Nucleon Structure from the lattice

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Outline

- Some basics about lattice QCD
- nucleon DAs and GPDs on the lattice
- Exploratory studies for TMDs
- $\langle x(u-d) \rangle$ a non-trivial observable
  The issues:
  - non-perturbative renormalization
  - chiral extrapolation
- Conclusion: QCD is the most difficult QFT. Progress is slow and tedious but also continuous.
  But, hadron phenomenology which ignores lattice results does not longer make sense.

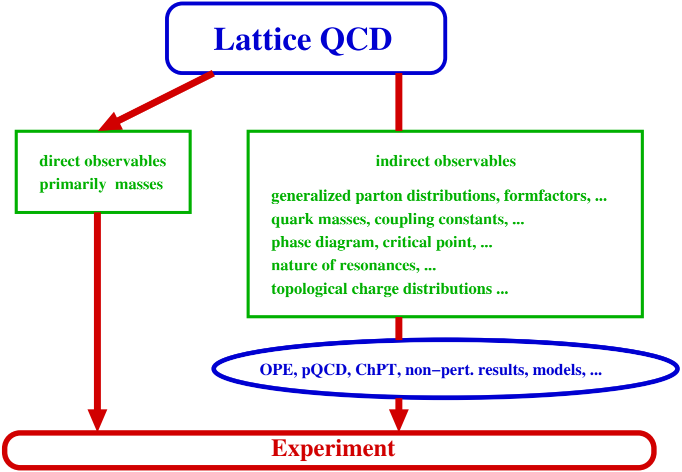
OPE and pQCD allows to link experimental observations to correlators in a well-defined manner:

$$\langle \, \mathit{Hadron} \mid \text{quark and gluon field operators} \mid \mathit{Hadron}' \, \rangle$$

$\langle P(p)\vert\bar{q}(x)\gamma_\mu D_{\mu_1}...D_{\mu_n}q(x)\vert P(p)\rangle$	momentum distribution of quarks
$\langle P(p')\vert\bar{q}(x)\gamma_\mu q(x)\vert P(p)\rangle$	form factors of a proton
$\langle P(p)\vert\bar{q}(x)\Gamma_\mu q(x)\bar{q}'(x)\Gamma'_\nu q'(x)\vert P(p)\rangle$	diquark correlations in a proton
$\langle P(p,s)\vert\bar{q}(x)\gamma_\mu \tilde{G}_{\nu\lambda}(x)q(x)\vert P(p,s)\rangle$	color magnetic field in a proton

These can and be calculated on the lattice

Lattice QCD needs pQCD and as many experimental checks as possible. There are many open theoretical problems.



Lattice QCD

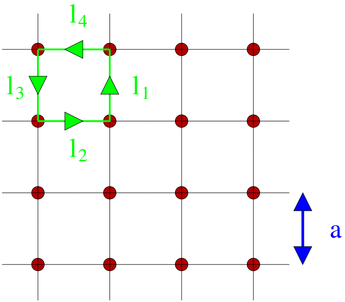
QCD is contained in the generating functional:

$$
\begin{aligned}
Z[J^a_\mu, \bar{\eta}^i, \eta^i] &= \int \mathcal{D}[A^{a\mu}, \bar{\psi}^i, \psi^i] \\
&\quad \exp\left(\mathrm{i} \int d^4x \left[\mathcal{L}_{\mathrm{QCD}} - J^a_\mu A^a_\mu - \bar{\psi}^i \eta^i - \bar{\eta}^i \psi^i\right]\right)
\end{aligned}
$$

A numerical integration is made possible by analytic continuation to imaginary time:

$$
\begin{aligned}
t &\leftrightarrow -\mathrm{i}\tau \\
S = \int d^4x (T - V) &\leftrightarrow \mathrm{i} \int d^4x_E (T + V) = \mathrm{i} S_E \\
e^{\mathrm{i}S} &\leftrightarrow e^{-S_E}
\end{aligned}
$$

Discretized space time ⇒ e.g. the Wilson action



$$U(l_1) = \exp\left(-igA^b(l_1)\,\frac{\lambda^b}{2}\,a\right)$$

$$W_\Box = \mathrm{Tr}\{U(l_1)U(l_2)U(l_3)U(l_4)\}$$

$$\sum_\Box \frac{2}{g^2}(3 - \mathcal{R}e\, W_\Box) = \frac{1}{4}\int d^4x\left(F^a_{\mu\nu}F^a_{\mu\nu} + O(a^2)\right)$$

Hadronic 2- and 3- Point functions

One needs combinations of field operators which have the wanted quantum numbers, e.g. for the nucleon ($C = i\gamma^2\gamma^4 = C^{-1}$):

$$\hat{B}_\alpha(t,\vec{p}) = \sum_{\vec{x}} e^{\mathrm{i}\vec{p}\cdot\vec{x}} \epsilon_{ijk} \hat{u}^i_\alpha(x)\ \hat{u}^j_\beta(x)(C^{-1}\gamma_5)_{\beta\gamma}\hat{d}^k_\gamma(x)$$

$$\begin{aligned}\langle 0|T\left\{\hat{B}(y_4)\hat{A}(x_4)\right\}|0\rangle &= e^{-(T-y_4+x_4)E_B}\langle B|\hat{B}(0)|0\rangle\langle 0|\hat{A}(0)|B\rangle \\ &+ e^{-(y_4-x_4)E_A}\langle 0|\hat{B}(0)|A\rangle\langle A|\hat{A}(0)|0\rangle\end{aligned}$$

$\hat{B}$ generates the antiparticle of $\hat{A}$. One has (anti)periodic boundary conditions.

To get the hadron masses one simply has to determine the slopes.

$$e^{-(y_4-x_4)M_N}\langle 0|\hat{N}^\dagger(0)|N\rangle\langle N|\hat{N}(0)|0\rangle$$

$$\begin{aligned} |B\rangle \quad &\sim \quad c_0|N\rangle \ + \ c_1|N'\rangle \ + \ c_2|N\pi\rangle \ + \ \ldots \\ &\Rightarrow \quad c_0 e^{-E_N t}|N\rangle \ + \ c_1 e^{-E_{N'} t}|N'\rangle \ + \ c_2 e^{-E_{N\pi} t}|N\pi\rangle \ + \ \ldots \end{aligned}$$

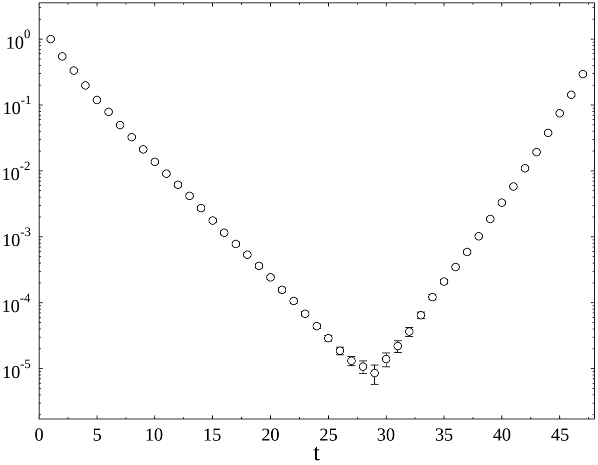
Note: A quark propagator is the inverse of the Dirac operator on the lattice, which is just a large matrix.

$$\langle B_\alpha(t,\vec{p})\bar{B}_\beta(0,\vec{p})\rangle$$

$$= \sum_{\substack{x \\ x_4=t}} \sum_{\substack{y \\ y_4=0}} \mathrm{e}^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})}\epsilon_{ijk}\epsilon_{i'j'k'}(C^{-1}\gamma_5)_{\alpha'\alpha''}(\gamma_5 C)_{\beta'\beta''}$$

$$\left\langle G^{ki'}_{\alpha''\beta'}(x,y)\left(G^{jj'}_{\alpha'\beta''}(x,y)G^{ik'}_{\alpha\beta}(x,y) - G^{ij'}_{\alpha\beta''}(x,y)G^{jk'}_{\alpha'\beta}(x,y)\right)\right\rangle_g$$

A nucleon 2-point function



Once the propagation in imaginary time has projected the original source onto the physical wave function on can calculate physical correlators from

$$\frac{\tilde{\Gamma}_{\alpha\beta}\langle B_\beta(t,\vec{p})\mathcal{O}\bar{B}_\alpha(0,\vec{p})\rangle}{\Gamma_{\alpha\beta}\langle B_\beta(t,\vec{p})\bar{B}_\alpha(0,\vec{p})\rangle}$$

The main problems of LQCD calculations

- The quartic limit

  chiral limit: $\lim_{m_q \to m_q(phys)}$

  statistics: $\lim_{num.conf. \to \infty}$

  infinite volume: $\lim_{V \to \infty}$

  continuum limit: $\lim_{a \to 0}$

- Renormalization:

  The Feynman rules on a discrete lattice are different from the continuum ones $\Rightarrow$ all radiative corrections are different

  The hypercubic group is smaller than that of continuum rotations $\Rightarrow$ operator mixing, e.g.

  $\langle p|\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q|p'\rangle$ can mix with $\frac{1}{a^2}\langle p|\bar{q}q|p'\rangle$

DA's and GPD's

Lets define a nucleon wave-function by

$$\Psi(x_i, k_{i\perp}) = \langle 0| T\,[q(x_1, k_{1\perp})q(x_2, k_{2\perp})q(x_3, k_{3\perp})]\,|p\rangle$$

Distribution Amplitude $\Phi(x_i, \mu) = Z(\mu) \int^{|k_\perp| \leq \mu} \mathrm{d}^3 k_{i,\perp} \Psi(x_i, k_{i\perp})$

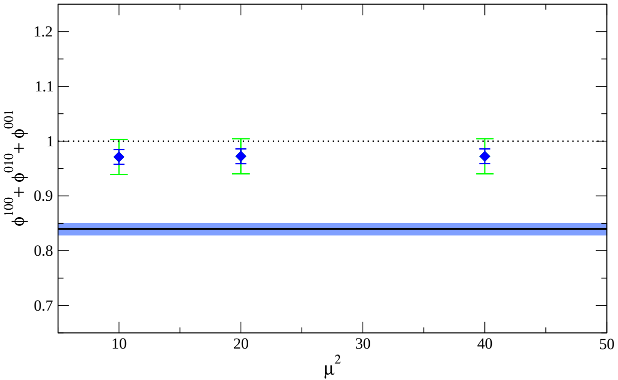
Distribution Function $Z(\mu) \int^{|k_\perp| \leq \mu} \mathrm{d}^3 k_{i\perp} \|\Psi(x, k_\perp)\|^2$

The proton state can be written as

$$
\begin{aligned}
|p,\uparrow\rangle &= \int_0^1 [\mathrm{d}x] \frac{\Phi_N(x_i)}{\sqrt{96 x_1 x_2 x_3}} \left| u^\uparrow(x_1) \left[u^\downarrow(x_2) d^\uparrow(x_3) - d^\downarrow(x_2) u^\uparrow(x_3) \right] \right\rangle \\
\Phi_N^{lmn} &= 2\phi^{lmn} - \phi^{nml} \\
\phi^{lmn} &= \frac{1}{f_N} \int_0^1 [\mathrm{d}x] x_1^l x_2^m x_3^n \phi(x_1, x_2, x_3) \\
1 &= x_1 + x_2 + x_3 \\
\phi^{lmn} &= \phi^{(l+1)mn} + \phi^{l(m+1)n} + \phi^{lm(n+1)} \\
\phi^{000} &\equiv 1
\end{aligned}
$$

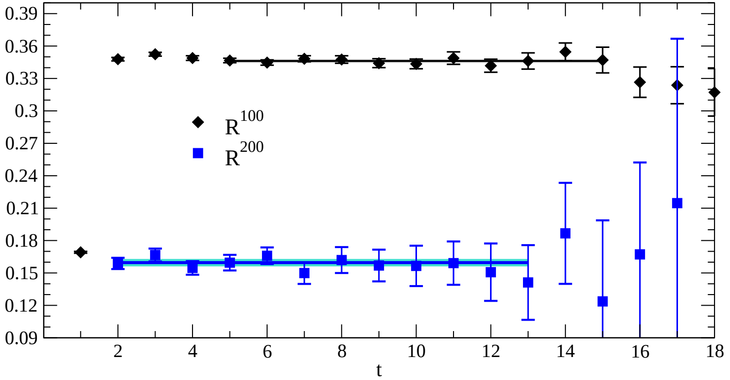
Test of sum rules

$$S_1 = \phi^{100} + \phi^{010} + \phi^{001}$$



Errors: statistical and chiral extrapolation

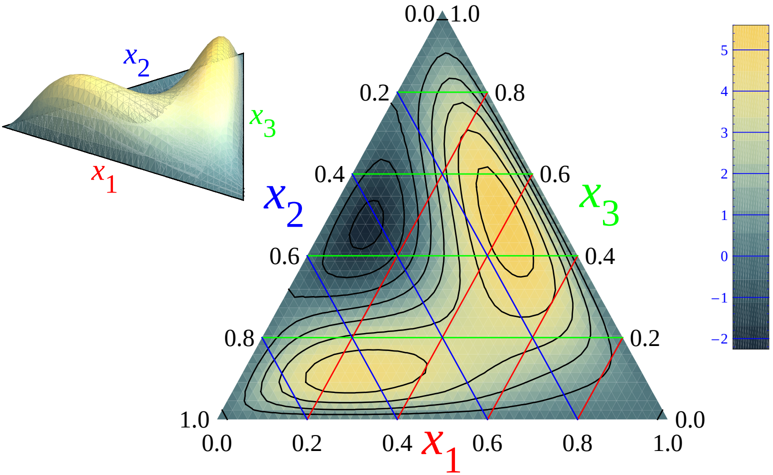
DA correlator ratios: $R^{lmn} = \langle \frac{\phi^{lmn}}{S_{(l+m+n)}} \rangle$



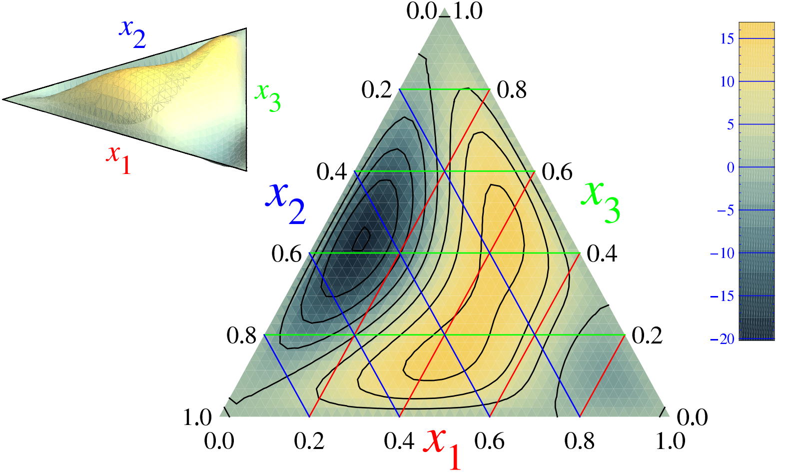
Old results [$\mu^2 = 1$ GeV^2, errors (stat.)(chiral extr.)(renorm.)]

	Asy	QCD-SR	KS	BK	BLW	Latt.
φ^{100}	$\frac{1}{3} \approx 0.333$	0.560(60)	0.55	0.38	0.415	0.3999(13)(122)(4)
φ^{010}	$\frac{1}{3} \approx 0.333$	0.192(12)	0.21	0.31	0.285	0.2986(22)(105)(6)
φ^{001}	$\frac{1}{3} \approx 0.333$	0.229(29)	0.24	0.31	0.300	0.3015(9)(17)(1)
φ^{200}	$\frac{1}{7} \approx 0.143$	0.350(70)	0.35	$0.18^\star$	0.212	0.1792(26)(85)(72)
φ^{020}	$\frac{1}{7} \approx 0.143$	0.084(19)	0.09	$0.13^\star$	0.123	0.1459(66)(42)(21)
φ^{002}	$\frac{1}{7} \approx 0.143$	0.109(19)	0.12	$0.13^\star$	0.132	0.1354(42)(180)(90)
φ^{011}	$\frac{2}{21} \approx 0.095$	$-0.030(30)$	0.02	$0.08^\star$	0.053	0.0491(54)(233)(118)
φ^{101}	$\frac{2}{21} \approx 0.095$	0.102(12)	0.10	$0.10^\star$	0.097	0.1171(21)(37)(29)
φ^{110}	$\frac{2}{21} \approx 0.095$	0.090(10)	0.10	$0.10^\star$	0.093	0.1037(34)(170)(96)

With next-to-next-to-leading conformal spin (ϕ^{101}, ϕ^{200}, ϕ^{002})



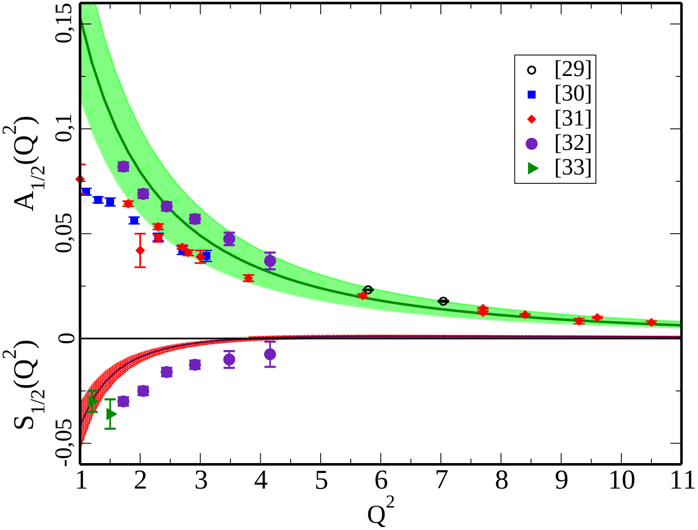
N^*



x2
x3
x1
0.0
1.0
0.2
0.8
0.4
0.6
x2
x3
0.6
0.4
0.8
0.2
1.0
0.0
0.0
0.2
0.4
x1
0.6
0.8
1.0
15
10
5
0
−5
−10
−15
−20

V. Braun et al., PRL 103(2009)072001

$\gamma^* N \to N^*(1535)$ transition formfactor from light-cone sumrules. The helicity amplitudes $A_{1/2}(Q^2)$ and $S_{1/2}(Q^2)$.



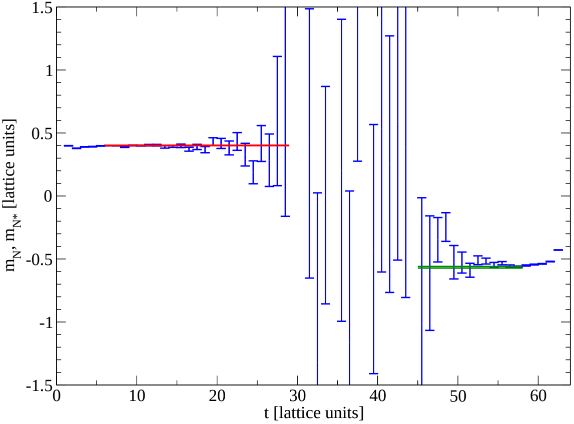
The shape parameters are defined through

$$\begin{aligned}
\varphi(x_i;\mu^2) &= 120x_1x_2x_3\Big\{1+c_{10}(x_1-2x_2+x_3)L^{\frac{8}{3\beta_0}} \\
&+c_{11}(x_1-x_3)L^{\frac{20}{9\beta_0}} \\
&+c_{20}\left[1+7(x_2-2x_1x_3-2x_2^2)\right]L^{\frac{14}{3\beta_0}} \\
&+c_{21}\,(1-4x_2)\,(x_1-x_3)\,L^{\frac{40}{9\beta_0}} \\
&+c_{22}\left[3-9x_2+8x_2^2-12x_1x_3\right]L^{\frac{32}{9\beta_0}}+\dots\Big\}
\end{aligned}$$

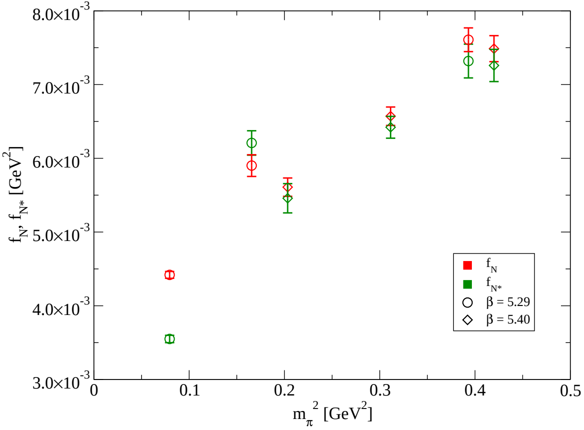
where

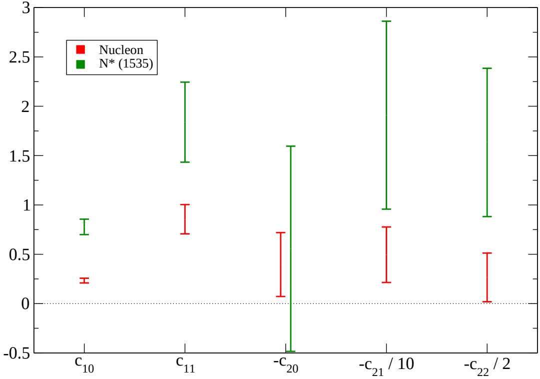
$$L=\alpha_s(\mu)/\alpha_s(\mu_0)$$

New data – with the help of QPACE



The nucleon quark coupling constants

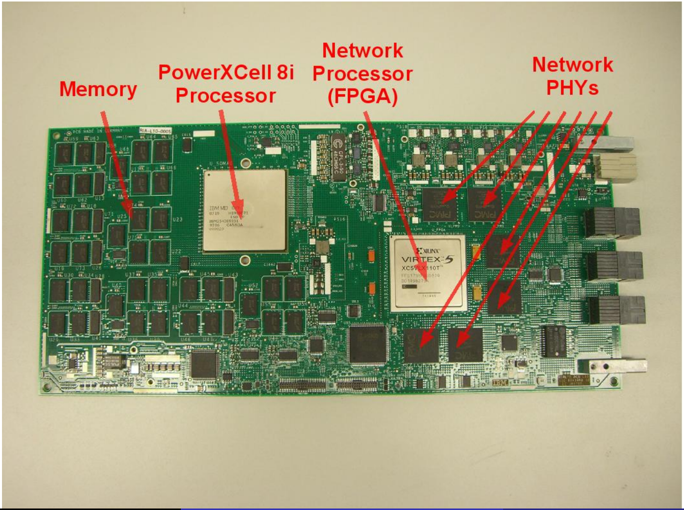


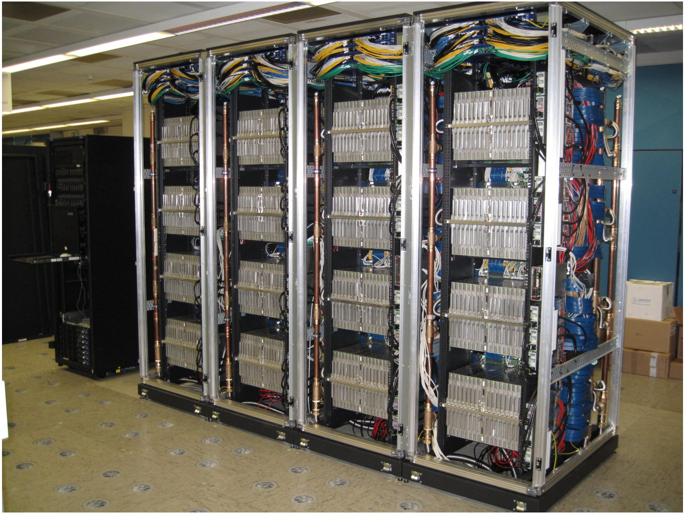


Nucleon
N* (1535)
3
2.5
2
1.5
1
0.5
0
-0.5
c_{10}
c_{11}
$-c_{20}$
$-c_{21}$ / 10
$-c_{22}$ / 2

QPACE

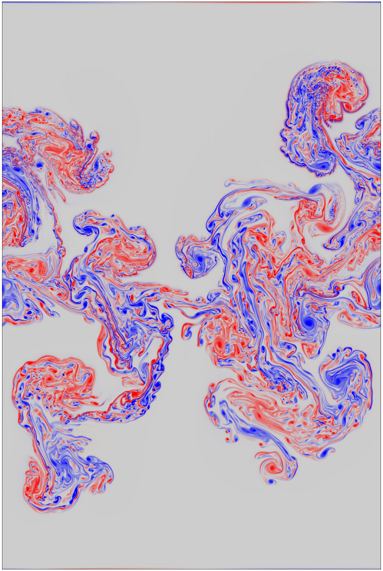
In spite of many problems, QPACE was completed within the schedule and is operational since beginning of 2010. The compute power is 200 TFlops peak double-precision



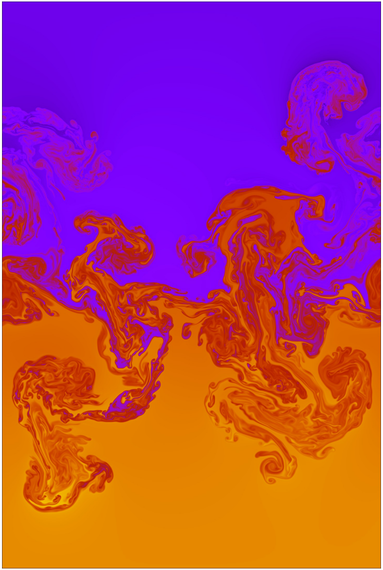


QPACE is NOT only a QCD machine

Rayleigh-Taylor instability, temperature



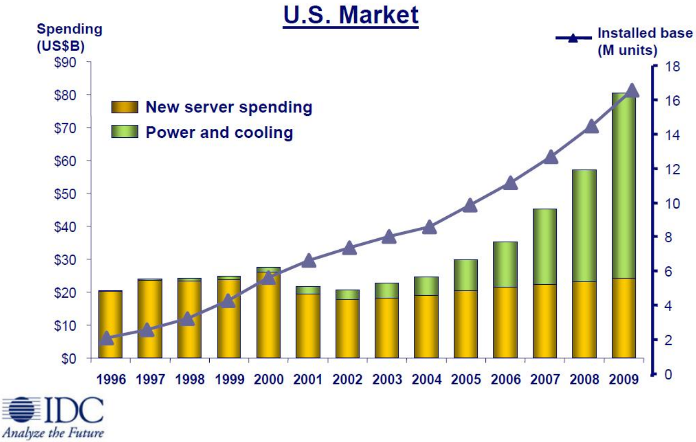
Rayleigh-Taylor instability, vorticity



green Top 500

Green500 Rank	MFLOPS/W	Site*	Computer*	Total Power (kW)	TOP500 Rank*
1	722.98	Forschungszentrum Juelich (FZJ)	QPACE SFB TR Cluster, PowerXCell 8i, 3.2 GHz, 3D-Torus	59.49	110
1	722.98	Universitaet Regensburg	QPACE SFB TR Cluster, PowerXCell 8i, 3.2 GHz, 3D-Torus	59.49	111
1	722.98	Universitaet Wuppertal	QPACE SFB TR Cluster, PowerXCell 8i, 3.2 GHz, 3D-Torus	59.49	112
4	458.33	DOE/NNSA/LANL	BladeCenter QS22/LS21 Cluster, PowerXCell 8i 3.2 Ghz / Opteron DC 1.8 GHz, Infiniband	276	29
4	458.33	IBM Poughkeepsie Benchmarking Center	BladeCenter QS22/LS21 Cluster, PowerXCell 8i 3.2 Ghz / Opteron DC 1.8 GHz, Infiniband	138	78
6	444.25	DOE/NNSA/LANL	BladeCenter QS22/LS21 Cluster, PowerXCell 8i 3.2 Ghz / Opteron DC 1.8 GHz, Voltaire Infiniband	2345.5	2
7	428.91	National Astronomical Observatory of Japan	GRAPE-DR accelerator Cluster, Infiniband	51.2	445
8	379.24	National SuperComputer Center in Tianjin/NUDT	NUDT TH-1 Cluster, Xeon E5540/E5450, ATI Radeon HD 4870 2, Infiniband	1484.8	5

D. Pleiter, T. Wettig, ET AL.



next project: iDataCool, which is aiming at PUE$<$ 1 **!**

another application: LHCb physics

Fermilab claims a $\sim 3\sigma$ effect in B_s decays.
$\Rightarrow$ you want to study similar decays as precisely as possible.

$\Lambda_b \to \Lambda$ decays are especially interesting as they allow to isolate helicity dependent matrix elements.
Λ_b HQET, Λ DA's are needed.
We are interested in the correlation function

$$z^\nu \tilde{T}_\nu(P,q) = iz^\nu \int d^4x e^{-iq\cdot x} \langle 0| T\{j_{\Lambda_b}(0)\tilde{j}_\nu(x)\}|\Lambda(P)\rangle$$

$$\tilde{j}_\nu(x) = \bar{b}(x)\gamma_\nu(1-\gamma_5)s(x)$$

Definition of GPDs

$$h(P_1) + \Gamma^*(q_1) \to h(P_2) + \Gamma(q_2)$$

with $\Delta_\mu = q_{2\mu} - q_{1\mu}$, $t = \Delta^2$, $P_\mu = (P_{1\mu} + P_{2\mu})/2$
and $\xi = -Q^2/2P \cdot q$

Spin $\frac{1}{2}$ - the nucleon (modulo gauge links)

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P_2 | \, \bar{q}(-\frac{1}{2}z) \, \gamma^+ q(\frac{1}{2}z) \, | P_1 \rangle \Big|_{z^+=0,\, z_\perp=0}$$
$$= \frac{1}{P^+} \left[H_q(x,\xi,t) \, \bar{N}(P_2)\gamma^+ N(P_1) + E_q(x,\xi,t) \, \bar{N}(P_2) \frac{i\sigma^{+\alpha}\Delta_\alpha}{2M} N(P_1) \right]$$

Some properties of GPDs:

- relation to form factors and distribution functions

$$H_q(x,0,0) = q(x) \qquad \int_{-1}^{1} dx H_q(x,\xi,t) = F_{1q}(t)$$

$$\tilde{H}_q(x,0,0) = \Delta q(x) \qquad \int_{-1}^{1} dx H_q(x,\xi,t) = g_{Aq}(t)$$

- OPE

$$\int_{-1}^{1} dx\, x^{n-1}\, H(x,\xi,t) = \sum_{\substack{k=0 \\ \text{even}}}^{n-1} (2\xi)^k A_{n,k}(t) + \mathrm{mod}(n+1,2)\,(2\xi)^n C_n(t)$$

$$\int_{-1}^{1} dx\, x^{n-1}\, E(x,\xi,t) = \sum_{\substack{k=0 \\ \text{even}}}^{n-1} (2\xi)^k B_{n,k}(t) - \mathrm{mod}(n+1,2)\,(2\xi)^n C_n(t)$$

$$\langle P_2|\mathbf{Sym}\ \bar{q}\gamma^{\mu}\ \overleftrightarrow{D}^{\mu_1} \ldots \overleftrightarrow{D}^{\mu_n}\ q|P_1\rangle = \mathbf{Sym}\ \bar{u}\gamma^{\mu} u \sum_{i\ \text{even}}^{n} A^{q}_{n,i}(t)\Delta^{\mu_1}\ldots\Delta^{\mu_i} P^{\mu_{i+1}}\ldots P^{\mu_n}$$

• Ji's sumrule

$$\left\langle J_q^3 \right\rangle \; = \; \frac{1}{2}[A_{2,0}^q(0) + B_{2,0}^q(0)]$$

• Transverse structure of hadrons in the impact parameter plane

$$H_q(x, 0, b_\perp^2) = \frac{1}{(2\pi)^2} \int d^2\Delta_\perp \; e^{ib_\perp \Delta_\perp} H_q(x, 0, \Delta_\perp^2)$$

This is highly relevant for LHC due to multiple hard interactions in one $p + p$ collision.

• The transverse spin structure of the nucleon
Quarks with transverse polarization $\vec{s}$ are projected out by the operator

$$\frac{1}{2}\bar{q}\gamma^+[1 + (\vec{s}\vec{\gamma})\gamma_5]\, q = \frac{1}{2}\bar{q}\,[\gamma^+ - s^j i\sigma^{+j}\gamma_5]\, q$$

The corresponding density, expressed in terms of GPDs

$$\frac{1}{(2\pi)^2}\int d^2\Delta_\perp e^{ib_\perp\cdot\Delta_\perp}\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle P_2|\,\bar{q}(-\frac{1}{2}z)\,\gamma^+[1+\vec{s}\cdot\vec{\gamma})\gamma_5]q(\frac{1}{2}z)\,|P_1\rangle\Big|_{z^+=0}^{z_\perp=0}$$

$$= \frac{1}{2}\left[F+s^iF_T^i\right]$$

$$= \frac{1}{2}\Bigg[H-S^i\epsilon^{ij}b^j\frac{1}{m}E'-s^i\epsilon^{ij}b^j\frac{1}{m}\left(E_T'+2\tilde{H}_T'\right)$$

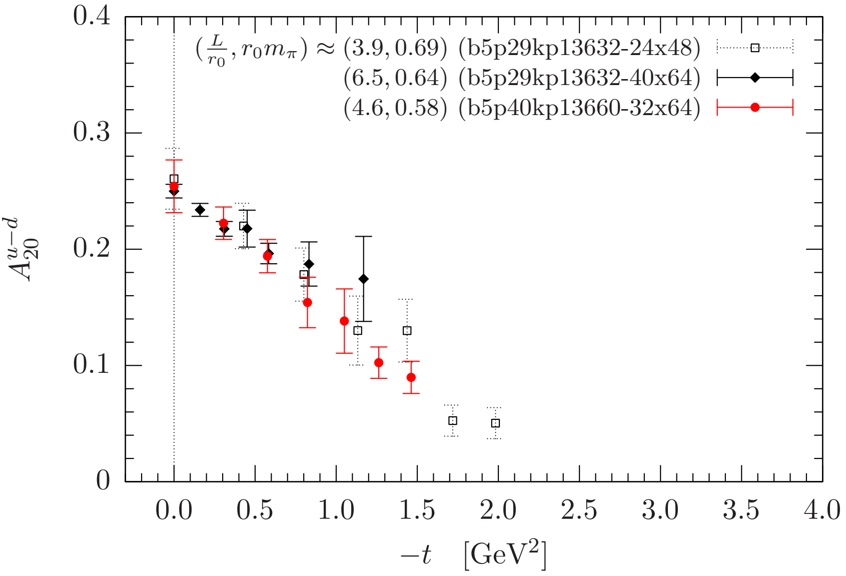
$$+s^iS^i\left(H_T-\frac{1}{4m^2}\Delta_b\tilde{H}_T\right)+s^i(2b^ib^j-b^2\delta^{ij})S^j\frac{1}{m^2}\tilde{H}_T''\Bigg]$$

has a simple interpretation:

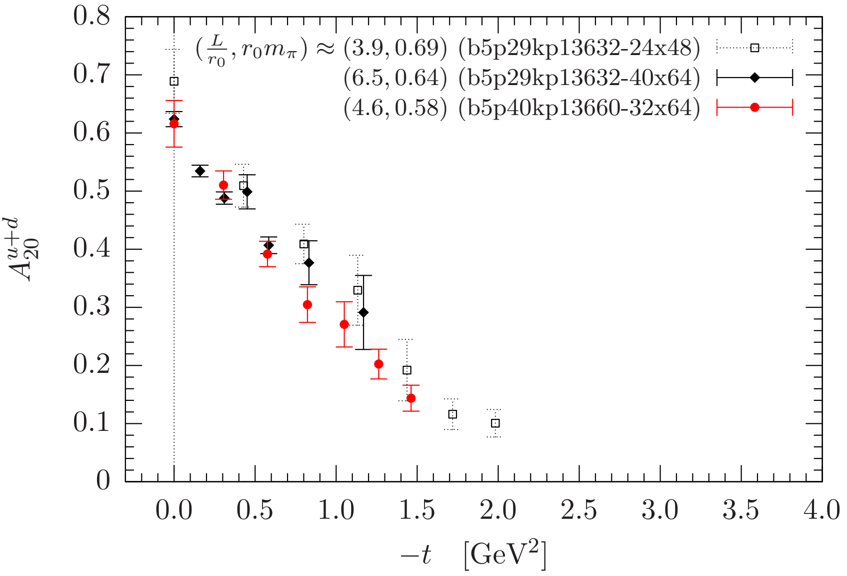
$S^i\epsilon^{ij}b^j$ coupling of proton spin to quark angular momentum

$s^i\epsilon^{ij}b^j$ coupling of quark spin to quark angular momentum

s^iS^i coupling of quark spin and proton spin



$(\frac{L}{r_0}, r_0 m_\pi) \approx (3.9, 0.69)$ (b5p29kp13632-24x48)
$(6.5, 0.64)$ (b5p29kp13632-40x64)
$(4.6, 0.58)$ (b5p40kp13660-32x64)
A_{20}^{u-d}
$-t$ [GeV^2]
0.4
0.3
0.2
0.1
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0



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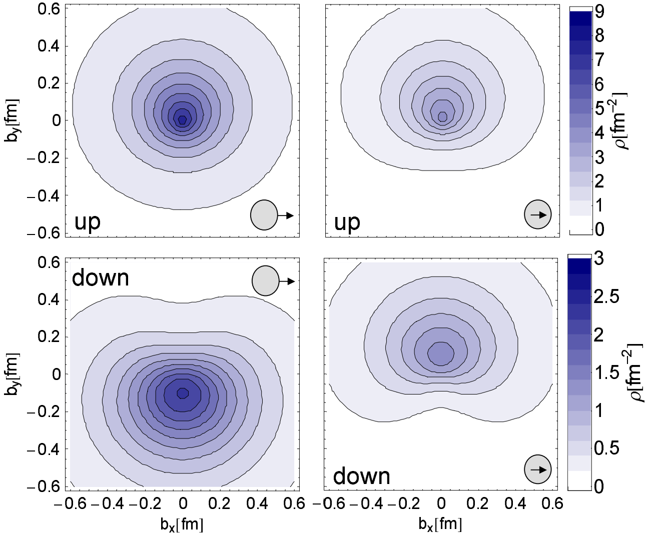
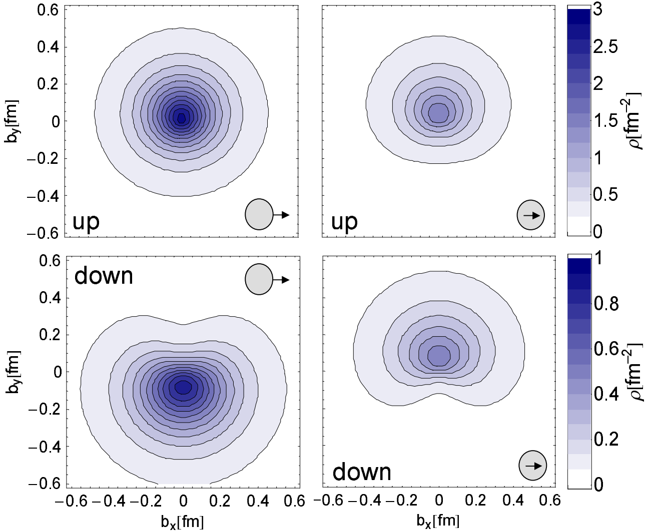
$(6.5, 0.64)$ (b5p29kp13632-40x64)
$(4.6, 0.58)$ (b5p40kp13660-32x64)
A_{20}^{u+d}
$-t$ [GeV^2]
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

The nucleon, first moment ($n = 1$)

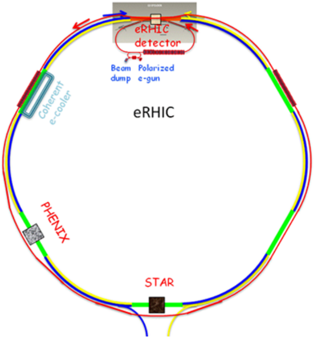


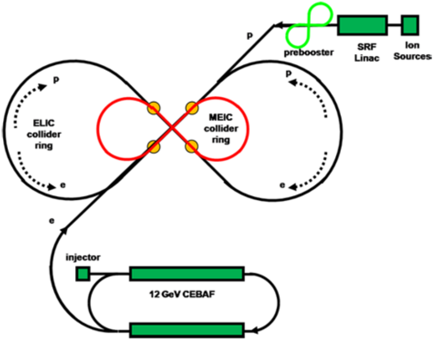
The nucleon, second moment ($n = 2$)



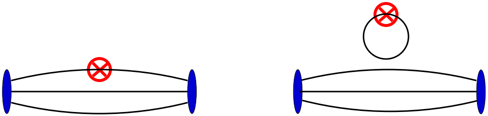
GPDs and related quantities are major motivations for

- New COMPASS proposal (CERN)
- JLab@12GeV
- EIC, ENC

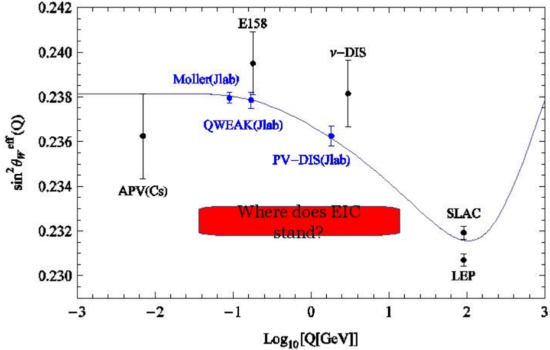


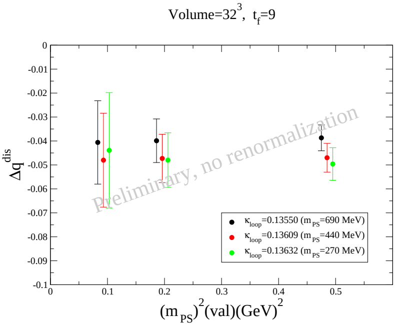


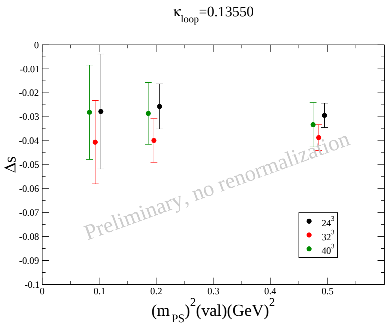
We meanwhile calculate systematically also disconnected contributions

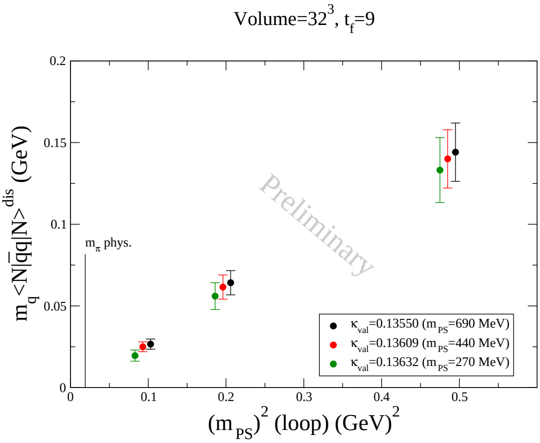


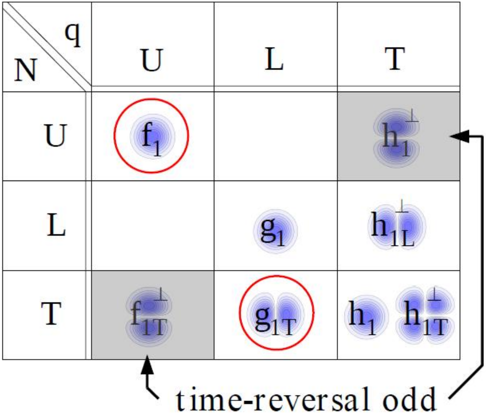
aim: clarifying the properties of the quark sea, especially the strange sea; NuTeV anomaly



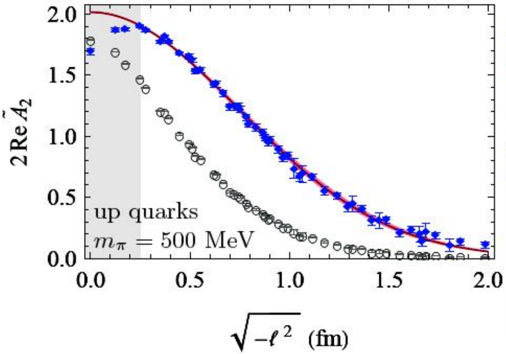

Volume=32^3, t_f=9
Δq^dis
(m_PS)^2(val)(GeV)^2
κ_loop=0.13550 (m_PS=690 MeV)
κ_loop=0.13609 (m_PS=440 MeV)
κ_loop=0.13632 (m_PS=270 MeV)
Preliminary, no renormalization


κ_loop=0.13550
Δs
(m_PS)^2(val)(GeV)^2
24^3
32^3
40^3
Preliminary, no renormalization


Volume=32^3, t_f=9
m_q <N|q̄q|N>^dis (GeV)
(m_PS)^2 (loop) (GeV)^2
m_π phys.
Preliminary
κ_val=0.13550 (m_PS=690 MeV)
κ_val=0.13609 (m_PS=440 MeV)
κ_val=0.13632 (m_PS=270 MeV)
0
0.05
0.1
0.15
0.2
0
0.1
0.2
0.3
0.4
0.5


q
N
U
L
T
U
f_1
$h_1^\perp$
L
g_1
$h_{1L}^\perp$
T
$f_{1T}^\perp$
g_{1T}
h_1
$h_{1T}^\perp$
time-reversal odd

$$f_1^{[1]}(\boldsymbol{k}_\perp^2) \equiv \int_{-1}^{1} dx\ f_1(x, \boldsymbol{k}_\perp^2) = \int \frac{d^2\ell_\perp}{(2\pi)^2}\ e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{\ell}_\perp}\ 2\,\tilde{A}_2(-\ell_\perp^2, 0)$$



fit function

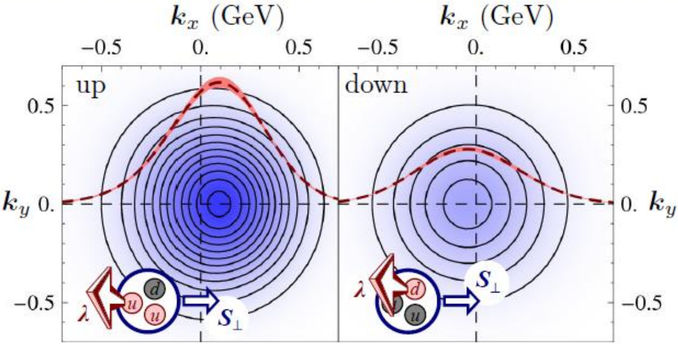
$C_1\ \exp(-|\ell|^2/\sigma_1^2)$

Z-factor

$Z^{-1}\ C_1^{\text{up-down}} \stackrel{!}{=} 1$

multiplicative renormalization based on quark counting

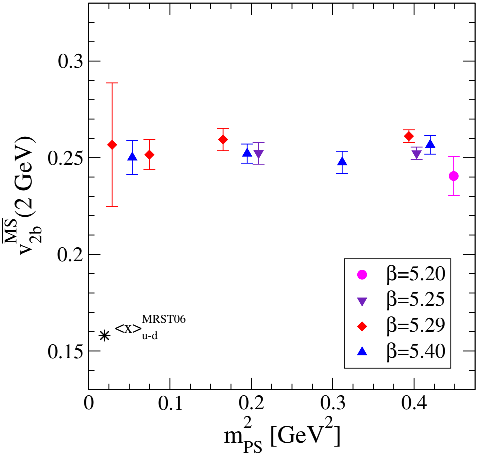
$$\rho^{[1]}_{TL}(\boldsymbol{k}_\perp; \boldsymbol{S}_\perp, \lambda) \equiv \frac{1}{2}\int dx \int dk^- \; \Phi^{[\gamma^+ \frac{1}{2}(\mathbb{1}+\gamma^5)]}(k, P, S_\perp)$$
$$= \frac{1}{2} f_1^{[1]}(\boldsymbol{k}_\perp^2) + \frac{\lambda}{2} \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{m_N} \; g_{1T}^{[1]}(\boldsymbol{k}_\perp^2)$$



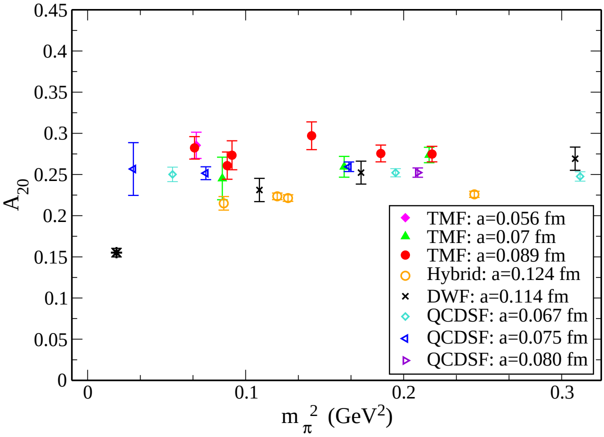
$\left(\begin{array}{c} m_\pi \approx 500 \text{ MeV, straight gauge link operator,} \\ \text{renormalization condition } C^{\text{ren}} = 0\text{, Gaussian fit} \end{array} \right)$

First QCDSF results, shown at LATTICE 2010, showed a problem:

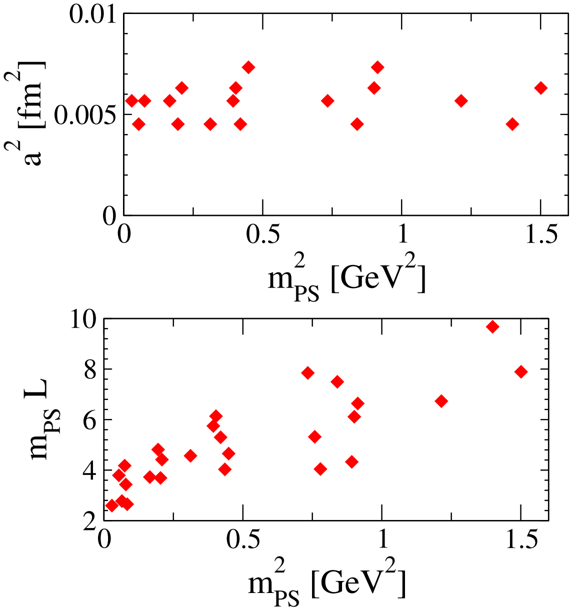
$$\int_0^1 dx\, x(u(x,Q^2) - d(x,Q^2))\Big|_{\overline{MS}}$$



we are not alone; C. Alexandrou, LATTICE 2010

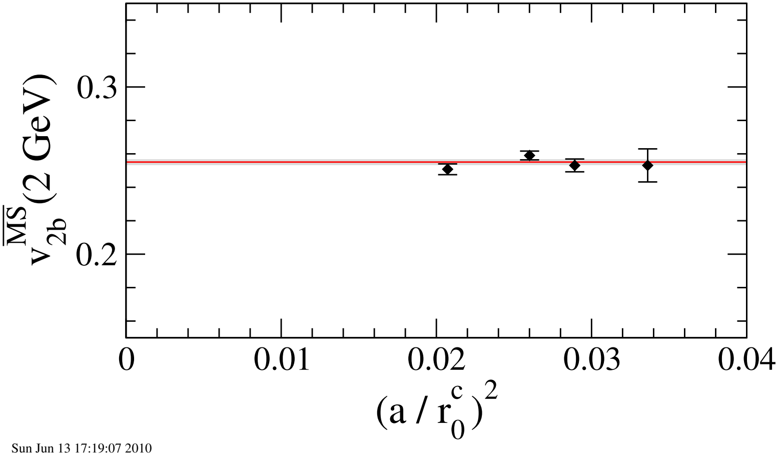


tests

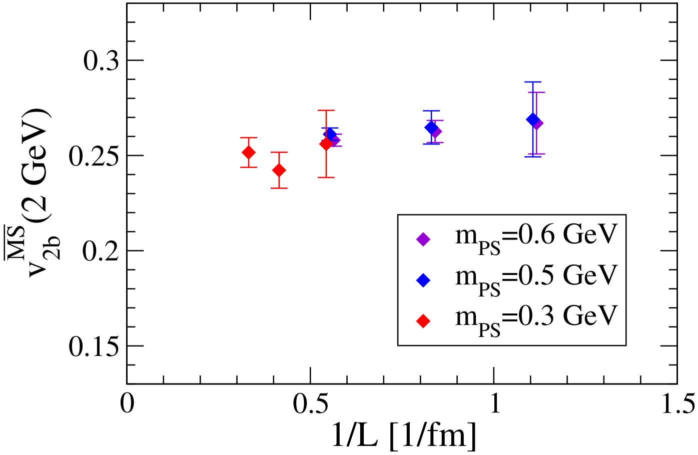


0.01
0.005
0
a^2 [fm^2]
0
0.5
1
1.5
m_{PS}^2 [GeV^2]
10
8
6
4
2
m_{PS} L
0
0.5
1
1.5
m_{PS}^2 [GeV^2]

continuum limit

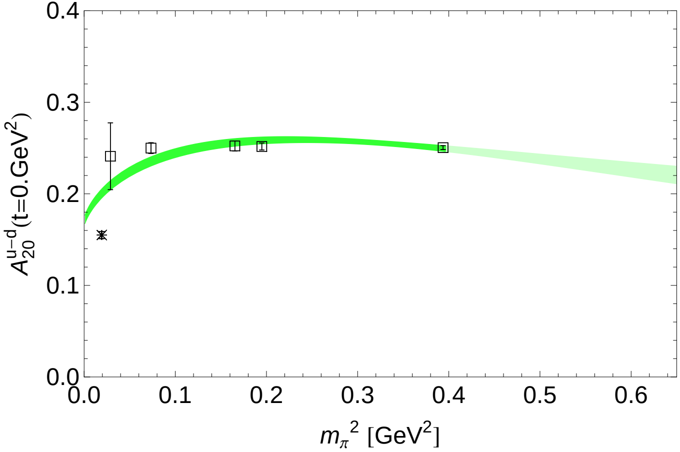


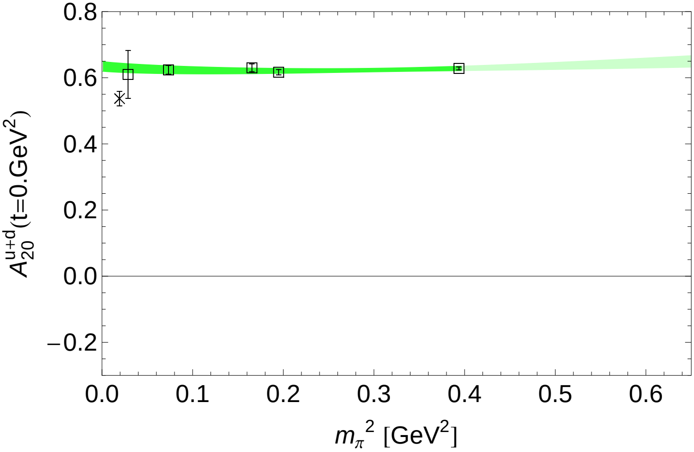
finite size



0.3
0.25
0.2
0.15
$v_{2b}^{\overline{MS}}(2\ \text{GeV})$
0
0.5
1
1.5
1/L [1/fm]
m_{PS}=0.6 GeV
m_{PS}=0.5 GeV
m_{PS}=0.3 GeV

chiral extrapolation (T. Hemmert) ?




0.8
0.6
0.4
0.2
0.0
−0.2
$A_{20}^{u+d}(t=0.GeV^2)$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
$m_\pi{}^2$ [GeV^2]

renormalization ?

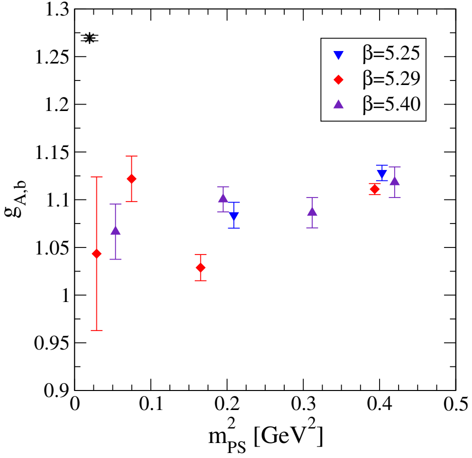
One of the technically and conceptually most difficult parts of LQCD.

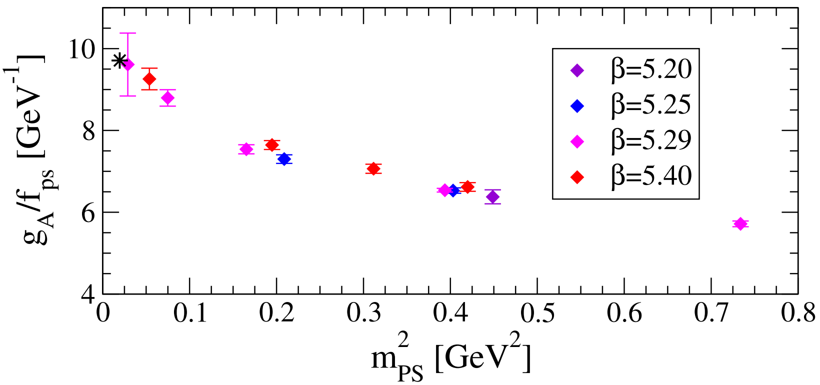
For details see M. Göckeler et al. 1003.5756

The basic idea

- Define non-perturbative renormalization factors by the requirement
  $\frac{1}{12}\mathrm{tr}_{DC}\left(\Gamma_{\mathrm{R}}(p)\Gamma_{\mathrm{Born}}(p)^{-1}\right)\Big|_{p^2=\mu^2} = 1$
- Relate this RI-MOM' scheme to the $\overline{MS}$ scheme in the continuum, i.e. by pQCD.
- Presently we and others develop alternative schemes to test the results

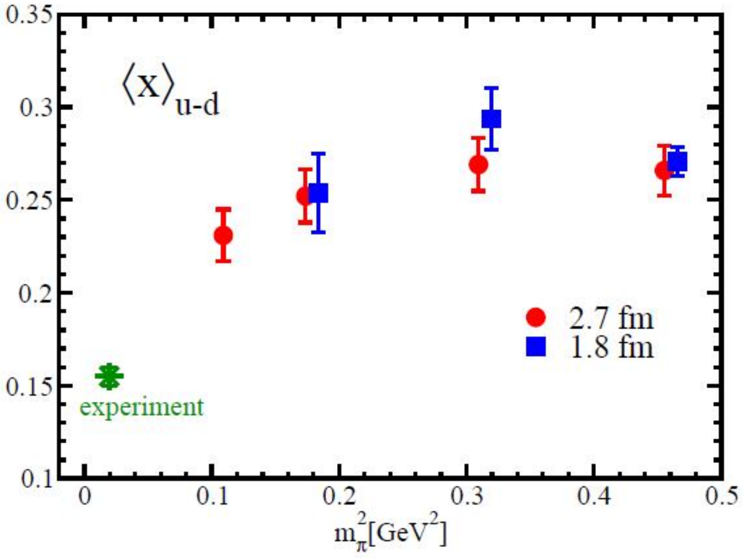
Some observables have the same renormalization factor, e.g. g_A and f_π

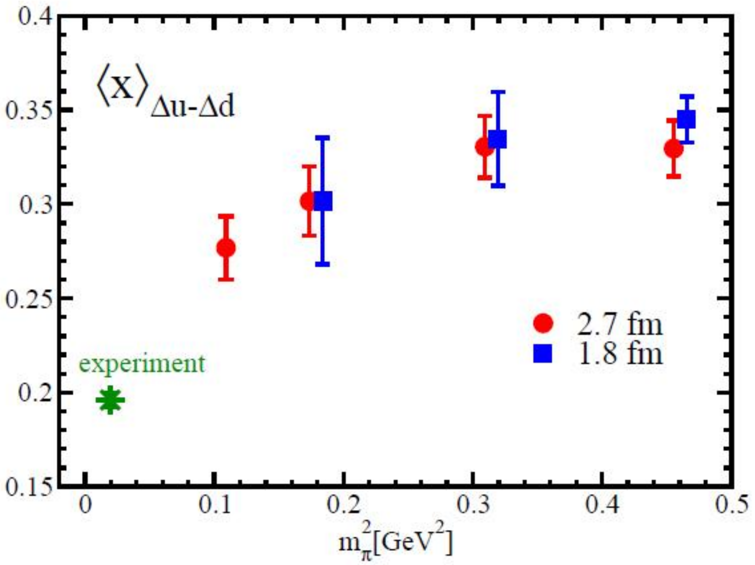




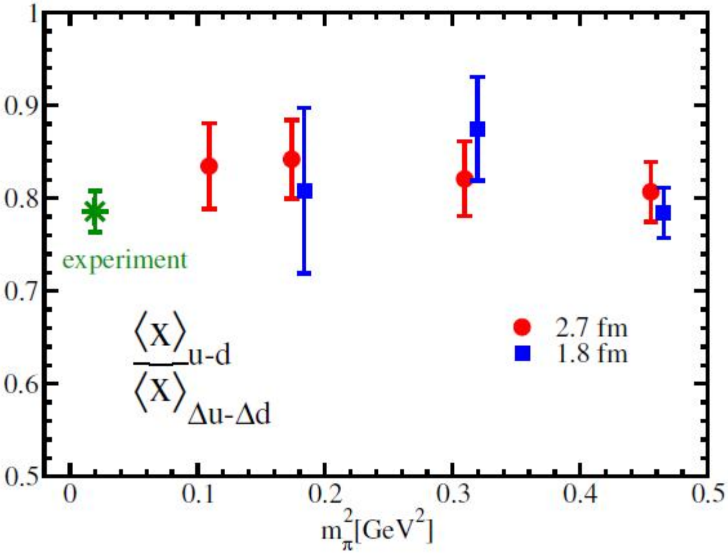
10
8
6
4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
g_A/f_{ps} [GeV^{-1}]
m^2_{PS} [GeV^2]
β=5.20
β=5.25
β=5.29
β=5.40

RBS-UKQCD: Domain wall fermions, $Z_A = Z_V$; arXiv:1003.3387

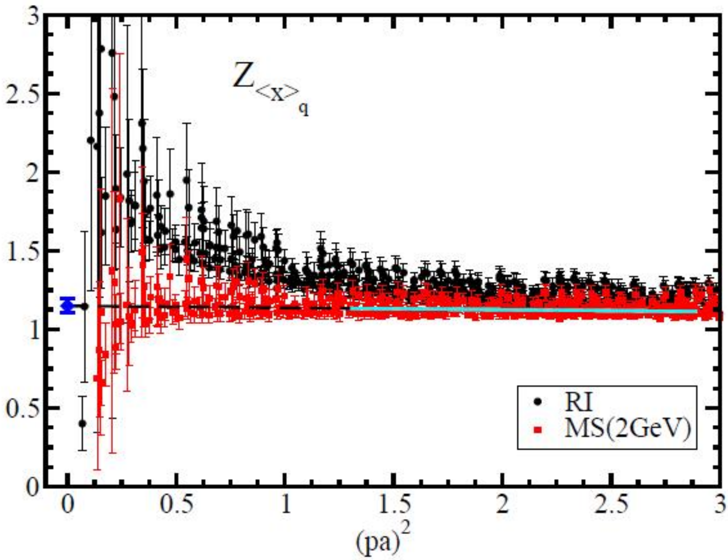




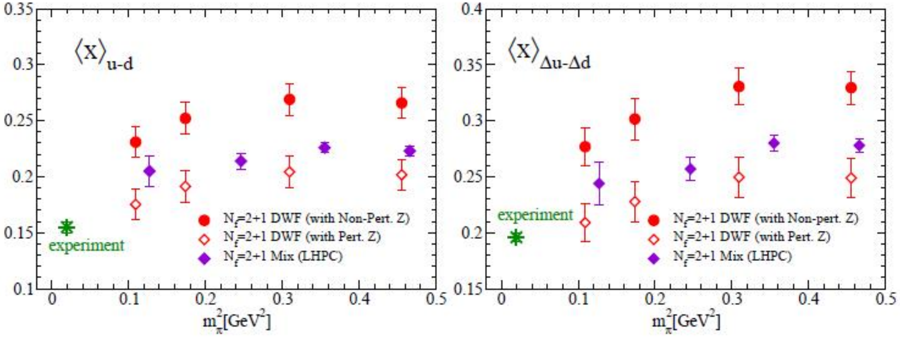
$\langle x \rangle_{\Delta u - \Delta d}$
0.4
0.35
0.3
0.25
0.2
0.15
experiment
2.7 fm
1.8 fm
0
0.1
0.2
0.3
0.4
0.5
$m_\pi^2 [GeV^2]$



1
0.9
0.8
0.7
0.6
0.5
0
0.1
0.2
0.3
0.4
0.5
$m_\pi^2[GeV^2]$
experiment
$\frac{\langle x \rangle_{u-d}}{\langle x \rangle_{\Delta u-\Delta d}}$
2.7 fm
1.8 fm



$Z_{\langle x \rangle_q}$
RI
MS(2GeV)
(pa)^2
0
0.5
1
1.5
2
2.5
3



⟨X⟩u-d
experiment
Nf=2+1 DWF (with Non-Pert. Z)
Nf=2+1 DWF (with Pert. Z)
Nf=2+1 Mix (LHPC)
m²π[GeV²]
⟨X⟩Δu-Δd
experiment
Nf=2+1 DWF (with Non-pert. Z)
Nf=2+1 DWF (with Pert. Z)
Nf=2+1 Mix (LHPC)
m²π[GeV²]

Conclusions

- LQCD+pQCD/OPE is an extremely powerful combination to study hadron structure.
- We have reached a point where many highly detailed questions can be answered.
- However, improving precision stays a hard, up-hill battle.
- One of the problems for phenomenology: Too little experienced pQCD manpower.

many thanks to:

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