Tetraquark resonances, flip-flop and cherry in a broken glass model

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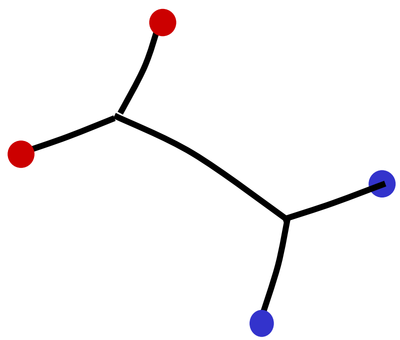
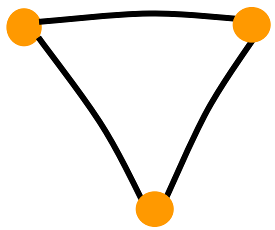
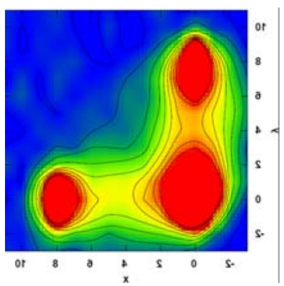
- Motivation for tetraquarks
- the triple flip-flop potential
- The cherry in a broken glass model
- Finite difference method
- Outgoing spherical wave method

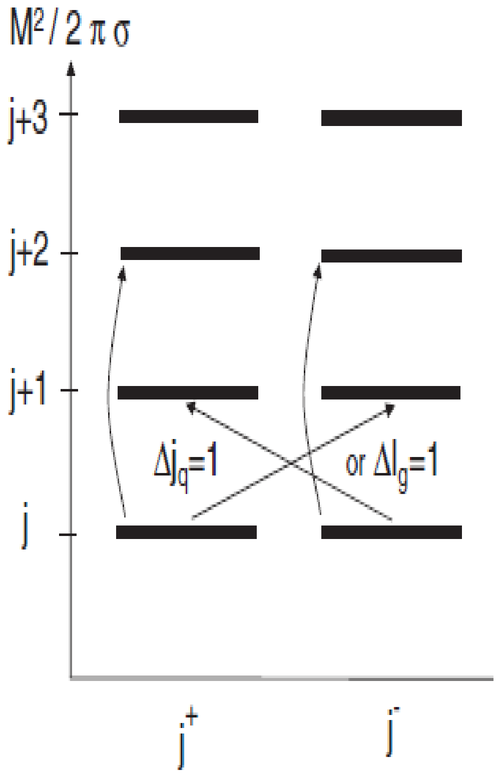
Motivation

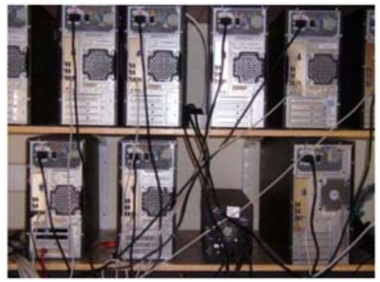
For ++ discussions, I also apply quark-gluon models and Lattice QCD to Temp=/=0, chiral symmetry, all sorts of exotics, molecules, excited hadrons











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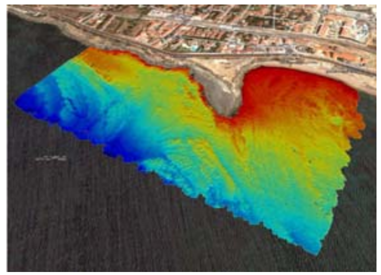
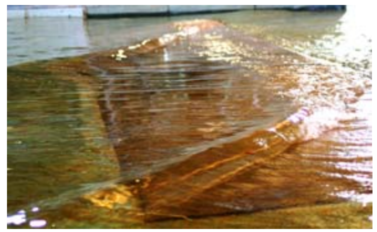




Motivation

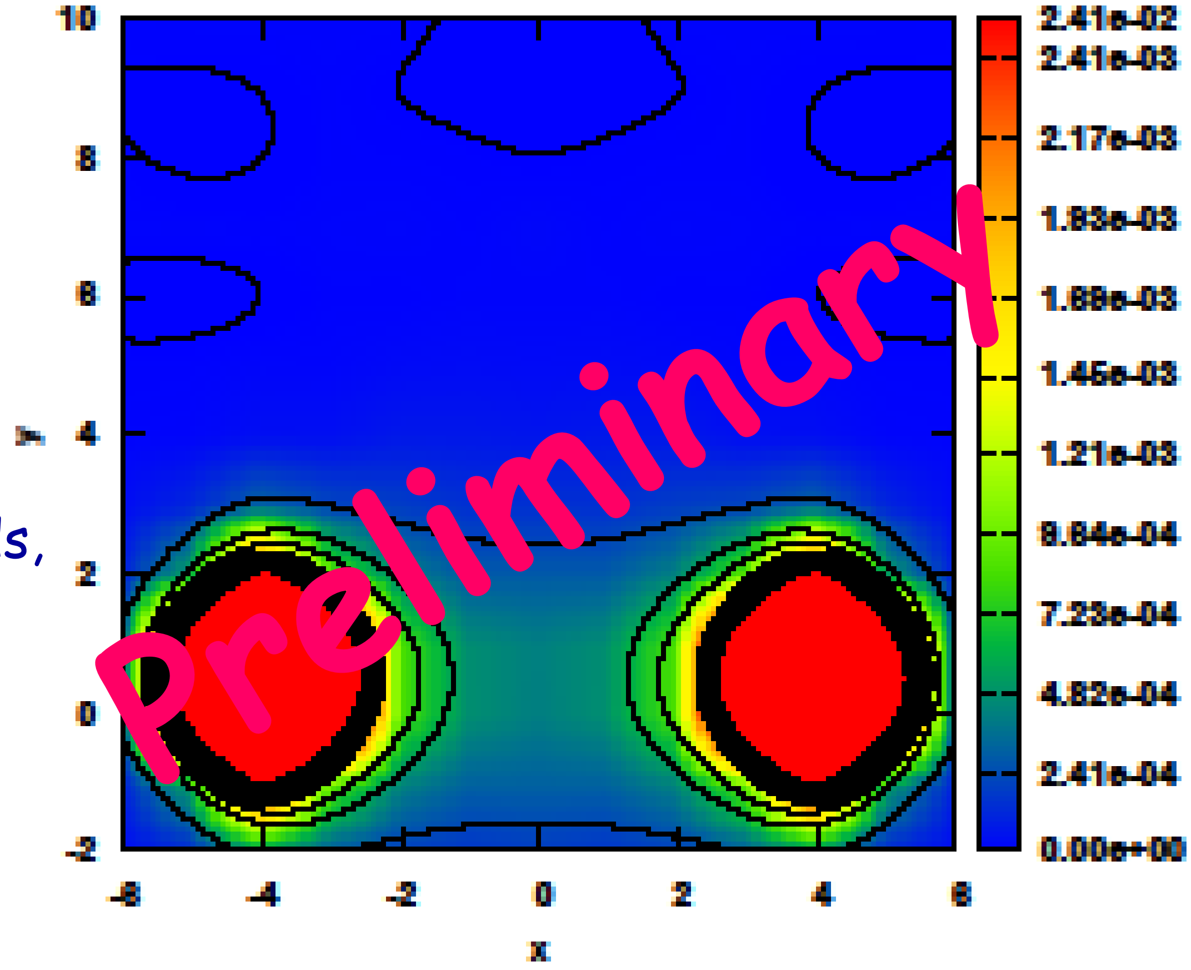
The main motivation is to contribute to understand whether exotic hadrons exit or not.

Although there is no QCD theorem rulling out exotics, they are so hard to find, that many friends even state that **?exotics don't exist? ?or that at least they should be very broad resonances?**

Observation of a J^PC = 1-+ exotic resonance in diffractive dissociation of 190 GeV/c pi- into pi- pi- pi+. COMPASS Collaboration Phys Rev Lett **104**, 241803 (2010). Candidates for different exotics exit! We specialize in tetraquarks, the less difficult multiquarks to compute beyond the baryons and hybrids.

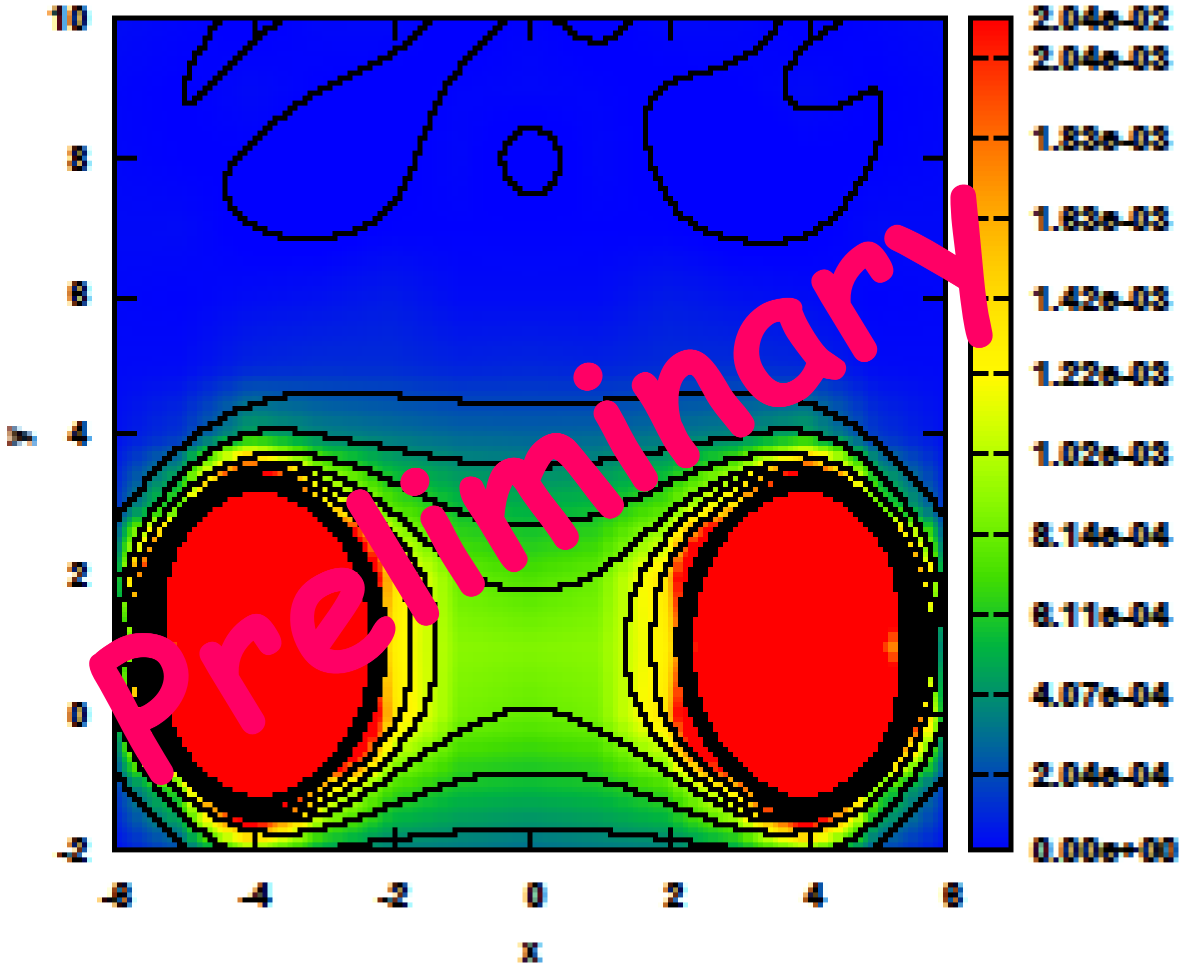
Notice that there are many possible sorts of tetraquarks:
- the borromean 3-hadron molecule
- the Heavy-Heavy-antilight-antilight
- the hybrid-like tetraquark
- the Jaffe-Wilczek diquark-antidiquark with a generalized Fermat string

As we just did for the hybrids in Lattice QCD computation of the colour fields for the static hybrid quark-gluon-antiquark system, and microscopic study of the Casimir scaling M. Cardoso, N. Cardoso, P. Bicudo Phys Rev **D81**, 034504 (2010), we are now studying the tetraquark fields,

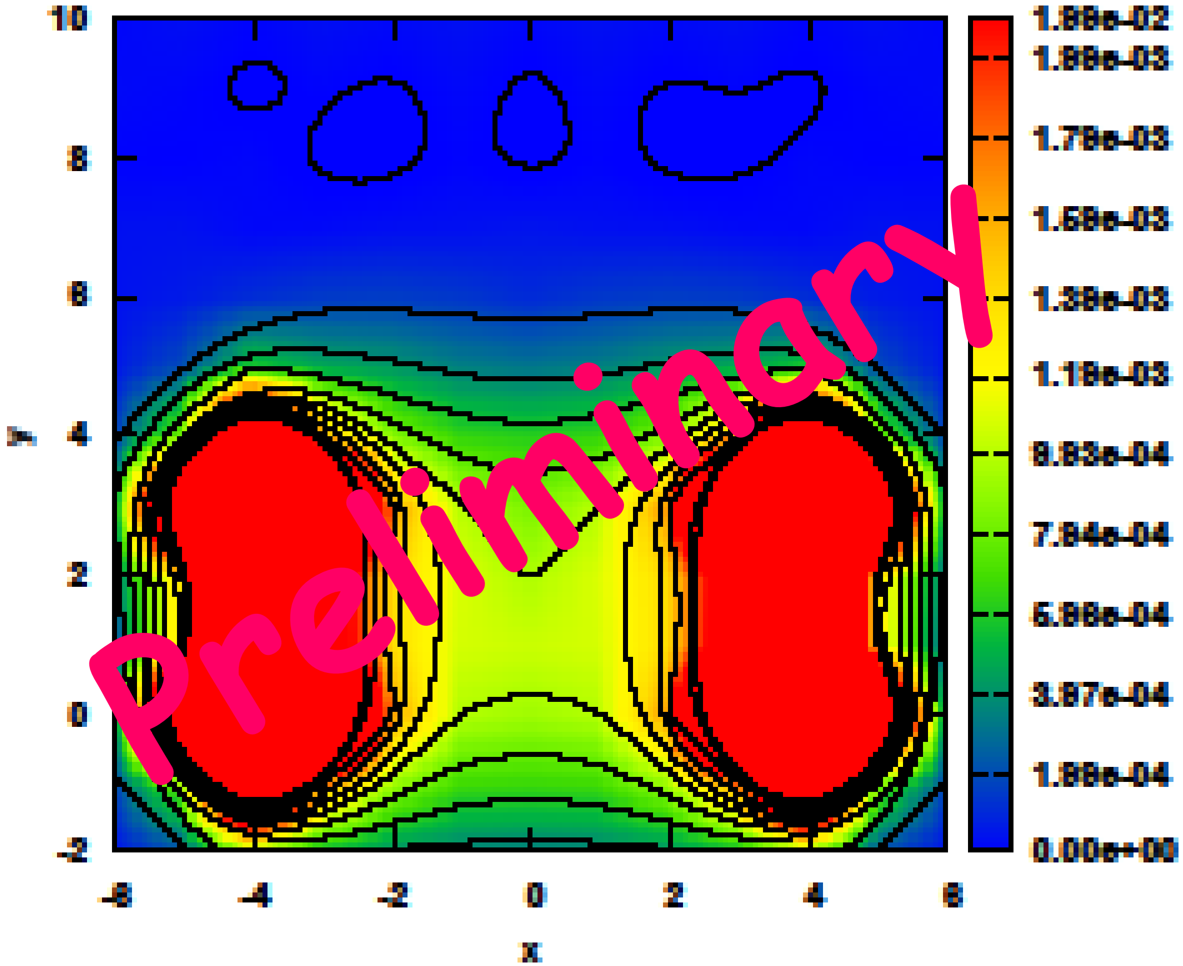


Motivation

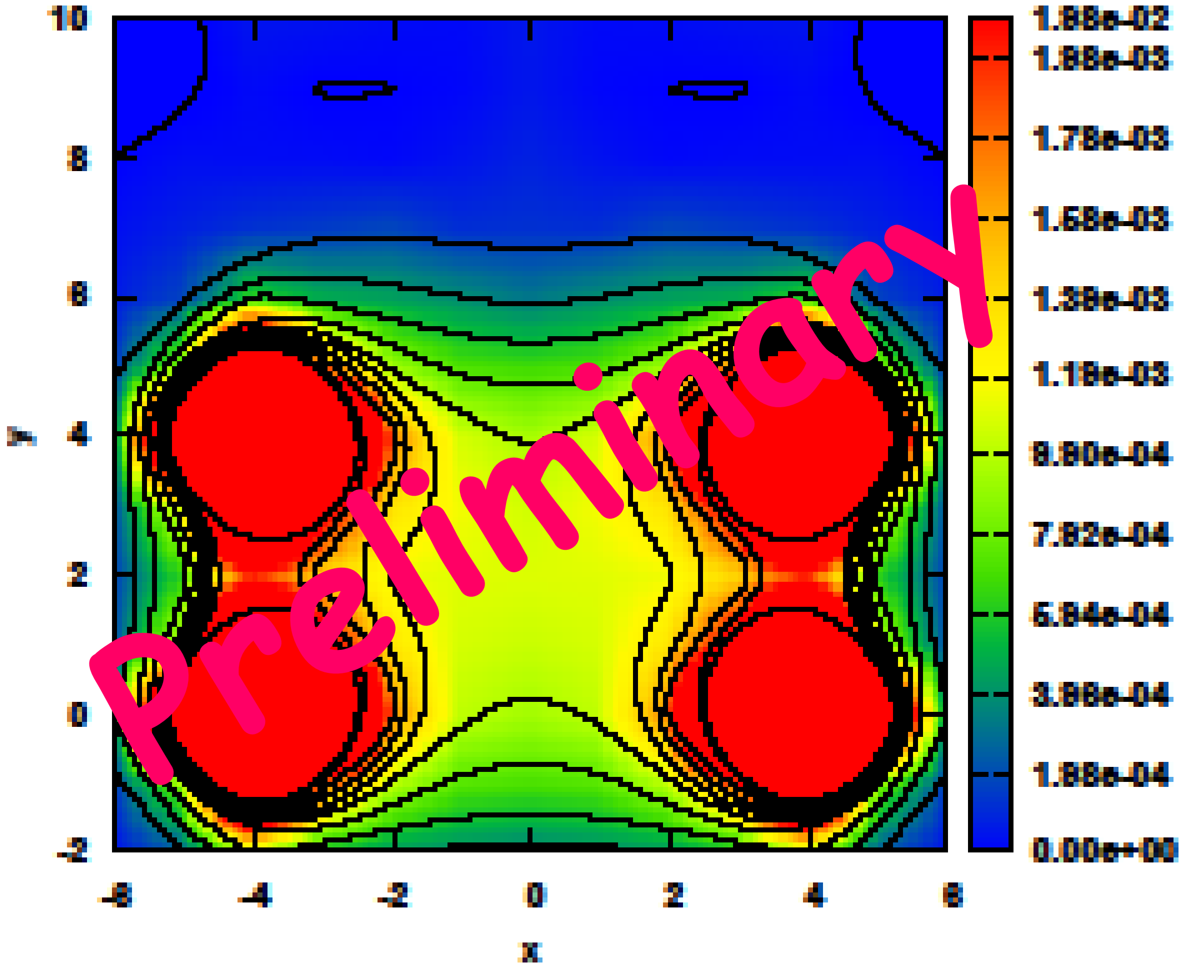
this is a preliminar series of plots of the chromo-electric field of a quark-quark- -antiquark-antiquark system, where we separate the quarks from the antiquarks,



Motivation



Motivation



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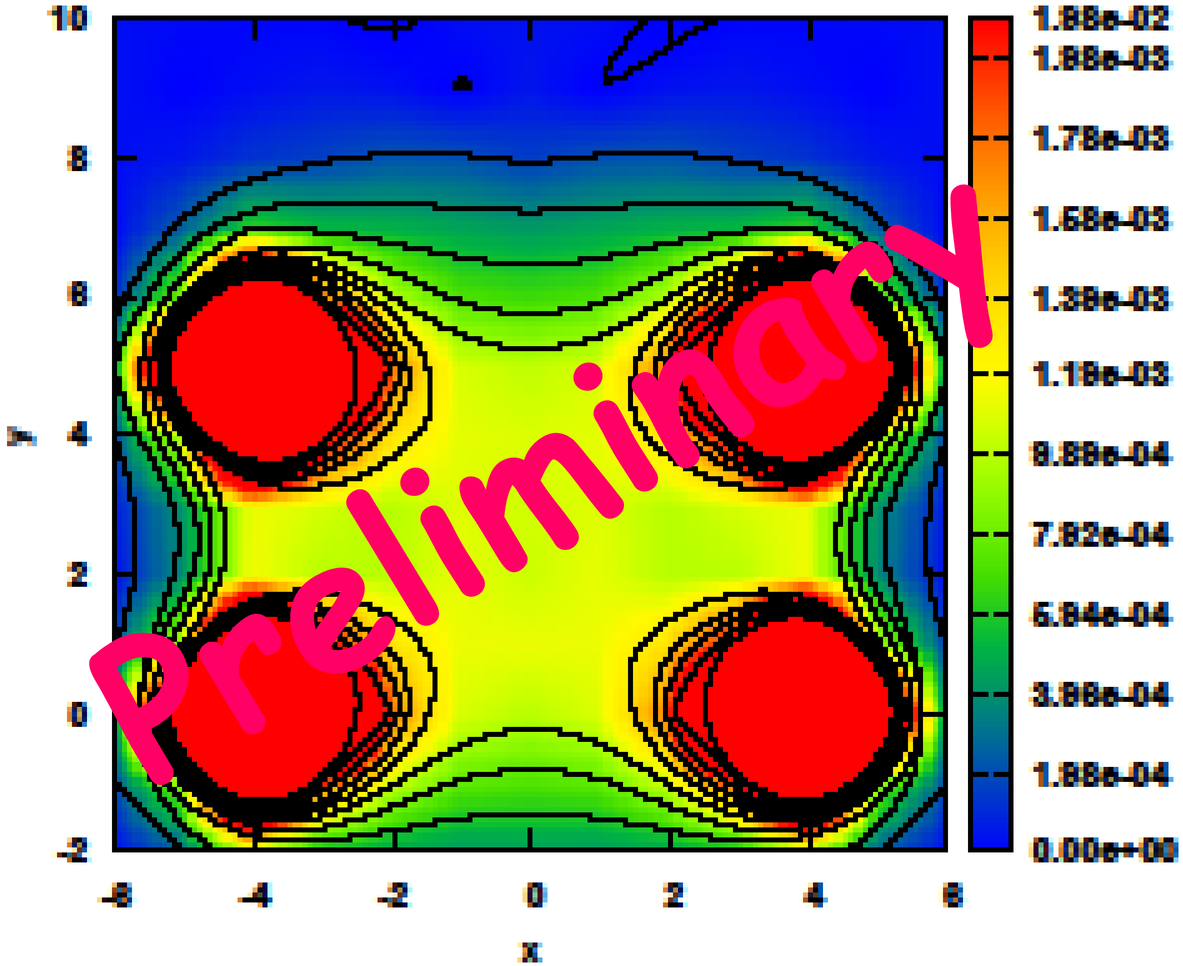
Preliminary

Motivation



Preliminary
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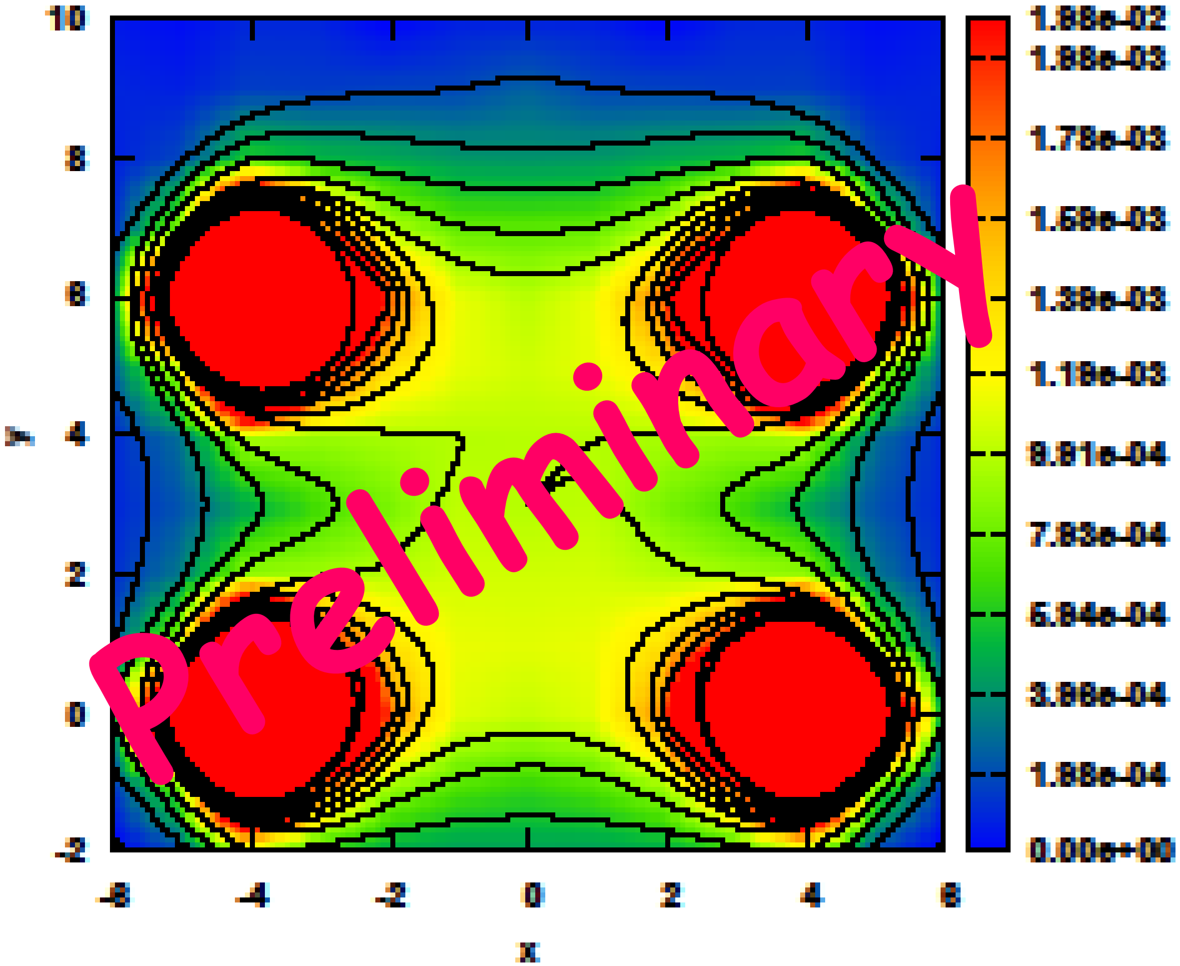
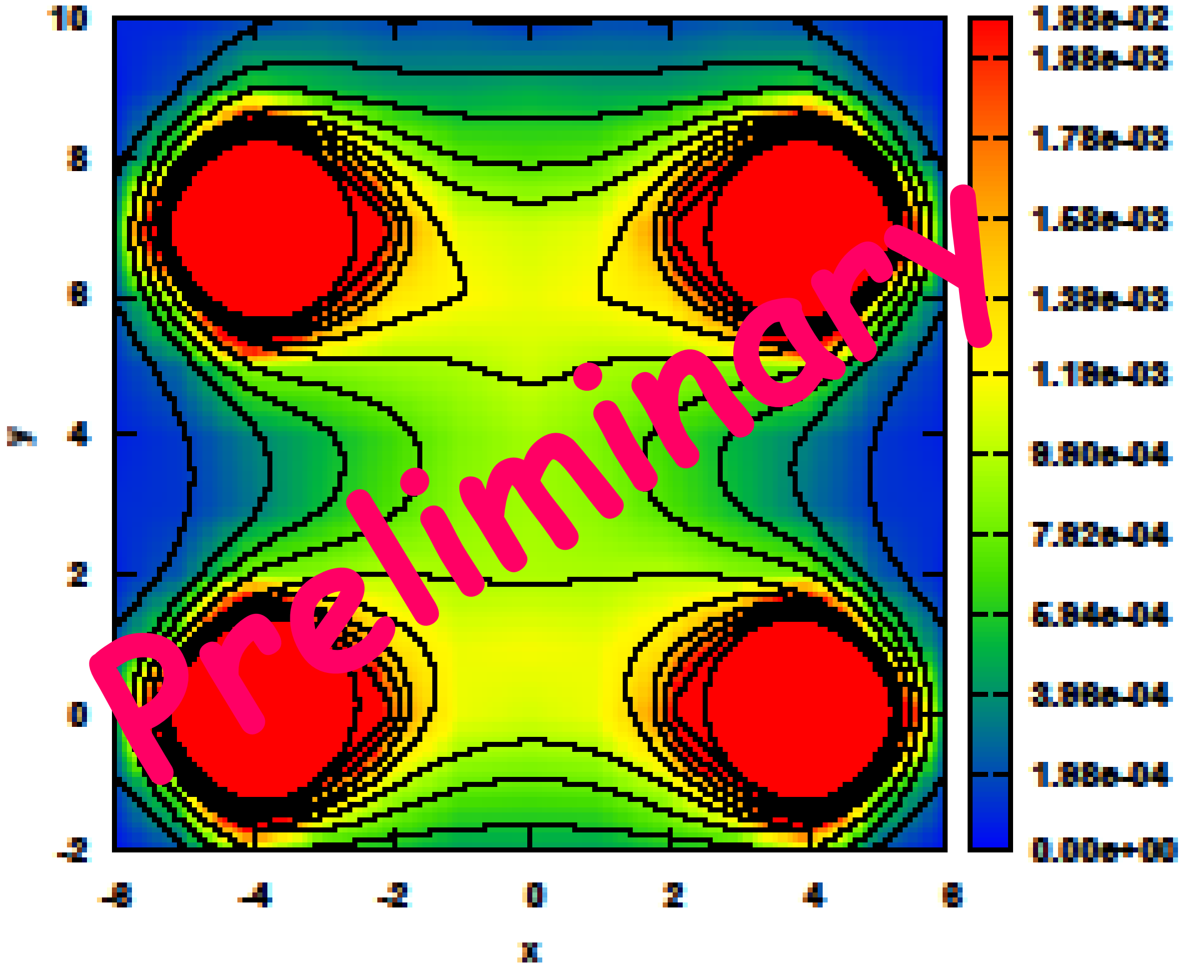
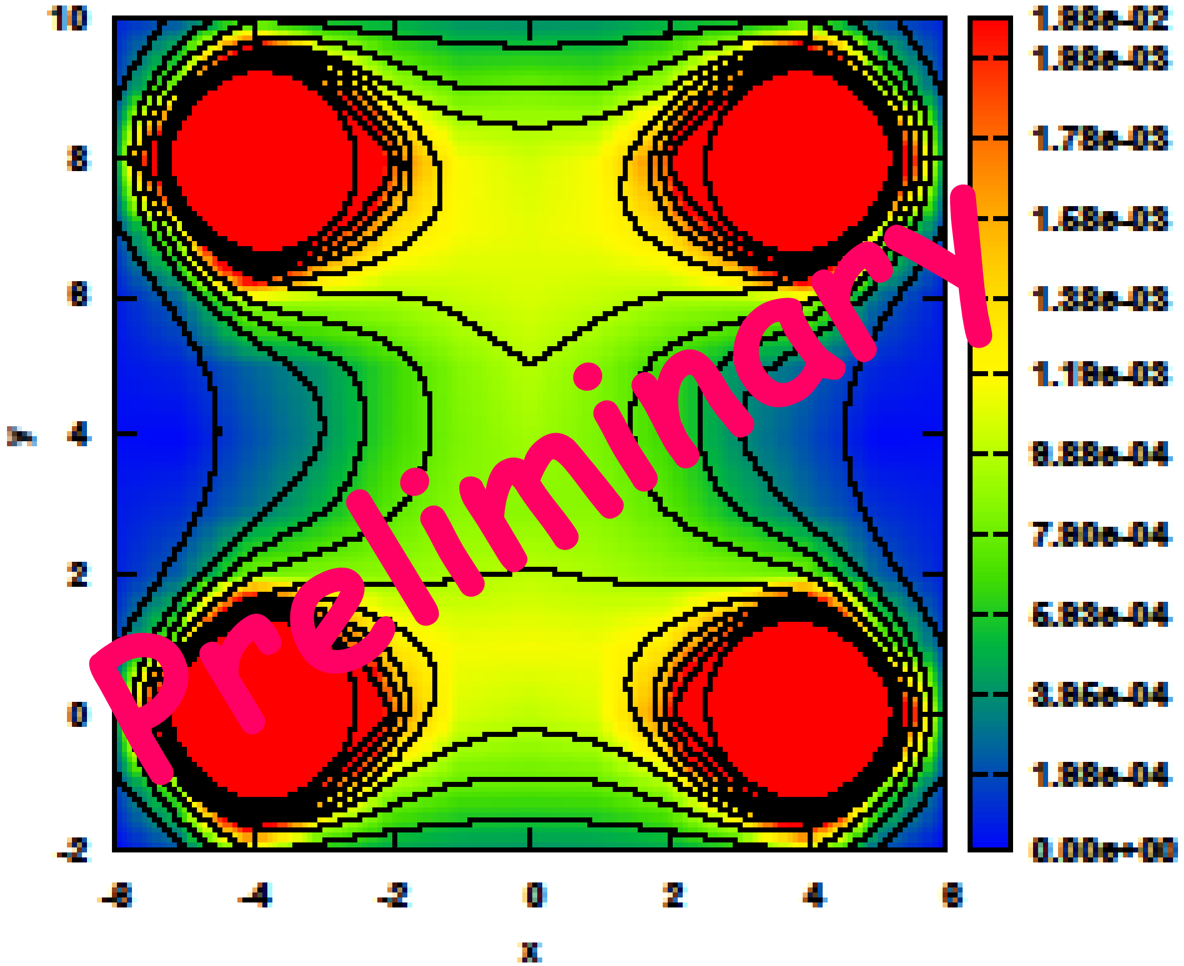
Motivation



Motivation



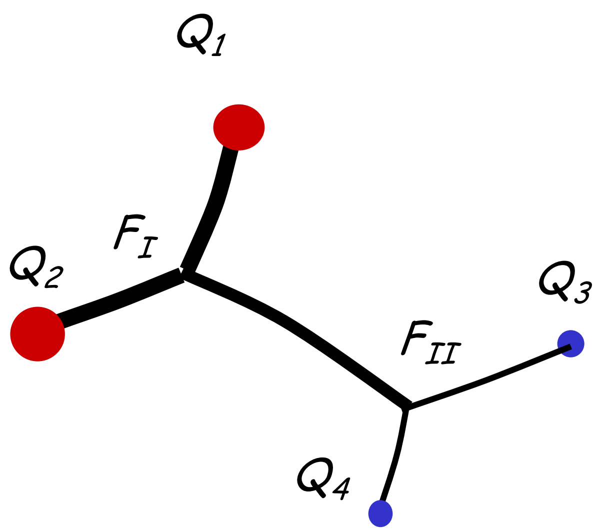
Motivation



Motivation

the Jaffe-Wilcsek diquark-antidiquark with a generalized Fermat string

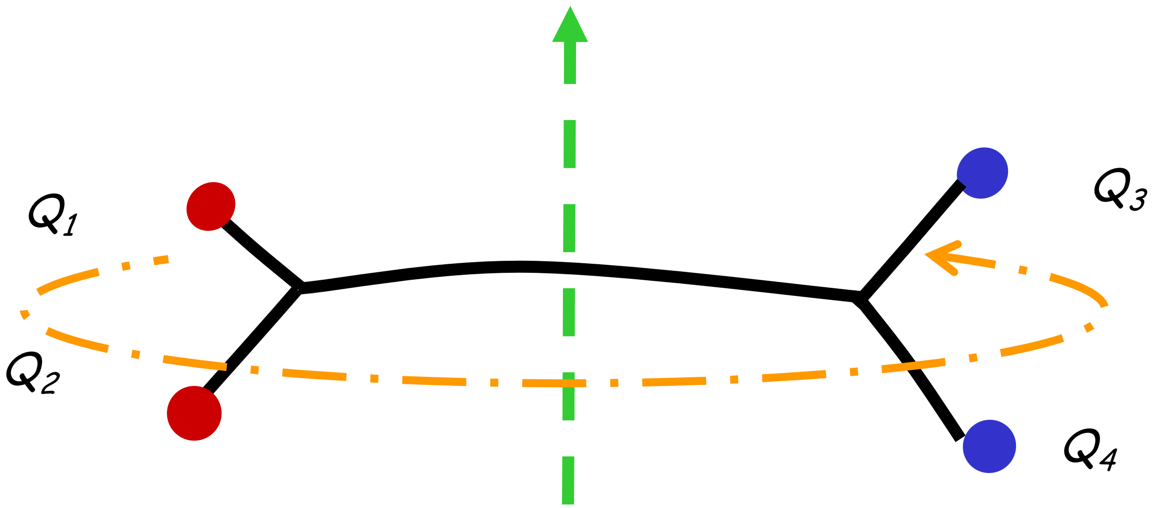
The flux tubes prefer to divide and link into fundamental flux tubes, and a possible configurationis in a H-like or butterfly-like flux tube, related to A Perspective on pentaquarks. R. Jaffe, F. Wilczek. Eur.Phys.J.C33:S38-S42,2004



The problem is that this tetraquark is open for the decay into a pair of mesons. We want to determine whether such resonances exist and study the decay width.

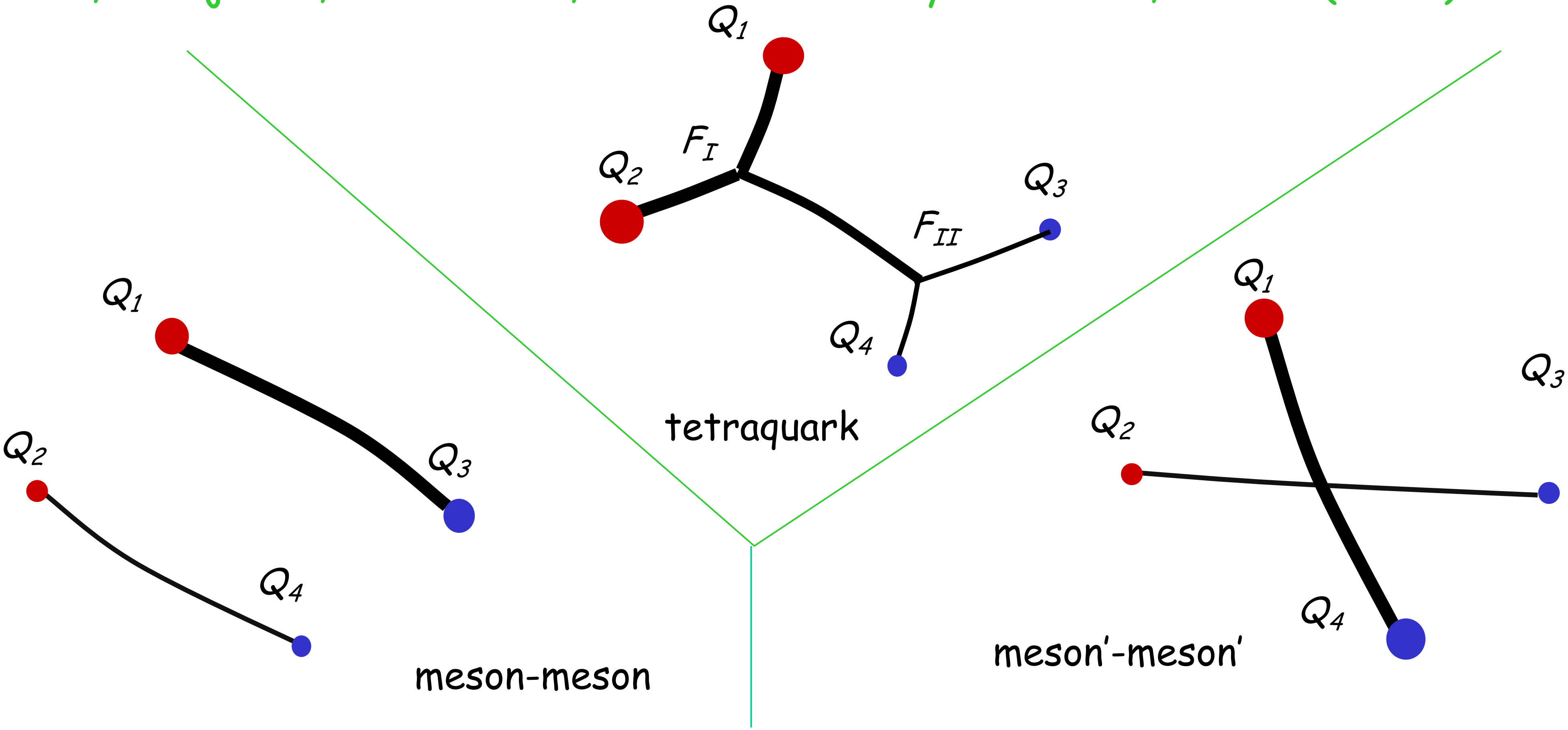
Motivation

A main ideia to stabilize this tetraquark, proposed by Karliner and Lipkin, was an that angular excitation in the radial coordinate of the diquarks, would partly prevent the quarks and antiquarks to recombine, producing a repulsive centrifugal barrier. Although this enhances the mass of teh tetraquarks, it decreases the decay width of the system.



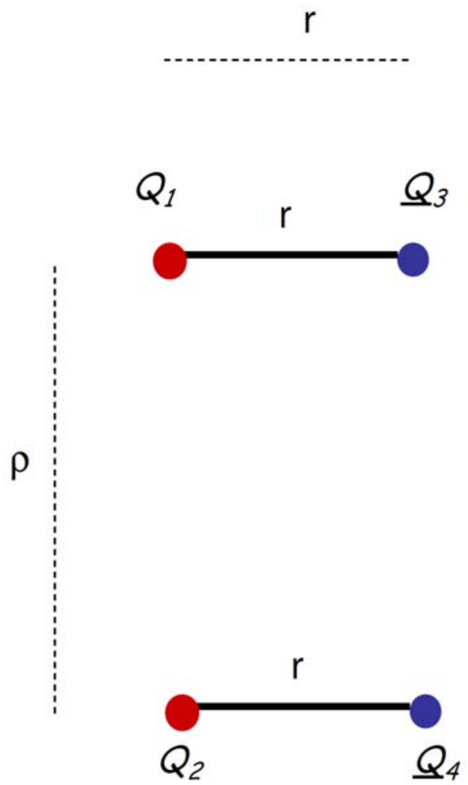
The triple flip-flop potential

2-body confinement produces Van der Waals forces. Van der Waals forces Are cured by string confinement. We arrive at the triple flipflop potential, extended with a tetraquark, as in Stability of multiquarks in a simple string model, J. Vijande, A. Valcarce, J.-M. Richard Phys Rev D76, 114013 (2007)*:*

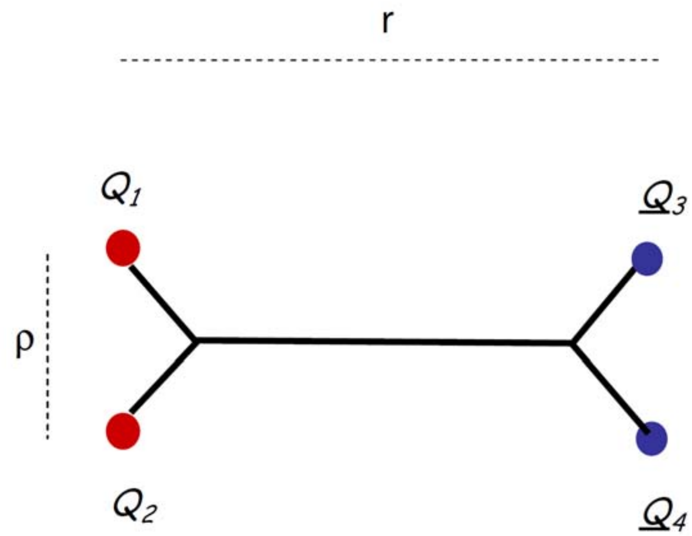


The triple flip-flop potential

We simplify the triple flipflop potential, using a single intermeson variable



meson-meson, $V = 2\,r$



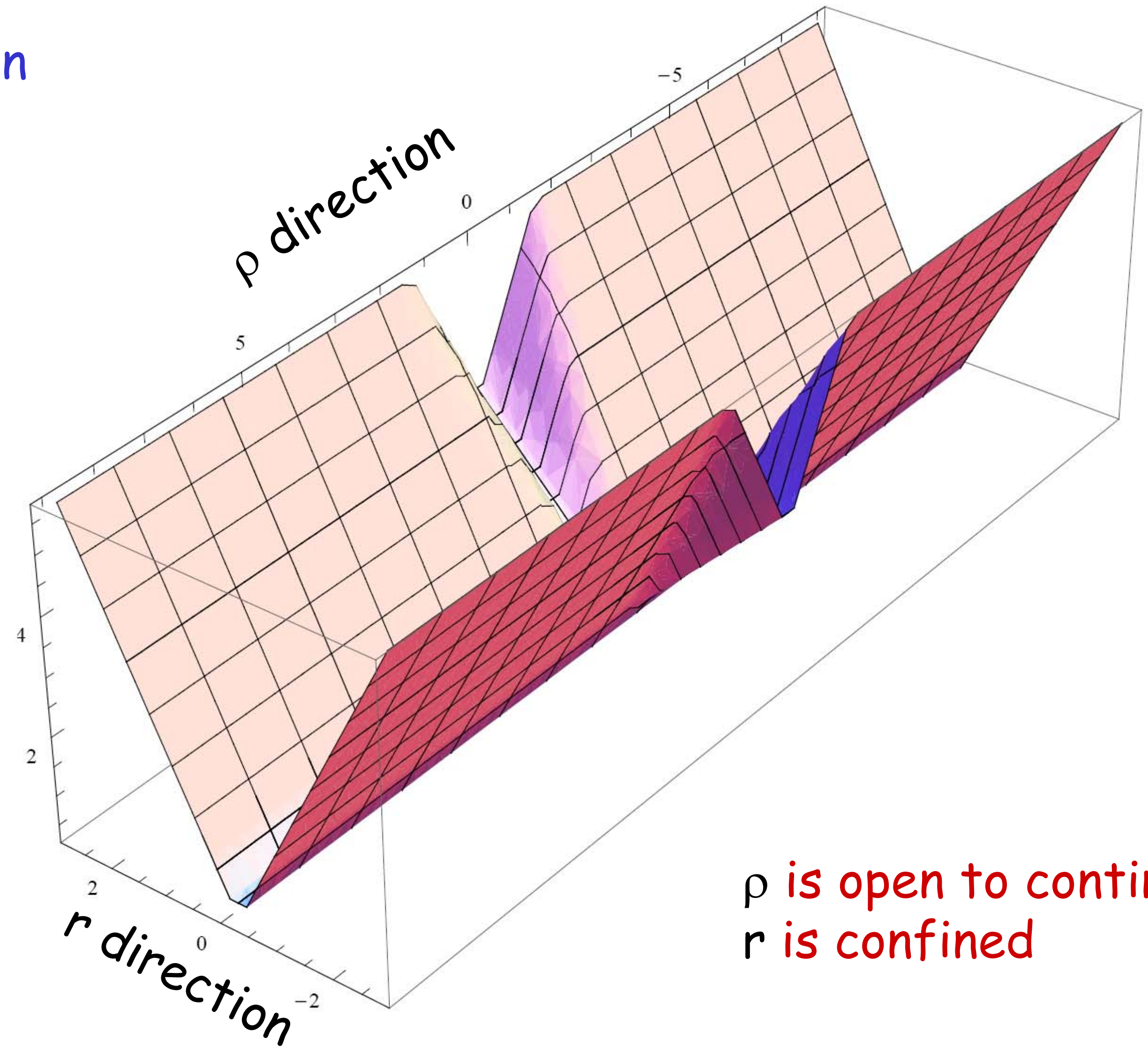
tetraquark, $V = r + \sqrt{3}\,\rho$

The triple flip-flop potential

Using this approximation

$r_{13}=r_{24}$

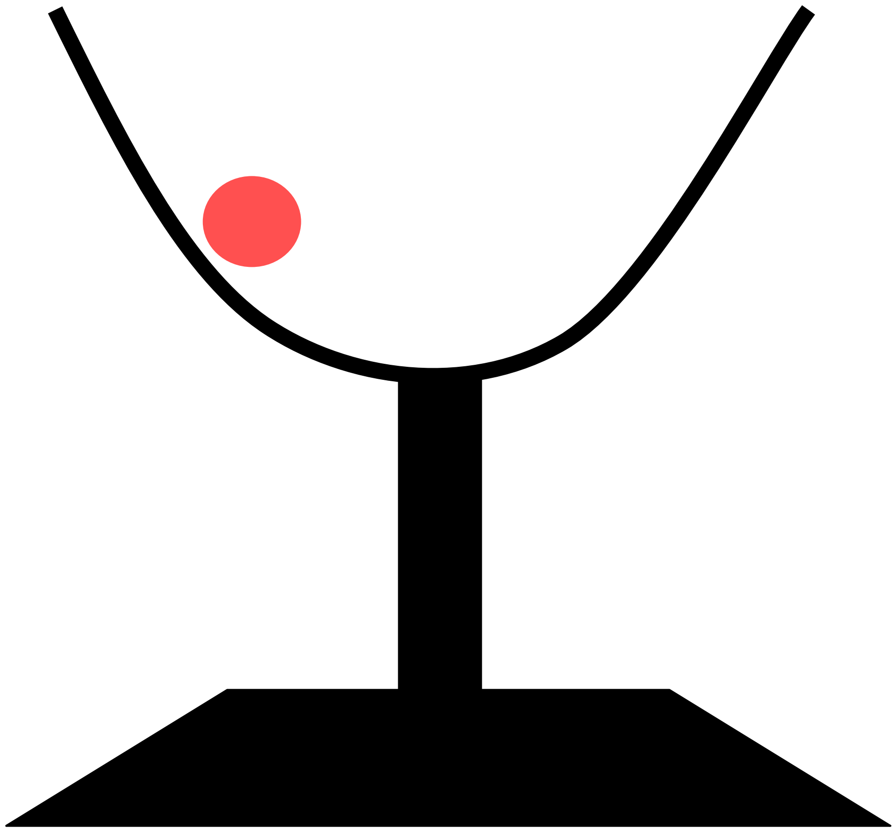
of having a single internal variable in the the mesons we get the potential in the case s-wave and s-wave :

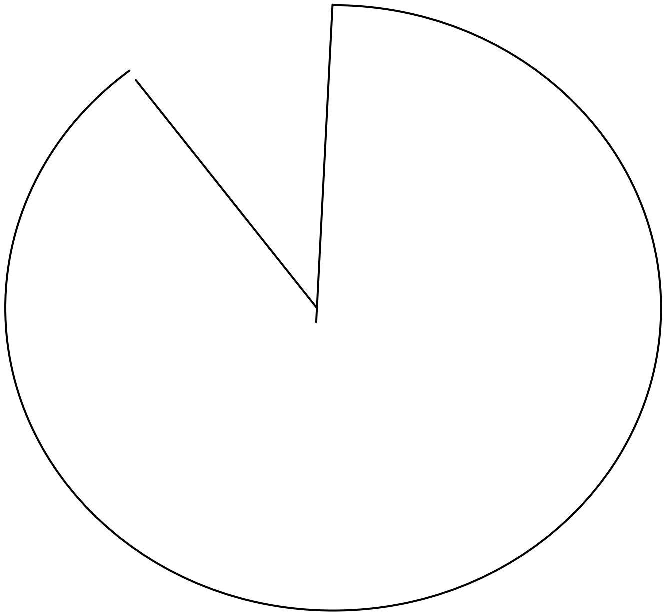


ρ is open to continuum
r is confined

The triple flip-flop potential

Our problem is similar to the classical school student problem of Cherry in a glass. However this is not simple a student's problem since glass is broken and the cherry may escape from the glass! In the quantum case, what is the width?





Finite Difference Method

Since there is a single scale in the potential and a single scale in the kinetic energy, we can rescale the energy and the coordinates, to get a dimensionless equation,

$$H\,\phi(r,\rho) = [-\,\Delta_r/_2 - \Delta_\rho\,/_2 + \mathit{min}\,(r+2\rho,\, 2r)]\,\phi(r,\rho) = E\,\phi(r,\rho)$$

that we first solve with the finite difference method.

This case is adequate to study equal mass quarks, where the mesons and the tetraquark have no constant energy shifts. For instance that would be ok for the system

u u <u>d</u> <u>d</u> (S=2) coupled to ρ+ ρ+

or the system

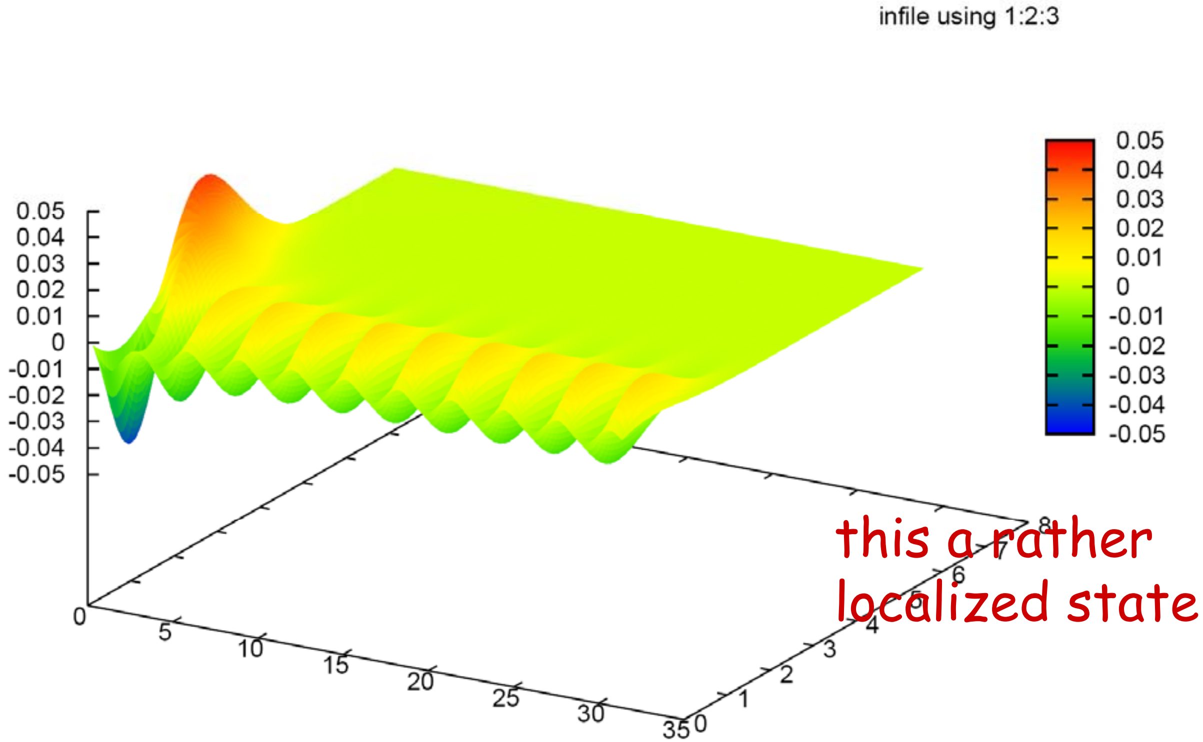
c c <u>c</u> <u>c</u> with any spin coupled to c <u>c</u> c <u>c</u>

Finite Difference Method

We discretize the space in anisotropic lattices and solve the finite difference Schrödinger equation, in up to 6000x6000 sparse matrices (equivalent to 40 points in the confined direction x 150 points in the radial continuum direction).

We first look for localized states, selecting among the 6000 eigenvalues the ones more concentrated close to the origin at

$\rho=0$



this a rather localized state

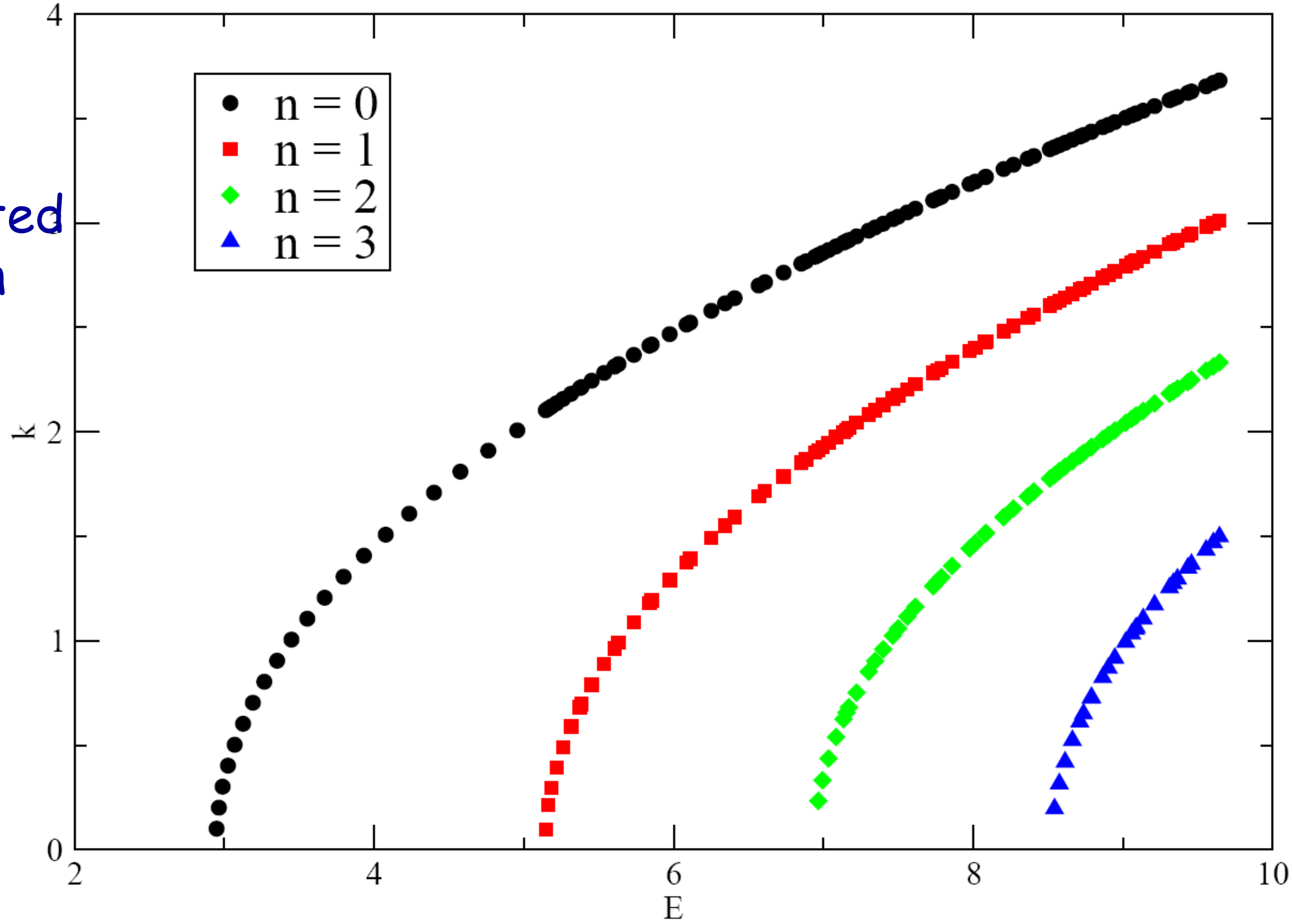
Finite Difference Method

Since the box quantization produces continuum states at large ρ, we can compute their wave length λ, and momentum k=λ^{-1}, by fitting the tail of the wave functions to A(sin k ρ),

we check that the momentum follows the expected dispersion relation

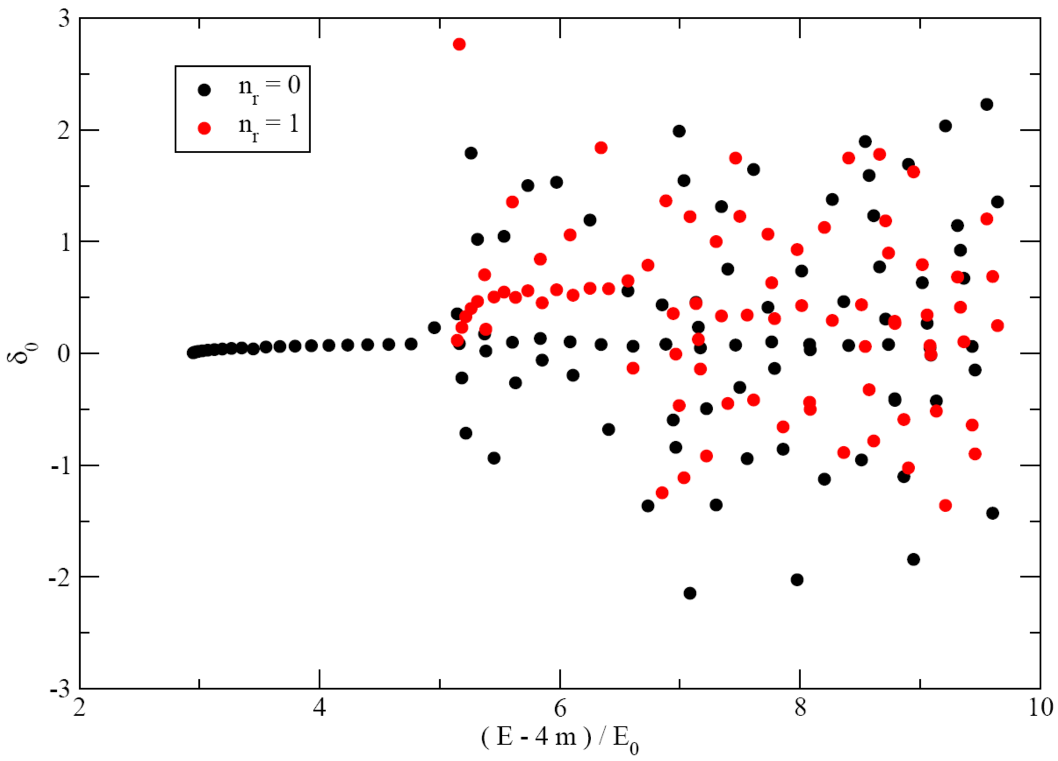
k= sqrt(E^2-M^2)

we find several channels, one per excitation of the confined, or compactified dimension, r



Finite Difference Method

Comparing the wavelength λ / momentum k with the box size L, we can compute the phase shifts, as a function of the energy. Unfortunately we get a discrete energy spectrum and there is no unique way to connect the phase shifts. When more compact channels, in r, open the phase shifts get unclear. A way out would be to use Luscher's method of changing the box size...



Outgoing spherical wave method

However the finite difference method is not entirely satisfactory, and we move to another method, consisting in studying the phase shifts of the outgoing spherical waves in the continuum.

Since the finite difference method shows clearly bands, or channels, for the different internal energies of the mesons, we project the confined coordinate r with eigenvalues of the meson equation, i.e. with Airy functions, and thus we are left with ordinary differential equations on the coordinate ρ .

In a coupled channel problem, we have the Schrödinger equation

$$-\frac{\hbar^2}{2m_i}\nabla^2\Psi_i + V_{ij}\Psi_j = (E-\epsilon_i)\Psi_i \qquad (17)$$

To compute the phase shifts and the cross sections, we have the following relations we consider the scattering to the channel i to the channel j.

Outgoing spherical wave method

Assimptoticaly, we have

$$\Psi_i \to e^{ik_i z} + f_{ii}(\hat{r})\frac{e^{ik_i r}}{r} \quad (18)$$

and, for $j \neq i$,

$$\Psi_j \to f_{ij}(\hat{r})\frac{e^{ik_j r}}{r} \quad (19)$$

if the channel j is open, otherwise vanisnhing.

With this and the conservation of the probability, we obtain the optical theorem

$$\sum_j \sigma_{i\to j} = \frac{4\pi}{k}\Im f_{ii}(0) \quad (20)$$

This could be formulated in terms of partial waves, in which case the equations (18) and (19) become

$$u_i^l \to \sin(k_i r + l\pi) + (2l+1)f_{ii}^l e^{ik_i r} \quad (21)$$

and

$$u_j^l \to (2l+1)f_{ij}^l e^{ik_j r} \quad (22)$$

and the optical theorem becomes

$$\sum_j \sigma_{i\to j}^l = \frac{1}{k}(2l+1)f_{ii}^l \quad (23)$$

The f_{ij}^l could be computed by considering the solutions of the eq. (17) of the form

$$\Psi_i = e^{ik_h r}\delta_{ih} + \psi_i^+ \quad (24)$$

We have the equation for ψ_i^+

$$-\frac{\hbar^2}{2m_i}\nabla^2\psi_i^+ + V_{ij}\psi_j^+ = (E-\epsilon_i)\psi_i - V_{ih}e^{ik_h r} \quad (25)$$

this equation could be simplified if V_{ij} is spherically symmetric, by writing ψ_i^+ as

$$\psi_i^+ = \frac{u_l(r)}{r}Y_{lm}(\theta,\varphi) \quad (26)$$

in which case the equation becomes

$$-\frac{\hbar^2}{2m_i}\frac{d^2u_l}{dr^2} + V_{ij}u = (E-\epsilon_i)u_l - V_{ih}j_l(kr)r \quad (27)$$

Outgoing spherical wave method

Finally we get extremely clear phase shifts and cross sections!
For s-waves we find no resonance

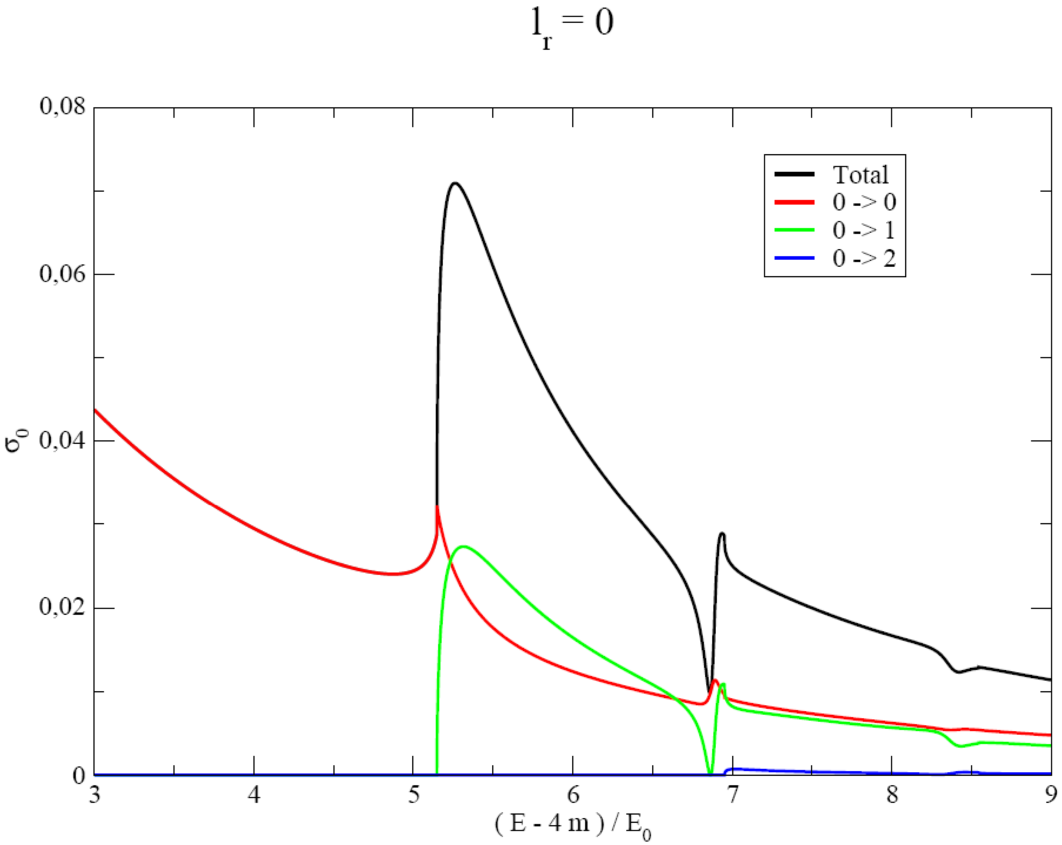


FIG. 7: S-wave scattering cross sections from the channel with $l_r = 0$ and $n_r = 0$.

Outgoing spherical wave method

Important: both angular momenta are conserved, $\mathbf{L}_r = \mathbf{r} \times \mathbf{p}_r$ and $\mathbf{L}_\rho = \boldsymbol{\rho} \times \mathbf{p}_\rho$

a tetraquark with an angular momentum in r indeed feels a centrifugal potential barrier as anticipated by Karliner and Lipkin.

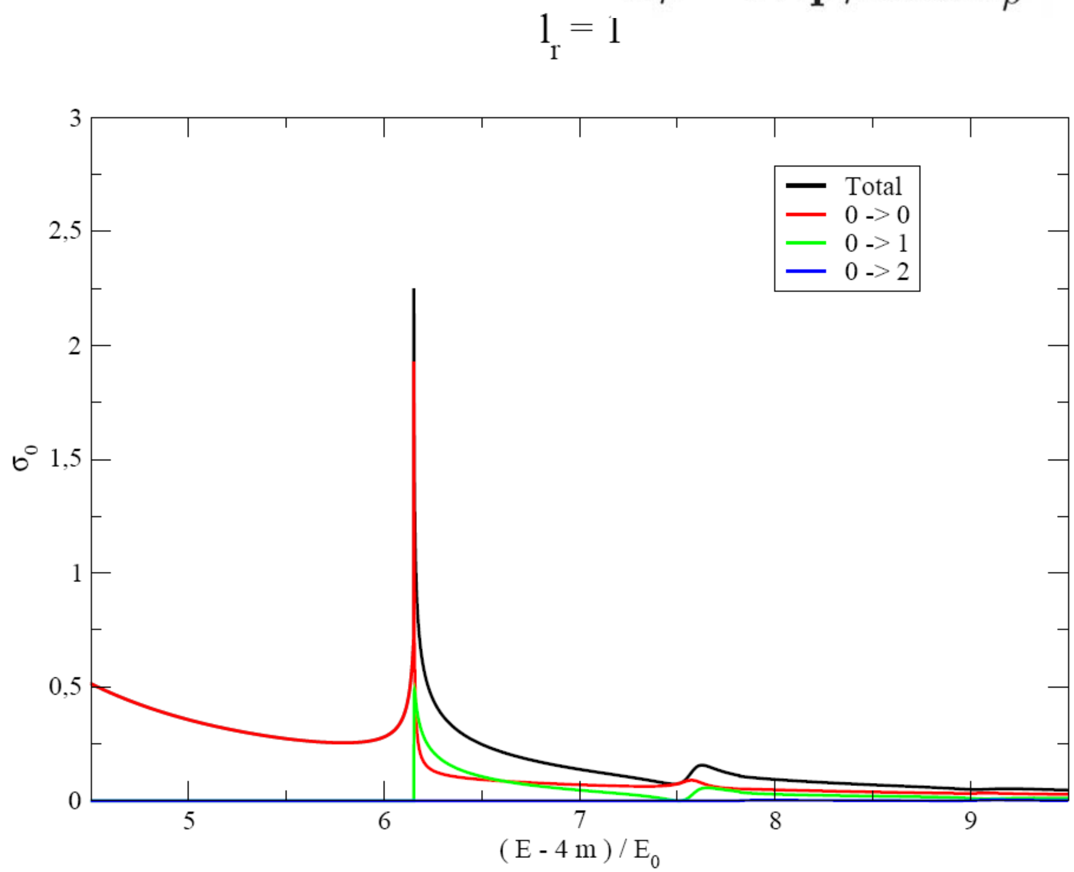


FIG. 8: S-wave scattering cross sections from the channel with $l_r = 1$ and $n_r = 0$.

Outgoing spherical wave method

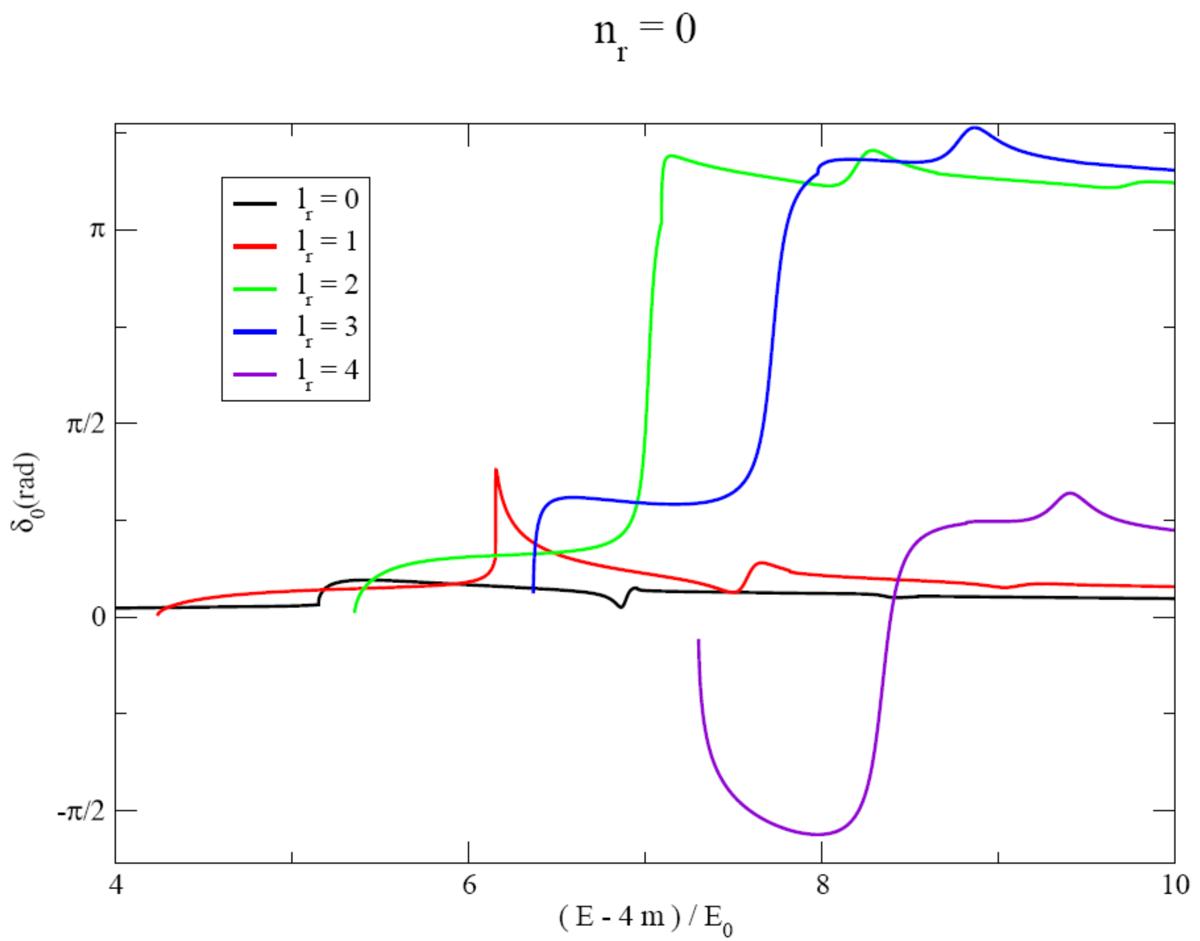


FIG. 9: Comparision of the phase shifts for $l_r = 0$ and $l_r = 1$, with $n_r = 0$.

Outgoing spherical wave method

We may also consider a radial excitation in r.

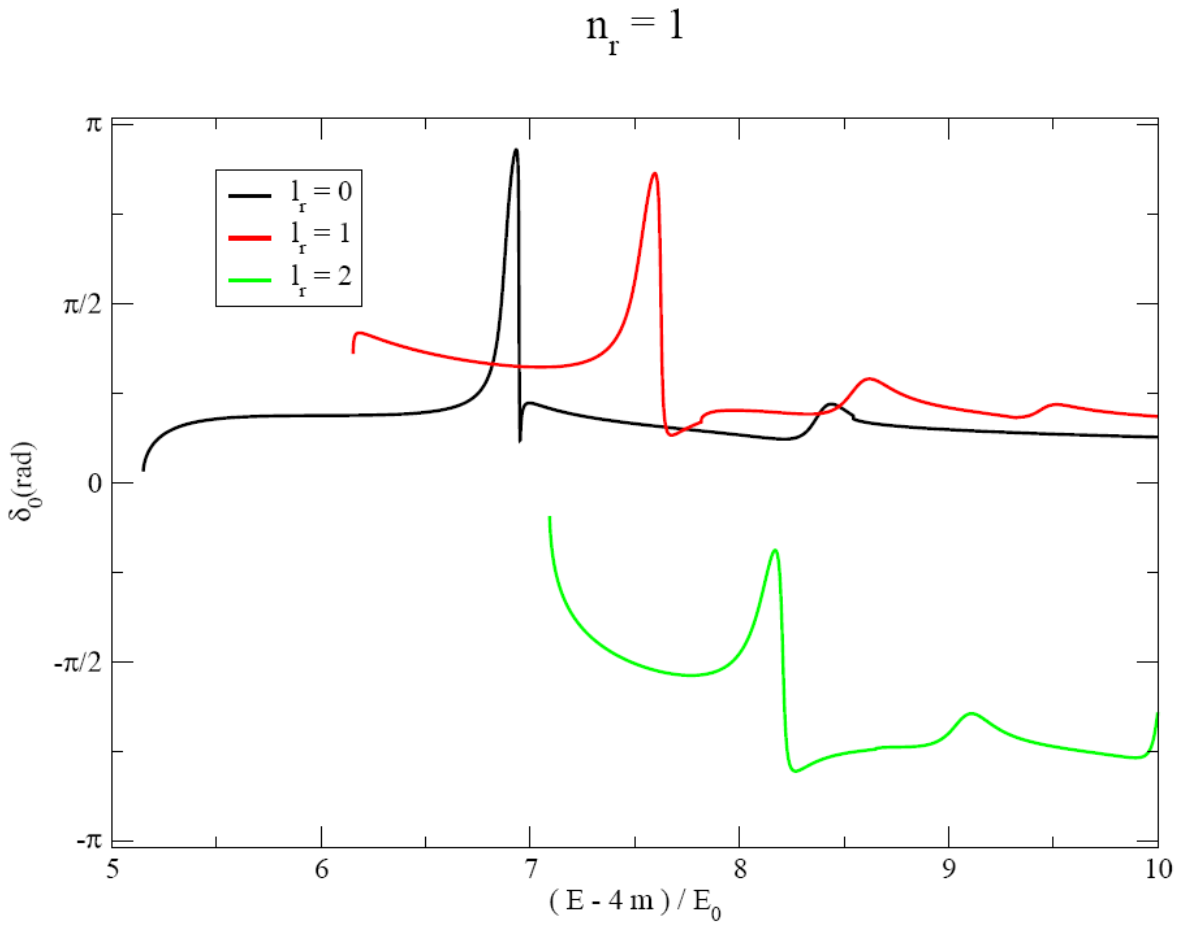


FIG. 10: Comparision of the phase shifts for different l_r, with $n_r = 1$.

Outgoing spherical wave method

Finally we can compute the decay width utilizing the phase shift derivative,

$\Gamma / 2 = (d \, \delta / d E)^{-1}$

and we get, say for l=2, a decay width of (in units of mq= sqrt σ =1),

$\Gamma \sim 2 \, (.2 / \pi) \sim 0.1$

for instance, for light quarks, say an exotic light tetraquark, since m ~ sqrt(σ) are both similar to 400 MeV this results in a width close to

$\Gamma \sim 40$ MeV

but one needs to add the decay widths of the two final meson resonances and these may add to the total decay width.

Conclusion & Outlook tetraquarks with a H/butterfly string

- ***We study tetraquarks in the Jaffe-Wilczek model, but include the open channels of decays to meson-meson pairs.***

- ***We consider an extended flip-flop model, where we add the tetraquark string to the two-meson strings.***

- ***We then utilize an approximate toy-model, simplifying the number of Jacobi variables. The model is similar to the model of a Cherry in a Broken Glass.***

- ***This allows the solution of the Schrödinger equation with finite differences in a box, where we look for localised states, and try to compute phase shifts.***

- ***To compute clearly the phase shifts we then solve the Schrödinger equation for the outgoing spherical waves. We compute de decay widths from the phase shifts. We find narrow tetraquarks, possibly widened by the width of the mesons in the meson-meson pair decay product.***

- ***In our next work we compare with phenomenology, using three Jacobi coordinates, and including the decay width of the final mesons.***