Gorkov-Green's function approach to open-shell nuclei

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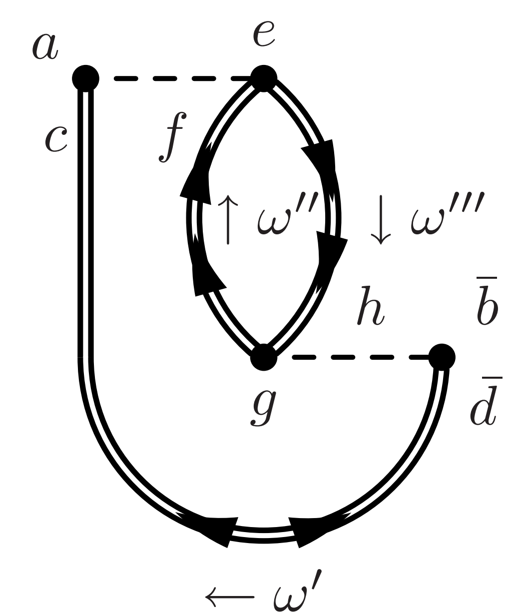


Based on:

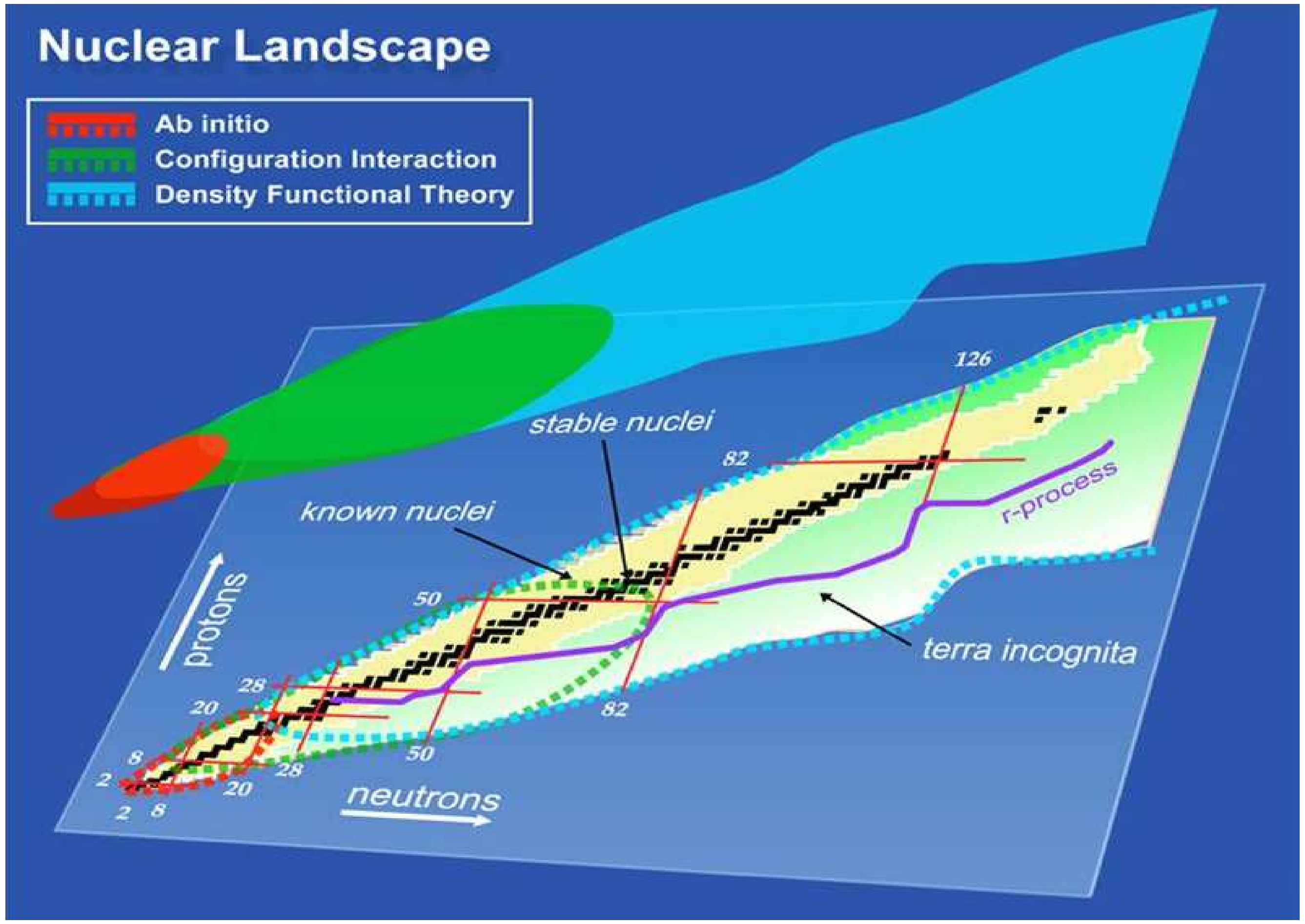
VS, Duguet, Barbieri PRC 84 064317 (2011)

“Facets of Strong-Interaction Physics”

Hirschegg 2012

Towards a unified description of nuclei

- ✤ How to extend to open-shell?
- ✤ How to link with EDF?
- ✤ How to calculate reactions?



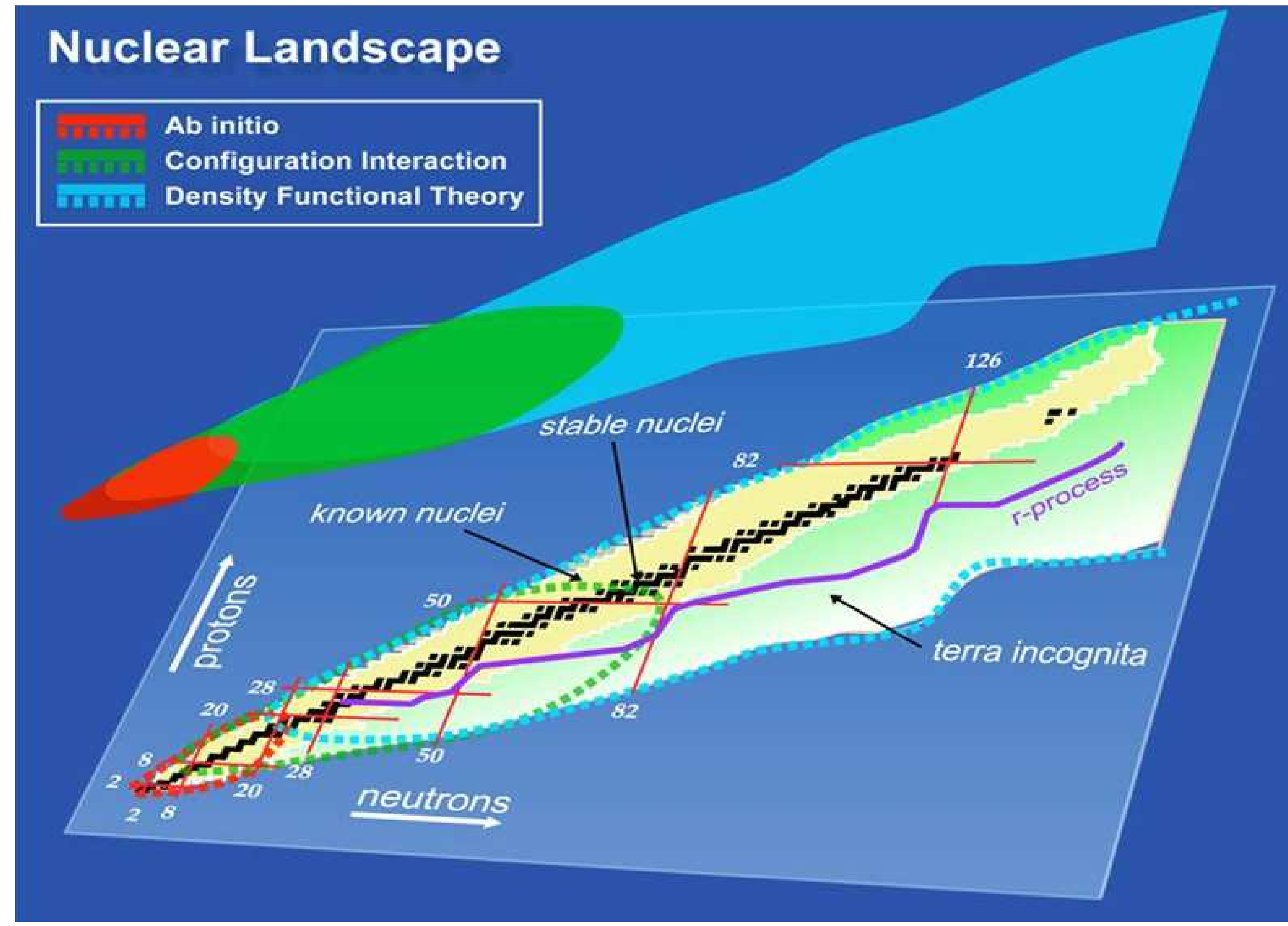
Towards a unified description of nuclei

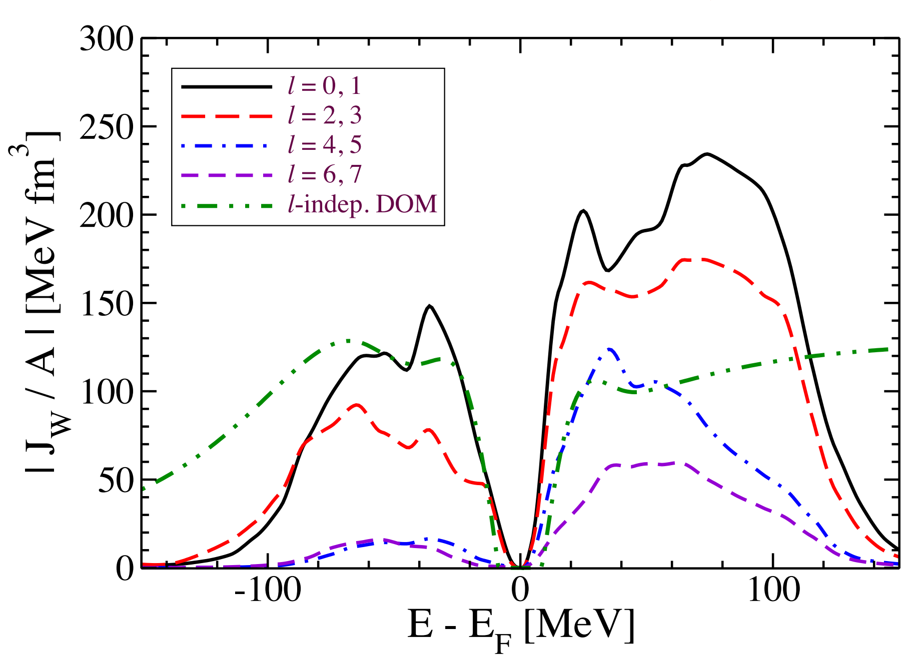
Ab-initio Green's functions

- ✤ How to extend to open-shell?
- ➟ this talk

- ✤ How to link with EDF?
- ➟ work in progress

- ✤ How to calculate reactions?
- ➟ TD-GF [Rios *et al.* 2011]
- ➟ link to DOM





[Waldecker, Barbieri, Dickhoff 2011]

State-of-the-art ab-initio nuclear structure theory

✷ Methods for an ab-initio description of medium-mass nuclei

(1) Self-consistent Dyson-Green's function [Barbieri, Dickhoff, ...]

(2) Coupled-cluster [Dean, Hagen, Hjorth-Jensen, Papenbrock, ...]

(3) In-medium similarity renormalization group [Tsukiyama, Bogner, Schwenk, ...]

→ Similar level of accuracy

→ But limited to to doubly-closed-shell ± 1 and ± 2 nuclei

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✵ Truly open-shell nuclei

(a) Multi-reference methods: IMSRG + CI, MR-CC

(b) Single-reference methods: explicit account of pairing mandatory

→ Dyson-Green's functions ⟶ Gorkov-Green's functions

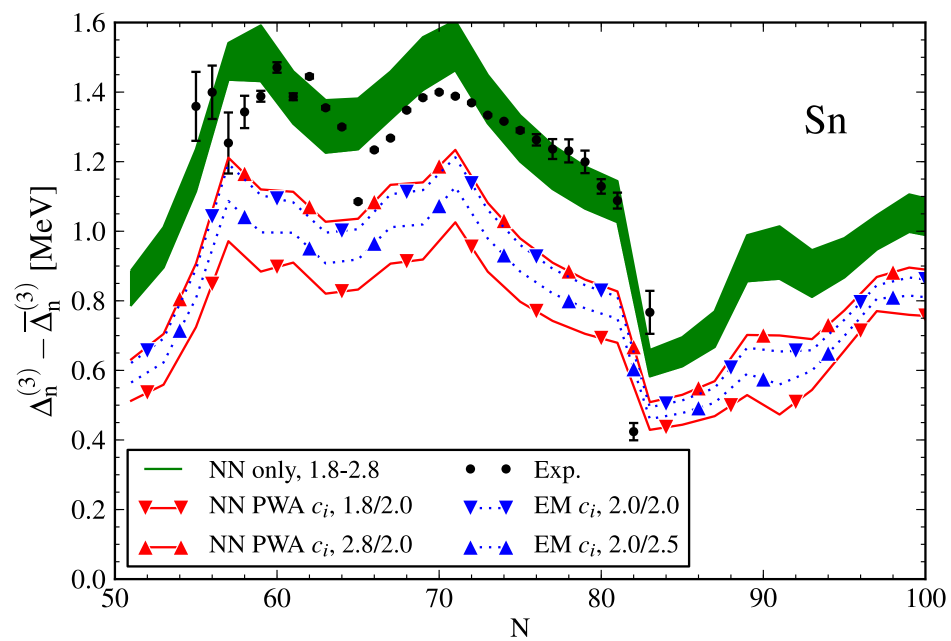
Connection to *non-empirical* EDF

- Standard EDF parameterizations (e.g. Skyrme, Gogny, relativistic)
  - Successful in major shell where adjusted
  - Lack predictive power in new regions of interest

Efforts to extend and connect with more fundamental approaches

- Non-empirical EDF from low-momentum interactions

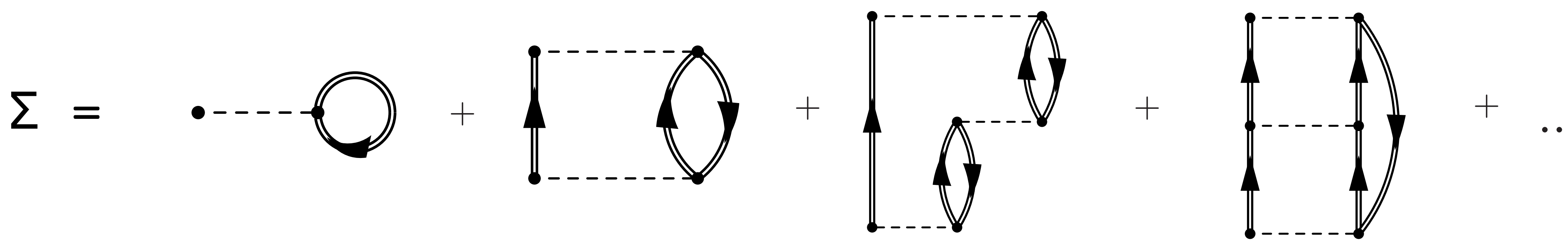
[Lesinski *et al.* 2011]



  - Pairing channel [Lesinski *et al.*]
  - Particle-hole channel [Gebremariam *et al.*]

Benchmarks needed from many-body methods that share the same features

Green's functions: essential features

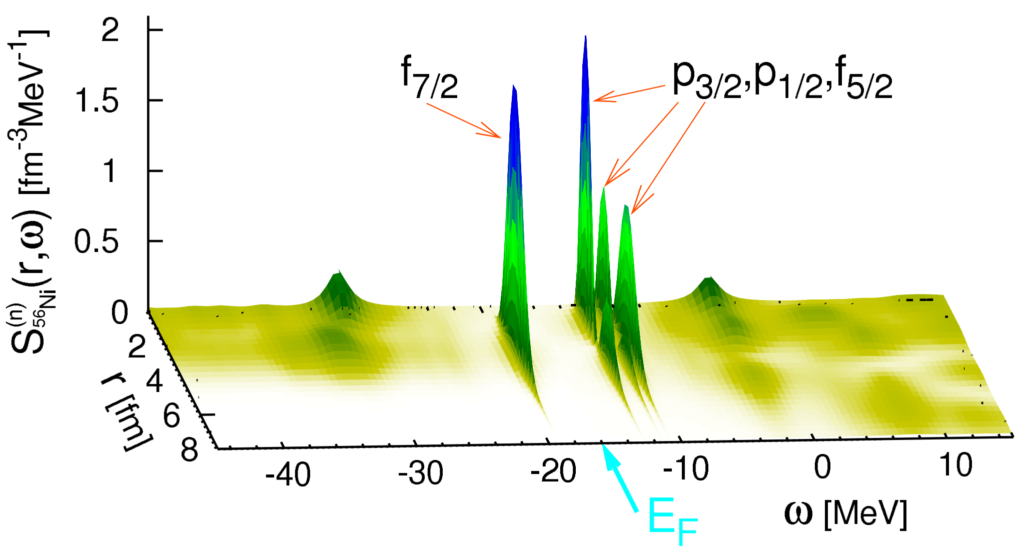


- ➠ Ab-initio, self-consistent approach
- ➠ Direct connection to observables
- ➠ Improvability (diagrammatic expansion)
- ➠ Control over many-body requirements (conserving approximations)

Dyson Green's functions

- Spectral function

$$S_a^-(\omega) \equiv \sum_k \left| \langle \psi_k^{N-1} | a_a | \psi_0^N \rangle \right|^2 \delta(\omega - (E_0^N - E_k^{N-1})) = \frac{1}{\pi} \operatorname{Im} G_{aa}(\omega)$$



[Barbieri 2009]

^{56}Ni

Dyson Green's functions

- Spectral function

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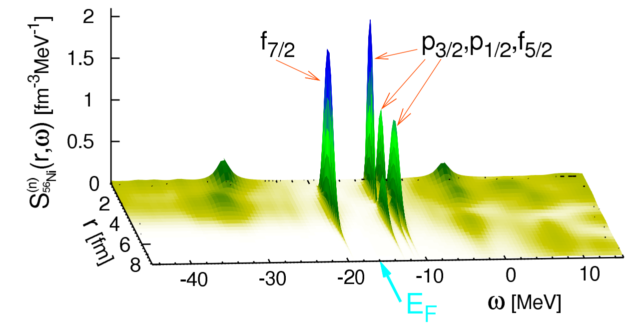
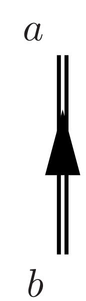


- Green's function

$$\boxed{i\, G_{ab}(t,t') \equiv \langle \Psi_0^N | T \left\{ a_a(t) a_b^\dagger(t') \right\} | \Psi_0^N \rangle} \equiv$$

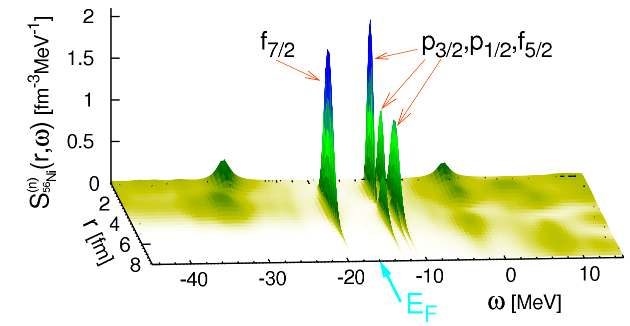


- N-particle ground state
- One nucleon addition and removal (N±1 systems)

Dyson Green's functions

✳ Spectral function

$$\mathrm{S}_a^-(\omega) \equiv \sum_k \left| \langle \psi_k^{N-1} | a_a | \psi_0^N \rangle \right|^2 \delta(\omega - (E_0^N - E_k^{N-1})) = \frac{1}{\pi} \, \mathrm{Im} \, G_{aa}(\omega)$$



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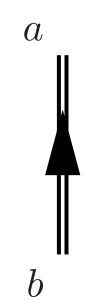


➟ N-particle ground state

➟ One nucleon addition and removal (N±1 systems)

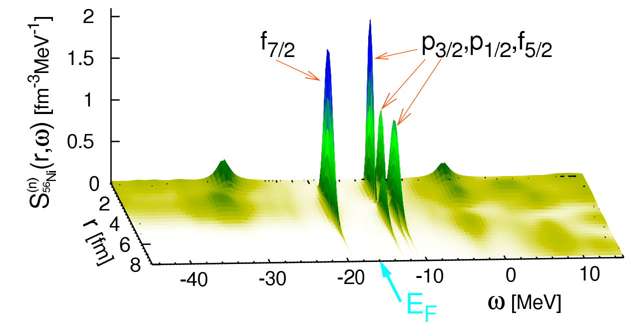
➟ Contains all structure information probed by nucleon transfer

$$G_{ab}(\omega) \equiv \sum_k \frac{\langle \psi_0^N | a_a | \psi_k^{N+1} \rangle \langle \psi_k^{N+1} | a_b^\dagger | \psi_0^N \rangle}{\omega - (E_k^{N+1} - E_0^N) + i\eta} + \sum_k \frac{\langle \psi_0^N | a_b^\dagger | \psi_k^{N-1} \rangle \langle \psi_k^{N-1} | a_a | \psi_0^N \rangle}{\omega - (E_0^N - E_k^{N-1}) - i\eta}$$

Dyson Green's functions

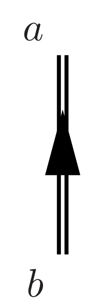
- Spectral function

$$S_a^-(\omega) \equiv \sum_k \left| \langle \psi_k^{N-1} | a_a | \psi_0^N \rangle \right|^2 \delta(\omega - (E_0^N - E_k^{N-1})) = \frac{1}{\pi} \, \mathrm{Im} \, G_{aa}(\omega)$$



- Green's function

$$\boxed{i \, G_{ab}(t,t') \equiv \langle \Psi_0^N | T \left\{ a_a(t) a_b^\dagger(t') \right\} | \Psi_0^N \rangle} \equiv$$



- N-particle ground state
- One nucleon addition and removal (N±1 systems)

- Dyson equation

$$\boxed{G_{ab}(\omega) = G_{ab}^{(0)}(\omega) + \sum_{cd} G_{ac}^{(0)}(\omega) \, \Sigma_{cd}^\star(\omega) \, G_{db}(\omega)}$$

Solution breaks down when pairing instabilities appear

Going open-shell: Gorkov ansatz

- Formulate the expansion scheme around a Bogoliubov vacuum

→ Zeroth order already incorporates pairing

- Auxiliary many-body state $|\Psi_0\rangle \equiv \sum_N^{\text{even}} c_N \, |\psi_0^N\rangle$

→ Mixes various particle numbers

→ Introduce a "grand-canonical" potential $\Omega = H - \mu N$

⇒ $|\Psi_0\rangle$ minimizes $\Omega_0 = \langle\Psi_0|\Omega|\Psi_0\rangle$

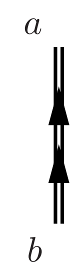
under the constraint $N = \langle\Psi_0|N|\Psi_0\rangle$

⇒ $\Omega_0 = \sum_{N'} |c_{N'}|^2 \Omega_0^{N'} \approx E_0^N - \mu N$

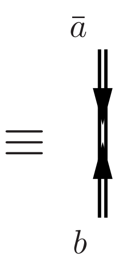
Gorkov Green's functions and equations

✺ Set of 4 Green's functions

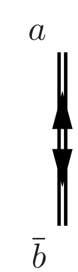
$$i\,G^{11}_{ab}(t,t') \equiv \langle \Psi_0 | T \left\{ a_a(t) a^\dagger_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$



$$i\,G^{21}_{ab}(t,t') \equiv \langle \Psi_0 | T \left\{ \bar{a}^\dagger_a(t) a^\dagger_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$



$$i\,G^{12}_{ab}(t,t') \equiv \langle \Psi_0 | T \left\{ a_a(t) \bar{a}_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$



$$i\,G^{22}_{ab}(t,t') \equiv \langle \Psi_0 | T \left\{ \bar{a}^\dagger_a(t) \bar{a}_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$

[Gorkov 1958]

$$\mathbf{G}_{ab}(\omega) = \mathbf{G}^{(0)}_{ab}(\omega) + \sum_{cd} \mathbf{G}^{(0)}_{ac}(\omega)\, \mathbf{\Sigma}^{\star}_{cd}(\omega)\, \mathbf{G}_{db}(\omega)$$

Gorkov equations

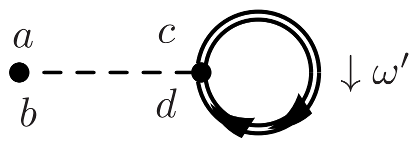
$$\mathbf{\Sigma}^{\star}_{ab}(\omega) \equiv \begin{pmatrix} \Sigma^{\star\,11}_{ab}(\omega) & \Sigma^{\star\,12}_{ab}(\omega) \\ \Sigma^{\star\,21}_{ab}(\omega) & \Sigma^{\star\,22}_{ab}(\omega) \end{pmatrix}$$

$$\mathbf{\Sigma}^{\star}_{ab}(\omega) \equiv \mathbf{\Sigma}_{ab}(\omega) - \mathbf{U}_{ab}$$

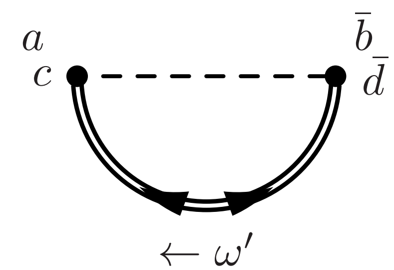
1st & 2nd order diagrams and eigenvalue problem

✴ 1st order ➟ energy-independent self-energy

$\Sigma^{11\,(1)}_{ab} =$

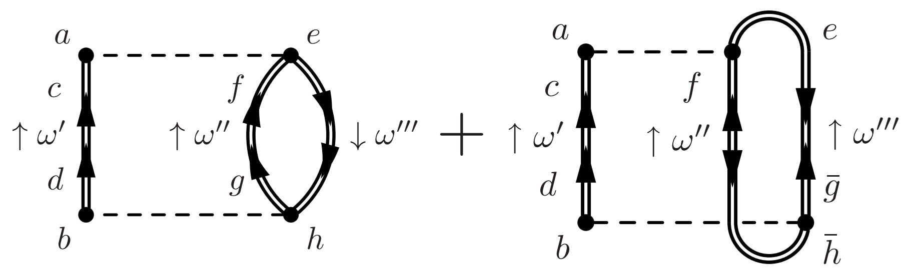


$\Sigma^{12\,(1)}_{ab} =$

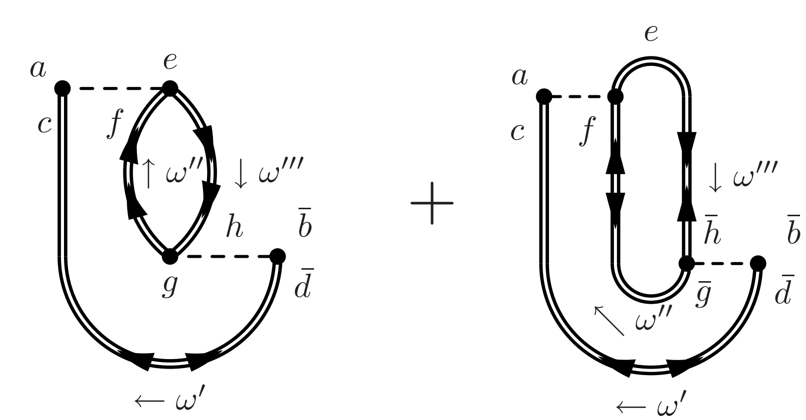


✴ 2nd order ➟ energy-dependent self-energy

$\Sigma^{11\,(2)}_{ab}(\omega) =$



$\Sigma^{12\,(2)}_{ab}(\omega) =$



✴ Gorkov equations $\longrightarrow$ eigenvalue problem

$$\boxed{\sum_b \begin{pmatrix} t_{ab} - \mu_{ab} + \Sigma^{11}_{ab}(\omega) & \Sigma^{12}_{ab}(\omega) \\ \Sigma^{21}_{ab}(\omega) & -t_{ab} + \mu_{ab} + \Sigma^{22}_{ab}(\omega) \end{pmatrix}\Bigg|_{\omega_k} \begin{pmatrix} \mathcal{U}^k_b \\ \mathcal{V}^k_b \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k_a \\ \mathcal{V}^k_a \end{pmatrix}}$$

$$\mathcal{U}^{k*}_a \equiv \langle \Psi_k | \bar{a}^\dagger_a | \Psi_0 \rangle$$
$$\mathcal{V}^{k*}_a \equiv \langle \Psi_k | a_a | \Psi_0 \rangle$$

Gorkov equations

$$\sum_b \begin{pmatrix} t_{ab} - \mu_{ab} + \Sigma^{11}_{ab}(\omega) & \Sigma^{12}_{ab}(\omega) \\ \Sigma^{21}_{ab}(\omega) & -t_{ab} + \mu_{ab} + \Sigma^{22}_{ab}(\omega) \end{pmatrix} \Bigg|_{\omega_k} \begin{pmatrix} \mathcal{U}^k_b \\ \mathcal{V}^k_b \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k_a \\ \mathcal{V}^k_a \end{pmatrix}$$

$$\boxed{\begin{pmatrix} T - \mu + \Lambda & \tilde{h} & \mathcal{C} & -\mathcal{D}^\dagger \\ \tilde{h}^\dagger & -T + \mu - \Lambda & -\mathcal{D}^\dagger & \mathcal{C} \\ \mathcal{C}^\dagger & -\mathcal{D} & E & 0 \\ -\mathcal{D} & \mathcal{C}^\dagger & 0 & -E \end{pmatrix} \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix}}$$

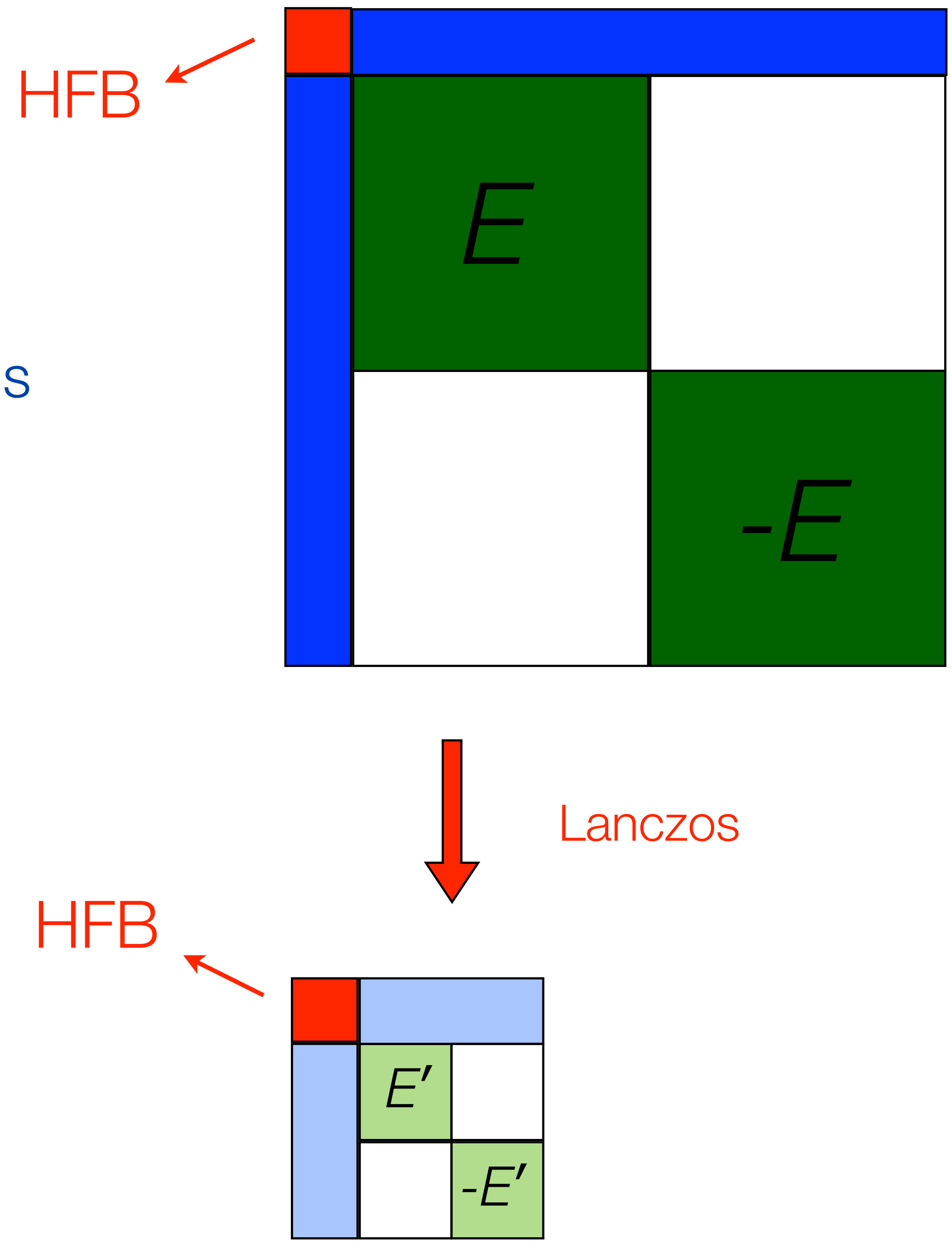
Energy *independent* eigenvalue problem

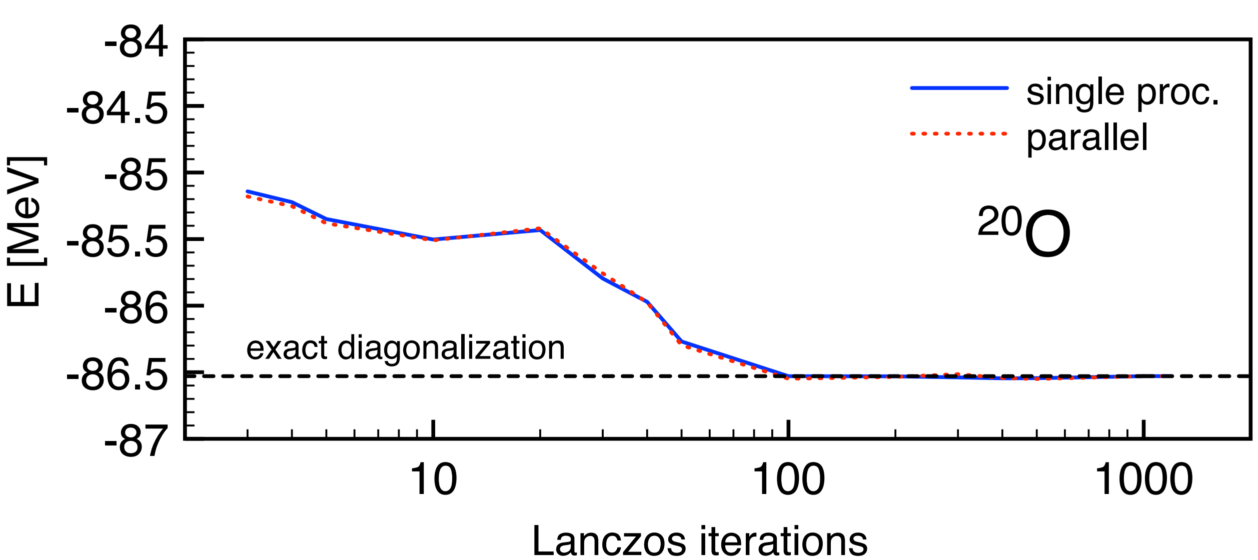
with the normalization condition $$\sum_a \left[\left|\mathcal{U}^k_a\right|^2 + \left|\mathcal{V}^k_a\right|^2 \right] + \sum_{k_1 k_2 k_3} \left[\left|\mathcal{W}^{k_1 k_2 k_3}_k\right|^2 + \left|\mathcal{Z}^{k_1 k_2 k_3}_k\right|^2 \right] = 1$$

Lanczos projection

$$\begin{pmatrix} T-\mu+\Lambda & \tilde{h} & \mathcal{C} & -\mathcal{D}^\dagger \\ \tilde{h}^\dagger & -T+\mu-\Lambda & -\mathcal{D}^\dagger & \mathcal{C} \\ \mathcal{C}^\dagger & -\mathcal{D} & E & 0 \\ -\mathcal{D} & \mathcal{C}^\dagger & 0 & -E \end{pmatrix} \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix}$$

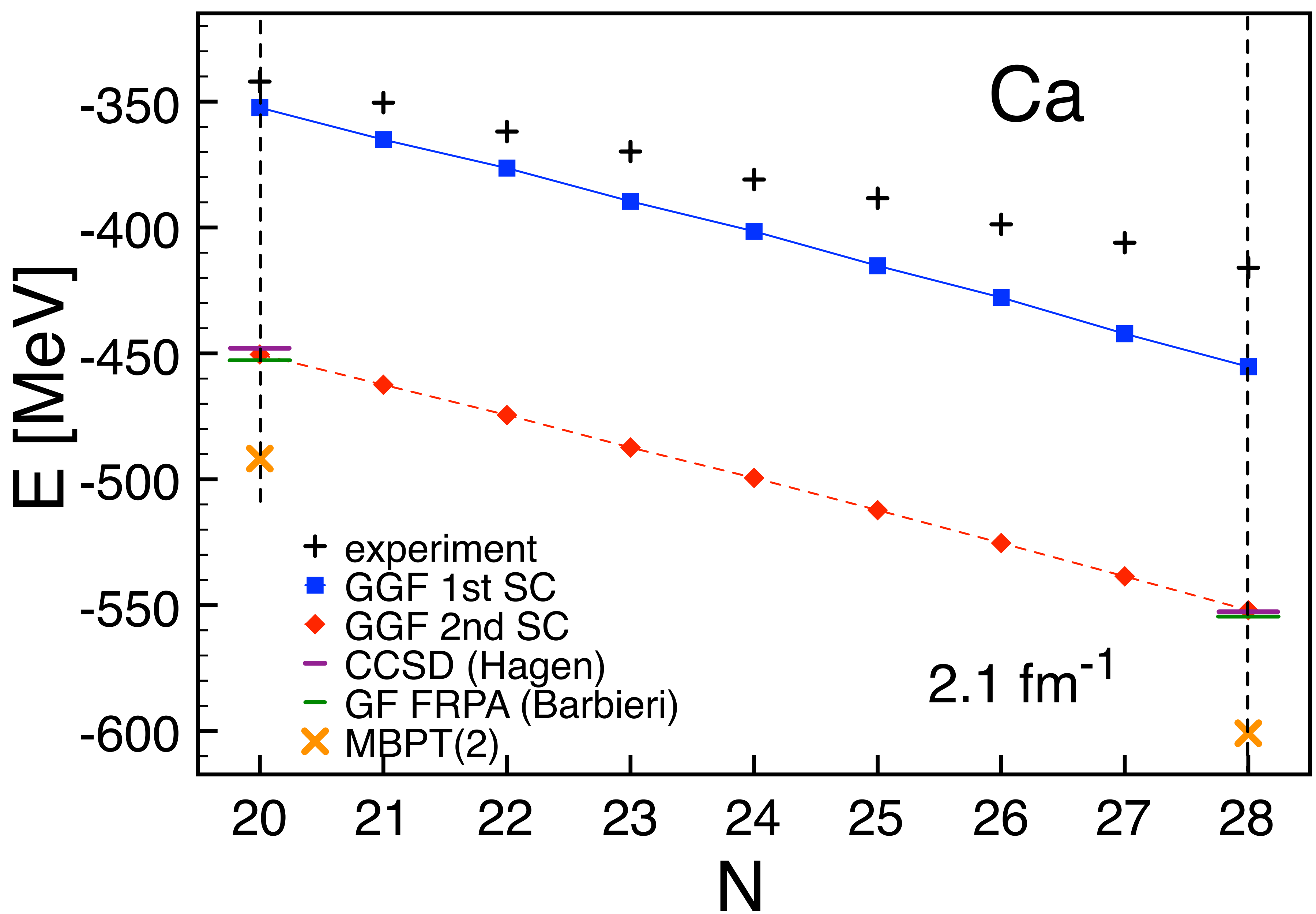
- Conserves moments of spectral functions
- Equivalent to exact diagonalization for $N_L \to \dim(E)$





Binding energies

- Systematic along isotopic/isotonic chains become available



Spectrum and spectroscopic factors

✷ Separation energy spectrum

$$G^{11}_{ab}(\omega) = \sum_k \left\{ \frac{\mathcal{U}^k_a \, \mathcal{U}^{k*}_b}{\omega - \omega_k + i\eta} + \frac{\bar{\mathcal{V}}^{k*}_a \, \bar{\mathcal{V}}^k_b}{\omega + \omega_k - i\eta} \right\}$$

Lehmann representation

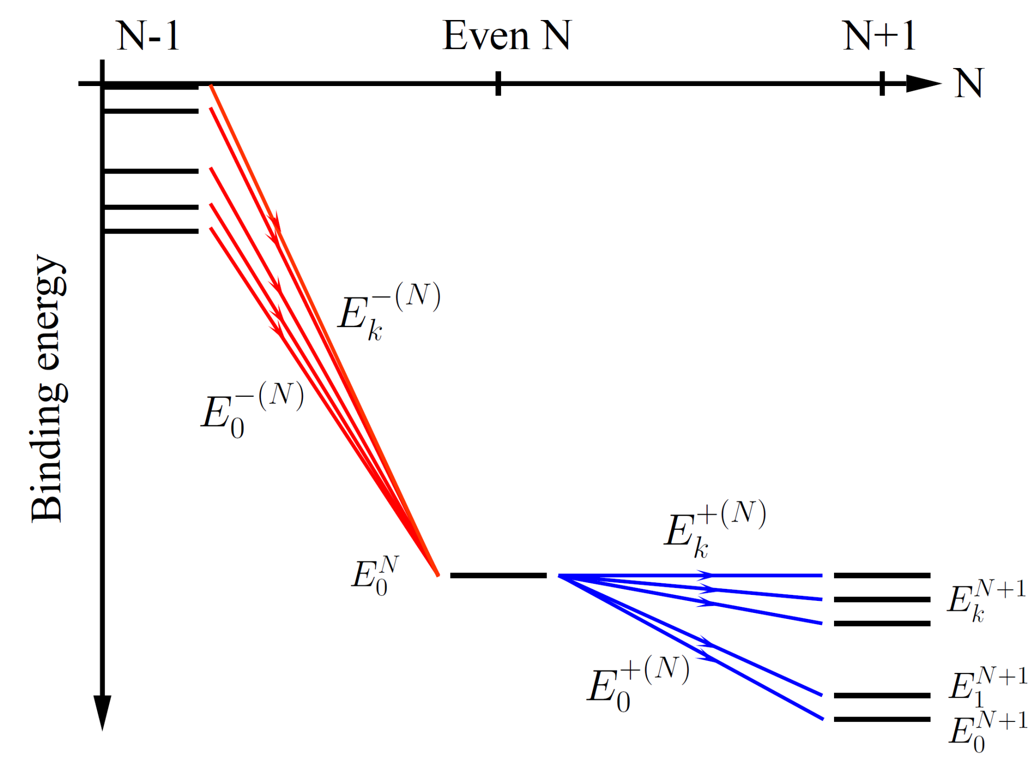
where
$$\begin{cases} \mathcal{U}^{k*}_a \equiv \langle \Psi_k | a^\dagger_a | \Psi_0 \rangle \\ \mathcal{V}^{k*}_a \equiv \langle \Psi_k | \bar{a}_a | \Psi_0 \rangle \end{cases}$$

and
$$\begin{cases} E^{+\,(N)}_k \equiv E^{N+1}_k - E^N_0 \\ E^{-\,(N)}_k \equiv E^N_0 - E^{N-1}_k \end{cases}$$

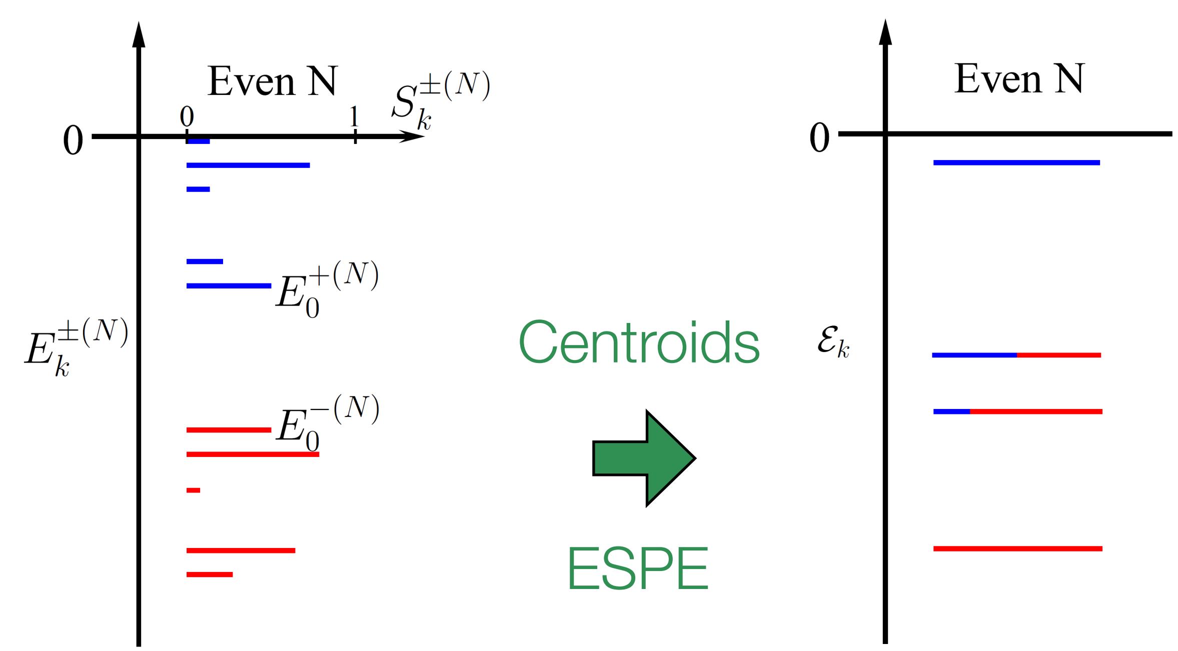
✷ Spectroscopic factors

$$\mathcal{S}^+_k \equiv \sum_a \left| \langle \psi_k | a^\dagger_a | \psi_0 \rangle \right|^2 = \sum_a \left| \mathcal{U}^k_a \right|^2$$

$$\mathcal{S}^-_k \equiv \sum_a \left| \langle \psi_k | a_a | \psi_0 \rangle \right|^2 = \sum_a \left| \mathcal{V}^k_a \right|^2$$

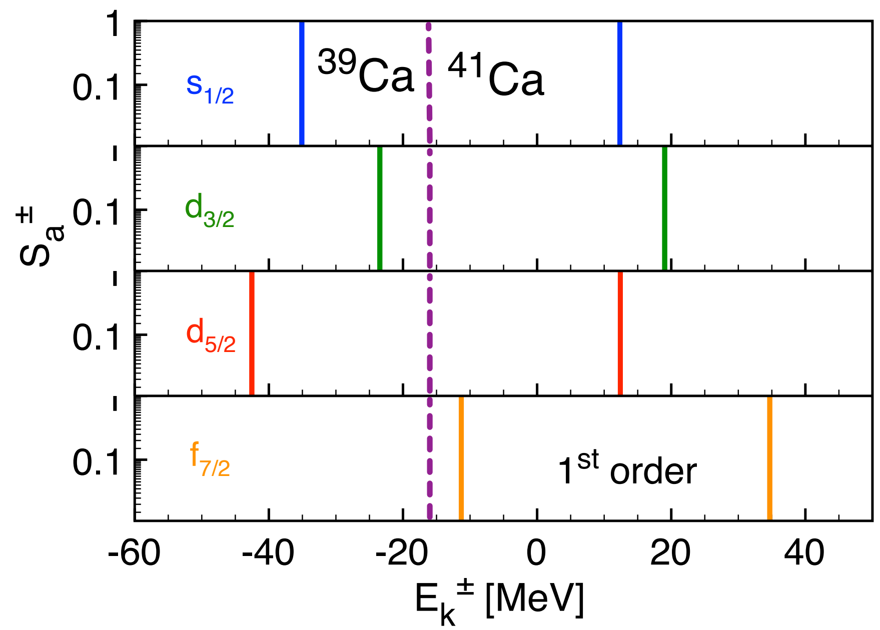


Separation energies + transfer strengths



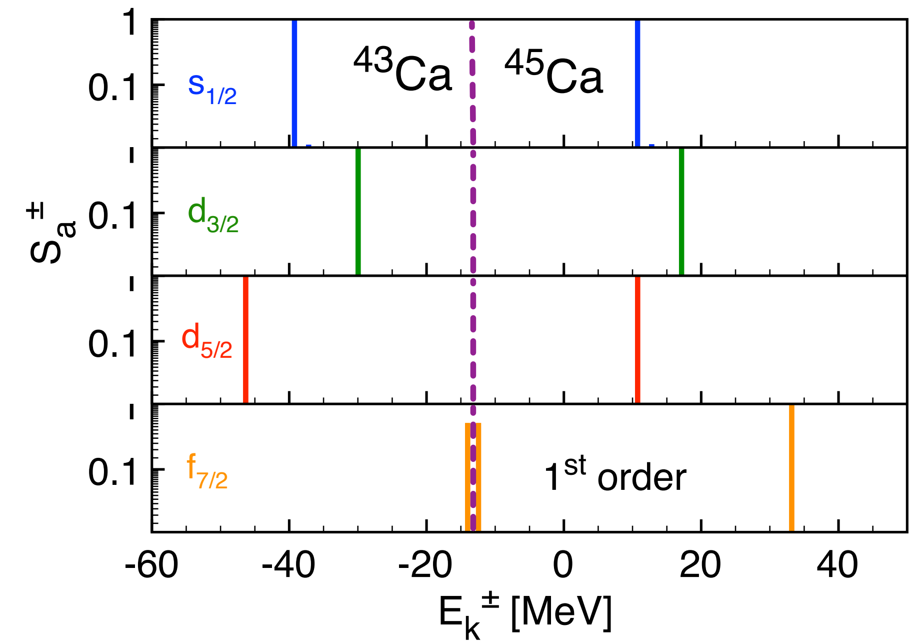
Spectral function

Dyson 1st order (HF)



Fragmentation

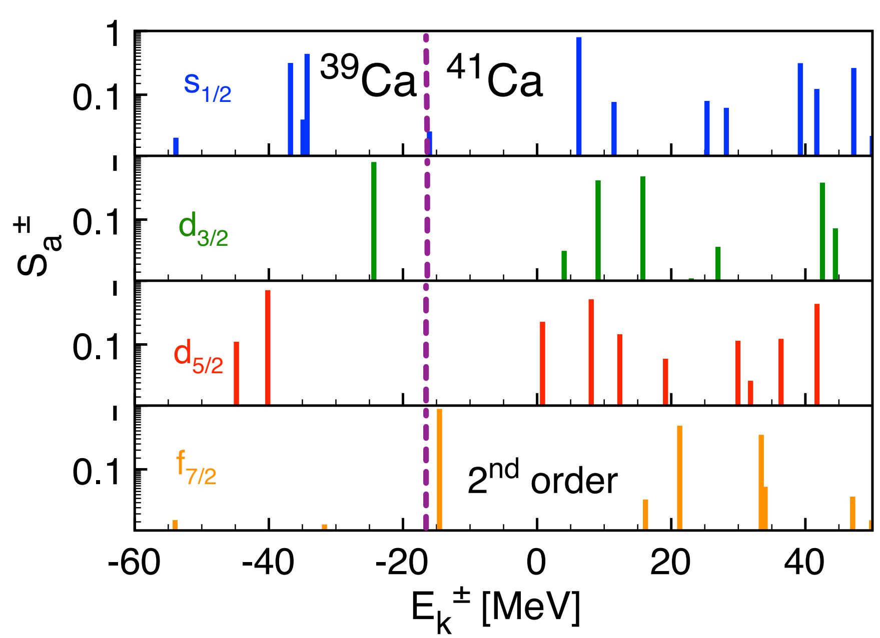
Gorkov 1st order (HFB)



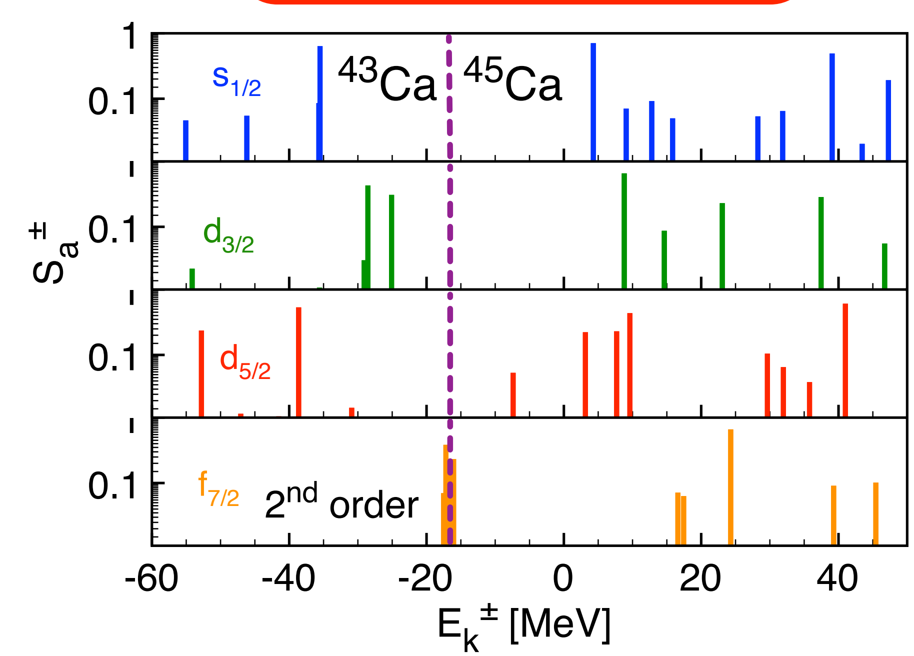
Static pairing

Dynamical fluctuations

Dyson 2nd order



Gorkov 2nd order



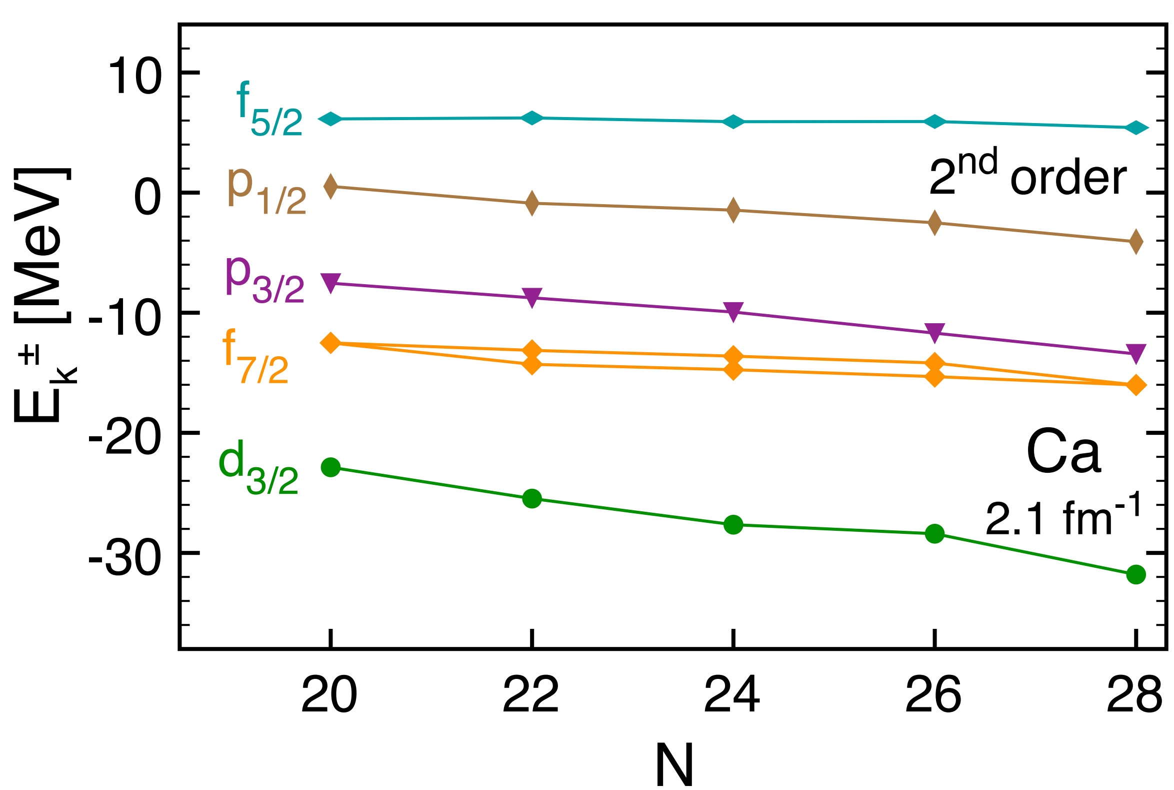
Shell structure evolution

✺ ESPE collect fragmentation of "single-particle" strengths from both N±1

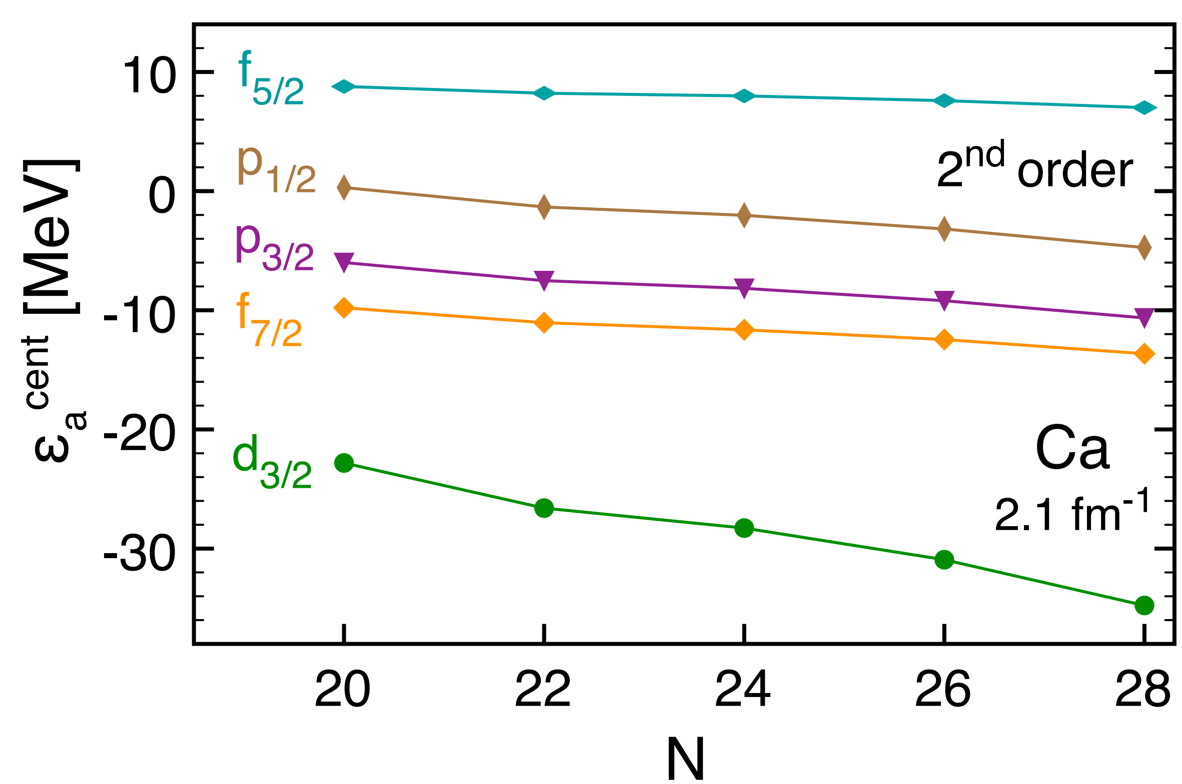
$$\epsilon_a^{cent} \equiv h_{ab}^{cent}\,\delta_{ab} = t_{aa} + \sum_{cd} \bar{V}^{NN}_{acad}\,\rho^{[1]}_{dc} + \sum_{cdef} \bar{V}^{NNN}_{acdaef}\,\rho^{[2]}_{efcd} \equiv \sum_k \mathcal{S}_k^{+a} E_k^+ + \sum_k \mathcal{S}_k^{-a} E_k^-$$

[Baranger 1970, Duguet *et al*. 2011]

Quasiparticle peaks

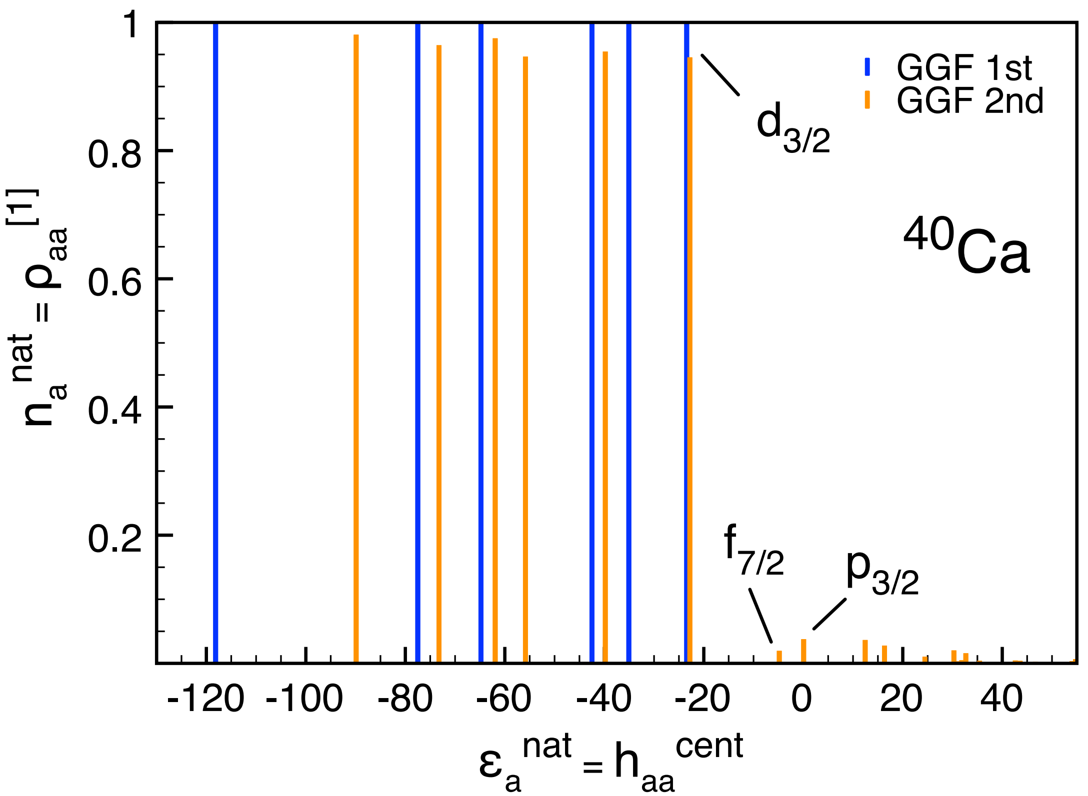


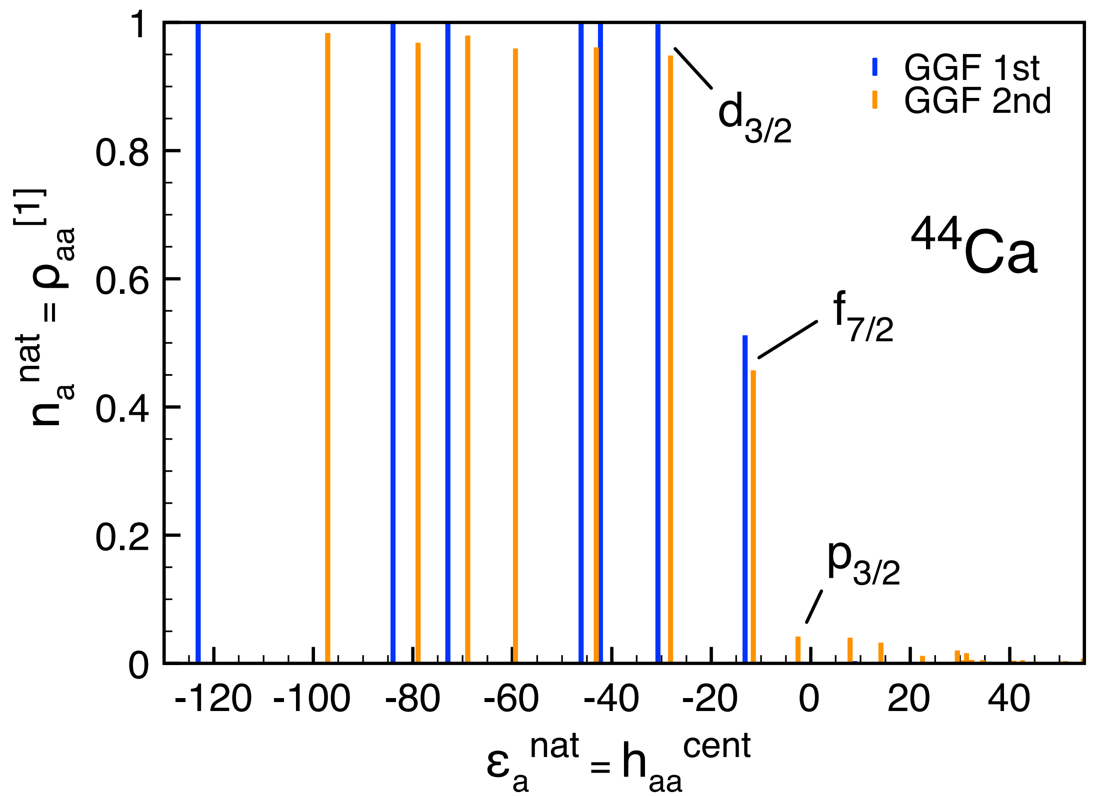
Centroids



Natural single-particle occupation

- Natural orbit *a*: $\rho_{ab}^{[1]} = n_a^{nat}\, \delta_{ab}$
- Associated energy: $\varepsilon_a^{nat} = h_{aa}^{cent}$





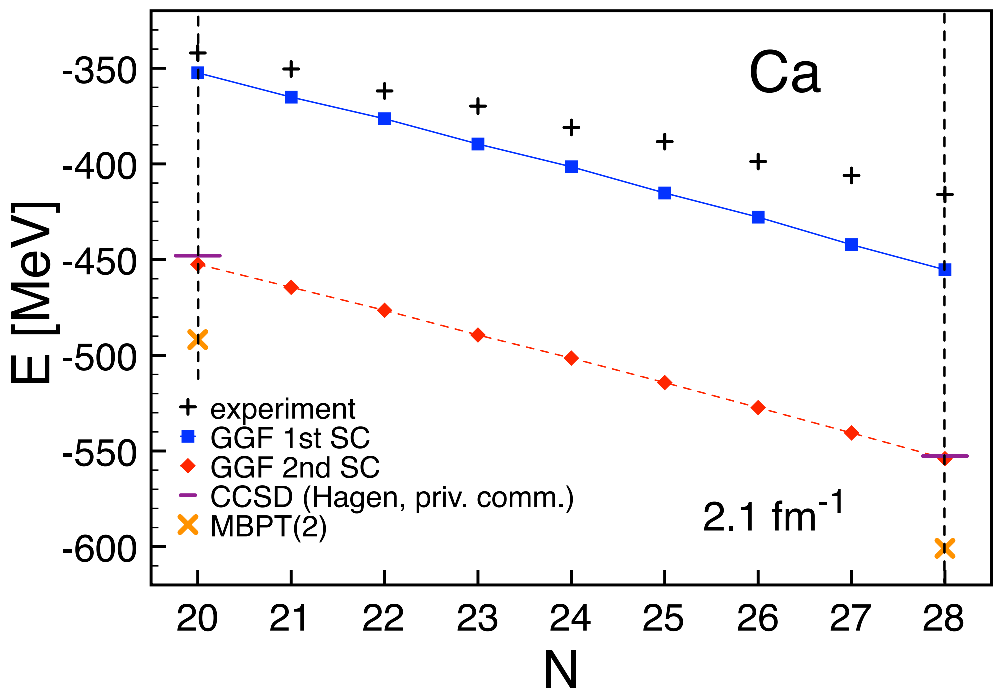
- Dynamical correlations similar for doubly-magic and semi-magic
- Static pairing essential to open-shells

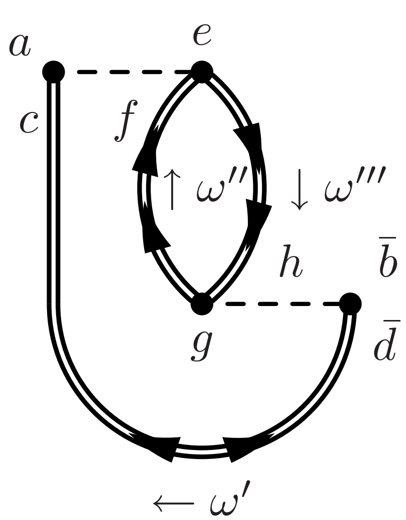
Conclusions & Outlook

- Gorkov-Green's functions:
  first ab-initio open-shell calculations

- Provide optical potentials
  for reaction models

- Provide constraints for next-generation
  Energy Density Functionals





- Implementation of three-body forces

- Formulation of particle-number restored Gorkov theory

- Improvement of the self-energy expansion