Ultrasoft fermionic modes at high temperature

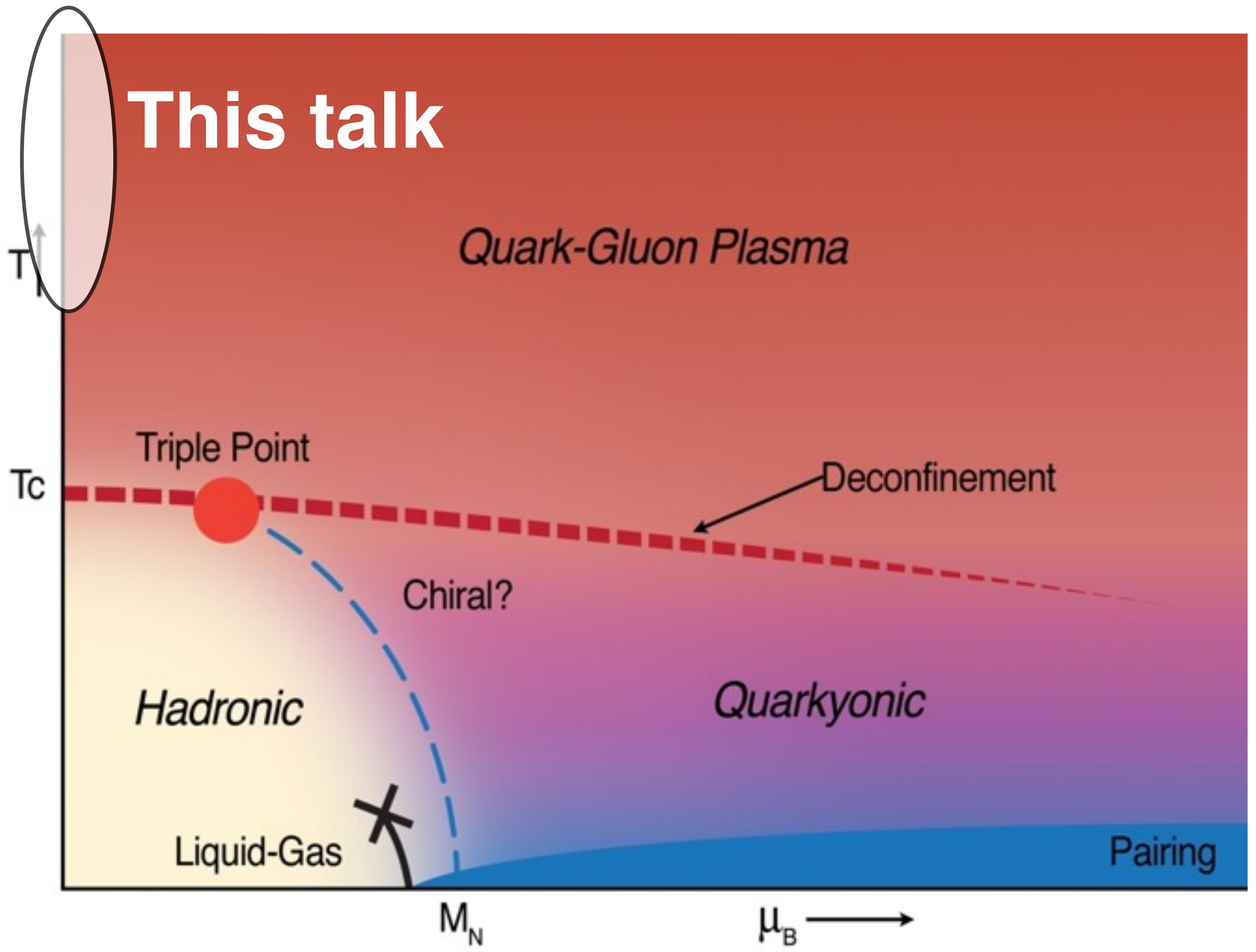
Yoshimasa Hidaka

Mathematical Physics Laboratory, RIKEN

Collaboration with Daisuke Satow and Teiji Kunihiro (Kyoto University)

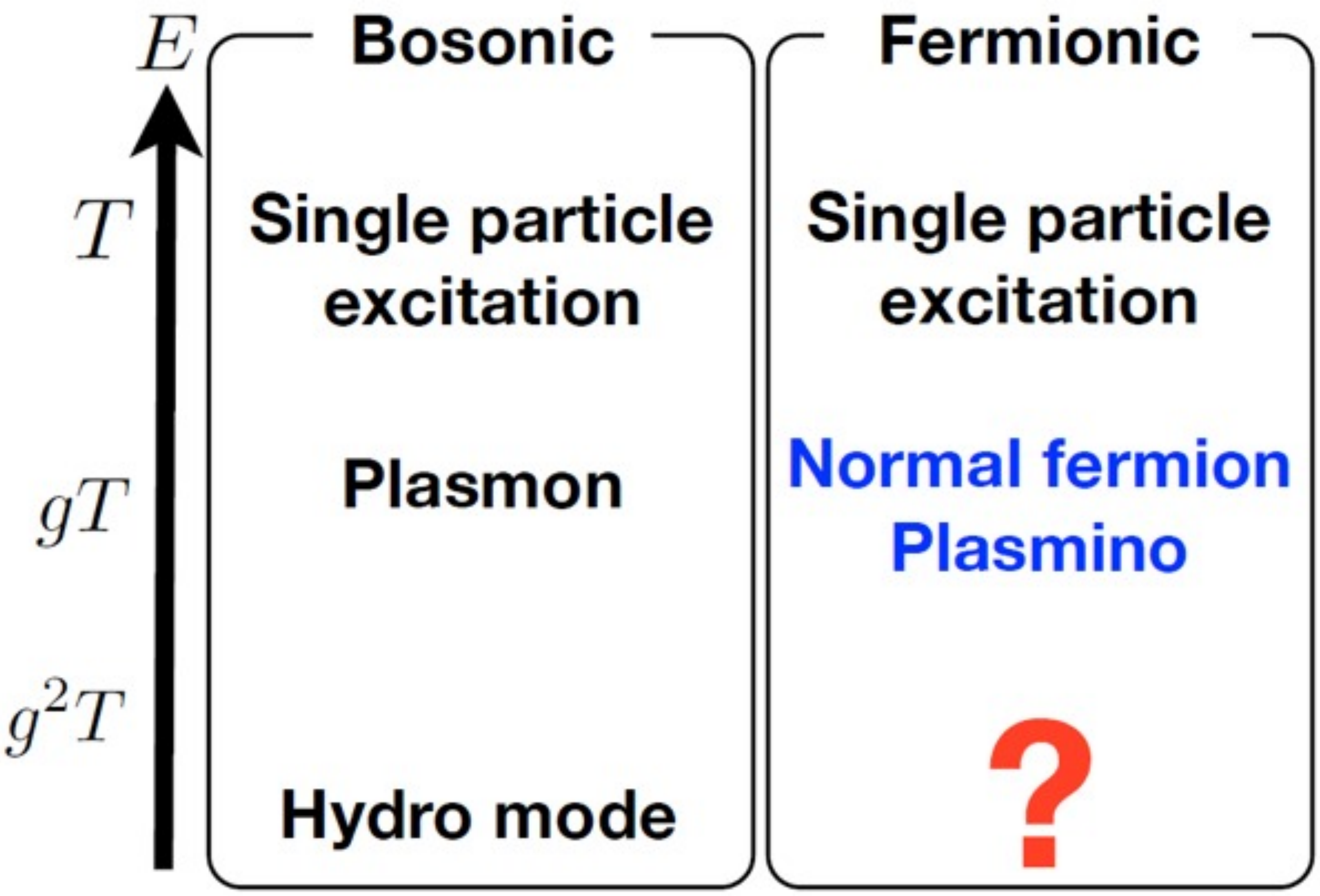
Based on arXiv:1105.0423 [hep-ph] and 1111.5015 [hep-ph], Nucl. Phys. A876, 93 (2012)

QCD Phase Diagram



Collective modes at high T

Massless fermion-boson system (Yukawa, QED, QCD)

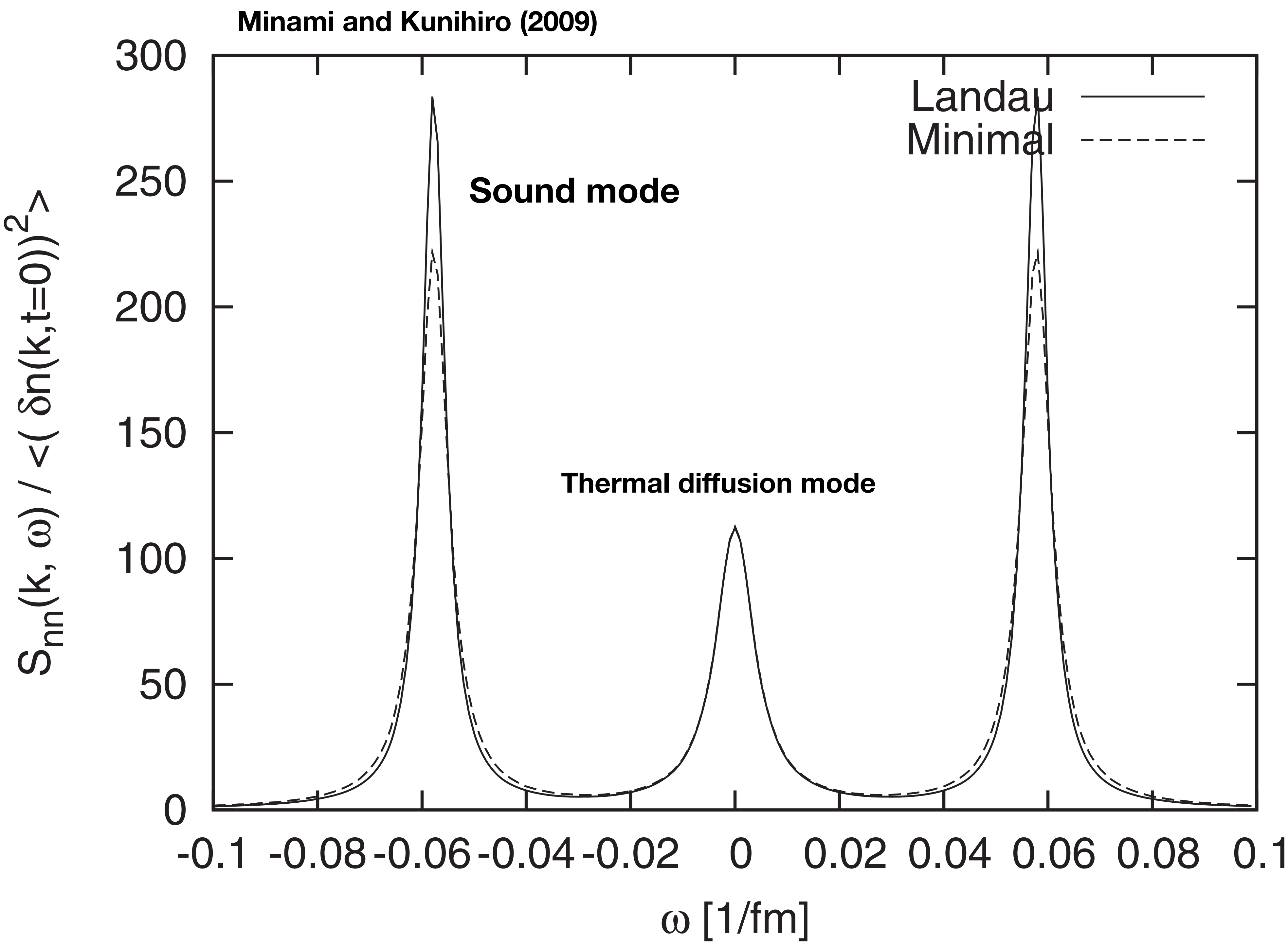


(*g*: coupling constant)

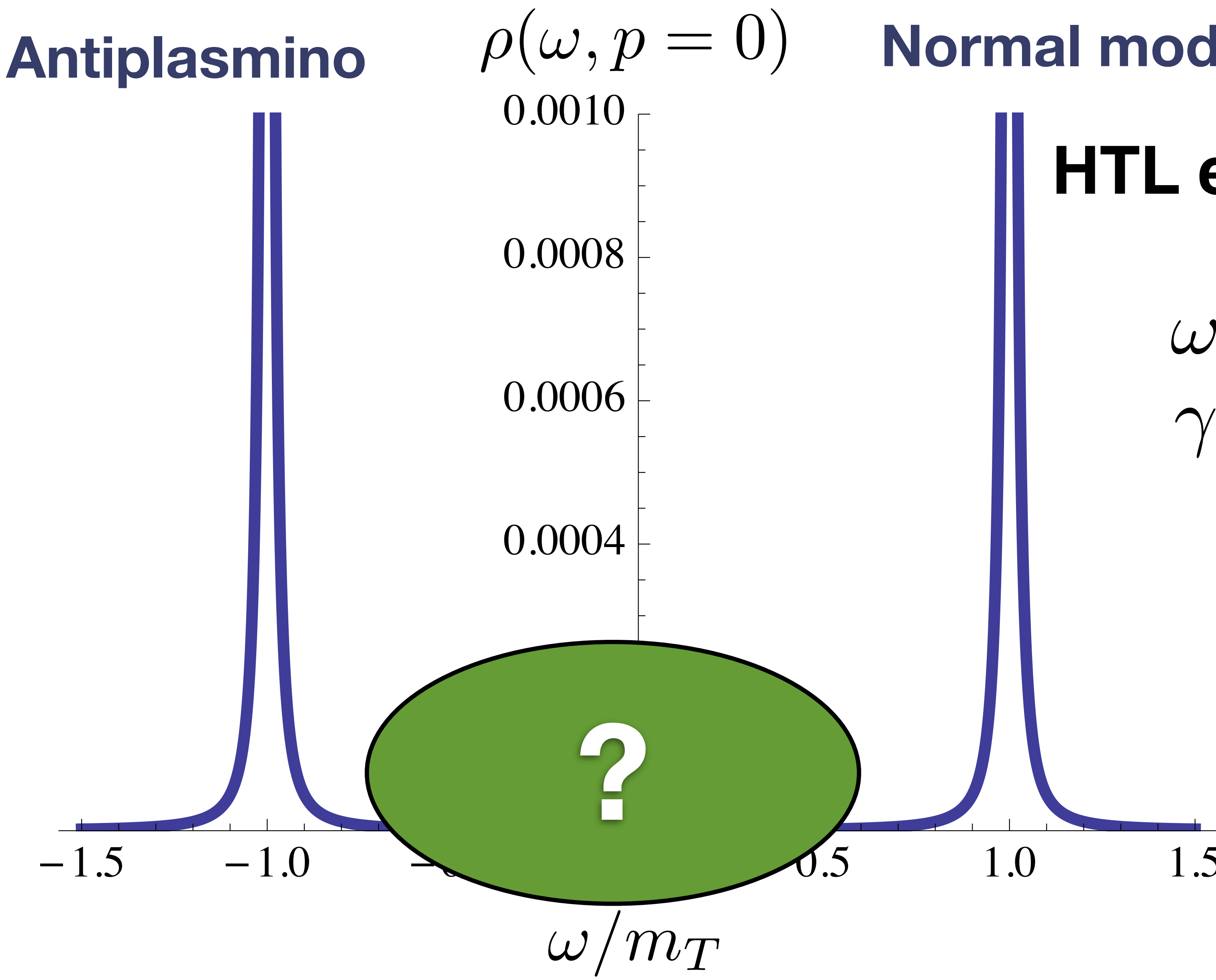
Low energy excitations are collective.

In bosonic sector, hydro mode exists as zero modes.

ex) Hydro mode:Density fluctuation



Fermionic ultrasoft-modes at high T?



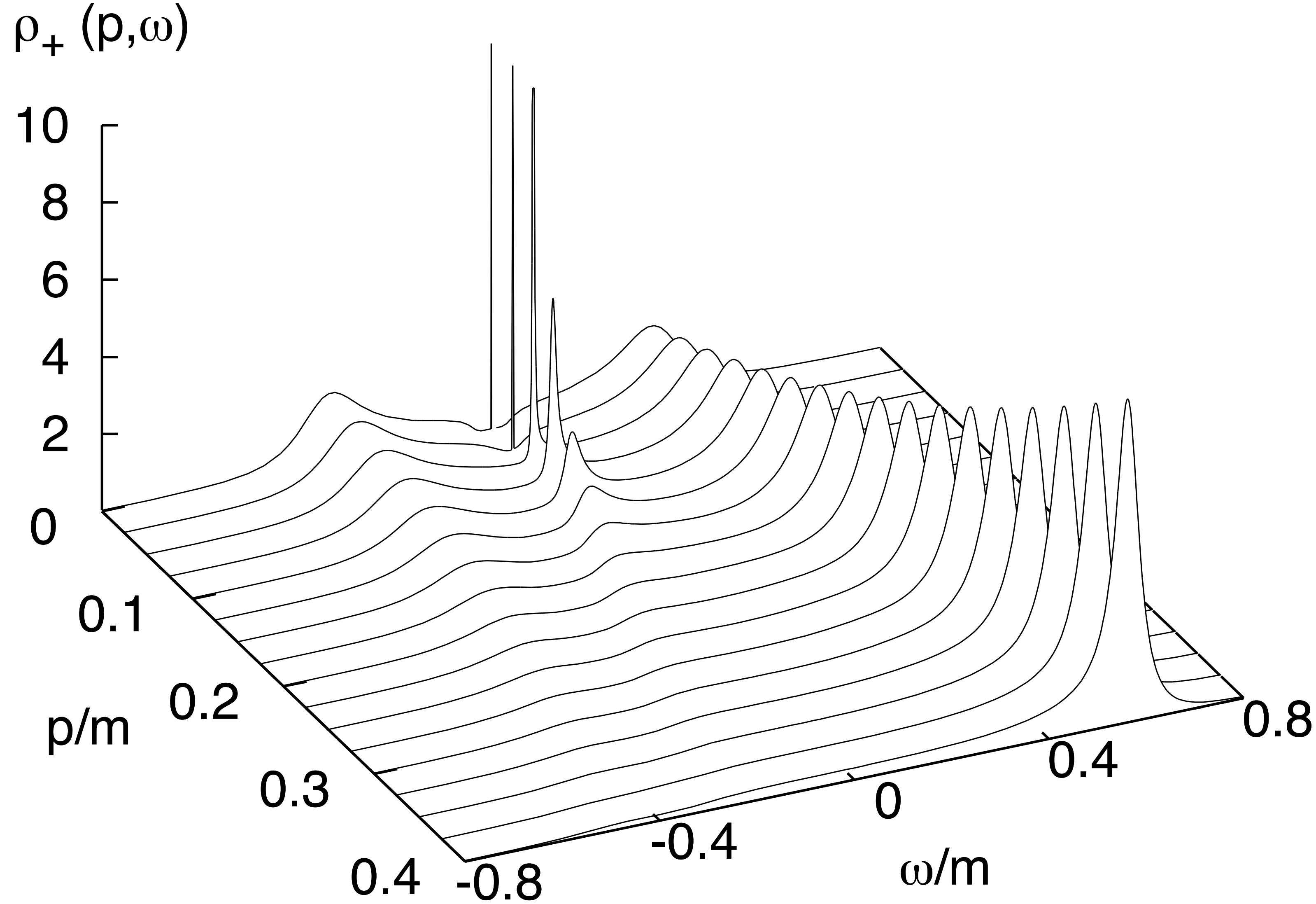
HTL effective theory works

$$\omega \sim gT$$
$$\gamma \sim g^2 T$$

cf: fermionic ultrasoft mode was suggested:
massive boson case: M. Kitazawa, T. Kunihiro and Y. Nemoto, PTP 117, 103 (2007),
QCD: V. V. Lebedev and A. V. Smilga, Annals Phys. 202, 229 (1990).

e.g., Yukawa model

$T \sim m$ **boson mass**



Kitazawa, Kunihiro and Nemoto ('06)

Does chiral symmetry imply the existence of zero modes?

Yes, in the vacuum.

Questionable at finite T.

Perturbation theory at high *T*

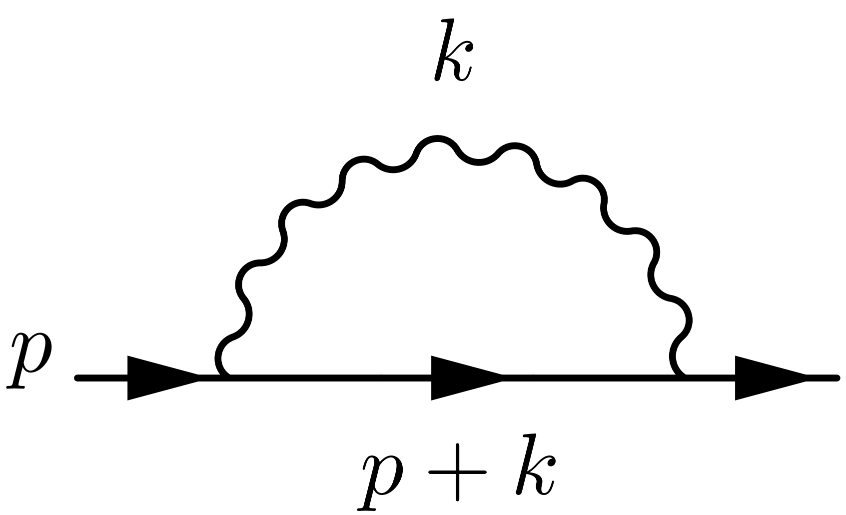
Naive perturbation does not work at ultrasoft momentum region

Bare propagators

Fermion: $D_R(k) = \dfrac{\not{k}}{k^2 + i\epsilon k^0}$ **Boson:** $G_A(k) = \dfrac{1}{k^2 - i\epsilon}$

(scalar, photon, gluon,..)

One-loop analysis



$$\simeq g^2 \int \frac{d^4k}{(2\pi)^4}\ \not{k}(n_F(k) + n_B(k)) G_A(k) D_R(k+p)$$

$$\simeq g^2 \int \frac{d^4k}{(2\pi)^4}\ \not{k}(n_F(k) + n_B(k)) \frac{1}{2p\cdot k}(2\pi)\delta(k^2)$$

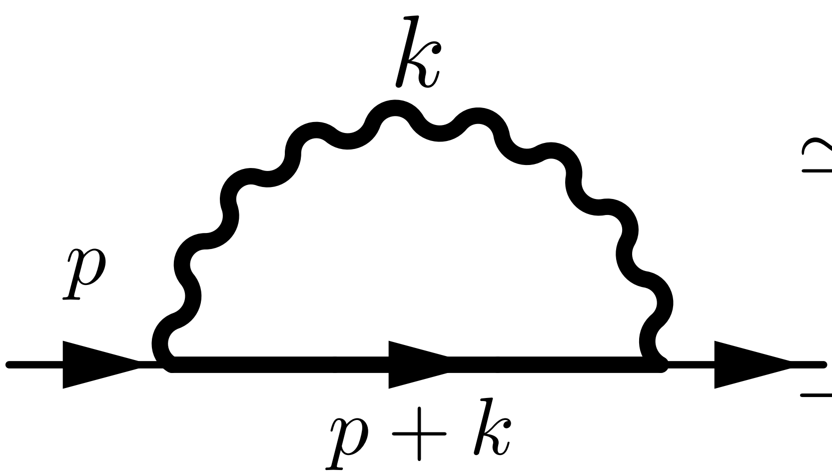
diverges as p→0. **Need improvement.**

Dressed perturbation theory

Dressed propagators

$$D_R(k) = \frac{\not{k}}{k^2 - m_f^2 + 2ik^0\gamma_f} \qquad G_A(k) = \frac{1}{k^2 - m_b^2 - 2ik^0\gamma_b}$$

m_f^2, m_b^2 : **thermal masses** $\qquad \gamma_f, \gamma_b$: **damping rates**



Fermion soft mode in QCD, Lebedev, Smilga ('90)

$$\simeq g^2 \int \frac{d^4k}{(2\pi)^4} \not{k}(n_F(k) + n_B(k)) \frac{1}{\delta m^2 + 2(p\cdot k + ik^0\gamma)}$$

$$\xrightarrow[p\to 0]{} g^2 \int \frac{d^4k}{(2\pi)^4} \not{k}(n_F(k) + n_B(k)) \frac{1}{\delta m^2 + 2(ik^0\gamma)}$$

Finite, improved!

where $\delta m^2 = m_b^2 - m_f^2 \quad \gamma = \gamma_b + \gamma_f$

One loop results (QED)

Propagator

$$D_R \simeq -\frac{Z}{2}\left(\frac{\gamma^0 - \hat{\mathbf{p}}\cdot\boldsymbol{\gamma}}{p^0 + v|\mathbf{p}| + i\gamma} + \frac{\gamma^0 + \hat{\mathbf{p}}\cdot\boldsymbol{\gamma}}{p^0 - v|\mathbf{p}| + i\gamma}\right).$$

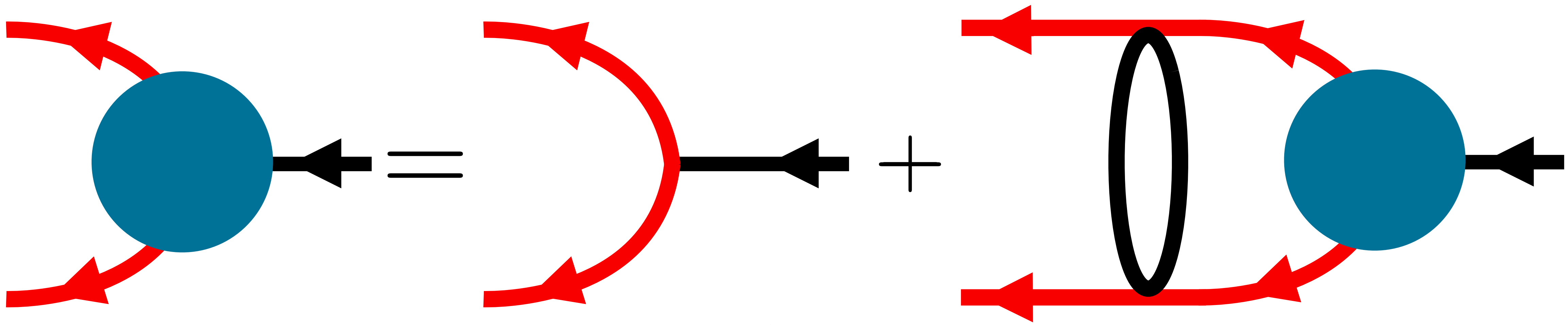
Pole $\omega = \pm\frac{1}{3}p + i\gamma$ **Residue** $Z = \frac{e^2}{16\pi^2}\left(\frac{8\delta m^2}{e^2T^2}\right)^2$

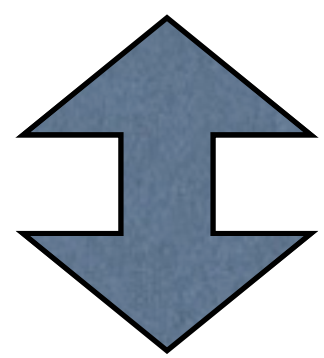
where $\delta m^2 = \frac{e^2}{12}$ $\gamma \sim \frac{e^2}{4\pi}\ln\frac{1}{e}$

But this is not the end of story....

Infinite numbers of higher order diagrams can contribute the leading order.

A similar situation: transport coefficients



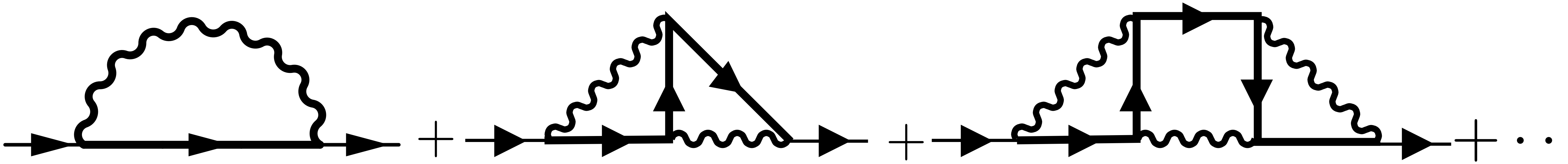


Boltzmann equation

Jeon ('94)

Resummation of ladder diagrams is necessary

Higher loop diagrams

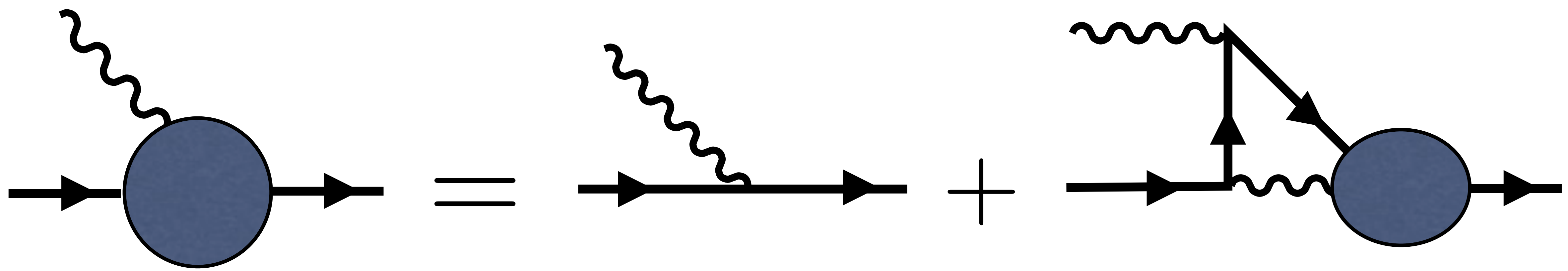


$\sim \frac{1}{g^2}$ $\qquad$ $\sim \left(\frac{1}{g^2}\right)^2 \times g^2 \sim \frac{1}{g^2}$ $\qquad$ $\sim \left(\frac{1}{g^2}\right)^3 \times g^4 \sim \frac{1}{g^2}$

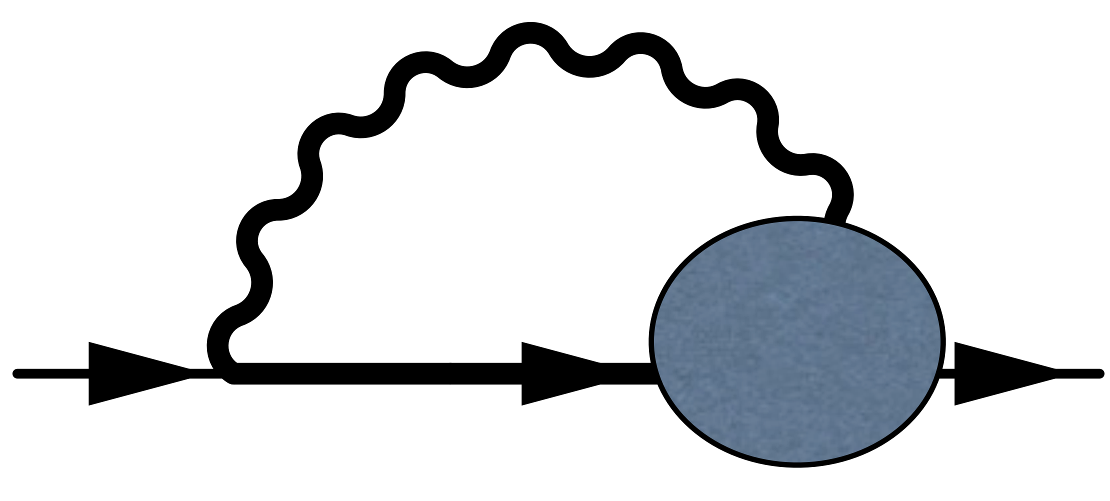
All ladder diagram contributes to the leading order.

Resummation

Self-consistent equation

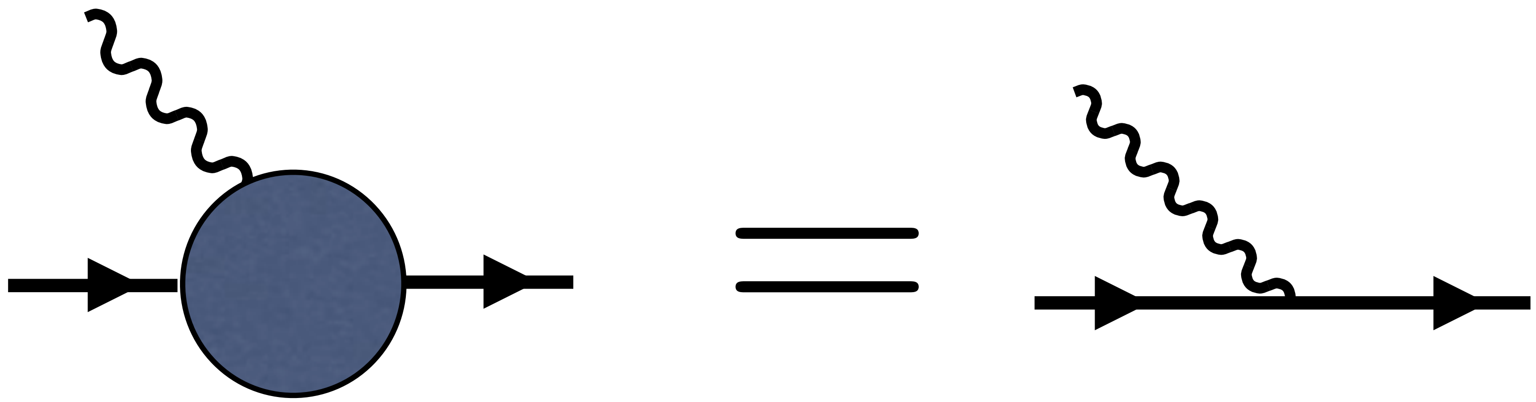


Self-energy in the leading order



Special case: Yukawa model

Yukawa model is a special case.



Ladder diagram is suppressed,
tree vertex function is leading.

The ‘wave function’ in the ladder diagram
$\sim m_f^2 \sim g^2$, so it is higher order correction.

Results

Pole

$$\omega = \pm\frac{1}{3}|\mathbf{p}| + i\gamma$$

Residue

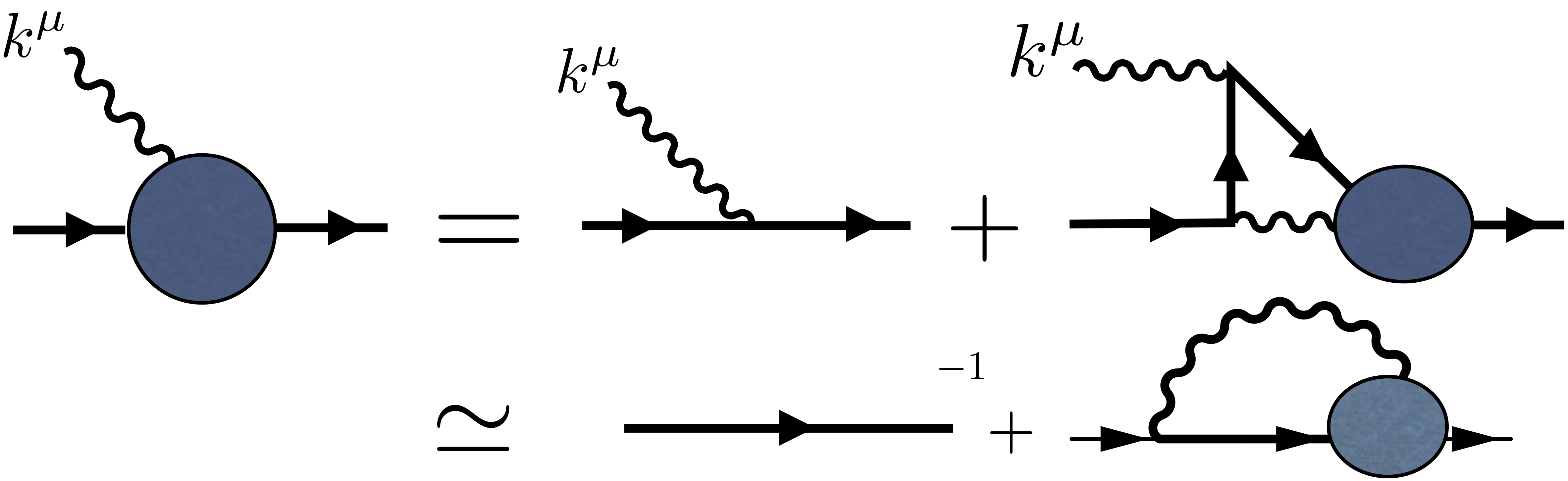
$$Z = \frac{g^2}{16\pi^2}c$$

The velocity is 1/3, the residue is of order g^2.

	γ	c
Yukawa model	$\sim g^4T$	$2/9$
QED	$\sim g^2T$	$1/9$
QCD	$\sim g^2T$	$4(N_f+5)/3$

Ward-Takahashi identity

$$k^\mu \Gamma_\mu(p,k) = \not{p} + \not{k} - \Sigma^R(p+k) - \not{p} + \Sigma^R(p).$$



Ward-Takahashi identity is satisfied in the leading order.

Origin of ultrasoft modes

Supersymmetry?

In a supersymmetric model:

Phonino as Goldstino due to the symmetry breaking of the SUSY at finite T.

Girardello, Grisaru, Salomonson ('81); Boyanovsky ('84); Aoyama, Boyanovsky ('84); Gudmundsdottir, Salomonson ('87). Lebedev, Smilga ('89).

For QCD:

At g=0, supersymmetry can be assigned to quarks and gluons, which is explicitly broken by the interaction.

Levedev and Smilga('90)

Origin of ultrasoft modes

Chiral symmetry no explicit mass

Time reversal

In vacuum

$$D^{-1}(\omega, 0) = -D^{-1}(-\omega, 0)$$ ➡ **pole at** $\omega = 0$

In medium

$$\mathrm{Re}D^{-1}(\omega, 0) = -\mathrm{Re}D^{-1}(-\omega, 0)$$

$$\mathrm{Im}D^{-1}(\omega, 0) = \mathrm{Im}D^{-1}(-\omega, 0)$$

If ReD^{-1} is continuous at ω=0, $\mathrm{Re}D^{-1}(0, 0) = 0$

and if Im D^{-1} is small, pole at $\omega = -i\gamma$

Summary

Ultrasoft fermionic mode

Pole

$$\omega = \pm\frac{1}{3}p + i\gamma$$

Residue

$$Z = \frac{g^2}{16\pi^2}c$$

Ward-Takahashi identity: OK

Outlook

Kinetic theory

for the ultrasoft fermionic modes.

(in preparation Satow and YH)

Observables

sensitive to the ultrasoft fermionic modes.