Dispersion Relations
and
Nucleon Form Factors

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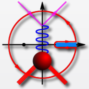
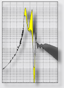
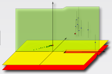


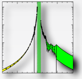
Baryon form factors and Dispersion Relations



The integral equation for G_M^p and G_M^n

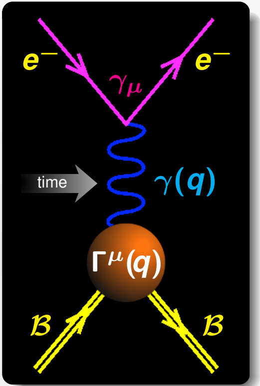


The ratio G_E^p / G_M^p



The sum rule for G_M^p

Baryon Form Factors definition
Space-like region ($q^2 < 0$)



- **Electromagnetic current** ($q = p' - p$)

$$J^\mu = \langle \mathcal{B}(p')|j^\mu|\mathcal{B}(p)\rangle = e\overline{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_2(q^2)\right]u(p)$$

- **Dirac and Pauli form factors F_1 and F_2 are real**
- **In the Breit frame**

$$\begin{cases} p = (E, -\vec{q}/2) \\ p' = (E, \vec{q}/2) \\ q = (0, \vec{q}) \end{cases} \qquad \begin{cases} \rho_q = J^0 = e\left[F_1 + \frac{q^2}{4M^2}F_2\right] \\ \vec{J}_q = e\,\overline{u}(p')\vec{\gamma}u(p)\,[F_1 + F_2] \end{cases}$$

- $2M\overline{u}(p')\gamma^\mu u(p) = \overline{u}(p')[(p + p')^\mu + i\sigma^{\mu\nu}q_\nu]u(p)$
- $\overline{u}(-\vec{p})u(\vec{p}) = E/M$
- $u^\dagger(-\vec{p})u(\vec{p}) = 1$

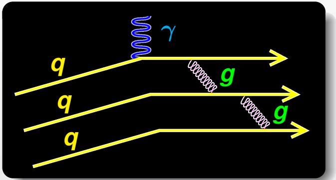
- **Sachs form factors**

$$G_E = F_1 + \frac{q^2}{4M^2}F_2$$
$$G_M = F_1 + F_2$$

- **Normalizations**

$$F_1(0) = Q_{\mathcal{B}} \qquad G_E(0) = Q_{\mathcal{B}}$$
$$F_2(0) = \kappa_{\mathcal{B}} \qquad G_M(0) = \mu_{\mathcal{B}}$$

pQCD asymptotic behavior
Space-like region



- **pQCD**: as $q^2 \to -\infty$, asymptotic behaviors of F_1 and F_2 must follow counting rules
- Quarks exchange gluons to distribute momentum

Dirac form factor F_1

- Non-spin flip
- Two gluon propagators
- $F_1(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$

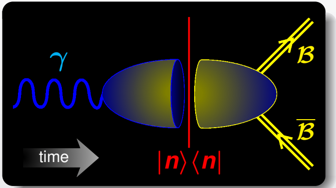
Pauli form factor F_2

- Spin flip
- Two gluon propagators
- $F_2(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-3}$

Sachs form factors G_E and G_M

- $G_{E,M}(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$
- Ratio: $\frac{G_E}{G_M} \underset{q^2 \to -\infty}{\sim}$ constant

Baryon form factors
Time-like region ($q^2 > 0$)



- **Crossing symmetry**:

$$\langle \mathcal{B}(p')|j^\mu|\mathcal{B}(p)\rangle \longrightarrow \langle \overline{\mathcal{B}}(p')\mathcal{B}(p)|j^\mu|0\rangle$$

- **Form factors are complex functions of** q^2

Optical theorem

$$\mathrm{Im}\langle \overline{\mathcal{B}}(p')\mathcal{B}(p)|j^\mu|0\rangle \sim \sum_n \langle \overline{\mathcal{B}}(p')\mathcal{B}(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle \quad \Longrightarrow \quad \begin{cases} \mathrm{Im} F_{1,2} \neq 0 \\ \text{for } q^2 > 4M_\pi^2 \end{cases}$$

$|n\rangle$ are on-shell intermediate states: $2\pi, 3\pi, 4\pi, \ldots$

Time-like asymptotic behavior

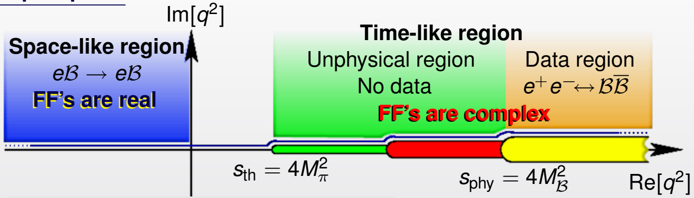
Phragmèn Lindelöf theorem:
If $f(z) \longrightarrow a$ as $z \longrightarrow \infty$ along a straight line, and $f(z) \longrightarrow b$ as $z \longrightarrow \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $a = b$ and $f(z) \longrightarrow a$ **uniformly in this angle**.

- $$\underbrace{\lim_{q^2\to-\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2\to+\infty} G_{E,M}(q^2)}_{\text{time-like}}$$

- $$G_{E,M} \underset{q^2\to+\infty}{\sim} (q^2)^{-2}$$ **real**

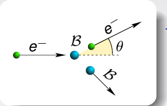
Cross sections and analyticity

q^2-complex plane



Crossing: tot. helicity $= \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$

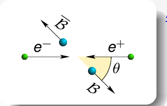
$$G_E(4M_{\mathcal{B}}^2) = G_M(4M_{\mathcal{B}}^2)$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2\frac{\theta}{2}}{4E_e^3 \sin^4\frac{\theta}{2}} \left[G_E^2 - \tau\left(1 + 2(1-\tau)\tan^2\frac{\theta}{2}\right)G_M^2\right]\frac{1}{1-\tau}$$

$$\tau = \frac{q^2}{4M_{\mathcal{B}}^2}$$

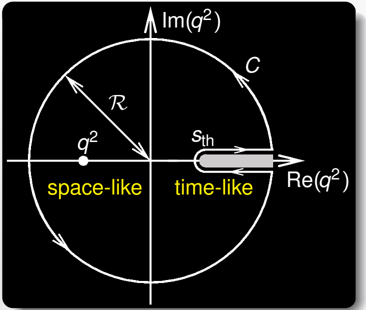


Annihilation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2\right]$$

$$\beta = \sqrt{1 - \frac{1}{\tau}}$$

Dispersion relations



- The form factors are **analytic** on the q^2-plane with a **multiple cut** $(s_{th} = 4M_\pi^2, \infty)$

- **Dispersion relation for the imaginary part** ($q^2 < 0$)

$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i} \oint_C \frac{G(z)dz}{z - q^2} = \frac{1}{\pi} \int_{s_{th}}^{\infty} \frac{\mathrm{Im}G(s)ds}{s - q^2}$$

- **Dispersion relation for the logarithm** ($q^2 < 0$)
  B.V. Geshkenbein, Yad. Fiz. 9 (1969) 1232.

$$\ln G(q^2) = \frac{\sqrt{s_{th} - q^2}}{\pi} \int_{s_{th}}^{\infty} \frac{\ln|G(s)|ds}{(s - q^2)\sqrt{s - s_{th}}}$$

Experimental inputs

- Space-like data on the real values of FF's from: $e^-\mathcal{B} \longrightarrow e^-\mathcal{B}$ and $e^{-\uparrow}\mathcal{B} \longrightarrow e^-\mathcal{B}^\uparrow$, with polarization
- Time-like data on moduli of FF's from: $e^+e^- \longrightarrow \mathcal{B}\overline{\mathcal{B}}$
- Time-like data on G_E-G_M relative phase from: $e^+e^- \longrightarrow \mathcal{B}^\uparrow\overline{\mathcal{B}}$ (pol.)

Theoretical ingredients

- Analyticity $\Rightarrow$ dispersion relations
- Normalization and threshold values
- Asymptotic behavior $\Rightarrow$ super-convergence relations

Dispersive approach: advantages and drawbacks

Advantages

- DR's are based on unitarity and analyticity $\Rightarrow$ **model-independent approach**
- DR's relate data from different processes in different energy regions

$$\left[\begin{array}{c}\textbf{space-like}\\ \textbf{form factor}\\ e\mathcal{B}\longrightarrow e\mathcal{B}\end{array}\right] = \int \left[\begin{array}{c}\textbf{Im(form factor) or ln|form factor|}\\ \textbf{over the time-like cut } (s_{\rm th},\infty)\\ e^+e^-\longrightarrow \mathcal{B}\overline{\mathcal{B}} + \textbf{theory}\end{array}\right]$$

- Normalizations and theoretical constraints can be directly implemented
- Form factors can be computed in the whole q^2-complex plane

Drawbacks

- Very long-range integration
  - **Remedy #1**
    pQCD power laws
  - **Remedy #2**
    Subtracted DR's
- **No data in the unphysical region, crucial in dispersive analyses**

Integral equation

- Dispersion relation for the logarithm
  - Regularization to stabilize solutions
    - Model-independent approach
      - No time-like $|G_E| - |G_M|$ separation

$\Longrightarrow$ $|G_M^p|$ and $|G_M^n|$ in the unphysical region

The integral equation for G_M

Dispersion relation subtracted at $t = 0$

$$\ln G(t) = \frac{t\sqrt{s_{th} - t}}{\pi} \int_{s_{th}}^{\infty} \frac{\ln|G(s)|ds}{s\sqrt{s - s_{th}}(s - t)}$$

- Less dependent on the asymptotic behavior of the FF
- $\ln G(0) = 0 \Longrightarrow$ no further terms have to be considered

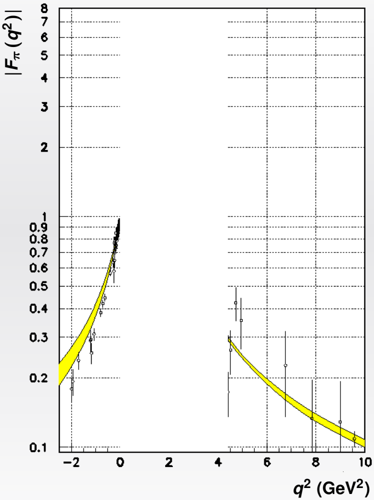
Splitting the integral $\int_{s_{th}}^{\infty}$ into $\int_{s_{th}}^{s'_{phy}} + \int_{s'_{phy}}^{\infty}$ we obtain the integral equation

$$\overbrace{\ln G(t) - I^{\infty}_{phy}(t)}^{\text{Data and Theory}} = \frac{t\sqrt{s_{th} - t}}{\pi} \int_{s_{th}}^{s'_{phy}} \frac{\overbrace{\ln|G(s)|}^{\text{Unknown}}\, ds}{s\sqrt{s - s_{th}}(s - t)}$$

- To avoid instabilities around $s_{phy} = 4M_N^2$, the upper boundary has been shifted to $s'_{phy} = s_{phy} + \Delta$, with $\Delta \simeq 0.5\ \text{GeV}^2$
- We impose continuity of the FF at s'_{phy} and s_{th}, in addition, at the upper boundary s'_{phy}, continuity of the first derivative is also required
- A regularization, depending on a free parameter τ, is introduced by requiring the FF total curvature in the unphysical region to be limited

Pion FF to fix the regularization parameter τ

Space-like (DR) and time-like data (yellow bands) have been used as input in the integral equation to retrieve the time-like FF in the nucleon unphysical region.

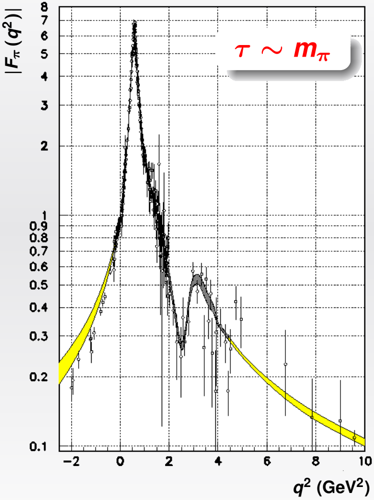


Testing the method

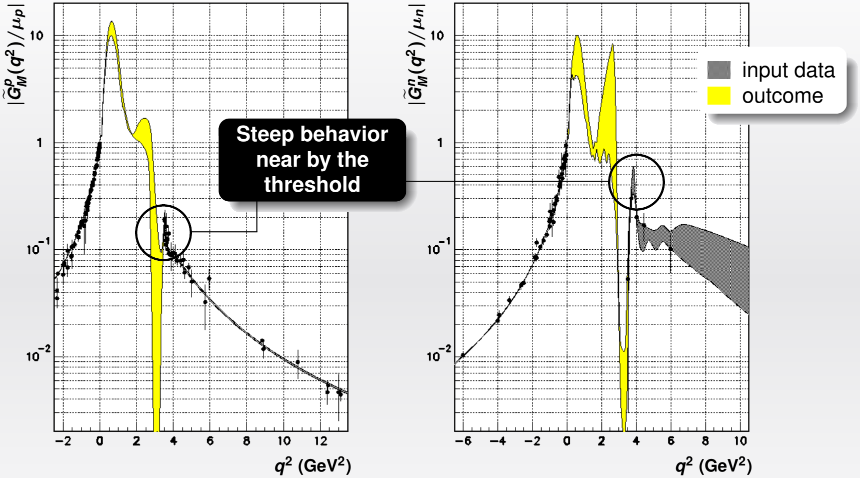
EPJC11 709

Pion FF to fix the regularization parameter τ

Space-like (DR) and time-like data (yellow bands) have been used as input in the integral equation to retrieve the time-like FF in the nucleon unphysical region (gray band).



Nucleon magnetic form factors



$M_1 \sim 770$ MeV $\quad \Gamma_1 \sim 350$ MeV

$M_2 \sim 1600$ MeV $\quad \Gamma_2 \sim 350$ MeV

The ratio $R = \mu_p G_E^p / G_M^p$

- Dispersion relation for the imaginary part
  - Model-independent approach
    - First time-like $|G_E| - |G_M|$ separation

$\Longrightarrow$ Ratio in the whole q^2 complex plane

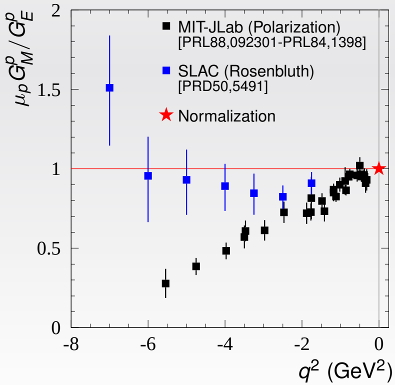
Data on $R = \mu_p G_E^p / G_M^p$

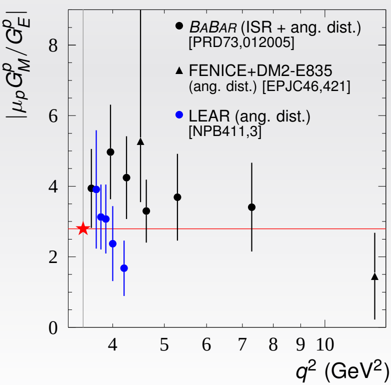
Space-like region

- Old Rosenbluth data in agreement with space-like **scaling** $G_E^p \simeq G_M^p / \mu_p$
- New data from polarization techniques show unexplained **increasing behavior**
- Only polarization data have been used in the dispersive analysis

Time-like region

- Only two sets of data from *BABAR* and LEAR otained studying angular distributions
- **Unique attempts** to perform a time-like $|G_E^p|$-$|G_M^p|$ **separation**
- Only *BABAR* data have been used in the dispersive analysis





We start from the imaginary part of the ratio $\boldsymbol{R(q^2)}$, written in the most general and model-independent way as

$\mathrm{Im}[\boldsymbol{R(q^2)}]$ = series of orthogonal polynomials

Theoretical constraints can be applied directly on the imaginary part

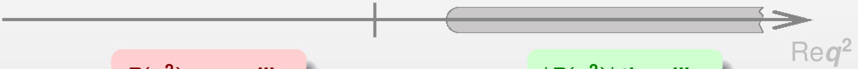
Dispersion Relations

The function $R(q^2)$ is reconstructed in time and space-like regions

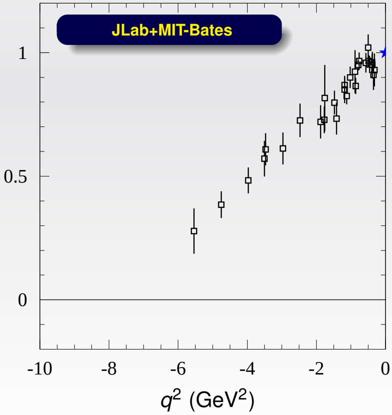
Additional theoretical conditions as well as experimental constraints are finally imposed on the obtained analytic expression of $\boldsymbol{R(q^2)}$

$R(q^2)$

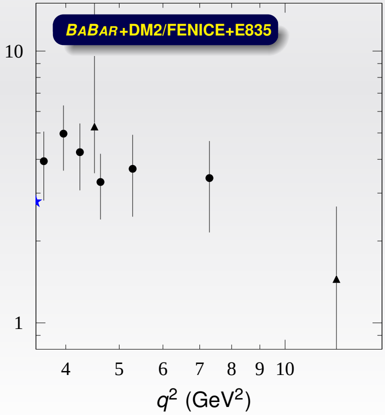
$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$



$R(q^2)$ space-like



$|R(q^2)|$ time-like



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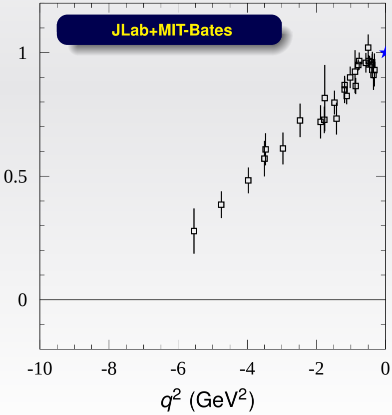
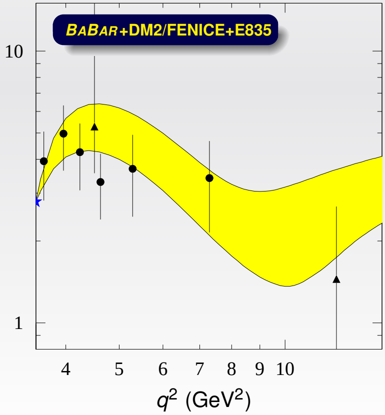


$|R(q^2)|$ time-like



$R(q^2)$

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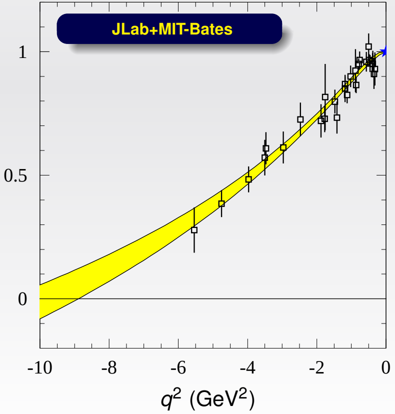
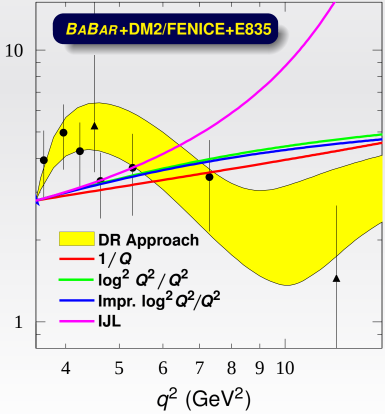
$R(q^2)$ space-like

$|R(q^2)|$ time-like

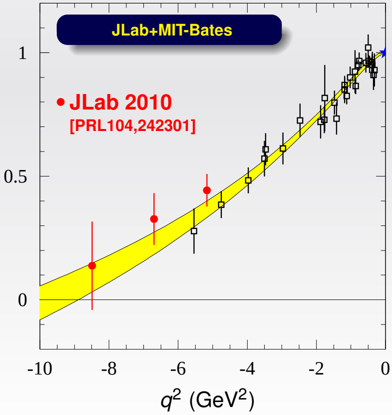




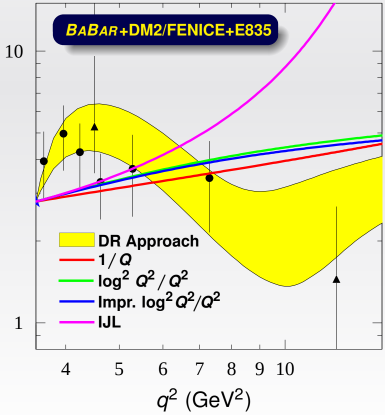
$R(q^2)$

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$

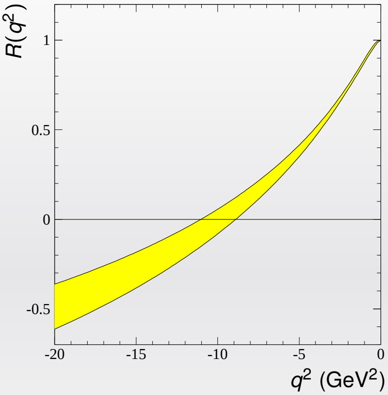
$R(q^2)$ space-like

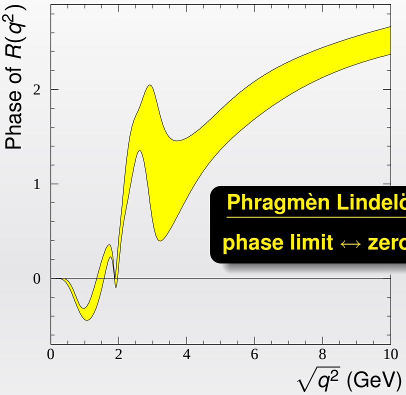


$|R(q^2)|$ time-like



$R(q^2)$: space-like zero and phase





Phragmèn Lindelöf

phase limit ↔ zeros

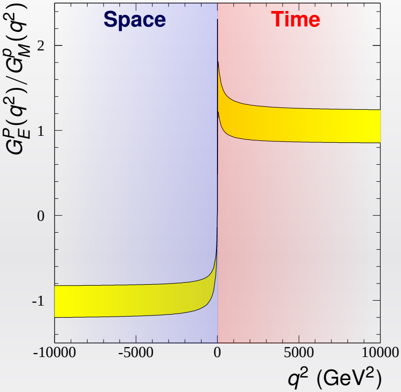
Space-like zero

$$t_0^{BABAR} = (-10 \pm 1)\ \text{GeV}^2$$

Phase from DR

$$\phi(q^2) = -\frac{\sqrt{q^2 - s_0}}{\pi}\ \text{Pr}\int_{s_0}^{\infty} \frac{\ln|R(s)|ds}{\sqrt{s-s_0}(s-q^2)}$$

Asymptotic $G_E^P(q^2)/G_M^p(q^2)$



- **Real asymptotic values for G_E^p/G_M^p**

$$\frac{G_E^p}{G_M^p} \xrightarrow[|q^2|\to\infty]{} -1.0 \pm 0.2$$

- **Asymptotic behavior of F_2/F_1**

$$\frac{q^2}{4M_N^2}\left|\frac{F_2}{F_1}\right| \xrightarrow[|q^2|\to\infty]{} \left|\frac{G_E^p}{G_M^p} - 1\right| = 2.0 \pm 0.2$$

pQCD prediction

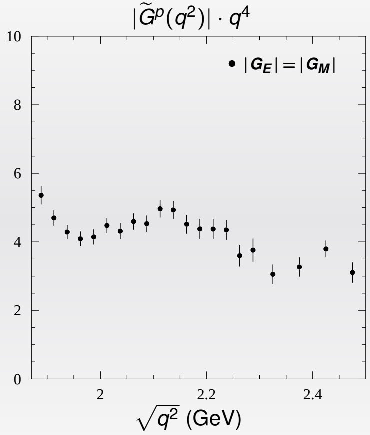
$$\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right| \xrightarrow[|q^2|\to\infty]{} 1$$

A sum rule for G_M^p

- Dispersion relation for the logarithm
  - Unphysical region suppression
    - Low-energy data $\longrightarrow$ asymptotic behavior

$\Longrightarrow$ Check for the asymptotic power law

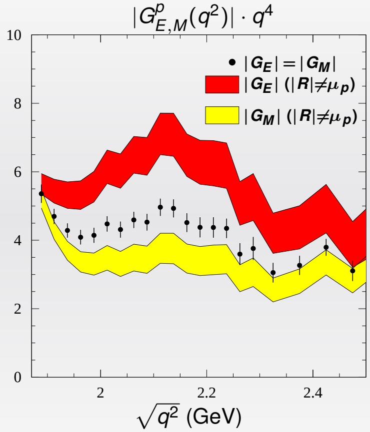
$|G_E^p(q^2)|$ and $|G_M^p(q^2)|$ from $\sigma_{p\bar{p}}$ and DR



$$|\widetilde{G}^p(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}} \left(1 + \frac{1}{2\tau}\right)^{-1}$$

- Usually what is extracted from the cross section $\sigma(e^+e^- \longrightarrow p\bar{p})$ is the effective time-like form factor $|\widetilde{G}^p|$ obtained assuming $|G_E^p| = |G_M^p|$ i.e. $|R| = \mu_p$
- Using our parametrization for R and the BABAR data on $\sigma(e^+e^- \longrightarrow p\bar{p})$, $|G_E^p|$ and $|G_M^p|$ may be disentangled

$|G_E^p(q^2)|$ and $|G_M^p(q^2)|$ from $\sigma_{p\bar{p}}$ and DR



$$|G_M(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}} \left(1 + \frac{|R(q^2)|}{2\mu_p\tau}\right)^{-1}$$

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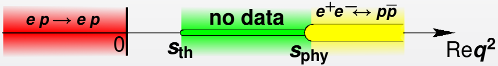
Dispersion relations and sum rules

Geshkenbeĭn, Ioffe, Shifman Yad. Fiz. 20, 128 (1974)

- **DR's connect space and time values of a form factor $G(q^2)$**

$$G(q^2) = \frac{1}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{Im}G(s)ds}{s-q^2}$$



Drawbacks

- **The imaginary part is not experimentally accessible**
- **There are no data in the unhysical region $[s_{th}, s_{phy}]$**
- **We need to know the asymptotic behavior**

- **They applied the DR for the imaginary part to the function**

$$\phi(z) = f(z)\frac{\ln G(z)}{z\sqrt{s_{th}-z}} \quad \text{with} \quad \int_0^{s_{phy}} f^2(z)dz << 1$$

Advantages

- **The DR integral contains the modulus $|G(s)|$**
- **The unhysical region contribution is suppressed**

Drawback

- **Zeros of $G(z)$ are poles for $\phi(z)$**

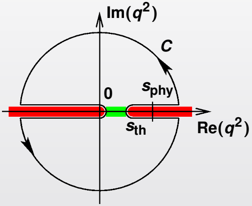
Attenuated DR and sum rule

Assuming $G(q^2) \neq 0$ and using the Cauchy theorem, we have the new DR

$$\oint_C \phi(z)dz = 0$$

$$\Downarrow$$

$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)]\ln G(t)}{t\sqrt{s_{\rm th}-t}}dt}_{\text{Space-like}} = \underbrace{\int_{s_{\rm th}}^{\infty} \frac{f(s)\ln|G(s)|}{s\sqrt{s-s_{\rm th}}}ds}_{\text{Time-like}}$$



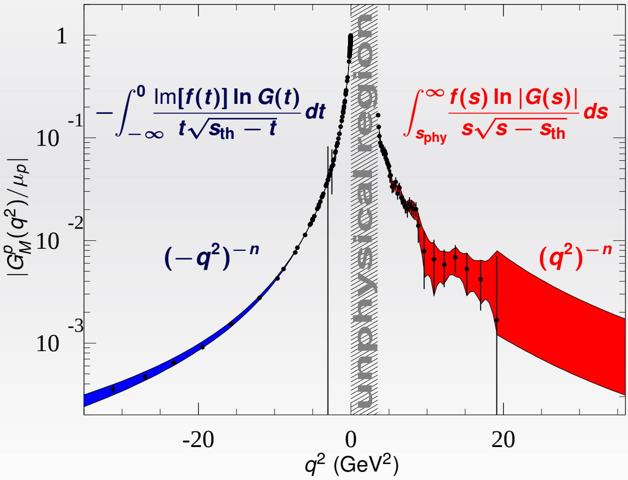
Convergence relation to find the asymptotic power-law behavior of G_M^p

$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)]\ln G(t)}{t\sqrt{s_{\rm th}-t}}dt}_{\text{Space-like data + }(-t)^{-n}} = \int_{s_{\rm th}}^{\infty} \frac{f(s)\ln|G(s)|}{s\sqrt{s-s_{\rm th}}}ds \approx \underbrace{\int_{s_{\rm phy}}^{\infty} \frac{f(s)\ln|G(s)|}{s\sqrt{s-s_{\rm th}}}ds}_{\text{Time-like data + }s^{-n}}$$

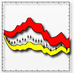
n is the free parameter

Sum rule: result for G_M^p

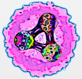
$$G_M^p(q^2) \underset{|q^2|\to\infty}{\propto} |q^2|^{-(2.27\pm0.36)}$$



"To do" list

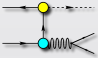


Time-like $|G_E| - |G_M|$ Separation: DR and data



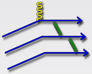
Understand threshold effect(s):

Dispersive analyses: integral equation, sum rule,...

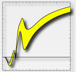


Experimental observation in $p\bar{p} \longrightarrow \pi^0 l^+ l^-$

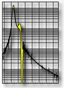
[C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas, Phys. Rev. C75, 045205 (2007)]



Asymptotic behavior: DR and data for the phase



Zeros $\leftrightarrow$ phases: DR and data



Unphysical region, VMD contributions:

integral equation, sum rule, data on $p\bar{p} \longrightarrow \pi^0 l^+ l^-$

. . . On the same topic. . .

Tuesday at 17:30
Yue Ma
“Time-like Form Factor from PANDA”

Wednesday at 9:00
Diego Bettoni
“Time-like Electromaganetic Form Factors (Overview)”

Wednesday at 11:00
Carl Carlson
“Two Photon Physics in the Time-like and Spacelike Regions”

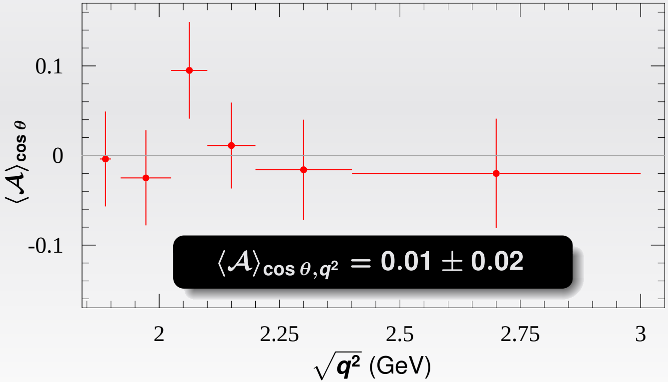
Wednesday at 17:30
Marco Maggiora
“Hadron Structure at BESIII”

Additional slides

$\gamma\gamma$ exchange from $e^+e^- \to p\bar{p}\gamma$ BABAR data

PLB659, 197

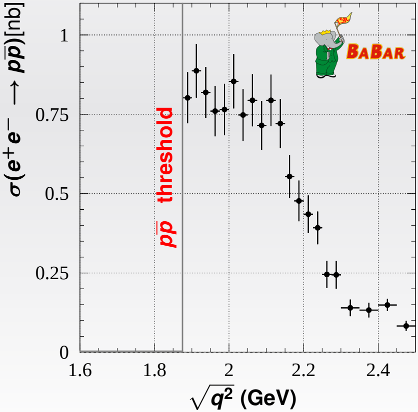
$$\mathcal{A}(\cos\theta, q^2) = \frac{\frac{d\sigma}{d\Omega}(\cos\theta, q^2) - \frac{d\sigma}{d\Omega}(-\cos\theta, q^2)}{\frac{d\sigma}{d\Omega}(\cos\theta, q^2) + \frac{d\sigma}{d\Omega}(-\cos\theta, q^2)}$$



$e^+e^- \longrightarrow p\bar{p}$
The incredible threshold value

PRD73, 012005

$$\sigma(e^+e^- \to p\bar{p}) = \frac{4\pi\alpha^2\beta C}{3q^2}\left[|G_M^p|^2 + \frac{2M_p^2}{q^2}|G_E^p|^2\right] \xrightarrow[q\to 2M_p]{} \frac{\pi\alpha^2\beta C}{2M_p^2}|G^p|^2$$



At threshold
$\sigma(e^+e^- \to p\bar{p}) = 0.80 \pm 0.05$ nb

$e^+e^- \to p\bar{p}$ is the only endothermic process that shows this peculiarity

$\Downarrow$

Where does the factor C come from?

$\beta C = \sqrt{1 - \frac{4M_p^2}{q^2}}\, C = \{$ finite at threshold

$e^+e^- \longrightarrow p\bar{p}$
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PRD73, 012005

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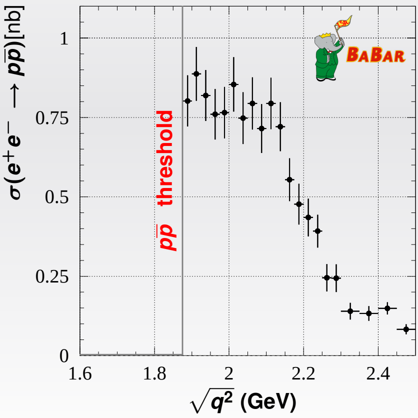

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$\Downarrow$

Where does the factor C come from?

$$\beta C = \sqrt{1 - \frac{4M_P^2}{q^2}}\, C = \begin{cases} \text{finite at} \\ \text{threshold} \end{cases}$$

Höler, Mergell, Meissner, Hammer procedure

- Optical theorem
  - Dispersion relation for the imaginary part
    - No time-like $|G_E| - |G_M|$ separation

$\Longrightarrow$ $G_E^{p,n}$ and $G_M^{p,n}$ in space and time-like region

H_2M_2: Höler, Mergell, Meissner, Hammer,. . .

PRC75 035202

Spectral decomposition

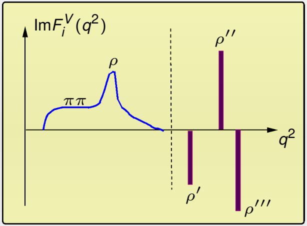
$$\mathrm{Im}\langle\overline{\mathcal{B}}(p')\mathcal{B}(p)|j^\mu|0\rangle \sim \sum_n \langle\overline{\mathcal{B}}(p')\mathcal{B}(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle \implies \begin{cases} \mathrm{Im}F_{1,2} \neq 0 \\ \text{for } q^2 > 4M_\pi^2 \end{cases}$$

H_2M_2: Höler, Mergell, Meissner, Hammer,. . .

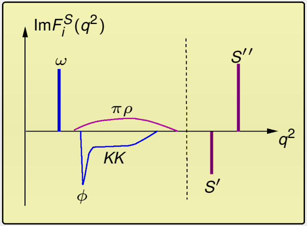
PRC75 035202

Spectral decomposition

$$\mathrm{Im}\langle\overline{\mathcal{B}}(p')\mathcal{B}(p)|j^\mu|0\rangle \sim \sum_n \langle\overline{\mathcal{B}}(p')\mathcal{B}(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle \implies \begin{cases} \mathrm{Im}F^{V,S}_{1,2} \neq 0 \\ \text{for } q^2 > 4M_\pi^2 \end{cases}$$



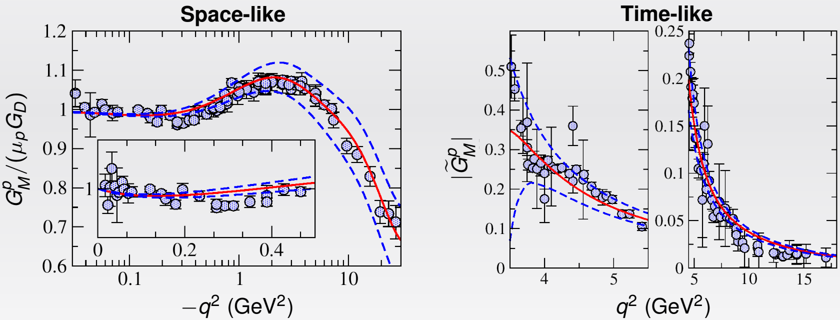
- 2π continuum is knowm for $q^2 \in [4M_\pi^2, \sim 40M_\pi^2]$
- The singularity on the second Riemann sheet in $\pi N \to \pi N$ amplitude gives the strong shoulder at threshold
- Poles for higher mass states



- *KK* continuum from analytic continuation of *KN* scattering amplitude
- Further contribution in the ϕ-region is due to $\pi\rho$ exchange
- Anomalous threshold behavior is masked because the pole in the second Riemann sheet is not close to $(3M_\pi)^2$
- Poles for higher mass states

H_2M_2: Theoretical constraints and result for G_M^p

- Superconvergence relations: $\int_{4M_\pi^2}^{\infty} \mathrm{Im}\, F_{1,2}(q^2)dq^2 = \int_{4M_\pi^2}^{\infty} q^2 \mathrm{Im}\, F_2(q^2)dq^2 = 0$
- Asymtpotic behaviors from perturbative QCD



$$F(q_{SL}^2) = \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im} F(q_{TL}^2)}{q_{TL}^2 - q_{SL}^2} dq_{TL}^2$$

The Lomon Model

- VMD + quark form factors
  - DRs $\longrightarrow$ analytic VM propagators
    - Time-like $|G_E| - |G_M|$ separation

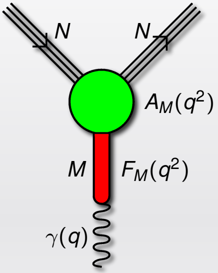
$\Longrightarrow$ $G_E^{p,n}$ and $G_M^{p,n}$ in space and time-like region

Analyticitization of phenomenological models
The Lomon Model

PRC66 045501

The **Lomon** parameterization for nucleon FF's is based on the Gari-Krümpelmann model, and it includes:

- coupling to the photons through vector meson exchange [VMD in terms of **propagators $F_M(q^2)$**, $M = \rho, \omega, \phi, \rho', \omega'$]
- **hadron/quark form factors $A_M(q^2)$** at vector meson-nucleon (quark) vertices to control transition to perturbative QCD at high momentum transfers

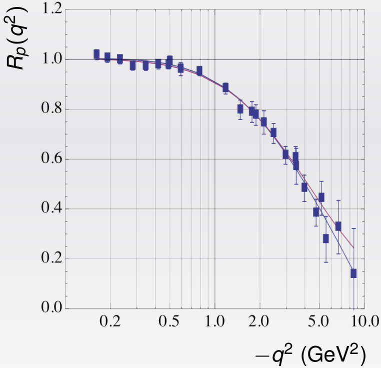


Analytic extension: space-like ⟶ time-like

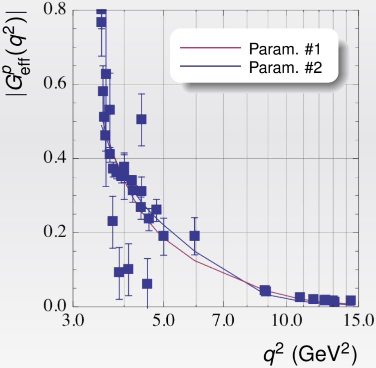
- ***F_M* for broad mesons**:
  simple poles ⟶ poles with finite energy-dependent widths
- **Dispersion realtions**:
  rigorous analytic continuation of F_M from time-like to space-like region

Lomon Model: Results for the proton

Space-like region: $R_p = \mu_p \dfrac{G_E^p}{G_M^p}$

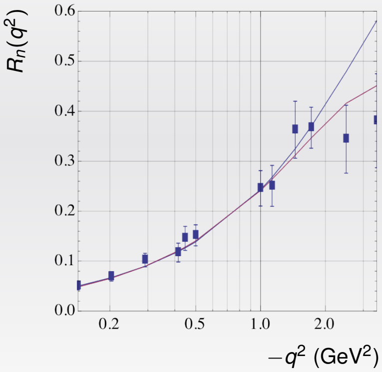


Time-like region: $|G_{\rm eff}^p(q^2)|$

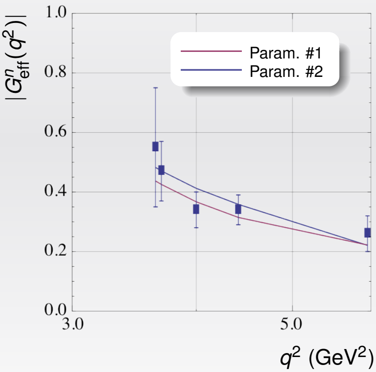


Lomon Model: Results for the neutron

Space-like region: $R_n = \mu_n \frac{G_E^n}{G_M^n}$

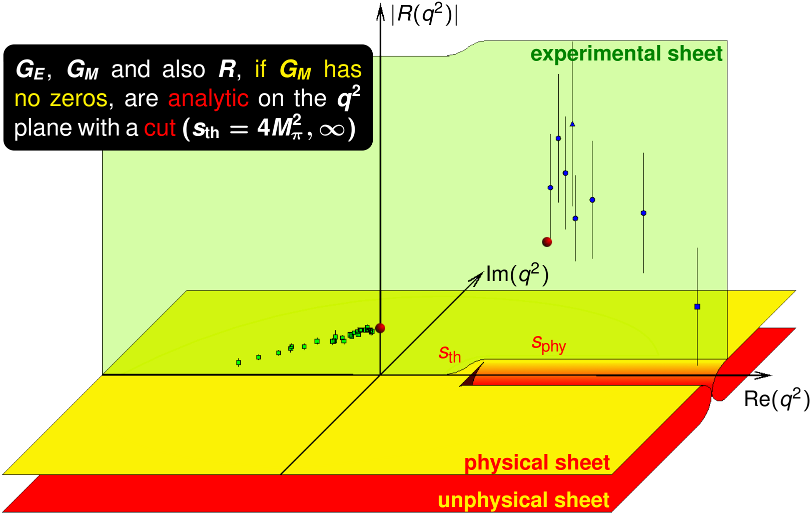


Time-like region: $|G_{\rm eff}^n(q^2)|$



$R(q^2)$ in the complex plane

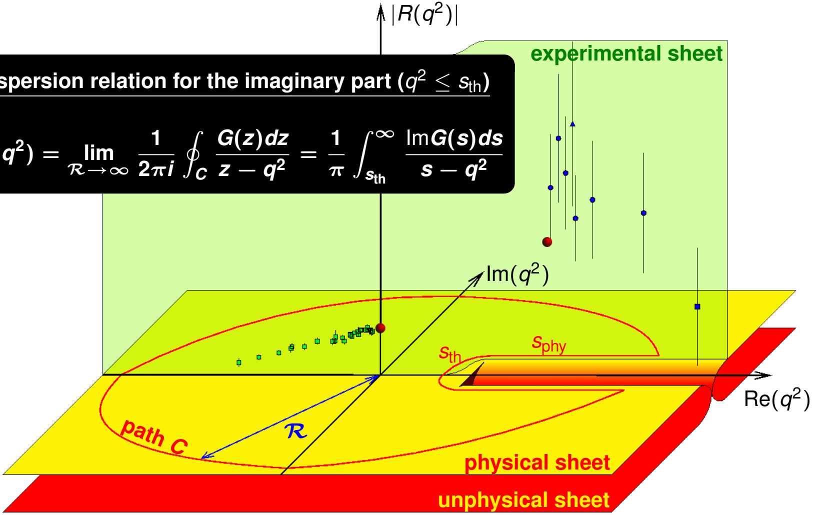
G_E, G_M and also **R**, if **G_M** has no zeros, are analytic on the **q^2** plane with a cut **$(s_{\rm th} = 4M_\pi^2, \infty)$**



$R(q^2)$ in the complex plane

Dispersion relation for the imaginary part ($q^2 \leq s_{\rm th}$)

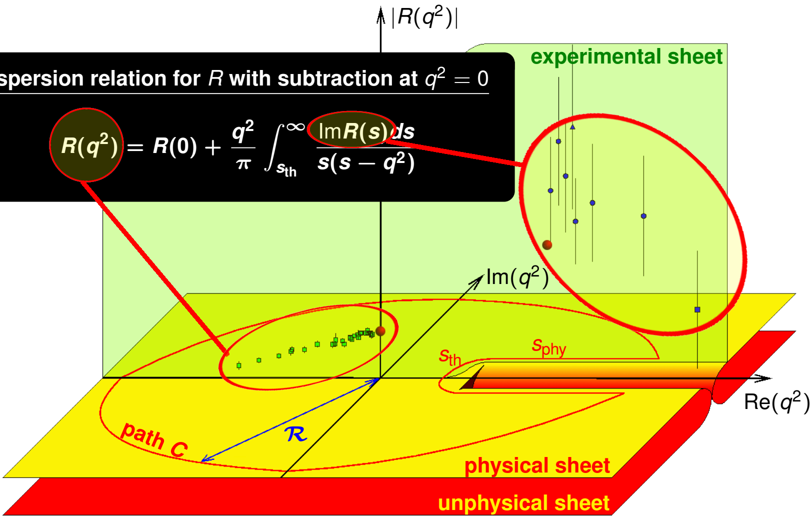
$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i} \oint_C \frac{G(z)dz}{z-q^2} = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} \frac{{\rm Im}G(s)ds}{s-q^2}$$



$R(q^2)$ in the complex plane

Dispersion relation for R with subtraction at $q^2 = 0$

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{s_{th}}^{\infty}\frac{\mathrm{Im}R(s)\,ds}{s(s-q^2)}$$



Parameterization and constraints

The imaginary part of R is parameterized by two series of orthogonal polynomials $T_i(x)$

$$\text{Im}R(q^2) \equiv I(q^2) = \begin{cases} \sum_i C_i T_i(x) & x = \frac{2q^2 - s_{\text{phy}} - s_{\text{th}}}{s_{\text{phy}} - s_{\text{th}}} & s_{\text{th}} \le q^2 \le s_{\text{phy}} \\ \sum_j D_j T_j(x') & x' = \frac{2s_{\text{phy}}}{q^2} - 1 & q^2 > s_{\text{phy}} \end{cases}$$

$s_{\text{th}} = 4M_\pi^2$

$s_{\text{phy}} = 4M_N^2$

Theoretical conditions on $ImR(q^2)$

- $R(4M_\pi^2)$ is real $\Longrightarrow I(4M_\pi^2) = 0$
- $R(4M_N^2)$ is real $\Longrightarrow I(4M_N^2) = 0$
- $R(\infty)$ is real $\Longrightarrow I(\infty) = 0$

Theoretical conditions on $R(q^2)$

- Continuity at $q^2 = 4M_\pi^2$
- $R(4M_N^2)$ is real and $\text{Re}R(4M_N^2) = \mu_p$

Experimental conditions on $R(q^2)$ and $|R(q^2)|$

- Space-like region ($q^2 < 0$) data for R from JLab and MIT-Bates
- Time-like region ($q^2 \ge 4M_N^2$) data for $|R|$ from FENICE+DM2, *BABAR*, E835 and Lear