   

Generalised Beth-Uhlenbeck description for the hadron-to-quark matter transition

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Facets of Strong-Interaction Physics

roadmap

- introduction
- model
- quarks
- diquarks
- mesons
- baryons

introduction

Beth-Uhlenbeck bound + scattering states: phase shifts

generalised medium modifications of bound states and phase shifts: Mott effect, …

very general approach

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generalised medium modifications of bound states and phase shifts: Mott effect, …

very general approach

- **atomic gases**

Beth, Uhlenbeck: Physica **3** (1936) 729 & **4** (1937) 915

introduction

Beth-Uhlenbeck **bound + scattering states: phase shifts**

generalised **medium modifications of bound states and phase shifts: Mott effect, …**

very general approach

- **atomic gases**
- **Coulomb plasmas**

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Ebeling et al.: Metal-to-Nonmetal Transitions, Springer (2010)

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Beth-Uhlenbeck — **bound + scattering states: phase shifts**

generalised — **medium modifications of bound states and phase shifts: Mott effect, …**

very general approach

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- **Coulomb plasmas**
- **solid state physics**

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Zimmermann, Stolz: Phys. Stat. Sol. **B131** (1985), 151

introduction

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very general approach

- **atomic gases**
- **Coulomb plasmas**
- **solid state physics**
- **nuclear physics**

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Zimmermann, Stolz: Phys. Stat. Sol. **B131** (1985), 151

Weinhold, Friman, Nörenberg: Phys. Lett. **B433** (1998) 236

introduction

Beth-Uhlenbeck — **bound + scattering states: phase shifts**

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- **atomic gases** — Beth, Uhlenbeck: Physica **3** (1936) 729 & **4** (1937) 915
- **Coulomb plasmas** — Ebeling et al.: Metal-to-Nonmetal Transitions, Springer (2010)
- **solid state physics** — Zimmermann, Stolz: Phys. Stat. Sol. **B131** (1985), 151
- **nuclear physics** — Weinhold, Friman, Nörenberg: Phys. Lett. **B433** (1998) 236
- **quark-hadron phase transition**

model

$$
\begin{aligned}
\mathscr{L} &= \mathscr{L}_0 + \mathscr{L}_\mathrm{S} + \mathscr{L}_\mathrm{V} + \mathscr{L}_\mathrm{D} \\
\mathscr{L}_0 &= \bar{q}(\mathrm{i}\not{\partial} - m_0 + \mu\gamma_0)q \\
\mathscr{L}_\mathrm{S} &= G_\mathrm{S}\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma_5\tau q)^2\right] \\
\mathscr{L}_\mathrm{V} &= -G_\mathrm{V}(\bar{q}\gamma_\mu q)^2 \\
\mathscr{L}_\mathrm{D} &= G_\mathrm{D} \sum_{A=2,5,7} (\bar{q}\mathrm{i}\gamma_5\tau_2\lambda_A q^C)(\bar{q}^C\mathrm{i}\gamma_5\tau_2\lambda_A q)
\end{aligned}
$$

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\end{aligned}
$$

$$\mathscr{L}_{\mathrm{B}} = G_{\mathrm{B}} d^\dagger \bar{q} q d$$

Wang[2], Rischke: Phys. Lett. **B704** (2011) 347

Reinhardt: Phys. Lett. **B244** (1990) 316

gen. Beth-Uhlenbeck

$$\Omega_{\mathrm{X}}(T,\mu) = \mathrm{Tr}\ln S_{\mathrm{X}}^{-1}(\mathrm{i}z_n,\mathbf{p}) = d_{\mathrm{X}} T \sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \ln S_{\mathrm{X}}^{-1}(\mathrm{i}z_n,\mathbf{p})$$

$$X = \{\mathrm{D},\mathrm{M},\mathrm{B}\}$$

Abuki: Nucl. Phys. **A791** (2007) 117

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$$S_{\mathrm{X}}^{-1}(\mathrm{i}z_n,\mathbf{p}) = G_{\mathrm{X}}^{-1} - \Pi_{\mathrm{X}}(\mathrm{i}z_n,\mathbf{p})$$

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$$S_{\mathrm{X}}(\mathrm{i}z_n,\mathbf{p}) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\nu}{2\pi} \frac{\rho_{\mathrm{X}}(\nu,\mathbf{p})}{\mathrm{i}z_n - \nu}$$

Abuki: Nucl. Phys. **A791** (2007) 117

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$$\Omega_{\mathrm{X}}(T,\mu) = -d_{\mathrm{X}} T \sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\nu}{2\pi} \frac{2\Phi_{\mathrm{X}}(\nu,\mathbf{p})}{\mathrm{i}z_n - \nu}$$

Abuki: Nucl. Phys. **A791** (2007) 117

quarks

$$
\begin{aligned}
\mathscr{Z} &= \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi \int \mathcal{D}\sigma\mathcal{D}\vec{\pi}\mathcal{D}\omega_\mu\mathcal{D}\vec{\rho}_\nu\mathcal{D}\Delta_A\mathcal{D}\Delta_A^* \; e^{-\int \mathrm{d}^4x \left\{ \frac{\sigma^2+\vec{\pi}^2}{4G_\mathrm{S}} - \frac{\omega_\mu^2+\vec{\rho}_\nu^2}{4G_\mathrm{V}} + \frac{\Delta_A\Delta_A^*}{4G_\mathrm{D}} - \bar{\Psi}S^{-1}\Psi \right\}} \\
&= \int \mathcal{D}\sigma\mathcal{D}\vec{\pi}\mathcal{D}\omega_\mu\mathcal{D}\vec{\rho}_\nu\mathcal{D}\Delta_A\mathcal{D}\Delta_A^* \; e^{-\int \mathrm{d}^4x \left\{ \frac{\sigma^2+\vec{\pi}^2}{4G_\mathrm{S}} - \frac{\omega_\mu^2+\vec{\rho}_\nu^2}{4G_\mathrm{V}} + \frac{\Delta_A\Delta_A^*}{4G_\mathrm{D}} - \ln\det S^{-1} \right\}}
\end{aligned}
$$

quarks

$$\mathscr{Z} = \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi \int \mathcal{D}\sigma\mathcal{D}\vec{\pi}\mathcal{D}\omega_\mu\mathcal{D}\vec{\rho}_\nu\mathcal{D}\Delta_A\mathcal{D}\Delta_A^* \; e^{-\int \mathrm{d}^4x \left\{ \frac{\sigma^2+\vec{\pi}^2}{4G_\mathrm{S}} - \frac{\omega_\mu^2+\vec{\rho}_\nu^2}{4G_\mathrm{V}} + \frac{\Delta_A\Delta_A^*}{4G_\mathrm{D}} - \bar{\Psi}S^{-1}\Psi \right\}}$$

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$$\begin{aligned}
\ln\det S^{-1} &= \mathrm{Tr}\ln S^{-1} \\
&= \mathrm{Tr}\ln\left(S_\mathrm{MF}^{-1} + \Sigma\right) \\
&= \mathrm{Tr}\ln S_\mathrm{MF}^{-1} + \mathrm{Tr}\ln\left(1 + S_\mathrm{MF}\Sigma\right) \\
&\approx \mathrm{Tr}\ln S_\mathrm{MF}^{-1} + \mathrm{Tr}\left(S_\mathrm{MF}\Sigma - \frac{1}{2}S_\mathrm{MF}\Sigma S_\mathrm{MF}\Sigma\right)
\end{aligned}$$

quarks

$$\mathscr{Z} = \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi \int \mathcal{D}\sigma\mathcal{D}\vec{\pi}\mathcal{D}\omega_\mu\mathcal{D}\vec{\rho}_\nu\mathcal{D}\Delta_A\mathcal{D}\Delta_A^* \; e^{-\int d^4x \left\{\frac{\sigma^2+\vec{\pi}^2}{4G_S} - \frac{\omega_\mu^2+\vec{\rho}_\nu^2}{4G_V} + \frac{\Delta_A\Delta_A^*}{4G_D} - \bar{\Psi}S^{-1}\Psi\right\}}$$

$$= \int \mathcal{D}\sigma\mathcal{D}\vec{\pi}\mathcal{D}\omega_\mu\mathcal{D}\vec{\rho}_\nu\mathcal{D}\Delta_A\mathcal{D}\Delta_A^* \; e^{-\int d^4x \left\{\frac{\sigma^2+\vec{\pi}^2}{4G_S} - \frac{\omega_\mu^2+\vec{\rho}_\nu^2}{4G_V} + \frac{\Delta_A\Delta_A^*}{4G_D} - \ln\det S^{-1}\right\}}$$

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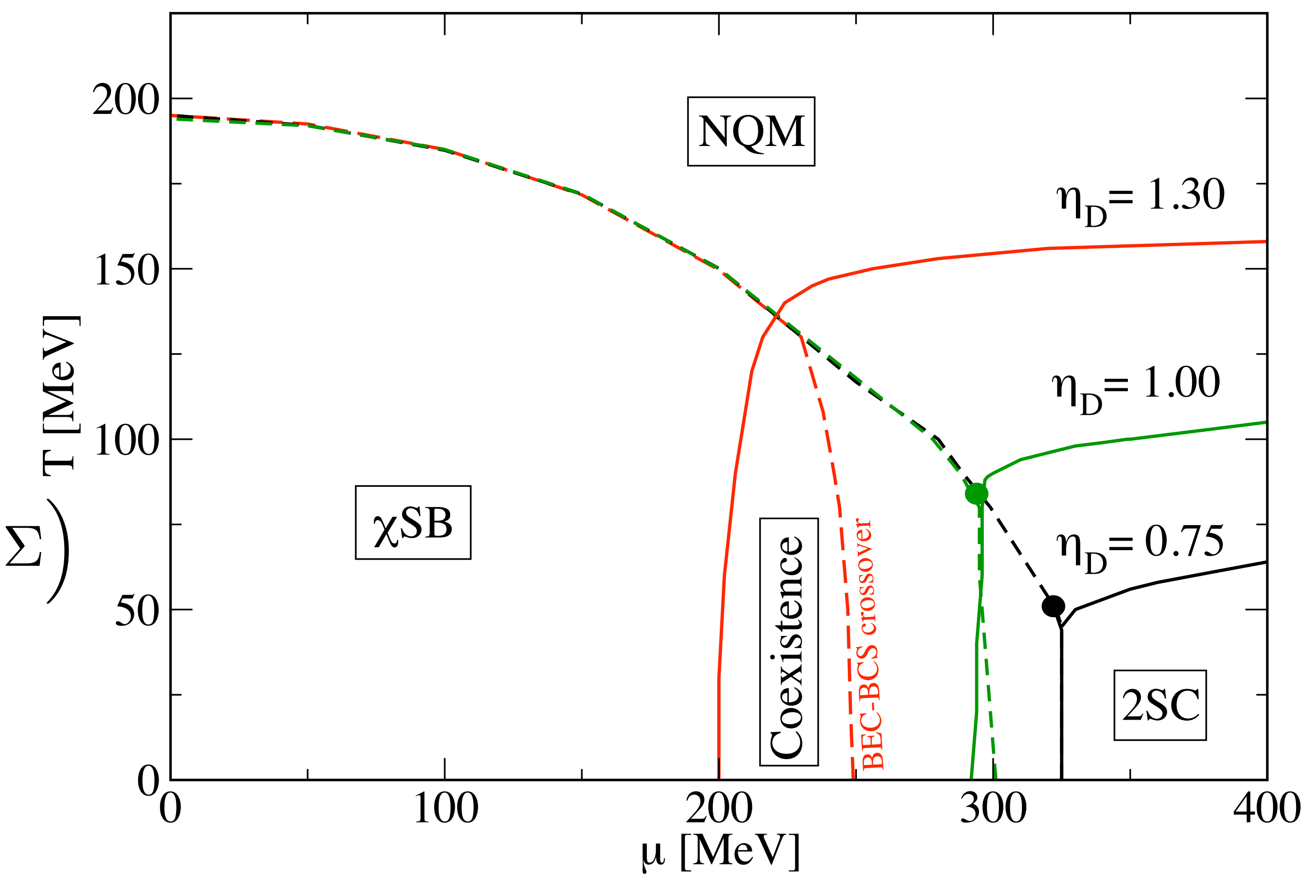

DZ, Anglani, Blaschke: AIP Conf. Proc. **1038** (2008) 159

diquarks

$$\Pi_{\mathrm{D}} = \frac{1}{2}\,\mathrm{Tr}\left(S_{\mathrm{MF}}\Sigma_{\mathrm{D}}S_{\mathrm{MF}}\Sigma_{\mathrm{D}}\right)$$

Kunihiro: Nucl. Phys. **B351** (1991) 593
Kitazawa, Koide, Kunihiro, Nemoto: Phys. Rev. **D65** (2002) 091504

diquarks

$$\Pi_{\mathrm{D}} = \frac{1}{2}\,\mathrm{Tr}\left(S_{\mathrm{MF}}\Sigma_{\mathrm{D}}S_{\mathrm{MF}}\Sigma_{\mathrm{D}}\right)$$

$$S_{\mathrm{D}}^{-1}(k_0,\mathbf{k}) = \frac{1}{2G_{\mathrm{D}}} - \Pi_{\mathrm{D}}(k_0,\mathbf{k})$$

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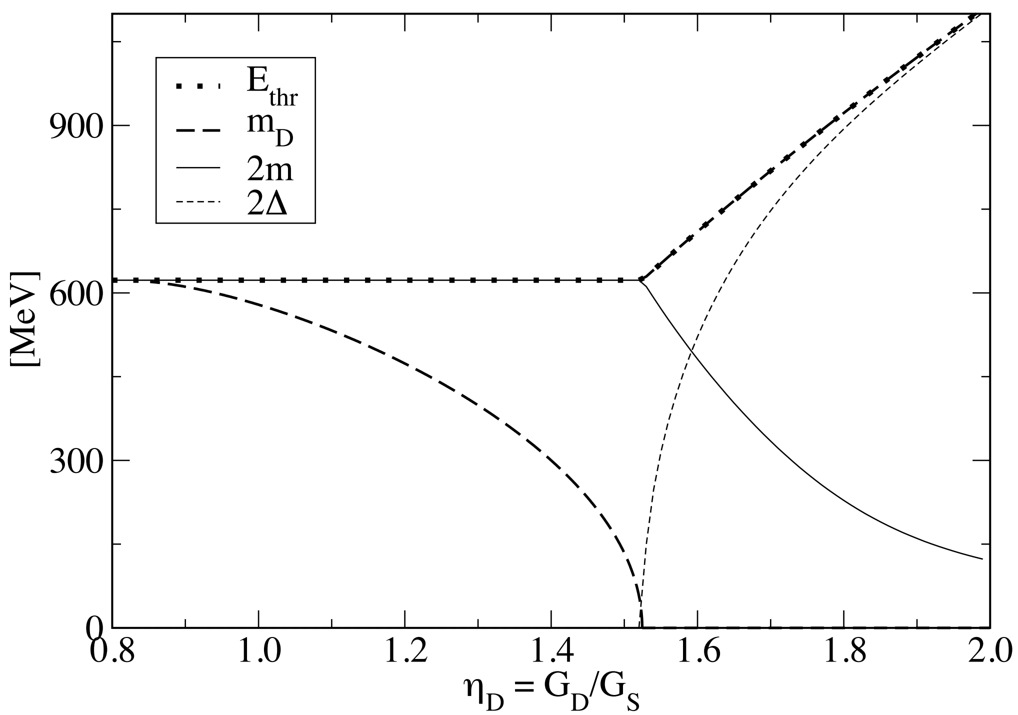

DZ, Blaschke, Anglani, Kalinovsky: APPB Suppl. **3** (2010) 771

Kunihiro: Nucl. Phys. **B351** (1991) 593

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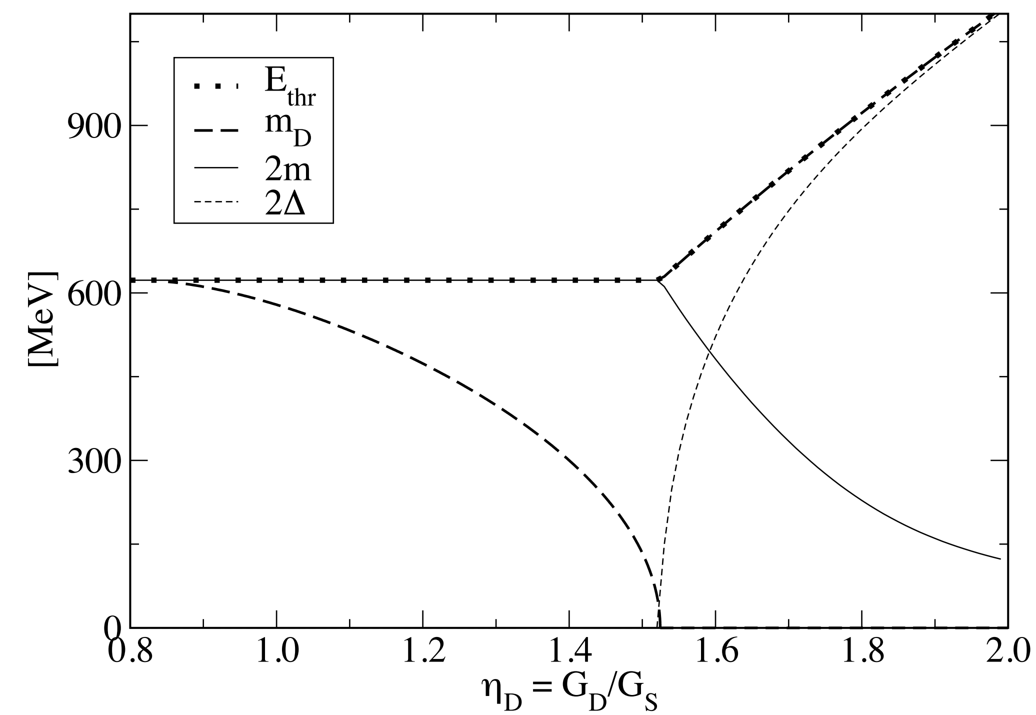
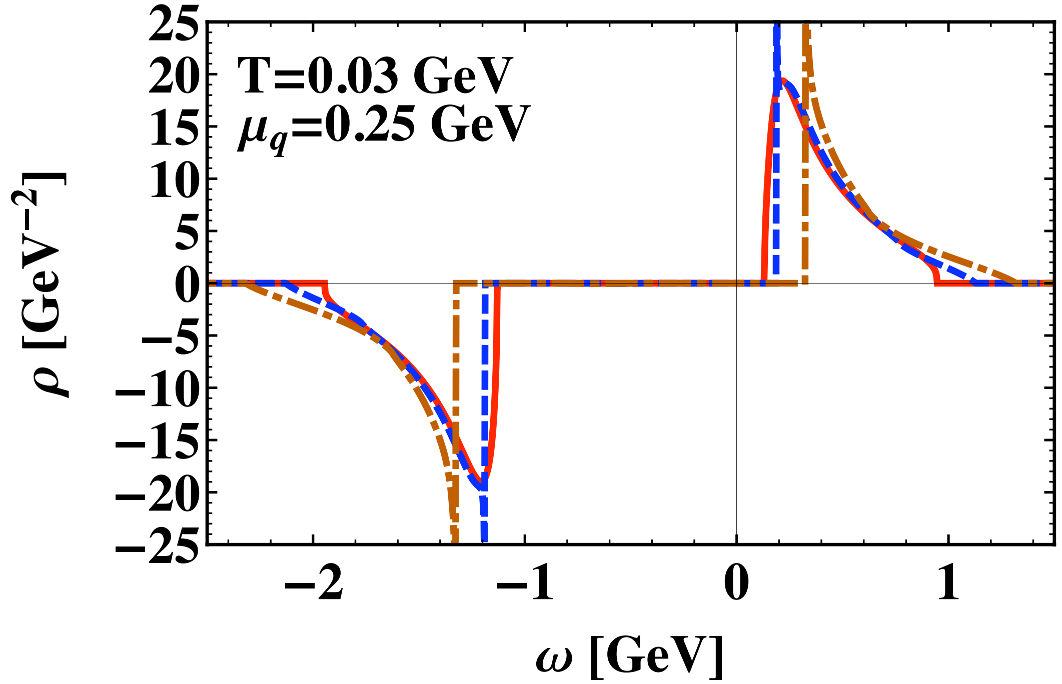
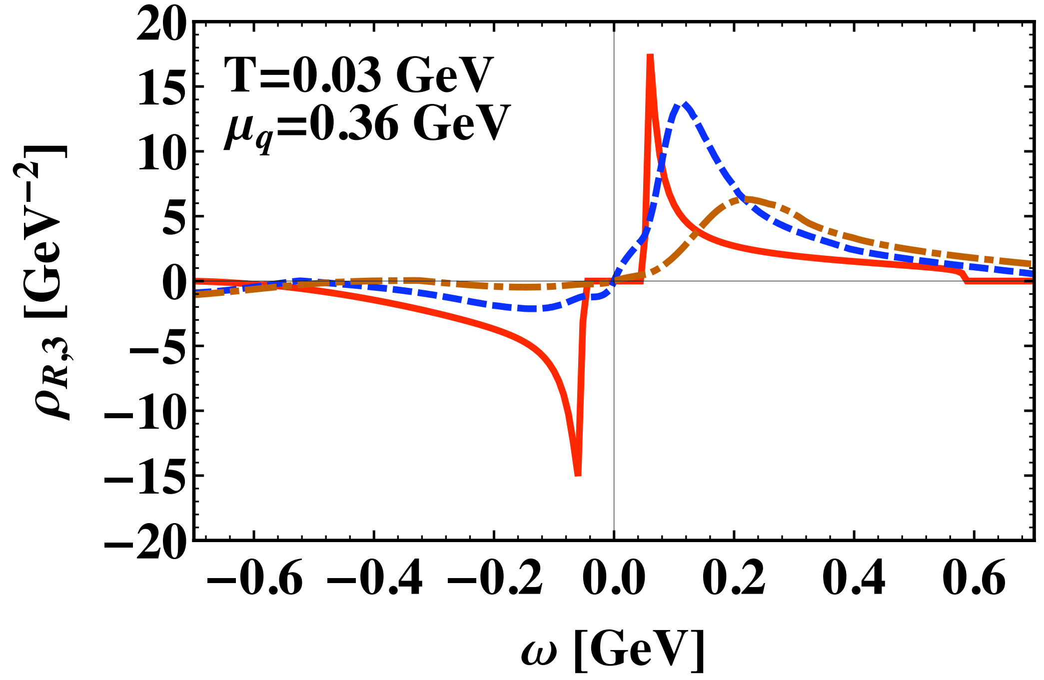


keep diquark coupling low enough
to avoid bound diquarks

DZ, Blaschke, Anglani, Kalinovsky: APPB Suppl. **3** (2010) 771
Kunihiro: Nucl. Phys. **B351** (1991) 593
Kitazawa, Koide, Kunihiro, Nemoto: Phys. Rev. **D65** (2002) 091504

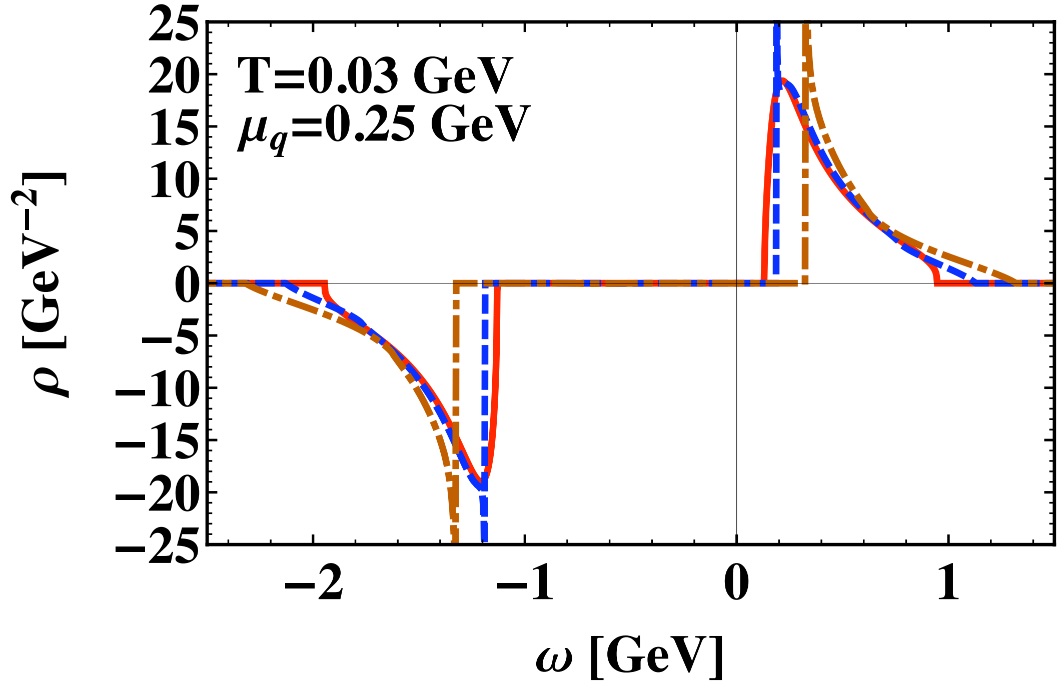
diquarks: spectral

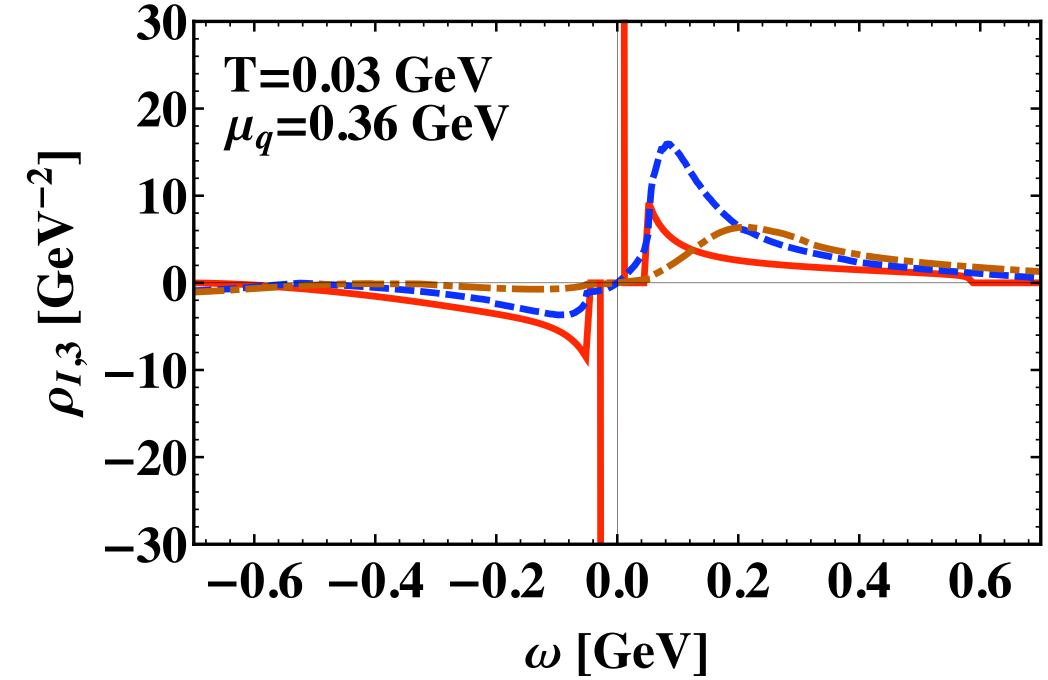


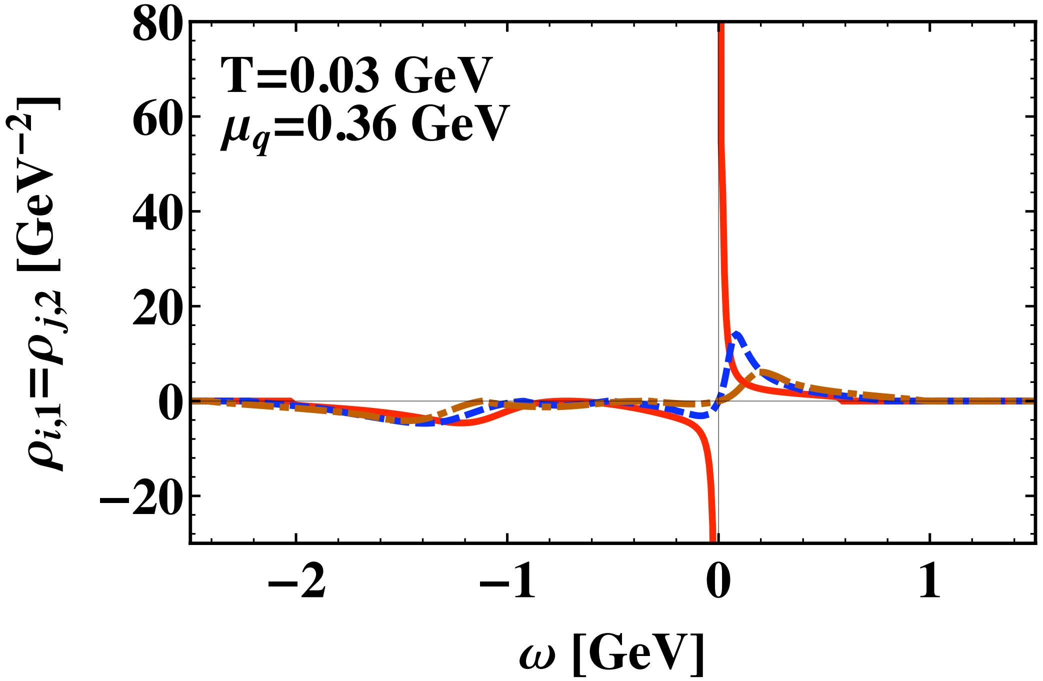


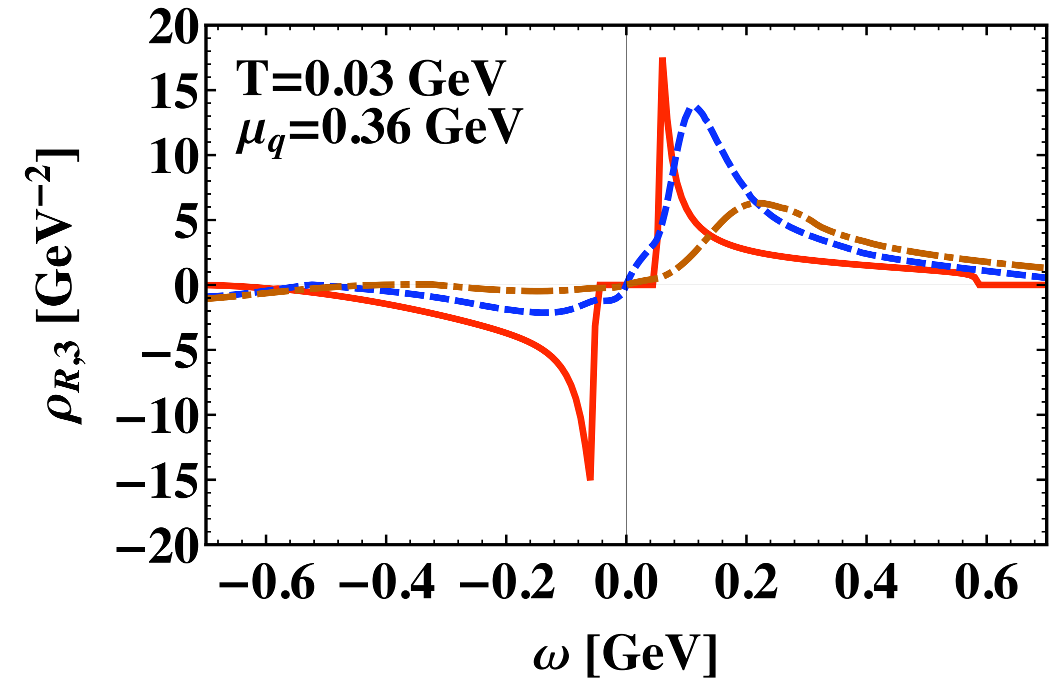
Wang[2], Rischke: Phys. Lett. **B704** (2011) 347

diquarks: spectral









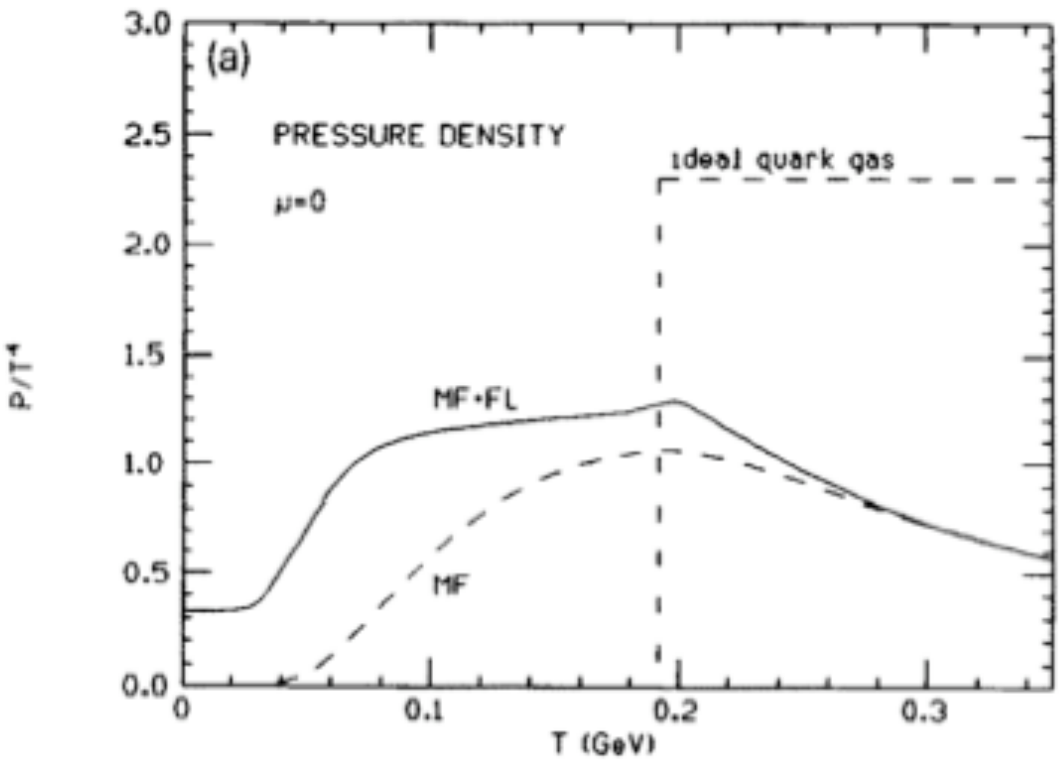
Wang[2], Rischke: Phys. Lett. **B704** (2011) 347

mesons

$$\delta\Omega = \nu \Bigg(\int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, \Theta(2m_{\mathrm{q}} - m_{\mathrm{pole}})\, T \ln(1 - e^{-E_q/T}) + B(T) + \\
+ \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \coth\left(\frac{\omega}{2T}\right) \arctan\left[\frac{G\,\mathrm{Im}\,\Pi(\omega, q)}{1 - G\,\mathrm{Re}\,\Pi(\omega, q)}\right] \Bigg)$$

mesons

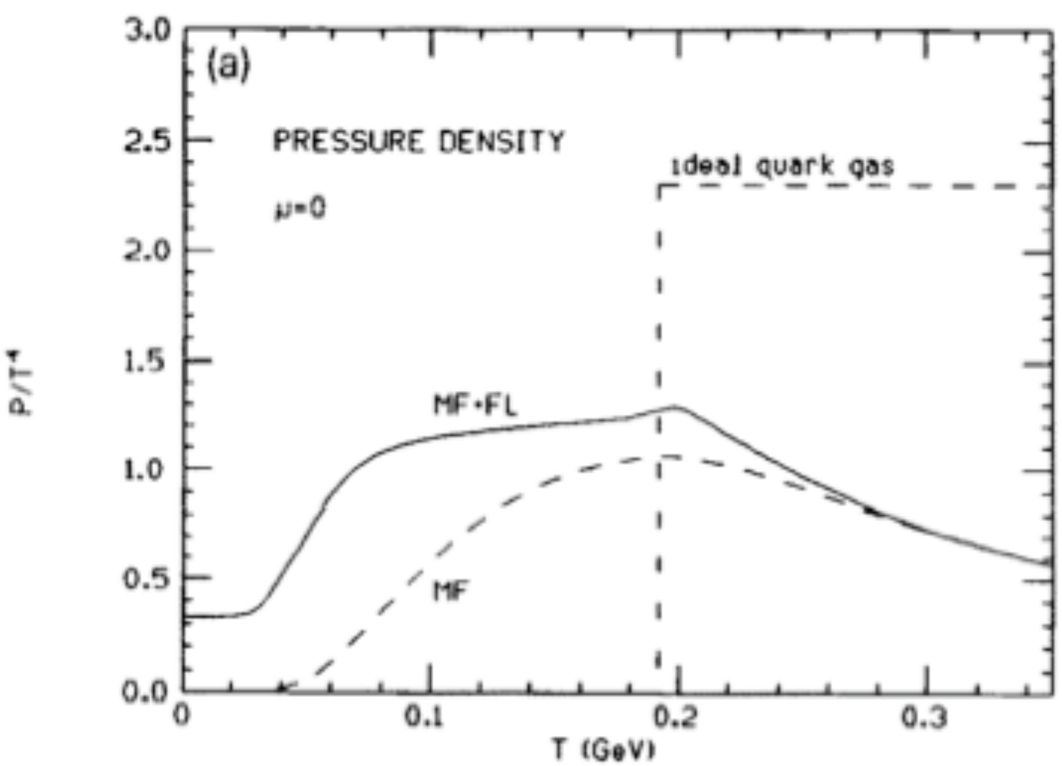
$$\delta\Omega = \nu \left(\int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, \Theta(2m_\mathrm{q} - m_\mathrm{pole})\, T \ln(1 - e^{-E_q/T}) + B(T) + \right.$$
$$\left. + \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \coth\left(\frac{\omega}{2T}\right) \arctan\left[\frac{G\,\mathrm{Im}\,\Pi(\omega,q)}{1 - G\,\mathrm{Re}\,\Pi(\omega,q)}\right] \right)$$

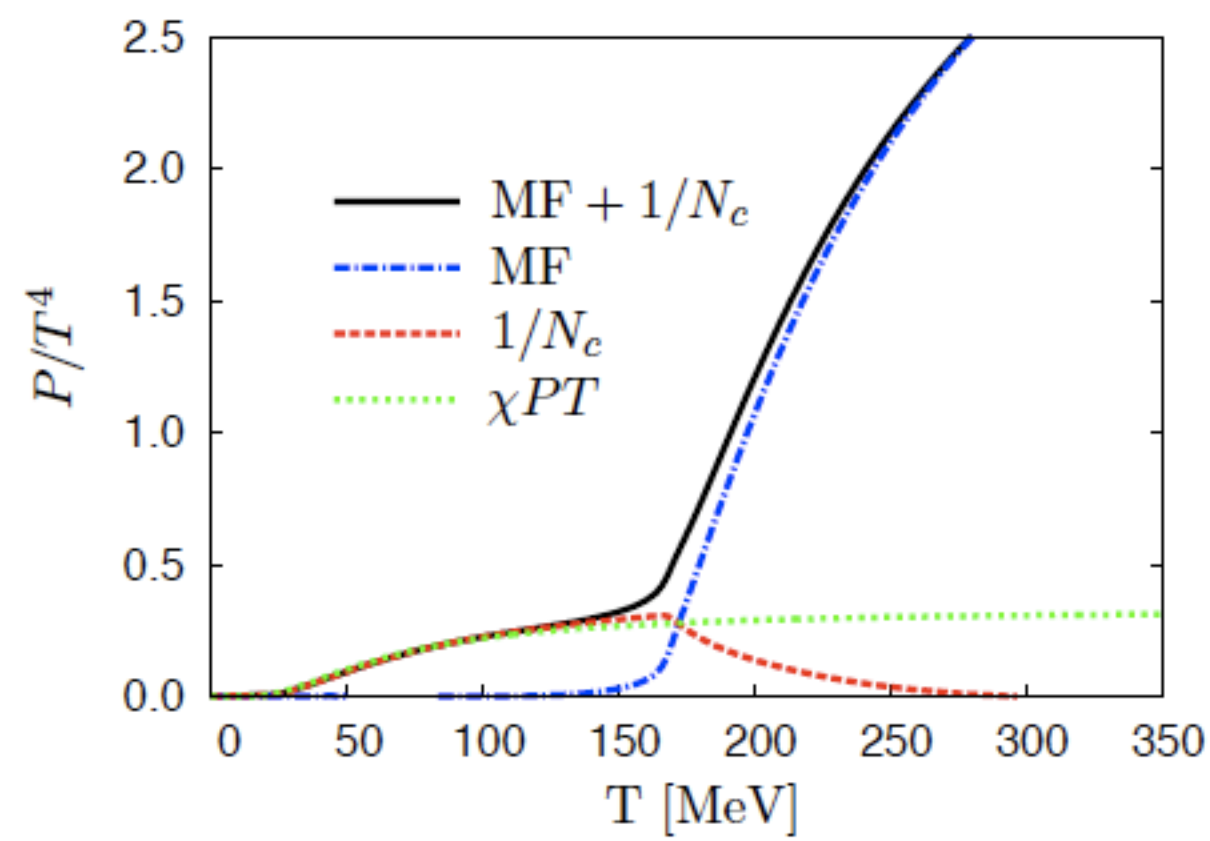


Zhuang, Hüfner, Klevansky: Nucl. Phys. **A576** (1994) 525
Hüfner, Klevansky, Zhuang, Voß: Ann. Phys. **234** (1994) 225

mesons

$$\delta\Omega = \nu\left(\int\frac{d^3q}{(2\pi)^3}\,\Theta(2m_q - m_{\text{pole}})\,T\ln(1-e^{-E_q/T}) + B(T) + \right.$$
$$\left. + \int\frac{d^3q}{(2\pi)^3}\int_0^\infty\frac{d\omega}{\pi}\coth\left(\frac{\omega}{2T}\right)\arctan\left[\frac{G\,\mathrm{Im}\,\Pi(\omega,q)}{1-G\,\mathrm{Re}\,\Pi(\omega,q)}\right]\right)$$





Zhuang, Hüfner, Klevansky: Nucl. Phys. **A576** (1994) 525
Hüfner, Klevansky, Zhuang, Voß: Ann. Phys. **234** (1994) 225

Radzhabov, Blaschke, Buballa, Volkov: Phys. Rev. **D83** (2011) 116004
Rößner, Hell, Ratti, Weise: Nucl. Phys. **A814** (2008) 118 & arXiv: 0712.3152v1

baryons = quark + diquark

$$Q = P - k$$

$$S_{\mathrm{B}}^{-1}(P_0, \mathbf{P}) = \frac{1}{2G_{\mathrm{B}}} - \int \frac{\mathrm{d}^4 k}{(2\pi)^4} S_{\mathrm{Q}}(Q_0, \mathbf{Q}) S_{\mathrm{D}}(k_0, \mathbf{k})$$

baryons = quark + diquark

$$Q = P - k$$

$$S_{\mathrm{B}}^{-1}(P_0, \mathbf{P}) = \frac{1}{2G_{\mathrm{B}}} - \int \frac{\mathrm{d}^4 k}{(2\pi)^4} S_{\mathrm{Q}}(Q_0, \mathbf{Q}) S_{\mathrm{D}}(k_0, \mathbf{k})$$

$$S_{\mathrm{B}}^{-1}(P) = \frac{1}{2G_{\mathrm{B}}} - \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{\rho_{\mathrm{D}}(\omega, \mathbf{k})}{\omega - k_0} \frac{\not{Q} + m}{Q_0^2 - E_{\mathbf{Q}}^2}$$

baryons: pole approximation

$$\Omega_{\mathrm{B}}^{(2)} = d_{\mathrm{B}} \int \frac{\mathrm{d}^3 P}{(2\pi)^3} \int_0^\infty \frac{\mathrm{d}\nu}{\pi} \left\{ \nu + T \ln \left[1 + \mathrm{e}^{-\beta(\nu - 3\mu)} \right] + T \ln \left[1 + \mathrm{e}^{-\beta(\nu + 3\mu)} \right] \right\} \frac{\mathrm{d}\Phi_{\mathrm{B}}(\nu, \mathbf{P})}{\mathrm{d}\nu}$$

baryons: pole approximation

$$\Omega_{\rm B}^{(2)} \quad = \quad d_{\rm B} \int \frac{\mathrm{d}^3 P}{(2\pi)^3} \int_0^\infty \frac{\mathrm{d}\nu}{\pi} \left\{ \nu + T \ln \left[1 + \mathrm{e}^{-\beta(\nu - 3\mu)} \right] + T \ln \left[1 + \mathrm{e}^{-\beta(\nu + 3\mu)} \right] \right\} \frac{\mathrm{d}\Phi_{\rm B}(\nu, \mathbf{P})}{\mathrm{d}\nu}$$

$$1 - G_{\rm B} \Pi_{\rm B}(\nu, \mathbf{P}) = (\nu^2 - E^2_{\mathbf{P},\rm B}) \frac{\mathrm{d}\Pi_{\rm B}}{\mathrm{d}\nu^2}\Big|_{\nu^2 = E^2_{\mathbf{P},\rm B}} = \mathrm{const} \cdot (\nu^2 - E^2_{\mathbf{P},\rm B})$$

baryons: pole approximation

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$$1 - G_{\mathrm{B}} \Pi_{\mathrm{B}}(\nu, \mathbf{P}) = (\nu^2 - E_{\mathbf{P},\mathrm{B}}^2) \frac{\mathrm{d}\Pi_{\mathrm{B}}}{\mathrm{d}\nu^2} \Big|_{\nu^2 = E_{\mathbf{P},\mathrm{B}}^2} = \mathrm{const} \cdot (\nu^2 - E_{\mathbf{P},\mathrm{B}}^2)$$

$$\frac{\mathrm{d}\Phi_{\mathrm{B}}}{\mathrm{d}\nu} = \pi \delta(\nu - E_{\mathbf{P},\mathrm{B}})$$

baryons: pole approximation

$$\Omega_{\mathrm{B}}^{(2)} \quad = \quad d_{\mathrm{B}} \int \frac{\mathrm{d}^3 P}{(2\pi)^3} \int_0^\infty \frac{\mathrm{d}\nu}{\pi} \left\{ \nu + T \ln \left[1 + \mathrm{e}^{-\beta(\nu - 3\mu)} \right] + T \ln \left[1 + \mathrm{e}^{-\beta(\nu + 3\mu)} \right] \right\} \frac{\mathrm{d}\Phi_{\mathrm{B}}(\nu, \mathbf{P})}{\mathrm{d}\nu}$$

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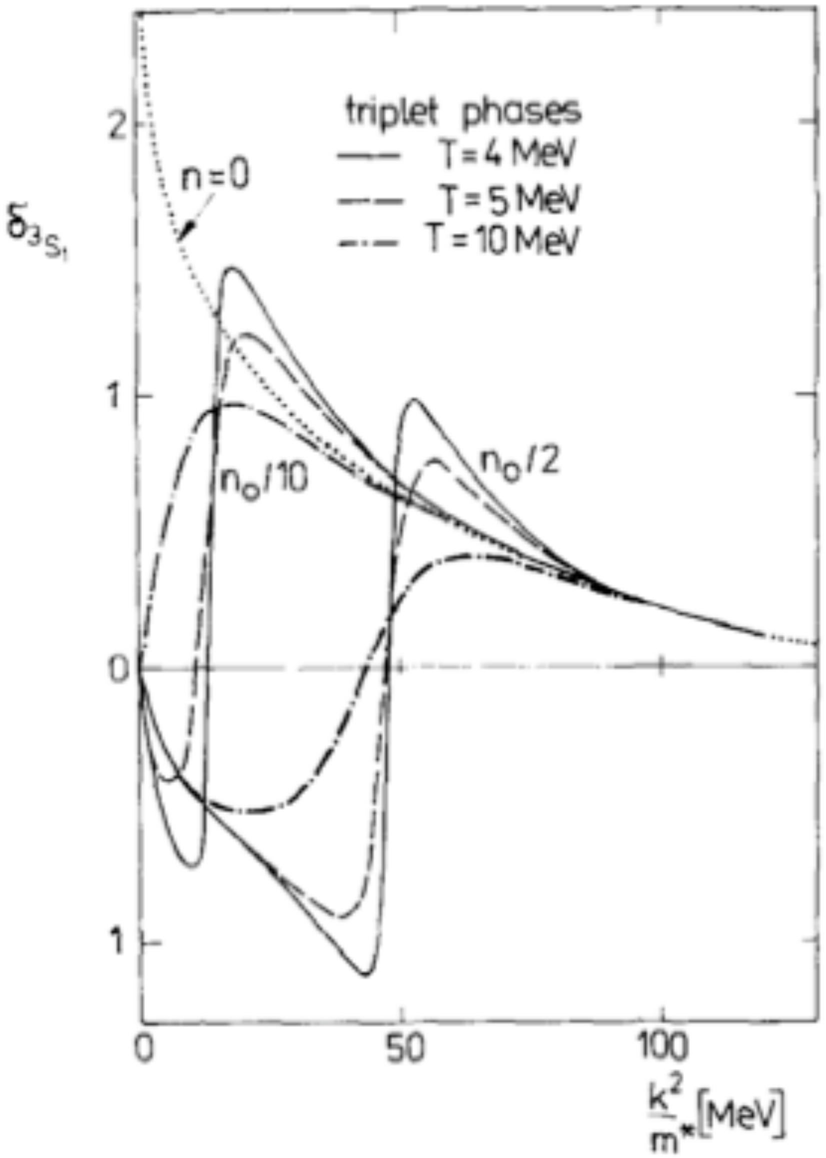
$$\frac{\mathrm{d}\Phi_{\mathrm{B}}}{\mathrm{d}\nu} = \pi \delta(\nu - E_{\mathbf{P},\mathrm{B}})$$

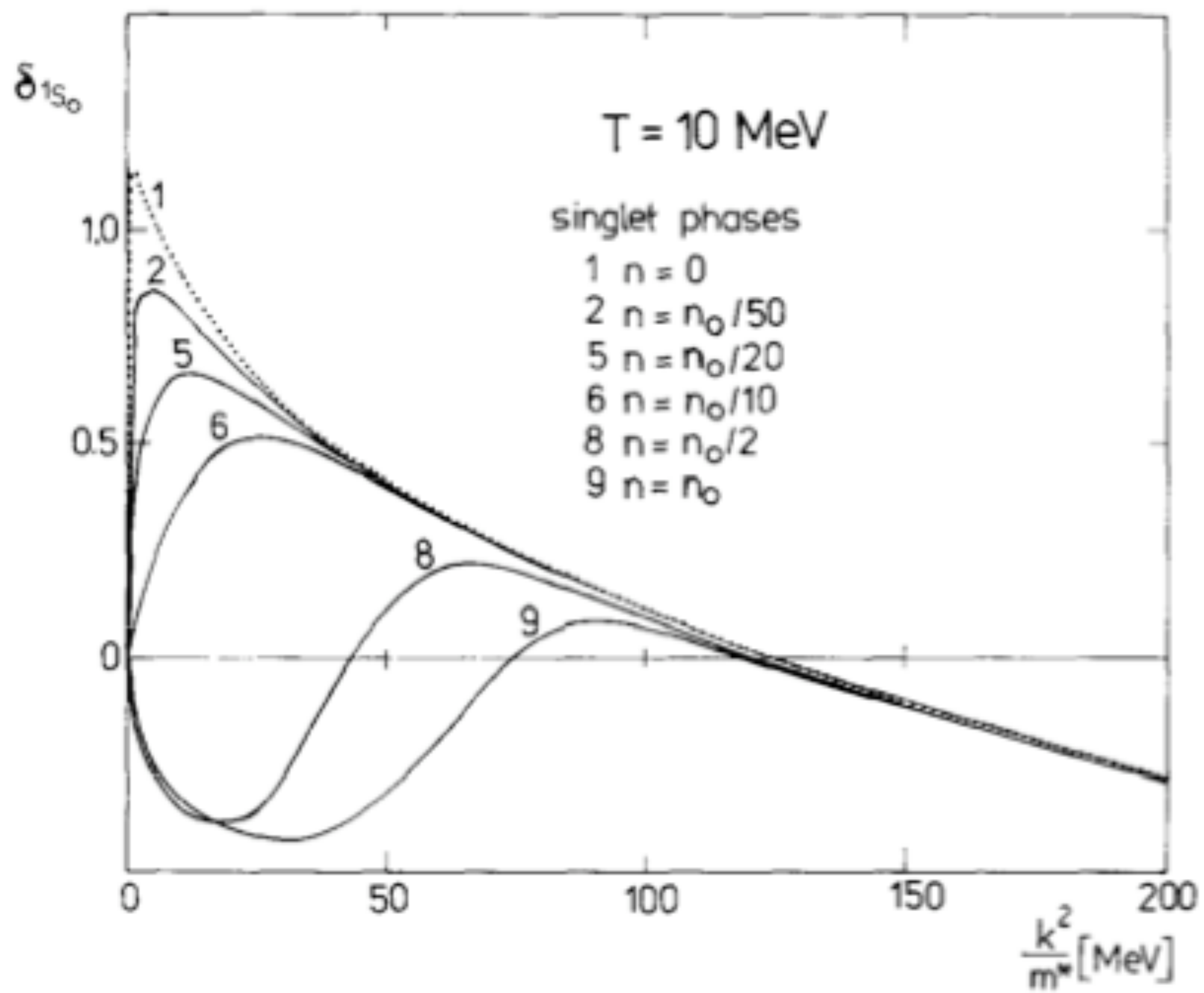
$$\Omega_{\mathrm{B}}^{(2)} \quad = \quad d_{\mathrm{B}} \int \frac{\mathrm{d}^3 P}{(2\pi)^3} \left\{ E_{\mathbf{P},\mathrm{B}} + T \ln \left[1 + \mathrm{e}^{-\beta(E_{\mathbf{P},\mathrm{B}} - 3\mu)} \right] + T \ln \left[1 + \mathrm{e}^{-\beta(E_{\mathbf{P},\mathrm{B}} + 3\mu)} \right] \right\}$$

baryons: bey. pole approximation

$$
\begin{aligned}
\Omega_{\mathrm{MF}} &= \frac{(m-m_0)^2}{4G_{\mathrm{S}}} - \frac{(\mu-\mu^*)^2}{4G_{\mathrm{V}}} + \frac{\Delta^2}{4G_{\mathrm{D}}} - 4I_{\Omega} \ , \\
\Omega_{\mathrm{D}}^{(2)} &= T\sum_m \int \frac{\mathrm{d}^3k}{(2\pi)^3} \ln\left[\frac{1}{2G_{\mathrm{D}}} - \Pi_{\mathrm{D}}(\mathrm{i}\Omega_m, \mathbf{k})\right] \\
&= \int \frac{\mathrm{d}^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \left\{\omega + T\ln\left[1-\mathrm{e}^{-\beta(\omega-2\mu)}\right] + T\ln\left[1-\mathrm{e}^{-\beta(\omega+2\mu)}\right]\right\} \frac{\mathrm{d}\Phi_{\mathrm{D}}(\omega,\mathbf{k})}{\mathrm{d}\omega} \ , \\
\Omega_{\mathrm{B}}^{(2)} &= d_{\mathrm{B}} \int \frac{\mathrm{d}^3P}{(2\pi)^3} \int_0^{\infty} \frac{\mathrm{d}\nu}{\pi} \left\{\nu + T\ln\left[1+\mathrm{e}^{-\beta(\nu-3\mu)}\right] + T\ln\left[1+\mathrm{e}^{-\beta(\nu+3\mu)}\right]\right\} \frac{\mathrm{d}\Phi_{\mathrm{B}}(\nu,\mathbf{P})}{\mathrm{d}\nu}
\end{aligned}
$$

phase shifts





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summary & next steps

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- many extensions
  - colour superconductivity
  - Polyakov loop
- ...