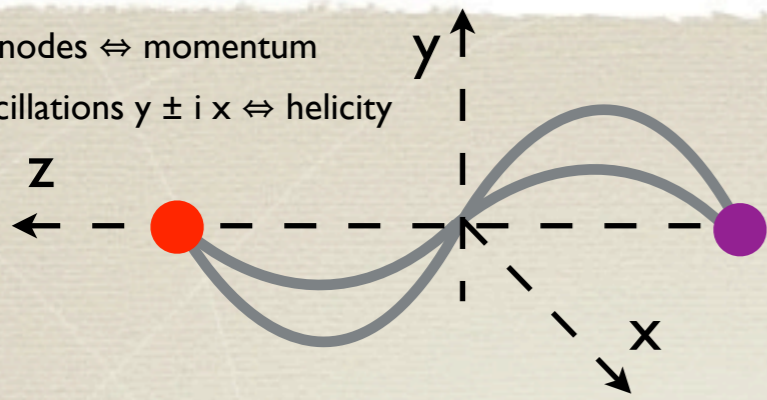


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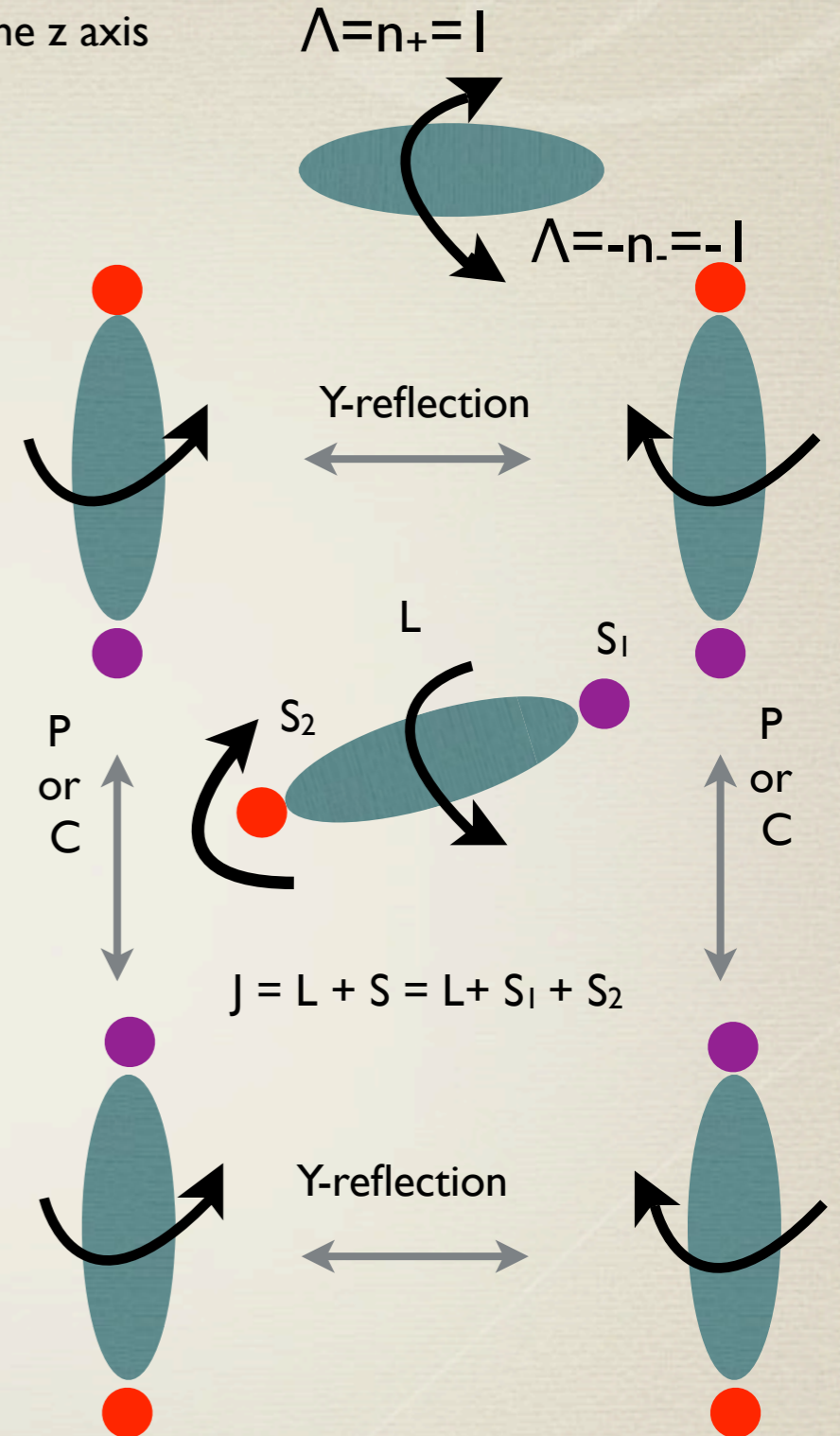


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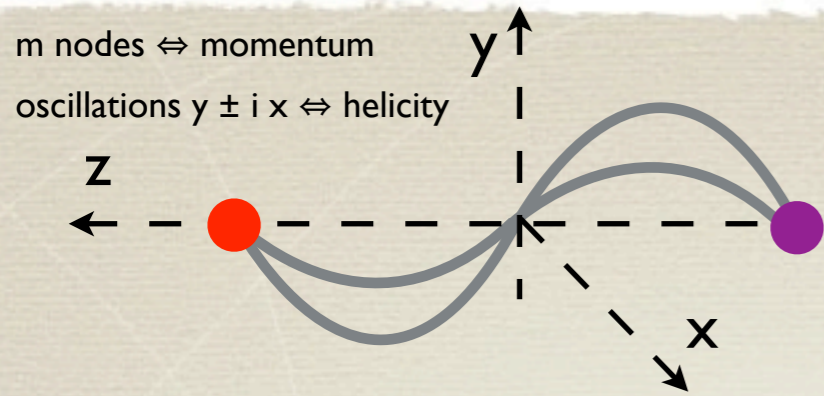


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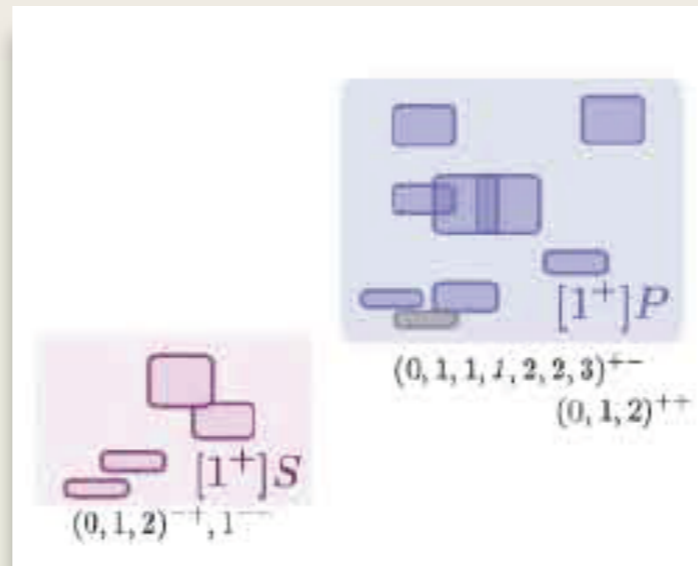


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degenerate in FT

$P = +1 \quad C = -1$   
 $P = -1 \quad C = +1$

$S = 0 \quad J^{PC} = 1^{--} \quad 1^{++}$   
 $S = 1 \quad J^{PC} = (0, 1, 2)^{-+} \quad (0, 1, 2)^{+-}$

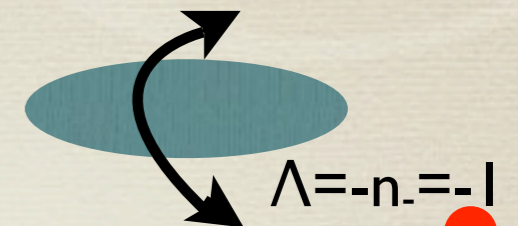


lattice and FT  
do not agree !

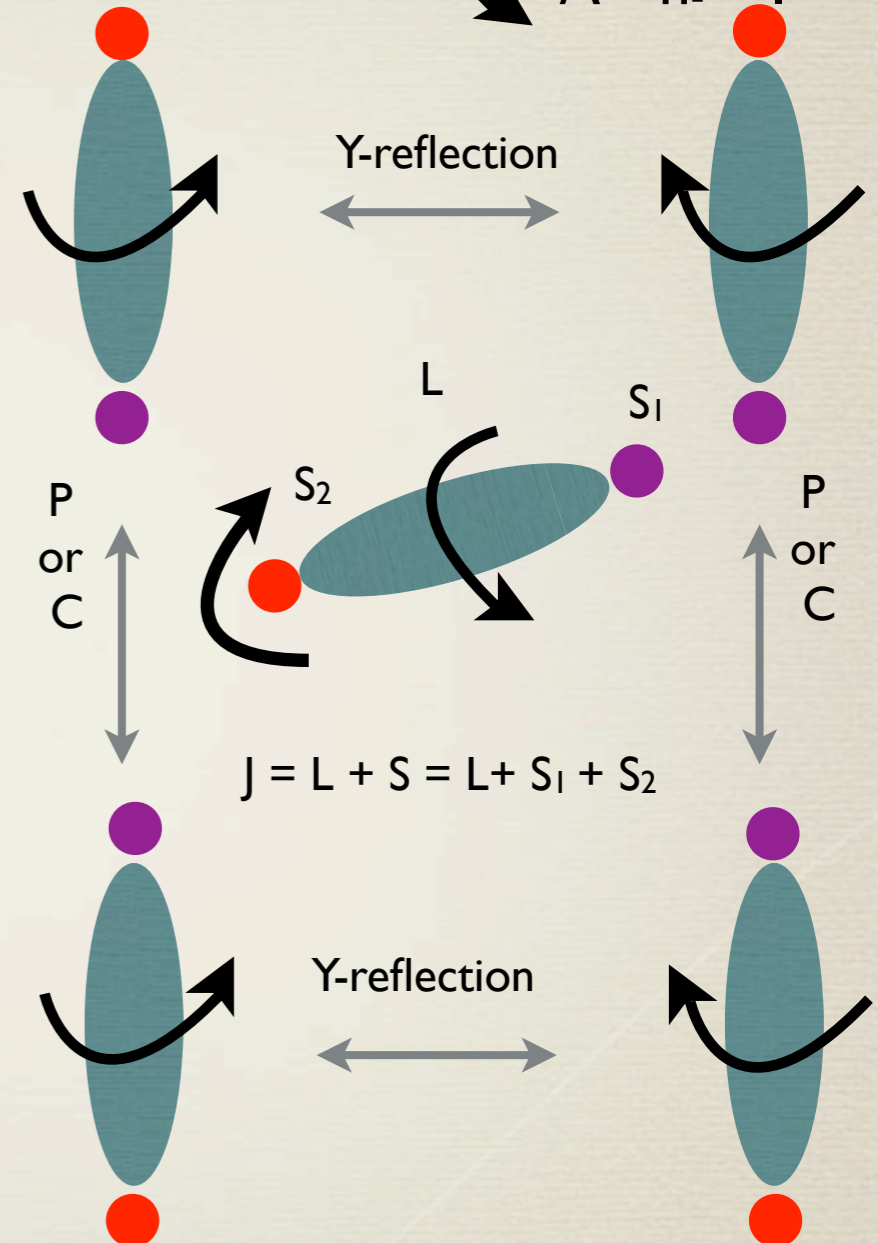
problem with the rigid  
rotor

for example the lowest energy  
mode  $N=1 \quad |\Lambda|=1$  (0-nodes)

$\Lambda = n_+ = 1$

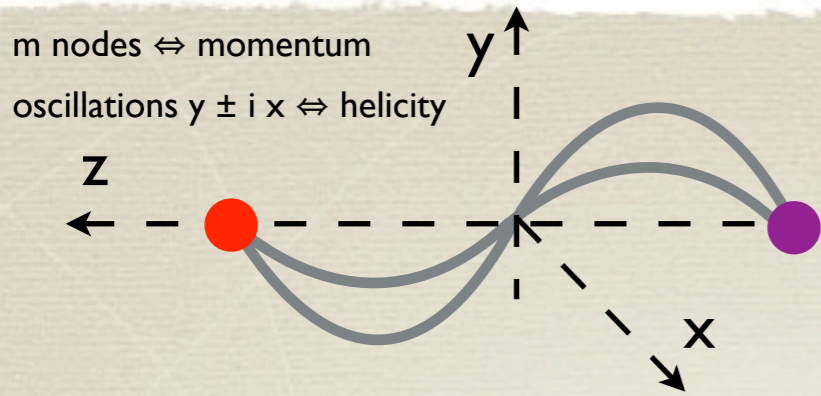


Y-reflection



$J = L + S = L + S_1 + S_2$

Y-reflection

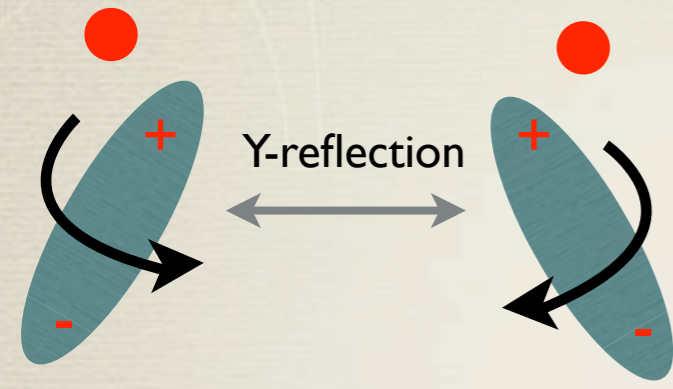


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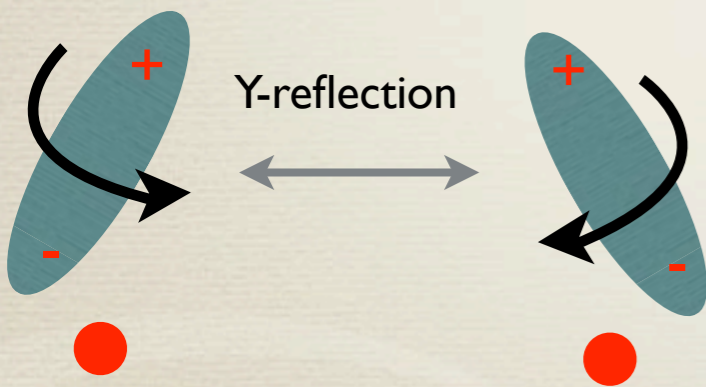
degenerate in FT

$P = +1 \quad C = -1$   
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Top two states have the same energy.



.. but energy of top states NEED NOT be the same as energy of the bottom states

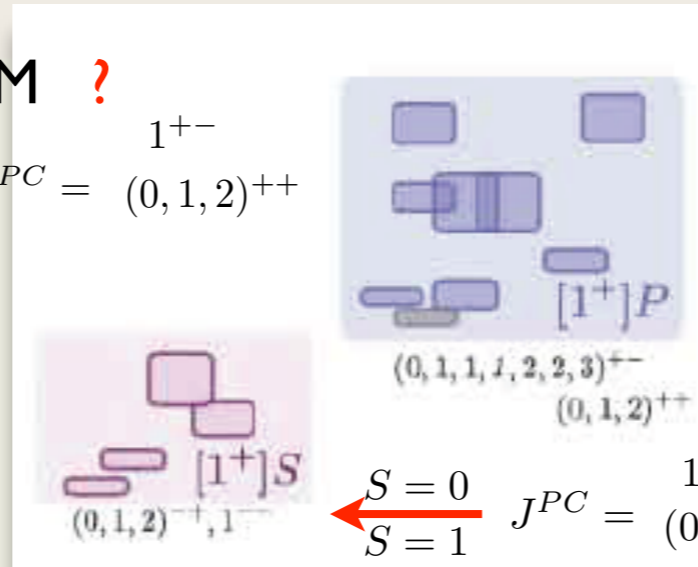


Bottom two states have the same energy,

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 $S = 1 \quad J^{PC} = (0, 1, 2)^{-+} \quad (0, 1, 2)^{+-}$

TM ?

$S = 0 \quad J^{PC} = 1^{+-}$   
 $S = 1 \quad J^{PC} = (0, 1, 2)^{++}$



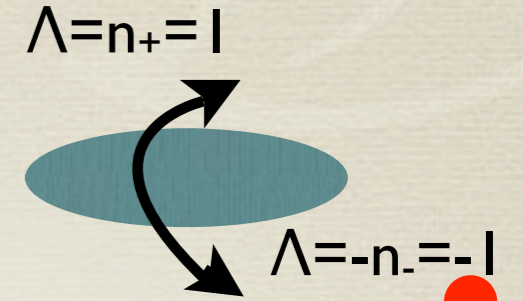
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TE

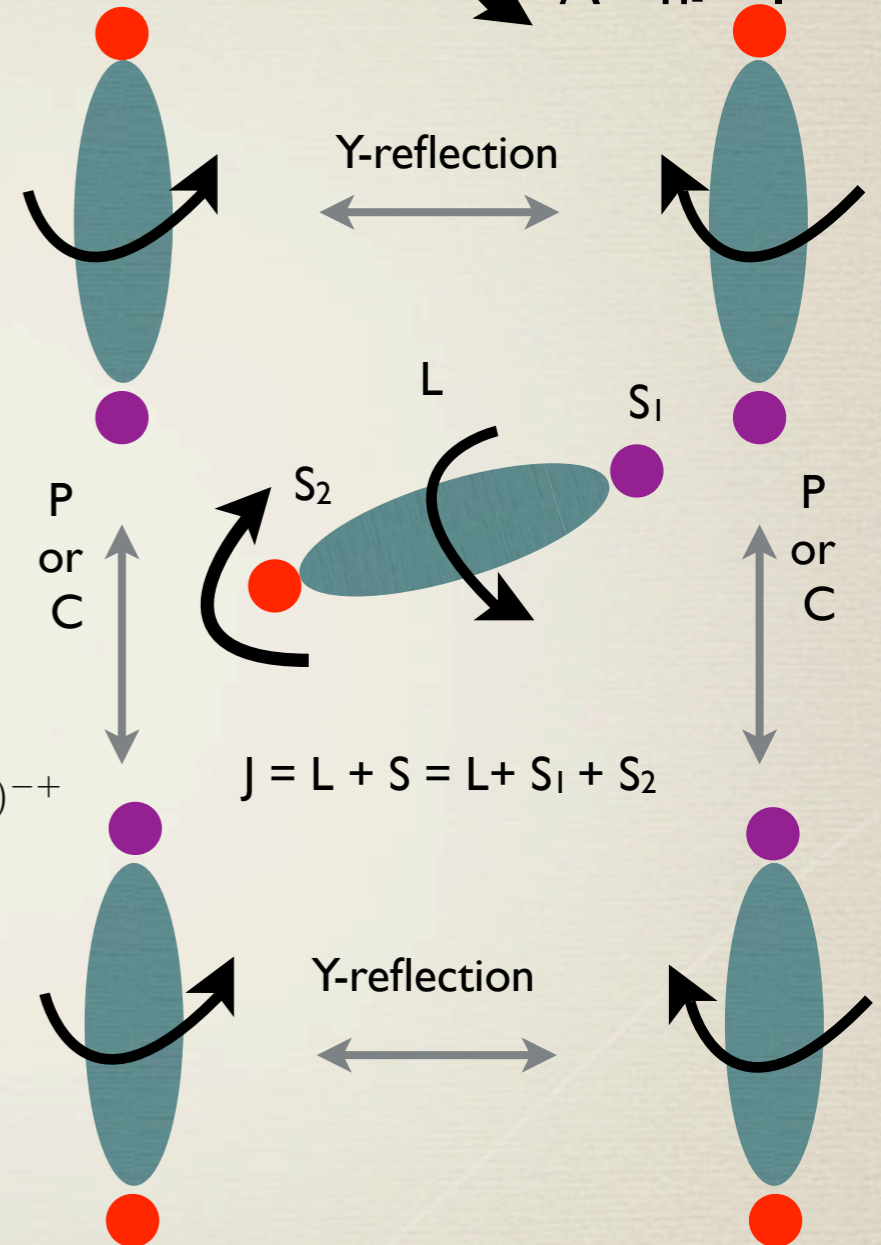
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# Solving for hadrons QCD

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$$H_{QCD}[p, q]\Psi_n(q) = E_n\Psi_n(q)$$

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AS (Indiana)  
H.Reinhard (Tuebingen)

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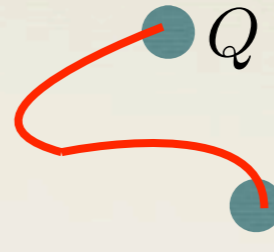
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in QCD gauge invariant variables are Wilson  
lines => periodicity (center symmetry)


$$P_l e^{ig \int d\vec{l} \vec{A}^a(l) T^a}$$

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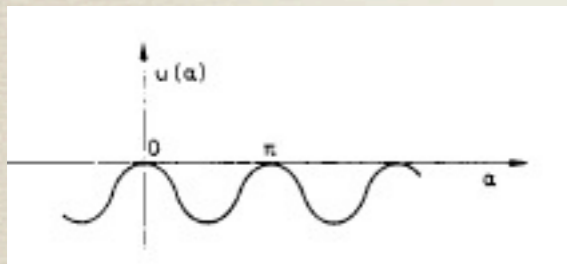
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tunneling between  
equivalent potential minima  
instantons -> vortices,  
monopoles



$$P_l e^{ig \int d\vec{l} \vec{A}^a(l) T^a}$$

vacuum = monopole gas

quark and gluons propagate  
in a monopole background

-> screening

(H.Metevosyan, AS)

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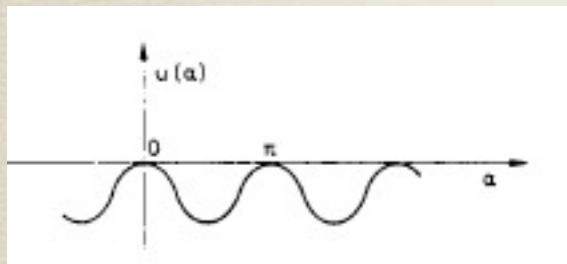
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(btw: "one gluon  
exchange kernel" is not  
responsible for  
confinement  
the monopole gas is!)



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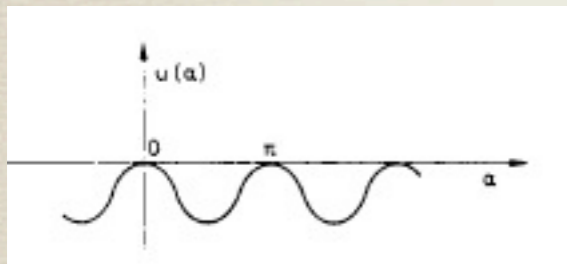
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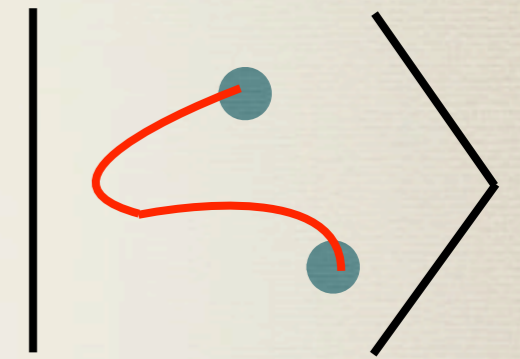
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(H.Metevosyan, AS)

$$P_l e^{ig \int d\vec{l} \vec{A}^a(l) T^a}$$

$$|Q\bar{Q}\rangle_{true} = \sum$$

from interaction between string and monopoles (dual super conductor)



# Solving for hadrons QCD

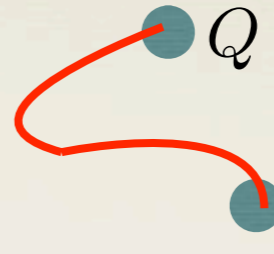
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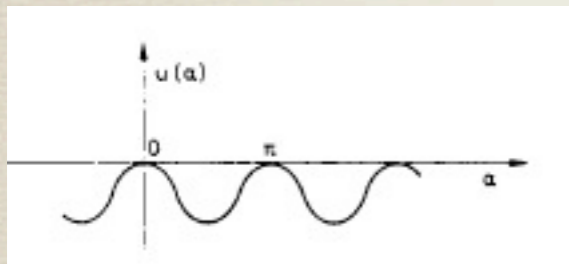
AS (Indiana)  
H.Reinhard (Tuebingen)

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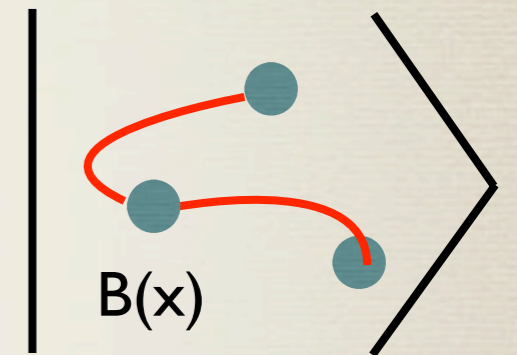


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tunneling between equivalent potential minima  
instantons -> vortices, monopoles



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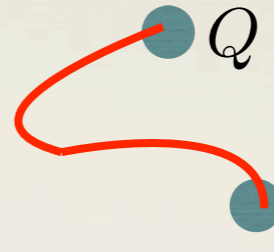
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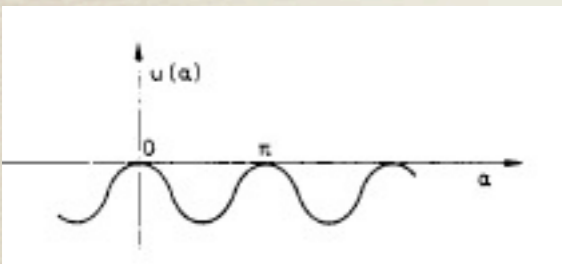
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$$|Q\bar{Q}g\rangle_{true} = \sum_{\text{B(x)}} \left[ \text{Diagram of Wilson loop with monopoles} \right]$$

vacuum = monopole gas

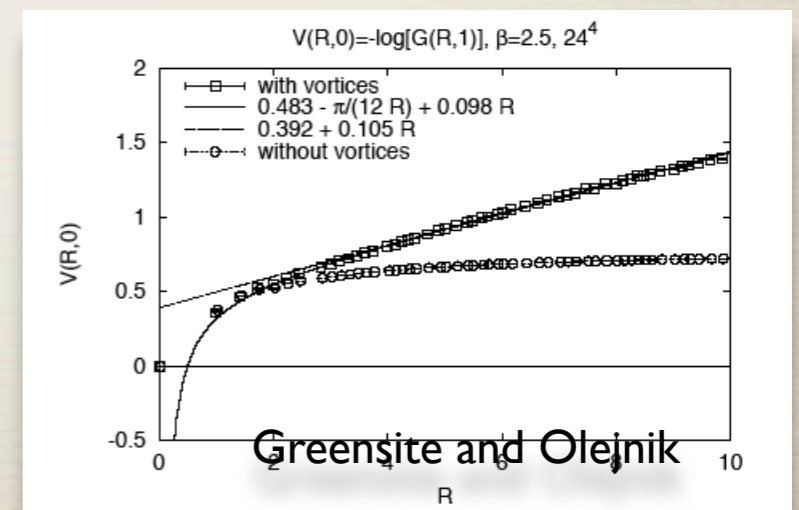
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(H.Metevosyan, AS)

$$|Q\bar{Q}\rangle_{var} = \left[ \text{Diagram of quark-antiquark pair with monopoles} \right]$$

Coulomb energy is overconfining and it also because of the monopole gas.



# Solving for hadrons QCD

$$H_{QCD}[p, q]\Psi_n(q) = E_n\Psi_n(q)$$

$$q \rightarrow \vec{A}_T^a(\vec{x}) \quad a = 1 \dots N_C^2 - 1 \quad \Psi_n(q) \rightarrow \Psi_n(A)$$

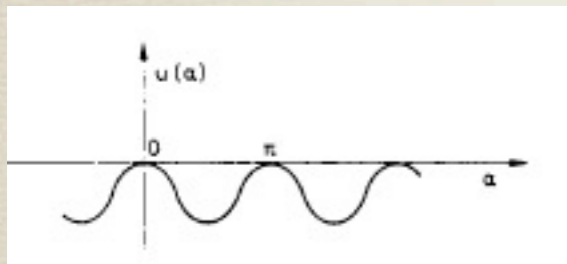
$$\Psi_{vac}(q) = e^{-\frac{1}{2} \int B(x)K(x,y)B(y)} \rightarrow e^{-\frac{1}{2} \int B \frac{1}{\sqrt{-\nabla^2}} B}$$

AS (Indiana)  
H.Reinhard (Tuebingen)

in QCD gauge invariant variables are Wilson lines => periodicity (center symmetry)

$$P_l e^{ig \int d\vec{l} \vec{A}^a(l) T^a}$$

tunneling between equivalent potential minima  
instantons -> vortices, monopoles



$$|Q\bar{Q}g\rangle_{true} = \sum_{\text{Wilson loops}} \left| \begin{array}{c} \text{Wilson loop} \\ B(x) \end{array} \right.$$

vacuum = monopole gas

quark and gluons propagate in a monopole background

-> screening

(H.Metevosyan, AS)

$$|Q\bar{Q}g\rangle_{var} = \left| \begin{array}{c} Q \\ g \\ \bar{Q} \end{array} \right.$$

# Gluon propagator and Monopoles

$$\langle A(\vec{k})A(-\vec{k}) \rangle = D(|\vec{k}|) \rightarrow \frac{1}{2|\vec{k}|}$$

IR suppression  
from monopole  
screening

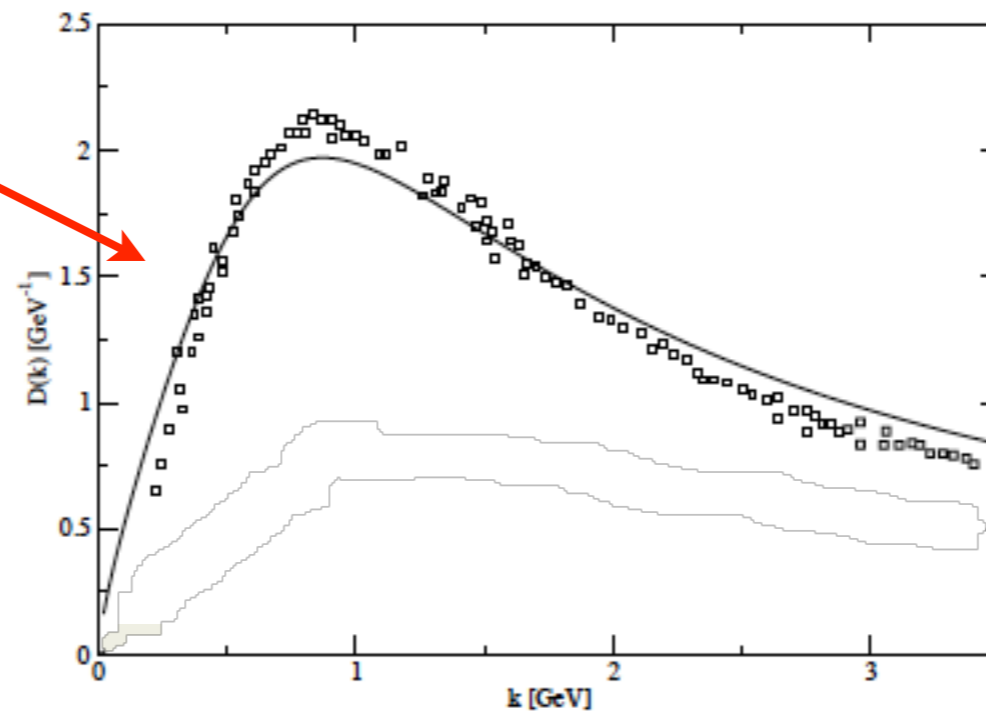
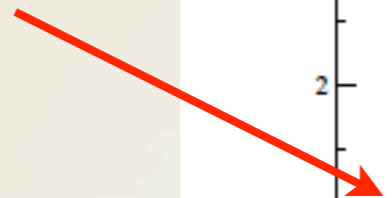


FIG. 1: Comparison of our gluon propagator with that obtained from lattice computations [52]

(H.Metevosyan, AS)



# non-relativistic hybrids

expected degeneracies

- \*  $Q\bar{Q}$  in  $L=0, S=0, I$
- \* coupled to  $I^-$  glue in the relative  $L=1$  state  $\Rightarrow J_g^{PC} = 1^{+-}$

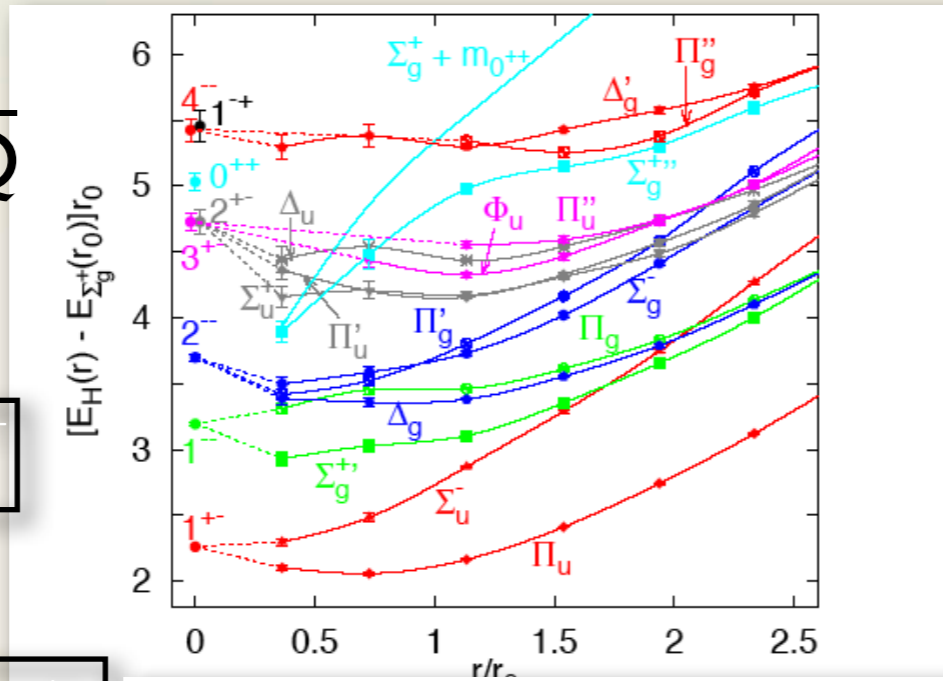
$J^{PC}$  glue

$J^{PC} Q\bar{Q}$

$$1^{+-} \times 0_{S_{Q\bar{Q}}}^{-+} = 1^{--}$$

$$1^{+-} \times 1_{S_{Q\bar{Q}}=1}^{--} =$$

$$0^{-+}, 1^{-+}, 2^{+-}$$

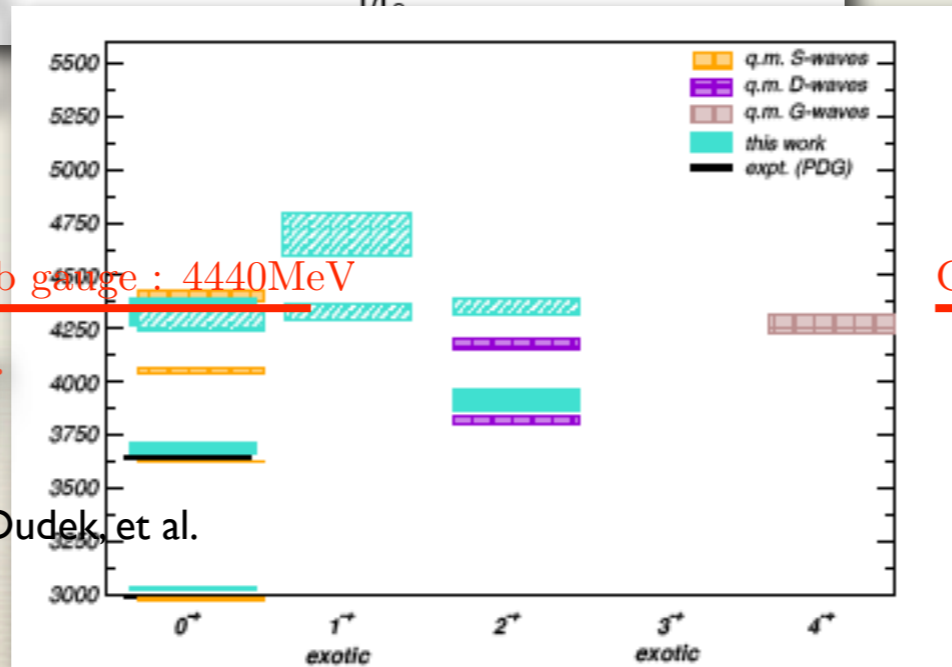


\* experiment  
Y(4260) (Belle, BaBar)

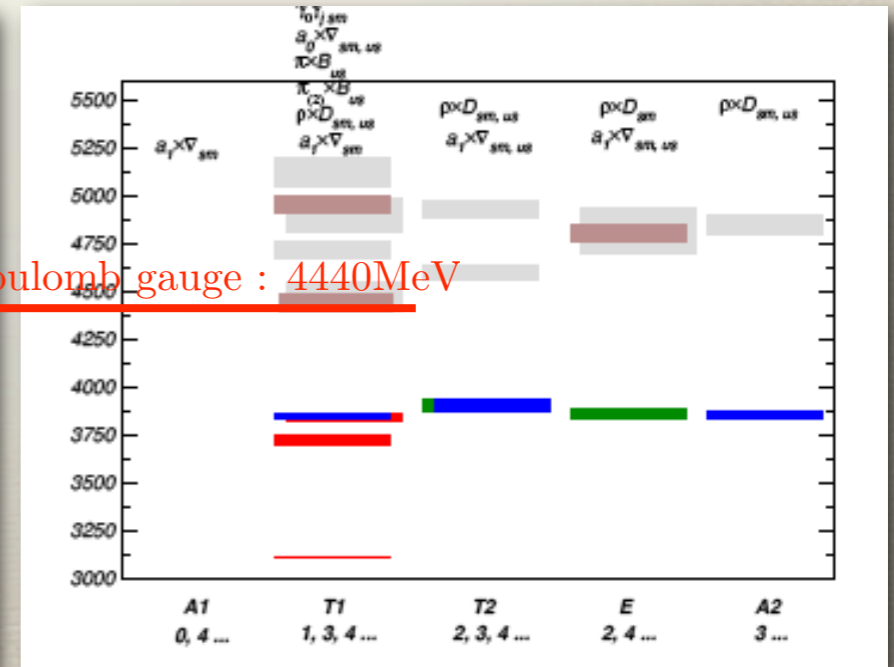
Coulomb gauge : 4440MeV

P.Guo et al., Phys.Rev.  
D78 (2008) 056003

J.Dudek, et al.



Coulomb gauge : 4440MeV



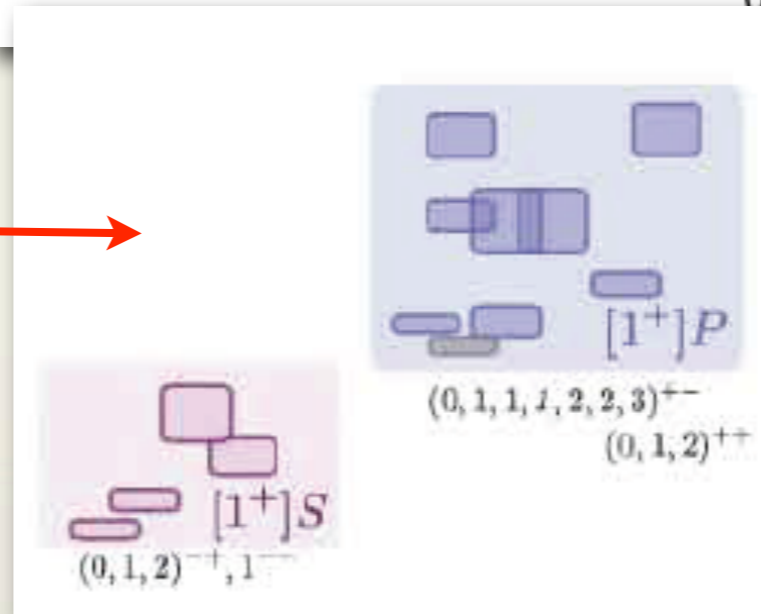
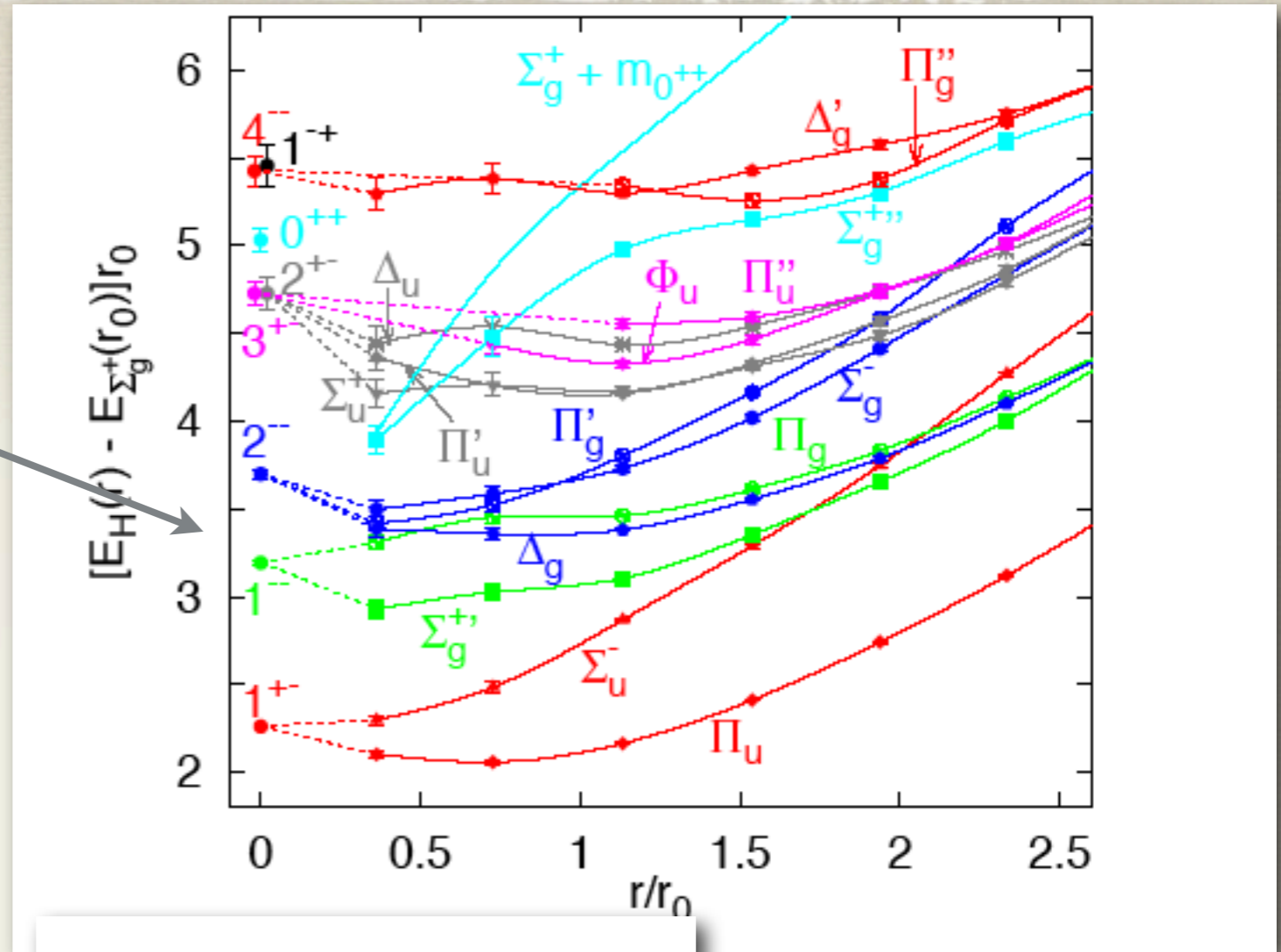
$$|hybrid\rangle = |JJ_g L_{Q\bar{Q}} S\rangle$$

$$J_g^{PC} = 1^{--}$$

	$S = 0$	$S = 1$
$L = 0$	$1^{+-}$	$(0, 1, 2)^{++}$
$L = 1$	$(0, 1, 2)^{-+}$	$(0, 1^3, 2^2, 3)^{--}$

$$J_g^{PC} = 1^{+-}$$

	$S = 0$	$S = 1$
$L = 0$	$1^{--}$	$(0, 1, 2)^{-+}$
$L = 1$	$(0, 1, 2)^{++}$	$(0, 1^3, 2^2, 3)^{+-}$



observed lattice pattern in perfect agreement with QCD CG even for light quarks



Exotic story  $\pi^- p \rightarrow \eta \pi^0 N$  ( $\eta\pi^0$ ) in P-wave has  $J^{PC}=1^-+$ !  
 $\rightarrow \eta \pi^- p$

$$\pi^- p \rightarrow \eta \pi^- p$$

$$M = 1370 \pm 16_{-30}^{+50} \text{ MeV} / c^2$$

$$\Gamma = 385 \pm 40_{-105}^{+65} \text{ MeV} / c^2$$

$$\pi^- p \rightarrow \eta \pi^0 n$$

New results: No consistent B-W interpretation possible but a weak  $\eta\pi$  interaction exists and can reproduce the exotic wave

$$\pi^- p \rightarrow \eta' \pi^- p$$

$$M = 1597 \pm 10_{-10}^{+45} \text{ MeV} / c^2$$

$$\Gamma = 340 \pm 40_{-50}^{+50} \text{ MeV} / c^2$$

$$\pi^- p \rightarrow \rho^0 \pi^- p$$

$$M = 1593 \pm 8_{-47}^{+29} \text{ MeV} / c^2$$

$$\Gamma = 168 \pm 20_{-12}^{+150} \text{ MeV} / c^2$$

$$\pi^- p \rightarrow b_1 \pi p$$

$$\pi^- p \rightarrow f_1 \pi p$$

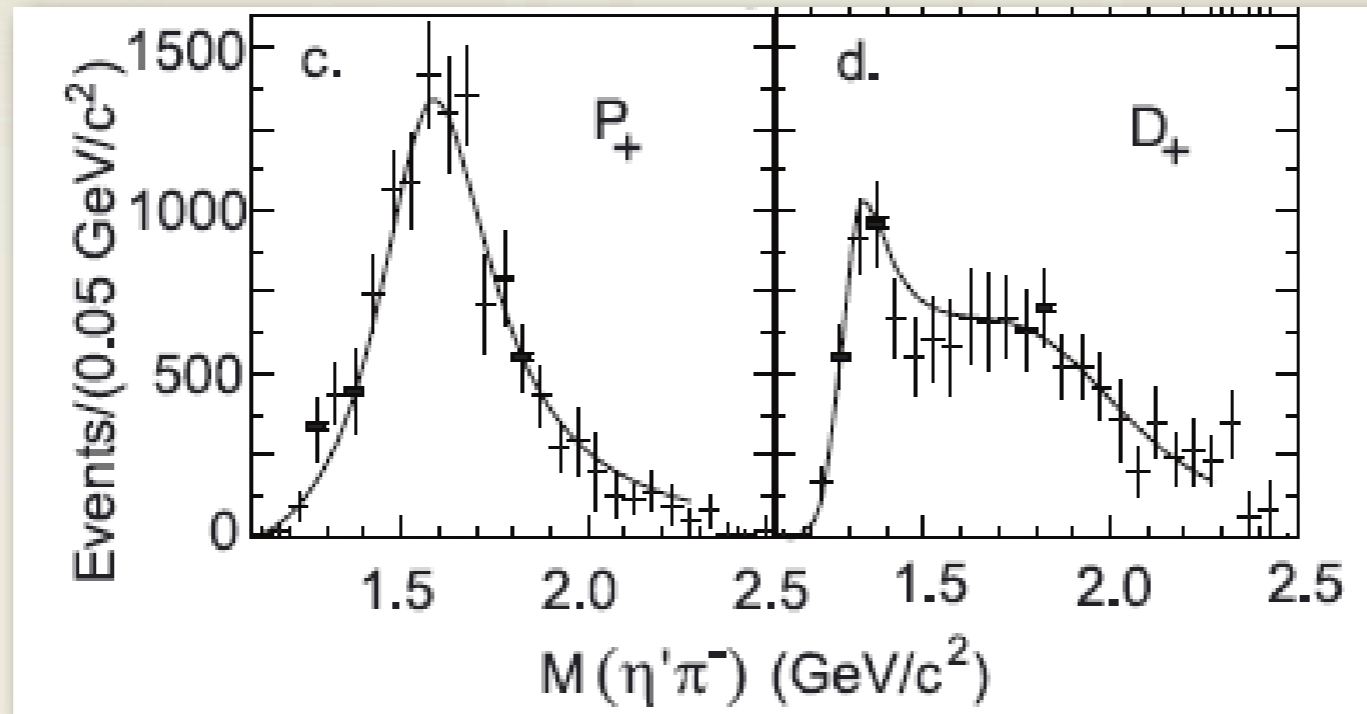
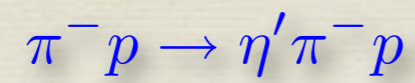
BNL (E852)

new analysis reduces the strength but COMPASS find the signal again

not every bump is a resonance  
(and even if, it may not be a BW)

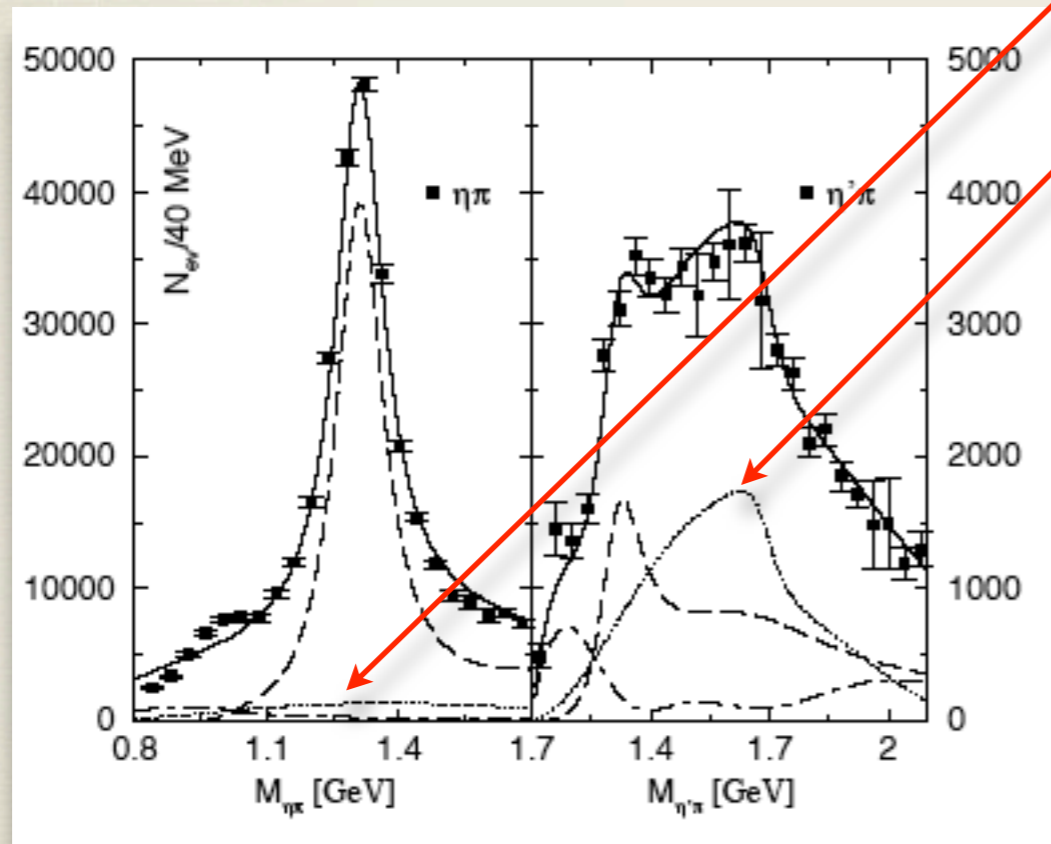
it is important to understand  
production mechanisms

$J^{PC} = 1^{-+}$  exotic wave signals (E852 data)



(other signals  
identified by E852,  
CB,VES)

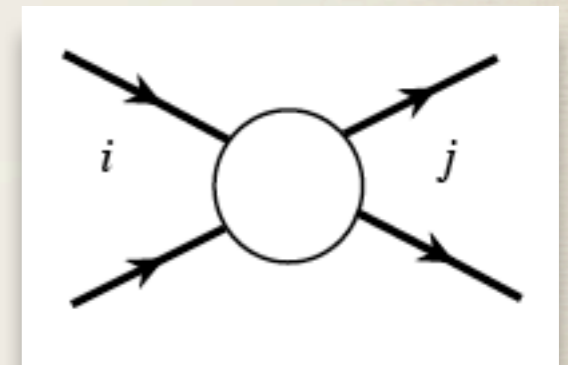
Fitting the E852 the  $\eta\pi$  and  $\eta'\pi$  spectra using eft give a good description of the exotic wave (APS et al.)



P-wave

P -wave  $\eta\pi, \eta'\pi$   
2 coupled channels

S, D -wave  $K\bar{K}, \eta\pi, \eta'\pi$   
3 coupled channels

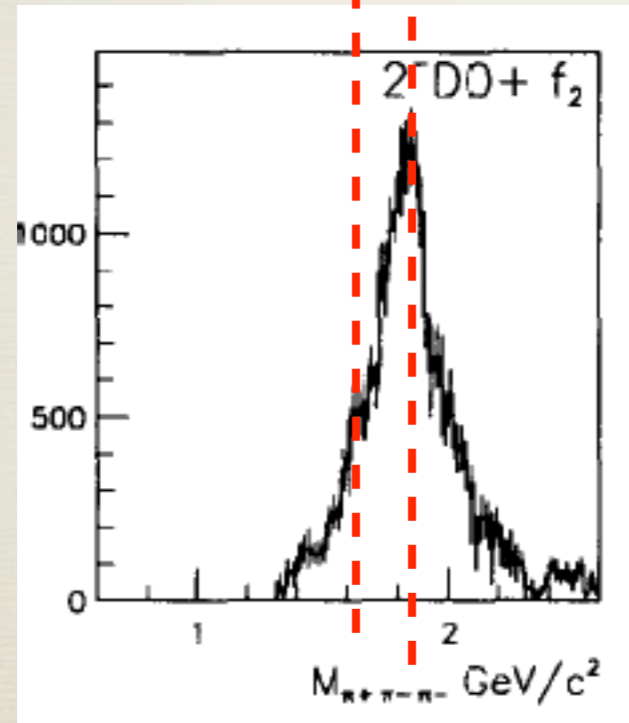
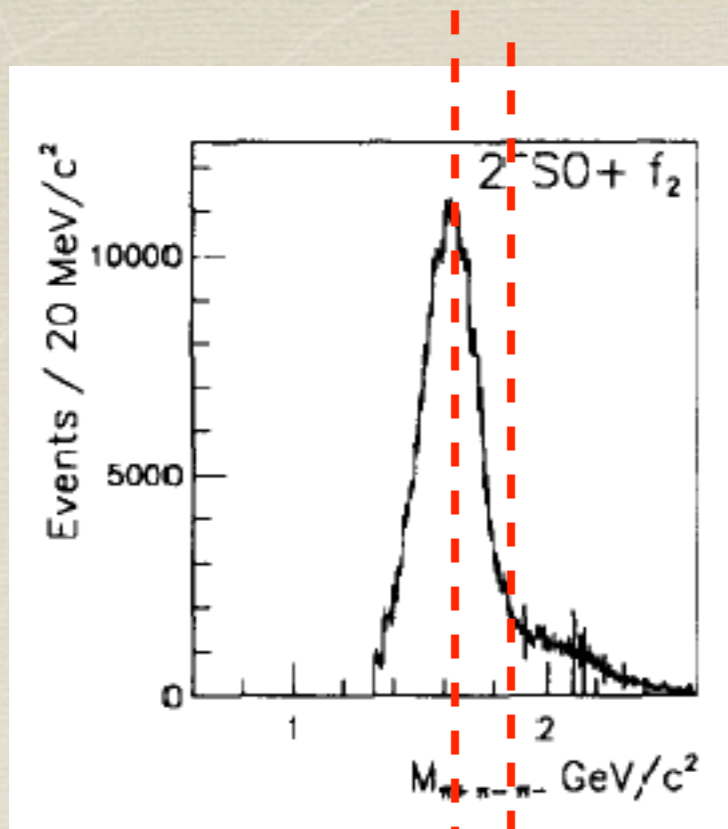


$$t(s) = \frac{1}{\text{Re } V^{-1} - i\rho}$$

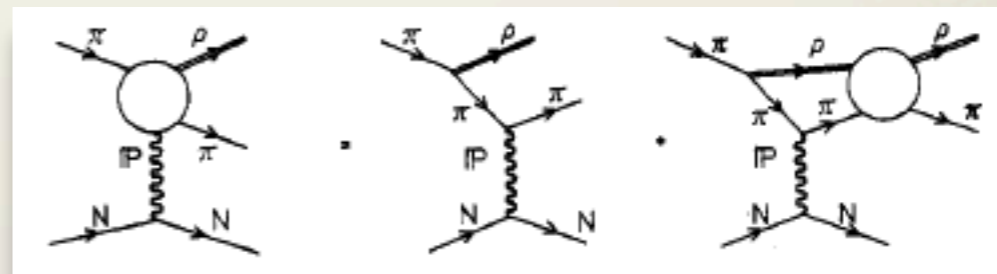
there are no long range forces in  
between  $\eta$  and  $\pi$

to fit the data  $V$  needs to have short  
range interactions

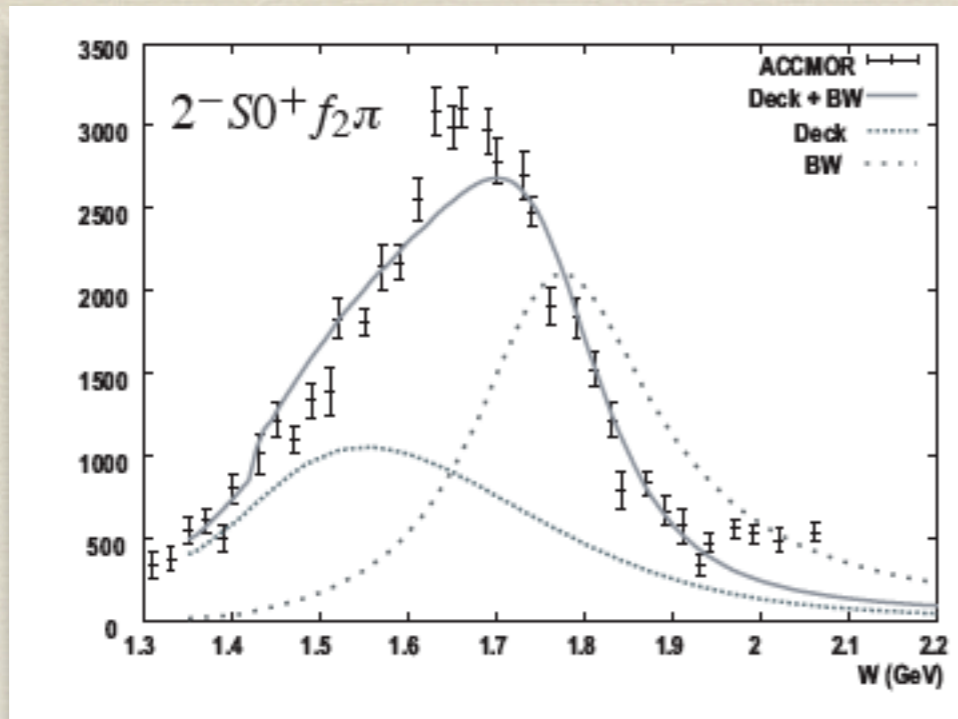
# Moving $\pi_2(1670)$ peak



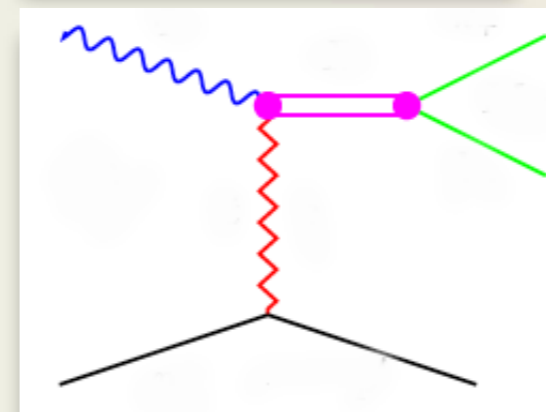
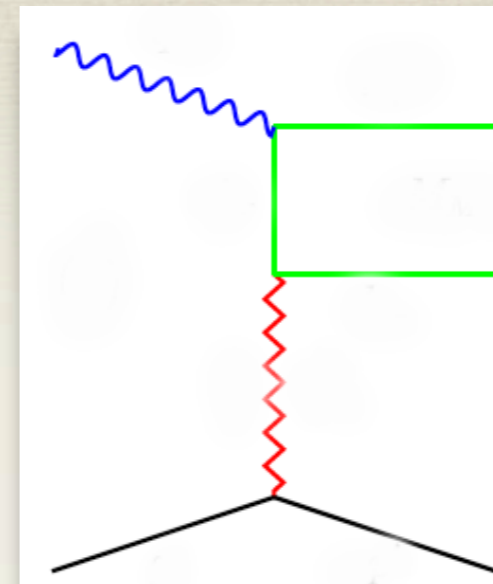
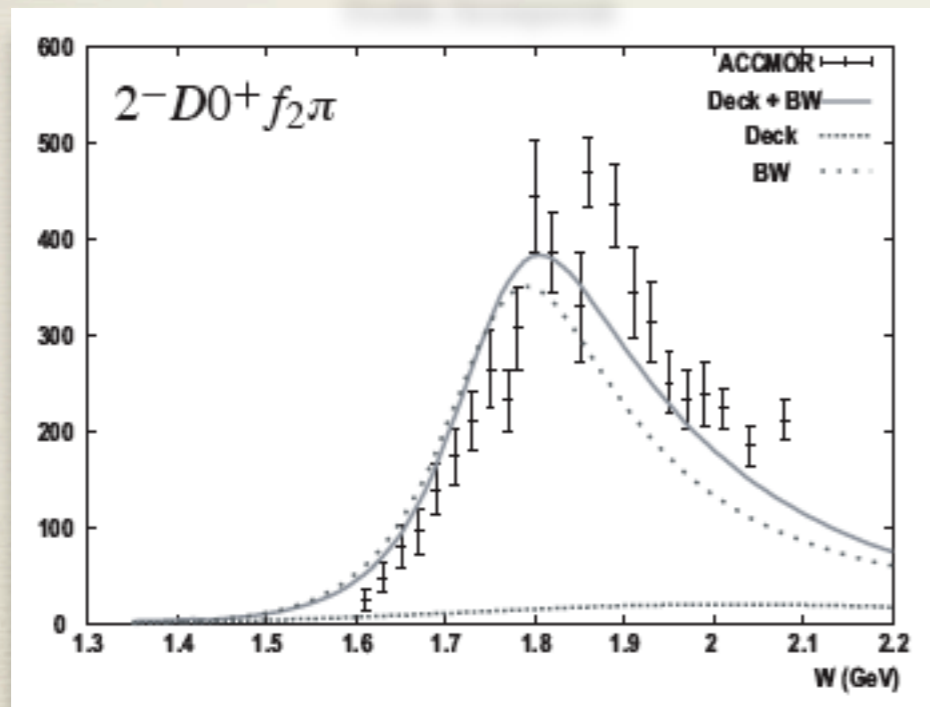
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
<b>1672.4 ± 3.2 OUR AVERAGE</b>		Error includes scale factor of 1.4. See the ideogram below.			
1749 ± 10 ± 100	145k	LU	05	E852	18 $\pi^- p \rightarrow \omega \pi^- \pi^0 p$
1676 ± 3 ± 8		1 CHUNG	02	E852	18.3 $\pi^- p \rightarrow \pi^+ \pi^- \pi^- p$
1685 ± 10 ± 30		2 BARBERIS	01		450 $p p \rightarrow p_f 3\pi^0 p_s$
1687 ± 9 ± 15		AMELIN	99	VES	37 $\pi^- A \rightarrow \omega \pi^- \pi^0 A^+$
1669 ± 4		BARBERIS	98B		450 $p p \rightarrow p_f p \pi p_s$
1670 ± 4		BARBERIS	98B		450 $p p \rightarrow p_f f_2(1270) \pi p_s$
1730 ± 20		3 AMELIN	95B	VES	36 $\pi^- A \rightarrow \pi^+ \pi^- \pi^- A$
1690 ± 14		4 BERDNIKOV	94	VES	37 $\pi^- A \rightarrow K^+ K^- \pi^- A$
1710 ± 20	700	ANTIPOV	87	SIGM	- 50 $\pi^- Cu \rightarrow \mu^+ \mu^- \pi^- Cu$
1676 ± 6		4 EVANGELISTA	81	OMEG	- 12 $\pi^- p \rightarrow 3\pi$
1657 ± 14		4,5 DAUM	80D	SPEC	- 63-94 $\pi p \rightarrow 3\pi X$
1662 ± 10	2000	4 BALTAY	77	HBC	+ 15 $\pi^+ p \rightarrow p 3\pi$
*** We do not use the following data for averages, fits, limits, etc. ***					
1742 ± 31 ± 49		ANTREASYAN	90	CBAL	$e^+ e^- \rightarrow e^+ e^- \pi^0 \pi^0 \pi^0$
1624 ± 21		1 BELLINI	85	SPEC	40 $\pi^- A \rightarrow \pi^- \pi^+ \pi^- A$
1622 ± 35		6 BELLINI	85	SPEC	40 $\pi^- A \rightarrow \pi^- \pi^+ \pi^- A$
1693 ± 28		7 BELLINI	85	SPEC	40 $\pi^- A \rightarrow \pi^- \pi^+ \pi^- A$
1710 ± 20		8 DAUM	81B	SPEC	- 63,94 $\pi^- p$
1650 ± 10		4 ASCOLI	73	HBC	- 5-25 $\pi^- p \rightarrow p \pi_2$



# Duality @ work ?



Dudek, Szczepaniak



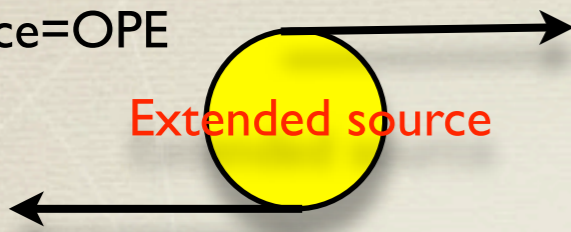
Isobar-type fits could involve spurious resonances

Resonances do not have to show up as peaks or can be skewed

(S-wave)

force=OPE

Extended source



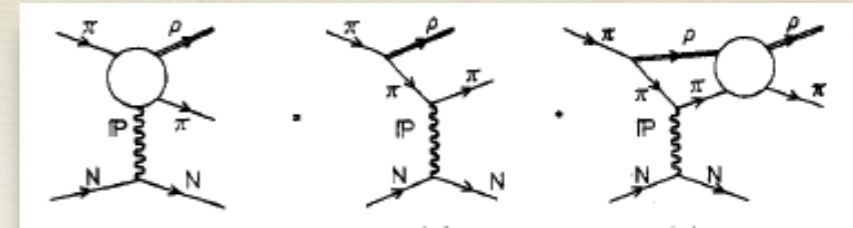
$$e^{i\delta(E)} \cos \delta(E)$$

QCD, CDD

Compact source

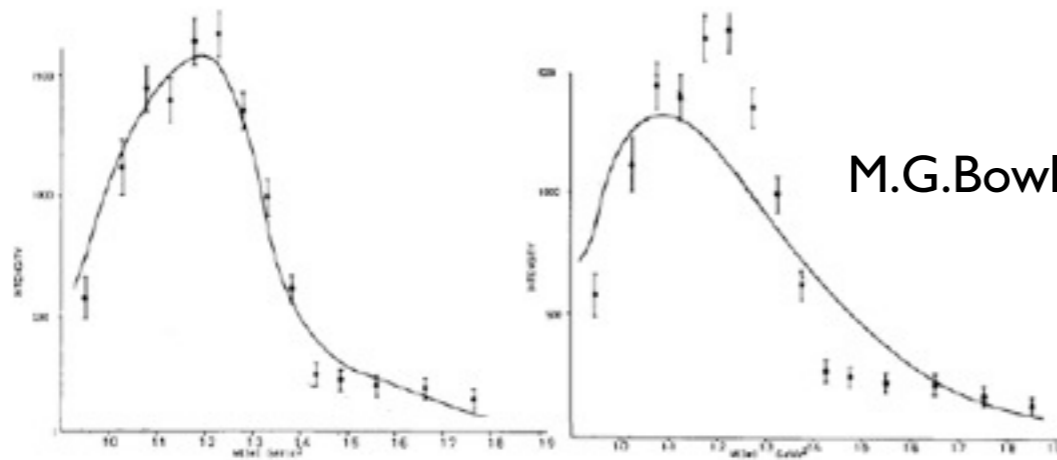


$$e^{i\delta(E)} \frac{\sin \delta(E)}{k}$$



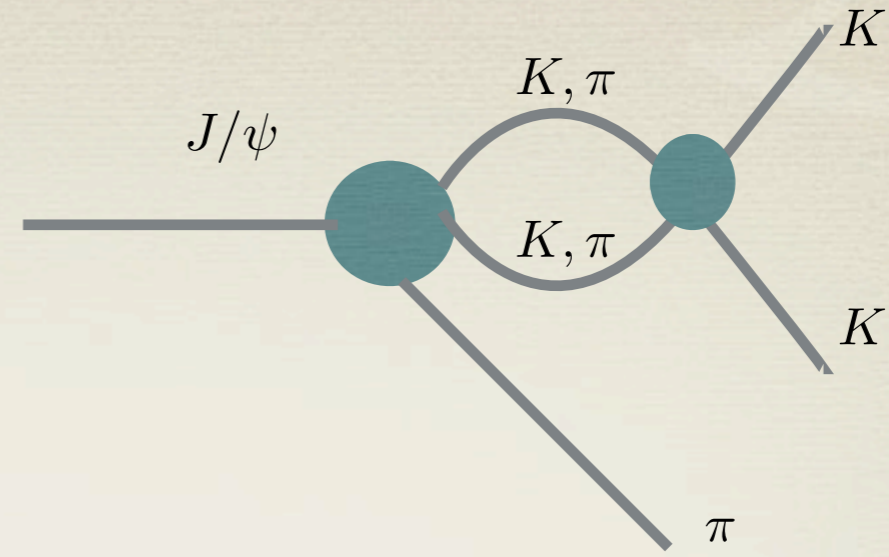
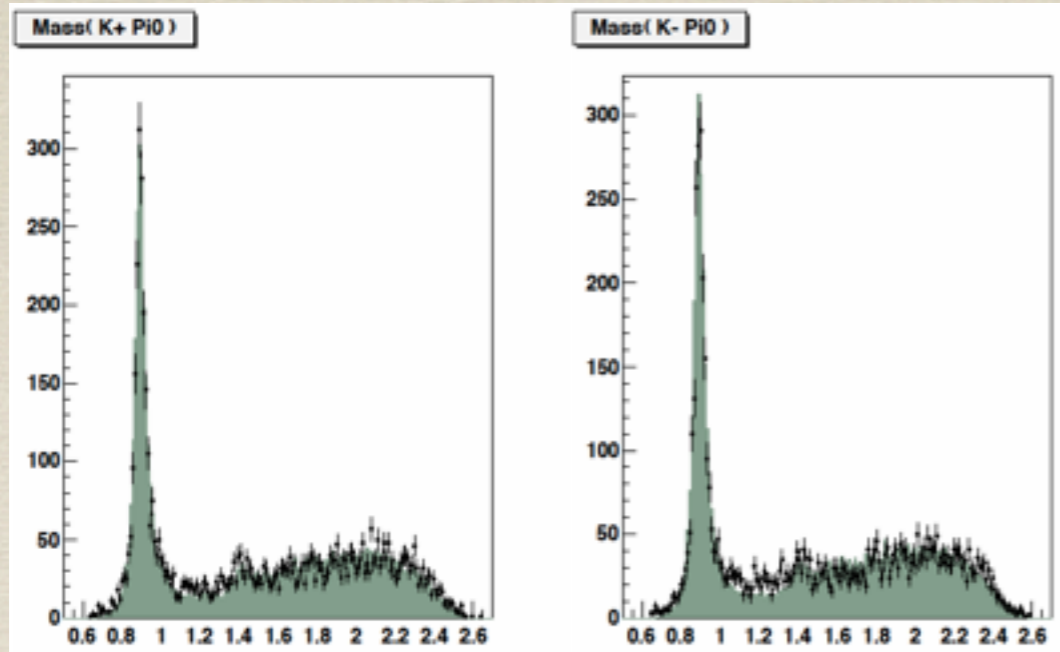
force=OPE+CDD

CDD

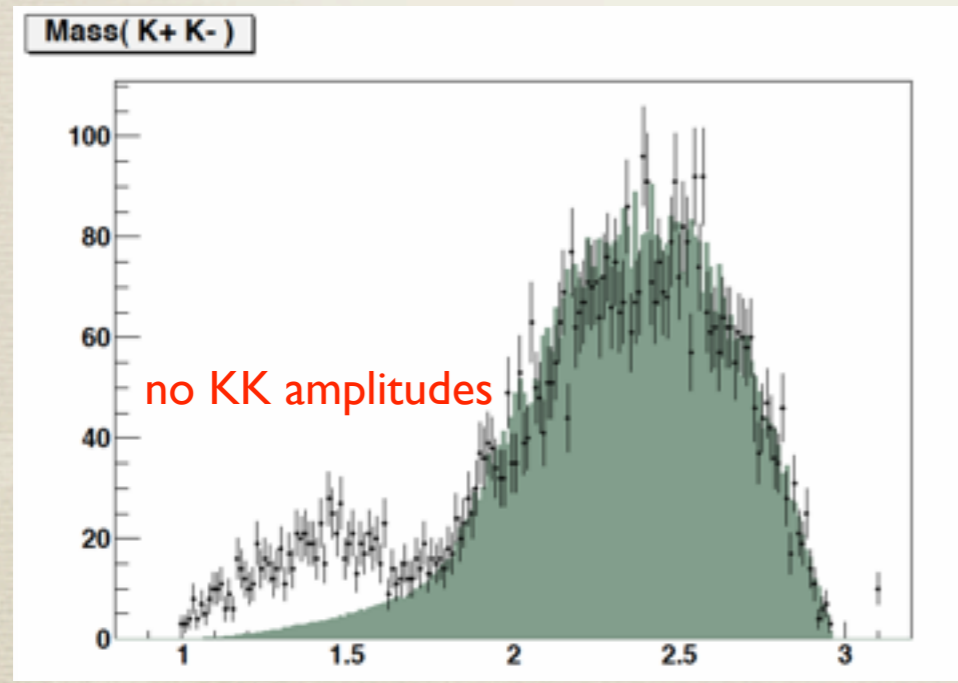
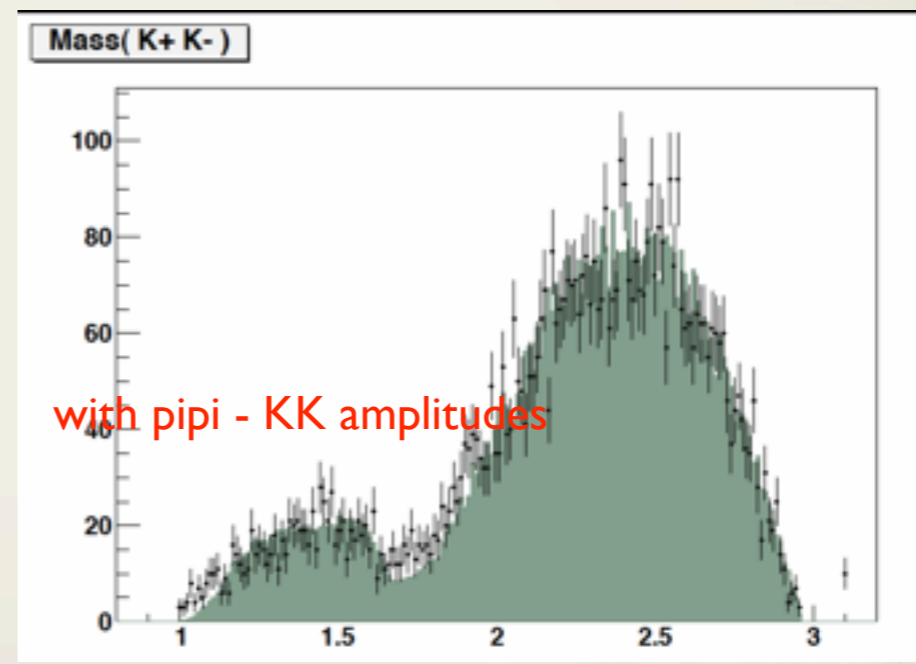
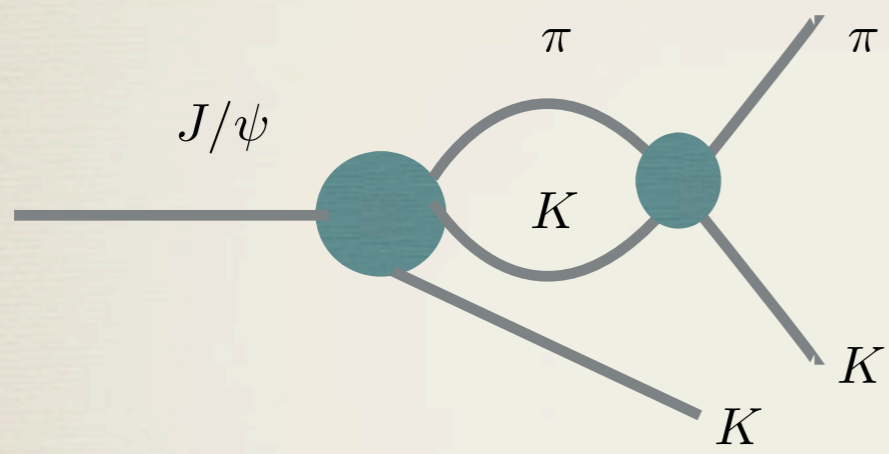


M.G.Bowler,(1975!)

Figure 11: Fit to the  $1^+ \rho\pi$  intensity from  $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$  at  $E_\pi = 25$  and  $E_\pi = 40$  GeV, CERN data [70], with (left) both long-range production from one pion exchange and short-range direct production and (right) short-range direct production only [63].



P.Guo,R.Mitchel,M.Shepherd,APS



PRELIMINARY



\* General idea

$$\text{Im}A(s) = R(s)\rho(s)|A(s)|^2$$

$$A(s) = \frac{1}{\pi} \int_{-\infty}^0 ds' \frac{\text{Im}A(s')}{s' - s} + \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\text{Im}A(s')}{s' - s}$$

integral equation for  
the amplitude

\* output : through unitarity related to  
measured x-section

\* input (“potential”) : through crossing lhc is  
related to other physical amplitudes

caveats

\* potential not known everywhere

\* in principle many ( $\infty$ ) channels contribute

\* x-sections known over limited energy  
range

\* solutions are not unique (CDD)

recent  
improvements and  
(1960's vs 2000)

\* QCD: interpretation of the ambiguities

\* chiral symmetry: low energy constraints

From dispersion relations



CDD pole required

FIG. 1:  $P$ -wave phase shift (upper panel) and inelasticity (lower panel). Data from [34–36], dashed-dotted (solid) line solution of dispersion relation without (with) a CDD pole. Dashed line is the fit of the quark model from Eq.(33).



bootstrap failed

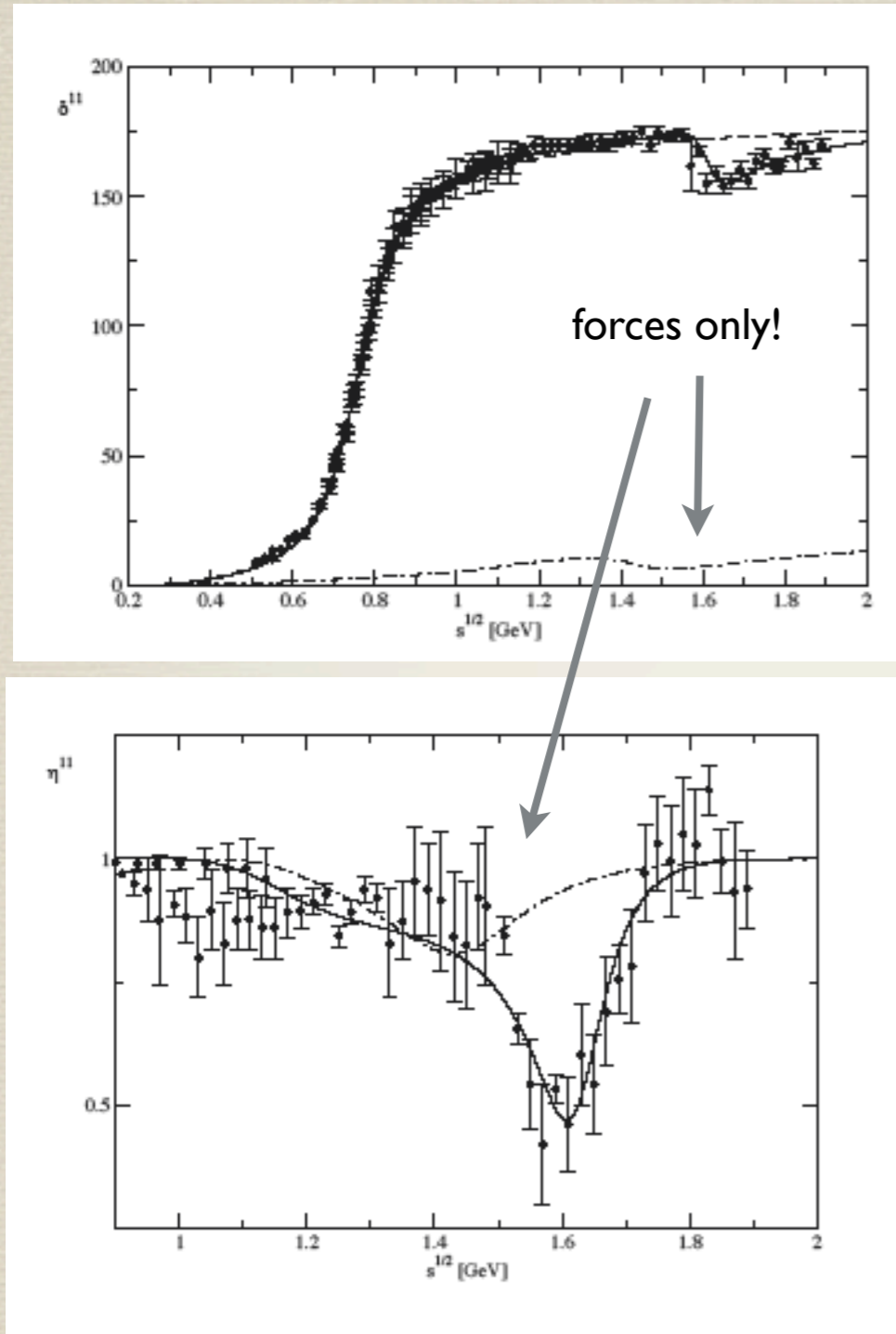
M.Battaglieri,R.de Vita,P.Guo,AS



resonances are not generated dynamically from interactions between other resonances



or as lattice suggests there are single hadron states in the spectrum



From dispersion relations

\* CDD pole required

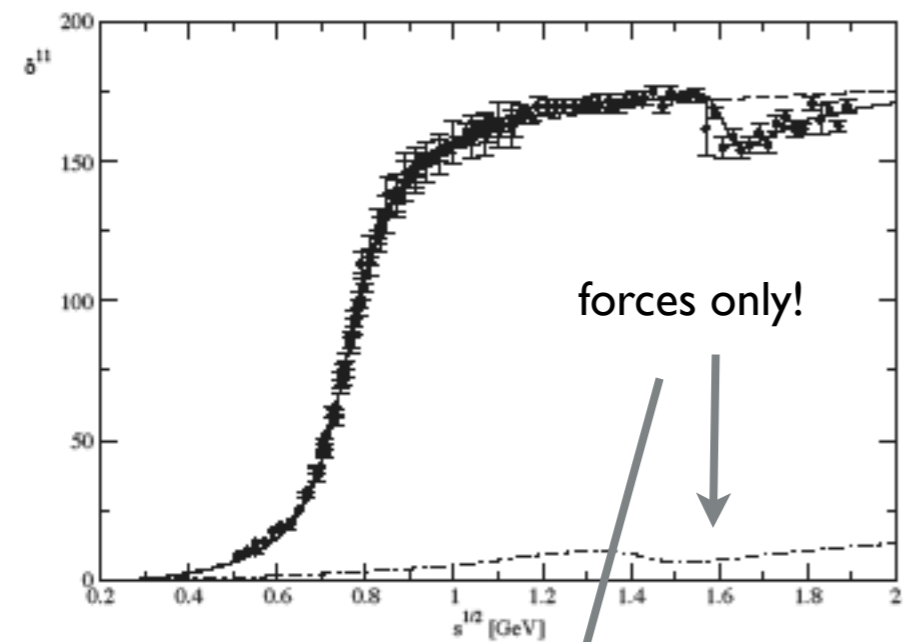
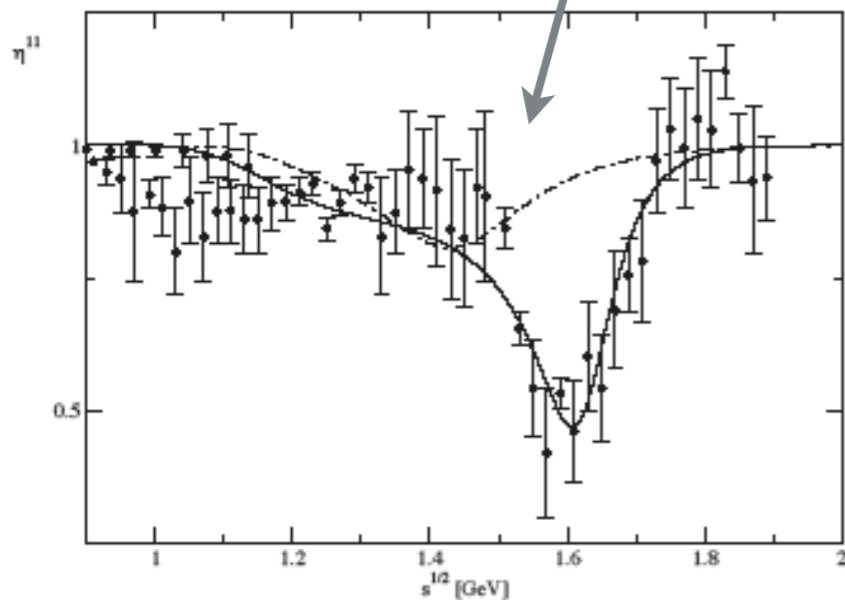


FIG. 1:  $P$ -wave phase shift (upper panel) and inelasticity (lower panel). Data from [34–36], dashed-dotted (solid) line solution of dispersion relation without (with) a CDD pole. Dashed line is the fit of the quark model from Eq.(33).



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- \* bootstrap failed
- \* resonances are not generated dynamically from interactions between other resonances
- \* or as lattice suggests there are single hadron states in the spectrum
- \* how does it fit in with the success of dynamically generated resonance program from a unitarized chi-PT approach ?

do the Uch-PT poles move?

# Summary

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\* Lattice : solid evidence for single-hadron QCD states (CDD poles) **including hybrids**

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- \* solid evidence for Mandelstam, Nambu,t'Hooft,Polyakov superconductor model of QCD vacuum

“dressed” gluon exchange does not generate hadrons  
but a correct phenomenology can be developed **including hybrids**

## Summary

\* Lattice : solid evidence for single-hadron QCD states (CDD poles) **including hybrids**

\* solid evidence for Mandelstam, Nambu,t'Hooft,Polyakov superconductor model of QCD vacuum

“dressed” gluon exchange does not generate hadrons

but a correct phenomenology can be developed **including hybrids**

\* bootstrap was abandoned because it discovered hadrons are not dynamically generated, S-matrix was abandoned because CDD poles could not be excluded

**these, however, are based on model-independent constrains which should not be forgotten in modern amplitude analyses.**