# Global theories of nuclear structure

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1. HFB theory and its extensions
2. CEA/DAM survey of ground state properties and even-parity excited states

J-P. Delaroche, M. Girod, J. Libert, H. Goutte, S. Hilaire, S. Peru, N. Pillet, and G.F. Bertsch,
Phys. Rev. C 81 014303 (2010)

Other related work:
D.Vretenar
P.-G. Reinhard
M. Bender, P.-H. Heenen

## From model to theory

Characteristics of good theories

- need only a small set of parameters
- have wide predictive power
- have intrinsic criteria for limits of validity

Goals in this work

- apply theory globally (but with cuts generated by internal criteria)
- quantitative assessment of performance
- for theorists: report weaknesses as well as strengths
- for experimenters: predictions to be tested

## The Hamiltonian

We would like to have an effective Hamiltonian:

$$H = t a^{\dagger} a + v^{(2)} a^{\dagger} a^{\dagger} a a + v^{(3)} a^{\dagger} a^{\dagger} a^{\dagger} a a a$$

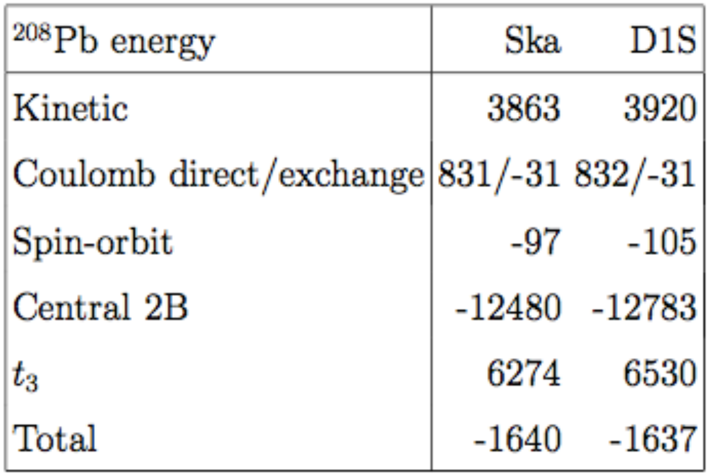
but all we actually have to work with is an energy functional:

$$H = t a^{\dagger} a + v^{(2)}(r - r') a_r^{\dagger} a_{r'}^{\dagger} a_{r'} a_r + t_3 \rho(r)^{1/3} v^{(3)} a_r^{\dagger} a_r^{\dagger} a_r a_r$$

## Self-consistent Mean-Field Theory

See the textbook (Ring and Schuck, 1980).

Example of Pb-208



| $^{208}$Pb energy | Ska | D1S |
|---|---|---|
| Kinetic | 3863 | 3920 |
| Coulomb direct/exchange | 831/-31 | 832/-31 |
| Spin-orbit | -97 | -105 |
| Central 2B | -12480 | -12783 |
| $t_3$ | 6274 | 6530 |
| Total | -1640 | -1637 |

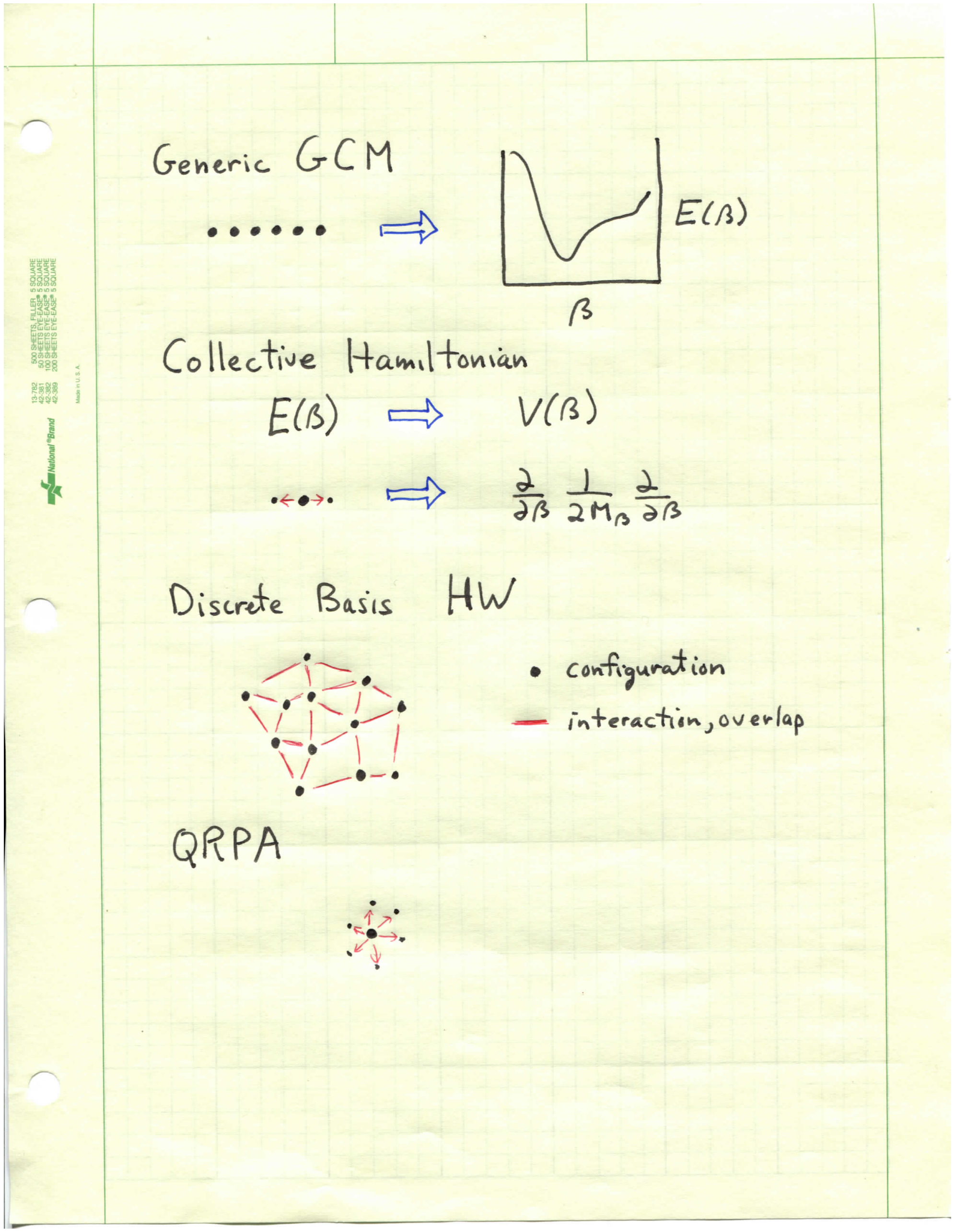
# Extensions of self-consistent mean-field theory for spectroscopy

Generator Coordinate Methods

Collective Hamiltonian
GOA

Discrete-basis Hill-Wheeler

Quasiparticle RPA



Generic GCM

$\Rightarrow$ E(β)

β

Collective Hamiltonian

$E(\beta) \Rightarrow V(\beta)$

$\Rightarrow \frac{\partial}{\partial\beta}\frac{1}{2M_\beta}\frac{\partial}{\partial\beta}$

Discrete Basis HW

● configuration

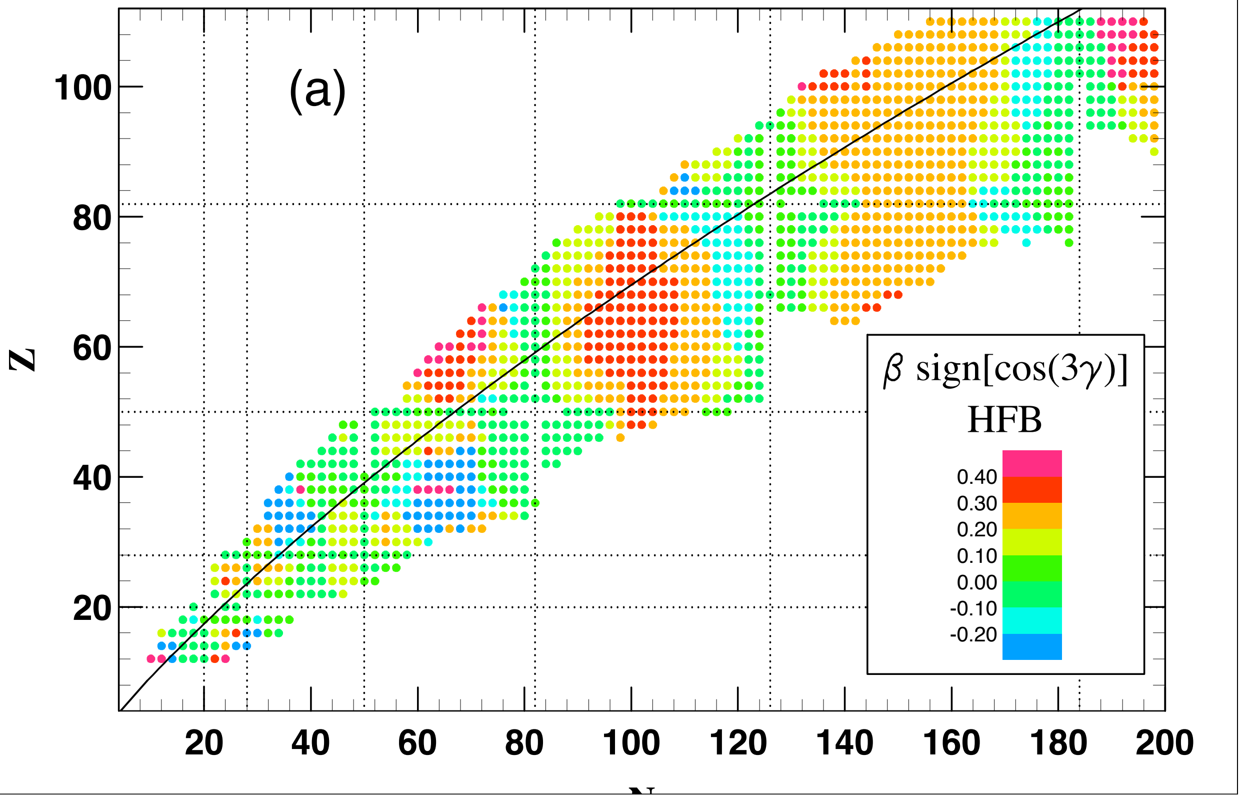
— interaction, overlap

QRPA

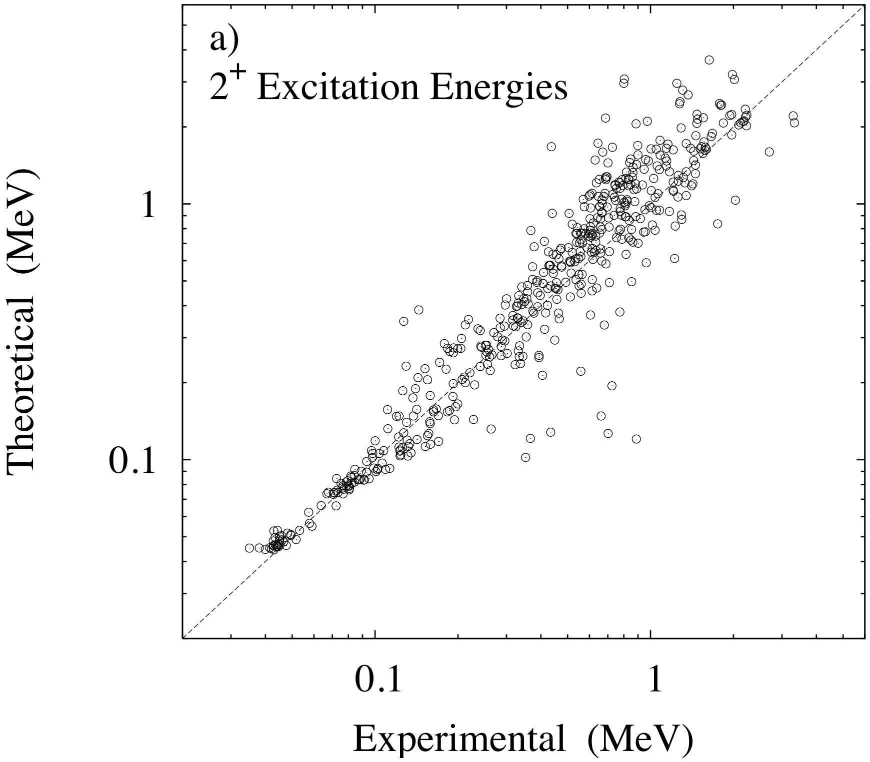
# The CEA/DAM global survey of even-even spectroscopy

Mapped collective Hamiltonian method
Gogny D1S interaction:  CPC 63 (1991), 13 parameters

Computed spectroscopic observables for 1712 nuclei:
- -yrast energies up to J=6
- -excited 0+, first and second yrare J=2
- -B(E2) values for many of the transitions
- -E0 matrix elements
- -deformations, including triaxiality



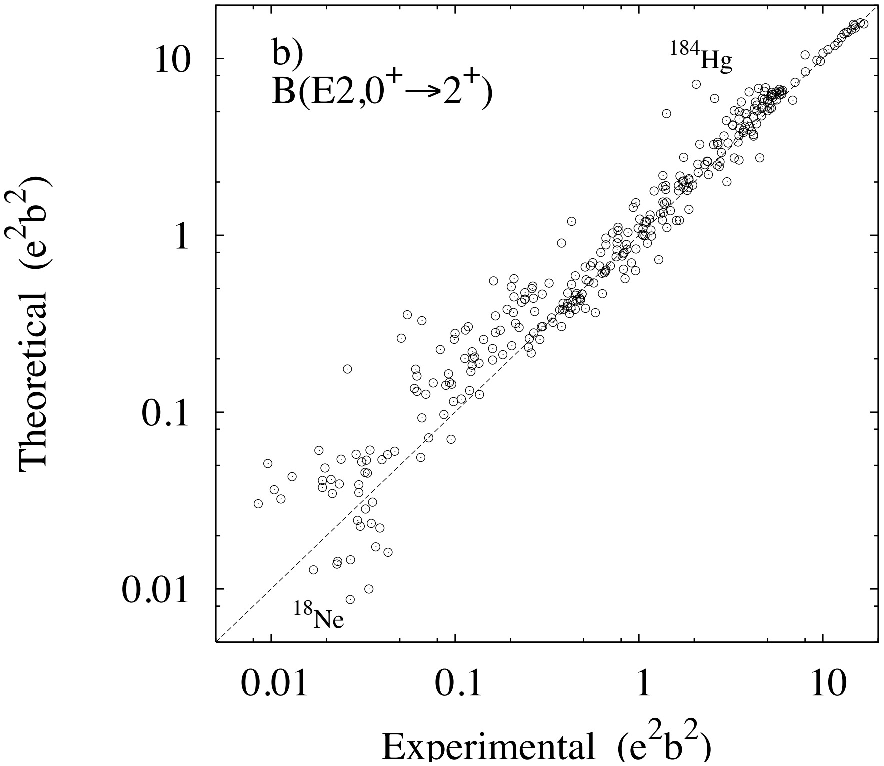
## The first excited 2+ state



$E_{theory} = (1.12 \pm 40\%) \times E_{exp}$ over 2 orders of magnitudes

Dispersion from < log(E_t/E_x)>

# Transition strengths



$$B(E2)_{theory} = (1.20 \pm 45\%) \times B(E2)_{exp}$$

# An indicator of deformation:  R_42

$$R_{42} = \frac{E(4_1^+)}{E(2_1^+)}.$$

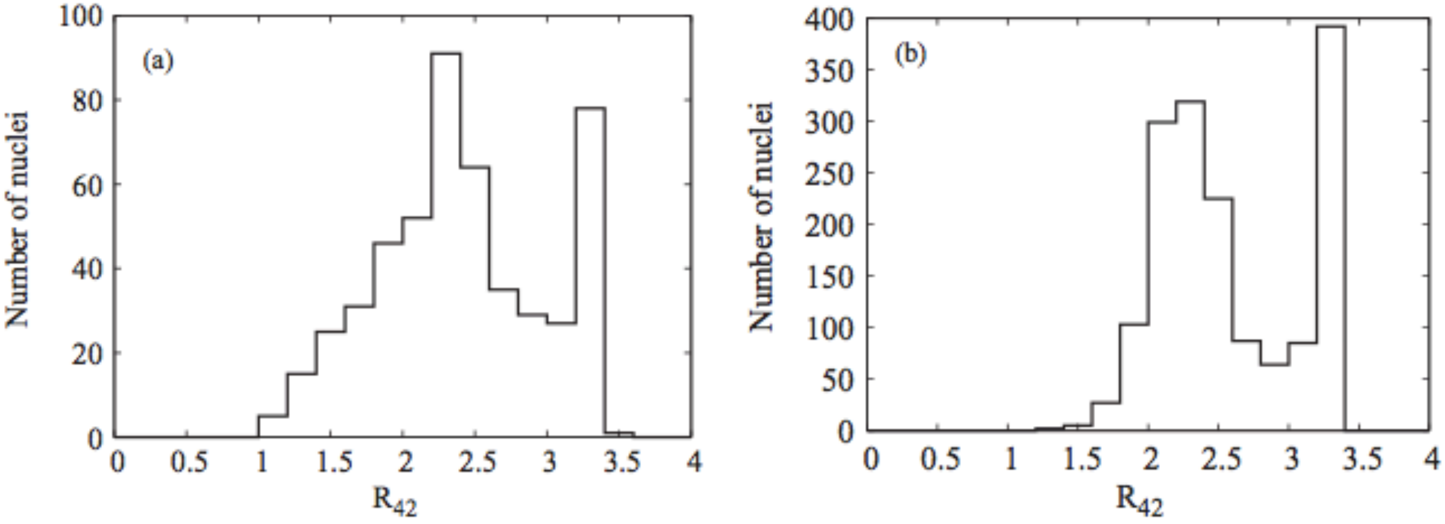


FIG. 12. (a) Histogram of experimental $R_{42}$ ratios, Eq. (26), for 501 even-even nuclei, with data from Ref. [24]. (b) Histogram of calculated $R_{42}$ ratios for 1609 even-even nuclei calculated in the CHFB+5DCH theory.

$$\frac{R_{42}(theory)}{R_{42}(exp.)} = 1.03 \pm 0.15$$

## Predictive power for deformed nuclei

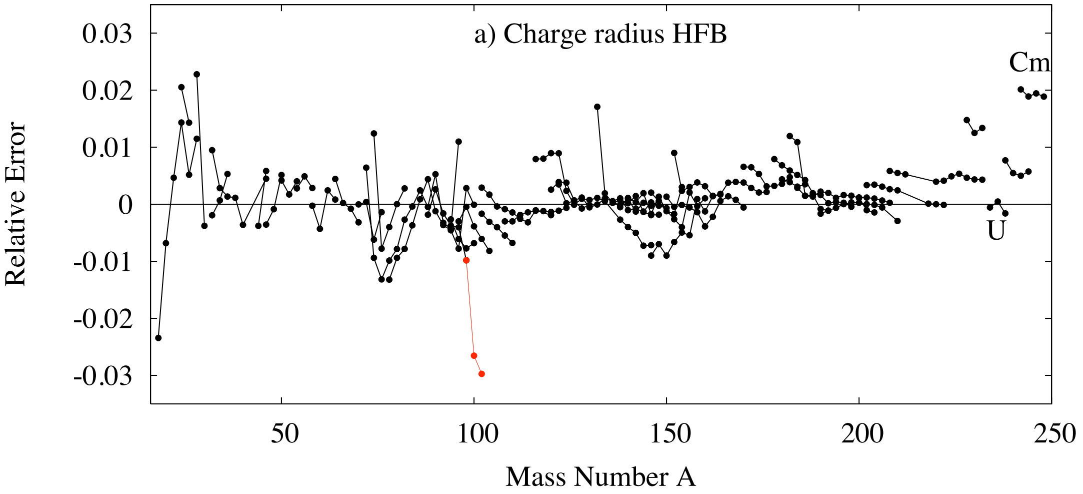
$$E_{theory} = (1.00 \pm 11\%) \times E_{exp}$$ 95 nuclei

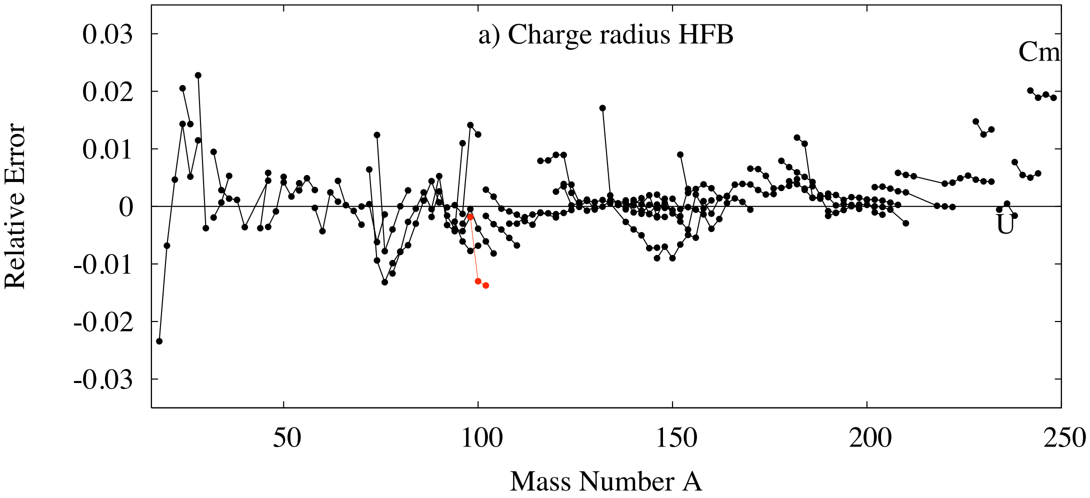
$$B(E2)_{theory} = (1.10 \pm 14\%) \times B(E2)_{exp}$$ 59 nuclei

# Charge radii

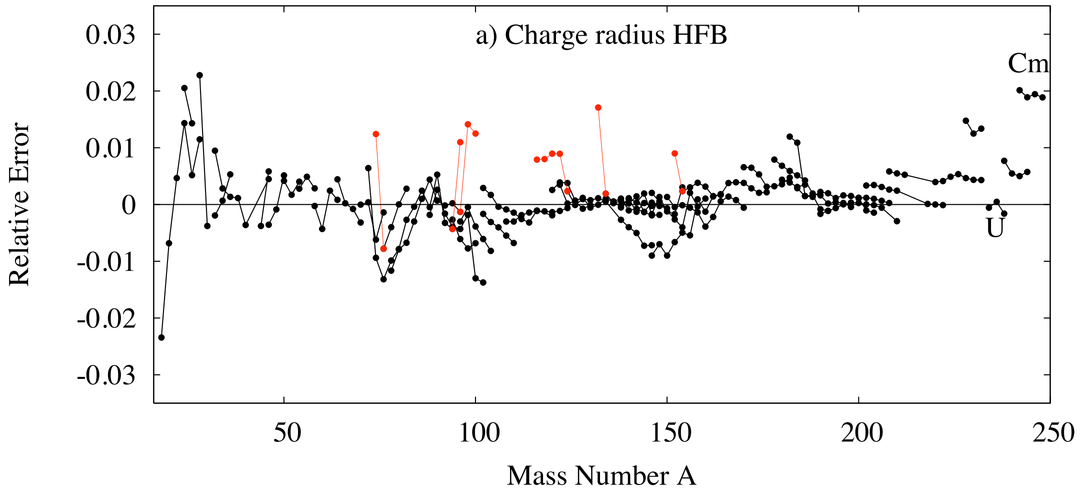
Experimental data from Angeli, ADNDT 87 (2004)

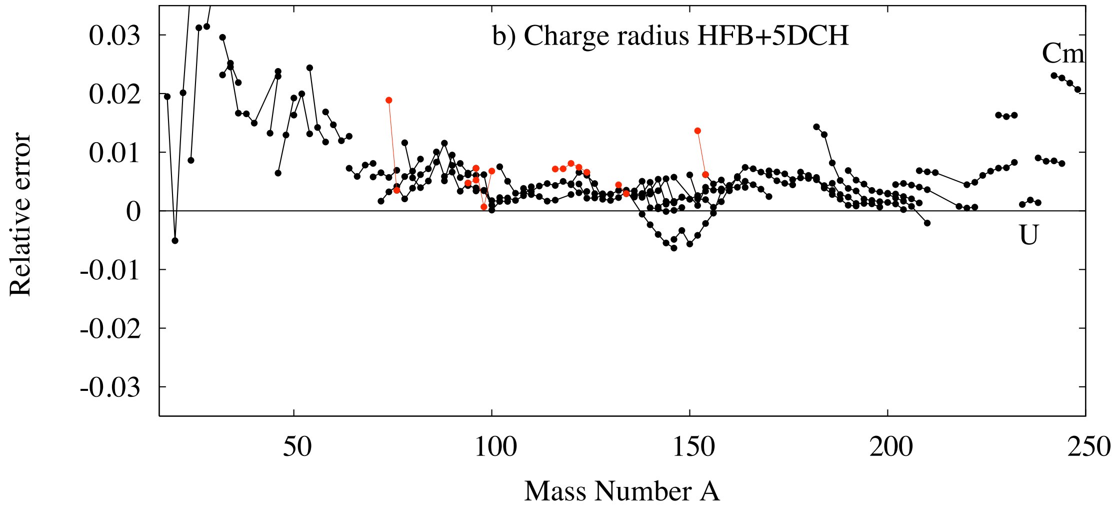
HFB





5DCH


a) Charge radius HFB
Relative Error
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
50
100
150
200
250
Mass Number A
Cm
U


b) Charge radius HFB+5DCH
Relative error
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
50
100
150
200
250
Mass Number A
Cm
U

# Performance on charge radius

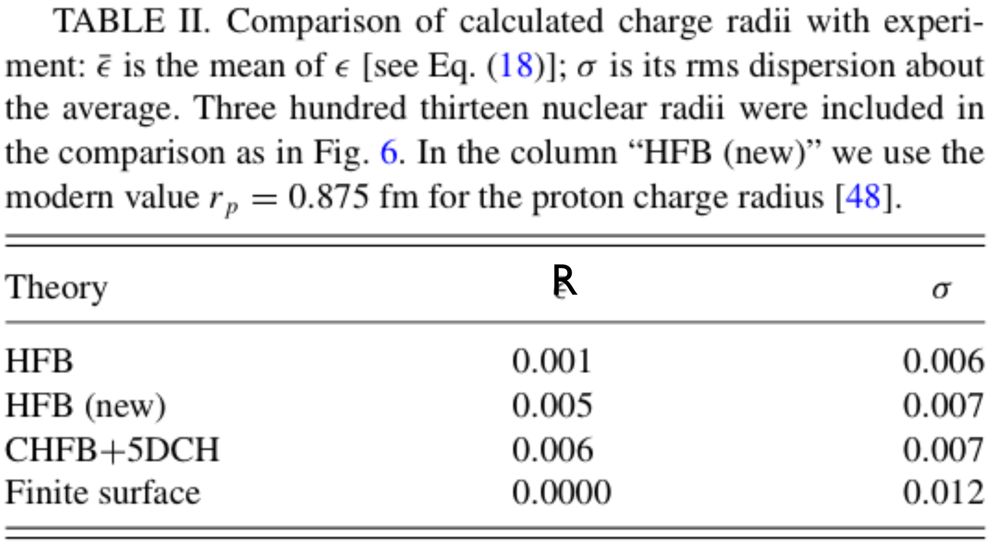


TABLE II. Comparison of calculated charge radii with experiment: $\bar{\epsilon}$ is the mean of $\epsilon$ [see Eq. (18)]; $\sigma$ is its rms dispersion about the average. Three hundred thirteen nuclear radii were included in the comparison as in Fig. 6. In the column "HFB (new)" we use the modern value $r_p = 0.875$ fm for the proton charge radius [48].

| Theory | R | $\sigma$ |
|---|---|---|
| HFB | 0.001 | 0.006 |
| HFB (new) | 0.005 | 0.007 |
| CHFB+5DCH | 0.006 | 0.007 |
| Finite surface | 0.0000 | 0.012 |

“Mutually enhanced magicity”   See Lunney, et al. RMP 75 (2003)

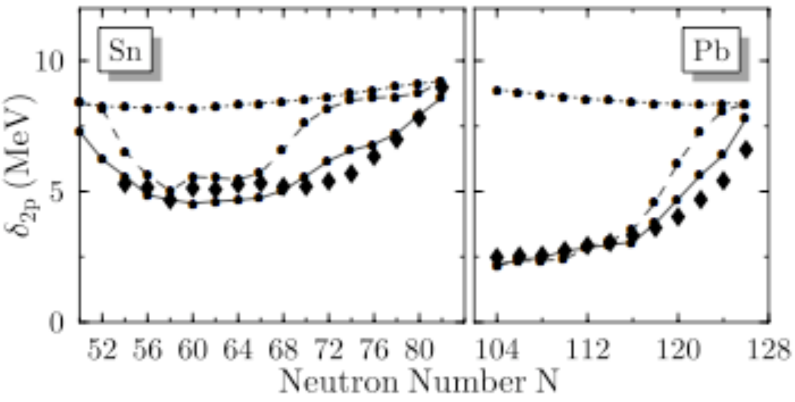
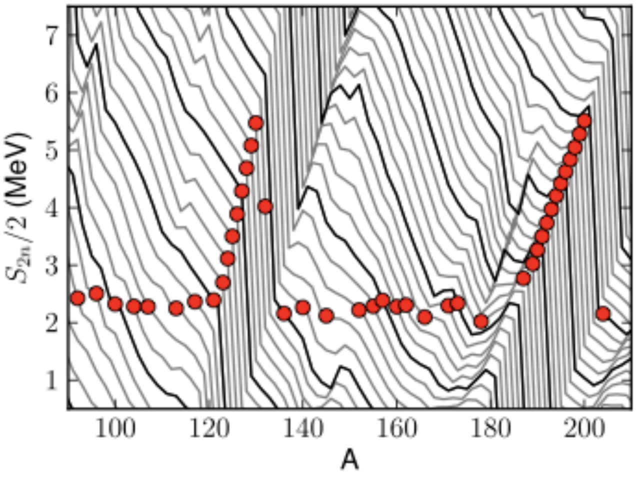


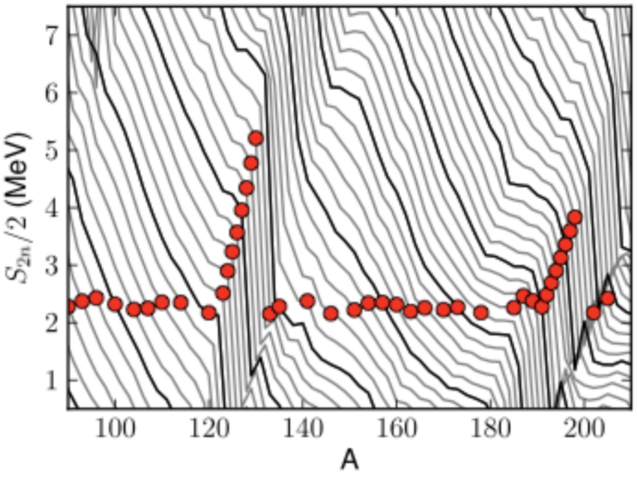
FIG. 2. Two-proton gaps, Eq. (3), for Pb and Sn isotopic chains. Theoretical curves are the following: spherical mean field (short dashed lines); mean field allowing for static deformations (long dashed lines); present theory (solid lines). Experimental values [1] are shown as diamonds.

Bender, et al. PRL 94 (2005)

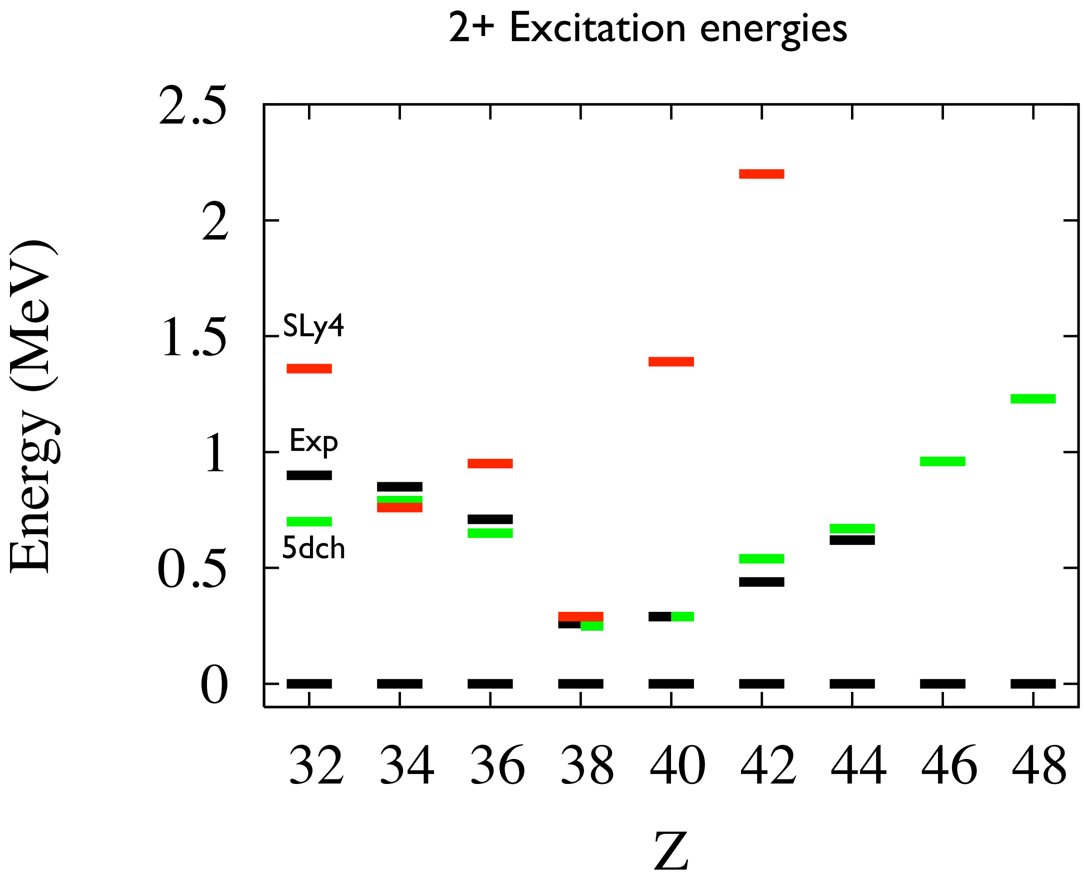
# r-Process nucleosynthesis

Arcones and Martinez-Pinedo, PRC 83

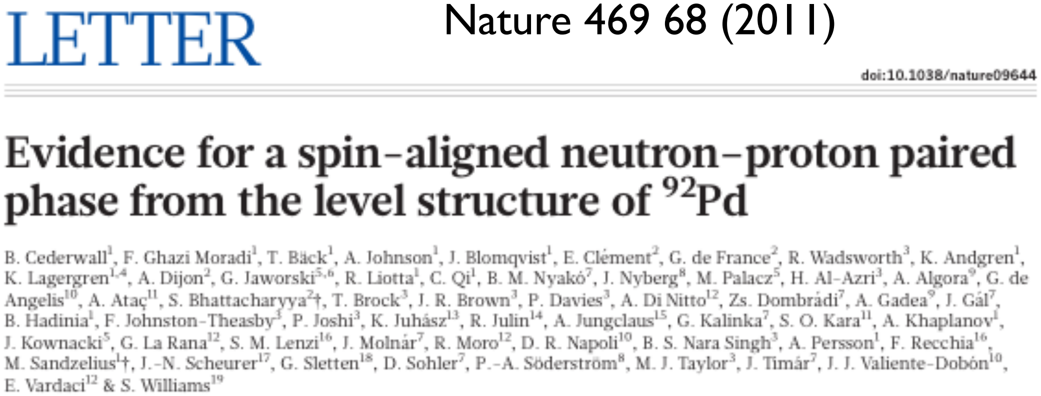




# The CEA/DAM global survey, on the N=Z line



Nature 469 68 (2011)

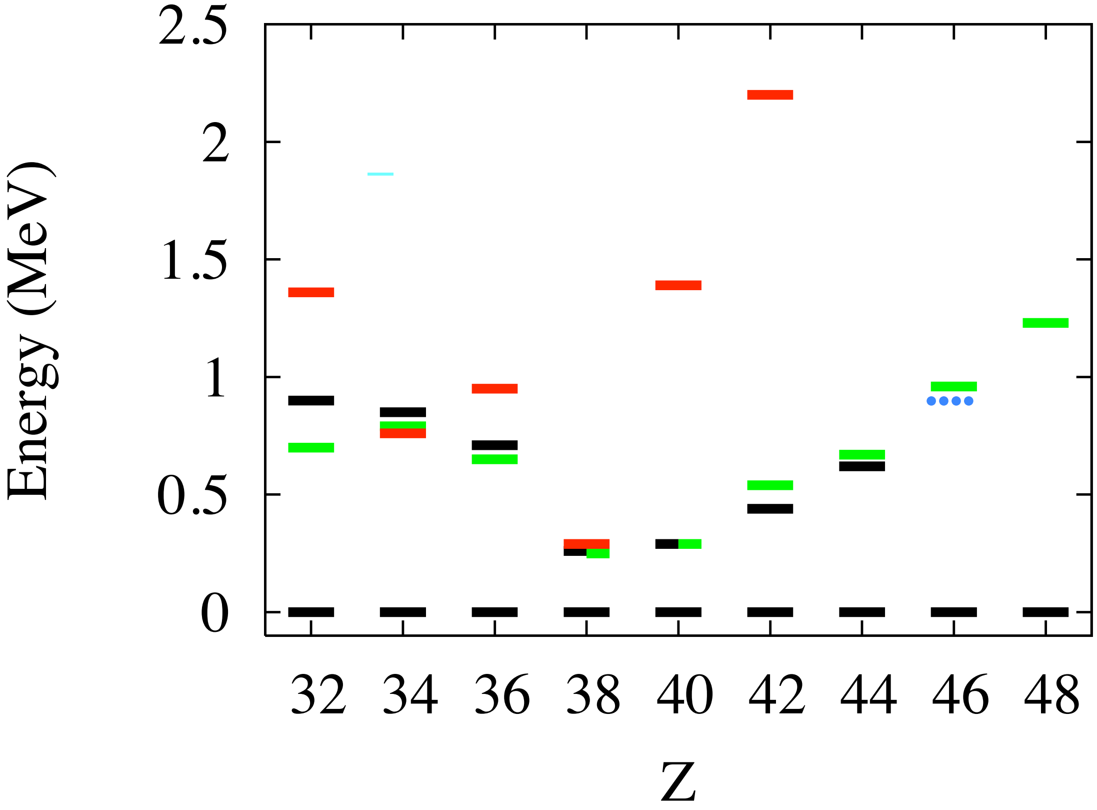


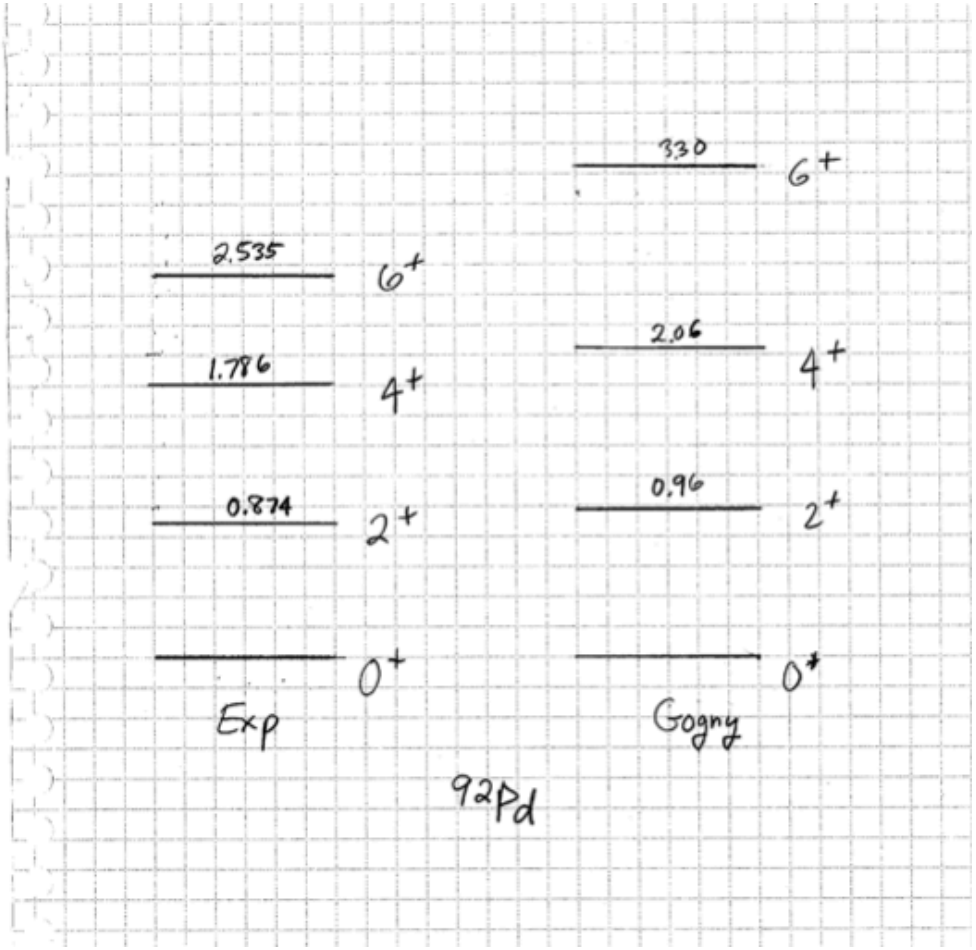
LETTER

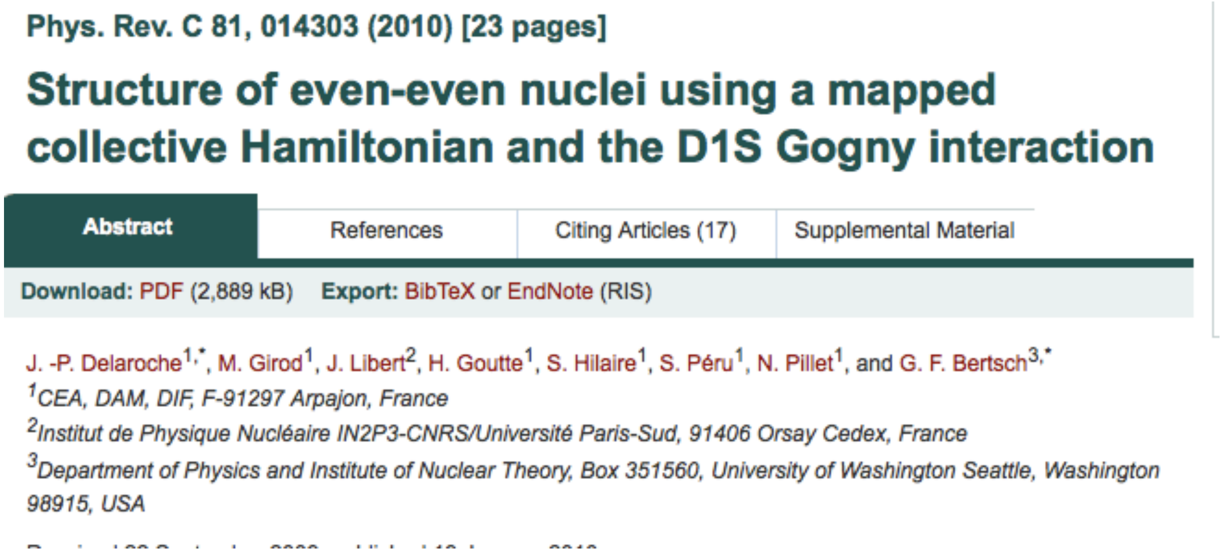
doi:10.1038/nature09644

# Evidence for a spin-aligned neutron–proton paired phase from the level structure of $^{92}$Pd

B. Cederwall[1], F. Ghazi Moradi[1], T. Bäck[1], A. Johnson[1], J. Blomqvist[1], E. Clément[2], G. de France[2], R. Wadsworth[3], K. Andgren[1], K. Lagergren[1,4], A. Dijon[2], G. Jaworski[5,6], R. Liotta[1], C. Qi[1], B. M. Nyakó[7], J. Nyberg[8], M. Palacz[5], H. Al-Azri[3], A. Algora[9], G. de Angelis[10], A. Ataç[11], S. Bhattacharyya[2]†, T. Brock[3], J. R. Brown[3], P. Davies[3], A. Di Nitto[12], Zs. Dombrádi[7], A. Gadea[9], J. Gál[7], B. Hadinia[1], F. Johnston-Theasby[3], P. Joshi[3], K. Juhász[13], R. Julin[14], A. Jungclaus[15], G. Kalinka[7], S. O. Kara[11], A. Khaplanov[1], J. Kownacki[5], G. La Rana[12], S. M. Lenzi[16], J. Molnár[7], R. Moro[12], D. R. Napoli[10], B. S. Nara Singh[3], A. Persson[1], F. Recchia[16], M. Sandzelius[1]†, J.-N. Scheurer[17], G. Sletten[18], D. Sohler[7], P.-A. Söderström[8], M. J. Taylor[3], J. Timár[7], J. J. Valiente-Dobón[10], E. Vardaci[12] & S. Williams[19]




3.30
6+
2.535
6+
2.06
4+
1.786
4+
0.96
2+
0.874
2+
0+
0+
Exp
Gogny
92Pd



**Phys. Rev. C 81, 014303 (2010) [23 pages]**

## Structure of even-even nuclei using a mapped collective Hamiltonian and the D1S Gogny interaction

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J. -P. Delaroche[1,*], M. Girod[1], J. Libert[2], H. Goutte[1], S. Hilaire[1], S. Péru[1], N. Pillet[1], and G. F. Bertsch[3,*]

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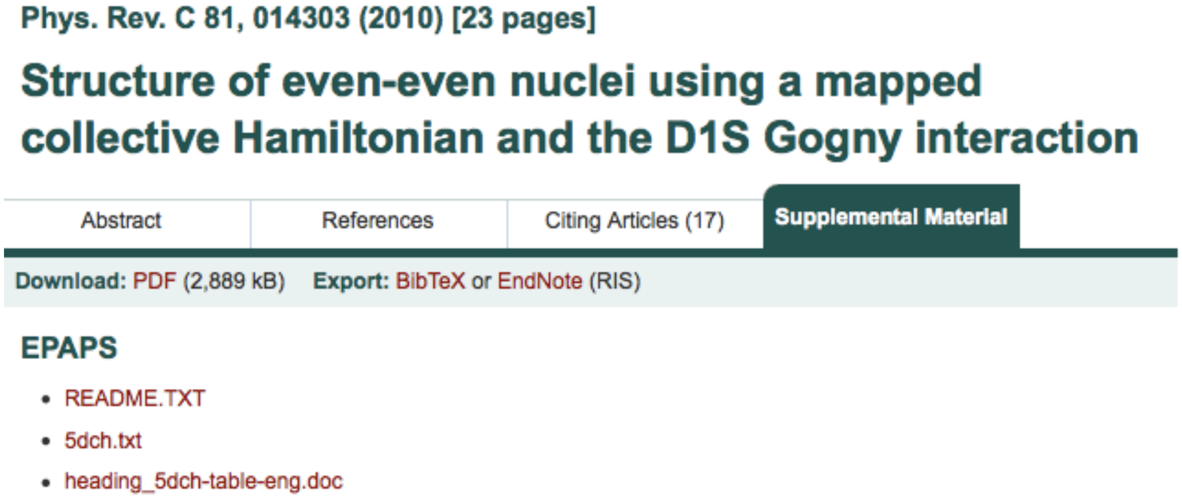

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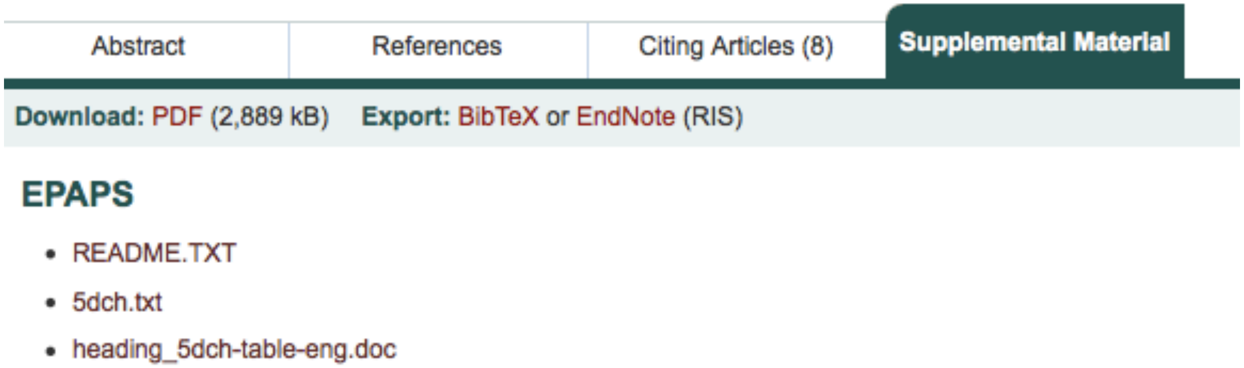
### EPAPS

- README.TXT
- 5dch.txt
- heading_5dch-table-eng.doc

# The CEA/DAM global survey (HFB/GCM/5DCH)

PRC 81, 014303 (2010)



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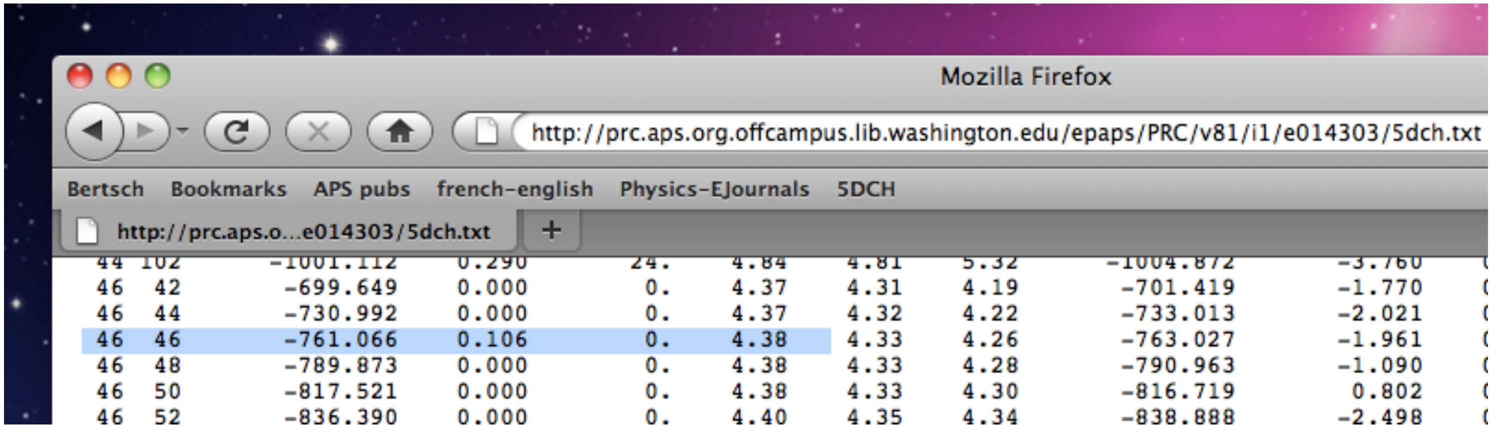


```
 44 102     -1001.112   0.290      24.     4.84    4.81    5.32     -1004.872      -3.760
 46  42      -699.649   0.000       0.     4.37    4.31    4.19      -701.419      -1.770
 46  44      -730.992   0.000       0.     4.37    4.32    4.22      -733.013      -2.021
 46  46      -761.066   0.106       0.     4.38    4.33    4.26      -763.027      -1.961
 46  48      -789.873   0.000       0.     4.38    4.33    4.28      -790.963      -1.090
 46  50      -817.521   0.000       0.     4.38    4.33    4.30      -816.719       0.802
 46  52      -836.390   0.000       0.     4.40    4.35    4.34      -838.888      -2.498
```

# How to do GCM

$\hat{H}$ = the many-body Hamiltonian, usually approximated by an EDF.

$\hat{Q}_i$ = a set of one-body operators

I) Minimize $\langle\psi_\lambda|\,\hat{H}-\sum_i\lambda_i\hat{Q}_i\,|\psi_\lambda\rangle$ to find $\psi_\lambda$

II find expectation values $q_i = \langle\psi_\lambda|\hat{Q}_i|\psi_\lambda\rangle$

$$V(q) \equiv V(\lambda(q)) = \langle\psi_\lambda|\,\hat{H}\,|\psi_\lambda\rangle$$

This is the potential energy surface.

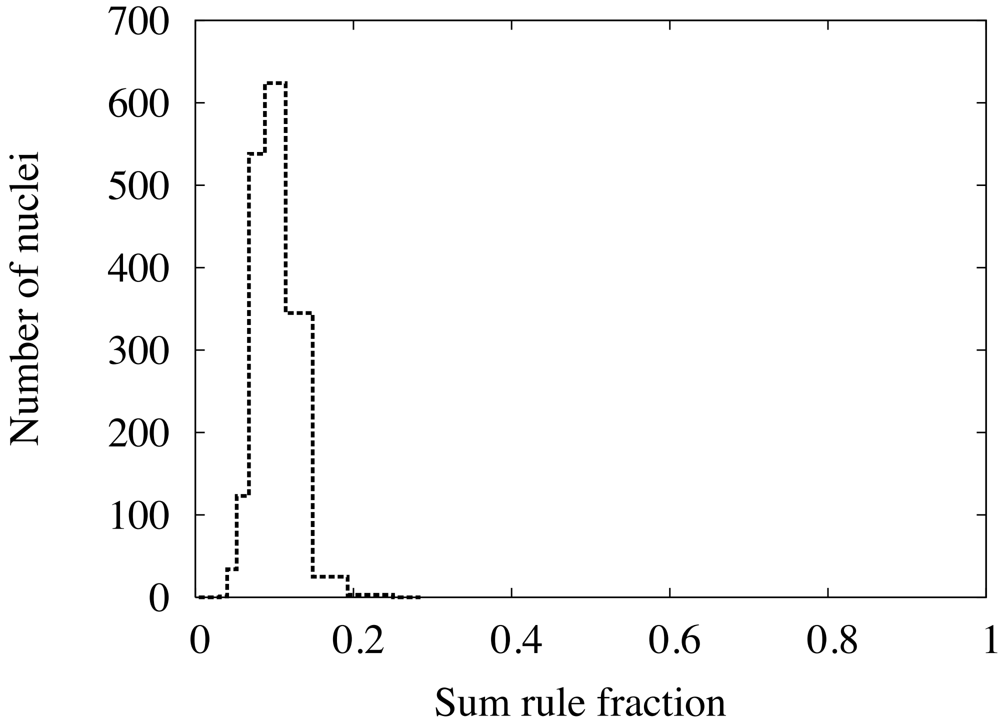
5DCH work: $\hat{N}, \hat{Z}, r^2Y_{20}, r^2Y_{2\pm2}, \hat{J}_x$

octupole study: $\hat{N}, \hat{Z}, r^2Y_{20}, r^3Y_{30}$

## Sum rule

$$S = \sum_i E(2_i^+) B(E2; 0_1^+ \to 2_i^+) = \frac{25}{4\pi} (\frac{\hbar^2}{m}) Z^2 \langle r^2 \rangle$$

The fraction of the sum rule in the lowest excitation is ~10%.



# Separation Energies

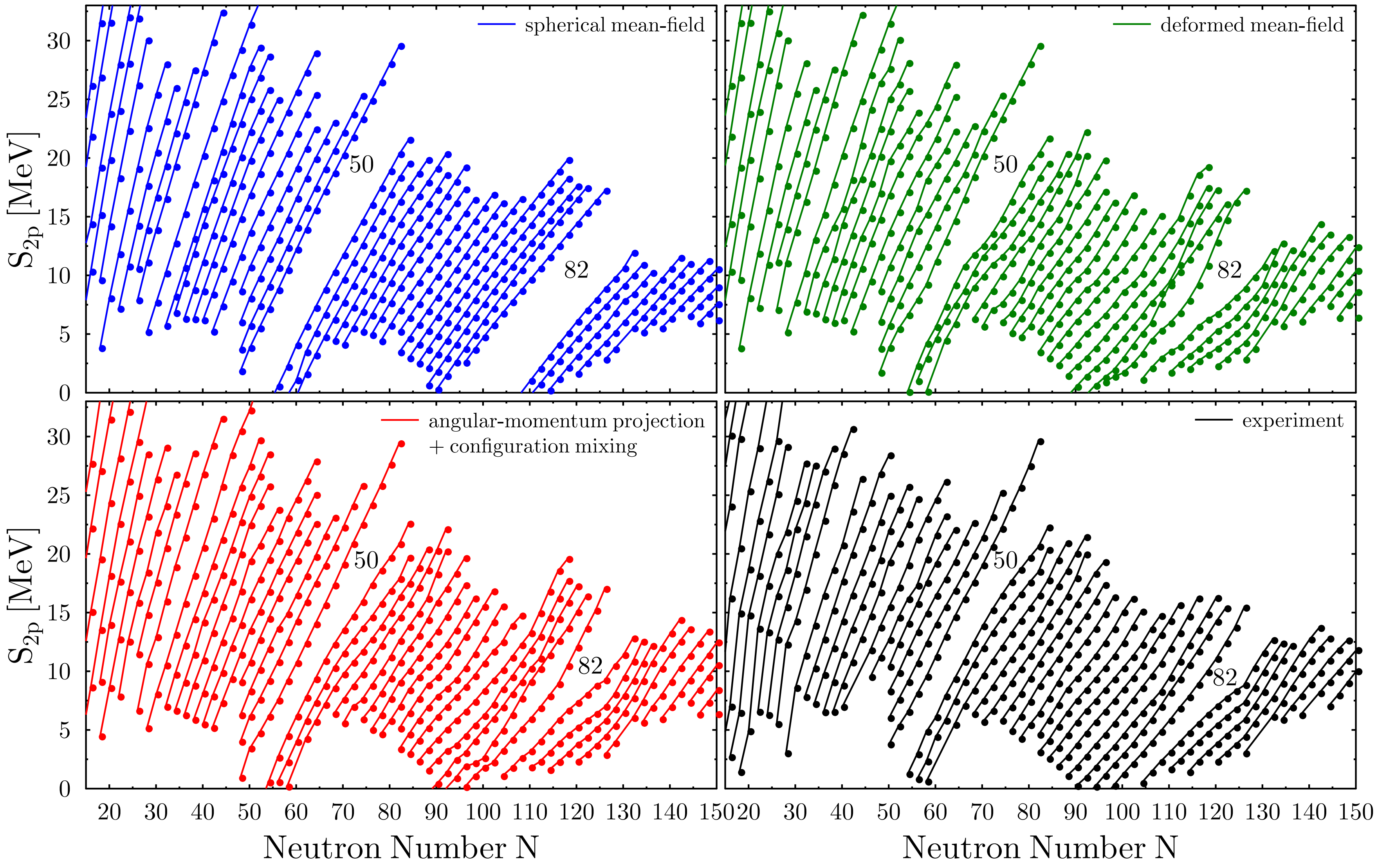


FIG. 2: Two-proton separation energies for isotonic chains.