Ginzburg-Landau approach to inhomogeneous chiral condensates

Hiroaki Abuki (Science University of Tokyo)
in collaboration with *Daisuke Ishibashi, Katsuhiko Suzuki*

Ref: Abuki, Ishibashi, Suzuki, arXiv: 1109.1615

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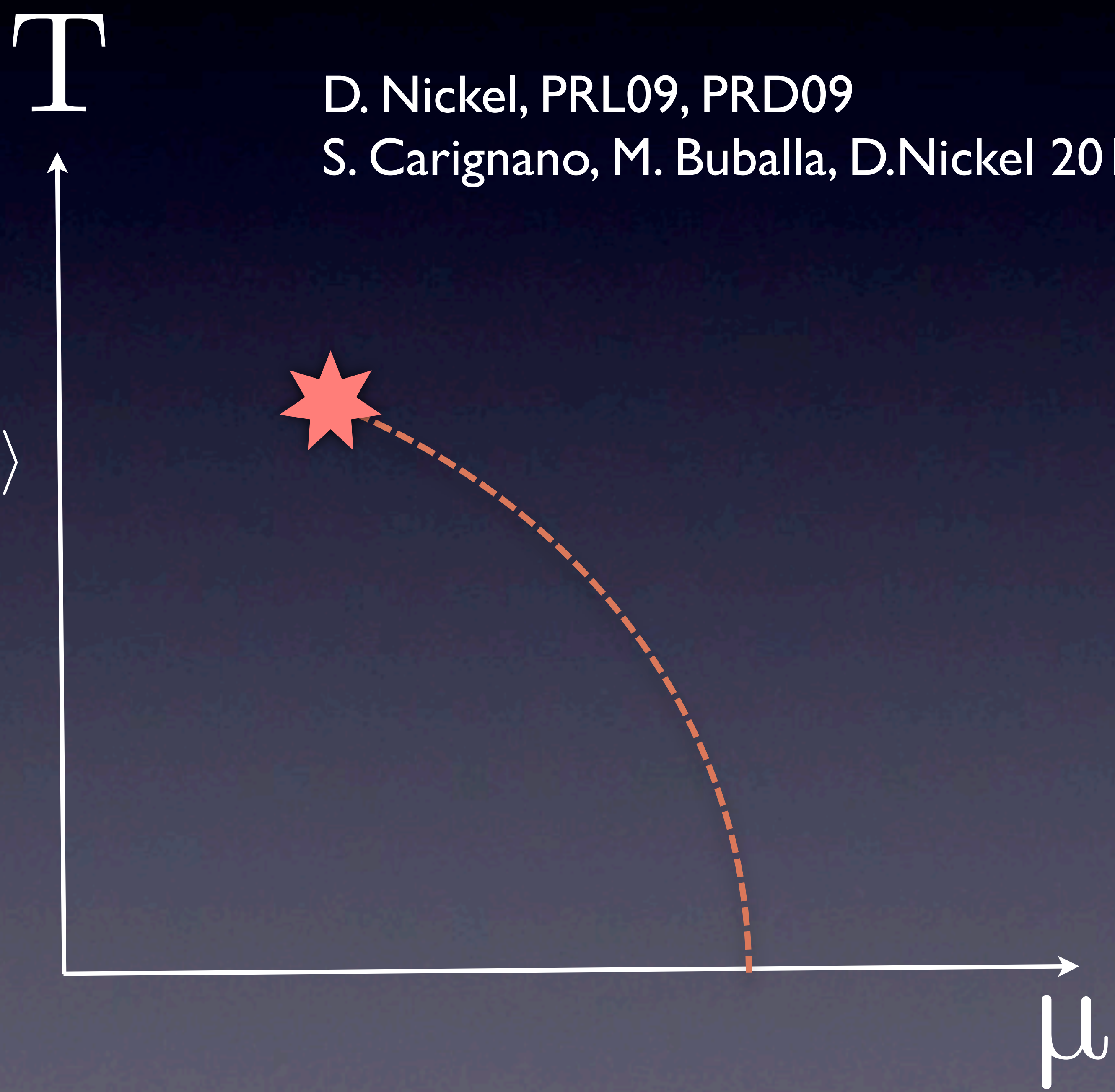
v2 will be coming soon...

Inhomogeneous chiral condensate

- A "*halfway*" state produced as a compromise b/w two conflicting effects:
  - ★ *net quark density* $\langle q^+ q \rangle$
  - ★ *q-qbar pair density* $\langle \bar{q} q \rangle$
- "*Hierarchical*" chiral restoration
- Similar example: FFLO states

D. Nickel, PRL09, PRD09
S. Carignano, M. Buballa, D.Nickel 2010



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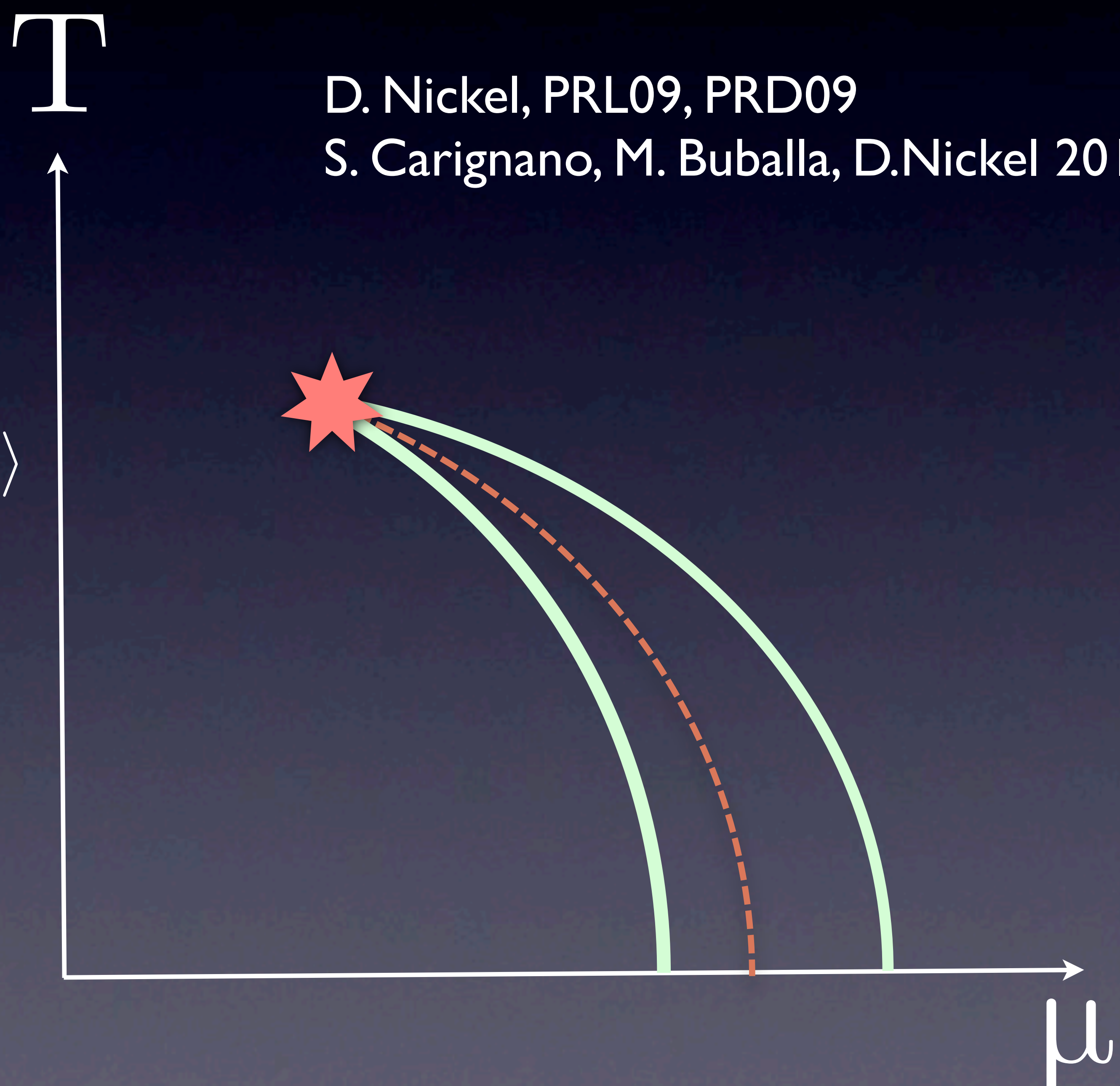
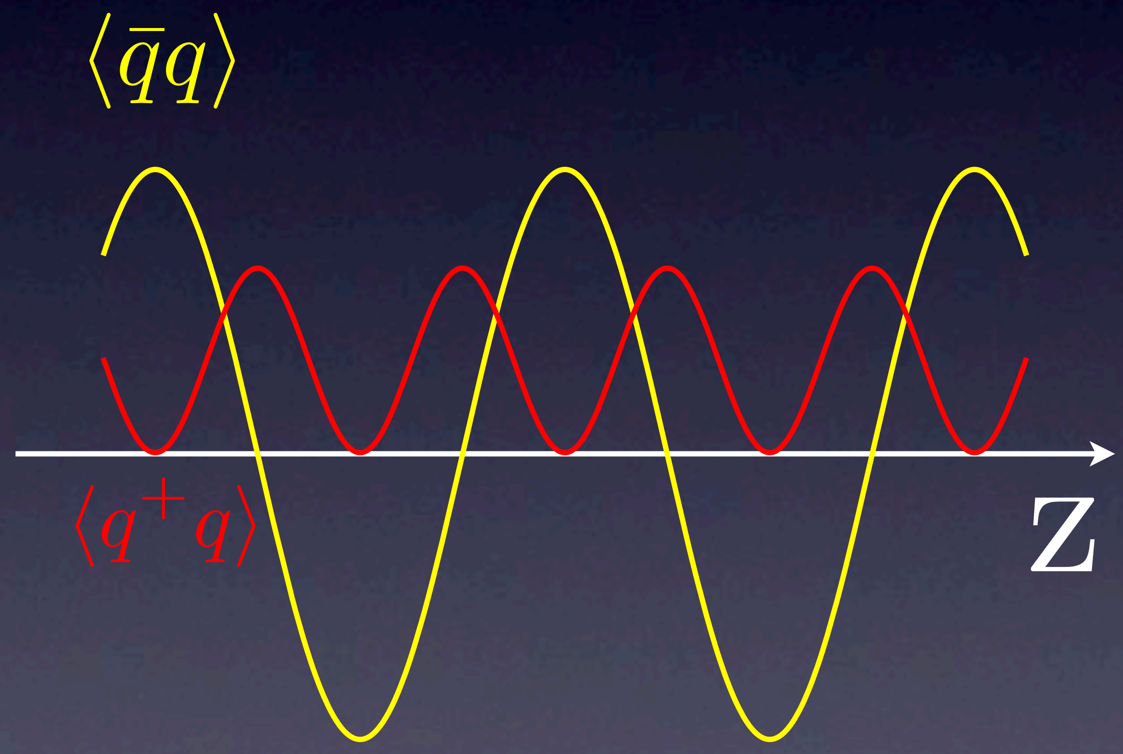


Two types of 1D Chiral Crystal

- Periodical chiral restoration in the real spatial space
  - ⋆ Larkin-Ovchinnikov (LO) type
- Chiral condensate forms partially in the momentum space
  - ⋆ Flude-Ferrel (FF) type

c.f. chiral spiral in quarkyonic matter
Kojo, Hidaka, McLerran, Pisarski (2010)

Spatial modulation in one direction (LO)



c.f. for explicit demonstration, see
Carignano, Buballa, Nickel 2010

This Talk

1. Multidimensional modulations near CP?
   - Low dimensional modulation in 3D might be unstable at finite T

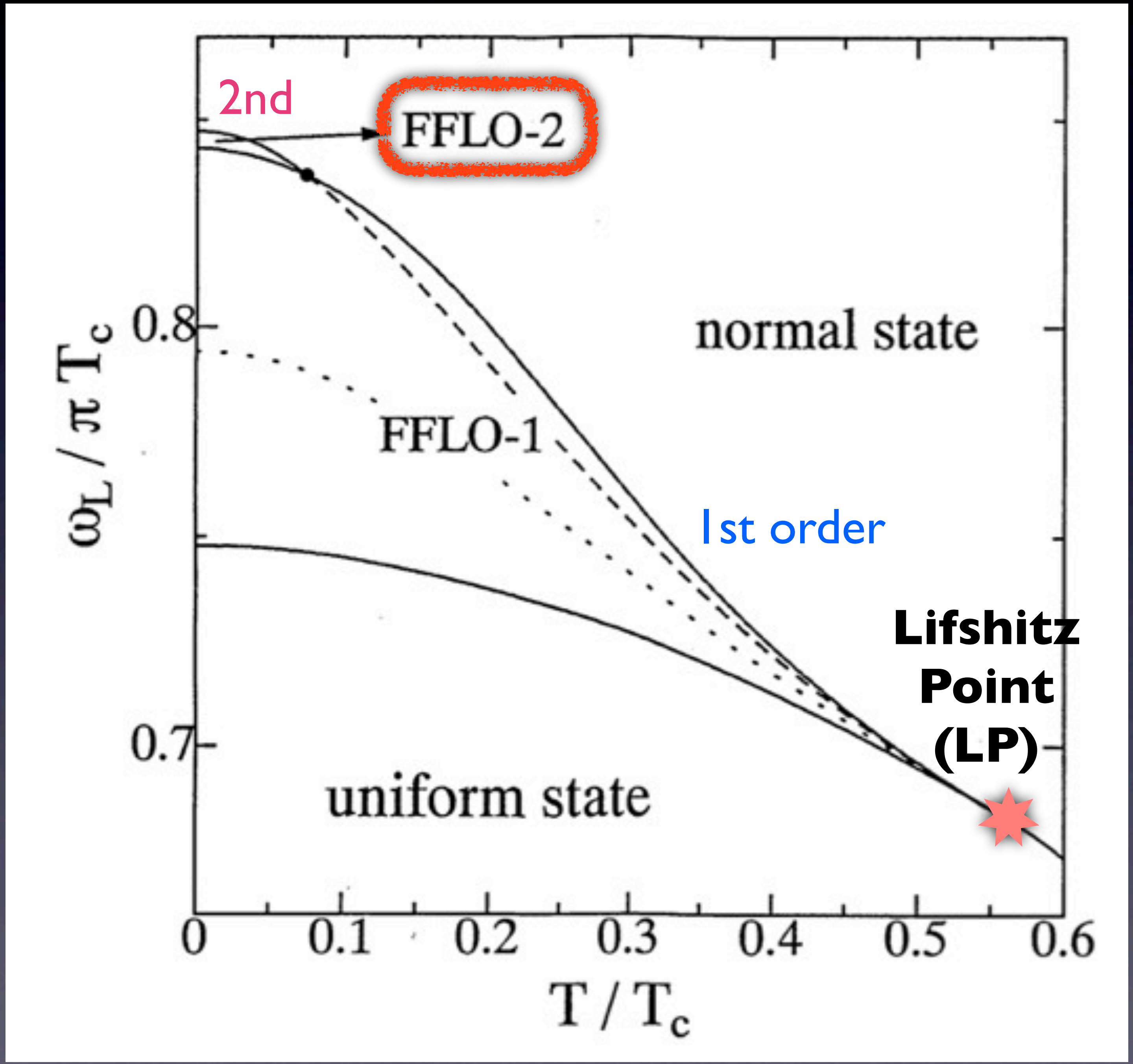
     Landau, Peierls Theorem: Baym, Friman, Grinstein, NPB1982

     Several works related:
     Quarkyonic chiral spiral; Kojo, Hidaka, Fukushima, McLerran, Pisarski (2011)
     NJL analysis at T=0; Carignoano, Buballa (2011)

2. How does the phase structure change going away from the critical point?
   - Richer phase structure even in the chiral limit?
   - Need to go beyond the minimal GL framework

Richer phase structure?

- New state may appear at low T region far from LP
- two types of FFLO states compete & New critical endpoint located away from LP
- 8th order terms in GL is needed!



Matsuo, Higashitani, Nagato, Nagai, JPSJ 1997
Quasi-classical green function method was employed

Phase Diagram of the Fulde-Ferrell-Larkin-Ovchinnikov State in a Three-Dimensional Superconductor

Shigemasa MATSUO, Seiji HIGASHITANI, Yasushi NAGATO and Katsuhiko NAGAI

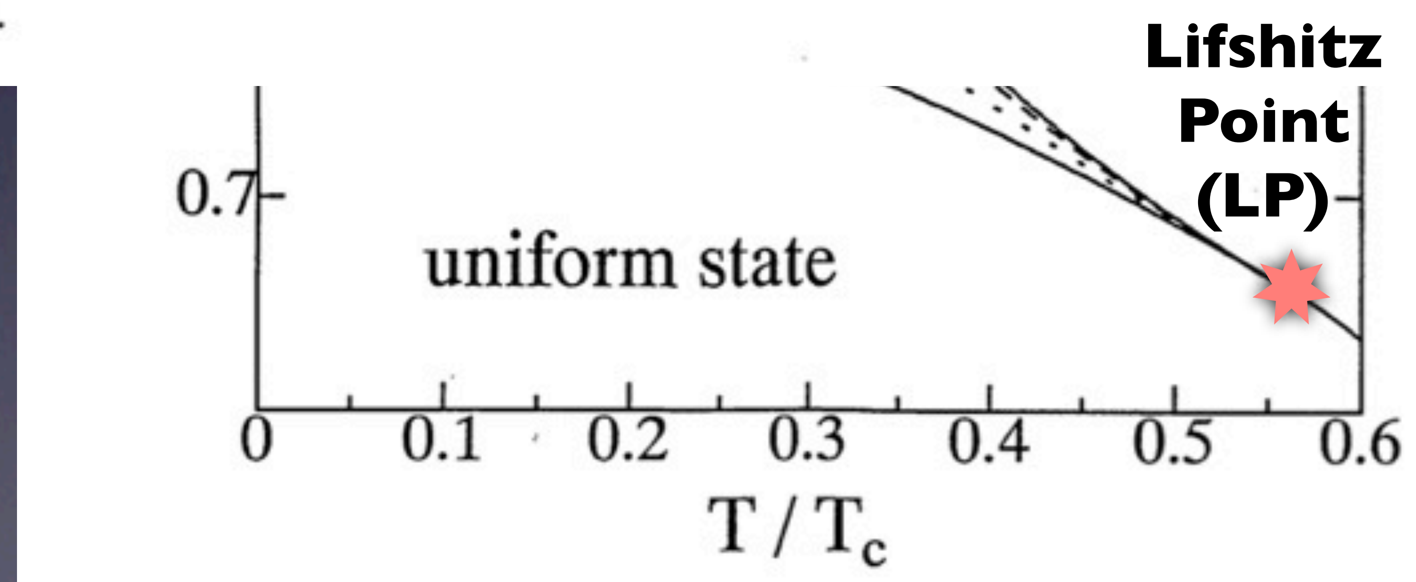
Faculty of Integrated Arts and Sciences, Hiroshima University, Higashi-hiroshima 739

(Received August 7, 1997)

The phase diagram of the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state in a 3-dimensional superconductor is discussed. We use the quasi-classical Green's function to calculate the free energy. It is shown that the phase transition from the normal state is of first order at high temperatures but is of second order near $T = 0$. To describe the phase transition in the Ginzburg-Landau theory, one has to take into account up to eighth order term with respect to the order parameter. The phase transition from the FFLO state to the uniform superconducting state is of second order in all the temperature range $T < 0.561T_c$ in accordance with Burkhardt and Rainer's result in a 2-dimensional system.

endpoint located away from LP

- 8th order terms in GL is needed!



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This Talk

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2. How does the phase structure change going away from the critical point?

This Talk

1. Multidimensional modulations near CP?

Ginzburg-Landau approach

- Expansion of Free energy w.r.t. the condensate and its spatial derivative

D. Nickel, PRL09

$$\Omega_{\rm GL} = \frac{\alpha_2}{2}M(\mathbf{x})^2 + \frac{\alpha_4}{4}M(\mathbf{x})^4 + \frac{\alpha_6}{6}M(\mathbf{x})^6$$
$$\boxed{+\frac{\alpha_4}{4}(\nabla M)^2 + \frac{5\alpha_6}{6}M^2(\nabla M)^2 + \frac{\alpha_6}{12}(\nabla\Delta M)^2}$$

★ Based on the symmetry & model independent in the vicinity of CP; Also applicable for multi-dimensional modulations

Dimensional analysis and Scaling

- Introducing dimensionless variables:

$$\alpha_2 = \eta_2\,[\alpha_4^2/\alpha_6] \qquad \Omega = \omega\,[|\alpha_4|^3/\alpha_6^2]$$

$$M = m\,[\sqrt{|\alpha_4|/\alpha_6}] \qquad \mathbf{x} = \tilde{\mathbf{x}}\,[\sqrt{\alpha_6/|\alpha_4|}]$$

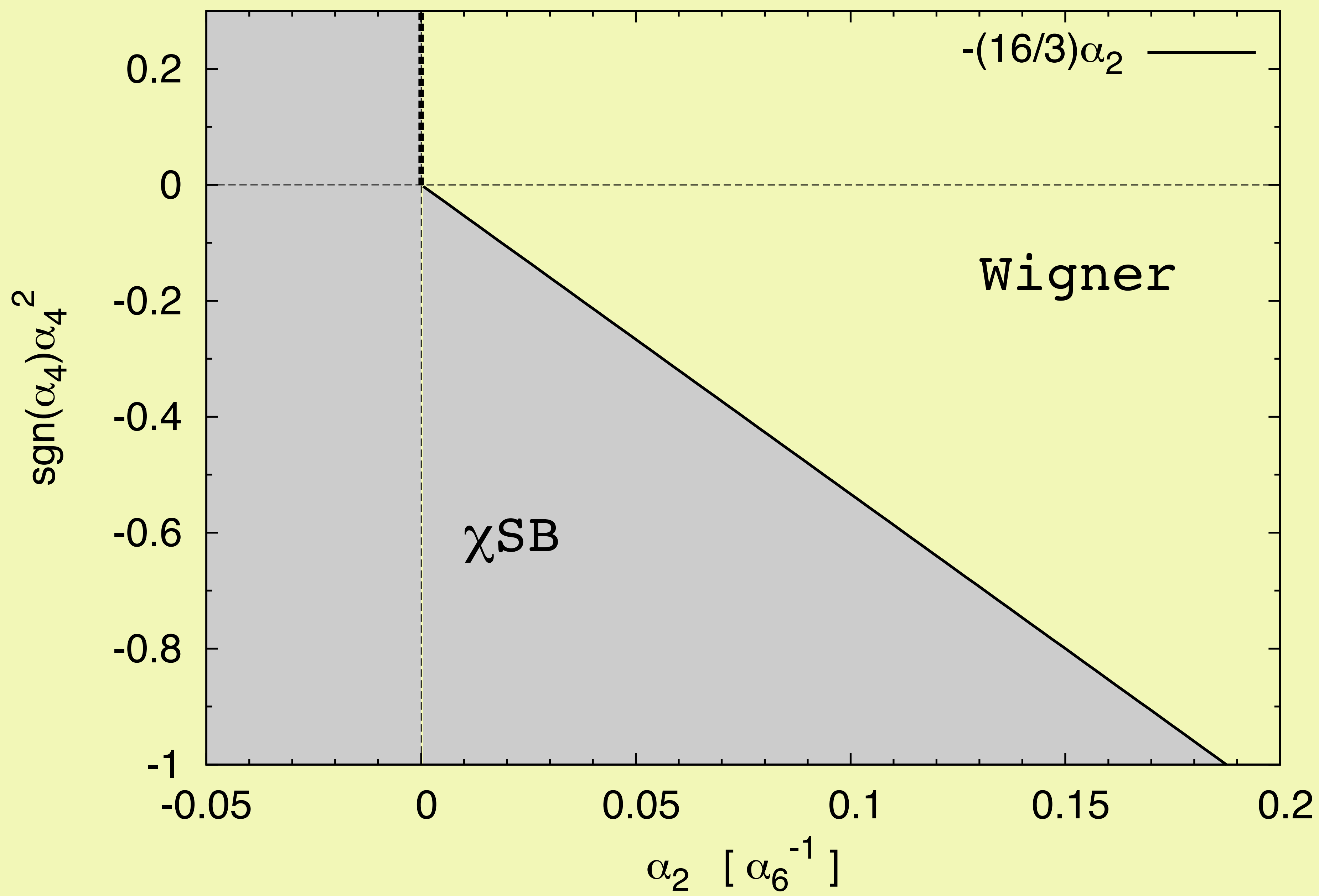
- Relevant parameters are reduced:

$$\begin{aligned}\omega &= \frac{\eta_2}{2}m^2 + \frac{\mathrm{sgn}(\alpha_4)}{4}(m^4 + (\tilde{\nabla}m)^2) \\ &\quad + \tfrac{1}{6}\left(m^6 + 5m^2(\tilde{\nabla}m)^2 + \tfrac{1}{2}(\tilde{\nabla}\tilde{\Delta}m)^2\right)\end{aligned}$$

- Inhomogeneous phase appears when α_4 becomes negative

GL in the vicinity of critical point

D. Nickel, PRL09



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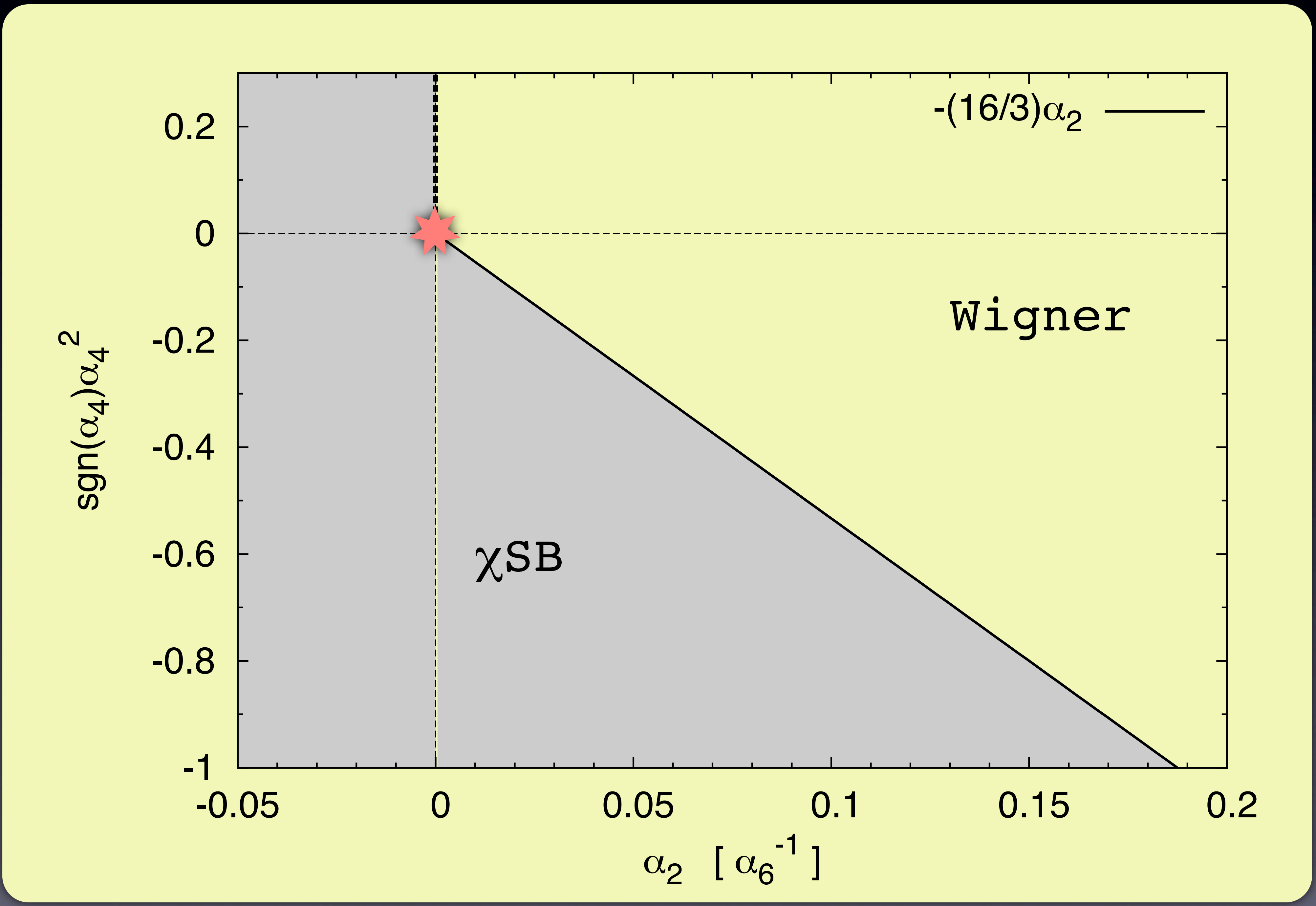

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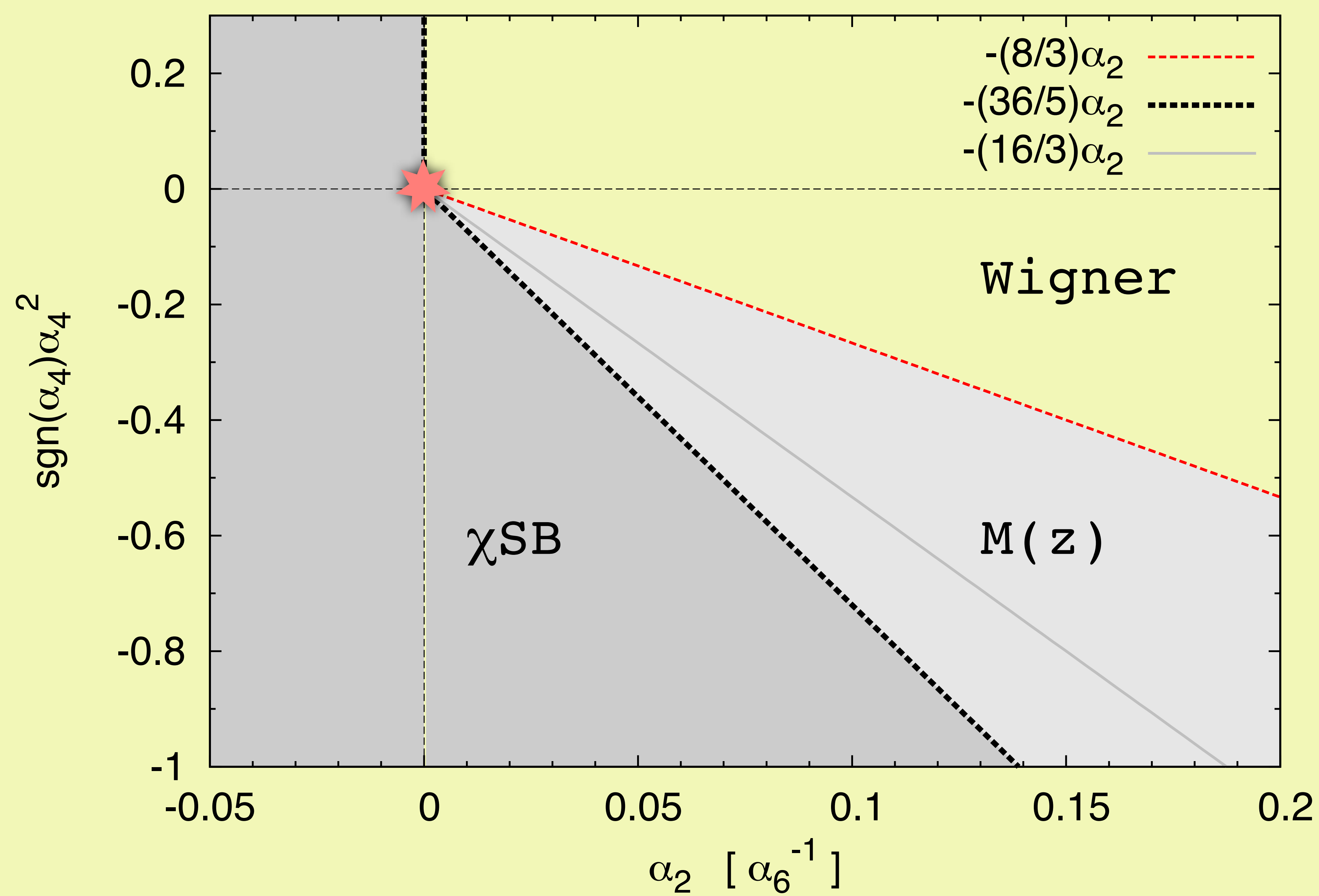

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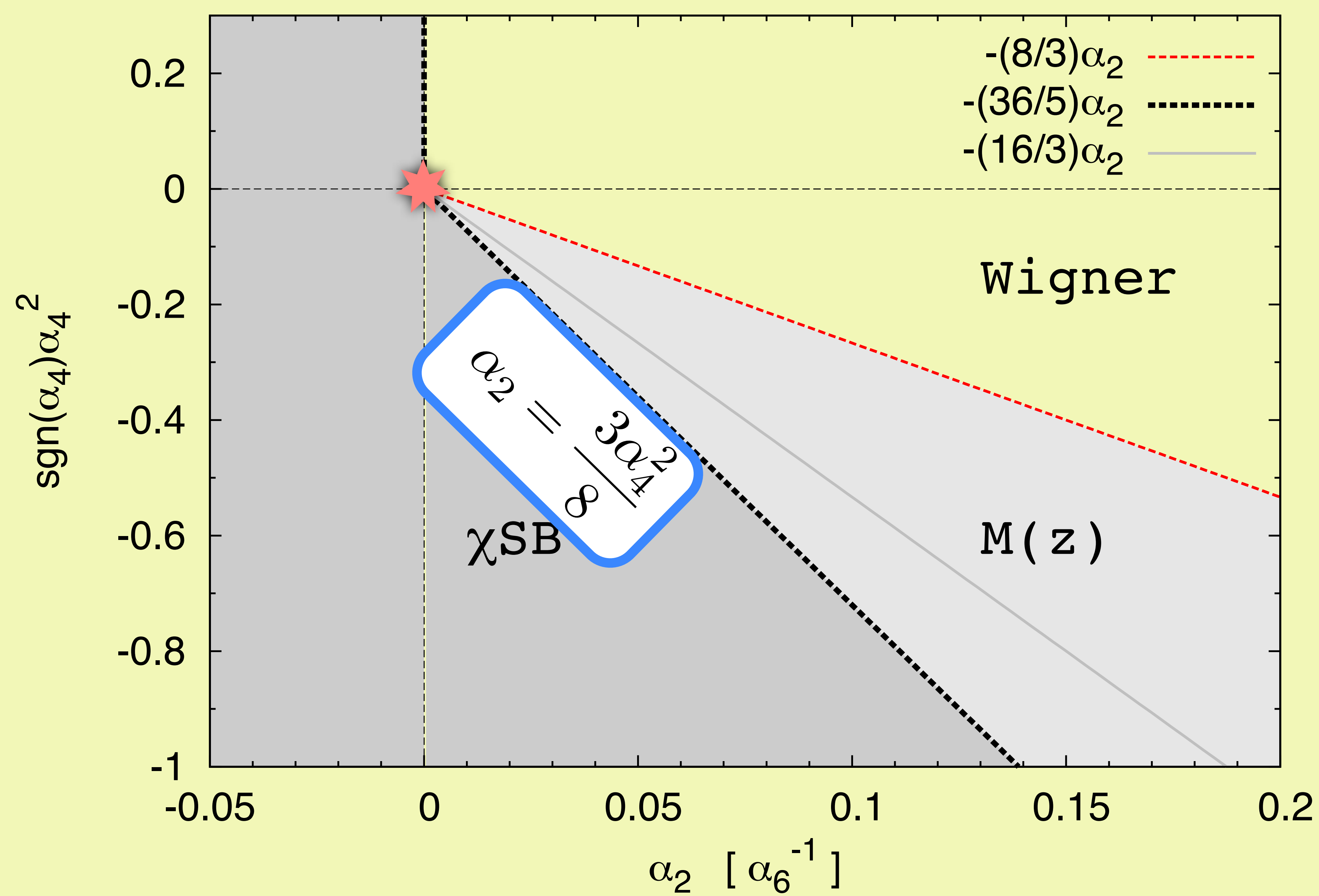

GL in the vicinity of critical point

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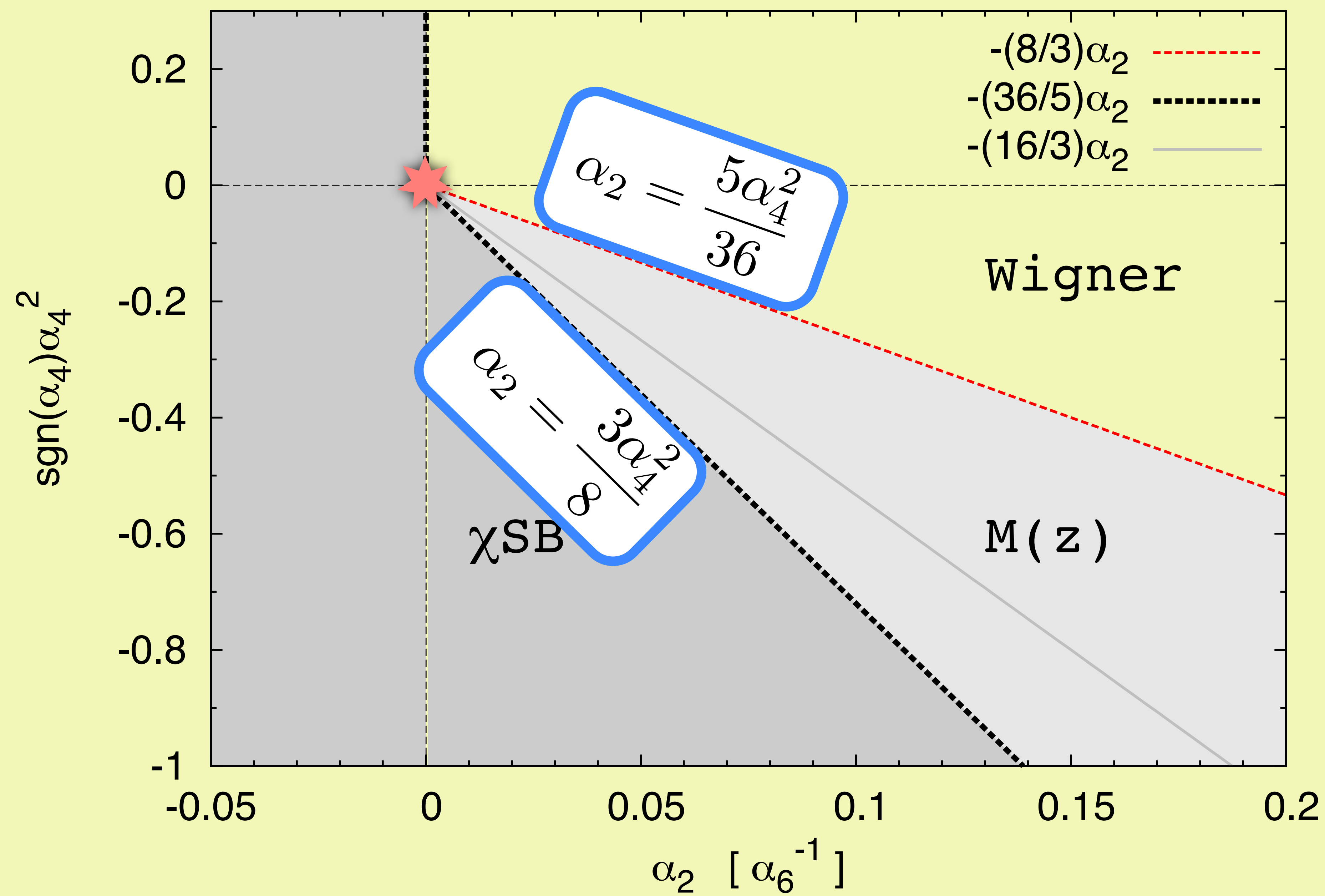
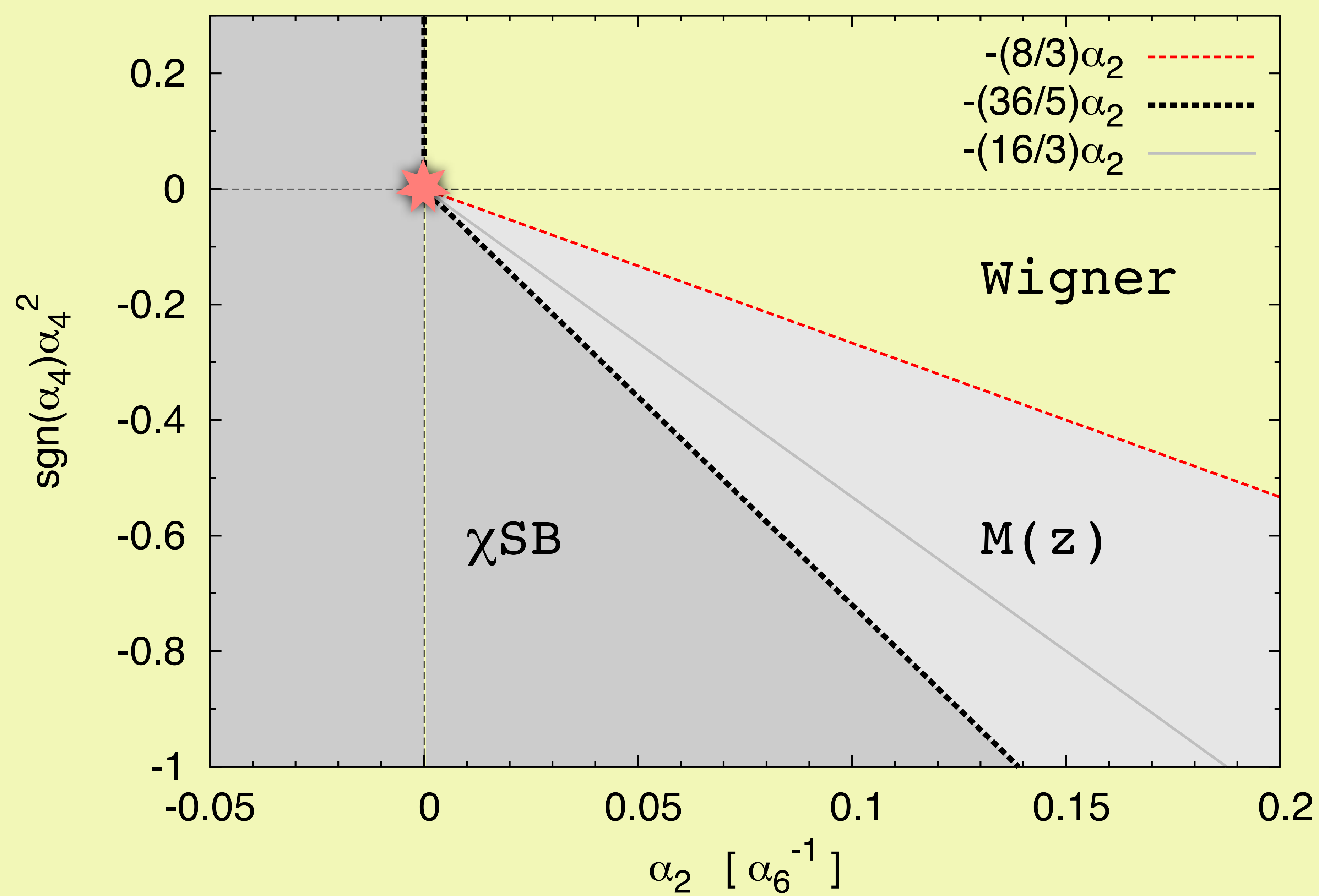


One dimensional modulations

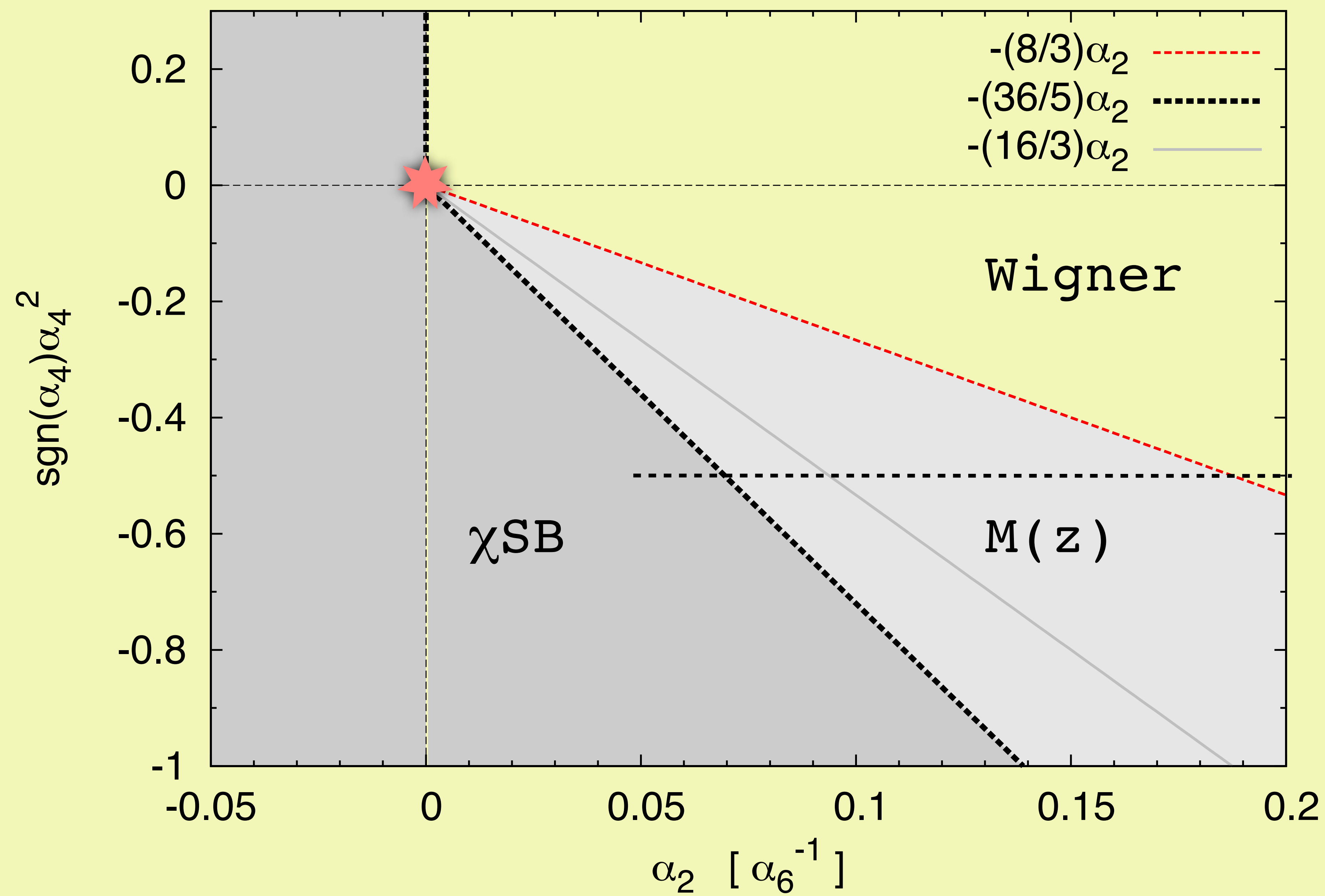
Chiral Condensate: $M(\mathbf{x}) = -2G\left(\langle\bar{q}q\rangle + i\langle\bar{q}i\gamma_5\tau_3 q\rangle\right)$

Fulde-Ferrell (1964) Chiral spiral Nakano-Tatsumi (2005)	$M_{\mathrm{FF}}(z) = \Delta e^{iqz}$	**Complex**
Larkin-Ovchinnikov (1964) CDW; Nickel (2009)	$M_{\mathrm{LO}}(z) = \Delta \sin(qz)$	**Real**
Solitonic chiral condensate Buzdin, Kachkachi (1997) Thies (2006), Nickel (2009)	$M_{\mathrm{sn}}(z) = \sqrt{\nu}q\,\mathrm{sn}(qz,\nu)$	**Real**
Higher harmonics truncated at N=5	$M_{\mathrm{hh}}(z) = \sum_{-N}^{N} \Delta_n e^{inqz}$	**Complex**

Comparison of free energies



Comparison of free energies



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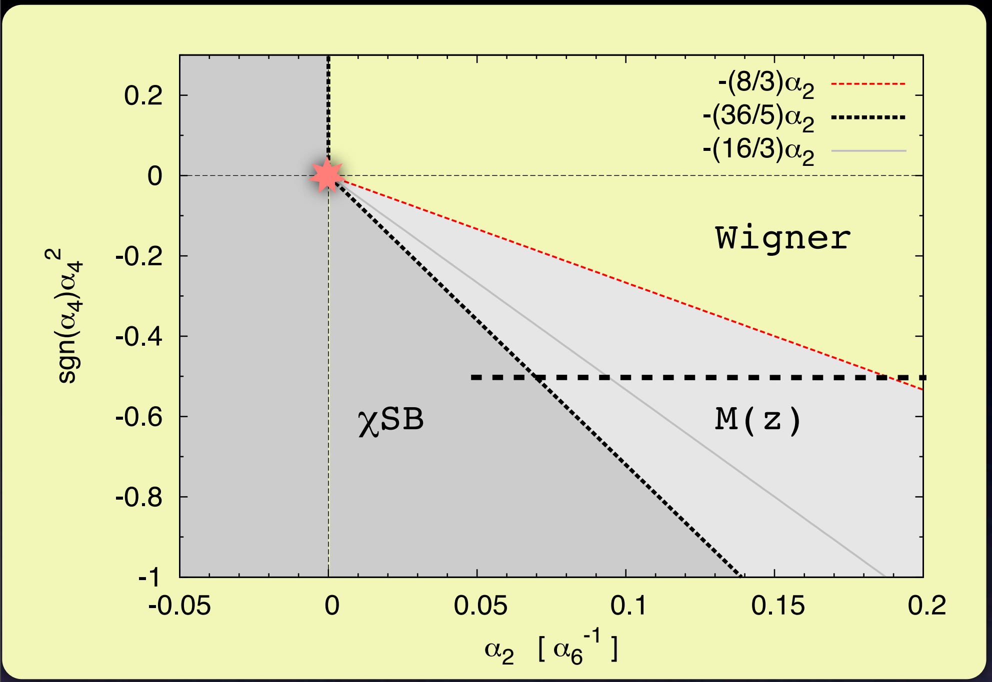
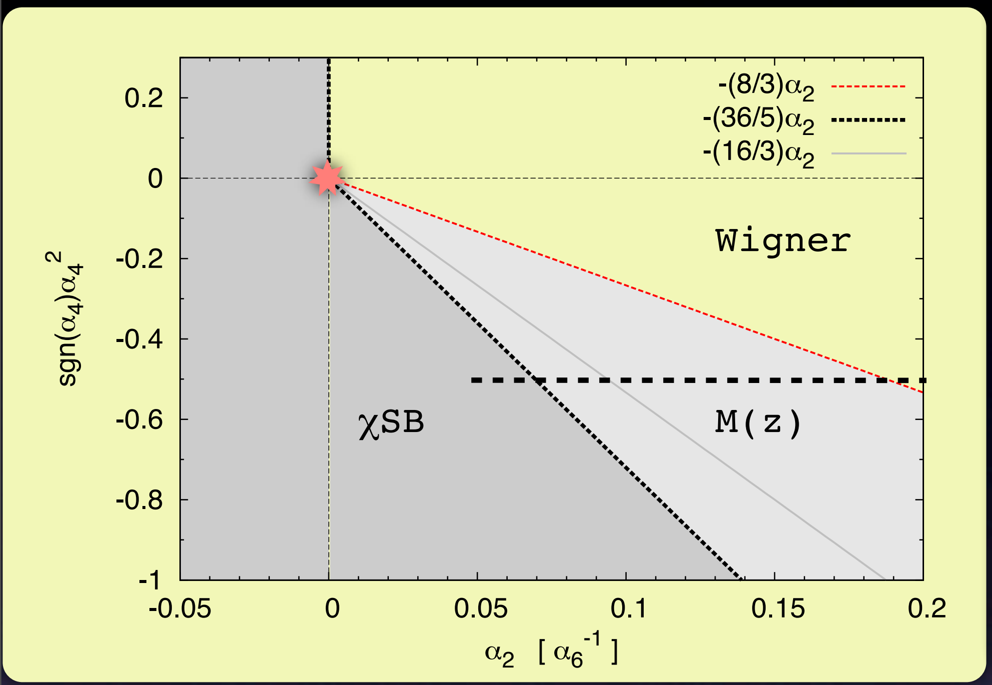
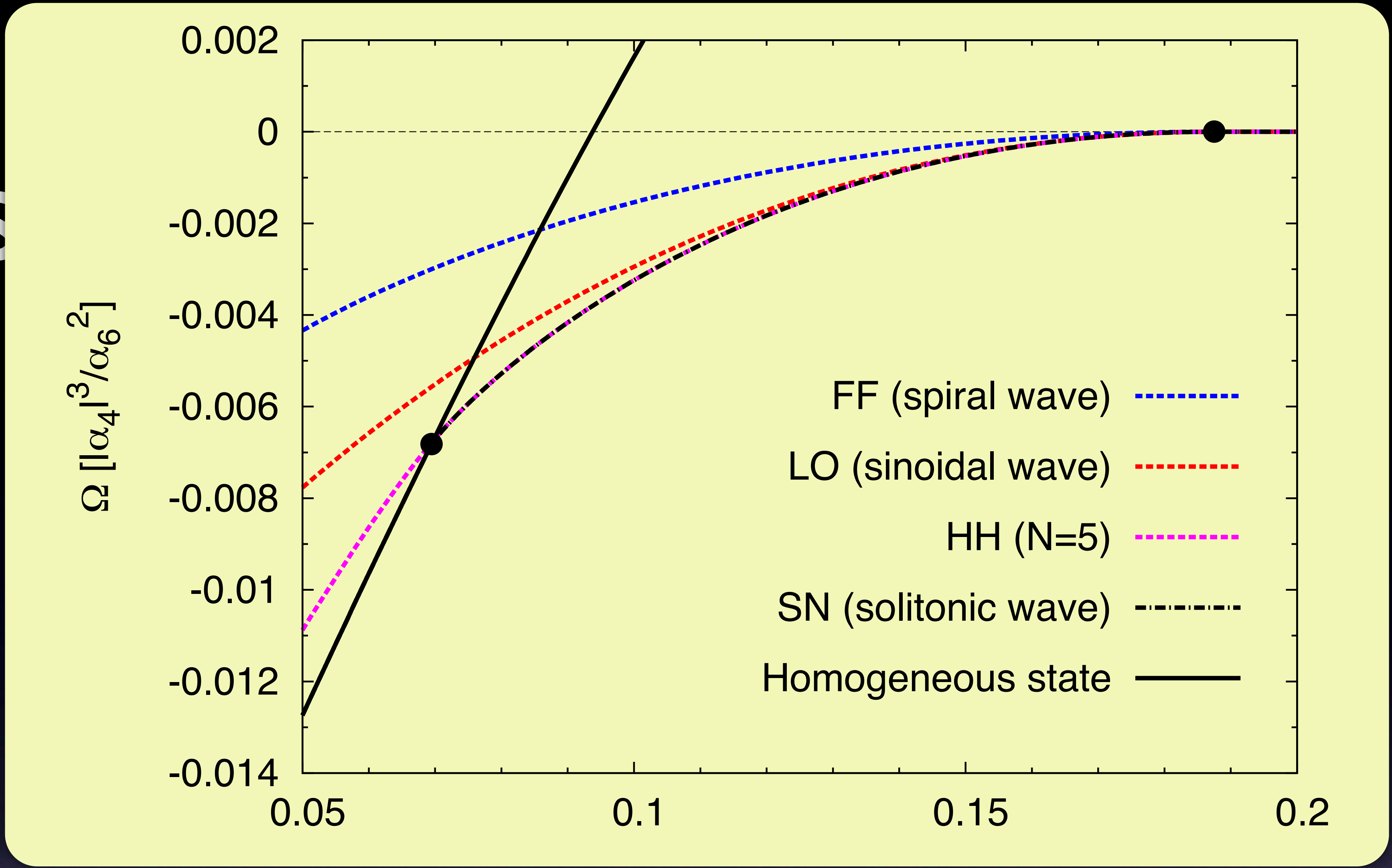


Comparis

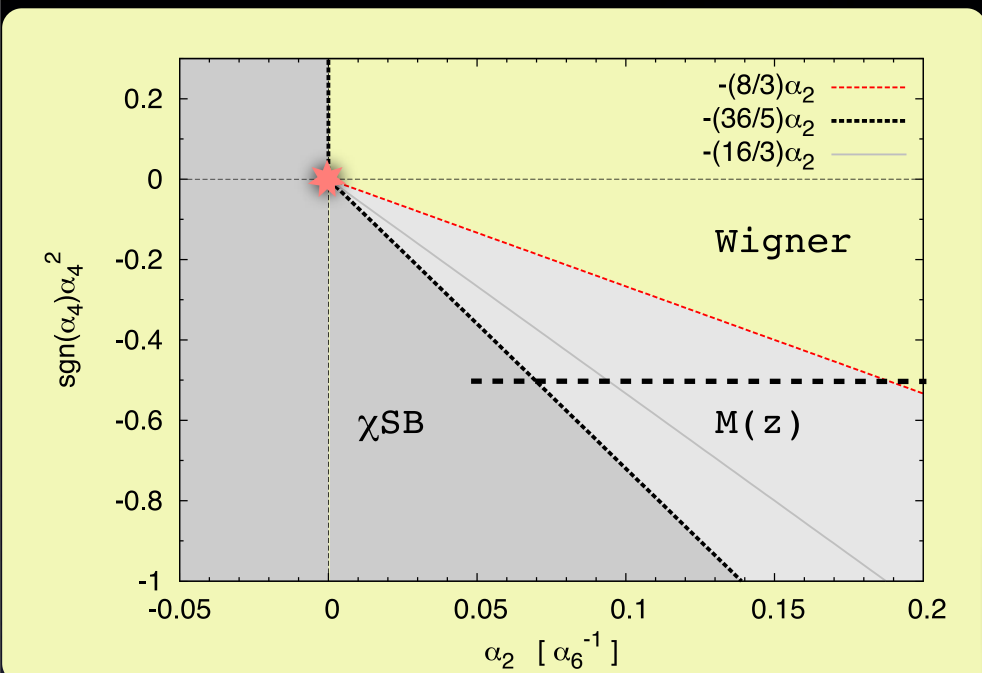


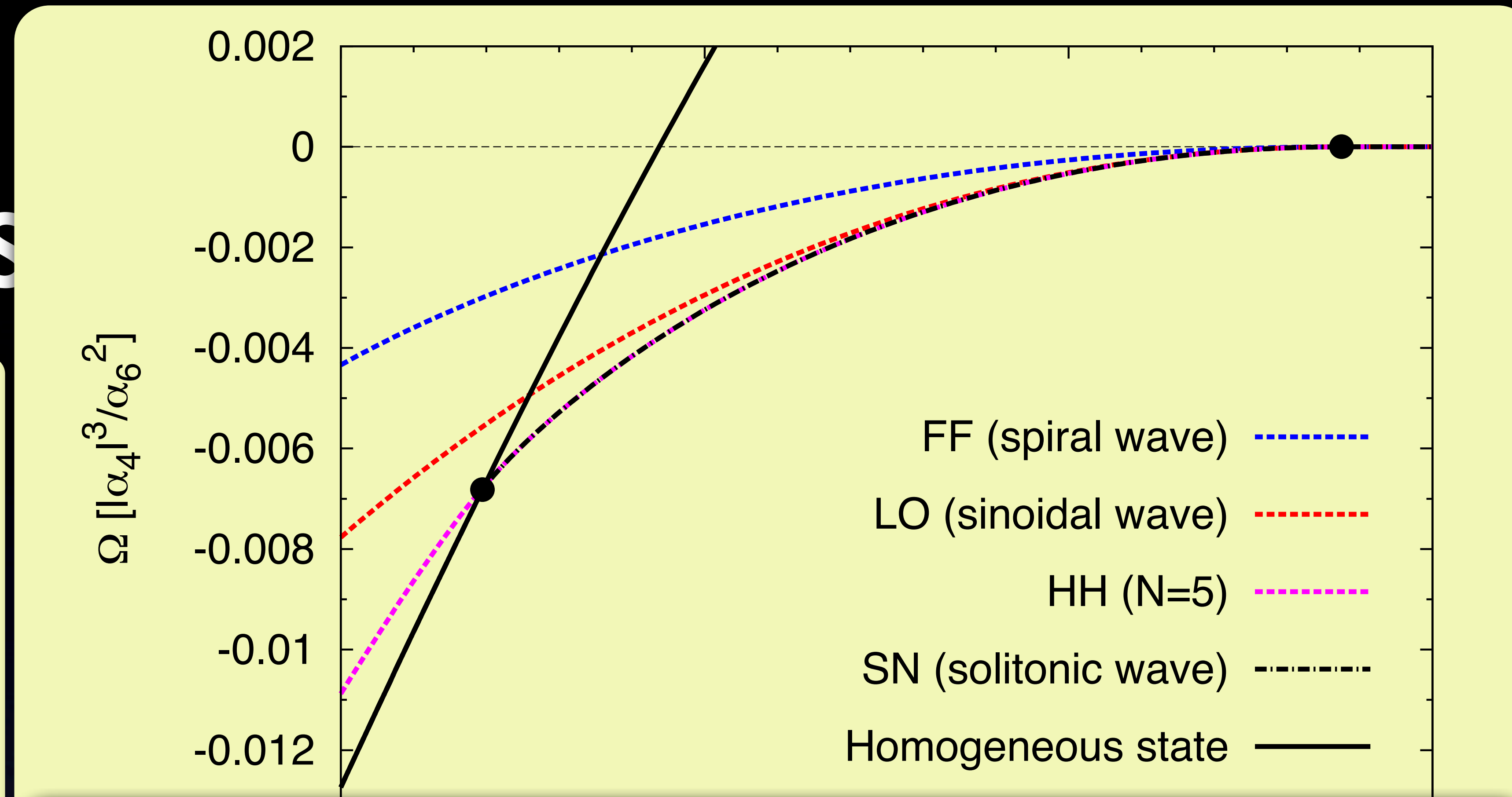
-(8/3)α_2
-(36/5)α_2
-(16/3)α_2
Wigner
χSB
M(z)
sgn(α_4)α_4^2
α_2 [α_6^{-1}]

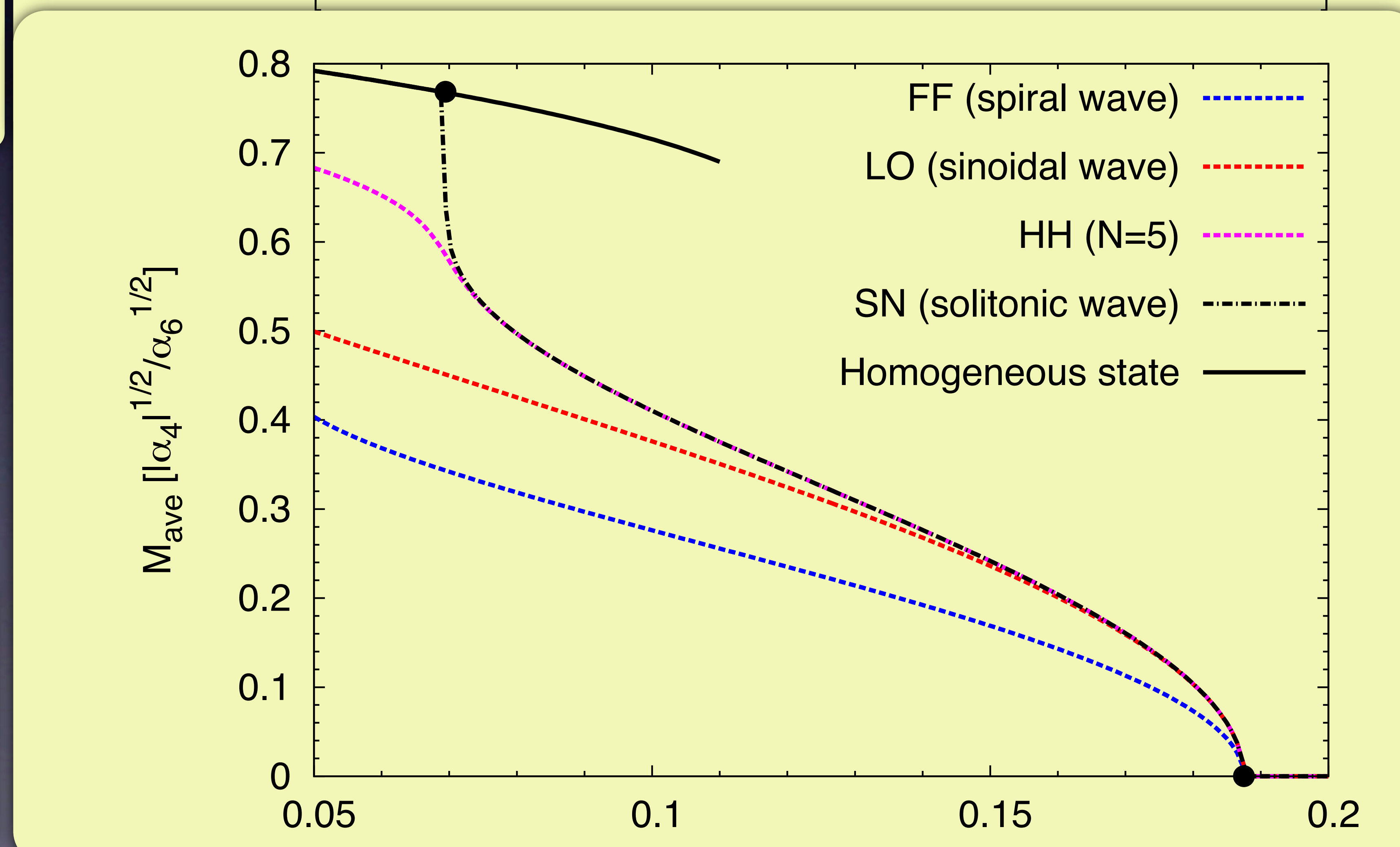


Ω [$|\alpha_4|^3/\alpha_6^2$]
FF (spiral wave)
LO (sinoidal wave)
HH (N=5)
SN (solitonic wave)
Homogeneous state

Comparis







Averaged mass as an order parameter

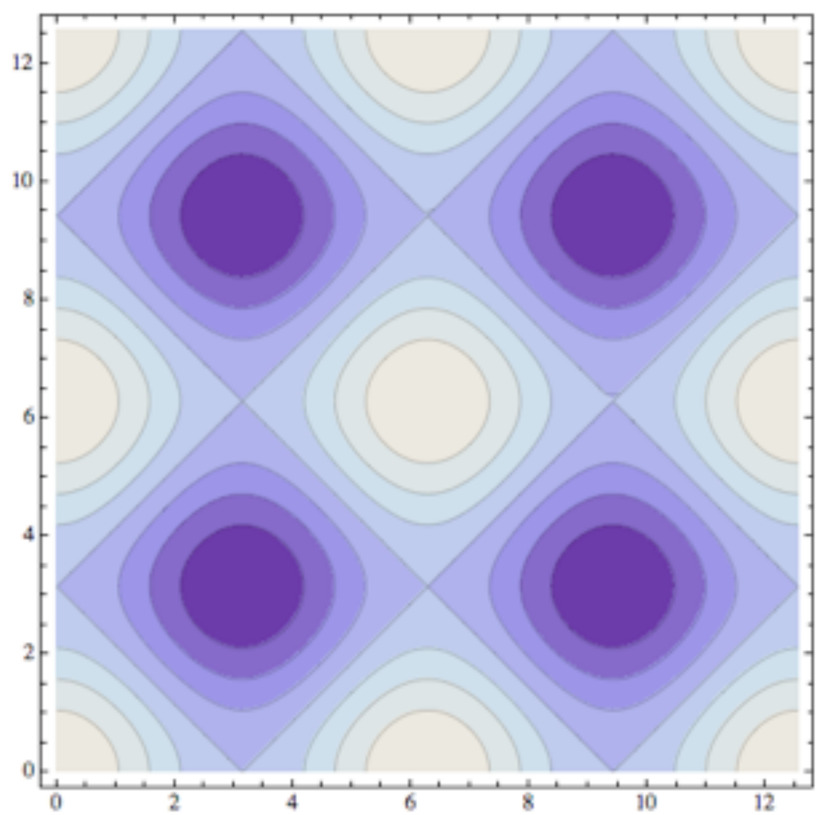
$$M_{\mathrm{ave}} = \sqrt{\langle M(z)^2 \rangle}$$

Multidimensional modulations?

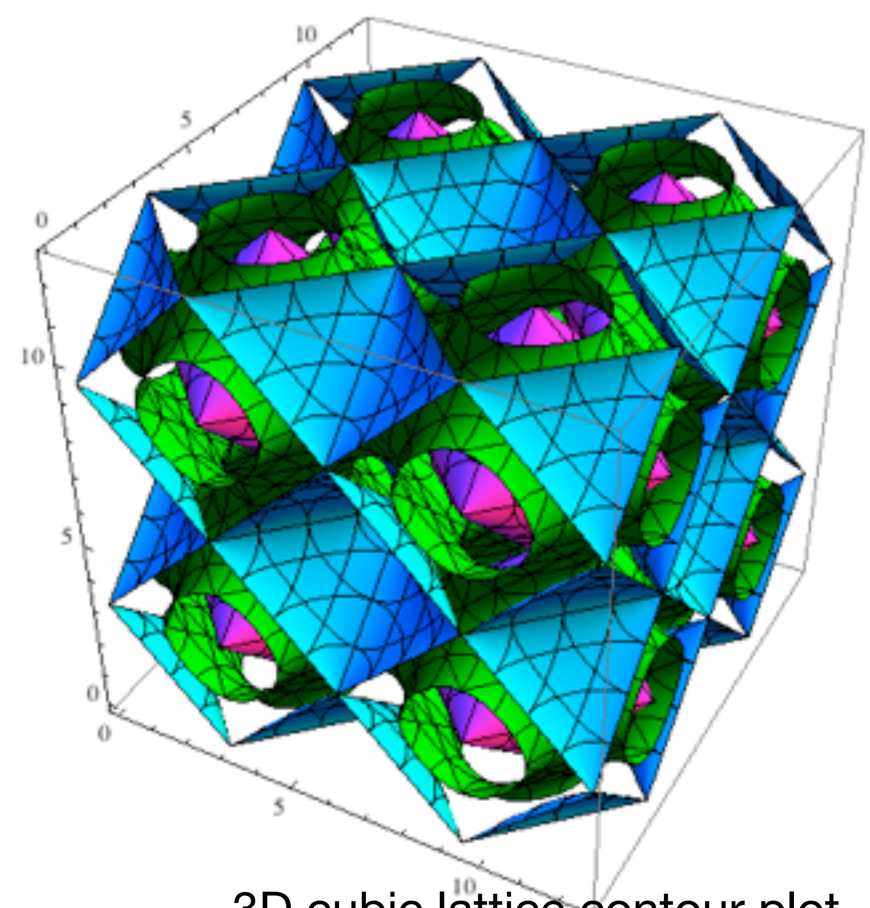
2D square lattice : $M_{\text{2D-LO}} = M_{ave}(\sin(qx) + \sin(qy))$

"egg-carton ansatz"; Carignano, Buballa (2011)

3D cubic lattice : $M_{\text{3D-LO}} = \sqrt{\frac{2}{3}} M_{ave}(\sin(qx) + \sin(qy) + \sin(qz))$

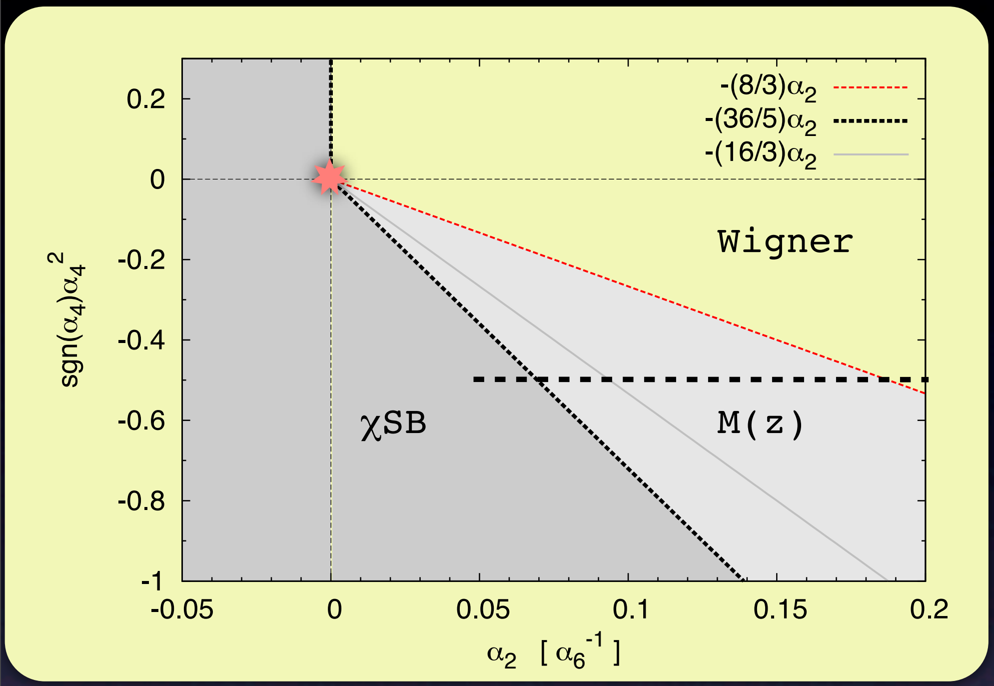


2D square lattice contour plot

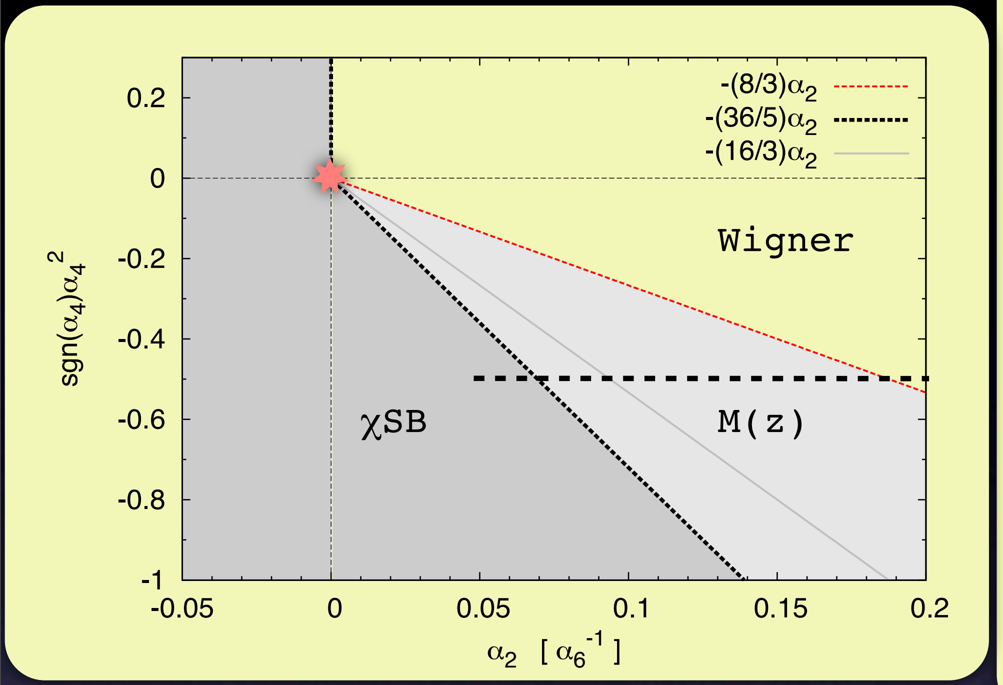


3D cubic lattice contour plot

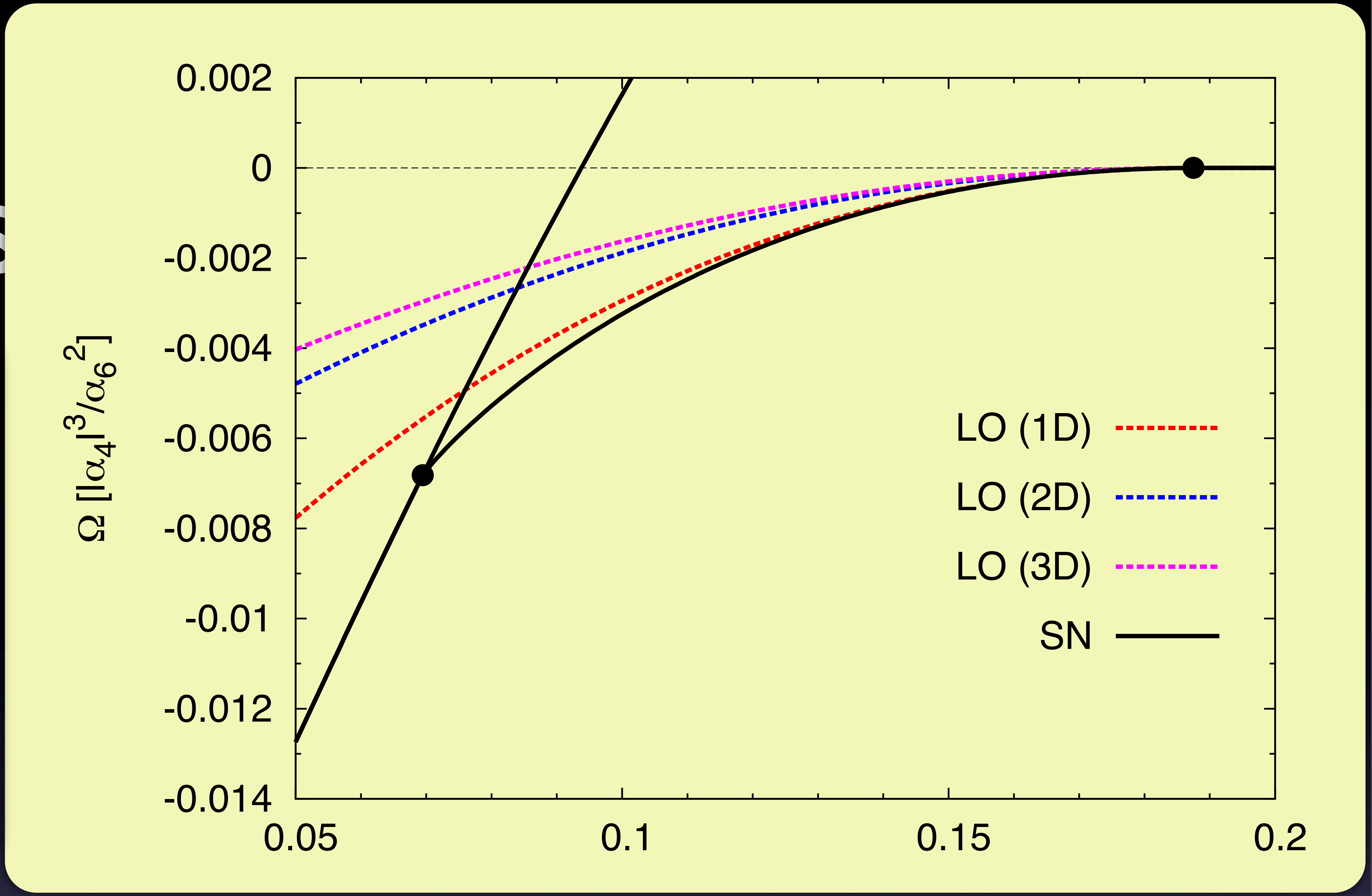
Comparison of free energies



Comparis

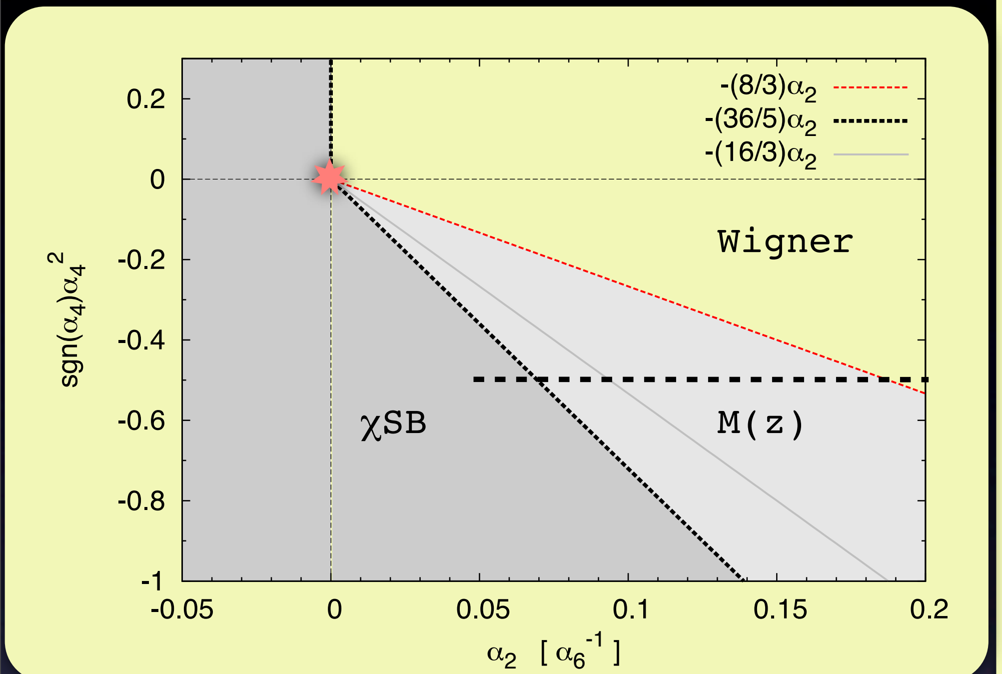


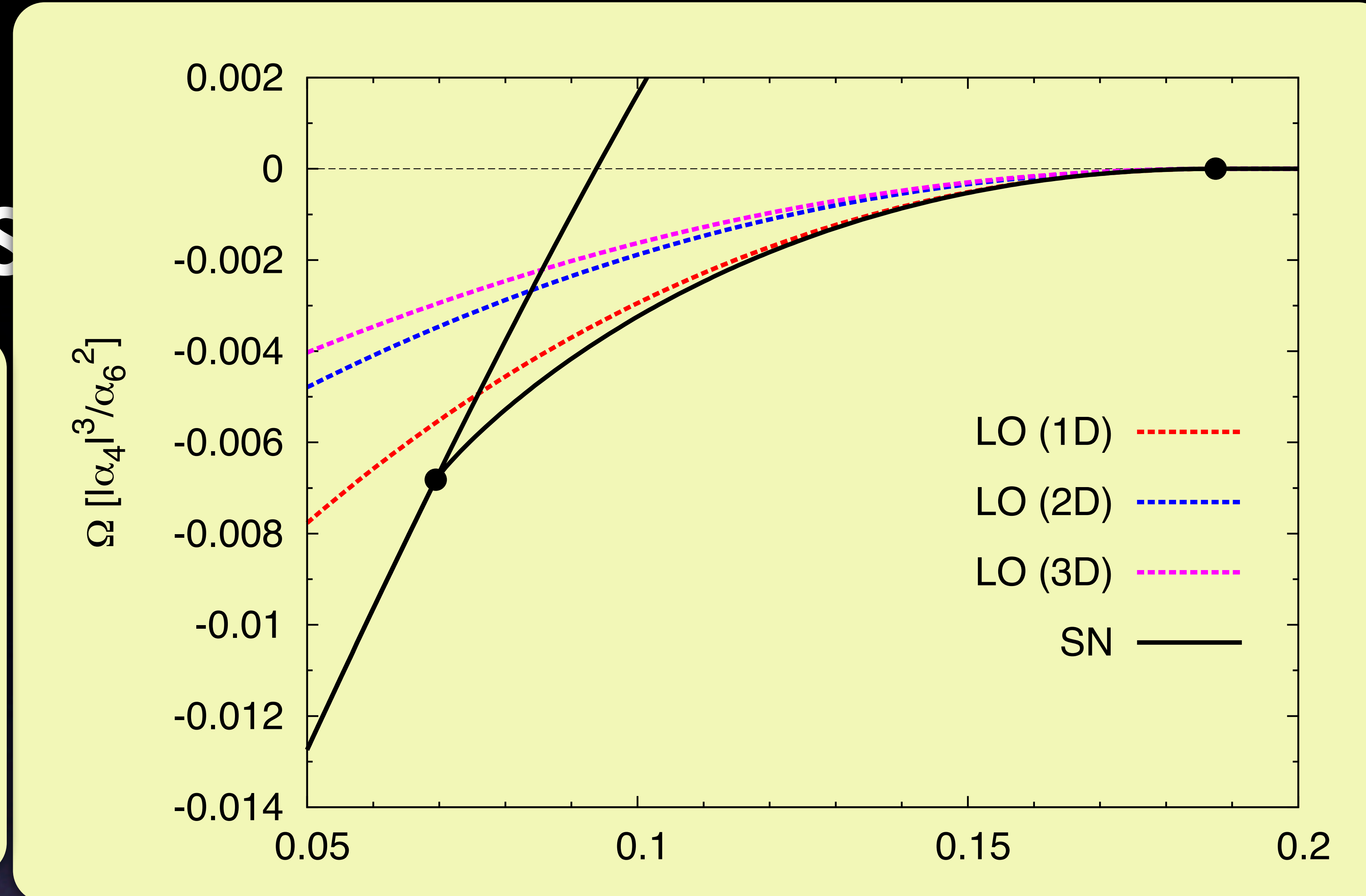
-(8/3)α_2
-(36/5)α_2
-(16/3)α_2
Wigner
χSB
M(z)
sgn(α_4)α_4^2
α_2 [α_6^{-1}]



Ω [$|\alpha_4|^3/\alpha_6^2$]
LO (1D)
LO (2D)
LO (3D)
SN

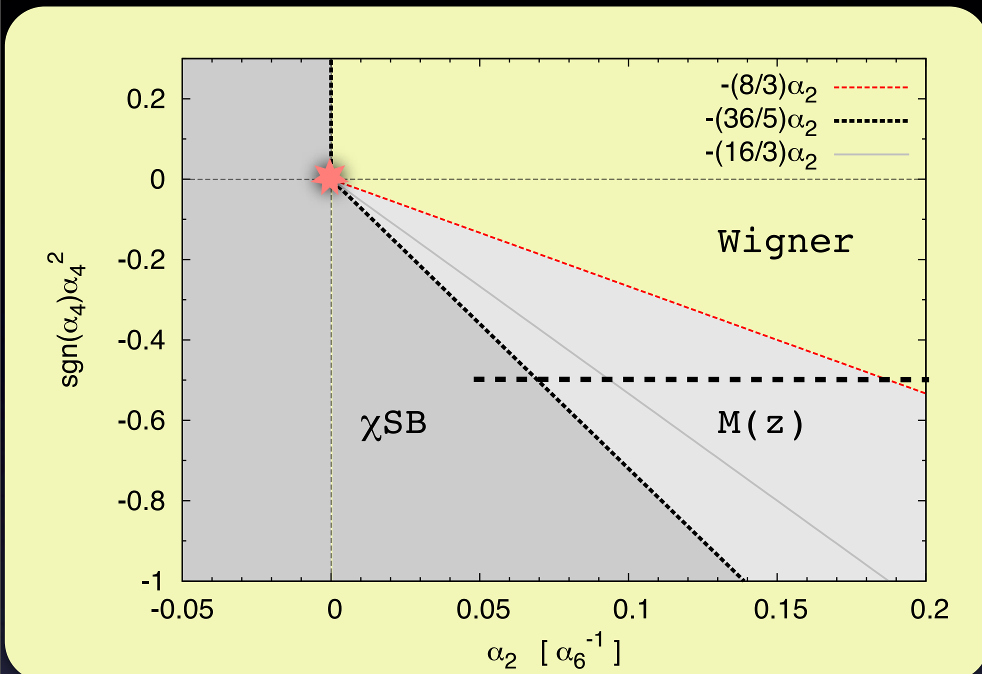
Comparis



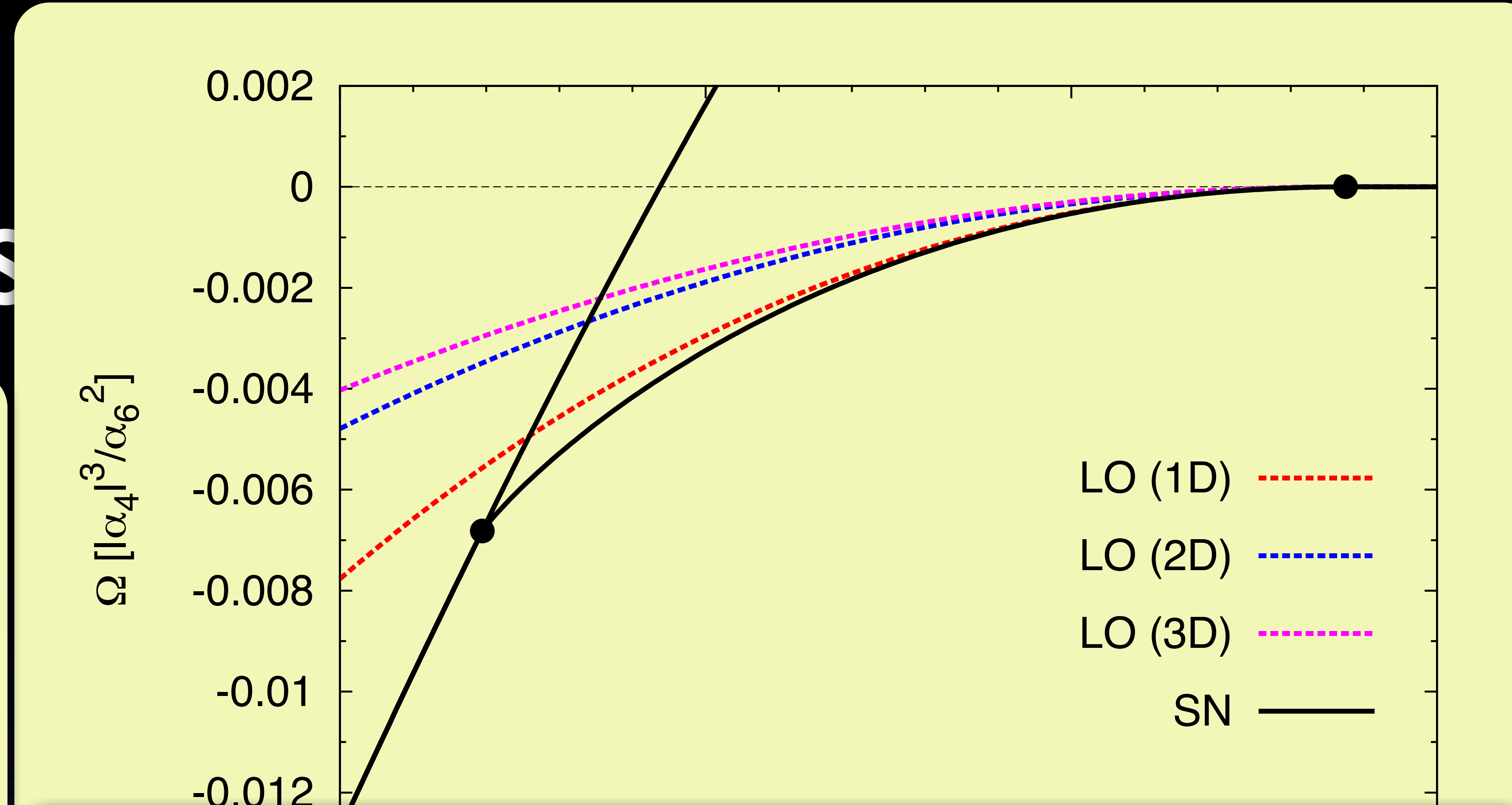


Similar results at T=0 in NJL analysis; Carignano, Buballa (2011)

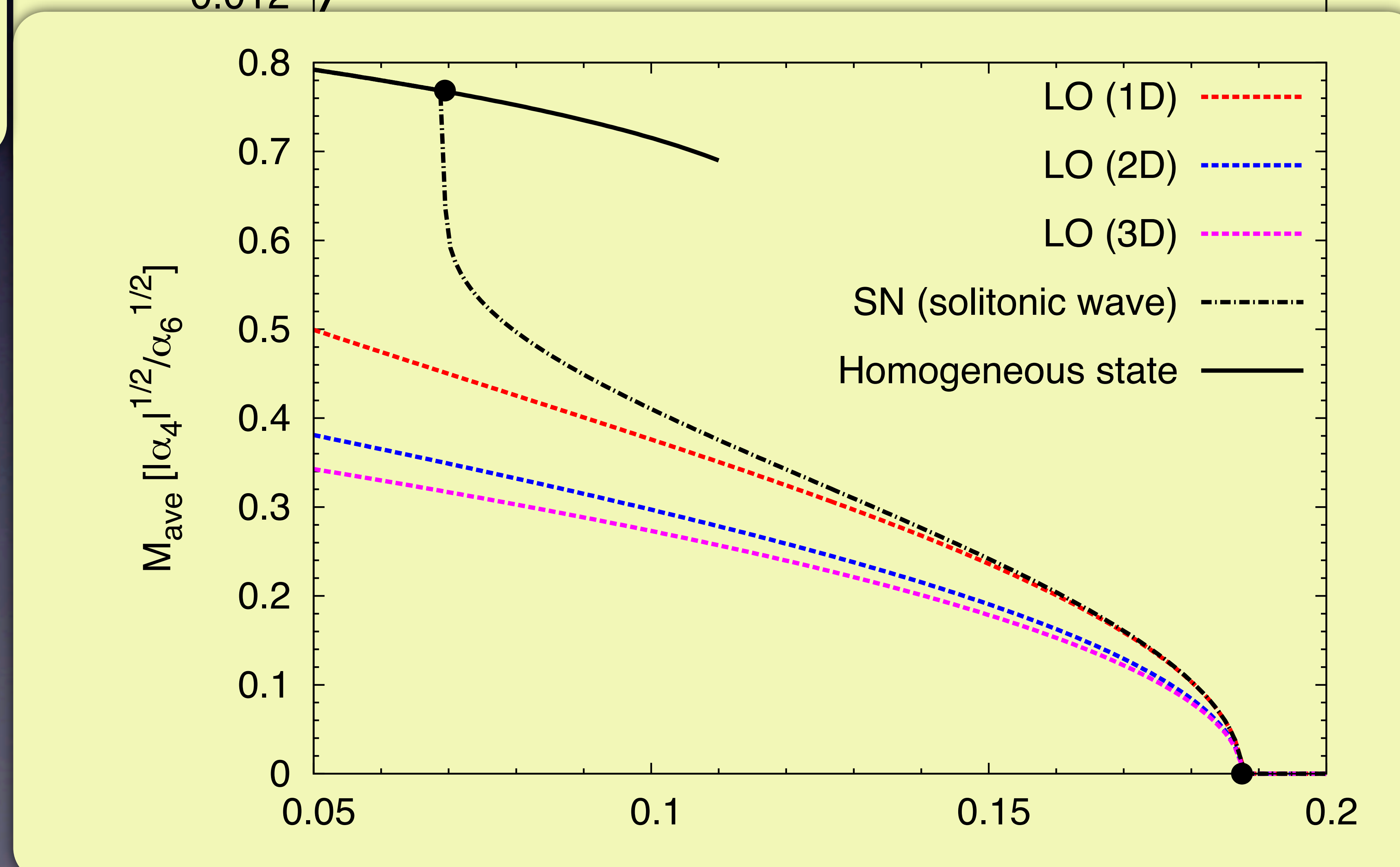
Comparis



-(8/3)α_2
-(36/5)α_2
-(16/3)α_2
Wigner
χSB
M(z)
sgn(α_4)α_4^2
α_2 [α_6^{-1}]
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.05
0
0.05
0.1
0.15
0.2



Ω [$|\alpha_4|^3/\alpha_6^2$]
LO (1D)
LO (2D)
LO (3D)
SN
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
-0.012



M_{ave} [$|\alpha_4|^{1/2}/\alpha_6^{1/2}$]
LO (1D)
LO (2D)
LO (3D)
SN (solitonic wave)
Homogeneous state
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.05
0.1
0.15
0.2

Analytical check

- Optimizing over wavevector q, the potential can be expanded in powers of M

$$\Omega_{1\mathrm{D}} = \left(\frac{a_2}{2} - \frac{3a_4^2}{16}\right) M_{ave}^2 + \frac{6|a_4|}{24} M_{ave}^4 + \mathcal{O}(M_{ave}^6)$$

$$\Omega_{2\mathrm{D}} = \left(\frac{a_2}{2} - \frac{3a_4^2}{16}\right) M_{ave}^2 + \frac{9|a_4|}{24} M_{ave}^4 + \mathcal{O}(M_{ave}^6)$$

$$\Omega_{3\mathrm{D}} = \left(\frac{a_2}{2} - \frac{3a_4^2}{16}\right) M_{ave}^2 + \frac{10|a_4|}{24} M_{ave}^4 + \mathcal{O}(M_{ave}^6)$$

Analytical check

- Optimizing over wavevector q, the potential can be expanded in powers of M

$$\Omega_{1\mathrm{D}} = \boxed{\left(\frac{a_2}{2} - \frac{3a_4^2}{16}\right) M_{ave}^2} + \frac{6|a_4|}{24} M_{ave}^4 + \mathcal{O}(M_{ave}^6)$$

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Analytical check

- Optimizing over wavevector q, the potential can be expanded in powers of M

$$\Omega_{1D} = \boxed{\left(\frac{a_2}{2} - \frac{3a_4^2}{16}\right) M_{ave}^2} + \frac{6|a_4|}{24} M_{ave}^4 + \mathcal{O}(M_{ave}^6)$$

$$\Omega_{2D} = \boxed{\left(\frac{a_2}{2} - \frac{3a_4^2}{16}\right) M_{ave}^2} + \frac{9|a_4|}{24} M_{ave}^4 + \mathcal{O}(M_{ave}^6)$$

$$\Omega_{3D} = \boxed{\left(\frac{a_2}{2} - \frac{3a_4^2}{16}\right) M_{ave}^2} + \frac{10|a_4|}{24} M_{ave}^4 + \mathcal{O}(M_{ave}^6)$$

quadratic coeff. $\longrightarrow$ critical point is shared

Analytical check

- Optimizing over wavevector q, the potential can be expanded in powers of M

$$\Omega_{1\mathrm{D}} = \boxed{\left(\frac{a_2}{2} - \frac{3a_4^2}{16}\right) M_{ave}^2} + \boxed{\frac{6|a_4|}{24} M_{ave}^4} + \mathcal{O}(M_{ave}^6)$$

$$\Omega_{2\mathrm{D}} = \boxed{\left(\frac{a_2}{2} - \frac{3a_4^2}{16}\right) M_{ave}^2} + \boxed{\frac{9|a_4|}{24} M_{ave}^4} + \mathcal{O}(M_{ave}^6)$$

$$\Omega_{3\mathrm{D}} = \boxed{\left(\frac{a_2}{2} - \frac{3a_4^2}{16}\right) M_{ave}^2} + \boxed{\frac{10|a_4|}{24} M_{ave}^4} + \mathcal{O}(M_{ave}^6)$$

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$$\Omega_{3\mathrm{D}} = \boxed{\left(\frac{a_2}{2} - \frac{3a_4^2}{16}\right) M_{ave}^2} + \boxed{\frac{10|a_4|}{24} M_{ave}^4} + \mathcal{O}(M_{ave}^6)$$

quadratic coeff. $\rightarrow$ critical point is shared

quartic coeff. $\rightarrow$ $\Omega_{1\mathrm{D}} < \Omega_{2\mathrm{D}} < \Omega_{3\mathrm{D}}$

This work

1. Multidimensional modulations near CP?

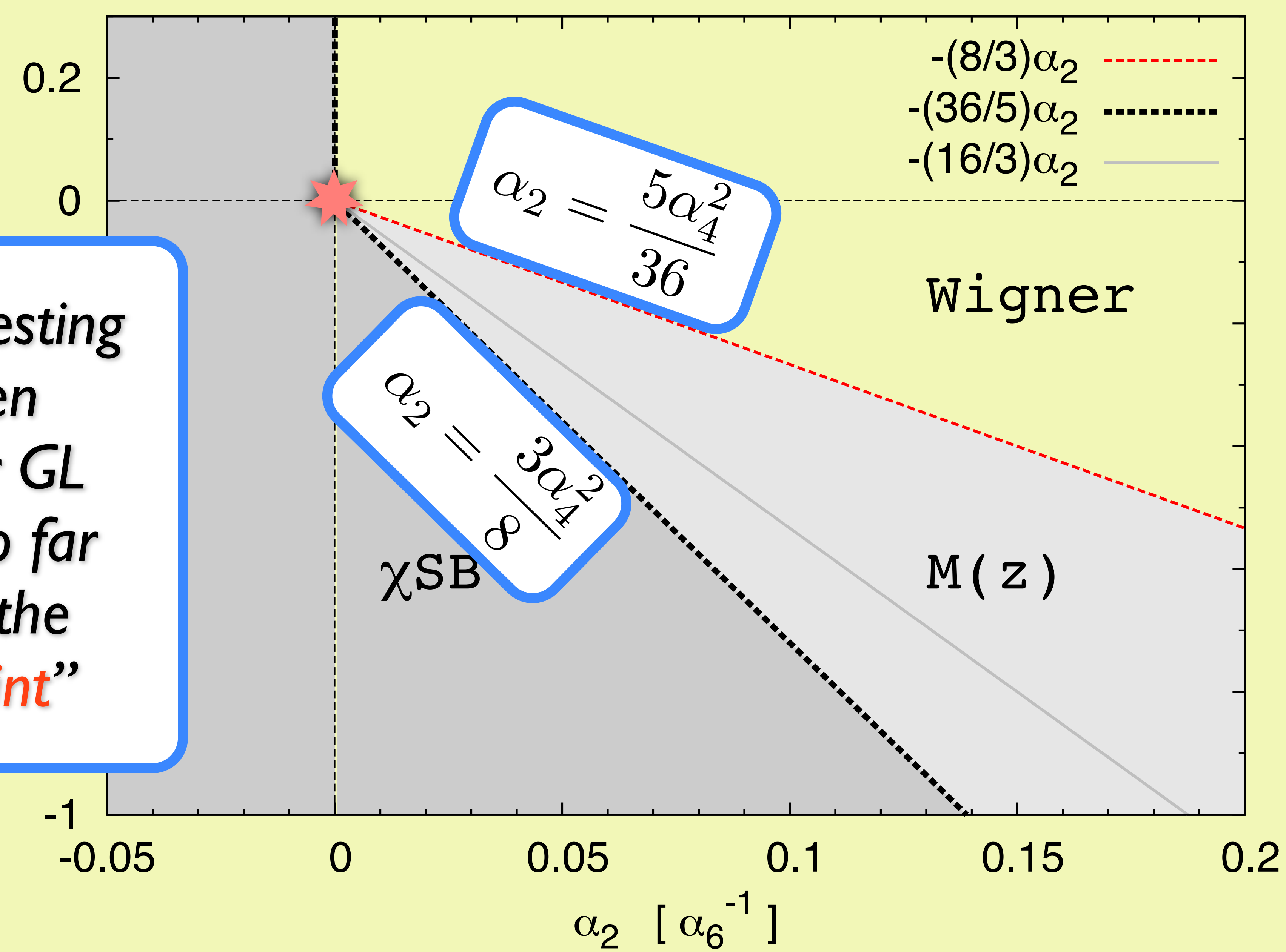
2. How does the phase structure change going away from the critical point?

This work

2. How does the phase structure change going away from the critical point?

GL in the vicinity of critical point

D. Nickel, PRL09



Nothing interesting can happen at 6th order GL even if we go far away from the "Lifshitz point"

GL in the vicinity of critical point

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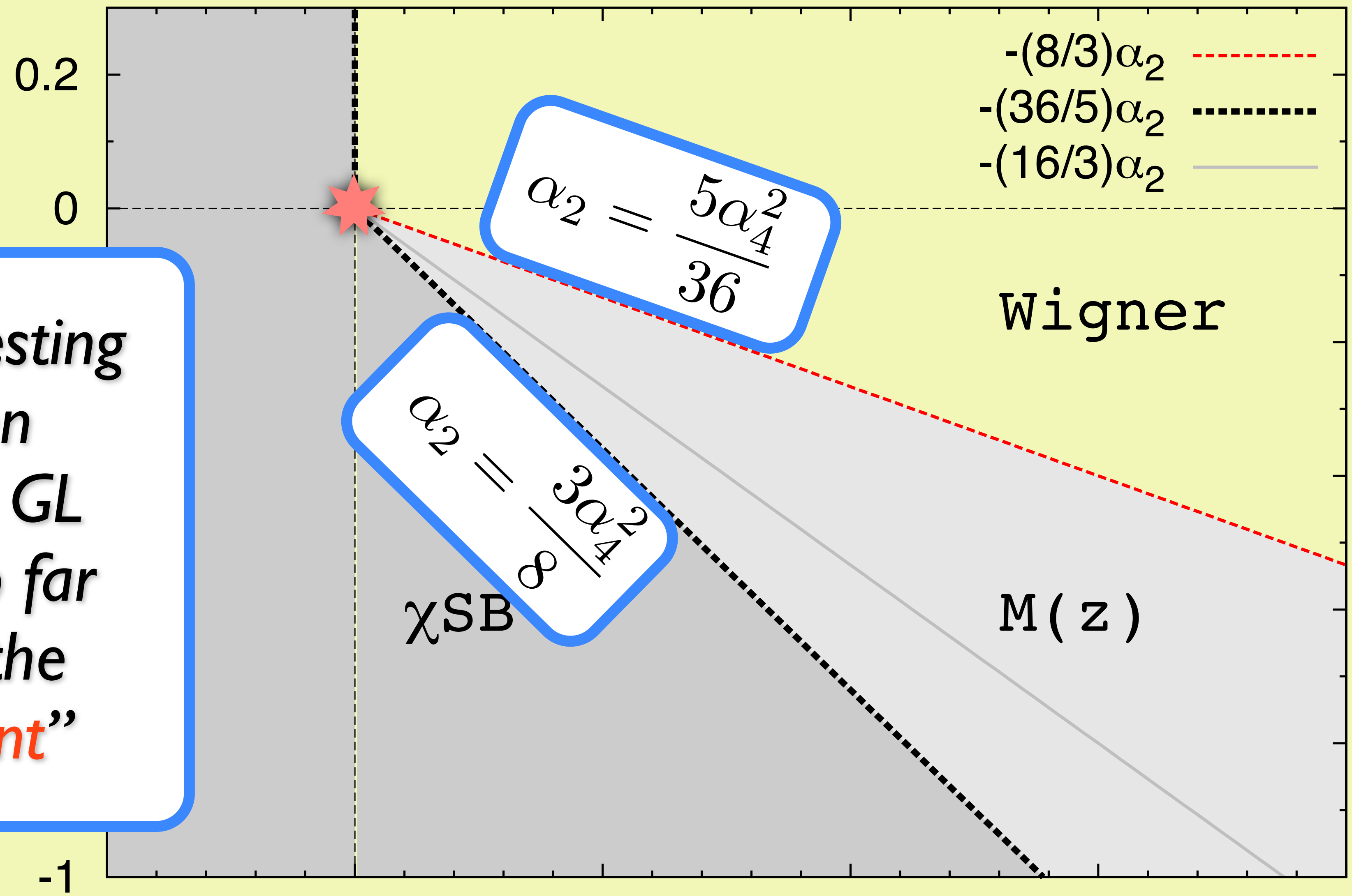


Nothing interesting can happen at 6th order GL even if we go far away from the "Lifshitz point"

Need to go beyond the minimal (6-th order) GL description

Going to 8th order

- Derivative expansion technique, straightforwardly applied

Abuki, Ishibashi, Suzuki (2011)

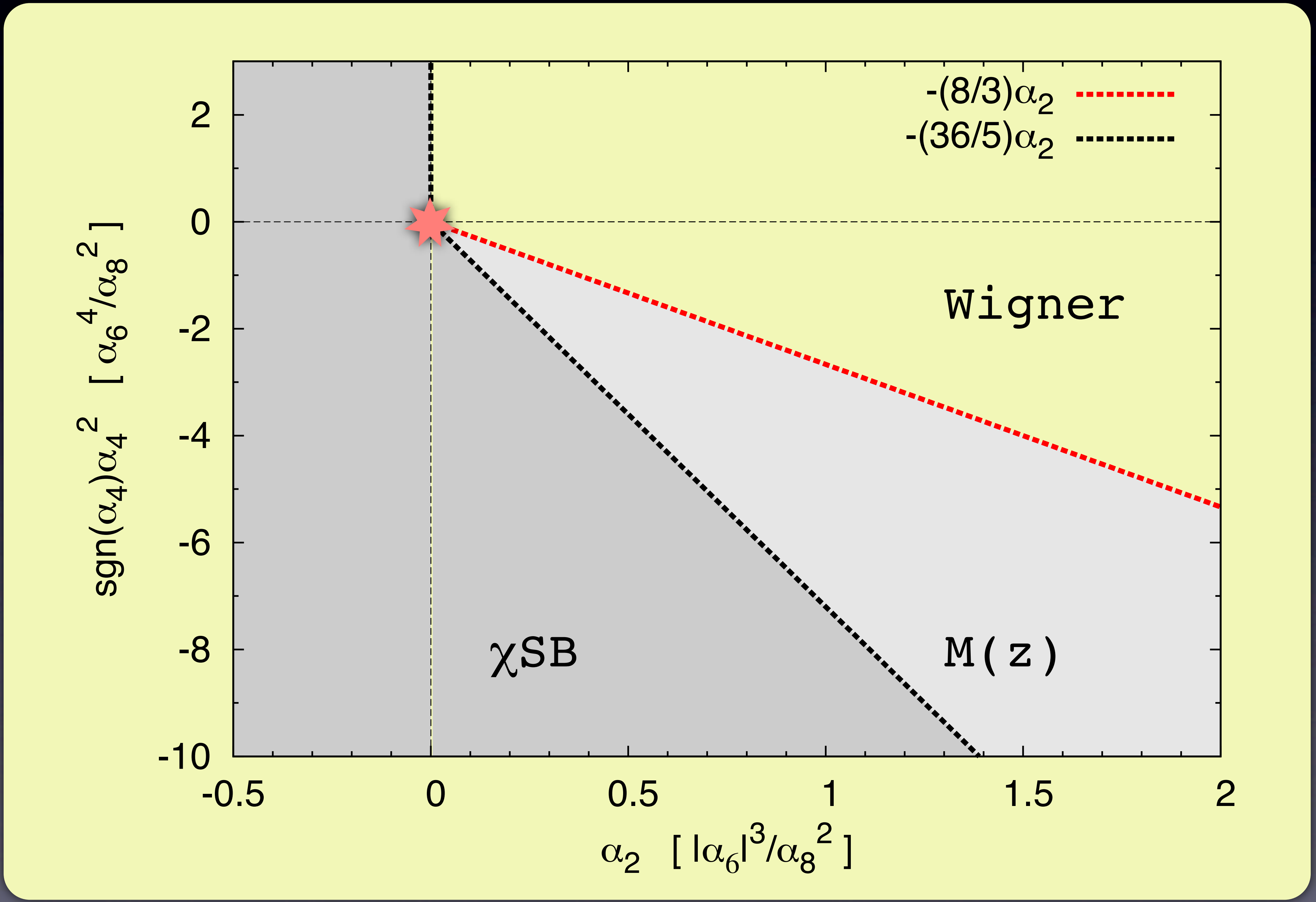
$$\Omega_{\rm GL} = \frac{\alpha_2}{2}M(\mathbf{x})^2 + \frac{\alpha_4}{4}M(\mathbf{x})^4 + \frac{\alpha_6}{6}M(\mathbf{x})^6 + \boxed{\frac{\alpha_8}{8}M(\mathbf{x})^8}$$
$$+\frac{\alpha_4}{4}(\nabla M)^2 + \frac{5\alpha_6}{6}M^2(\nabla M)^2 + \frac{\alpha_6}{12}(\nabla\Delta M)^2$$
$$\boxed{+\frac{\alpha_8}{8}\left(\xi_2 M^4(\nabla M)^2 + \xi_{4a}(\nabla M)^4 + \xi_{4b}M\Delta M(\nabla M)^2 + \xi_{4c}M^2(\Delta M)^2 + \xi_6(\nabla\Delta M)^2\right)}$$

★ Quark loop diagrams yield:

$$\xi_2 = 14, \quad \xi_{4a} = -\frac{1}{5}, \quad \xi_{4b} = \frac{18}{5}, \quad \xi_{4c} = \frac{14}{5}, \quad \xi_6 = \frac{1}{5}$$

★ Only one additional parameter $\alpha_8 > 0$!

Departing the critical point ($\alpha_6 > 0$)



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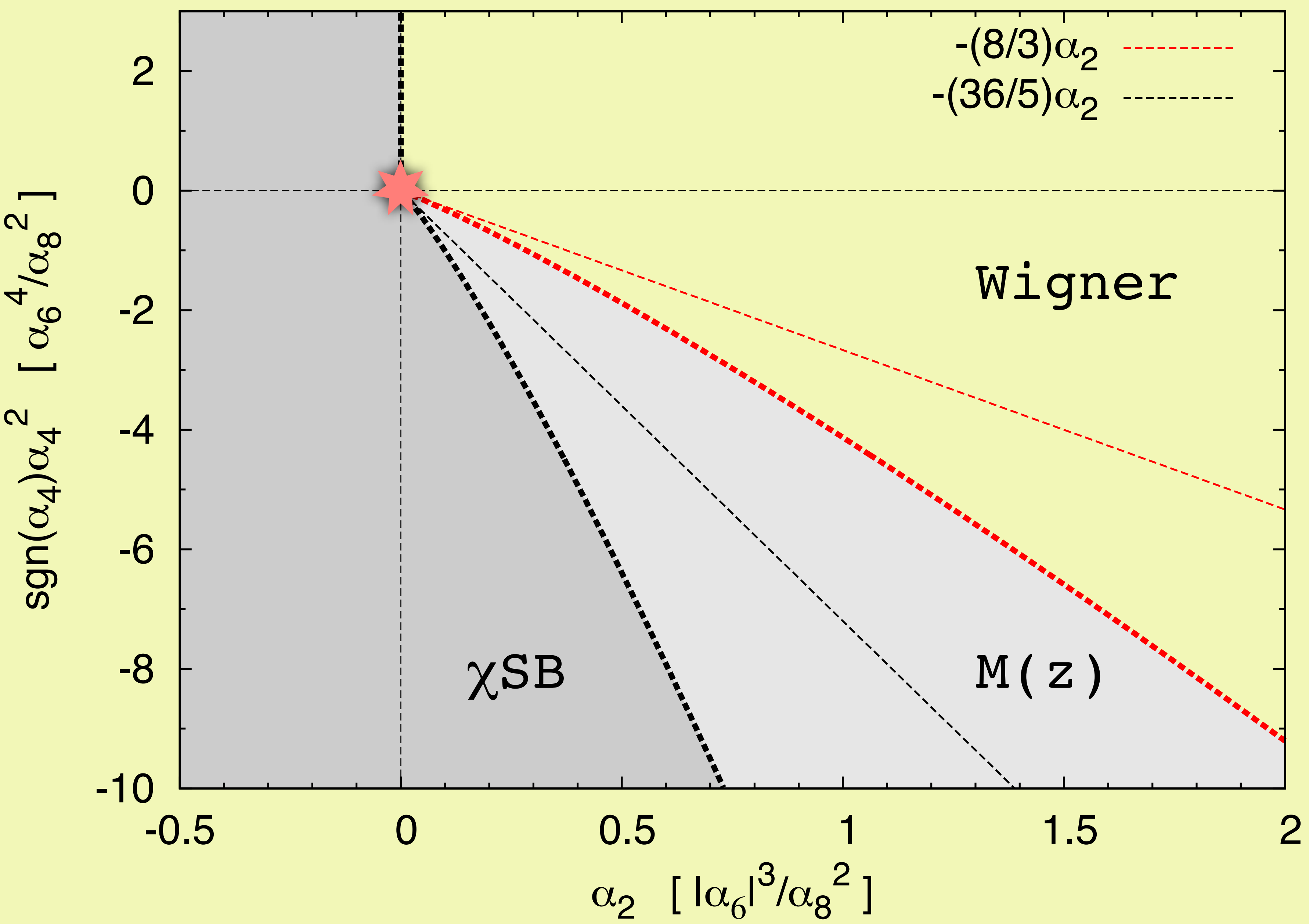

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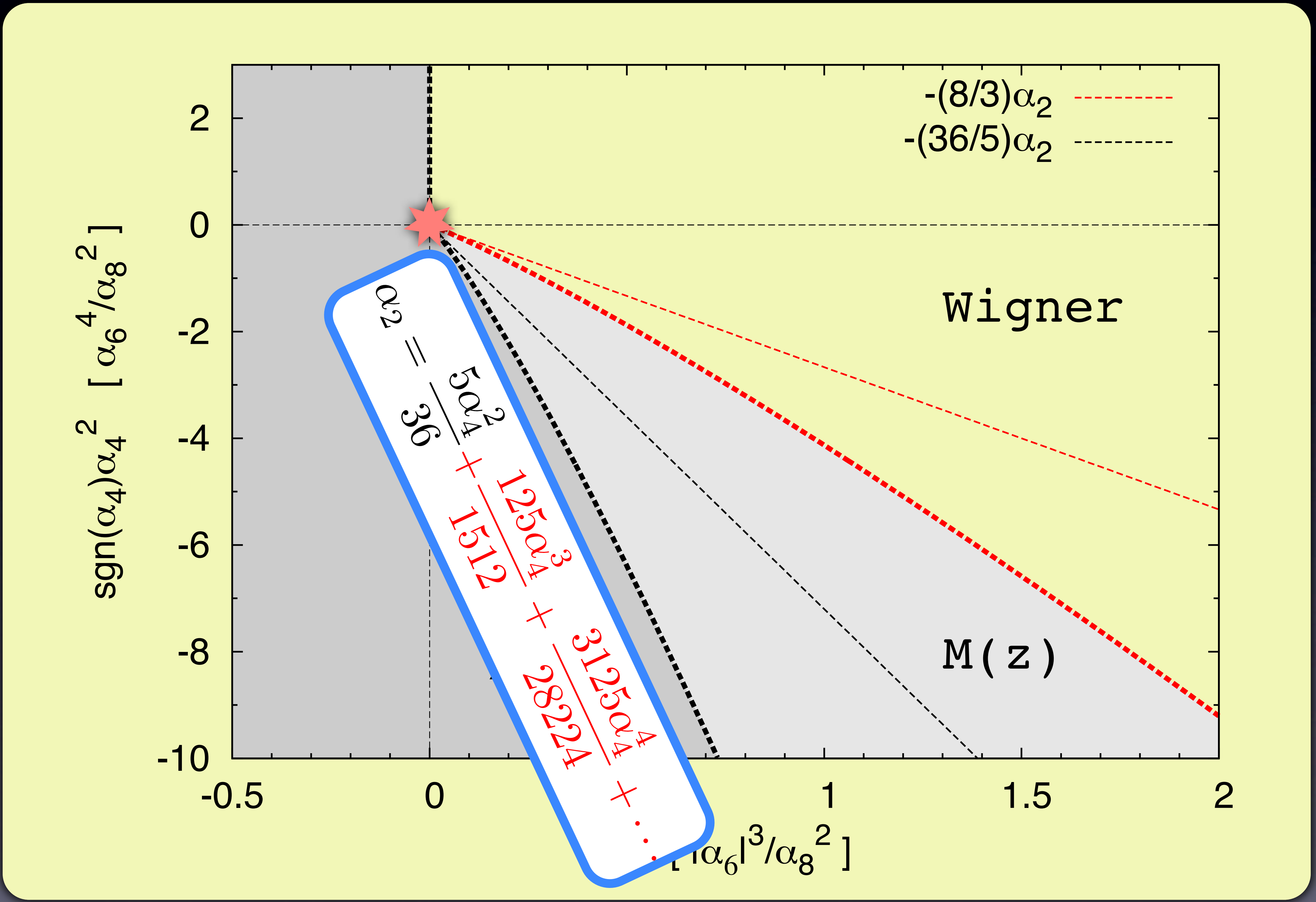
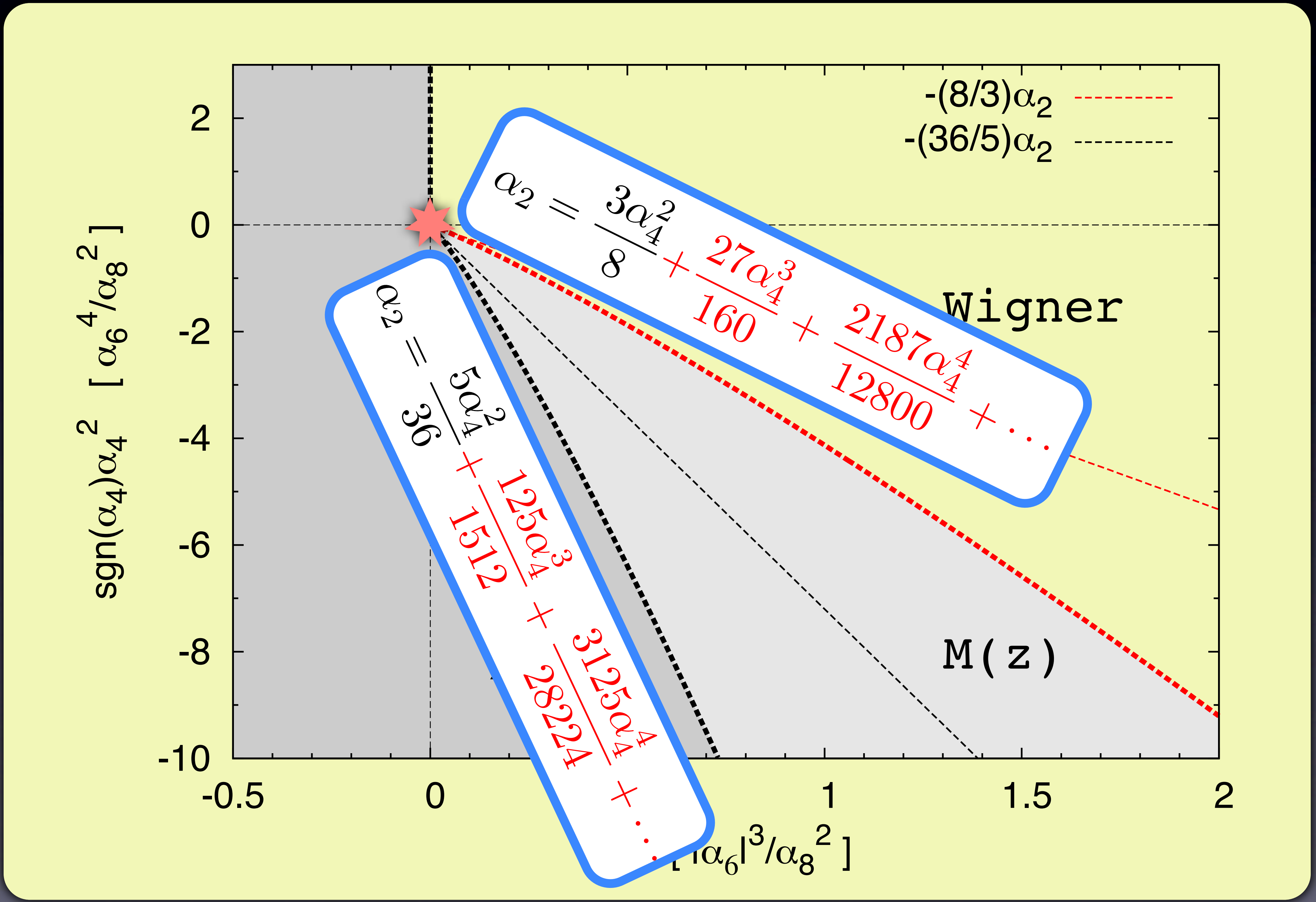
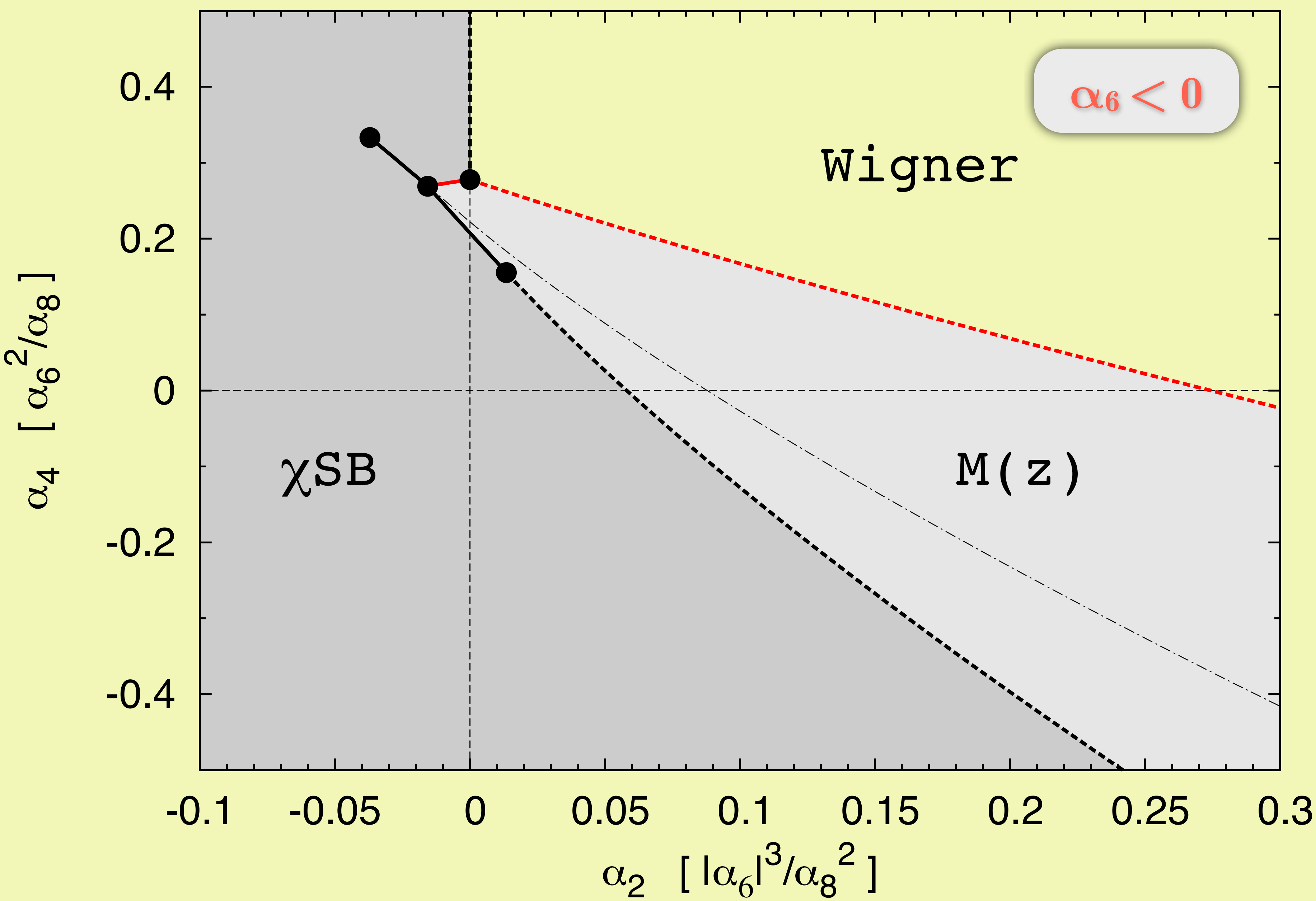


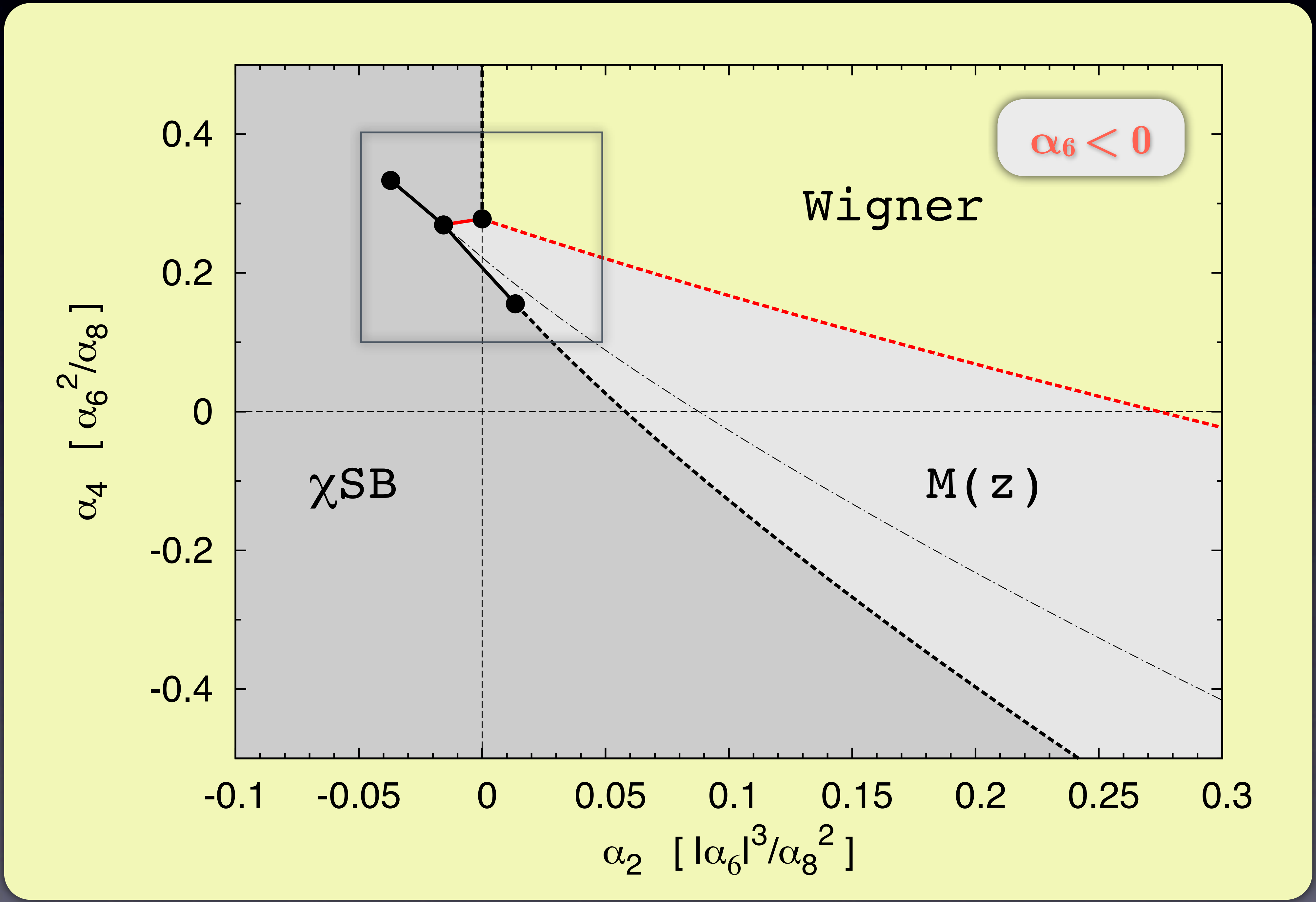
Departing the critical point ($\alpha_6 > 0$)



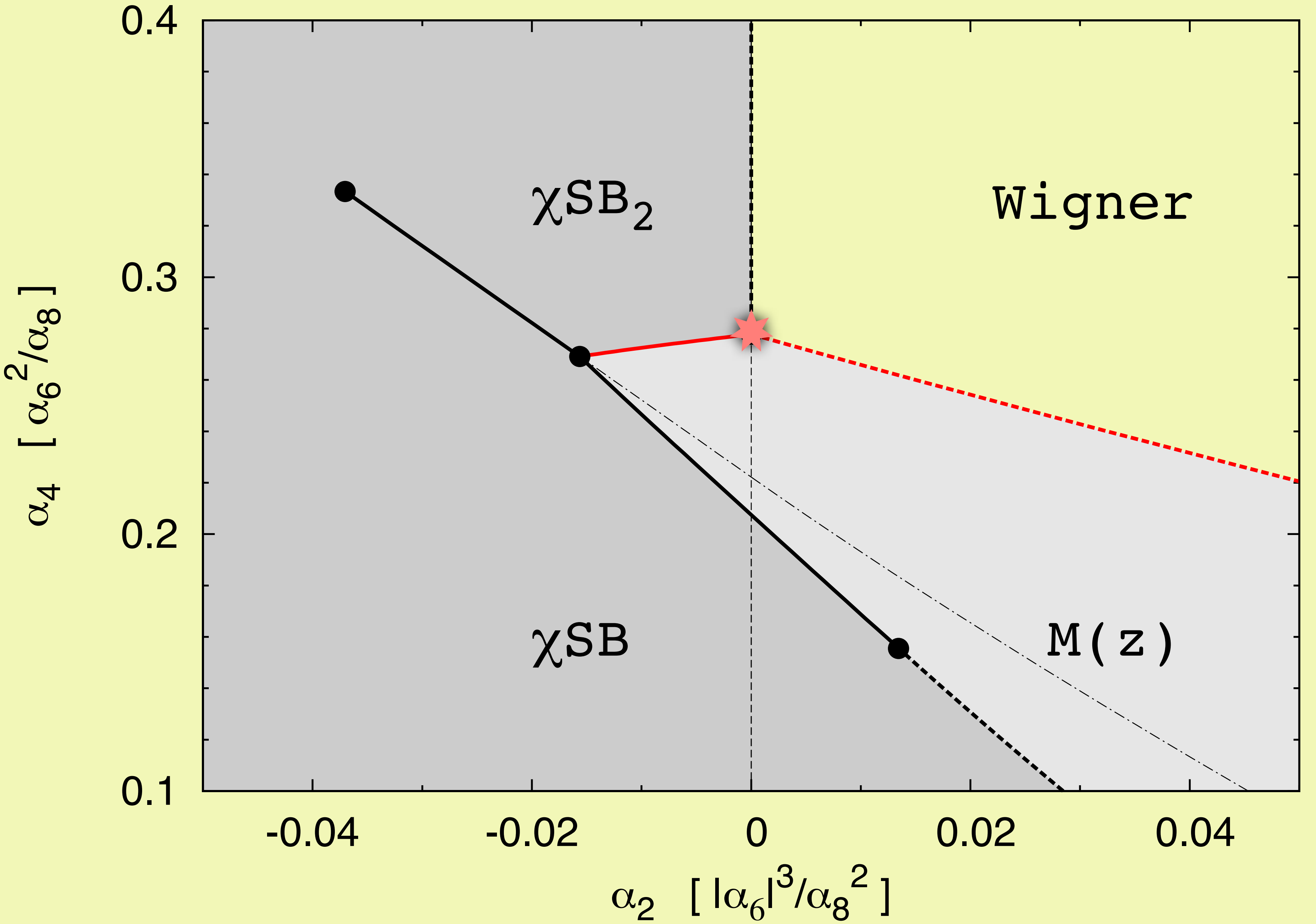
Going to new regime: $\alpha_6 < 0$



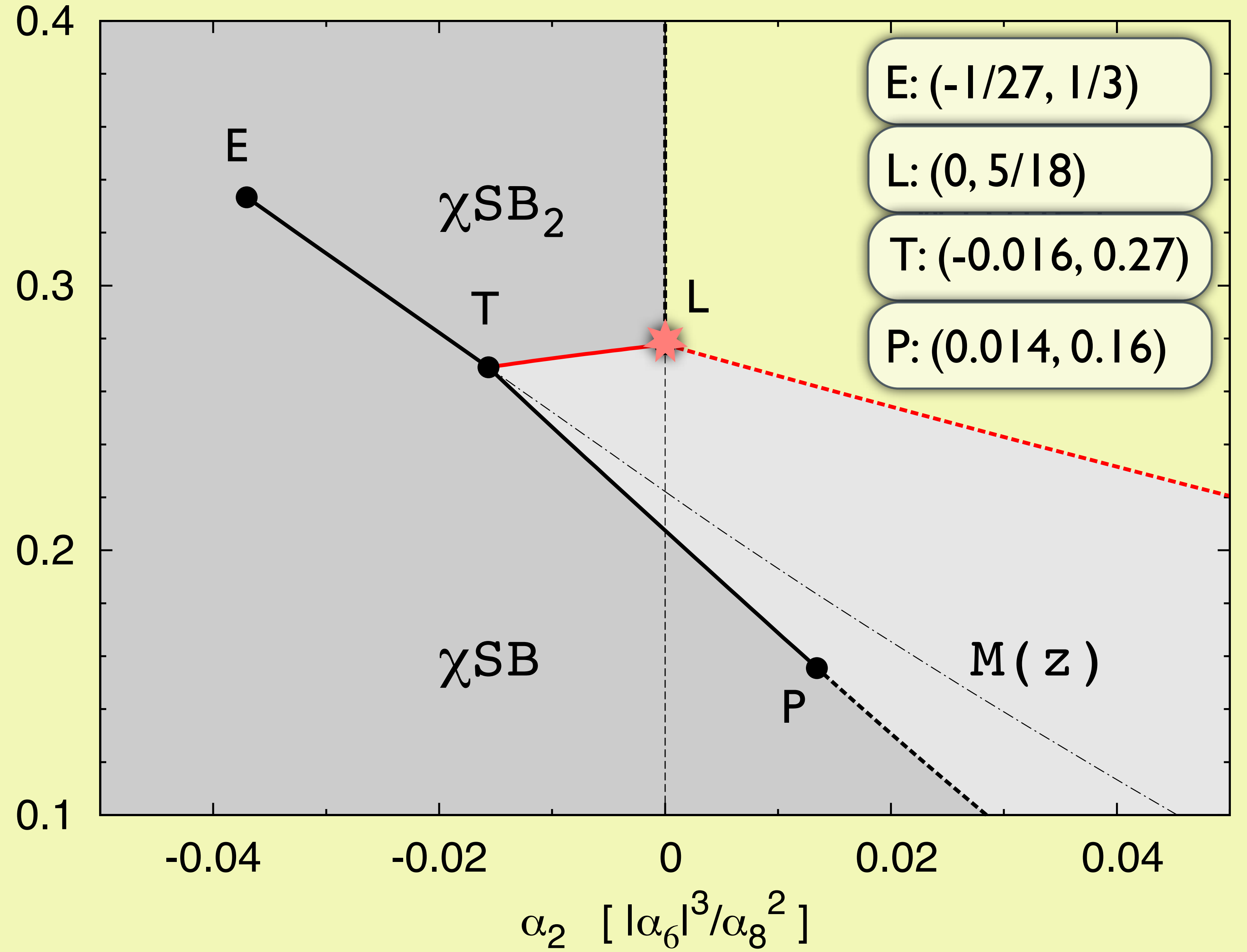
Going to new regime: $\alpha_6 < 0$



Going to new regime: $\alpha_6 < 0$



Going to new regime: $\alpha_6 < 0$



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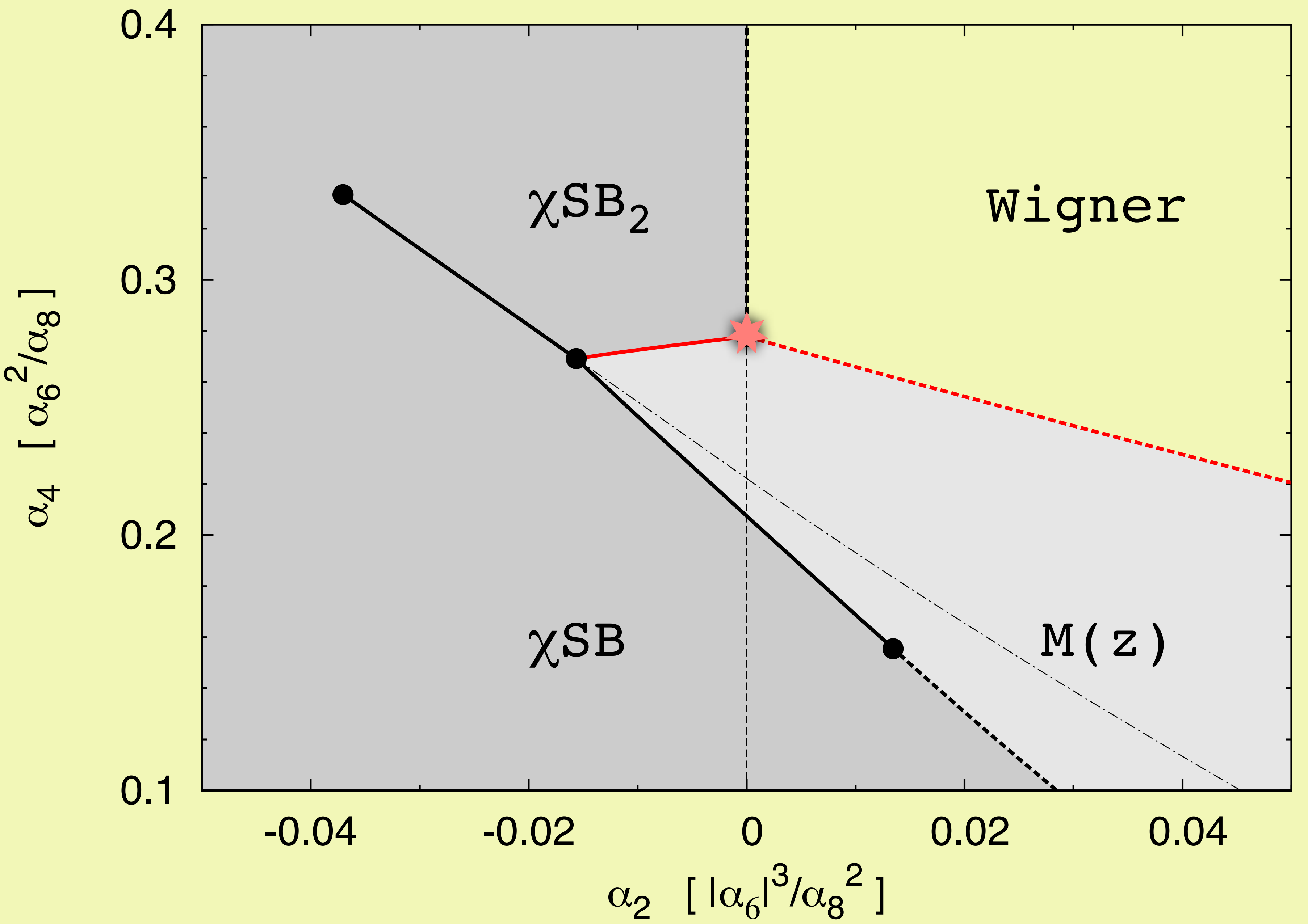
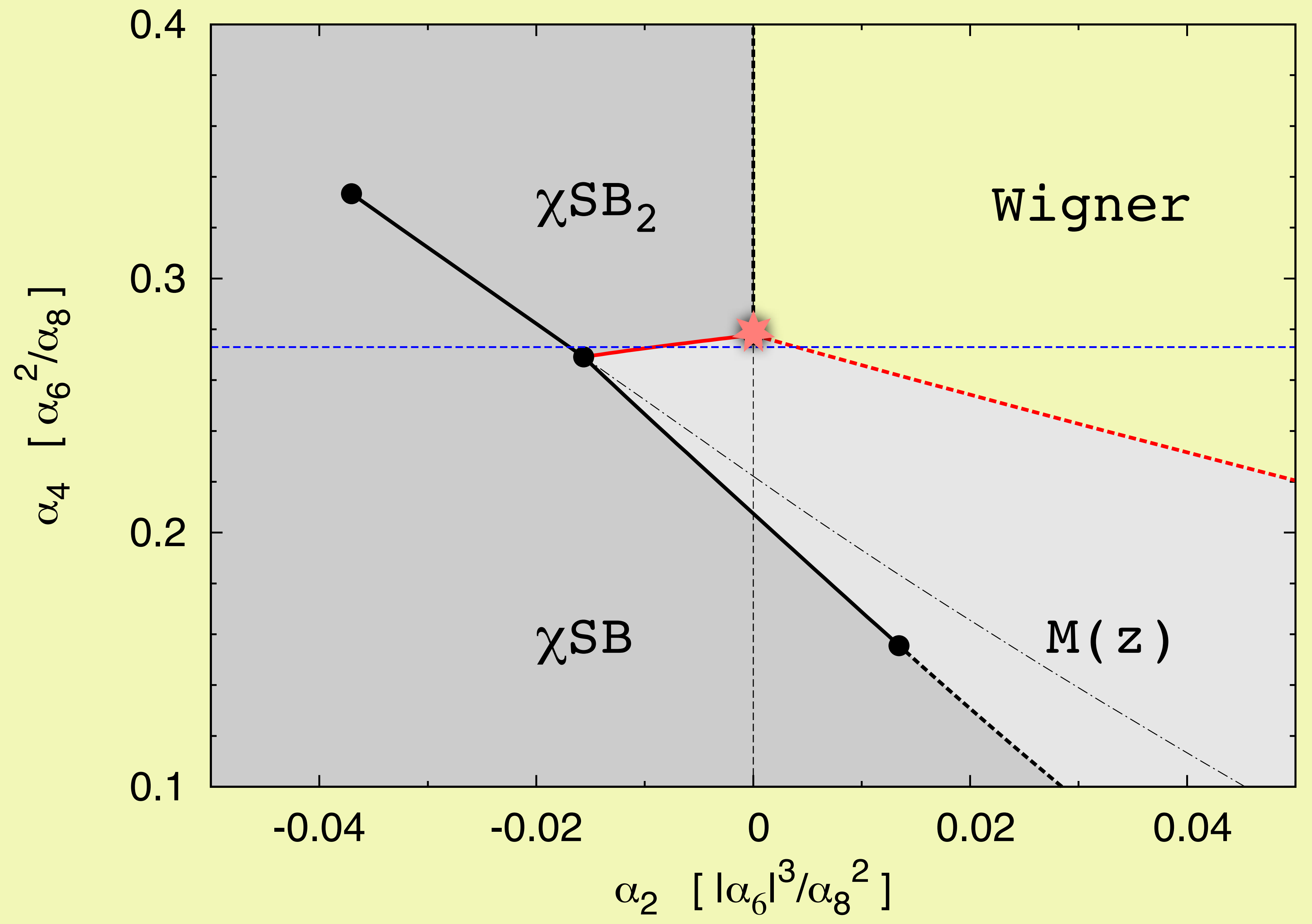
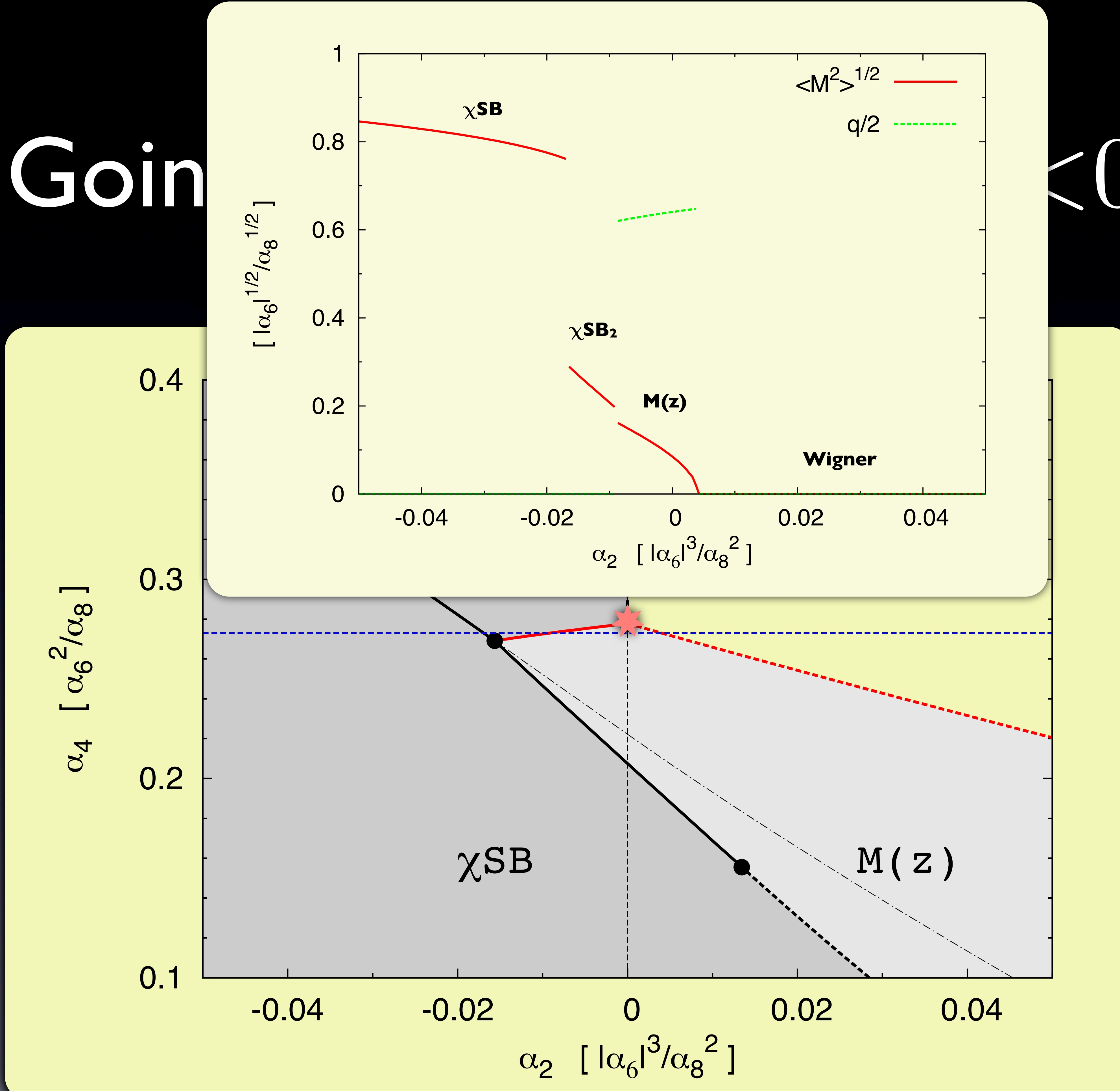


Going to new regime: $\alpha_6 < 0$



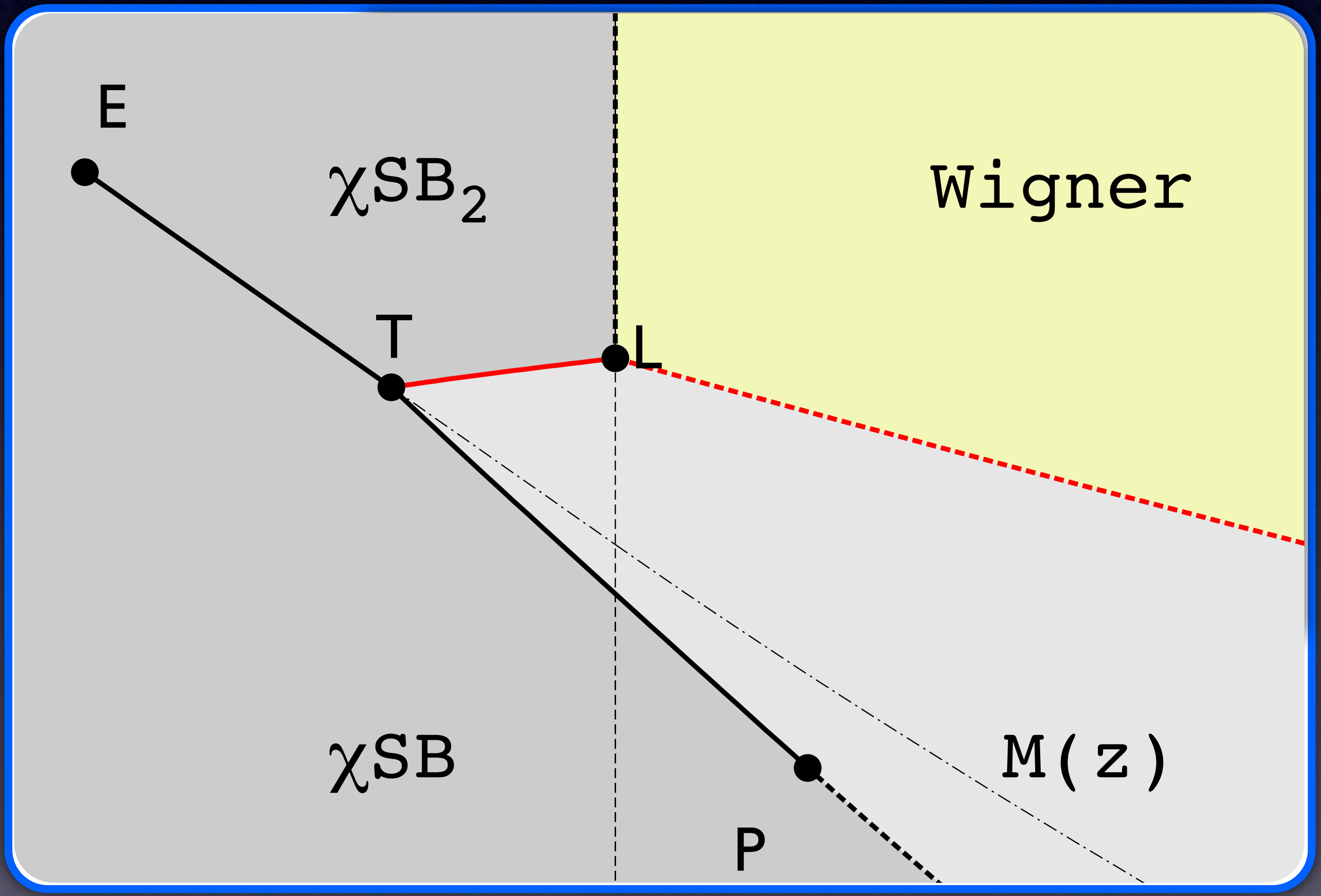
Goin … <0



1
0.8
0.6
0.4
0.2
0
-0.04
-0.02
0
0.02
0.04
[|α6|^1/2/α8^1/2]
α2 [|α6|^3/α8^2]
<M^2>^1/2
q/2
χSB
χSB2
M(z)
Wigner
0.4
0.3
0.2
0.1
α4 [α6^2/α8]
χSB
M(z)

Universal ratios at Point T

- Three different forms of chiral phase compete each other (and coexist) at the triple point (T)!



Universal ratios at Point T

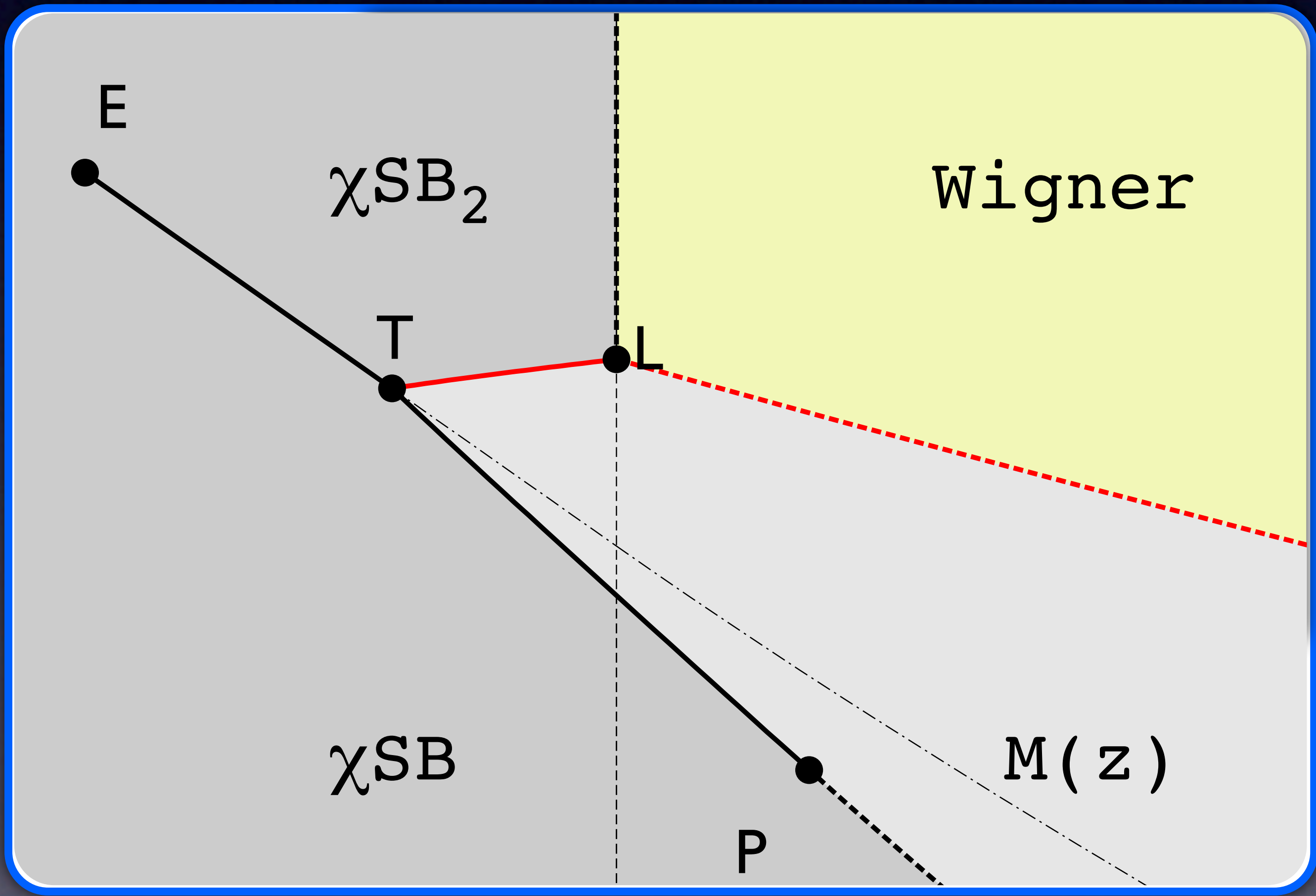
- Three different forms of chiral phase compete each other (and coexist) at the triple point (T)!

$$\frac{M_{\chi_2}}{M_\chi} \to 0.369..$$

$$\frac{\sqrt{\langle M(z)^2\rangle}}{M_\chi} \to 0.308..$$

$$\frac{q}{\sqrt{\langle M(z)^2\rangle}} \to 5.001..$$

$$\left(q \equiv \frac{2\pi}{L}\right)$$



How does GL map onto (μ, T)-phase diagram

D. Nickel, PRL09

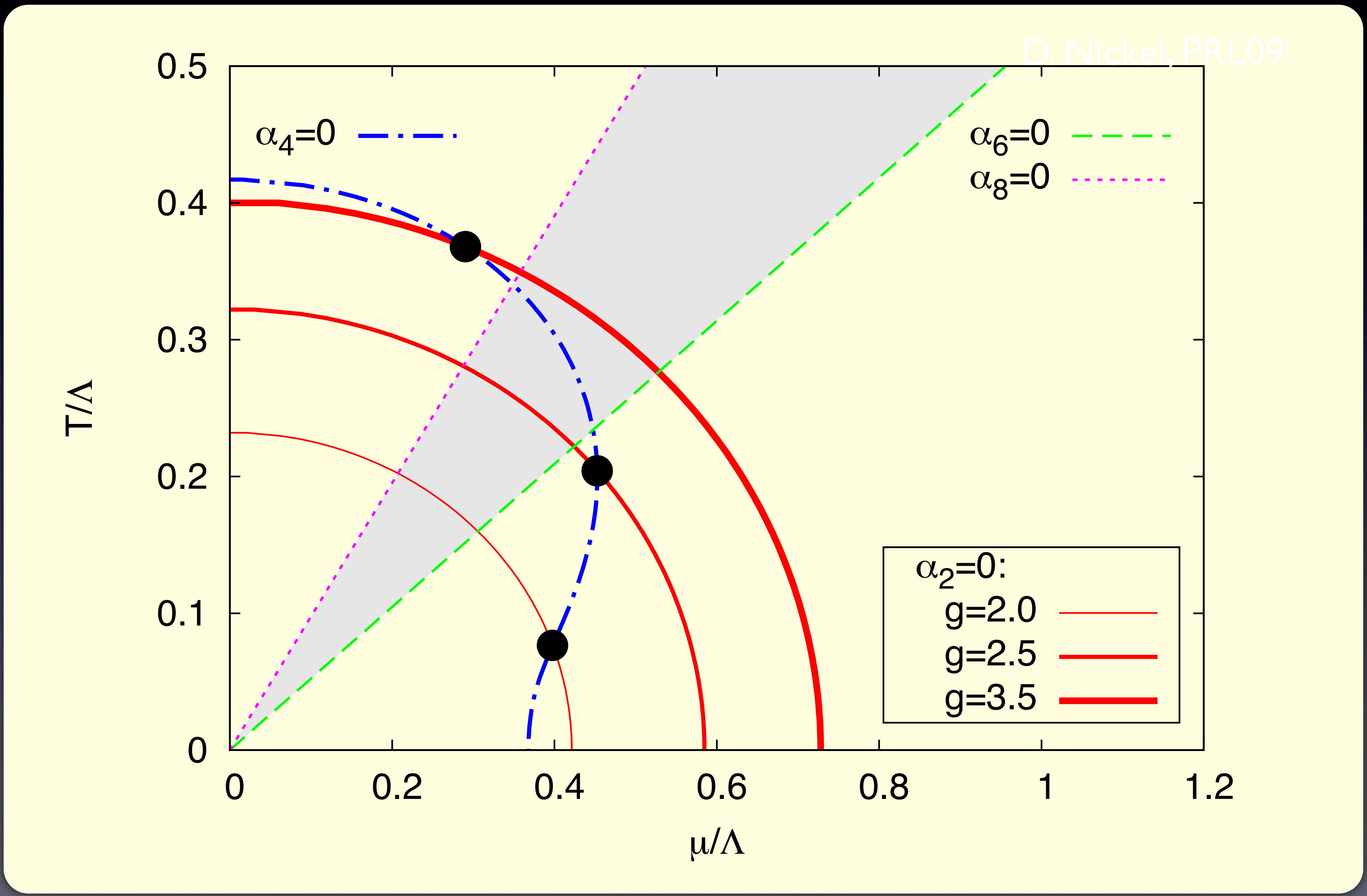
$$\alpha_2(\mu, T, \Lambda, G) = \frac{1}{2G} + 4N_cN_fT\sum_n \frac{d^3p}{(2\pi)^3}\frac{1}{[(i\omega_n+\mu)^2 - p^2]}$$

$$\alpha_4(\mu, T, \Lambda) = 4N_cN_fT\sum_n \frac{d^3p}{(2\pi)^3}\frac{1}{[(i\omega_n+\mu)^2 - p^2]^2}$$

$$\alpha_{6(8)}(\mu, T) = 4N_cN_fT\sum_n \frac{d^3p}{(2\pi)^3}\frac{1}{[(i\omega_n+\mu)^2 - p^2]^{3(4)}}$$

$$\rightarrow \begin{cases} \alpha_6(\mu, T) = f(T/\mu)/\mu^2 \\ \alpha_8(\mu, T) = g(T/\mu)/\mu^4 \end{cases}$$

How does GL map onto (μ, T)-phase diagram



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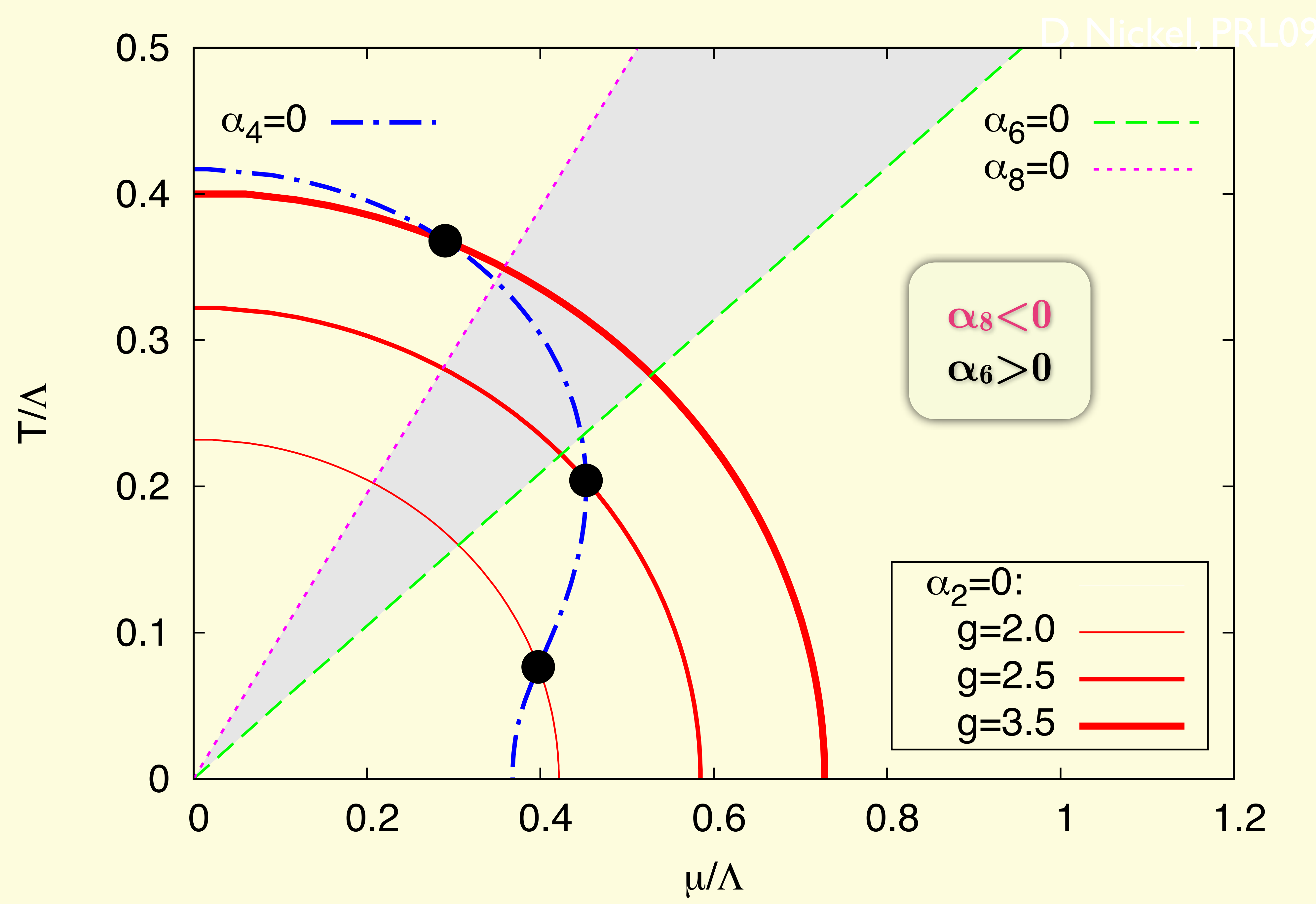

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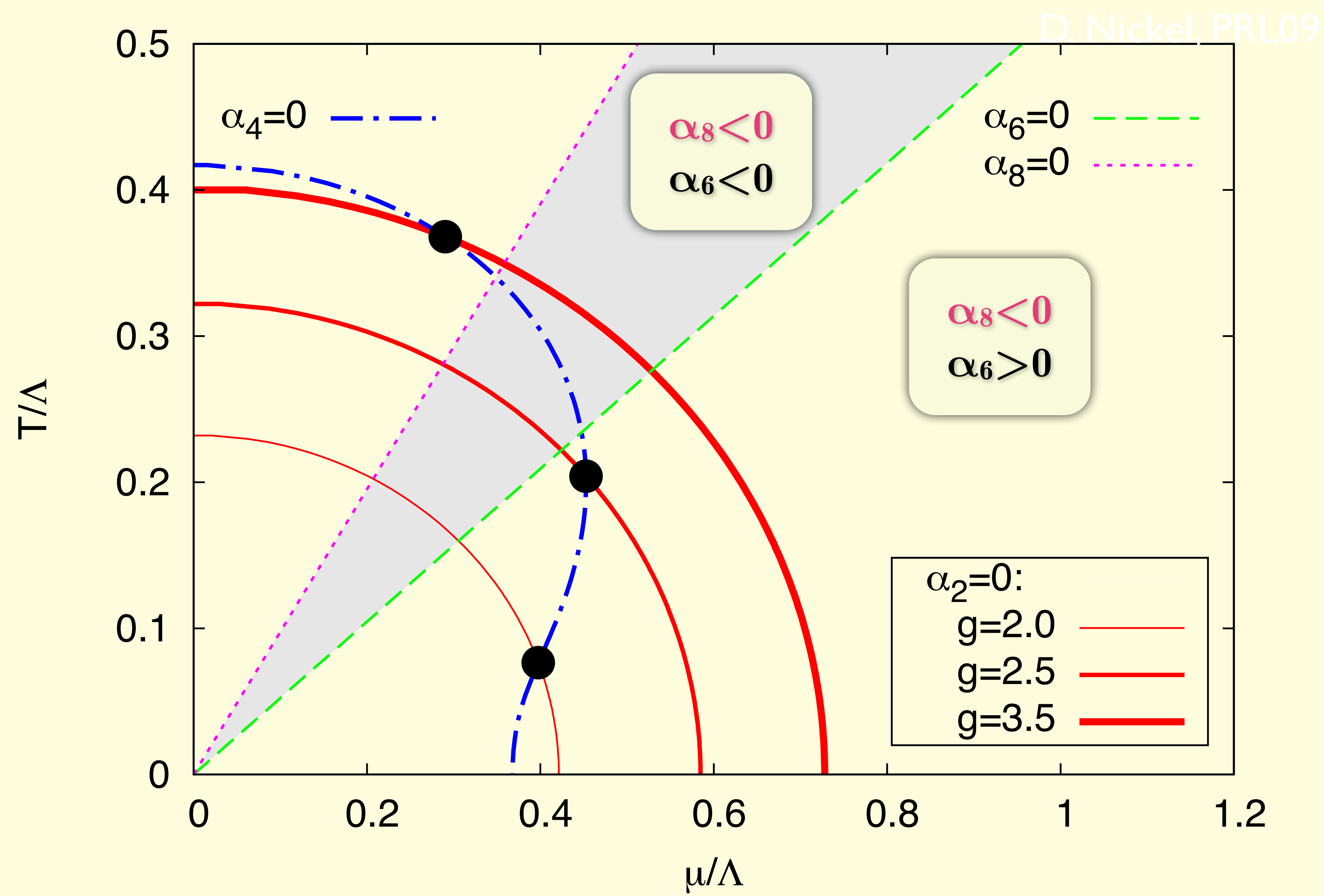

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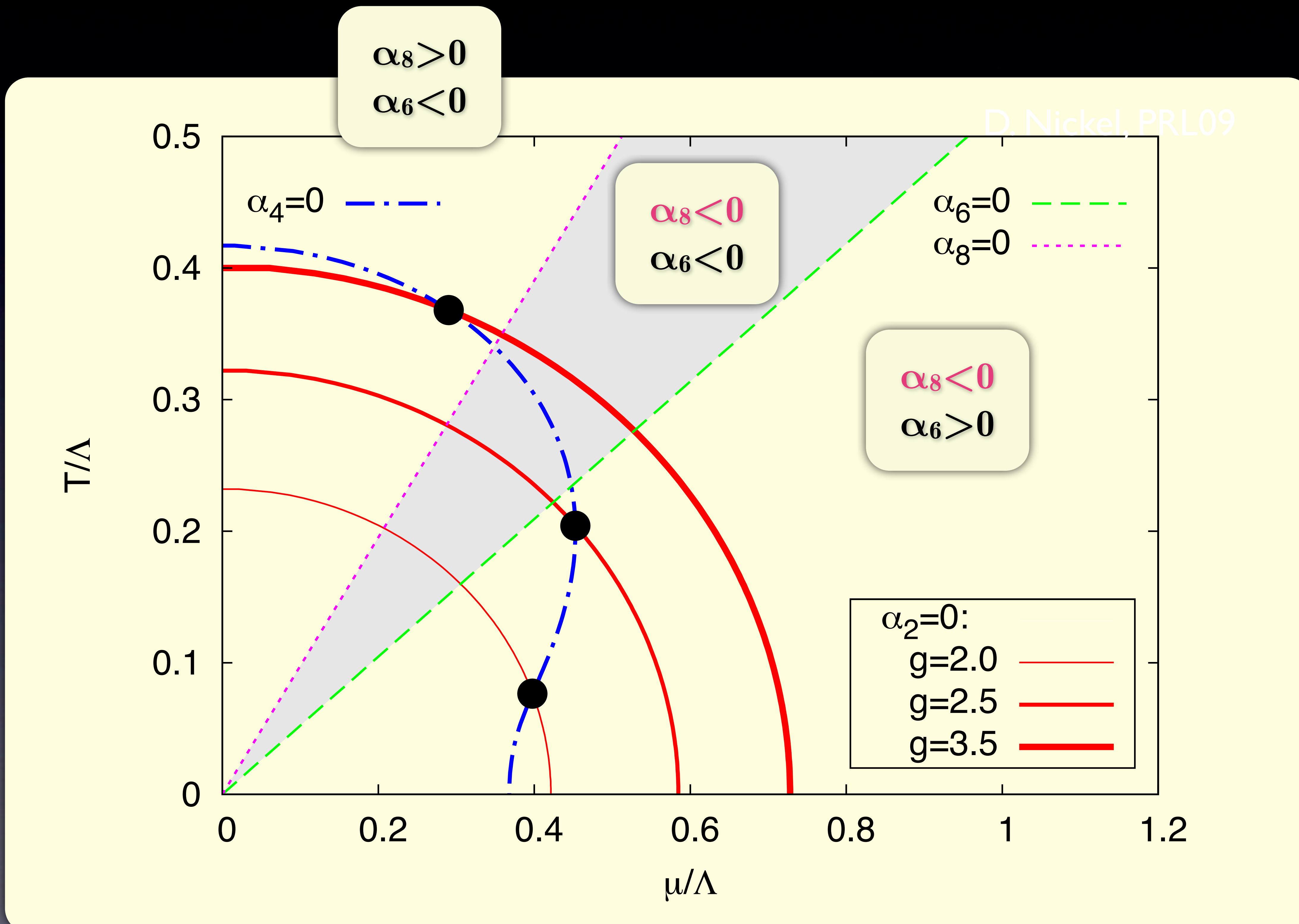
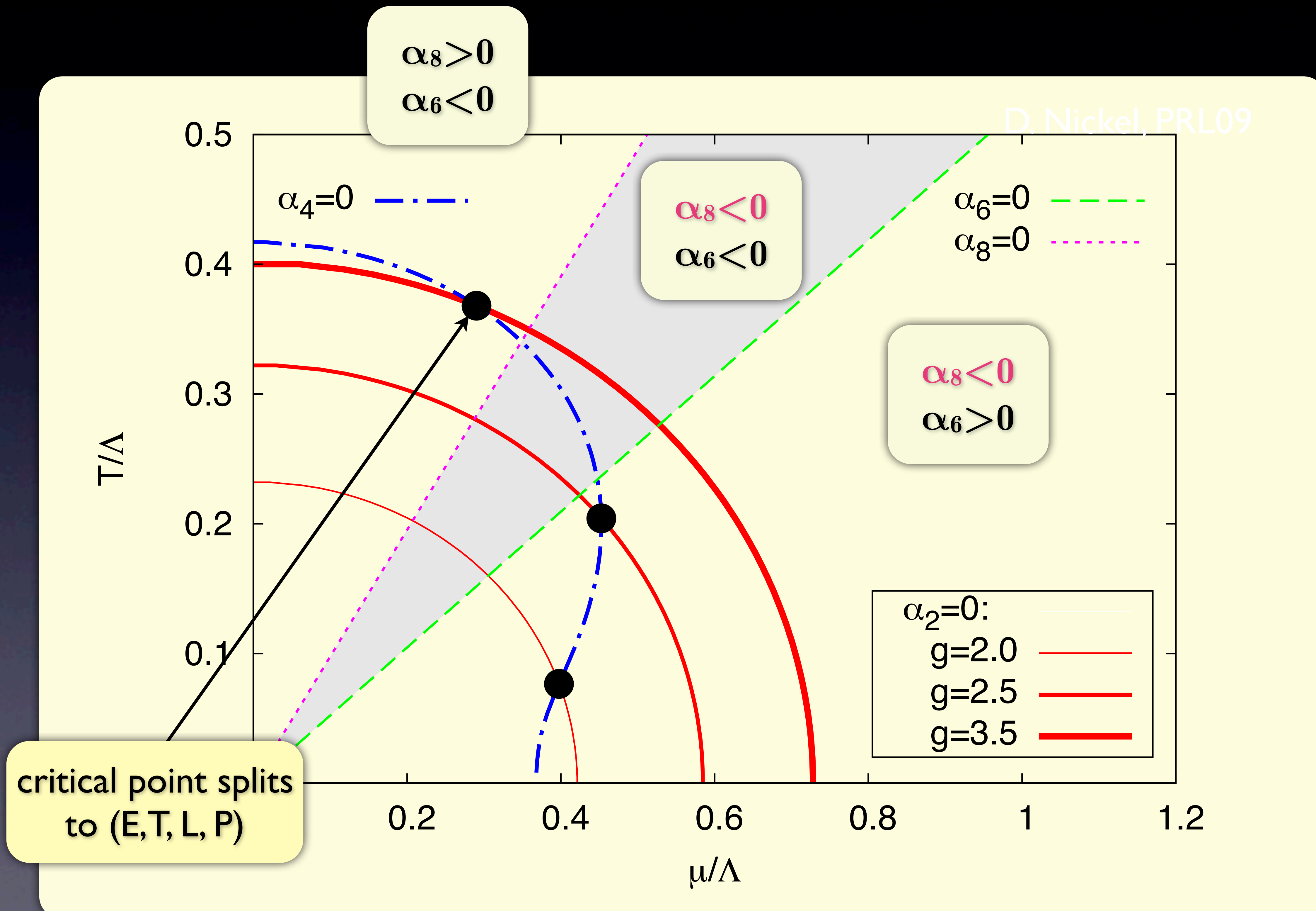


How does GL map onto (μ, T)-phase diagram



Summary

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- Multidimensional crystals are not favored
- Extended the previous GL analysis to higher order (8th order) and explored the phase structure away from the LP.
- if $\alpha_6 > 0$, qualitative phase structure remains the same.
- If $\alpha_6 < 0$, phase structure becomes rather rich; LP splits to 4-points; Existence of Triple point

Further questions

- Do we really have no suitable multi-dimensional chiral crystal? c.f. Landau, Peierls Theorem: Baym, Friman, Grinstein, NPB 1982
- If no, how could one dimensional structure be stabilized against thermal fluctuations?
- Can we have intriguing triple point (T) in QCD or other models?

Thank you very much
for your attention

Backups

Dimensional & Scaling analysis again

- Introducing dimensionless variables:

$$\alpha_2 = \eta_2 \left[|\alpha_6|^3/\alpha_8^2\right] \quad M = m \left[\sqrt{\alpha_6|/\alpha_8}\,\right]$$

$$\alpha_4 = \eta_4 \left[\alpha_6^2/\alpha_8\right] \quad \mathbf{x} = \tilde{\mathbf{x}} \left[\sqrt{\alpha_8/|\alpha_6|}\right]$$

$$\Omega = \omega \left[\alpha_6^4/\alpha_8^3\right]$$

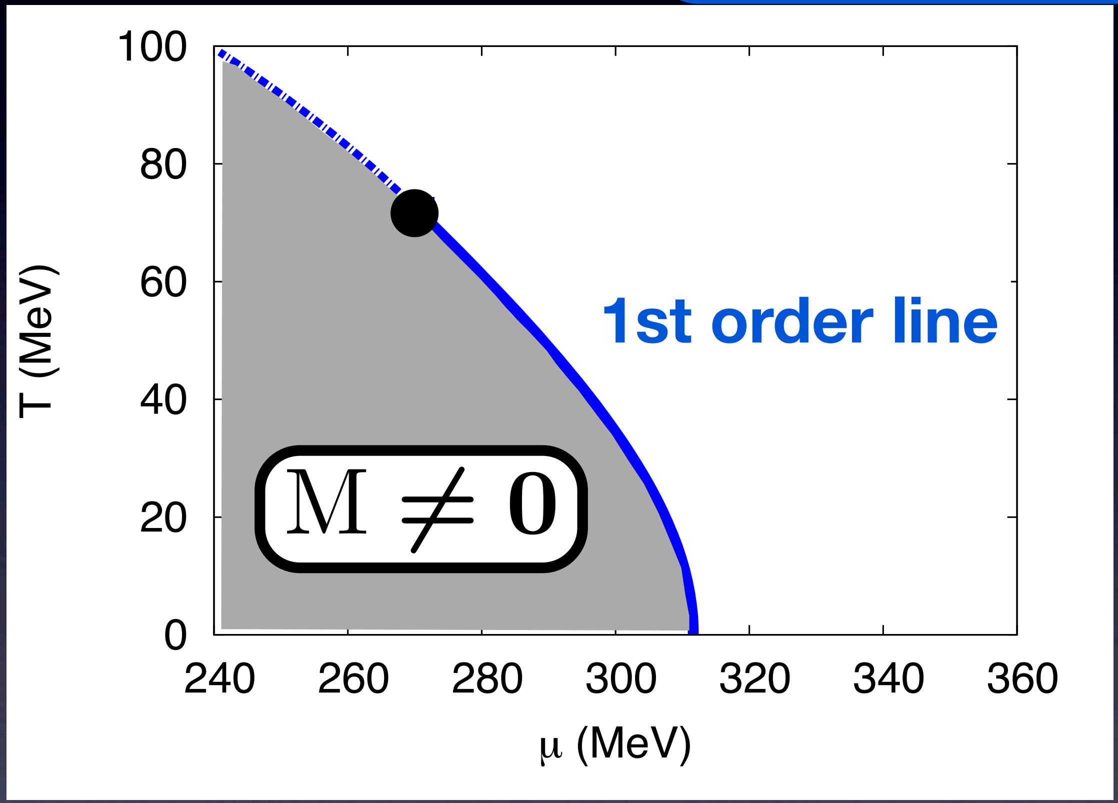
- Relevant parameters are reduced:

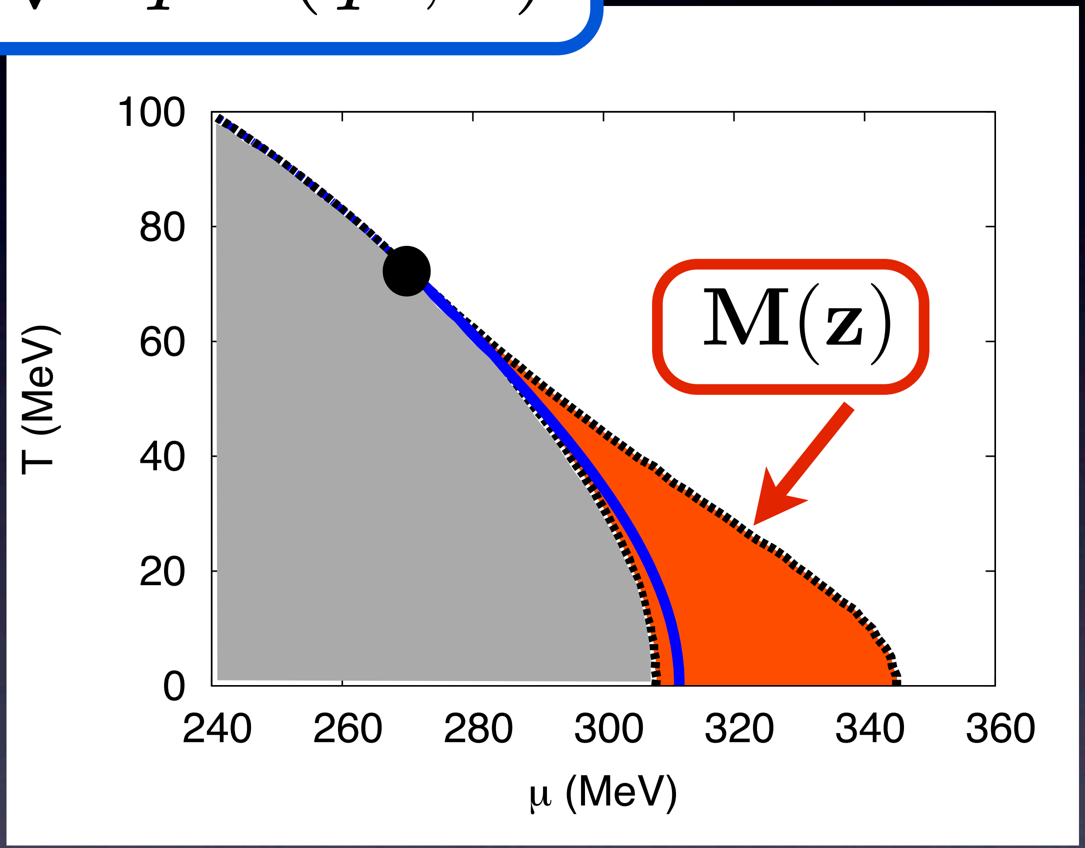
$$\begin{aligned}
\omega = {} & \frac{\eta_2}{2}m^2 + \frac{\eta_4}{4}(m^4 + (\tilde{\nabla} m)^2) \\
& + \frac{\mathrm{sgn}(\alpha_6)}{6}\left(m^6 + 5m^2(\tilde{\nabla} m)^2 + \tfrac{1}{2}(\tilde{\nabla}\tilde{\Delta} m)^2\right) \\
& + \tfrac{1}{8}\left(m^8 + 14m^4(\tilde{\nabla} m)^2 - \tfrac{1}{5}(\tilde{\nabla} m)^4\right. \\
& \left. + \tfrac{18}{5} m\tilde{\Delta} m(\tilde{\nabla} m)^2 + \tfrac{14}{5} m^2(\tilde{\Delta} m)^2 + \tfrac{1}{5}(\tilde{\nabla}\tilde{\Delta} m)^2\right)
\end{aligned}$$

NJL model phase diagram

$$M_{sn}(z) = \sqrt{\nu} q \, \mathrm{sn}(qz, \nu)$$

M. Thies, J. Phys. A2006
D. Nickel, PRL09, PRD09





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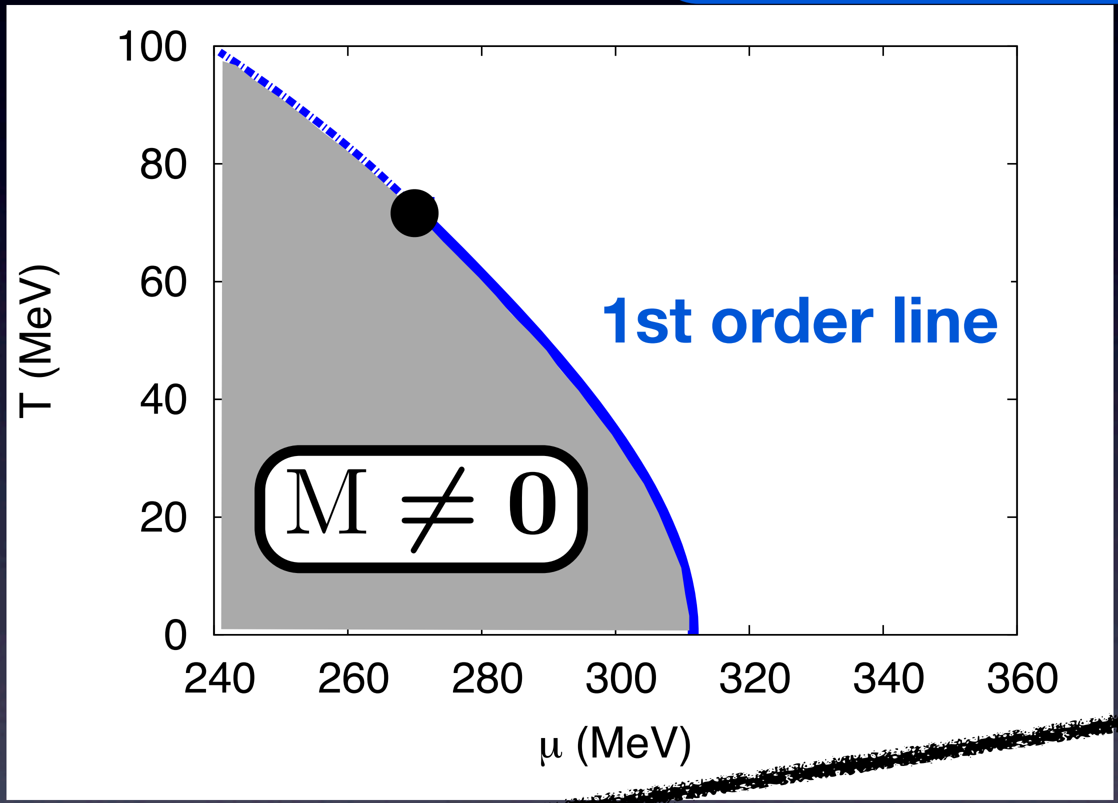

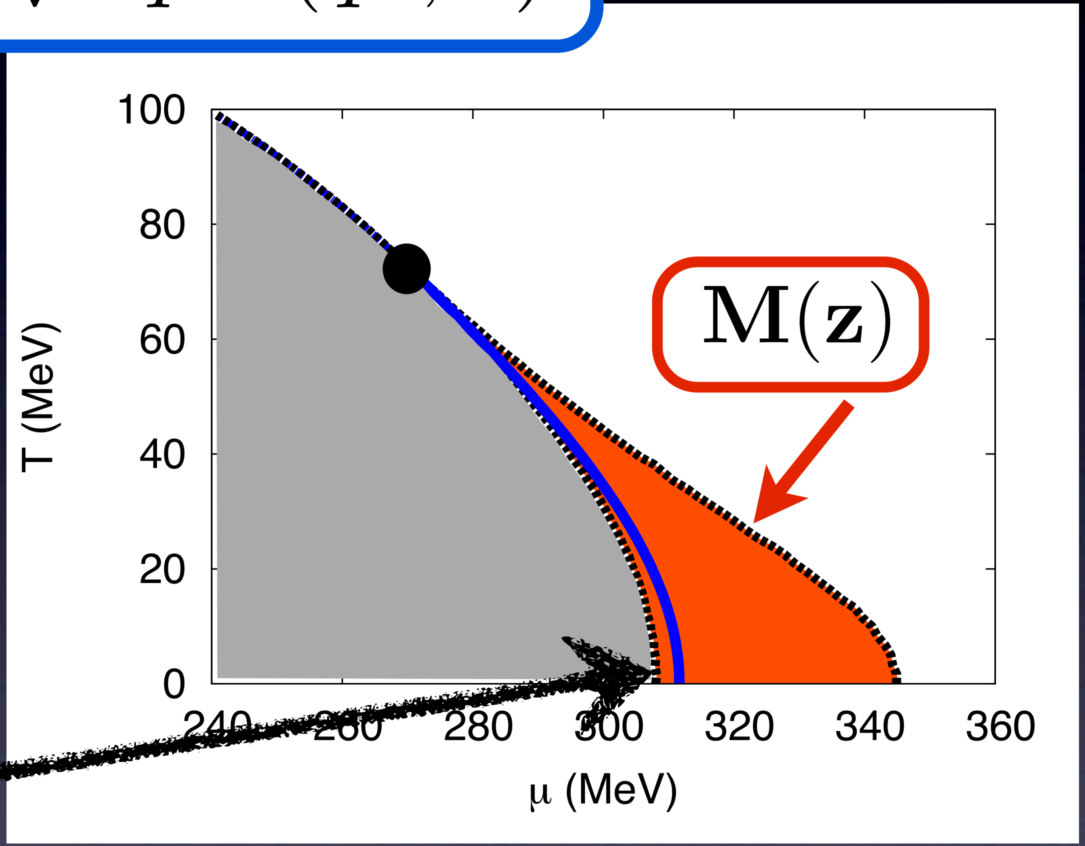

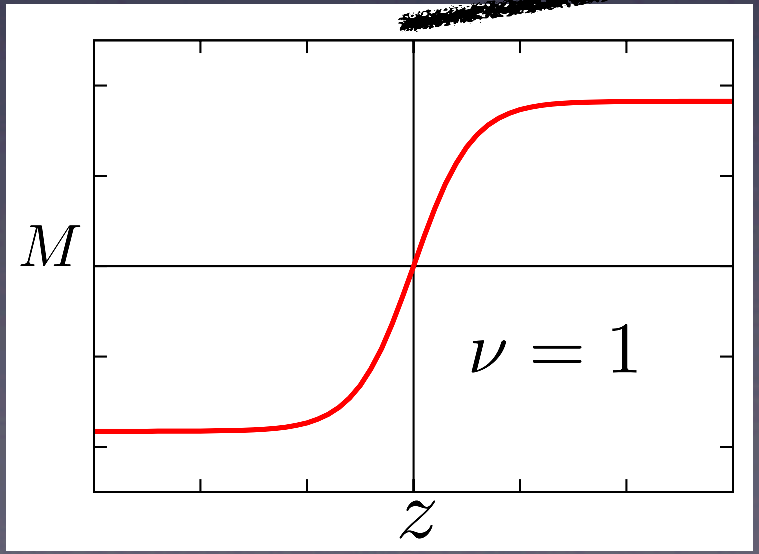

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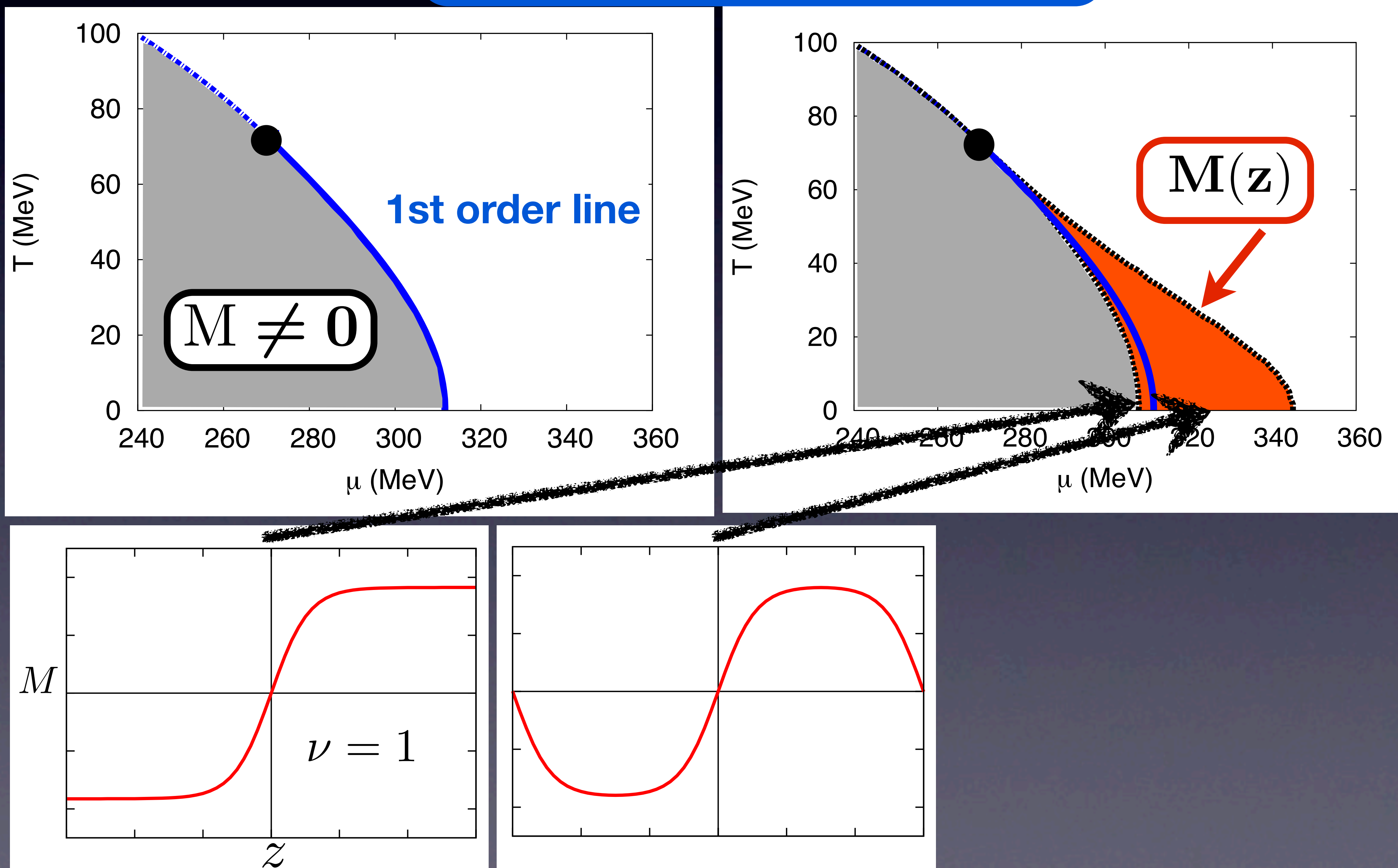

NJL model phase diagram

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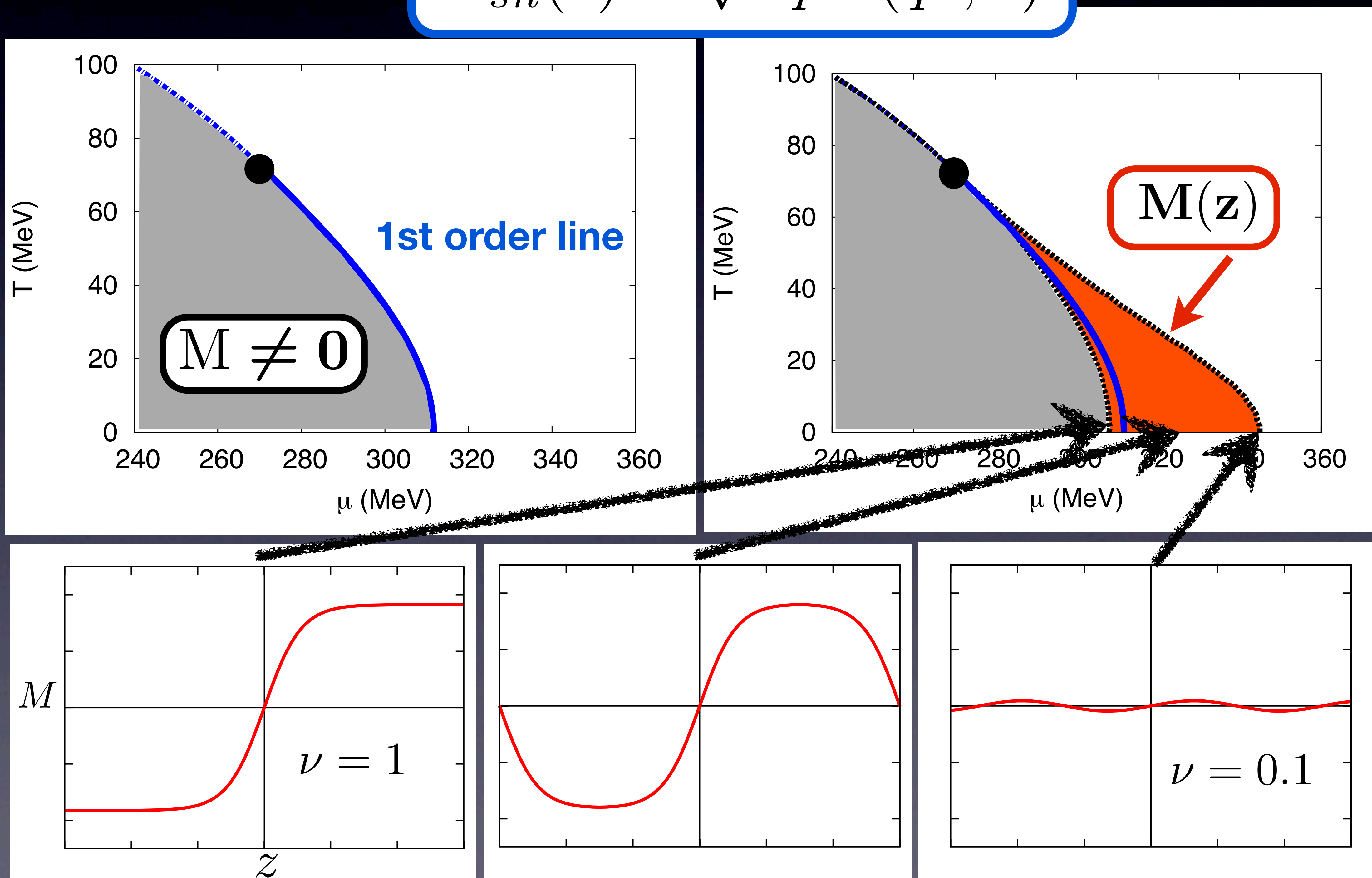
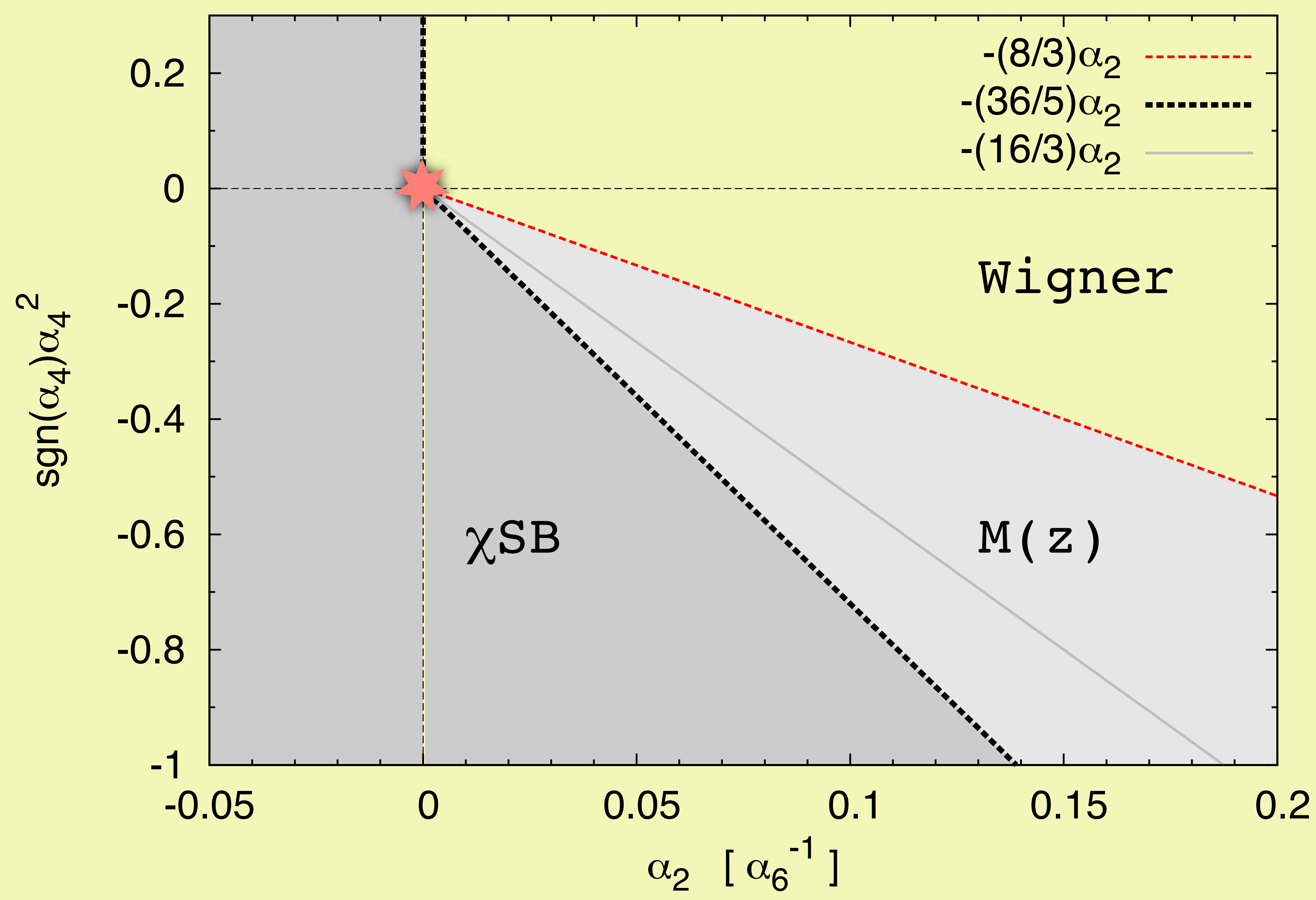
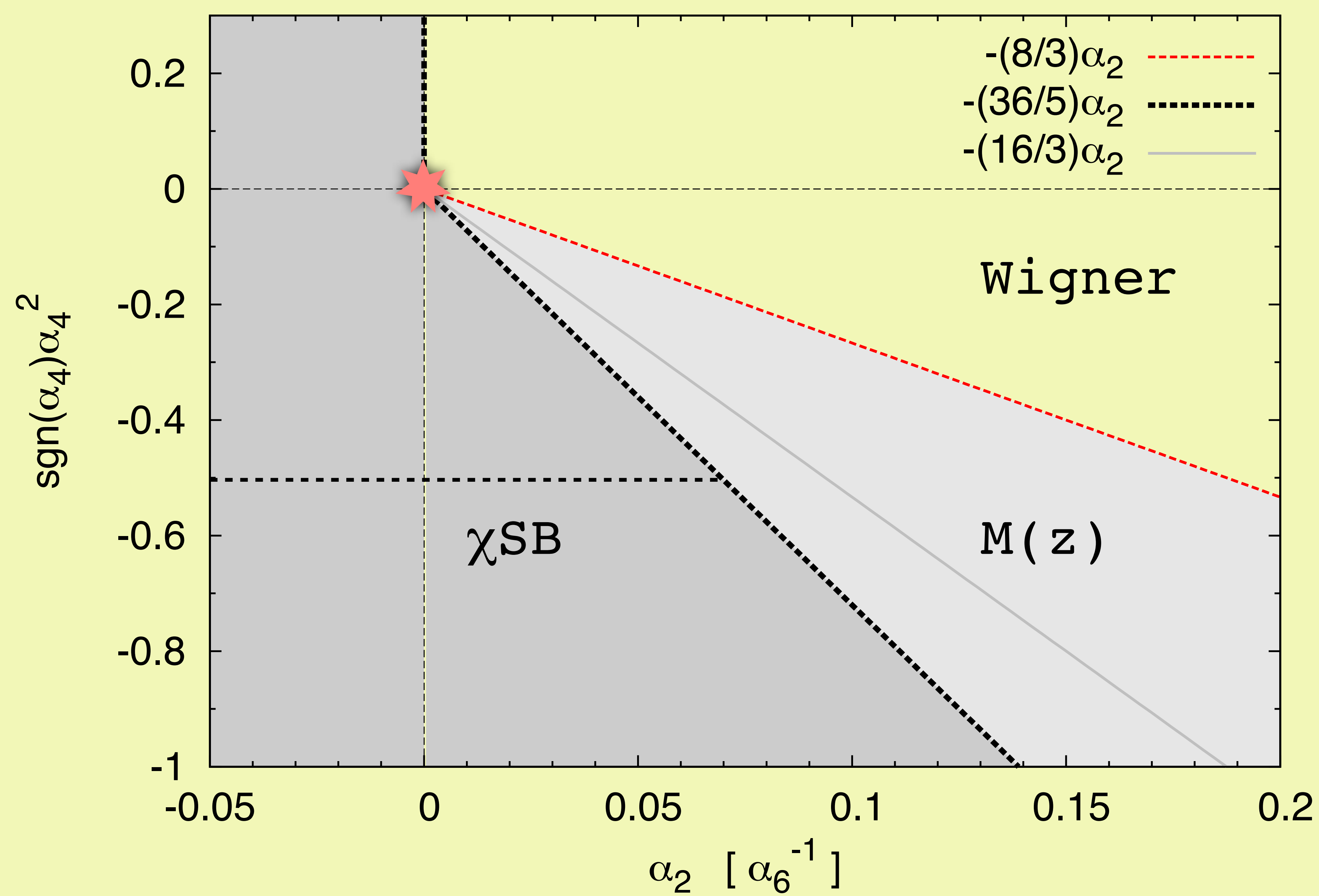


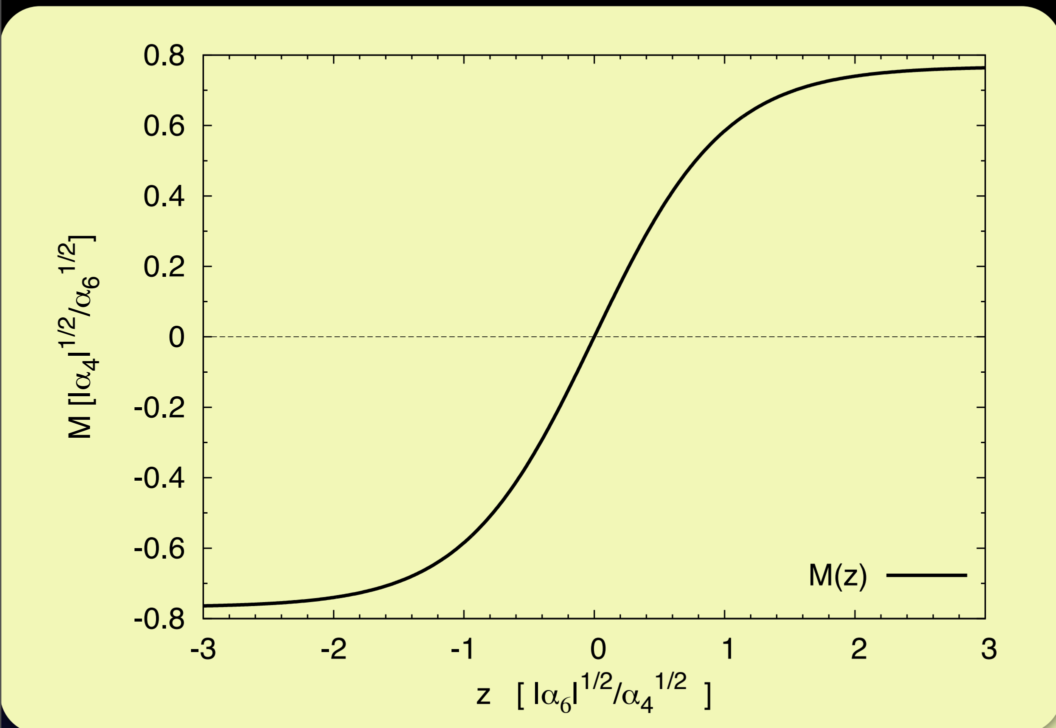
Soliton formation

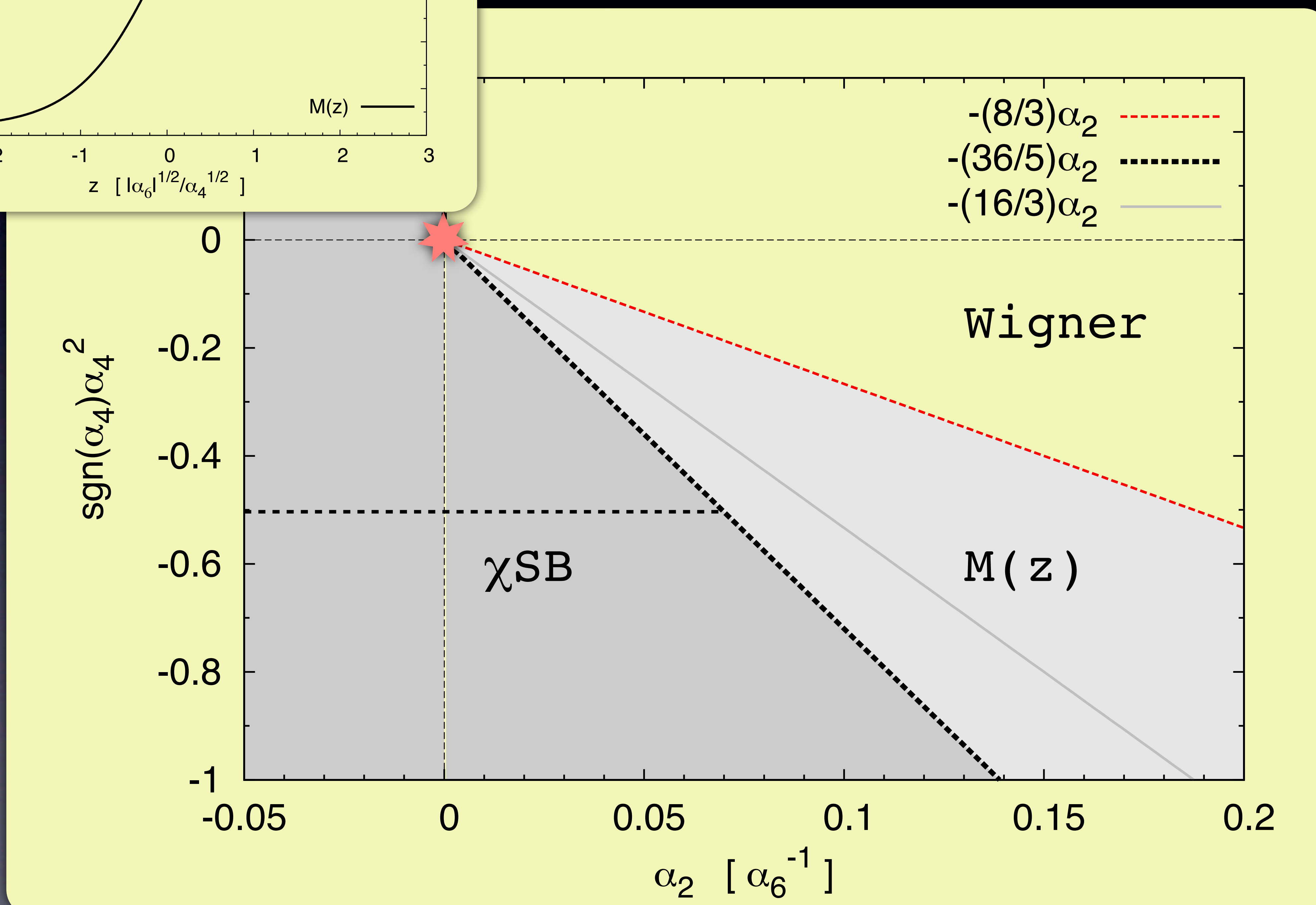


Soliton formation

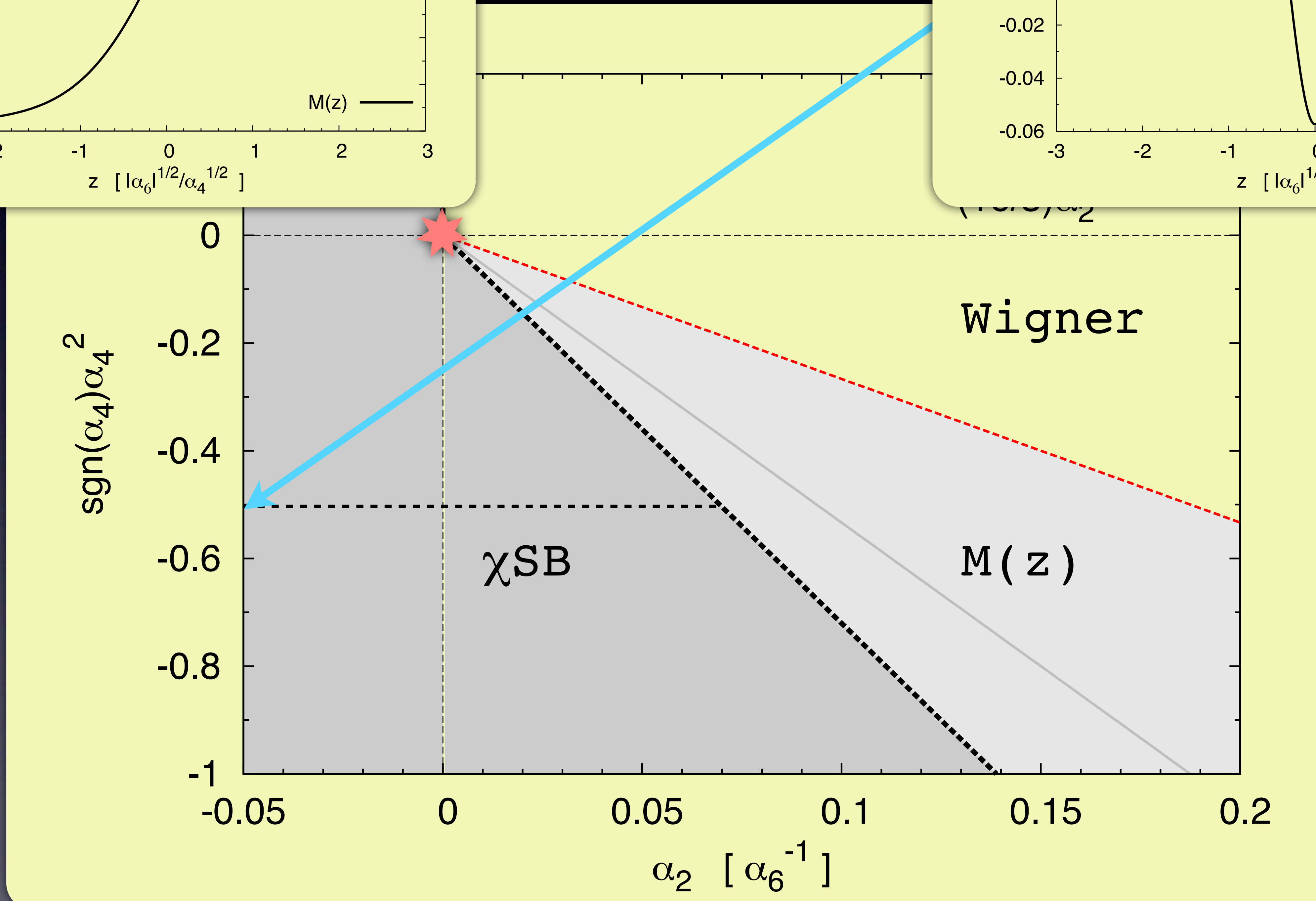


on formation

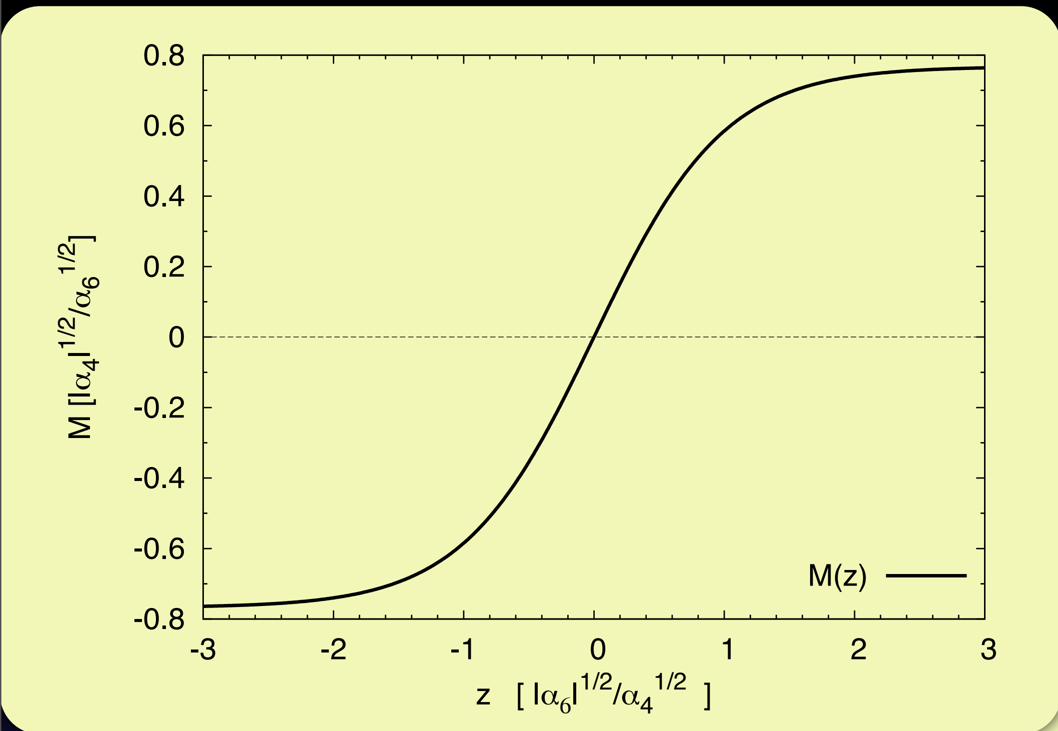




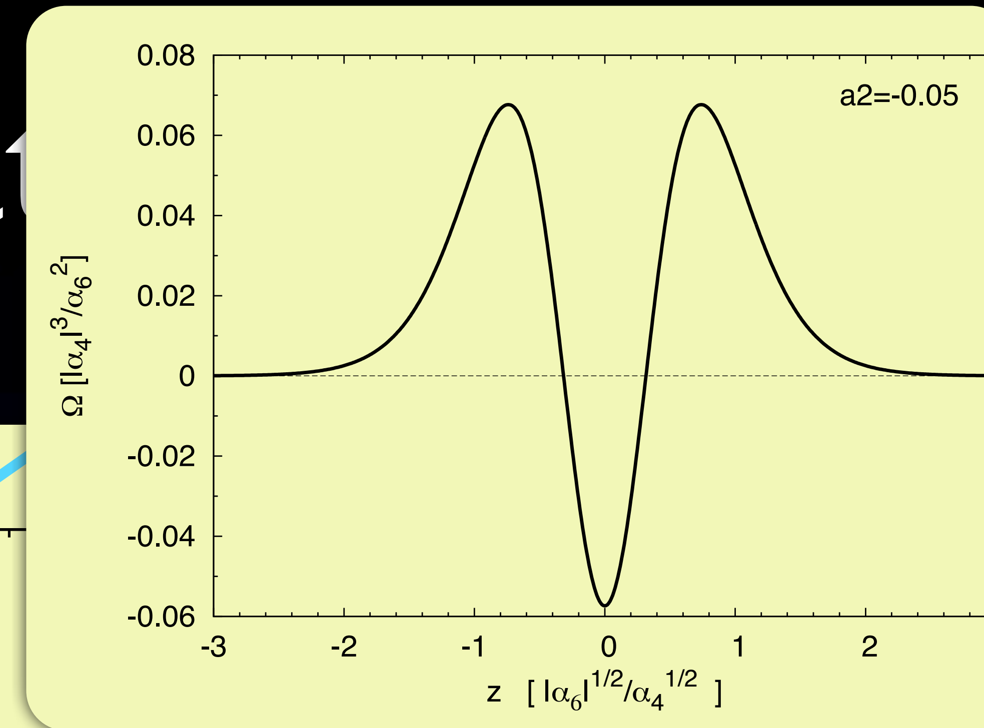
on format



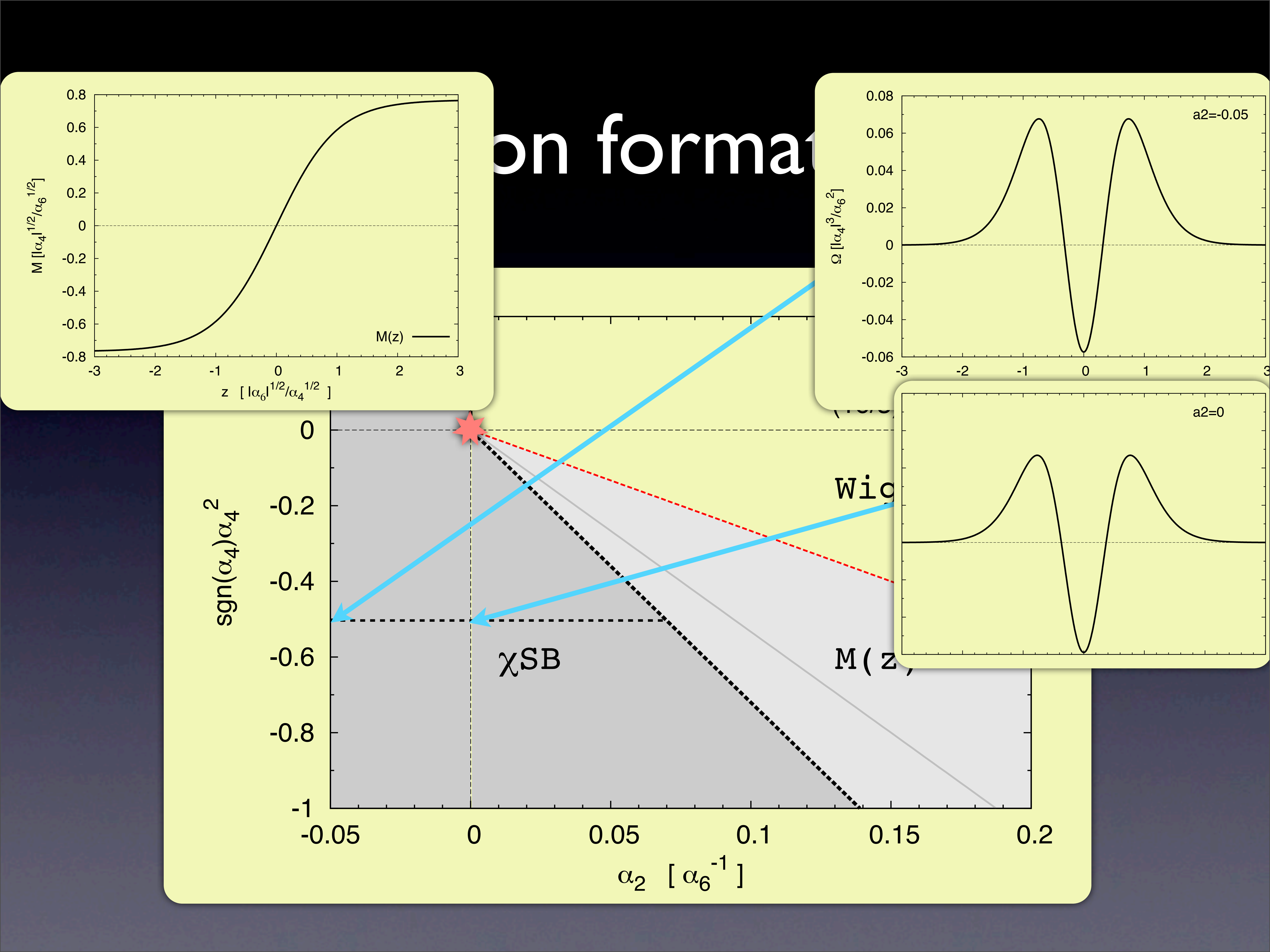
sgn(α4)α4 2
α2 [α6 -1]
Wigner
χSB
M(z)
(10/3)α2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.05
0
0.05
0.1
0.15
0.2

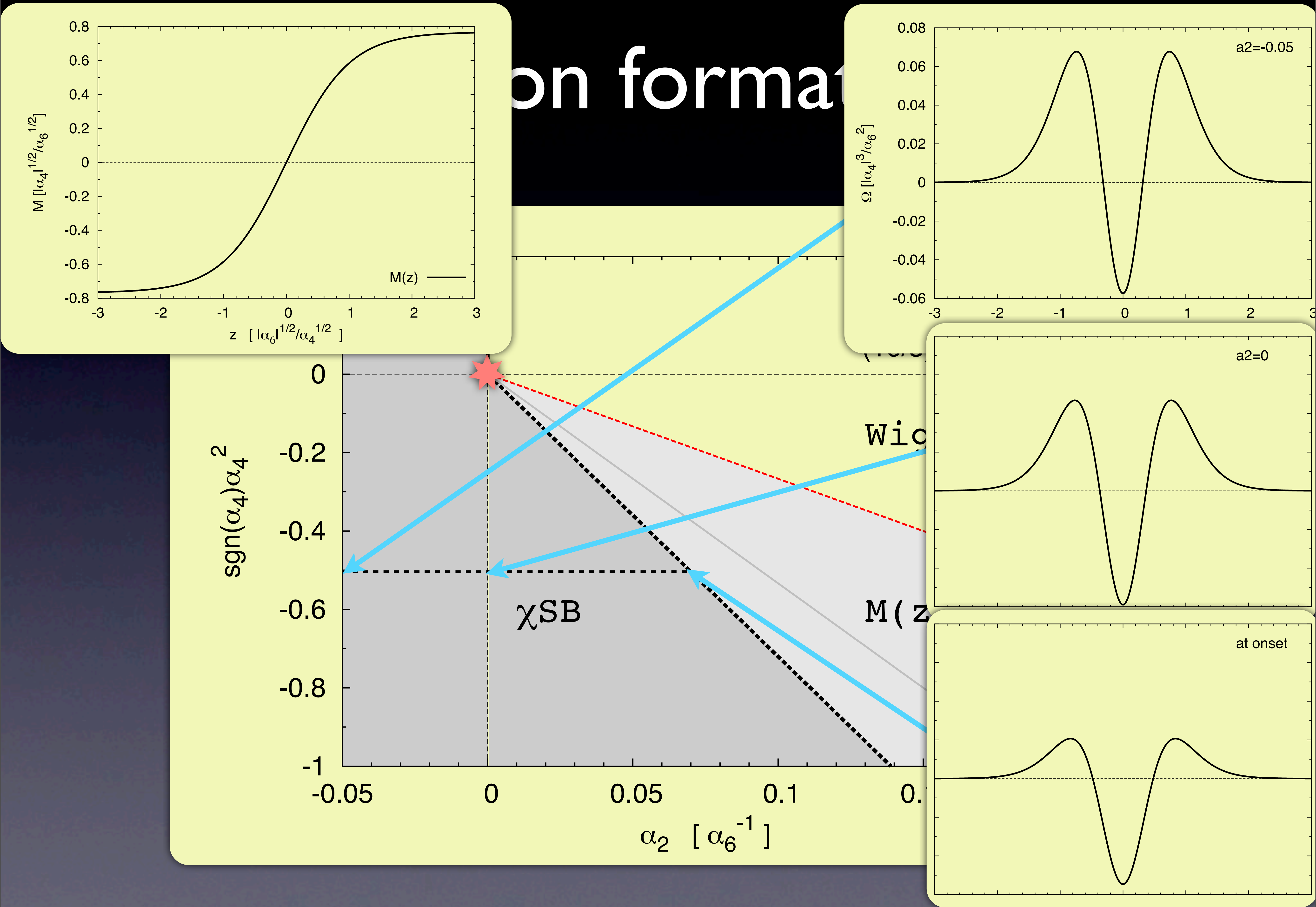


M [|α4|1/2/α6 1/2]
z [|α6|1/2/α4 1/2]
M(z)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-3
-2
-1
0
1
2
3



Ω [|α4|3/α6 2]
z [|α6|1/2/α4 1/2]
a2=-0.05
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-3
-2
-1
0
1
2
3


on format
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-3
-2
-1
0
1
2
3
M(z)
z [$|\alpha_6|^{1/2}/\alpha_4^{1/2}$]
M [$|\alpha_4|^{1/2}/\alpha_6^{1/2}$]
0.08
0.06
0.04
0.02
-0.02
-0.04
-0.06
a2=-0.05
Ω [$|\alpha_4|^3/\alpha_6^2$]
a2=0
Wig
χSB
M(z)
$\mathrm{sgn}(\alpha_4)\alpha_4^2$
-1
-0.05
0.05
0.1
0.15
0.2
α_2 [α_6^{-1}]



on format
M(z)
M [|α4|^1/2/α6^1/2]
z [|α6|^1/2/α4^1/2]
a2=-0.05
Ω [|α4|^3/α6^2]
a2=0
at onset
sgn(α4)α4^2
α2 [α6^-1]
Wig
χSB
M(z

Energy density profile in detail

$$\Omega(z) = \underbrace{\left(\frac{\alpha_2}{2}M(z)^2 + \frac{\alpha_4}{4}M(z)^4 + \frac{1}{6}M(z)^6\right) - \Omega_0}_{\text{1: no derivative terms}}$$

$$\underbrace{+\,\alpha_4\frac{\nabla M^2}{4}}_{\text{2: Derivative term at 4 th order}} + \underbrace{\left(\frac{5M^2(\nabla M)^2}{6} + \frac{(\Delta M)^2}{12}\right)}_{\text{3: Derivative terms at 6 th order}}$$

1: no derivative terms

2: Derivative term at 4 th order

3: Derivative terms at 6 th order