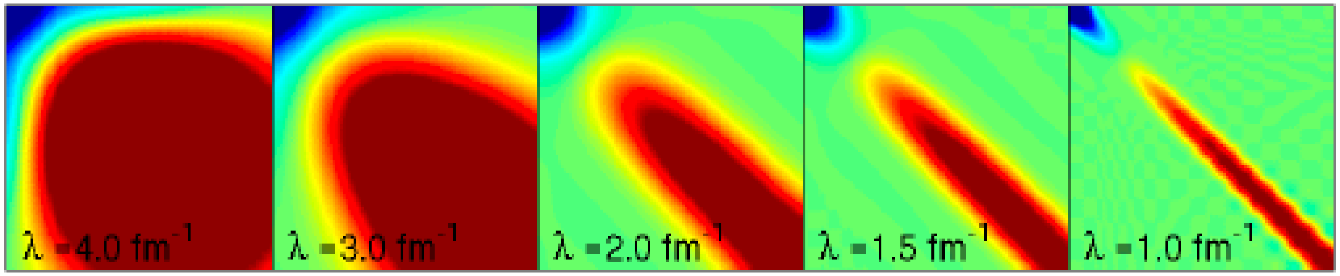

In-medium similarity renormalization group for nuclei and nuclear matter

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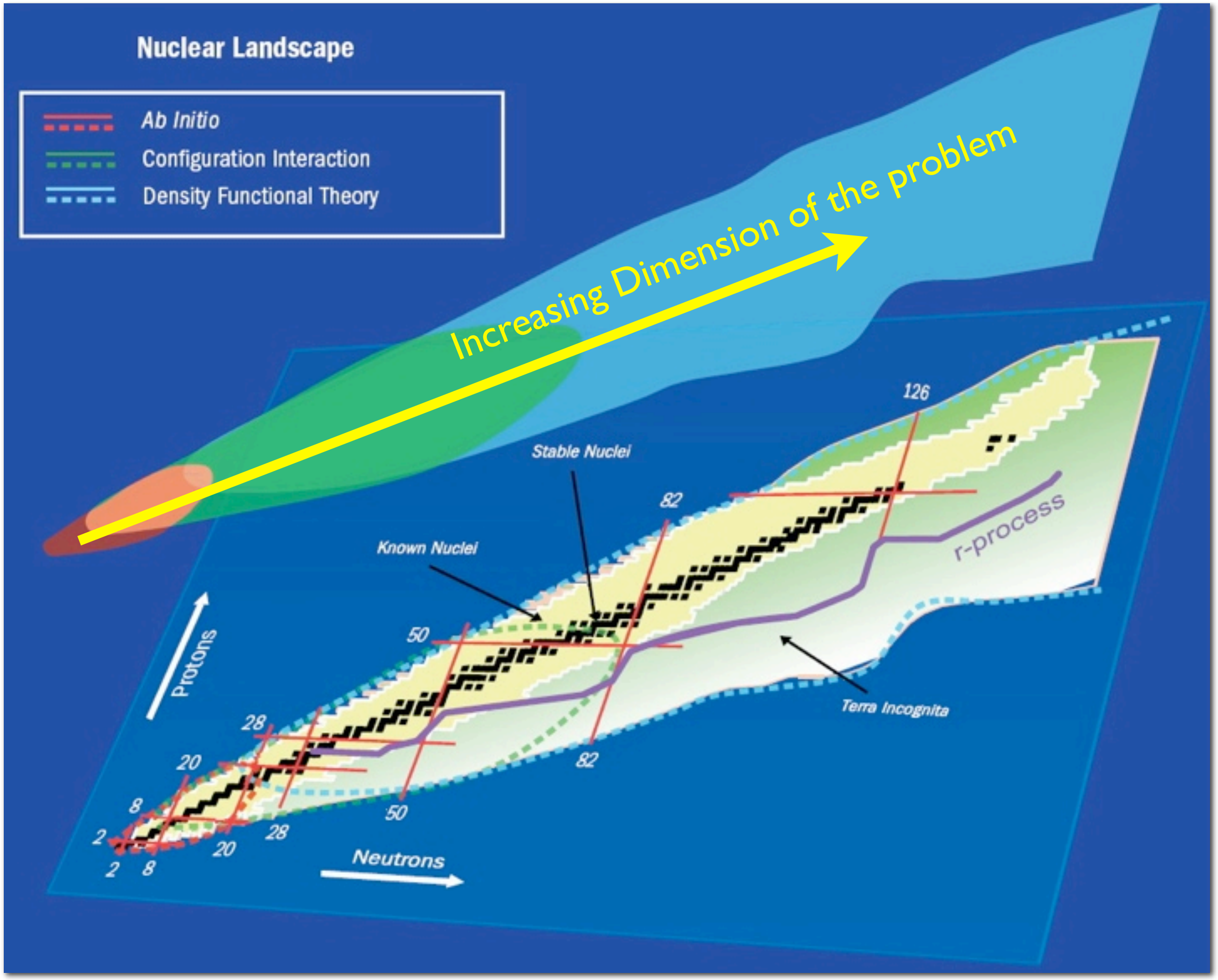
NSCL

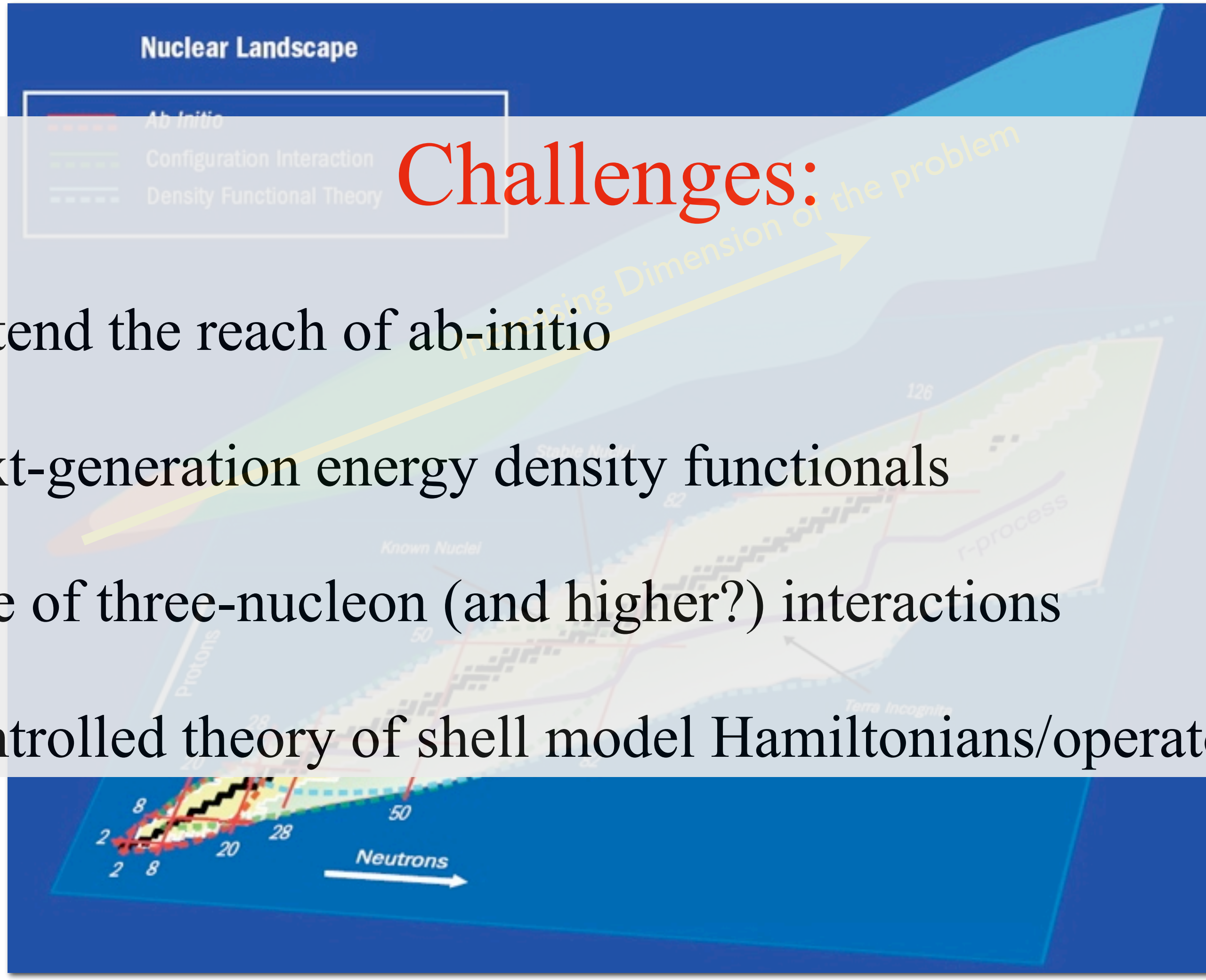
*In collaboration with K.Tsukiyama (Tokyo) and A. Schwenk (Darmstadt)

The nuclear many-body landscape



Calculate the properties of thousands of strongly-interacting nuclei rooted in the underlying QCD

The nuclear many-body landscape



Challenges:

1) Extend the reach of ab-initio

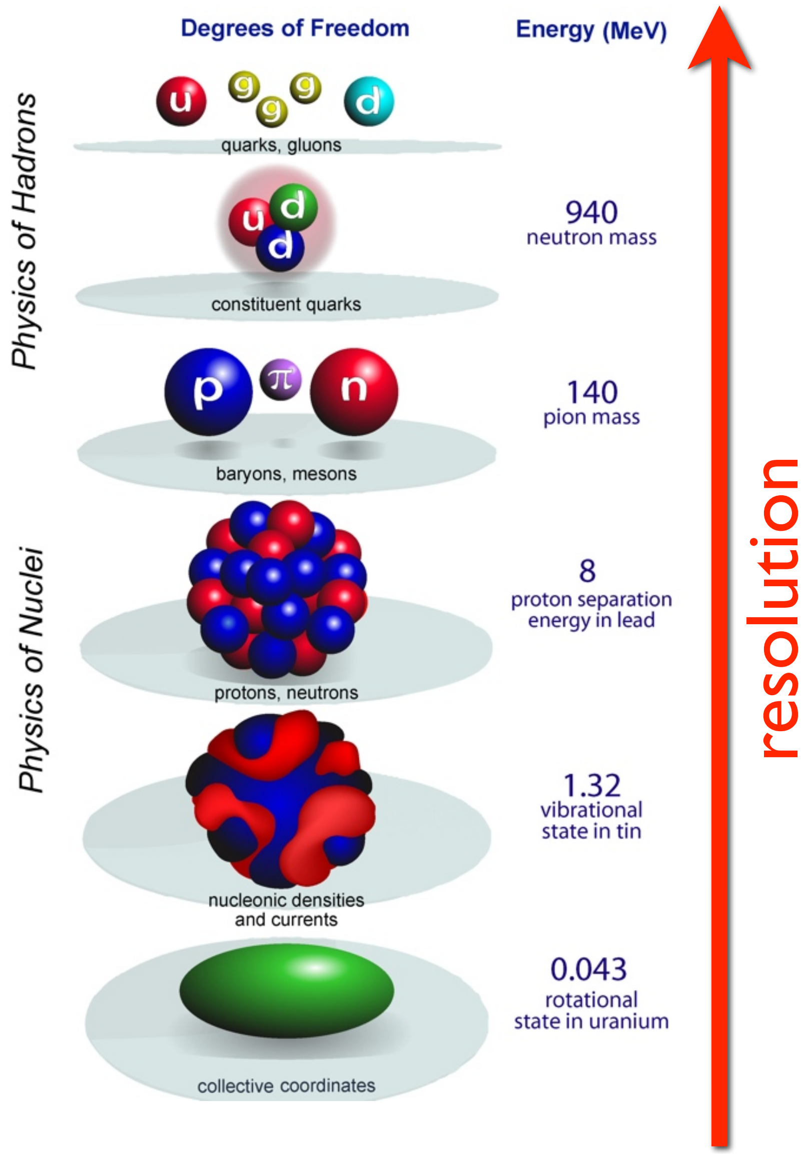
2) next-generation energy density functionals

3) role of three-nucleon (and higher?) interactions

4) controlled theory of shell model Hamiltonians/operators

Calculate the properties of thousands of strongly-interacting nuclei rooted in the underlying QCD

Multiple Scales in Nuclear Physics



Old View

- Multiple scales complicate life
- No easy way to connect them

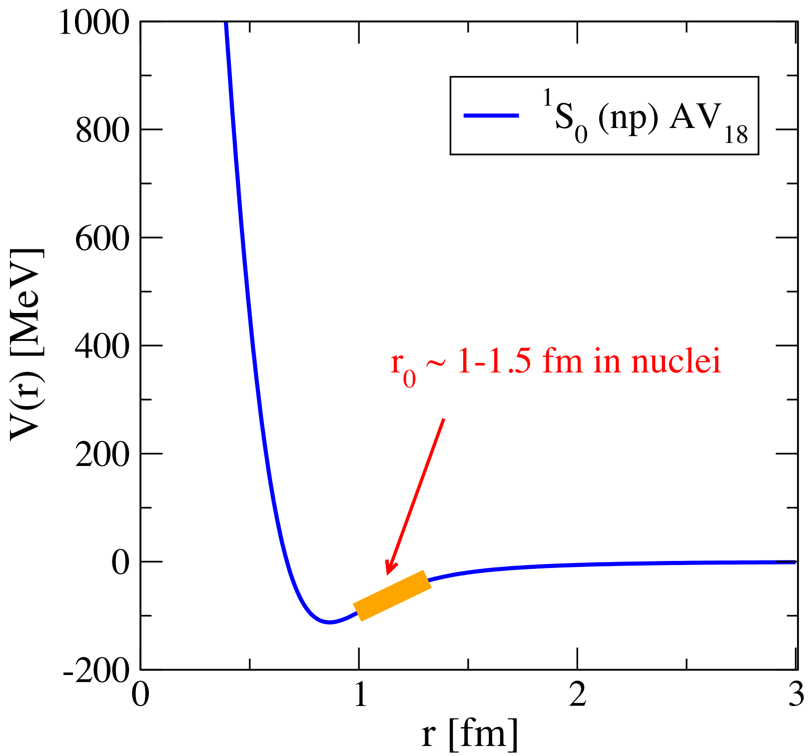
Modern View

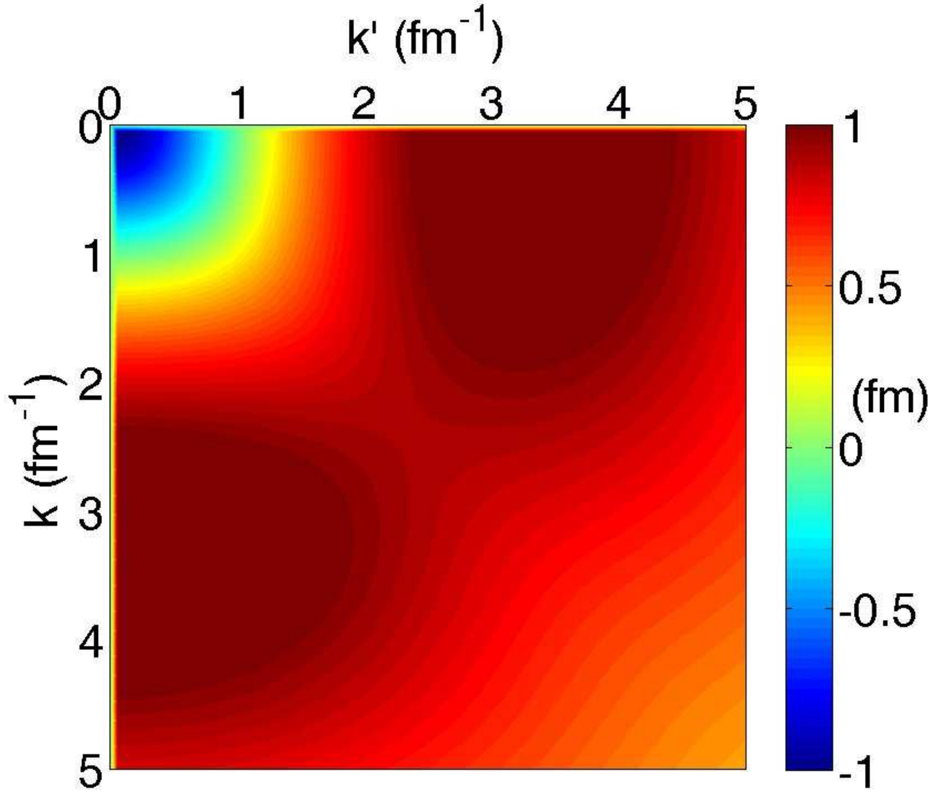
- Ratio of scales => small parameters!
- Effective theories at each scale connected by renormalization group

$$V(\Lambda) = V_{2N}(\Lambda) + V_{3N}(\Lambda) + \cdots$$

Use RG to pick a convenient Λ

Scale-dependent sources of non-perturbative physics





Repulsive core & strong tensor force => low and high k modes strongly coupled by the interaction

Complications: strong correlations, non-perturbative, poorly convergent basis expansions, ...

Example: Why large Λ's are painful

Assume we want to compute the binding energy of a nucleus with mass number A in a wave function based approach. Assume that the interaction has a momentum cutoff Λ.

Q: What are the minimum requirements for the model space?

A:

1. The basis must be sufficiently extended in position space to capture a nucleus with radius $R \approx 1.2\, A^{1/3}$ fm
2. The basis must be sufficiently extended in momentum space to capture the cutoff Λ.
3. THUS: we need approximately $K=(R\Lambda/(2\pi))^3$ single-particle states (phase space volume!) In practice $K \approx (R\Lambda/2)^3 \sim \Lambda^3 A$.

Computation of oxygen: Λ=4/fm and $R \approx 2.5$fm
Thus, our model space has about $K=5^3 = 125$ single-particle states.
Matrix dimension: $D=K!/(K-A)!/A! \approx (K/A)^A \approx 8^{16} \approx 2^{48} \approx 10^{14}$.

Thus, the matrix dimension is $D=K!/((K-A)!\ A!)$, with $K \approx (R\Lambda/2)^3$

Example: Why large Λ's are painful

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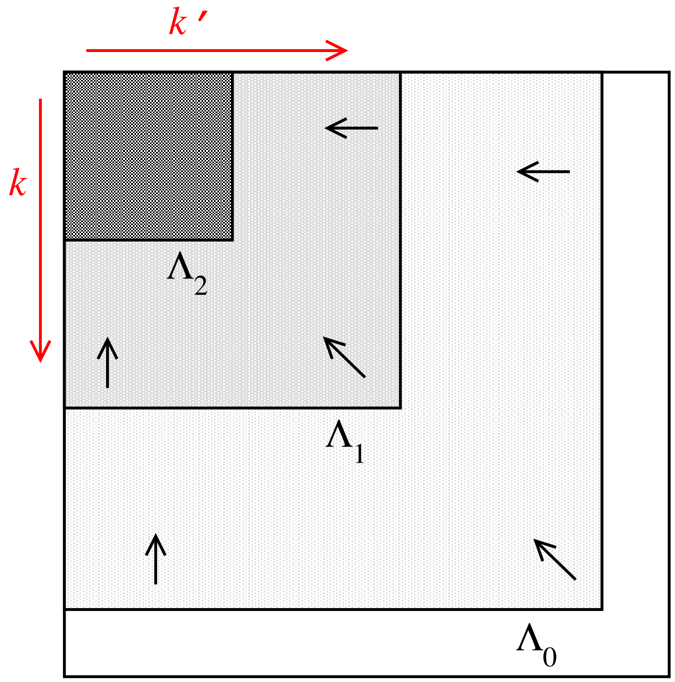
Easiest way to extend the reach of ab-initio to heavier nuclei
is to use lower resolutions (Λ)

"physical" scales k_F and $m_\pi \sim 1\ \mathrm{fm}^{-1}$ ...

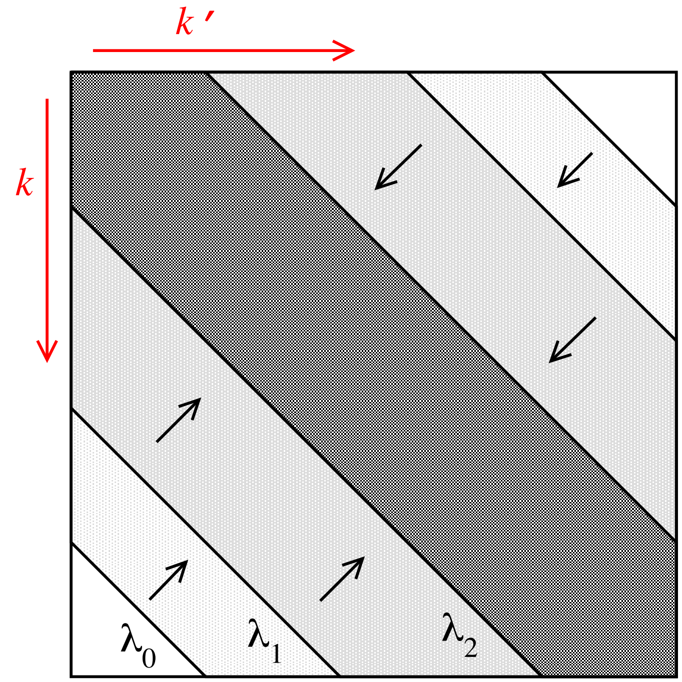
2 Types of Renormalization Group Transformations



“$V_{\text{low k}}$”

integrate-out high k states

preserves observables for $k < \Lambda$



“Similarity RG”

eliminate far off-diagonal coupling

preserves “all” observables

Very similar consequences despite differences in appearance
(low and high momentum decoupled)

The Similarity Renormalization Group

Wegner, Glazek and Wilson

Unitary transformation via flow equations:

$$\frac{dH_\lambda}{d\lambda} = [\eta(\lambda), H_\lambda] \qquad \text{with} \qquad \eta(\lambda) \equiv \frac{dU(\lambda)}{d\lambda} U^\dagger(\lambda)$$

Engineer η to do different things as λ => 0

$$\lambda \equiv s^{-1/4}$$

$$\eta(\lambda) = [\mathcal{G}_\lambda, H_\lambda]$$

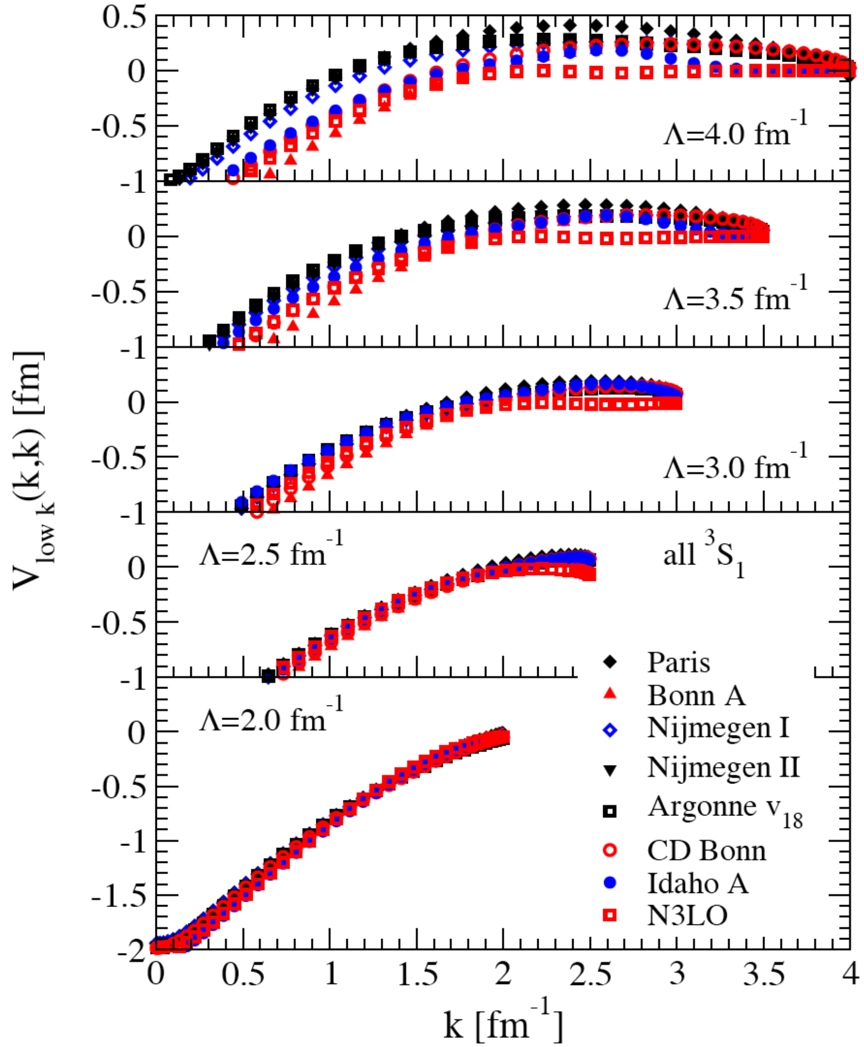
$$\mathcal{G}_\lambda = T \;\Rightarrow\; H_\lambda \text{ driven towards diagonal in k} - \text{space}$$

$$\mathcal{G}_\lambda = PH_\lambda P + QH_\lambda Q \;\Rightarrow\; H_\lambda \text{ driven to block} - \text{diagonal}$$

$$\vdots$$

Good things happen at lower resolution scales!

Example: universal low-momentum nuclear Hamiltonian

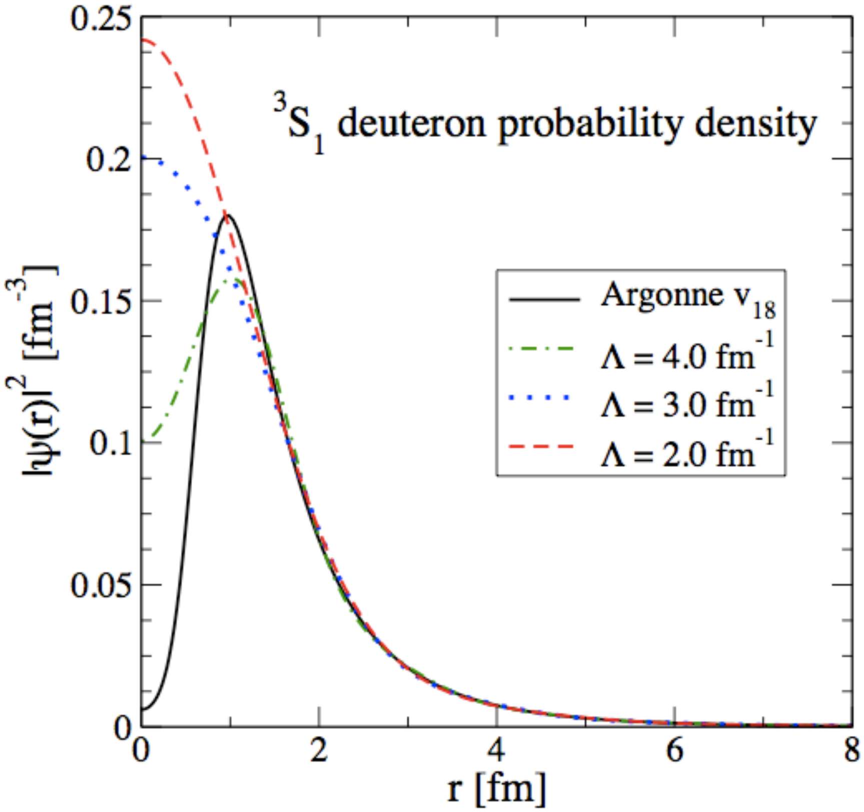


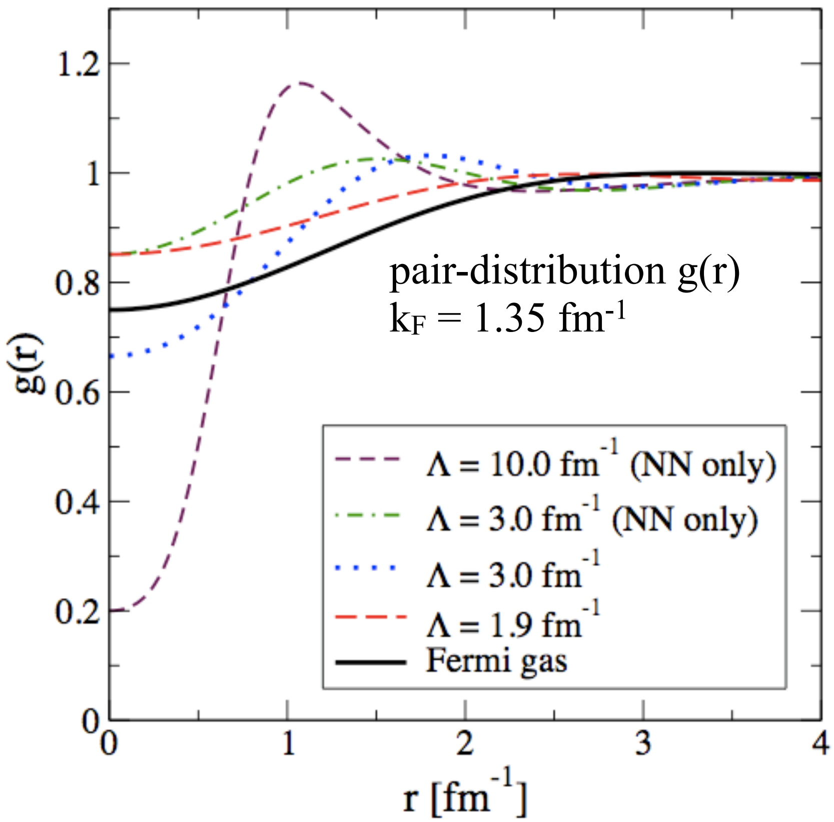
All nuclear force models have the same long-distance structure...

...but are totally model-dependent at short-distances

Under RG evolution, the different models "collapse" to a universal curve

Simplifications from lowering Λ

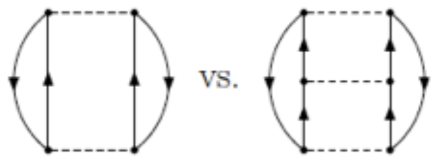




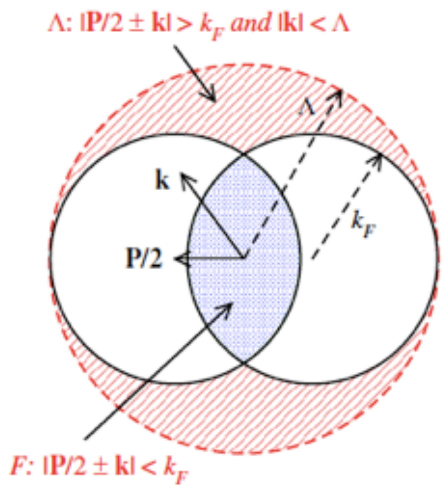
correlations in wave functions blurred out
more effective variational calcs.,
efficient basis expansions,
more perturbative...

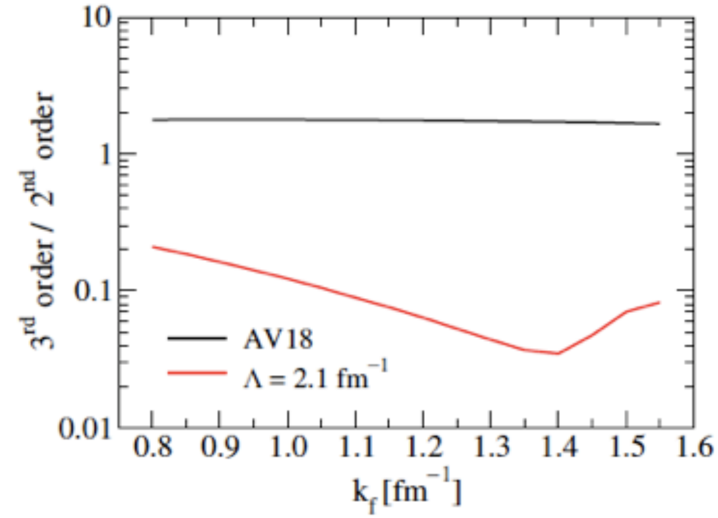
Lowering Λ increases "perturbativeness"

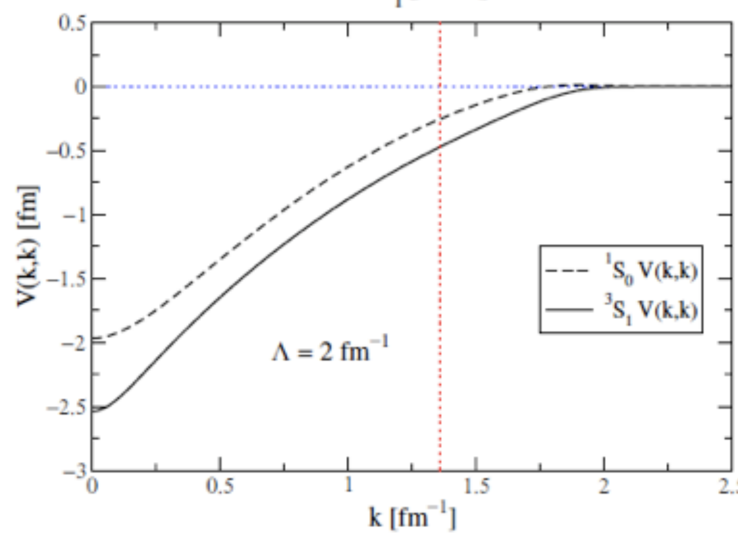
Strong short-range repulsion
$\Longrightarrow$ Sum V ladders $\Longrightarrow$ G



$V_{\mathrm{low}\,k}$ momentum
dependence + phase space
$\Longrightarrow$ perturbative

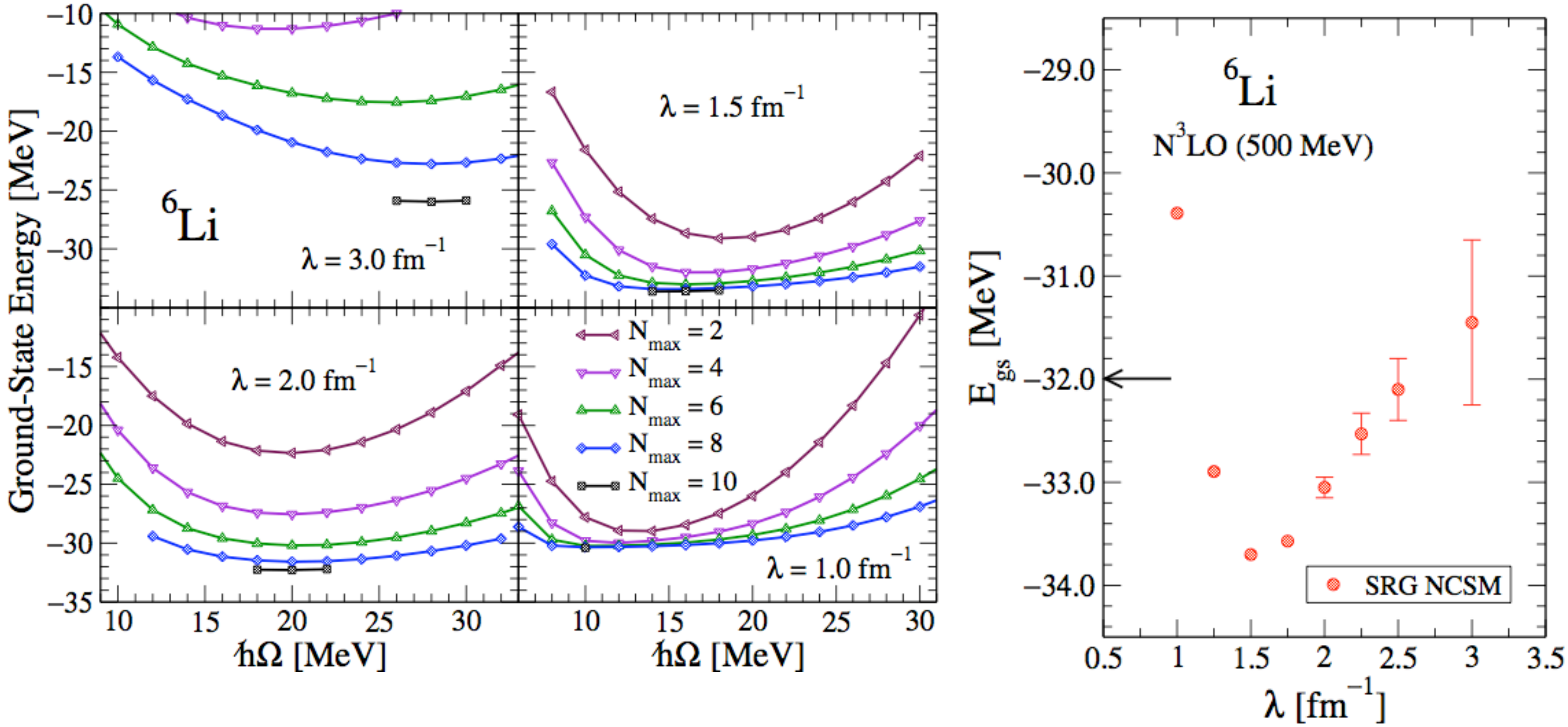






"nuclear matter" calculations relevant for neutron stars, nuclear equation of state, etc. become perturbative

Full Configuration Interaction (FCI) Calculations

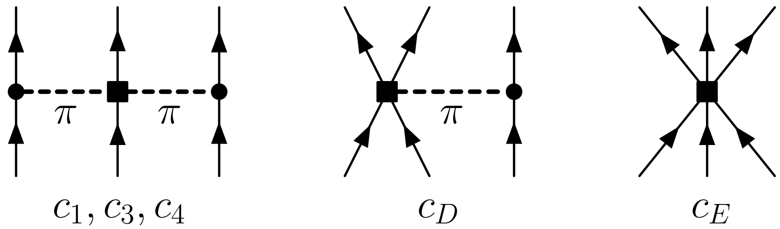


Decoupling high-k and low-k => accelerated convergence, more perturbative

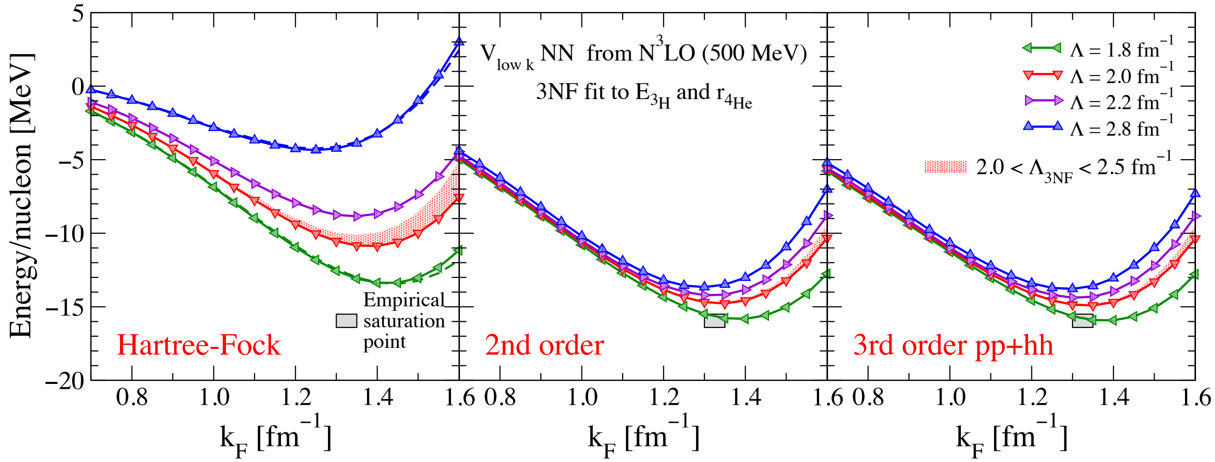
BUT note...

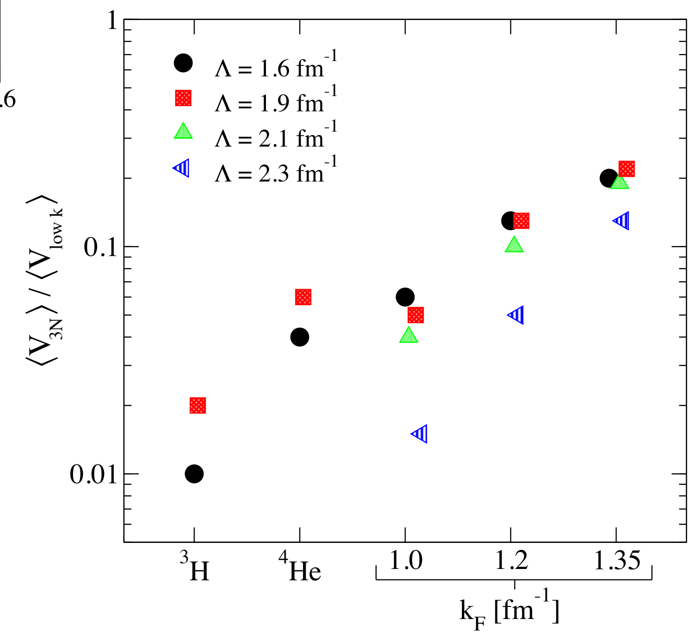
λ-dependent results (omitted induced 3...A-body forces)

Approximation to the full 3N evolution



- Chiral EFT complete operator basis
- Project onto N^{2}LO 3NF
- Fit c_E and c_D to A=3,4 at each Λ

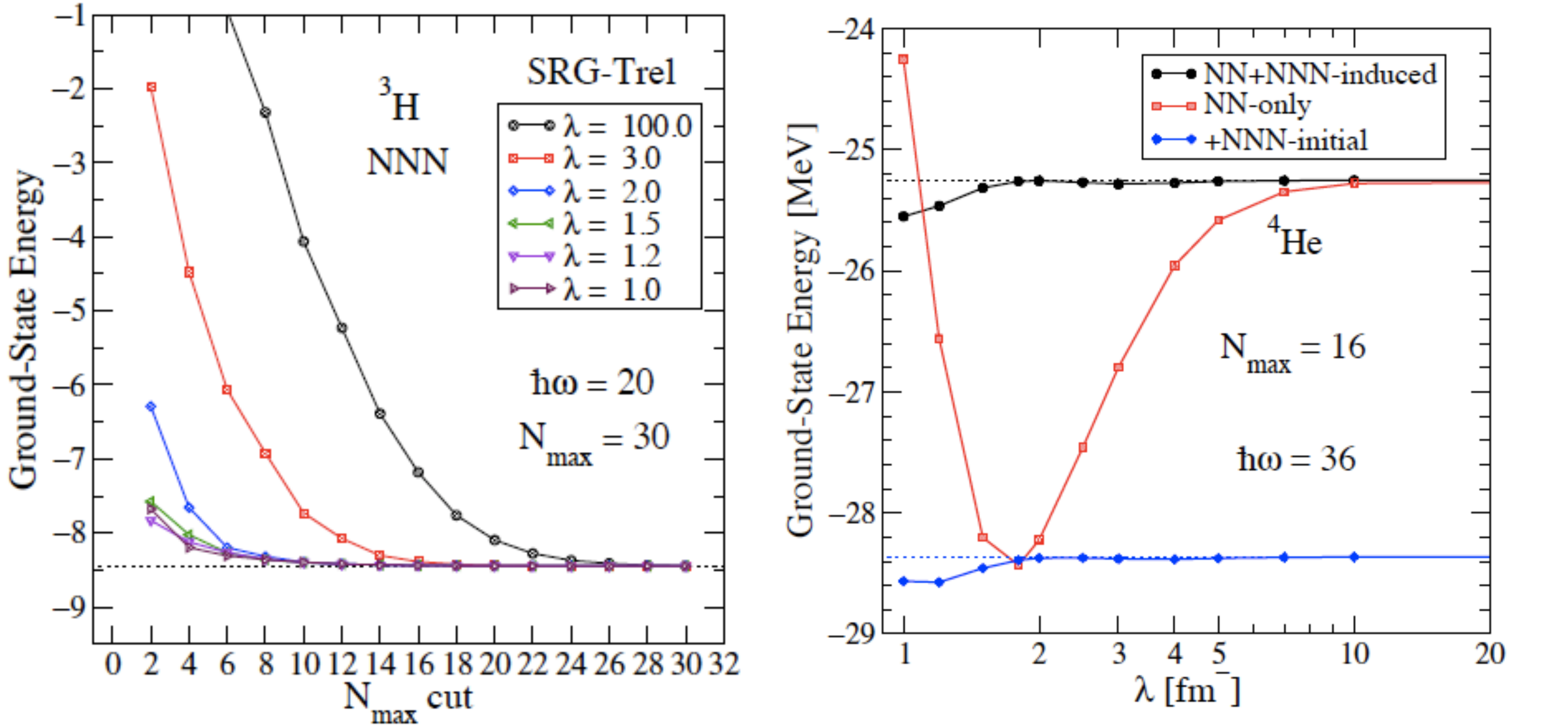




- <3N>/<2N> doesn't explode
- Need high order calculation of nuclei and/or nuclear matter and exact evolution to assess further

Consistent 3N SRG evolution

(Jurgenson, Navratil, Furnstahl)



It works! λ-independent ^{3}H, softer convergence

Induced 4N are small (but see R. Roth's talk for possible problems)

Free space versus in-medium evolution

Free space SRG: $V(\lambda)_{2N}$ fixed in 2N system
$V(\lambda)_{3N}$ fixed in 3N system

$V(\lambda)_{aN}$ fixed in aN system

Use $T + V(\lambda)_{2N} + V(\lambda)_{3N} + \ldots + V(\lambda)_{aN}$ in A-body system

In-medium SRG:
evolution done at finite density (i.e., directly in A-body system).

Different mass regions => different SRG evolutions

inconvenience outweighed (?) by simplifications allowed by normal-ordering

Normal Ordered Hamiltonians

Pick a reference state Φ (e.g., HF) and apply Wick's theorem to 2nd-quantized Hamiltonian

$$
\begin{aligned}
A_iA_jA_kA_l\cdots A_m &= N(A_iA_jA_kA_l\cdots A_m) \\
&+ N\Big((\overbracket{A_iA_j}A_kA_l\cdots A_m) + \text{all other single contractions}\Big) \\
&+ N\Big((\overbracket{A_iA_j}\,\overbracket{A_kA_l}\cdots A_m) + \text{all other double contractions}\Big) \\
&\vdots \\
&+ N\Big((\text{all fully contracted terms}\Big)
\end{aligned}
$$

$$\overbracket{a_i^\dagger a_j} = \delta_{ij}\theta(\epsilon_F - \epsilon_i) \qquad \overbracket{a_i a_j^\dagger} = \delta_{ij}\theta(\epsilon_i - \epsilon_F)$$

$$\langle\Phi|N(\cdots)|\Phi\rangle = 0$$

Normal Ordered Hamiltonians

$$H = \sum t_i a_i^\dagger a_i + \frac{1}{4}\sum V_{ijkl}^{(2)} a_i^\dagger a_j^\dagger a_l a_k + \frac{1}{36}\sum V_{ijklmn}^{(3)} a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l$$

Normal-order w.r.t. some reference state Φ (e.g., HF) :

$$H = E_{vac} + \sum f_i N(a_i^\dagger a_i) + \frac{1}{4}\sum \Gamma_{ijkl} N(a_i^\dagger a_j^\dagger a_l a_k) + \frac{1}{36}\sum W_{ijklmn} N(a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l)$$

$$\begin{aligned}
E_{vac} &= \langle\Phi|H|\Phi\rangle \\
f_i &= t_{ii} + \sum_h \langle ih|V_2|ih\rangle\, n_h + \frac{1}{2}\sum_{hh'} \langle ihh'|V_3|ihh'\rangle\, n_h n_{h'} \\
\Gamma_{ijkl} &= \langle ij|V_2|kl\rangle + \sum_h \langle ijh|V_3|klh\rangle\, n_h \\
W_{ijklmn} &= \langle ijk|V_3|lmn\rangle
\end{aligned}$$

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0-, 1-, 2-body terms contain some 3NF effects thru density dependence => Efficient truncation scheme for evolution of 3N?

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$$\begin{aligned}
E_{vac} &= \langle \Phi | H | \Phi \rangle \\
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\Gamma_{ijkl} &= \langle ij | V_2 | kl \rangle + \sum_h \langle ijh | V_3 | klh \rangle \, n_h \\
W_{ijklmn} &= \langle ijk | V_3 | lmn \rangle
\end{aligned}$$

$$\langle \Phi | N(\cdots) | \Phi \rangle = 0$$

0-, 1-, 2-body terms contain some 3NF effects thru density dependence => Efficient truncation scheme for evolution of 3N?

In-medium SRG for Nuclear matter

- Normal order H w.r.t. non-int. fermi sea
- Choose SRG generator to eliminate "off-diagonal" pieces

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$

$$\eta = [\hat{f}, \hat{\Gamma}]$$

$\Rightarrow$

$$\lim_{s\to\infty} \Gamma_{od}(s) = 0$$

$$\lambda \equiv s^{-1/4}$$

$$\langle 12|\Gamma_{od}|34\rangle = 0 \;\text{ if }\; \mathrm{f}_{12} = \mathrm{f}_{34}$$

- Truncate to 2-body normal-ordered operators "IM-SRG(2)"
  - dominant parts of induced many-body forces included implicitly

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  - dominant parts of induced many-body forces included implicitly

$$H(\infty) = E_{vac}(\infty) + \sum f_i(\infty) N(a_i^\dagger a_i) + \frac{1}{4}\sum [\Gamma_d(\infty)]_{ijkl} N(a_i^\dagger a_j^\dagger a_l a_k)$$

$$E_{vac}(\infty) \quad \rightarrow \quad E_{gs}$$

$$f_k(\infty) \quad \rightarrow \quad \epsilon_k \text{ (fully dressed s.p.e.)}$$

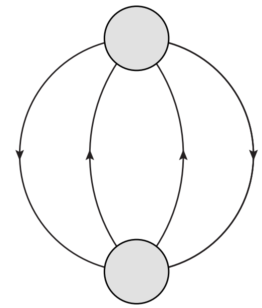
$$\Gamma_d(\infty) \quad \rightarrow \quad f(k', k) \text{ (Landau q.p. interaction)}$$

Microscopic realization of SM ideas: dominant MF + weak A-dependent NN_{eff}

In-medium SRG Equations Infinite Matter

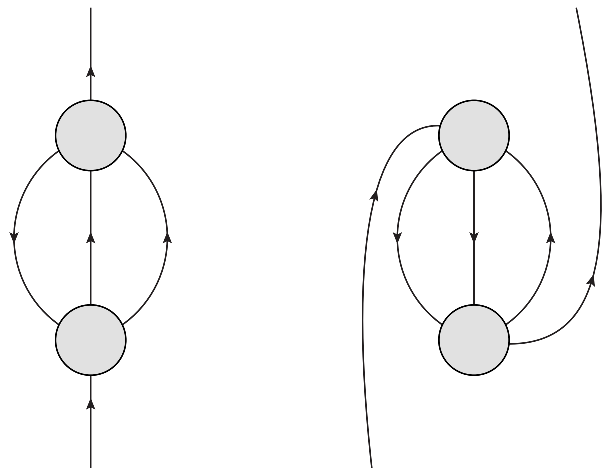
0-body flow

$$\frac{d}{ds}E_{vac} = \frac{1}{2}\sum_{ijkl}(f_{ij} - f_{kl})|\langle ij|\Gamma|kl\rangle|^2\, n_i n_j \bar{n}_k \bar{n}_l$$



1-body flow

$$\frac{d}{ds}f_a = \sum_{bcd}(f_{ad} - f_{bc})|\langle ad|\Gamma|bc\rangle|^2 \left(\bar{n}_b\bar{n}_c n_d + n_b n_c \bar{n}_d\right)$$

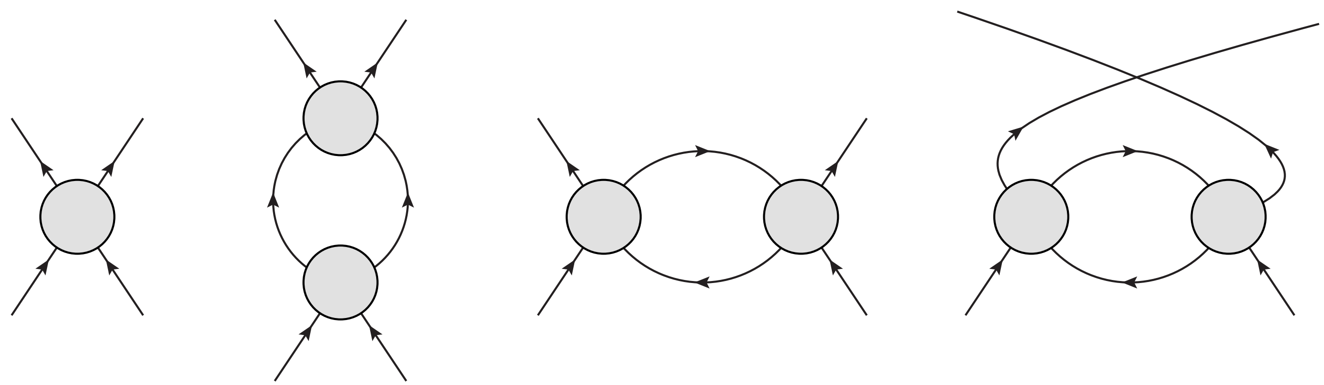


interference of 2p1h 2h1p
self-energy terms

In-medium SRG Equations Infinite Matter

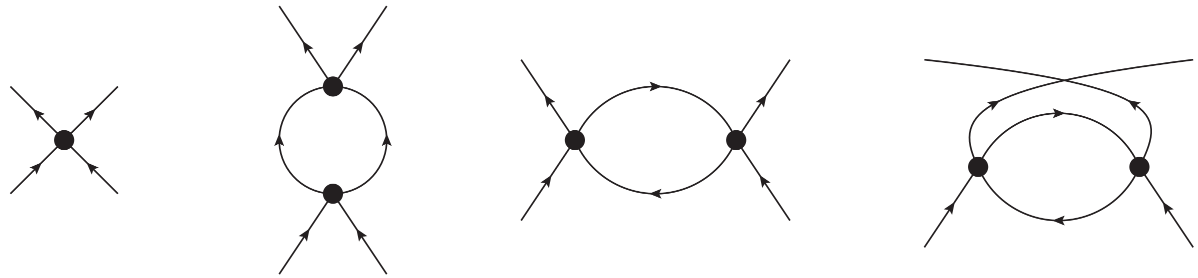
2-body flow

$$
\begin{aligned}
\langle 12|\frac{d\Gamma}{ds}|34\rangle &= -(f_{12}-f_{34})^2\langle 12|\Gamma|34\rangle \\
&+ \frac{1}{2}\sum_{ab}(f_{12}+f_{34}-2f_{ab})\,\langle 12|\Gamma|ab\rangle\langle ab|\Gamma|34\rangle\big(1-n_a-n_b\big) \\
&+ \sum_{ab}\big[(f_{1a}-f_{3b})-(f_{2b}-f_{4a})\big]\langle 1a|\Gamma|3b\rangle\langle b2|\Gamma|a4\rangle(n_a-n_b) \\
&- \sum_{ab}\big[(f_{2a}-f_{3b})-(f_{1b}-f_{4a})\big]\langle 2a|\Gamma|3b\rangle\langle b1|\Gamma|a4\rangle(n_a-n_b)
\end{aligned}
$$



Note the interference between s, t, u channels a-la Parquet theory

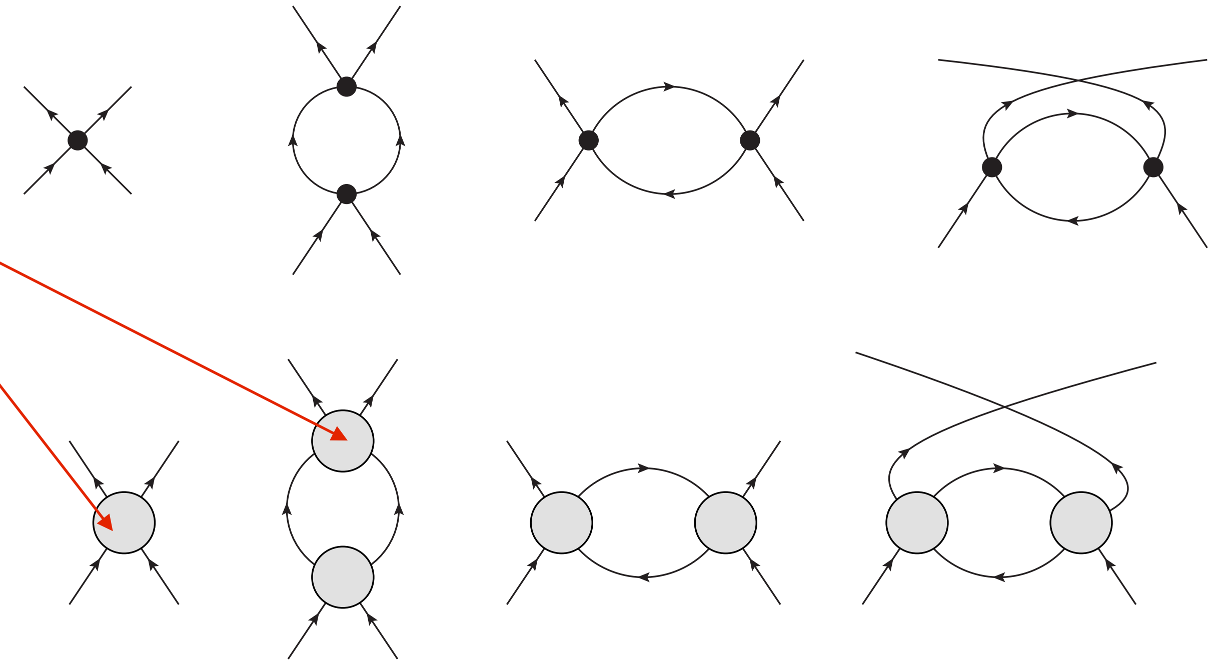
SRG is manifestly non-perturbative

$\Gamma(\delta s) \sim$ 

SRG is manifestly non-perturbative

$\Gamma(\delta s) \sim$

$\Gamma(2\delta s) \sim$



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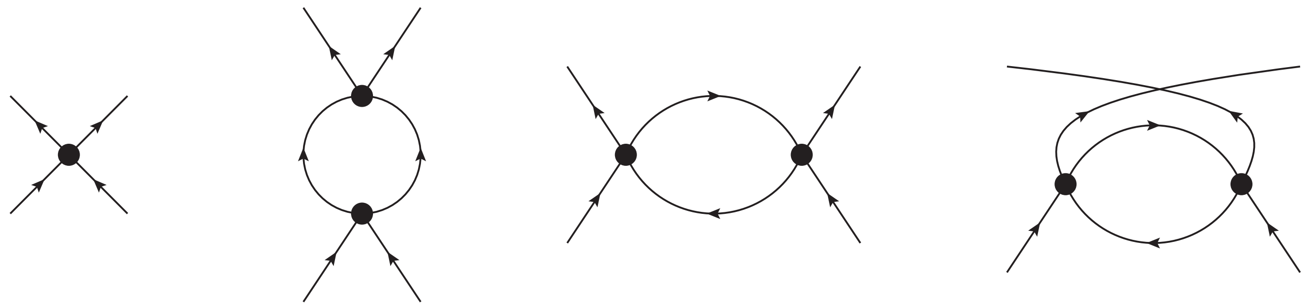

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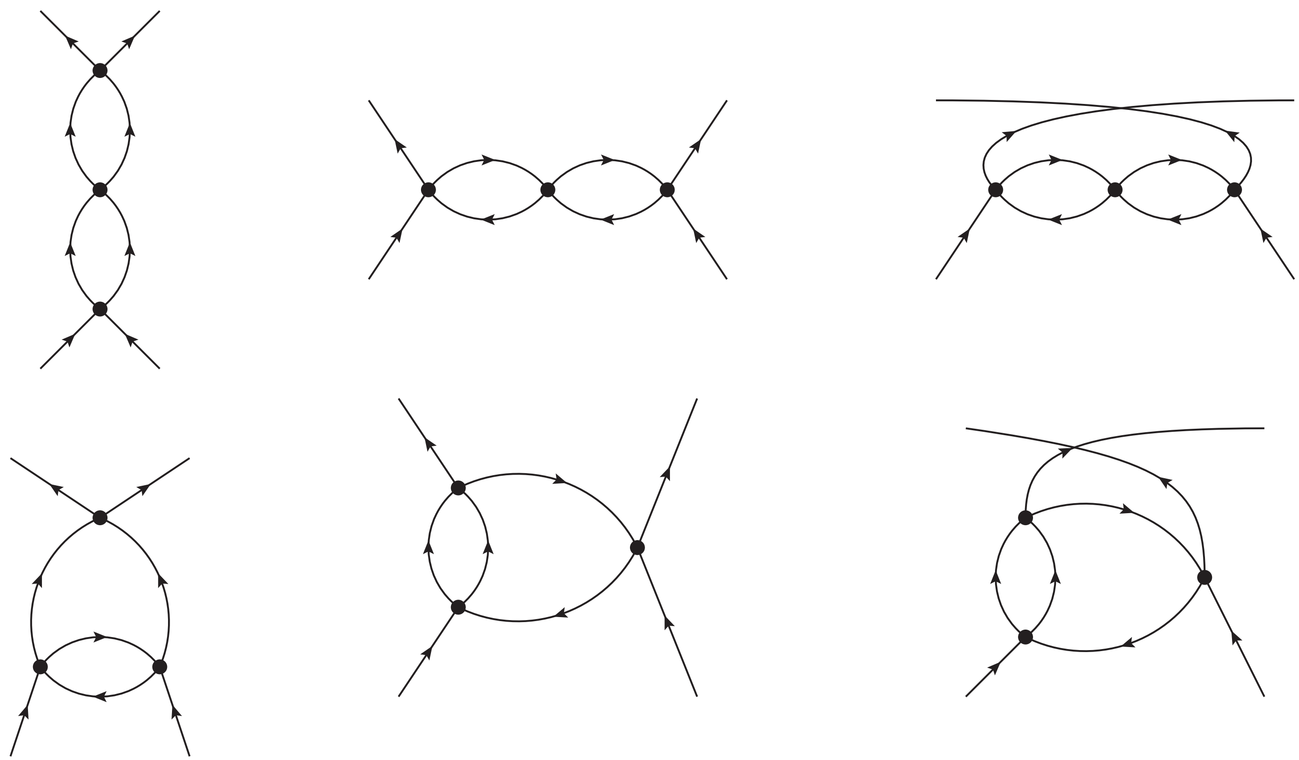


+ many more ...

Some observations

1) $\frac{d}{ds}\langle H\rangle_0 \leq 0$ for monotonic f_k correlations weakened, HF picks up more binding with increasing s.

2) pp channel + 2 ph channels treated on equal footing

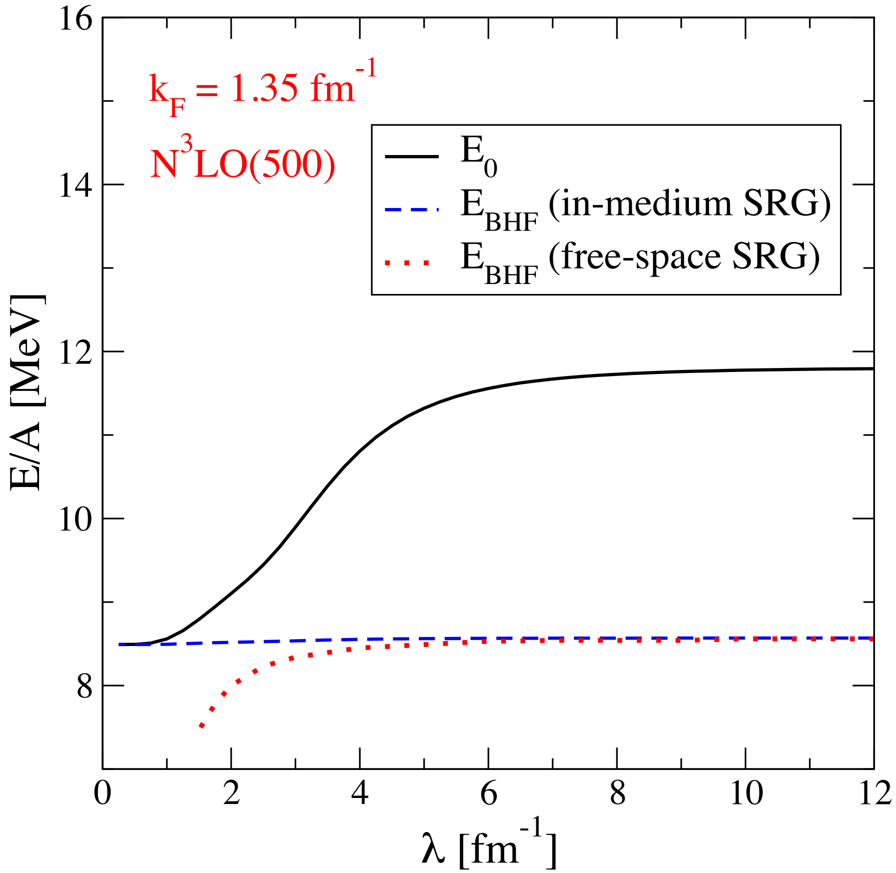
3) Intrinsically non-perturbative

4) no unlinked diagrams (size extensive, etc.)

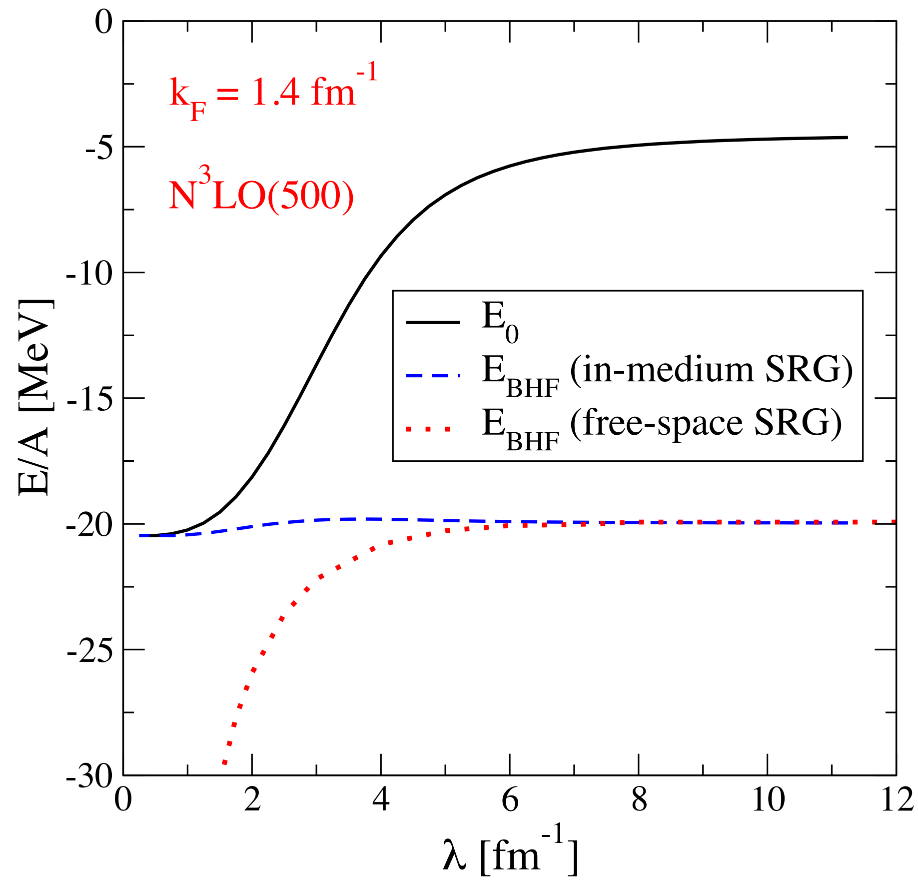
5) "3rd-order exact" a-la CCSD

6) Extension to effective operators/Shell model possible

Correlations "adiabatically" summed into H(λ)



PNM*



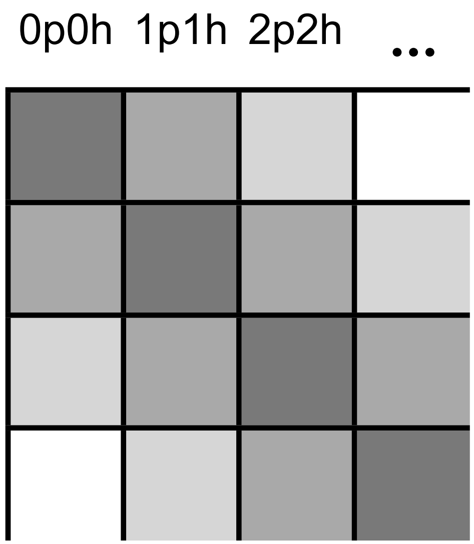
SNM*

Weak cutoff dependence over large range => dominant 3,4,...-body terms evolved implicitly

*Neglects ph-channel.

In-medium SRG for closed-shell nuclei (g.s.)

H =



Df. "offdiagonal" part of H as terms that mix 0p0h with higher ph sectors

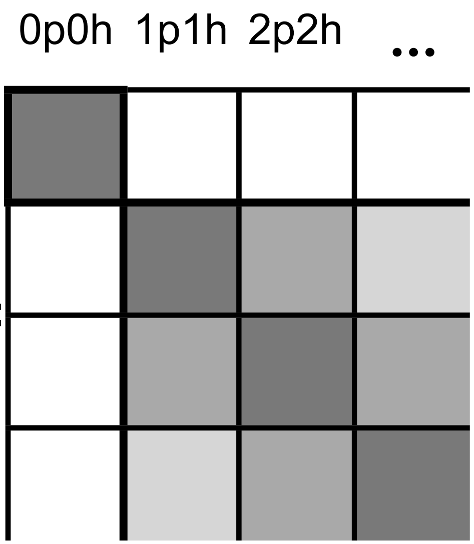
$$\eta(s) = [H(s), H^{od}(s)]$$

$$\Gamma^{od} = \sum_{pp'hh'} \Gamma_{pp'hh'} \, N(a_p^\dagger a_{p'}^\dagger a_h a_{h'}) + h.c$$

$$f^{od} = \sum_{ph} f_{ph} \, N(a_p^\dagger a_h) + h.c.$$

In-medium SRG for closed-shell nuclei (g.s.)

$H(\infty) =$



HF reference state decouples from higher npnh states

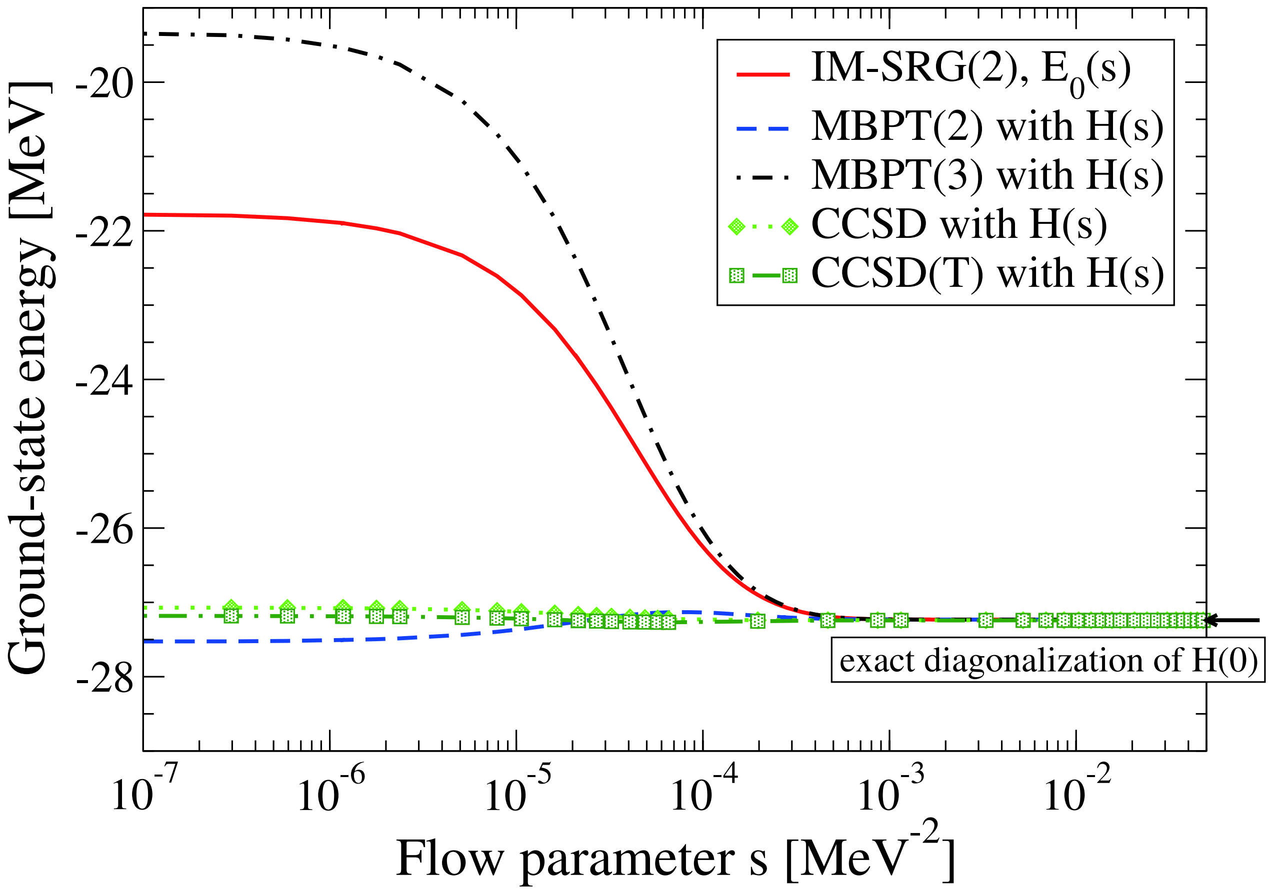
$$\lim_{s\to\infty} \langle\phi|H(s)|\phi\rangle = E_{gs}$$

$$\lambda \equiv s^{-1/4}$$

$$QH(\infty)P = 0, \quad PH(\infty)Q = 0,$$

where $P = |\Phi\rangle\langle\Phi|$ and $Q = 1 - P$.

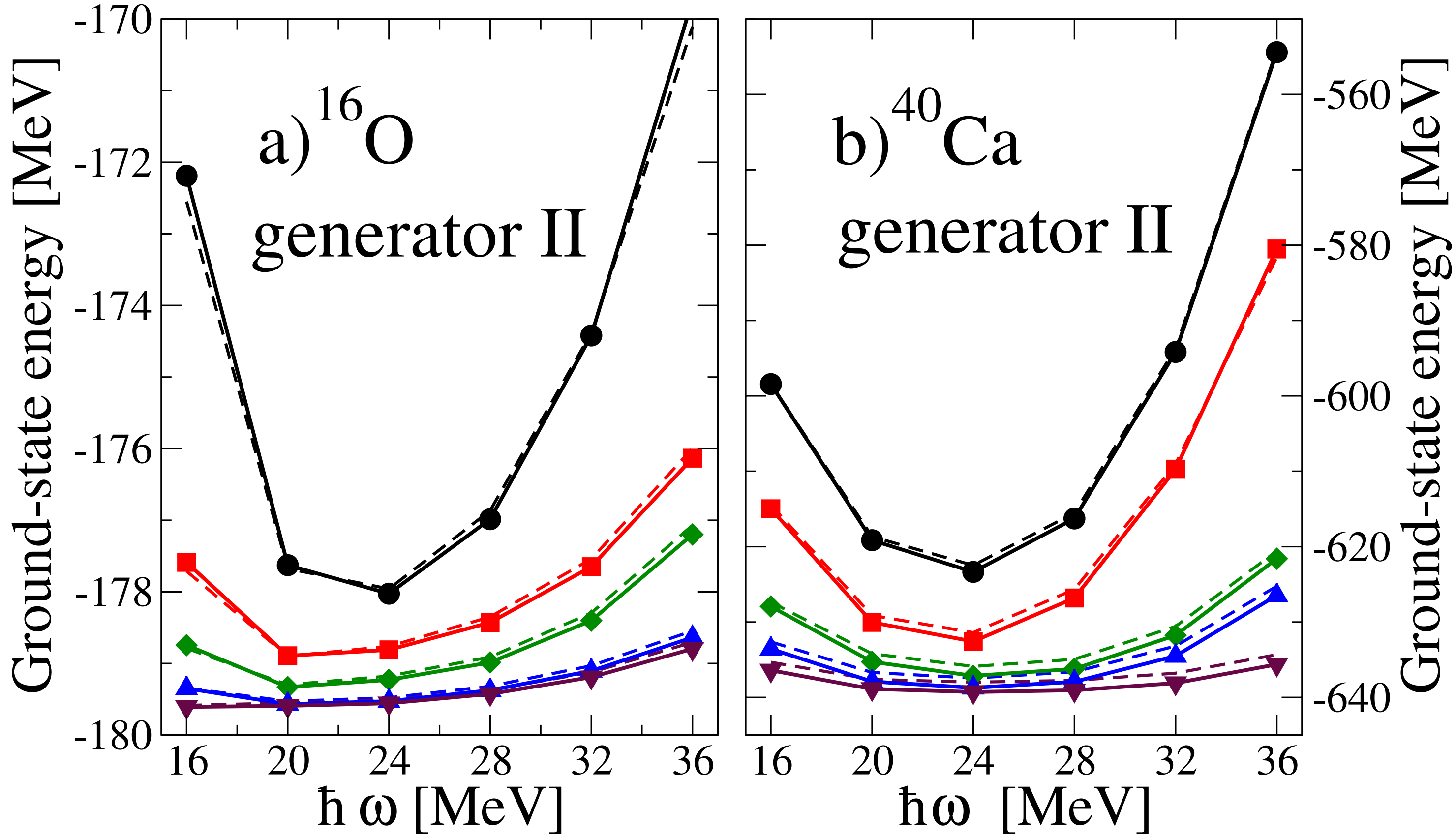
IM-SRG to build "soft" interactions



$\lambda \equiv s^{-1/4}$

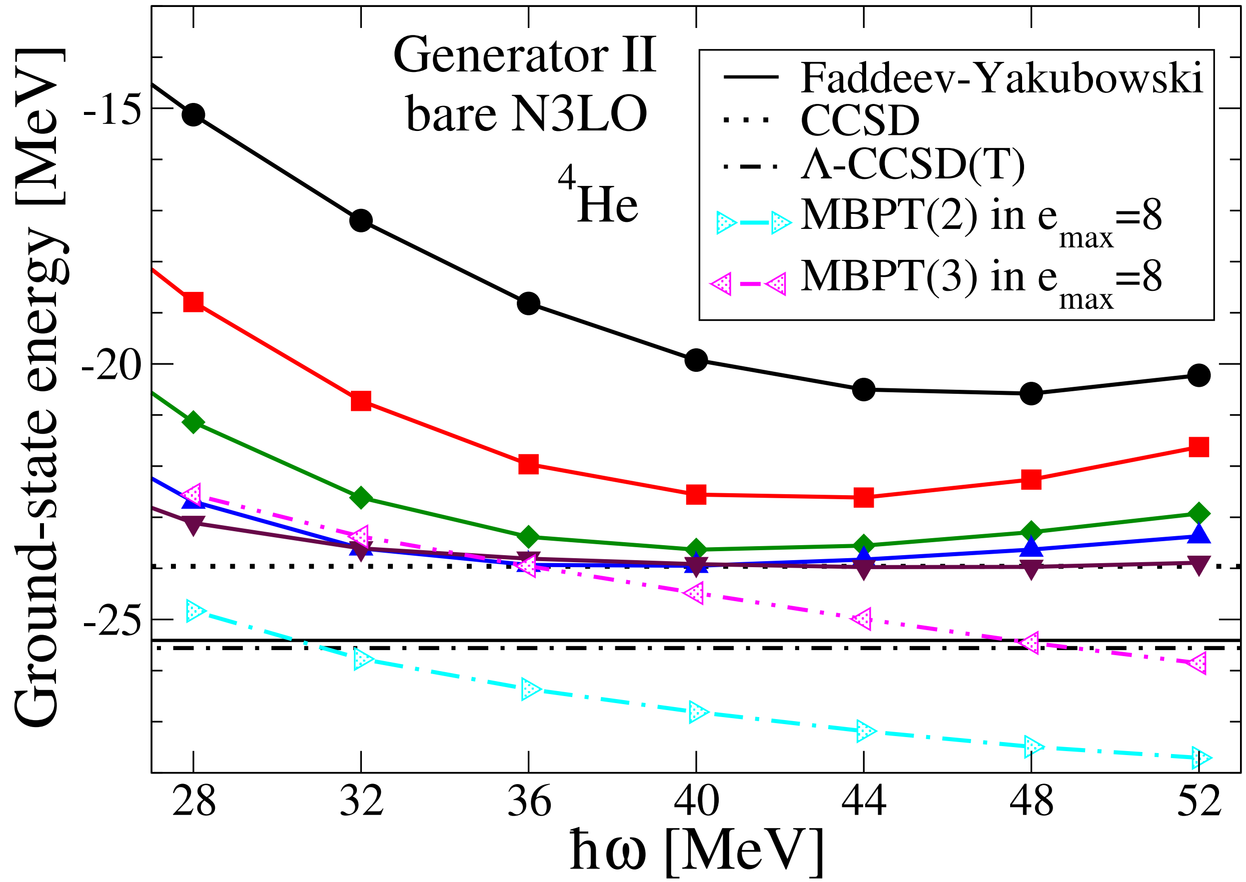
~ λ-independent CC results (dominant induced many-body interactions included)

IM-SRG to diagonalize many-body problems



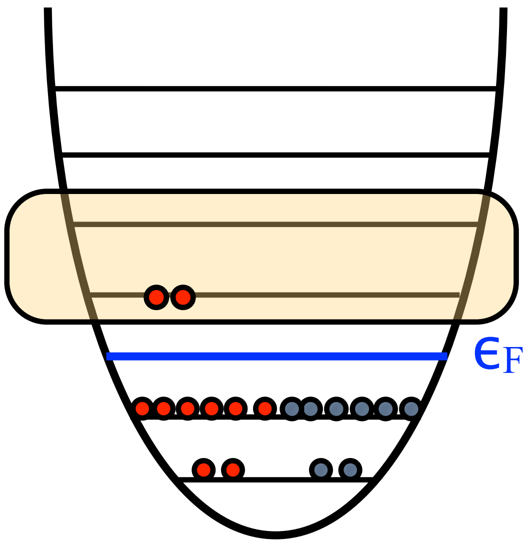
- good agreement with CCSD (dashed lines)
- N^6 scaling with number of s.p. orbitals

Also works well for harder interactions:



- agrees well w/CCSD using bare N^{3}LO (Entem/Machleidt)

In-medium SRG for open shell nuclei (Shell model)



inactive particle orbitals q_i

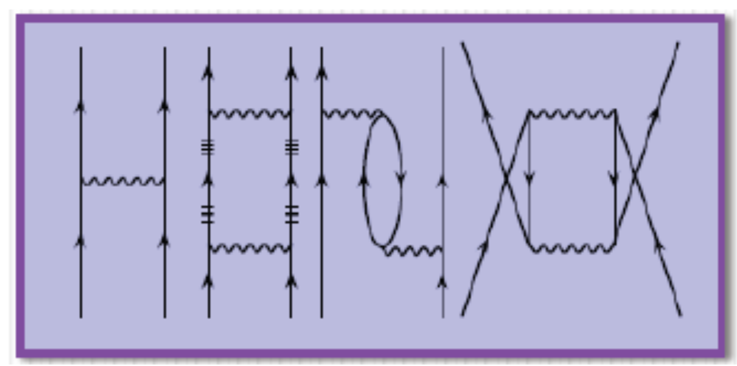
active valence orbitals v_i

inactive hole orbitals h_i

Decouple valence orbitals and diagonalize:

$$PH_{eff}P|\Psi\rangle = (E - E_c)P|\psi\rangle$$

Previously, H_{eff} from MBPT and empirical corrections



Can we use the IM-SRG to do this?

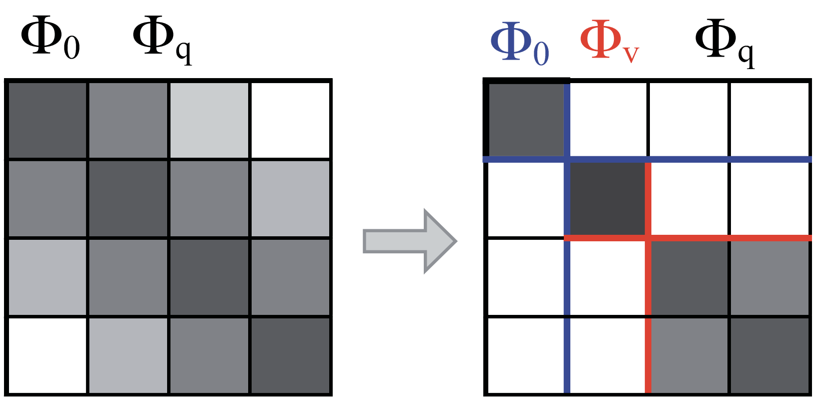
In-medium SRG recipe for shell model

1) Identify all terms in H that don't annihilate model-space states

$$\hat{O}_i\left(\mathrm{v}_1^\dagger \ldots v_{N_v}^\dagger |\phi\rangle\right) \neq 0$$

2) Solve flow equations

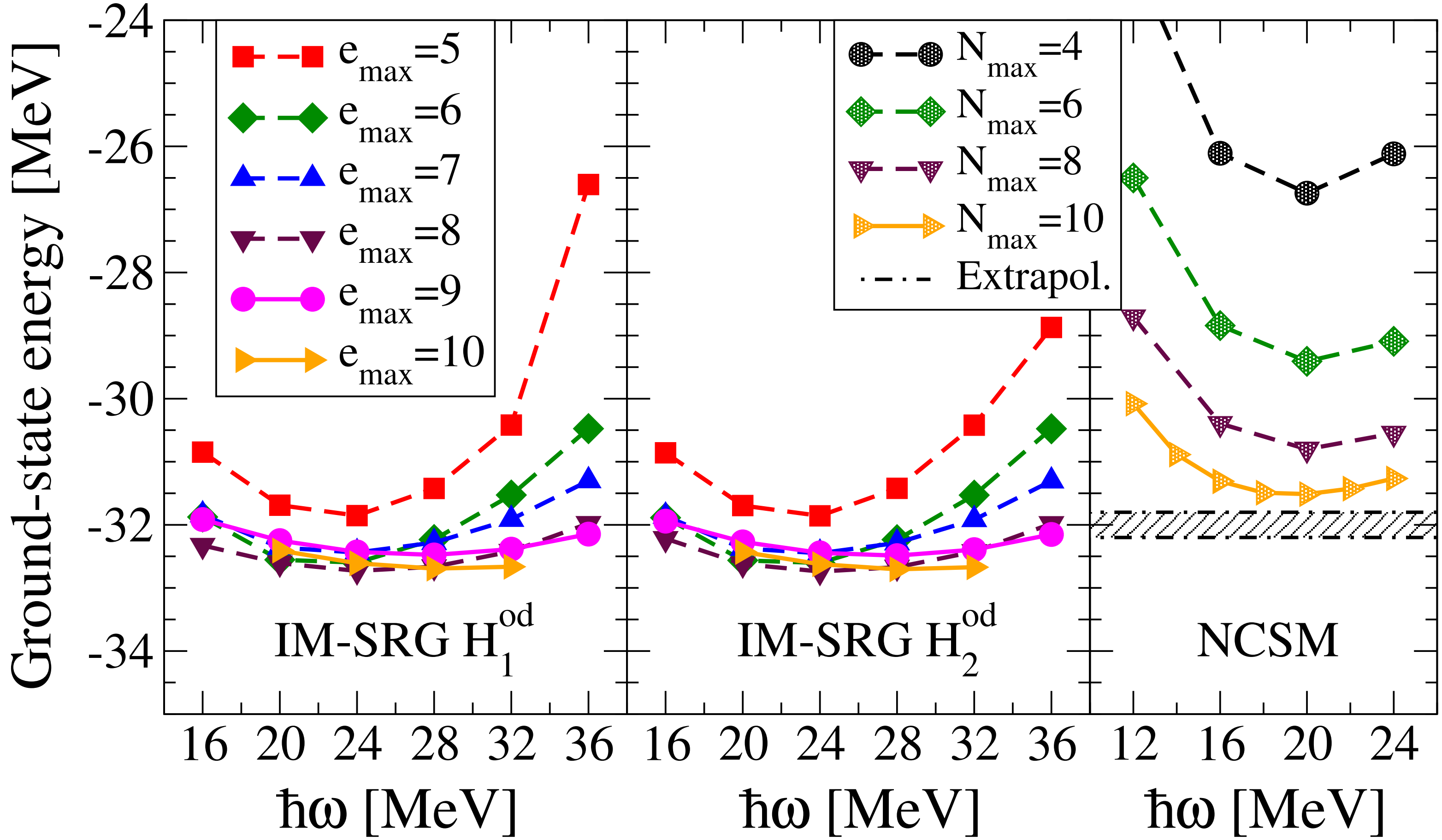
$$\begin{aligned} \frac{dH}{ds} &= [\eta, H] \\ \eta &= [H^{(od)}, H] \\ H^{(od)} &= \sum g_i \hat{O}_i \end{aligned}$$



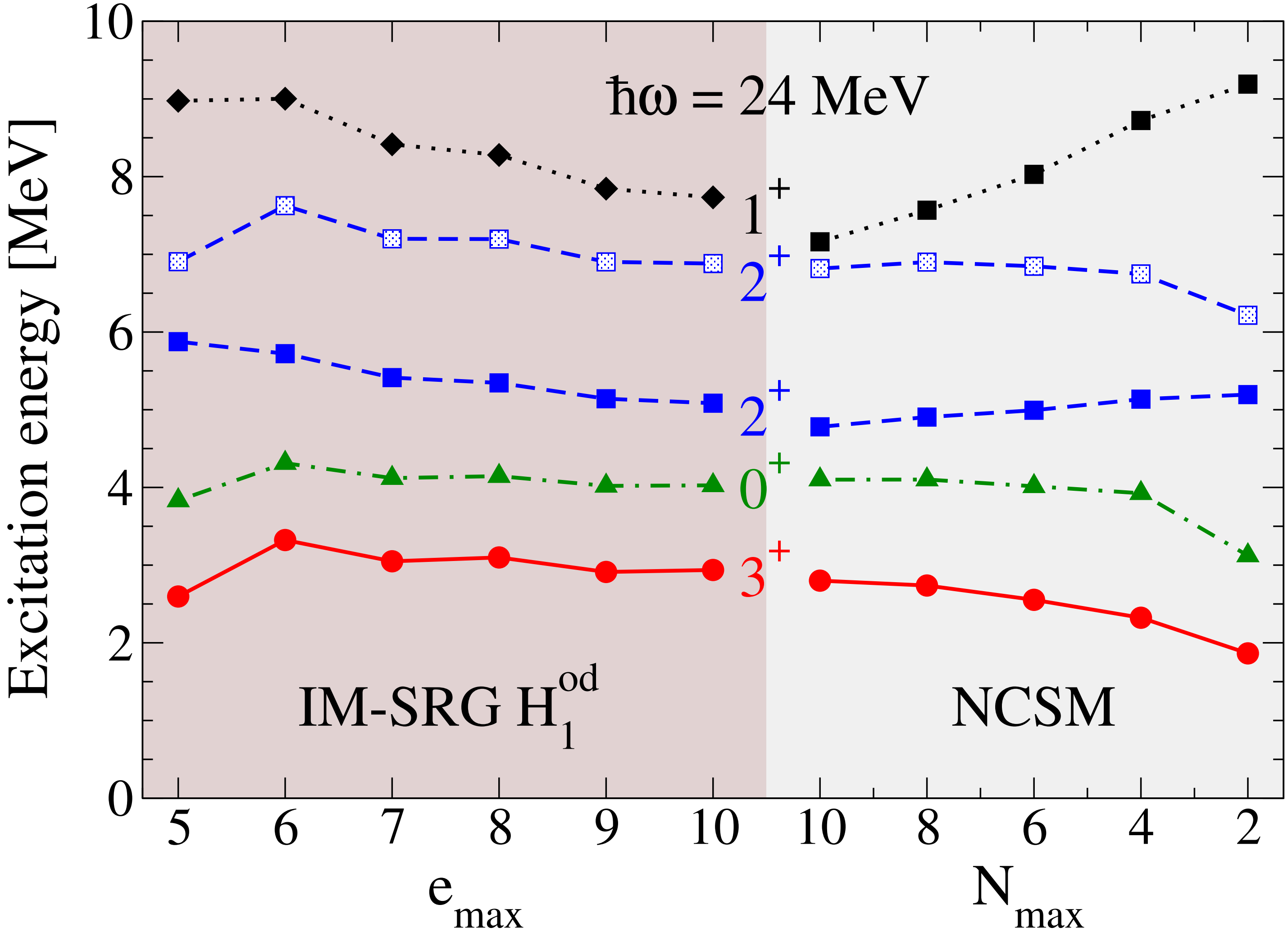
3) Diagonalize fully-evolved H in the reduced valence space

$$PH(\infty)P|\Psi\rangle = (E - E_c)P|\psi\rangle$$

^{6}Li ground state: comparison to exact NCSM



^{6}Li spectra: comparison to NCSM



Summary

- RG methods can simplify many-body calculations immensely **provided that induced many-body operators are under control**
- In-medium evolution + truncations based on normal-ordering => simple way to evolve dominant induced 3, 4, ...A-body interactions with 2-body machinery
- Can be used as ab-initio method in and of itself, or to construct soft interactions for other ab-initio methods
- Extensions to shell model H_{eff}/O_{eff} look promising

Ongoing work

I) Extension to open-shell nuclei using number-projected HFB reference states (Heiko Hergert, Ohio State)

II) Extension to shell model effective operators (e.g., $0\nu\beta\beta$ decay)
  - suitable for multi-shell model spaces and non-perturbative

III) Apply to neutron/nuclear matter in a box

IV) Next higher truncation (explicit N-ordered 3-body)