Meson Spectroscopy at COMPASS

Methods for Amplitude Analysis

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Technische Universität München

Hirschegg, January 2011

The COMPASS Experiment

Diffractive Pion Dissociation

Amplitude Analysis Formalism

PWA Model Selection
- Mass Independent Fit
- Bayesian Model Evaluation
- Waveset Exploration

Mass Dependent Fit

ROOTPWA Analysis Toolkit

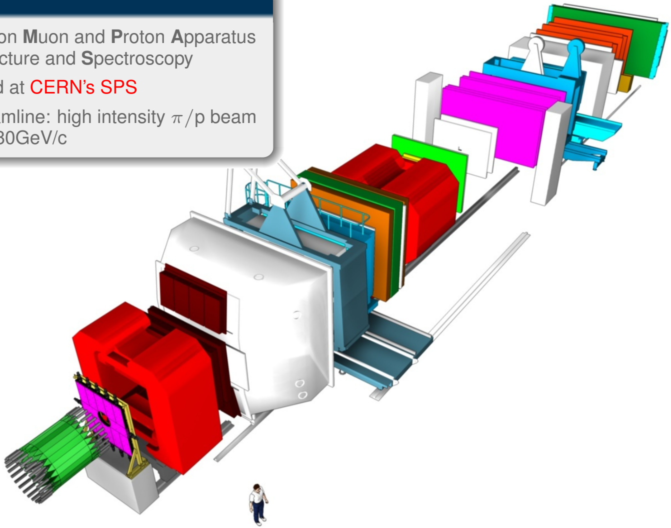
The COMPASS Experiment

Searching for Gluonic Contributions to the Meson Spectrum

Technische Universität München

Overview

- **CO**mmon **M**uon and **P**roton **A**pparatus for **S**tructure and **S**pectroscopy
- Located at CERN's SPS
- M2-beamline: high intensity π/p beam up to 280GeV/c



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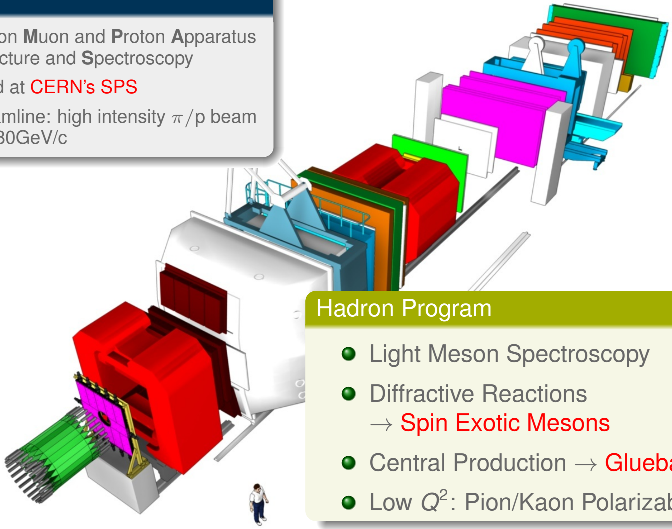
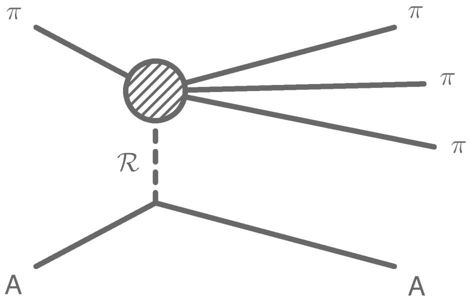


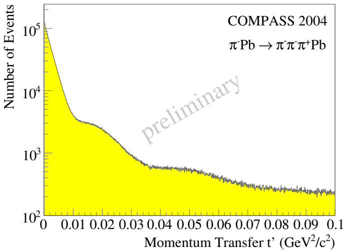
Hadron Program

- Light Meson Spectroscopy
- Diffractive Reactions
  $\rightarrow$ Spin Exotic Mesons
- Central Production $\rightarrow$ Glueballs
- Low Q^2: Pion/Kaon Polarizabilities

Diffractive Pion Dissociation

Example: 3 Pion Final State





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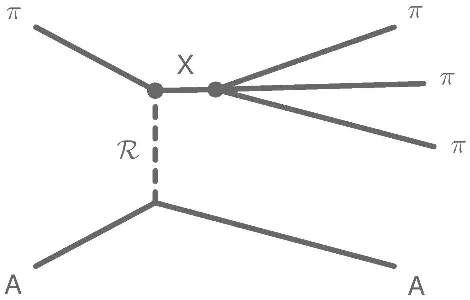

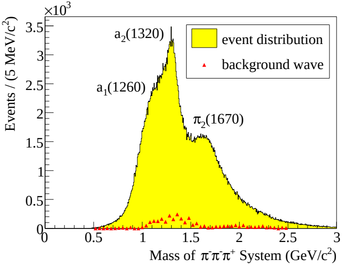

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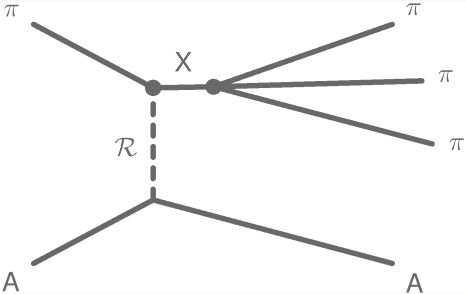

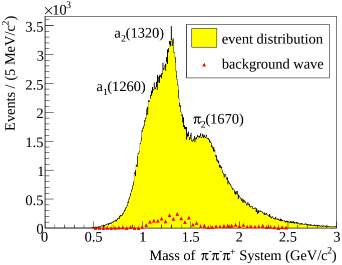

X: $J^{PC}M^{\epsilon}$ decay amplitude in *reflectivity base*

Implement parity conservation:

$$\psi^{\epsilon}_{JM} = c(M)\left[\psi_{JM}(\tau) - \epsilon P(-1)^{J-M}\psi_{J(-M)}(\tau)\right]$$

$$\epsilon = \pm 1 \qquad M \in [0..J] \qquad c(M>0) = \frac{1}{\sqrt{2}} \qquad c(M=0) = \frac{1}{2}$$

PWA Formalism Overview

2Stage Isobar-Model Fit

STEP 1: Mass-Independent PWA

- Fit angular distributions + isobar systems in independent mass bins

$$\mathcal{I}(\tau, m) = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_{\alpha} T_{\alpha r}^{\epsilon} \, \psi_{\alpha}^{\epsilon}(\tau, m) \right|^2$$

- Production amplitude
- Decay amplitude

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- Production amplitude
- Decay amplitude

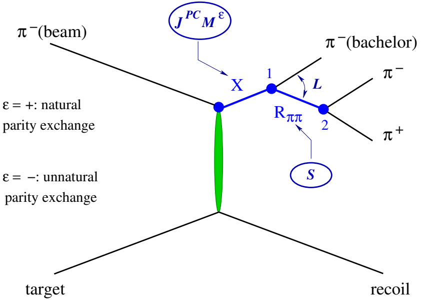
STEP 2: Mass-Dependent χ^2 fit → Extract Resonance Parameters

- Parameterization of spin-density matrix elements $\sum_r T^{\epsilon}_{ir} T^{\epsilon *}_{jr}(m_x)$
- Takes into account interference terms
- Coherent background for some waves

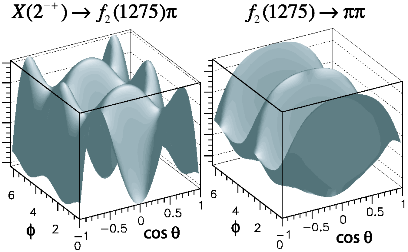
Decay Parameterization: The Isobar Model

Chain of successive 2-body decays

Model n-body decay by a chain of successive 2-body decays:



Example angular distributions:



- For fixed n-body mass m there are $3n - 4$ parameters (angles, intermediate state masses)
- Parameterization of isobar subsystems

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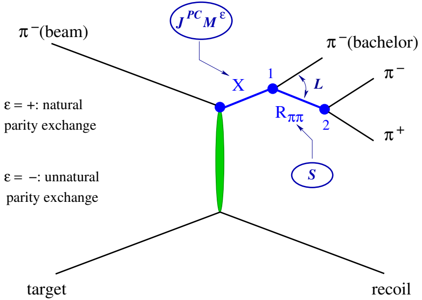
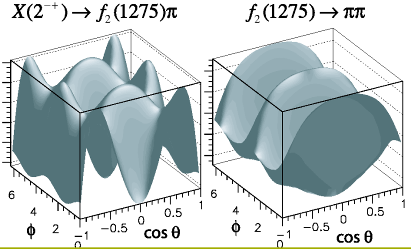


Example angular distributions:



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Known shortcomings

- Unitarity violation
- Rescattering effects
- → Potential for improvement
- Input from theory needed (see e. g. talk by B. Kubis)

Mass Independent Amplitude Fit

Intensity distribution parameterization

Intensity distribution $\mathcal{I}$ as a function of decay-kinematic variables τ:

$$\mathcal{I}(\tau) = \sum_{\epsilon=\pm1} \sum_{r} \left| \sum_{\alpha \in M} T^{\epsilon}_{\alpha r} \bar{\psi}^{\epsilon}_{\alpha}(\tau) \right|^2$$

- Finite *waveset M*
- Production amplitude
- Decay amplitude

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- Finite *waveset M* → $\alpha \in M$
- Production amplitude → $T^{\epsilon}_{\alpha r}$
- Decay amplitude → $\bar{\psi}^{\epsilon}_{\alpha}(\tau)$

The likelihood $\mathcal{L}$ to observe (a specific set of) N events in a bin with finite acceptance $\eta(\tau)$ (assuming a model M, parameters T^{ϵ}_{ir}) is:

$$P(\mathrm{Data}|T_{ir}, M) = \mathcal{L} = \left[\frac{\bar{N}^N}{N!} e^{-\bar{N}}\right] \prod_i^N \frac{\mathcal{I}(\tau_i)\eta(\tau_i) f(\tau_i)}{\underbrace{\int \mathcal{I}(\tau)\eta(\tau) d\rho(\tau)}_{=\bar{N}}} \quad \text{with} \quad d\rho(\tau) = f(\tau) d\tau$$

Mass Independent Amplitude Fit

Definition of LogLikelihood Function

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$$\mathcal{L} = \left[\frac{\bar{N}^N}{N!} e^{-\bar{N}}\right] \prod_i^N \frac{\mathcal{I}(\tau_i)}{\bar{N}} \eta(\tau_i) f(\tau_i) = \frac{1}{N!} \prod_i^N \mathcal{I}(\tau_i) \cdot \prod_i^N \eta(\tau_i) f(\tau_i) \cdot e^{-\bar{N}}$$

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Taking the logarithm leads to and inserting for $\bar{N}$

$$\ln \mathcal{L} = -N \ln N + \sum_i^N \eta(\tau_i) f(\tau_i) + \sum_i^N \ln \mathcal{I}(\tau_i) - \int \mathcal{I}(\tau) \eta(\tau) d\rho(\tau)$$

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drop $(-N \ln N + \sum_i^N \eta(\tau_i) f(\tau_i))$ and insert intensity parameterization

$$\ln \mathcal{L} = \sum_{n=1}^{N_{\mathrm{events}}} \ln \left[\sum_{\epsilon,r} \sum_{\alpha,\beta \in M} T^{\epsilon}_{\alpha r} T^{\epsilon *}_{\beta r} \bar{\psi}^{\epsilon}_{\alpha}(\tau_n) \bar{\psi}^{\epsilon}_{\beta}(\tau_n)^* \right] - \sum_{\epsilon,r} \sum_{\alpha,\beta \in M} T^{\epsilon}_{\alpha r} T^{\epsilon *}_{\beta r} I A^{\epsilon}_{\alpha\beta}$$

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With acceptance-corrected phase space integral

$$IA^{\epsilon}_{\alpha\beta} = \int \bar{\psi}^{\epsilon}_{\alpha}(\tau_n) \bar{\psi}^{\epsilon}_{\beta}(\tau_n)^* \eta(\tau) \mathrm{d}\tau$$

Which waves to include into the waveset?

Which waves to include into the waveset?

Avoid overfitting

Which waves to include into the waveset?

Avoid overfitting

$\rightarrow$ Data driven method

How to Measure the Goodness of a Model

Marginal Likelihood Definition

Bayes' Theorem (for the Model Probability after Observation)

$$P(M_k|\mathrm{Data}) = \frac{P(\mathrm{Data}|M_k)P(M_k)}{\sum_{k'} P(\mathrm{Data}|M_{k'})P(M_{k'})}$$

with model-priors $P(M_k)$ $\quad \sum_{k'} P(M_{k'}) = 1$

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Marginal Likelihood or Evidence

$$P(\mathrm{Data}|M_k) = \int \underbrace{P(\mathrm{Data}|T^k, M_k)}_{\mathcal{L}} \underbrace{P(T^k|M_k)}_{\mathrm{Prior}} dT^k$$

$P(T^k|M_k)$ contains any pre-knowledge on the model-parameters T

- Marginalization ($= \int dT$) is not trivial in high-dimensional spaces
- Numerically stable is only the LogLikelihood

The Occam Factor Approximation

David J. C. MacKay, 2003 “Information Theory, Inference and Learning Algorithms”

$$P(\mathrm{Data}|M_k) = \int \underbrace{P(\mathrm{Data}|T^k, M_k)}_{\mathcal{L}} \underbrace{P(T^k|M_k)}_{\mathrm{Prior}} \, dT^k$$

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Approximate with Laplace’s method:

$$P(\mathrm{Data}|M_k) \approx P(\mathrm{Data}|T^k_{\mathrm{ML}}, M_k) \cdot \underbrace{P(T^k_{\mathrm{ML}}|M_k) \cdot \sqrt{(2\pi)^d |\mathbf{C}_{T|\mathrm{Data}}|}}_{\textit{Occam factor}}$$

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- $P(\mathrm{Data}|T^k_{\mathrm{ML}}, M_k)$ LogLikelihood at maximum likelihood solution T_{ML}
- $|\mathbf{C}_{T|\mathrm{Data}}|$ determinant of covariance matrix
- Dimension of parameter space: d

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- $|\mathbf{C}_{T|\mathrm{Data}}|$ determinant of covariance matrix
- Dimension of parameter space: d

Logarithmic evidence:

$$\ln P(\mathrm{Data}|M_k) \approx \ln P(\mathrm{Data}|T^k_{\mathrm{ML}}, M_k) + \ln P(T^k|M_k) + \ln \sqrt{(2\pi)^d |\mathbf{C}_{T|D}|}$$

Final Definition

Log-Evidence

$$\ln P(Data|M_k) \approx \ln \mathcal{L}_{ML} + \ln \sqrt{(2\pi)^d |\mathbf{C}_{T|\mathrm{Data}}|} - \ln V_T^k + \sum_{i \in M} \ln S_i$$

where V_T^k is the (prior) volume of parameter space

- Models (=wavesets) compared through the Bayes-Factor

$$B_{12} = \frac{P(Data|M_1)}{P(Data|M_2)}$$

- Interpretation according to Kass&Raftery:

$2 \ln B_{12}$	B_{12}	Evidence
0 to 2	1 to 3	Not worth mentioning
2 to 6	3 to 20	Positive
6 to 10	20 to 150	Stong
> 10	> 150	Very strong

Kass, Raftery, *Bayes Factors*, J. Am. Stat. Assoc. 90 (1995) 773

Automatic Waveset Exploration

Genetic Algorithm

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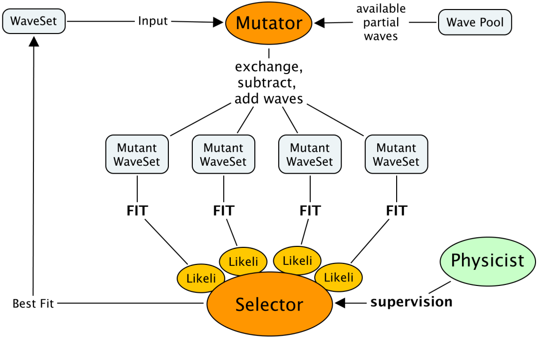
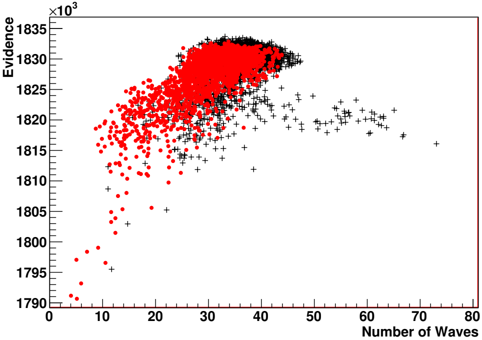


Strategies:

- Start with population of small (2-15 waves) wavesets (adding waves)
- Start with diverse population (10 - 80 waves)
- Start with population of large wavesets (not done yet)

Automatic Waveset Exploration

Genetic Algorithm – 50 generations, population size 50



Number of waves optimizes at around 35

Automatic Waveset Exploration

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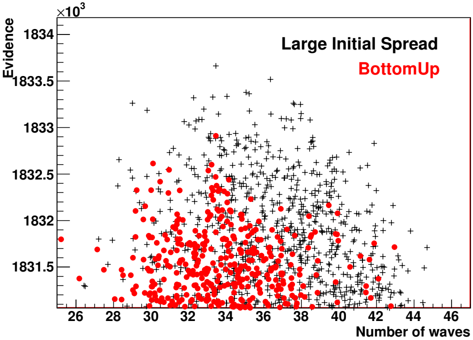

- Diverse initial population run → better results
- Typical log-Evidence differences: 30-100

Example: Top 20 Fits from Genetic Search

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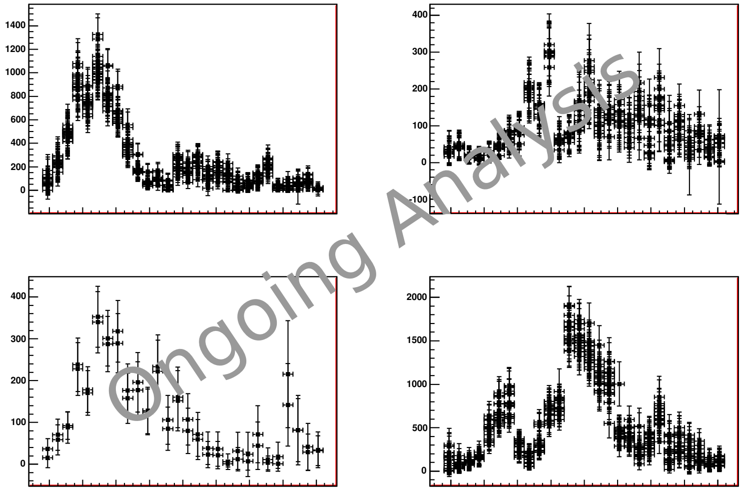

Example Mass Dependent Fit

$$T_i^\epsilon T_j^{\epsilon *} = \rho_{ij}^\epsilon(m) = \left(\sum_k C_{ik}^\epsilon BW_k(m)\sqrt{\int |\psi_i^\epsilon|^2 d\tau}\right)\left(\sum_l C_{jl}^\epsilon BW_l(m)\sqrt{\int |\psi_j^\epsilon|^2 d\tau}\right) \quad (1)$$

with Breit-Wigner amplitude:

$$BW_{ik}(m, M_0, \Gamma_0) = \frac{M_0\Gamma_0}{m^2 - M_0^2 + i\Gamma_{tot}(m)M_0} \quad (2)$$

and dynamic width:

$$\Gamma_{tot}(m) = \sum_n \gamma_n \frac{\rho_n(m)}{\rho_n(M_0)} \qquad \rho_n(m) \sim \int |\psi_i^\epsilon|^2 dq \qquad \sum \gamma_n = \Gamma_0 \quad (3)$$

and background terms:

$$bkg(m) = e^{-\alpha q} \qquad q - \text{Breakup momentum} \quad (4)$$

Fit Results Overview - Spin Density Matrix

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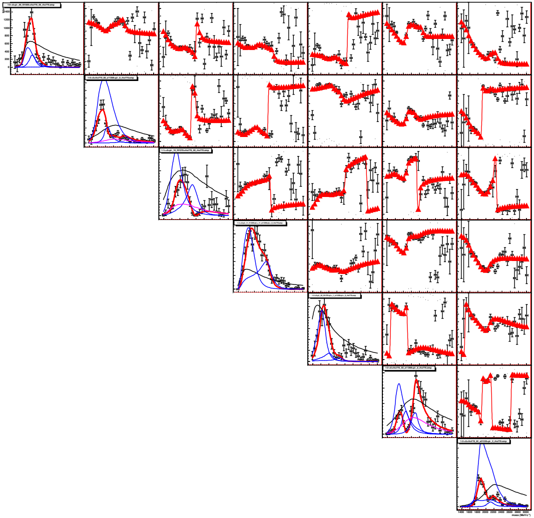
7 waves, 8 resonances



ROOTPWA: Open Source Analysis Toolkit

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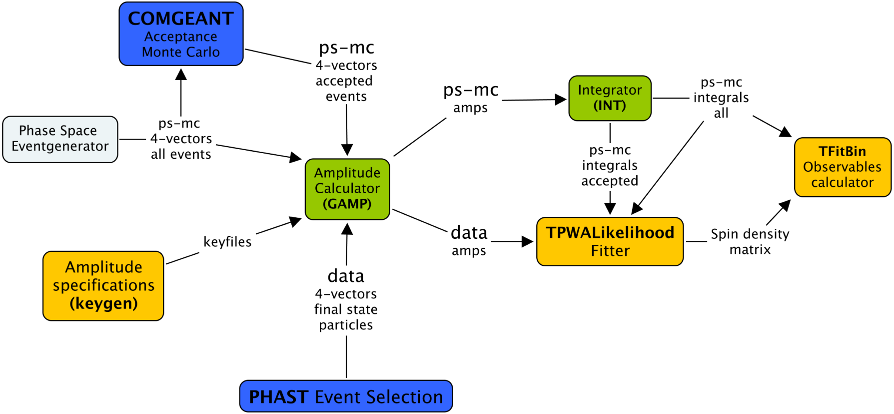
http://sourceforge.net/projects/rootpwa

- Based on BNL code "pwa2000"
- Largely rewritten
- Workflow for mass-dependent fit:



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Main Features:

- Amplitude calculator for diffractive production (Helicity Form.)
- General Amplitude Framework upcoming (B. Grube)
- MC generators (diffraction)
- Numerical tools
  - MC integrator
  - Fitters
  - Genetic Optimization
- Resonance parameterizations (under development)
- Visualization & Plotting tools (ROOT-based)
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ROOTPWA Graphical User Interface

Developed by P. Jasinski

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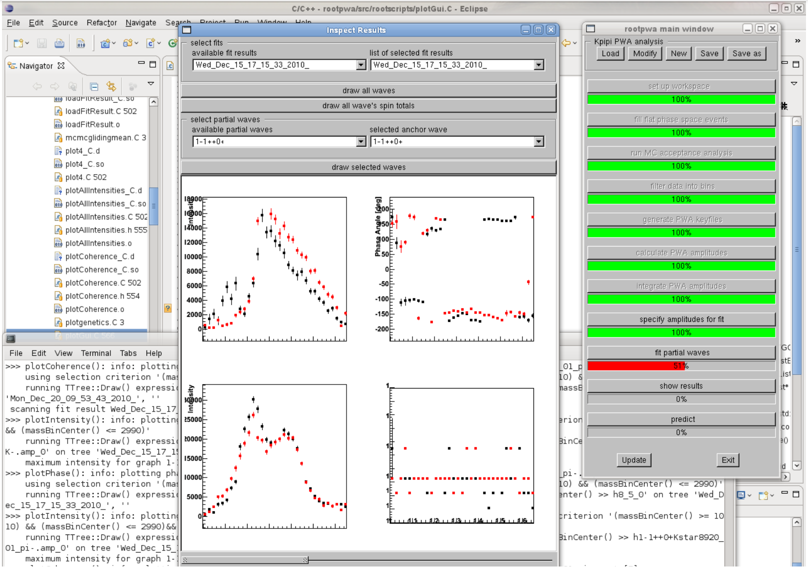

Summary and Conclusion

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- ROOTPWA is one of 2 PWA programs used at COMPASS
- 2 step analysis:
  1. Fit angular correlations with Isobar Model decay
  2. Parameterize dynamics $\rightarrow$ resonance extraction
- Genetic search for waveset exploration
- Open source toolkit `http://sourceforge.net/projects/rootpwa`

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Outlook

- Improvements in amplitude parameterizations
- Study of non-resonant contributions (Deck effect)
- Theory input needed (Rescattering etc.)
- Status of Analyses and Results → Talk by B. Ketzer tomorrow