Constraining mean-field models of the nuclear matter equation of state at low densities

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Motivation

EoS for astrophysical applications:

Supernova explosions
Structure of neutron stars

Many EoS, developed in the past

Challenge:

Cover full parameter space (n, T, Y_p) in a single model
Combine different approaches

In this work:

Constrain low-density behavior of the EoS
- Introduce bound and scattering states in Ω
- Require consistency with the Virial EoS

Virial EoS-Low densities

- Model-independent approach
- Two-body correlations are encoded in second virial coefficient b_{ij}
- Expansion of grand canonical partition function $\mathcal{Q}$ in powers of $z_i = \exp(\mu_i/T) \ll 1$
- limitation $n_i\lambda_i^3 \ll 1, \quad \lambda_i = \sqrt{2\pi/(m_i T)}$

The grand canonical potential

$$\Omega(T,V,\mu_i) = -T\ln\mathcal{Q}(T,V,\mu_i) =$$
$$-TV\left(\sum_i \frac{b_i}{\lambda_i^3} z_i + \sum_{ij} \frac{b_{ij}}{\lambda_i^{3/2}\lambda_j^{3/2}} z_i z_j + \sum_{ijk} \frac{b_{ijk}}{\lambda_i\lambda_j\lambda_k} z_i z_j z_k + \dots\right)$$

Virial coefficients

- The virial integral

$$I_l^{(ij)}(T) = \int_0^\infty \frac{dE}{\pi} \frac{d\delta_l^{(ij)}}{dE} \exp\left(-\frac{E}{T}\right)$$

- The second virial coefficient

$$\begin{aligned} b_{nn}(T) &= \frac{\lambda_n^3}{\lambda_{nn}^3} \sum_l g_l^{(nn)} I_l^{(nn)} - g_n 2^{-5/2} \,, \\ b_{np1}(T) &= \frac{1}{2} \frac{\lambda_n^{3/2} \lambda_p^{3/2}}{\lambda_{np}^3} \sum_l g_l^{(np1)} I_l^{(np1)} \,, \\ b_{np0}(T) &= \frac{1}{2} \frac{\lambda_n^{3/2} \lambda_p^{3/2}}{\lambda_{np}^3} \left[g_d \exp\left(\frac{B_d}{T}\right) + \sum_l g_l^{(np0)} I_l^{(np0)} \right] , \end{aligned}$$

$b_{np} = b_{np0} + b_{np1}$ with total isospin $\mathcal{T} = 0$ and $\mathcal{T} = 1$.

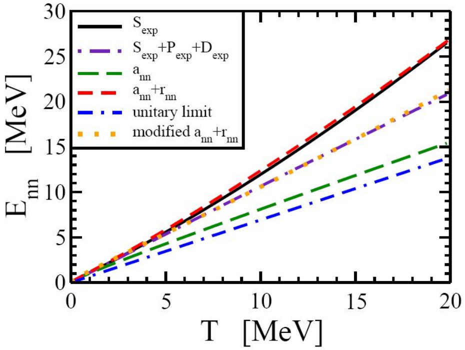
- With experimental bound state energies/phase shifts $\rightarrow$ model-independent EoS
(C. J. Horowitz, A. Schwenk, Nucl. Phys. A 776 (2006) 55)

Resonance Energies

- Introduce a bound state with effective resonance energy $E_{ij}(T)$

$$\sum_l g_l^{(ij)} I_l^{(ij)} = \int_0^\infty \frac{dE}{\pi} \frac{d\delta_{ij}}{dE} \exp\left(-\frac{E}{T}\right) = \hat{g}_{ij} \exp\left[-\frac{E_{ij}(T)}{T}\right]$$

$$\delta_{ij} = \sum_l g_l^{ij} \delta_l^{ij}, \quad \hat{g}_{ij} = \pm g_0^{(ij)}$$



- Use effective-range approximation for the s-wave phase shift

$$k \cot \delta_0^{(ij)} = -\frac{1}{a_{ij}} + \frac{1}{2} r_{ij} k^2$$

Generalized RMF model

(S. Typel et al., Phys. Rev. C 81 (2010) 015803)

The grand canonical potential for uniform nuclear matter

$$\Omega = \sum_i \Omega_i - V\left[\frac{1}{2}\left(m_\omega^2\omega^2 + m_\rho^2\rho^2 - m_\sigma^2\sigma^2 - m_\delta^2\delta^2\right) + \left(\Gamma'_\omega \omega n_\omega + \Gamma'_\rho \rho n_\rho - \Gamma'_\sigma \sigma n_\sigma - \Gamma'_\delta \delta n_\delta\right)(n_n + n_p)\right].$$

Contributions from nucleons and clusters

$$\Omega_i = \mp g_i VT \int \frac{d^3k}{(2\pi)^3} \ln\left[1 \pm \exp\left(-\frac{E_i - \tilde{\mu}_i}{T}\right)\right].$$

$$E_i = V_i + \sqrt{k^2 + (m_i - S_i)^2}$$

The mass of a cluster

$$m_i = N_i m_n + Z_i m_p - (1-\delta_{in})(1-\delta_{in})B_i^{vac}$$

Self-energies with the medium-dependent binding energy shift ΔB_i,

$$S_i = \Gamma_{i\sigma}\sigma + \Gamma_{i\delta}\delta - (1-\delta_{in})(1-\delta_{ip})\,\Delta B_i\,.$$

We assume that in general g_i can be temperature dependent.

Fugacity expansion

Idea:

Expand Ω up to second order in z_i and compare it with the VEoS.

The contribution of the individual nucleon can be approximated as

$$\Omega_i \approx -VT\frac{g_i}{\lambda_i^3}\left\{k_2\left(\frac{m_i}{T}\right)\left[1+\frac{S_i}{T}\left(1-\frac{k_2'\left(\frac{m_i}{T}\right)}{k_2\left(\frac{m_i}{T}\right)}-\frac{3}{2}\frac{T}{m_i}\right)-\frac{V_i}{T}\right]z_i -\frac{1}{2^{5/2}}k_2\left(2\frac{m_i}{T}\right)z_i^2\right\}.$$

For the deuteron contribution we have

$$\Omega_d = -VT\frac{g_d}{\lambda_d^3}k_2\left(\frac{m_d}{T}\right)z_n z_p \exp\left(\frac{B_d}{T}\right)$$

Without scattering correlations (continuum contributions) VEoS can not be reproduced even with density dependent couplings

Extension of the gRMF approach

arxiv:1201.1078

• Introduce two-body correlations as two-body clusters in Ω represented by resonance energy $\tilde{E}_{ij}(T)$ and total rest mass

$$\tilde{M}_{ij} = N_{ij} m_n + Z_{ij} m_p + \tilde{E}_{ij} + \Delta B_{ij}$$

• Compare fugacity expansions $\rightarrow$

- relativistic corrections in the first order of the fugacities

$$b_n = g_n k_2 \left(\frac{m_n}{T}\right) , \quad b_p = g_p k_2 \left(\frac{m_p}{T}\right)$$

- consistency conditions

$$-\frac{T}{\lambda_{np0}^3} k_2 \left(\frac{M_{np0}}{T}\right) \hat{g}_{np0} \exp\left(-\frac{E_{np0}}{T}\right) = -\frac{T}{\tilde{\lambda}_{np0}^3} k_2 \left(\frac{\tilde{M}_{np0}}{T}\right) g_{np0}^{(\mathrm{eff})} \exp\left(-\frac{\tilde{E}_{np0}}{T}\right)$$
$$+\frac{1}{2} \frac{g_n g_p}{\lambda_n^3 \lambda_p^3} \left[\left(C_\omega - 3C_\rho\right) k_2 \left(\frac{m_n}{T}\right) k_2 \left(\frac{m_p}{T}\right) - \left(C_\sigma - 3C_\delta\right) k_1 \left(\frac{m_n}{T}\right) k_1 \left(\frac{m_p}{T}\right) \right]$$

Temperature dependent degeneracy factors

- Resonance energies are chosen identical to the VEoS $\tilde{E}_{ij} = E_{ij}$.

$\rightarrow$ derive degeneracy factors $g_{ij}^{(\mathrm{eff})}(T)$

In the non-relativistic limit

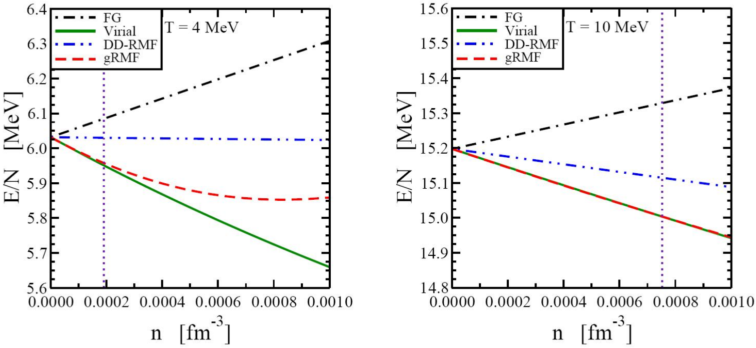
$$
\begin{aligned}
g_{nn}^{(\mathrm{eff})}(T) &= \hat{g}_{nn} + \frac{g_n^2}{2T}\frac{\lambda_{nn}^3}{\lambda_n^6} C_1 \exp\left[\frac{E_{nn}(T)}{T}\right] \\
g_{pp}^{(\mathrm{eff})}(T) &= \hat{g}_{pp} + \frac{g_p^2}{2T}\frac{\lambda_{pp}^3}{\lambda_p^6} C_1 \exp\left[\frac{E_{pp}(T)}{T}\right] \\
g_{np1}^{(\mathrm{eff})}(T) &= \hat{g}_{np1} + \frac{g_n g_p}{2T}\frac{\lambda_{np1}^3}{\lambda_n^3\lambda_p^3} C_1 \exp\left[\frac{E_{np1}(T)}{T}\right] \\
g_{np0}^{(\mathrm{eff})}(T) &= \hat{g}_{np0} + \frac{g_n g_p}{2T}\frac{\lambda_{np0}^3}{\lambda_n^3\lambda_p^3} C_0 \exp\left[\frac{E_{np0}(T)}{T}\right]
\end{aligned}
$$

with the relevant couplings in the two isospin channels

$$
\begin{aligned}
C_1 &= C_\omega - C_\sigma + C_\rho - C_\delta \, , \\
C_0 &= C_\omega - C_\sigma - 3\,(C_\rho - C_\delta)
\end{aligned}
$$

Thermodynamical quantities

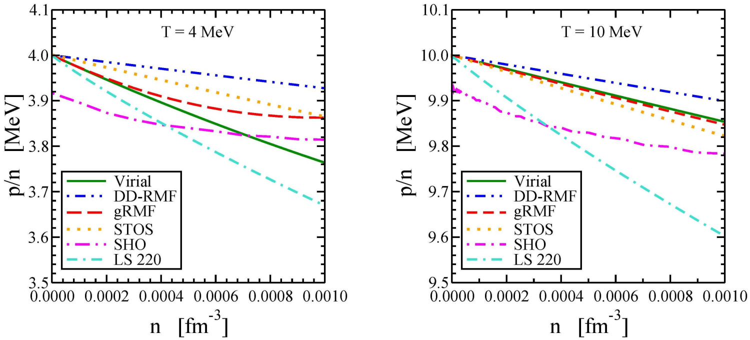
$$\varepsilon = \frac{1}{V}\left(\Omega + TS\right) + \sum_{i=n,nn} \tilde{\mu}_i n_i$$



Vertical dotted lines indicate the density where $n\lambda_n^3 = 1/10$.

Comparison with other approaches

- RMF model of G. Shen et al. (SHO) with the FSUGold parametrization
- RMF model of H. Shen et al. (STOS) with the parameter set TM1
- Non-relativistic Lattimer-Swesty EoS with $K = 220$ MeV (LS 220)



Conclusions

- An extension of the gRMF model was developed by requiring the consistency of the finite-temperature EoS at low densities with the VEoS.
- Two-nucleon correlations are introduced as new degrees of freedom in grand canonical potential
- They are characterized by temperature dependent resonance energies and degeneracy factors
- The proposed extension is rather general and can be applied to other mean-field approaches

Future steps:

- Include heavier nuclei
- Generate EoS tables for a broad range in n, T and Y_p